ECOLE POLYTECHNIQUE
CENTRE NATIONAL DE LA RECHERCHE SCIENTIFIQUE

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Cahier n° 2006-036

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Information, Competition, and (In)complete Discrimination*

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Résumé: Nous considérons un dupole dans lequel les valuations des consommateurs pour les qualités des produits offerts par les firms sont hétérogènes. Les firmes sont informées asymétriquement sur les valuations des consommateurs: chaque firme connaît les goûts des consommateurs pour son produit mais est non informée sur les goûts pour le produit offert par sa rivale. Dans ce contexte, nous étudions différents types d'équilibres suivant les restrictions imposées sur les outils de tarification à la disposition des firmes.

Abstract: We analyze a duopoly model in which consumers' values for the qualities of the firms products are heterogenous. Firms are asymmetrically informed about the consumers' taste parameters: while each firm knows the consumers' tastes for its own brand, it is uninformed about the taste for the rival brand. Competition under this type of asymmetry yields new features, namely upward distortions with respect to the first-best, and consumers' rent which is decreasing in their valuations. We also show that both firms are better off by pricing uniformly rather than screening along the unknown dimension of the consumer's characteristics. Finally, our model predicts qualitatively different equilibrium outcomes depending on whether consumers have the possibility to shop exclusively with one of the firms (delegated common agency) or not (intrinsic common agency).

Mots clés : Tarification non linéaire, Concurrence, Discrimination par les prix, Agence commune.

Key Words : Nonlinear Pricing, Competition, Price Discrimination, Common agency

Classification JEL: L13, D82

* Particular thanks to Claude d'Aspremont, Bernard Caillaud, Jacques Cr\'emer, Jean-Jacques Laffont, Maurice Marchand, Marc Ivaldi, David Martimort, Bernard Salani\'e and Jean Tirole for their support and help during this work. We gratefully acknowledge the help of Giacomo Calzolari at the early stages of the project. Part of this work has been done when the second author was research assistant at CORE-UCL (Marie Curie fellowship); he wishes to thank this institute for its hospitality

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1 Introduction

The literature on nonlinear pricing (or second degree price discrimination) in oligopolistic environments has mainly analyzed settings in which consumers are characterized by one dimension of heterogeneity. In a recent paper, Rochet and Stole (2003, first paragraph of the introduction) argue that "a minimally accurate model of imperfect competition between duopolists suggests including *two* dimensions of heterogeneity -vertical and horizontal." Indeed, any empirical estimation of a nonlinear price competition is likely to require describing the consumers as having different tastes for the various products they are offered.

Following early attempts in the eighties, models with two dimensions of heterogeneity are receiving increasing attention.[1] The existing works always assume that competing firms are symmetrically uninformed about the consumers' privately observed characteristics. However, in many situations, it is not difficult to believe that each of the competing firms may have an informational advantage over its rivals on some subsets of the consumers' information set that directly relates to this firm.[2] For instance, when firms compete over many periods, it becomes likely that a firm may be better informed than its rivals about the consumer's taste for its *own* brand.[3] Moreover, as documented by Liu and Serfes (2003), firms can acquire information in two ways: directly, by repeated interaction (transaction or after-sale), telemarketing or direct mail survey, or indirectly, by credit cards reports, or a marketing firm; while indirect information is available to all competitors, direct information can be obtained only by the concerned firm, which implies that a firm may better be informed than its competitors about consumers' preferences for its own product. How does this asymmetric informational (dis)advantage affect competition? Does it benefit or does it harm firms?

To answer these questions, we develop a model of duopolistic competition under information asymmetry in which consumers are characterized by two dimensions of heterogeneity: their distinct tastes for two (imperfectly substitutable) goods, each good being offered by one firm only.[4] We assume that, while each firm perfectly knows the consumers' taste for its own brand, it remains uninformed about the taste for the rival's brand. Hence, in our model, there is an information asymmetry not only between consumers and firms, but also between firms themselves. Strikingly enough, the equilibria that we derive exhibit interesting new features, that is, *upward distortions* –in the products attributes– with respect to the first-best, rents that may be *decreasing* in the consumers' valuation for one of the brands, and strong qualitative differences between the situation in which consumers must buy both goods or none and the case in which they can shop exclusively with one

[1] See Borenstein (1985), Spulber (1989), Borenstein and Rose (1994), Champsaur and Rochet (1989), Ivaldi and Martimort (1994), Stole (1995), Villas-Boas and Schmidt-Mohr (1999), Armstrong and Vickers (2001), and Rochet and Stole (2002, 2003) among others.

[2] In a paper that performs an empirical test of nonlinear pricing in the US cellular industry, Miravette and Röller (2003, p.20) allow a possible asymmetry (between firms) in the distribution of the consumer's taste parameters. This asymmetry, which they coined "as an important departure from Ivaldi and Martimort (1994)", "perhaps reflects a better knowledge [from one of the firms] of the communication behavior of customers in such market."

[3] Over time, a firm can see how consumers react to changes in its price or to the different qualities it offers. From the observation of its own demand, firms prices and total sales in the industry, a firm can learn about consumer's taste for its own products.

[4] In a sense, there is both horizontal and vertical differentiation.

firm.

We also compare two pricing policies, basically screening vs. uniform pricing along the unknown dimension. Under fully discriminating pricing (FDP), each firm screens (via nonlinear pricing) the consumers along the unknown dimension of heterogeneity. Hence, given that each firm is informed about one dimension of the consumer's characteristics, the FDP consists in a mix of perfect discrimination along the known dimension and second-degree price discrimination along the unknown dimension. By contrast, under partially discriminating pricing (PDP), firms practice a mix of perfect discrimination along the known dimension, and uniform pricing along the unknown dimension. We show that, *in terms of firms' profit, PDP dominates FDP*. In other words, when each firm has an informational advantage over its rival, then it is better off *not* trying to screen the consumer along the (unknown) dimension that the rival firm is informed about. In a nutshell, *strategic ignorance of a consumer's taste for the rival's brand softens competition.*

The recent developments in common agency theory have made it an attractive tool to analyze competition between firms sharing common consumers.[5] Within this theory, one has to distinguish between *intrinsic* common agency and *delegated* common agency. Under the first setting, firms make simultaneously their offers, and the consumer must either accept both offers or none of the offers. While this setting is more realistic in vertical relations between producers and distributors[6], it can also apply to market competition with substitute goods. Indeed, consider the following example by Martimort and Stole (2003b): software and CPU are complements in the sense that one is worth nothing without the other. Hence a user will acquire both of them or none. However, they are also substitutes in that, more CPU can compensate for low quality software and vice versa; this setting correspond exactly to the case of intrinsic common agency with substitute goods.[7] Under delegated common agency, the consumer may refuse one of the firms' offers, or even both. Hence, this latter setting appears to be more descriptive of market competition in general. In our paper, we nevertheless conduct the analysis under both delegated and intrinsic common agency.

In our framework, each firm knows the information that its rival doesn't know. It implies that a firm's preferences over the consumer's type is strongly affected by the price offered by the rival firm. Using the terminology of Miravette and Röller (2003), the consumer's "effective type" becomes endogenous. Given that a firm's ranking of the consumer's unknown type depends on the equilibrium price of the rival firm (which knows that type), we obtain some strategic effects that, to the best of our knowledge, are new in this literature. As a result, *quality/quantity distortions, as well as the variation of a consumer's rent with respect to his type may go in opposite direction than one would expect.*

While under delegated common agency (where the equilibrium is unique) a firm's

[5]See Bernheim and Winsthon (1986), Stole (1991), Martimort (1992), Martimort and Stole (2003a, 2003b) among others.

[6]Assume that there are two main brands competing. Typically, a supermarket will want to distribute both brands instead of one only, otherwise it may loose the consumers who like the missing brand. Hence the decision is about opening a supermarket (and distributing both brands), or not.

[7]Even in the case of producer-distributor relationship, the distributor's tastes for each brand is likely to reflect those of the consumers that it will ultimately serve. A supermarket will not acquire too many units of, say brand A, if its customers prefer brand B.

equilibrium ranking of the consumer's types is standard (that is, consumers with higher valuation for a given brand get higher rents), the equilibrium distortions in quality are not standard. With respect to the first-best (where both –non cooperating– firms are fully informed about both dimensions), oversupply of quality may occur, hence a socially wasteful allocation. The intuition is roughly the following: whatever the pricing policies adopted by the firms, firm i distorts the qualities of its product in order to ensure that consumers who value much the product of the rival firm j still buy its own product; the same reasoning holds for firm j. Remember now that each firm is informed on the consumers' valuations for its own product; hence, for some consumers types, firm i does not distort much the quality offered whereas firm j distorts a lot its own quality; since qualities are substitutes from the consumers' viewpoint, a feed-back effect arises at equilibrium: firm i sometimes over-supplies quality whereas firm j under-supplies it with respect to the complete information situation.

Under intrinsic common agency, the results are more surprising. Roughly speaking, in that context the participation constraint of the consumer from the perspective of firm i still depends on the price offered by firm j (this is not the case under delegated common agency). Hence, the ranking of the various consumers from firm i's viewpoint is *directly* affected by the firm j's offer since firm j knows the information unknown to firm i. We argue in the paper that the ranking of the consumers types becomes endogenous in this context and show that there exists two equilibria with the following feature: in each equilibrium, consumers' rent are increasing in their valuation for one brand, and decreasing in their valuation for the other brand! (the two equilibria differ in that brands are permuted). This means that in each of these equilibria, the consumer with the highest valuation for, say brand A, and the lowest valuation for, say Brand B, gets the highest rent while the consumer with reverse features gets the lowest rent. Hence, our model suggests that *brand loyalty is not beneficial for all consumers*[8].

The paper is organized as follows. We first briefly relate our work to the literature. Section 2, introduces the building blocks of the model, and briefly present the complete information benchmark. The partially-discriminatory and fully-discriminatory equilibria are derived in Sections 3 and 4 respectively. In Section 5, we compare the two pricing policies for a particular specification of the model. Section 6 concludes.

Related literature. Our paper is related to the recent work on competition in nonlinear pricing with multidimensional heterogeneity, e.g., Ivaldi and Martimort (1994), Stole (1995), Schmidt-Mohr and Villas-Boas (1999), Armstrong and Vickers (1999), and Rochet and Stole (2002). In the first two papers, although heterogeneity is two-dimensional, the adverse selection problem that each firm ultimately faces is uni-dimensional, as in our framework. The remaining papers do analyze competition in which adverse selection is purely multidimensional. Armstrong and Vickers (1999), and Rochet and Stole (2002) show that competition in nonlinear prices may take simple forms; when there is not full coverage (competition), then each firm acts as a monopoly. Remarkably, under full competition, all distortions disappear. Our paper belongs to the category of the first two papers,

[8]In the literature on horizontal differentiation, a high valuation to pay for a brand is usually interpreted as brand loyalty. Our results suggest that only consumers loyal to one of the two brands will get high rents. Consumers loyal to the competing brand will get little rent.

and is most closely related to Ivaldi and Martimort (1994), whose framework we use. They find empirical evidence of nonlinear pricing in the French Energy market. The patterns or rents (decreasing) and (upward) quality distortions that we obtain in our paper do not occur in the works cited above. An exception is Bond and Gresik (1997) who analyze a tax competition game using a (uni-dimensional) intrinsic common agency framework with asymmetrically informed principals. They also obtain that the rent of the agent *may* decrease in its cost efficiency parameter. While in their paper, this phenomena may not occur (as it is one of three possible equilibria), it occurs in all equilibria of our intrinsic common agency game.

Our result that the PDP dominates the FDP (which we obtain under delegated common agency) also contributes to the the literature on the welfare effects of uniform vs. discriminatory pricing (see, e.g., Thisse and Vives, 1988, Holmes, 1989, and Corts, 1998, among others), which is extensively surveyed in Stole (2003). One of the main findings of this literature is that duoplists may prefer uniform pricing to (third degree) price discrimination. We add to that literature a new comparison which incorporates different forms of price discrimination simultaneously.

2 The Model

Consider a duopoly in which two firms ($i = 1, 2$) sell one unit of variable quality products to a continuum of buyers.[9] The gross utility of a given consumer, who consumes quality q_1 of good 1 and quality q_2 of good 2, is (superscript 'g' stands for 'gross')

$$U^g(\theta, q_1, q_2) \equiv u_1(\theta_1, q_1) + u_2(\theta_2, q_2) + u(q_1, q_2), \tag{1}$$

where $\theta \equiv (\theta_1, \theta_2)$. Parameter θ_i relates to the consumer's intrinsic valuation for the quality of good i; that θ_i is (a priori) different from θ_j simply translates the fact that, for equal products' qualities, some consumers inherently have a stronger preference for one good. However, the buyers' utility derived from the consumption of the products also depends on the attributes attached to the different goods, which is embodied in the quality levels. Products are assumed to be imperfect substitutes, i.e., $U^g_{q_1q_2} = u_{q_1q_2} \leq 0$, for all quality levels: provided that a consumer buys both goods, a marginal increase in the quality of one product reduces the marginal valuation for the other good.

The following assumptions are assumed to hold for all strictly positive consumption levels: $U^g_{q_i} > 0$, $U^g_{q_iq_i} < 0$, $U^g_{\theta_i} > 0$, $U^g_{\theta_iq_i} > 0$. These conditions are interpreted as follows: for each good, marginal utility increases with quality, but at a decreasing rate; the stronger the preference for good i is, the higher are the consumer's utility and marginal utility levels. For tractability, we also assume that $u_i(\theta_i, 0) = 0$, $U^g(\theta, 0, 0) = 0$ and $|U^g_{q_iq_i}| \geq |U^g_{q_iq_j}|$.[10]

It is common knowledge that θ_i is independently distributed on the interval $\Theta_i \equiv [\underline{\theta}_i, \overline{\theta}_i]$

[9] Following Maskin and Riley (1984), another interpretation is that the consumers buy a variable quantity of the various goods.

[10] That $u_i(\theta_i, 0) = U^g(0, 0) = 0$ is assumed for analytical convenience only and does not affect our results as long as $u_i(\theta_i, 0)$ and $U^g(\theta, 0, 0)$ do not depend neither on θ_i nor on θ_j; the last condition, namely $|U^g_{q_iq_i}| \geq |U^g_{q_iq_j}|$, is standard and implies that, were firms offering purely linear prices, a consumer's demand function for one good would be more sensitive to the price of that good than to the rival price.

according to the (strictly positive) density $f_i(.)$ with c.d.f. $F_i(.)$, $i = 1, 2$.[11] However, only a consumer perfectly knows his valuation for both goods. As regards firms, we assume they are asymmetrically un-informed as defined next.

Definition 1. *Asymmetrically un-informed duopolists: firm i is perfectly informed about θ_i but only knows the distribution of θ_j, $i \neq j$, $i, j = 1, 2$.*

This information structure underlines the fundamental asymmetry between competitors: each firm is better informed on the consumers' marginal valuations for its product than on the consumers' valuations for the rival product.

Firm i's profit is defined as follows

$$V_i = p_i - C_i(q_i),$$

where $C_i(q_i)$ is the strictly increasing and convex cost of producing good i with quality level q_i and p_i is the price paid by the consumer to firm i.

A consumer's net utility is the difference between his gross utility and the prices paid to the firms,[12] or

$$U = -p_1 - p_2 + U^g(\theta, q_1, q_2).$$

Throughout the paper, we shall focus on a given consumer.

The timing goes as follows: first, firms make simultaneously and non-cooperatively their offers; second, consumers decide which offers they accept. As argued in the Introduction, two cases are worth considering:[13]

- *Intrinsic common agency.* In that case, the consumer attributes no value to product i without the purchase of good j. Therefore, to ascertain that the consumer will purchase its good, firm i must ensure that the consumer's utility when he accepts both firms' offers is larger that his utility if he decides to purchase none (which is normalized to 0), or
$$U \geq 0. \qquad (PC_i^I)$$

- *Delegated common agency.* In that setting, the consumer has the extra option to decide buying only one product. Therefore, from the perspective of firm i the participation constraint becomes
$$U \geq \max\left\{0; U_i^{out}(\theta)\right\}, \qquad (PC_i^D)$$
where $U_i^{out}(\theta)$ is the consumer's outside opportunity with respect to firm i: this represents the consumer's rent when he decides to buy only from the rival firm j. Notice that this outside opportunity is endogenous since it depends on the rival firm's offer.

[11] Notice that considering imperfectly correlated adverse selection parameters would not qualitatively change the main messages conveyed in this paper since this would only modify each firm's prior on the unknown piece of information.

[12] If $q_i = 0$ then the price paid by the consumer to firm i is null.

[13] The cases correspond to the intrinsic and delegated common agency settings, as coined by Bernheim and Whinston (1986).

Benchmark. As a useful benchmark, let us briefly analyze the case in which both firms perfectly know both consumer's preferences, i.e., θ_1 *and* θ_2. For future references, we refer to this case as the *complete information* case. In this case, each firm can perfectly discriminate the different types of consumers. Under complete information, firm i can restrict his offer to a pair $\{q_i(\theta), p_i(\theta)\}$.[14]

Under *intrinsic common agency*, firm i solves

$$\max_{\{q_i(\theta),p_i(\theta)\}} p_i(\theta) - C_i(q_i(\theta))$$
$$\text{s.t. } (PC_i^I) : U(\theta) = U^g(\theta, q_1(\theta), q_2(\theta)) - p_1(\theta) - p_2(\theta) \geq 0.$$

The participation constraint must bind at equilibrium since firm i seeks to set as high a price as possible; this defines the value of the optimal price set by firm i:

$$p_i(\theta) = U^g(\theta, q_1(\theta), q_2(\theta)) - p_j(\theta).$$

Substituting $p_i(\theta)$ in the objective of firm i and optimizing with respect to $q_i(\theta)$ yields the following first-order condition (which is necessary and sufficient in our context):

$$U^g_{q_i}(\theta, q_i^{FB}(\theta), q_j(\theta)) = C_{iq_i}(q_i^{FB}(\theta)) \quad i \neq j \quad i, j = 1, 2. \tag{2}$$

Equation (2) is a usual 'marginal benefit equals marginal cost' rule from firm i's perspective, which accounts for firm j non-cooperative offer. Solving the system formed by the two best-responses yield the complete information equilibrium qualities offered by the firms. It is worth mentioning that only the sum of the prices $p_1(\theta) + p_2(\theta)$ is defined in equilibrium of the intrinsic common agency game, implying that the way firms share the consumer' surplus is not defined at equilibrium.

Considering (2) evaluated for the equilibrium qualities and differentiating w.r.t. θ_i and θ_j, we obtain that $\frac{\partial q_i^{FB}}{\partial \theta_i}(\theta) \geq 0$ and $\frac{\partial q_i^{FB}}{\partial \theta_j}(\theta) \leq 0$, for $i, j = 1, 2$, $i \neq j$. Intuitions are straightforward. First, the lower the consumer's valuation θ_i for the quality of the good produced by firm i is, the lower is firm i's equilibrium quality level. Second, applying a similar argument, the smaller θ_j is, the smaller is quality q_j; such a decrease in q_j leads to an increase in q_i since the marginal utility of the consumer for firm i's good is increased.

Consider now the case of *delegated common agency*. With respect to the previous case, the unique difference is that the participation constraint becomes:

$$U(\theta) = U^g(\theta, q_i(\theta), q_j(\theta)) - p_i(\theta) - p_j(\theta) \geq \max\left\{0; U_i^{out}(\theta) = U^g(\theta, q_i = 0, q_j(\theta)) - p_j(\theta)\right\}.$$

In words, from the perspective of firm i, it must ensure that the consumer is better off accepting both firm's offers rather than consuming exclusively the rival firm's good.[15]

[14] In common agency games under adverse selection, firms are typically assumed to compete in nonlinear prices; see Martimort and Stole (2002) for a complete analysis of the role of the space of contracts in these games. In our context, under complete information, one can show that there is no loss of generality in restricting attention to direct contracts, i.e. to price-quality pairs based on the consumer's information.

[15] Implicit in this formulation of the outside opportunity gain for the consumer is that a firm's offer cannot be contingent on the consumer's decision to buy only the rival product. Such a contracting possibility is likely to violate antitrust rules and is therefore discarded from our analysis.

Later on, we shall spend time understanding the impact of the outside opportunity on the competition between firms. At this stage, we simply want to emphasize the following point: under complete information, if the outside opportunity is strictly positive then equilibrium quality levels are left unaffected. Indeed, suppose that $U_i^{out}(\theta)$ is positive; as previously, firm i will set the highest price consistent with the consumer buying its product, or

$$p_i(\theta) = U^g(\theta, q_i(\theta), q_j(\theta)) - U^g(\theta, q_i = 0, q_j(\theta)).$$

Putting this expression in firm i's objective, one can see that the quality chosen by firm i is the same as the one under intrinsic common agency: under complete information, the possibility for the consumer to shop exclusively with one of the competitors does not affect the equilibrium quality choices and only modifies the consumer's equilibrium rent.[16] The intuition is immediate: under full information, each firm perfectly (first-degree) discriminates the consumer, but might be constrained, in terms of surplus given up to the consumer (the outside opportunity), by its competitors. Under complete information on the firms' side, the competitive pressure which is channeled through the buyer's outside opportunities only leads to a reallocation of the total surplus between the consumer and the firms. Note that unlike in the intrinsic common agency case, firm i's price is now uniquely defined. Loosely speaking, the outside opportunity of the consumer with respect to firm i determines the latter's bargaining power in sharing the surplus with the other firm.

Remark. Note that the outcomes obtained in the previous benchmark would have been obtained had we assumed that both firms act cooperatively (multiproduct monopoly). Hence, for later comparison, the complete information benchmark also refers to the perfectly discriminating monopoly or the first-best from the viewpoint of firms.

3 Competition in Partially-Discriminatory Pricing

We are first going to analyze the case of competition in which firms price uniformly along the unknown dimension of heterogeneity. Under Partially-Discriminatory Pricing (hereafter PDP), firm i being informed about the consumer's valuation for its own product, its offer to the consumer still depends on this piece of information. Hence, the firms' pricing policy consists in a mix of first-degree price discrimination along the known dimension of the consumer's preference and uniform pricing along the unknown dimension: firms only partially discriminate the different types of buyer. This competition in uniform pricing allows to highlight many of the specificities of the interaction between asymmetrically un-informed firms.

Delegated common agency. Firm i offers a price-quality pair which depends only on the known information, $\{p_i(\theta_i), q_i(\theta_i)\}$,[17] in order to maximize its expected profit while

[16] In common agency models with symmetrically uninformed principals, this result also holds under incomplete information. As we shall see later, this results breaks down in our model when principals' information is incomplete and asymmetric.

[17] In a sense, firm i's offer consists in a price schedule which is degenerated to a single point.

ensuring that the buyer does not shop exclusively with its rival, or

$$\max_{\{q_i(\theta_i),p_i(\theta_i)\}} \mathbb{E}_{\theta_j}\left\{p_i(\theta_i) - C_i\left(q_i(\theta_i)\right)\right\}$$

$$\text{s.t. } (PC_i^D): \forall\theta_j \in \Theta_j, \quad U(\theta_i,\theta_j) \geq \max\left\{0; U_i^{out}(\theta_j) = -p_j(\theta_j) + U^g(\theta_j, q_i = 0, q_j(\theta_j))\right\}.$$

First, we need to check whether the consumer's 'threat' to shop exclusively with the rival firm is credible, i.e., whether outside opportunities are positive when firms compete in partially-discriminatory pricing policies. The following result is obtained.

Lemma 1. *When goods are substitutes and firms compete in partially-discriminatory pricing policies, both outside opportunities are positive.*

Proof. See Appendix A.1. □

The intuition is straightforward. Consider the extreme scenario in which qualities are highly substitutable. In that case, if the consumer decides not to acquire product i, then his loss of utility can be almost fully offset by increasing the level of quality chosen for good j; differently stated, the consumer can threaten firm i to shop exclusively with its rival and derive a strictly positive utility level. Lemma 1 shows that this reasoning holds even when products are weakly substitutable and firms adopt partially-discriminatory pricing strategies.

In our setting with asymmetrically informed firms, the main difficulty that arises in solving firm i's problem stems from the fact that, *a priori*, firm j's offer to the consumer depends on firm i's unknown information, θ_j. As we argue next, this implies that the behavior of the consumer's rent requires to be studied with particular attention. To highlight this point, we split the firms' optimization problem into two sub-problems: first, we study the duopolists' best-responses assuming that the buyer's rent behaves monotonically w.r.t. the adverse selection parameters; second, we focus on the variation of the buyer's rent w.r.t. the private information parameters.

Best-responses for given indifferent types. Given Lemma 1, a relevant variable in our context is the difference between the rent $U(\theta)$ and the outside opportunity $U_i^{out}(\theta_j)$ that we call the 'net rent'. The main issue here is to determine how the net rent of the consumer in relation to a given firm varies with respect to the information unknown by this firm. Assume in a first time that $\frac{\partial}{\partial\theta_j}[U(\theta) - U_i^{out}(\theta_j)]$ has a constant sign.[18] This implies that, from firm i's viewpoint, there exists a type $\theta_j^* \in \{\underline{\theta}_j, \overline{\theta}_j\}$ of buyer, called the *indifferent type for firm i*, such that if the participation constraint is satisfied for this type, then it is satisfied for all the other types of buyer. Since firm i dislikes giving up excessive rent to the buyer, it will offer the price $p_i(\theta_i)$ such that the individual rationality constraint binds at equilibrium for $\theta_j = \theta_j^*$, that is,

$$U^g(\theta_i,\theta_j^*, q_i(\theta_i), q_j(\theta_j^*)) - p_i(\theta_i) - p_j(\theta_j^*) = -p_j(\theta_j^*) + U^g(\theta_i,\theta_j^*, q_i = 0, q_j(\theta_j^*)), \quad (3)$$

[18] This property is satisfied at equilibrium.

or equivalently,

$$p_i(\theta_i) = U^g(\theta_i, \theta_j^*, q_i(\theta_i), q_j(\theta_j^*)) - U^g(\theta_i, \theta_j^*, q_i = 0, q_j(\theta_j^*)). \tag{4}$$

Two points are worth highlighting: first, since we consider participation of all the different types of consumers, everything happens as if firm i was considering that the buyer is of type θ_j^*; second, the price offered by firm i is uniquely defined by the binding participation constraint.

Replacing (4) in firm i's profit and optimizing w.r.t. $q_i(\theta_i)$ yields the following first-order condition[19]

$$U^g_{q_i}(\theta_i, \theta_j^*, q_i(\theta_i), q_j(\theta_j^*)) = C_{iq_i}(q_i(\theta_i)). \tag{5}$$

We can perform similar computations for firm j and obtain the following condition

$$U^g_{q_j}(\theta_i^*, \theta_j, q_i(\theta_i^*), q_j(\theta_j)) = C_{jq_j}(q_j(\theta_j)). \tag{6}$$

Considering (5) evaluated in $\theta_i = \theta_i^*$ and (6) in evaluated in $\theta_j = \theta_j^*$ forms a system of two equations, whose solution determines $q_i(\theta_i^*)$ and $q_j(\theta_j^*)$. Then, plugging back $q_i(\theta_i^*)$ and $q_j(\theta_j^*)$ in (5) and (6) respectively, and solving this new system yields the quality profiles $q_i(\theta_i)$ and $q_j(\theta_j)$ in the PDP equilibrium. We note that total differentiation of (5) with respect to θ_i yields (we omit arguments for simplicity)

$$q_i'(\theta_i) = -\frac{U^g_{\theta_i q_i}}{U^g_{q_i q_i}} > 0.$$

The indifferent types. It remains to determine θ_i^* and θ_j^*. To this purpose and to build the intuition, let us introduce the 'net prices' defined as follows: $\tilde{p}_i(\theta_i) \equiv p_i(\theta_i) - u_i(\theta_i, q_i(\theta_i))$, $i = 1, 2$. Since firm i is perfectly informed on the buyer's valuation for its product, the part $u_i(\theta_i, q_i(\theta_i))$ of the gross utility is perfectly observed by that firm and hence cannot be a source of rent for the consumer. This is the basic motivation to focus on net prices. With this new notation, the consumer's rent is

$$U(\theta) = -\tilde{p}_i(\theta_i) - \tilde{p}_j(\theta_j) + u(q_i(\theta_i), q_j(\theta_j)),$$

while his outside opportunity becomes

$$U_i^{out}(\theta_j) = -\tilde{p}_j(\theta_j) + u(0, q_j(\theta_j)).$$

The net rent can hence be rewritten as

$$\begin{aligned} U(\theta) - U_i^{out}(\theta_j) =& - \tilde{p}_i(\theta_i) + u(q_i(\theta_i), q_j(\theta_j)) - u(0, q_j(\theta_j)) \\ =& \tilde{p}_i(\theta_i) + \int_0^{q_i(\theta_i)} u_{q_i}(x, q_j(\theta_j))dx. \end{aligned}$$

[19] The associated second-order condition is satisfied under our assumptions.

Differentiating w.r.t. θ_j yields:

$$\frac{\partial}{\partial \theta_j}\left[U(\theta) - U_i^{out}(\theta_j)\right] = q_j'(\theta_j)\int_0^{q_i(\theta_i)} u_{q_i q_j}(x, q_j(\theta_j))dx \leq 0. \tag{7}$$

Hence, the price-quantity couple offered by firm i is such that the consumer's net rent is decreasing in the adverse selection parameter θ_j, and symmetrically for firm j. To see this from a different perspective, consider a hypothetical marginal increase $d\theta_j > 0$ in the consumer's valuation for good j; this increase has two effects:

- Firstly, the consumer is led to pay a larger net price to firm j. Indeed, using the equivalent of (4) for firm j, we get

  $$p_j(\theta_j) = u_i(\theta_i^*, q_i(\theta_i^*)) + u_j(\theta_j, q_j(\theta_j)) + u(q_i(\theta_i^*), q_j(\theta_j)) - u_i(\theta_i^*, q_i(\theta_i^*)) - u(q_i(\theta_i^*), 0).$$

  Hence, the equilibrium net price is defined by $\tilde{p}_j(\theta_j) = u(q_i(\theta_i^*), q_j(\theta_j)) - u(q_i(\theta_i^*), 0)$; the variation of the net price is then given by $\tilde{p}_j'(\theta_j)d\theta_j = u_{q_j}(q_i(\theta_i^*), q_j(\theta_j))q_j'(\theta_j)d\theta_j \geq 0$. Note that this increase affects in a similar way the consumer's rent and the outside opportunity, so that this effect is neutral for the net rent.

- Secondly, this increase also affects the part of the buyer's rent which cannot be fully captured by the firm; this in turn impacts differently the consumer's rent and his outside opportunity since the former increases by $u_{q_j}(q_i(\theta_i), q_j(\theta_j))q_j'(\theta_j)d\theta_j$ whereas the latter increases by $u_{q_j}(0, q_j(\theta_j))q_j'(\theta_j)d\theta_j$; since products are substitutes for the consumer, the former increase is weaker than the latter, thereby implying that the net rent is decreasing in θ_j.

An implication of (7) is that, for firm i, the indifferent type is $\theta_i^* = \overline{\theta}_i$; using symmetry we obtain $\theta_j^* = \overline{\theta}_j$. Another implication of (7) is that, at equilibrium, the consumer's rent can be expressed as follows

$$U(\theta) = U_i^{out}(\theta_j) + \int_{\theta_j}^{\overline{\theta}_j} q_j'(z)\int_0^{q_i(\theta_i)} -u_{q_i q_j}(x, q_j(z))dxdz. \tag{8}$$

Equation (8) illustrates that the consumer derives utility from two distinct sources. The first one is incomplete information on the firms' side, which prevents competitors from fully extracting the consumer' surplus. The second one stems from the implicit threat put by the consumer on each of the non-cooperating firms to shop exclusively with the rival firm. Firm i provides the consumer with the highest valuation for the rival good (i.e., $\overline{\theta}_j$) with a utility level equal to his outside opportunity; consumers with lower valuations for good j earn more than their outside opportunities. However, the $\overline{\theta}_j$-consumer gets more rent than consumers with lower valuation. Indeed, we have

$$\begin{aligned}\frac{\partial U}{\partial \theta_j}(\theta) &= -\tilde{p}_j'(\theta_j) + u_{q_j}(q_i(\theta_i), q_j(\theta_j))q_j'(\theta_j)\\ &= q_j'(\theta_j)\int_{q_i(\theta_i)}^{q_i(\overline{\theta}_i)} -u_{q_i q_j}(x, q_j(\theta_j))dx \geq 0,\end{aligned}$$

meaning that the larger the consumer's valuation for the rival (say firm j)'s product, the larger the consumer's rent given up by firm i at equilibrium. An intuition for this result is the following. In relation to firm i, the $\overline{\theta}_j$-consumer has the highest outside opportunity[20]. Indeed, if he was an exclusive consumer of firm j, such consumer would have the highest rent; firm j, uninformed about the private information of the consumer (θ_j) and pricing uniformly, would set its unique price in such a way that the high-valuation consumer gets the highest rent while the low valuation consumer gets no rent. Hence, firm i must provide the highest rent to the $\overline{\theta}_j$-consumer to make him buy its good. Since firm i dislikes leaving rents, it will set the rent exactly equal to the outside opportunity. Conversely, the $\underline{\theta}_j$ has the lowest outside opportunity. Given that, under uniform pricing, the *unique* price offered by each firm is set so low (to allow the $\overline{\theta}_j$-consumer to get at least his (high) outside opportunity), this benefits the $\underline{\theta}_j$-consumer, who in the end gets a rent higher than its (low) outside opportunity.

In sum, competition forces each firm to provide higher rents to consumers with higher valuations for the rival's product, although they get exactly their outside opportunity, contrarily to lower valuation buyers who get a smaller rent but larger than their outside opportunity. It is these two effects, which have not been analyzed by the literature, that our analysis wishes to emphasize. The profile of rents is illustrated in Figure 1.[21]

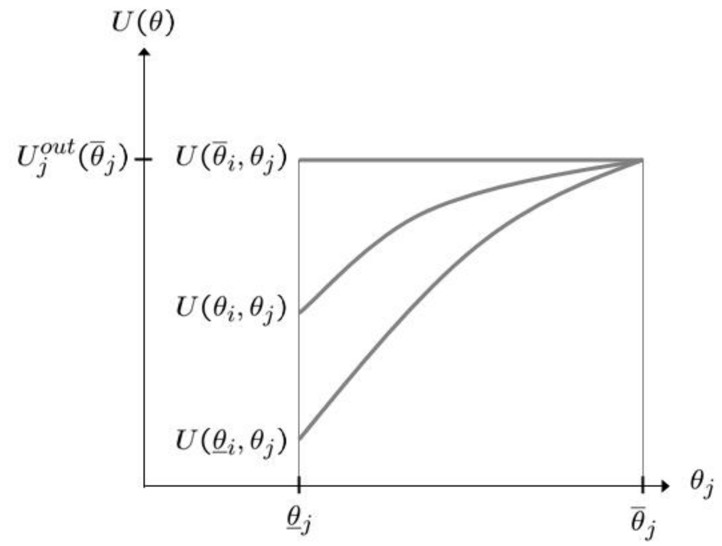

Figure 1: Rent profiles in the PDP equilibrium with the possibility of exclusive shopping.

To complete our analysis of the delegated common agency case, we now focus on the

[20]Formally, the fact that $\frac{\partial}{\partial \theta_j}\left[U(\theta) - U_i^{out}(\theta_j)\right] \leq 0$ and $\frac{\partial}{\partial \theta_j} U(\theta) \geq 0$ necessarily imply that $\frac{\partial}{\partial \theta_j} U_i^{out}(\theta_j) \geq 0$.

[21]To draw the utility curves in Figure 1, we use the fact that at equilibrium $\frac{\partial^2 U}{\partial \theta_i \partial \theta_j}(\theta) \leq 0$.

pattern of quality levels. The first-order conditions (5) and (6) can be rewritten as follows

$$U^g_{q_i}(\theta, q_i(\theta_i), q_j(\theta_j)) - C_{iq_i}(q_i(\theta_i)) = -\int_{q_j(\theta_j)}^{q_j(\overline{\theta}_j)} u_{q_iq_j}(q_i(\theta_i), y)dy \geq 0, \tag{9}$$

$$U^g_{q_j}(\theta, q_i(\theta_i), q_j(\theta_j)) - C_{jq_j}(q_j(\theta_j)) = -\int_{q_i(\theta_i)}^{q_i(\overline{\theta}_i)} u_{q_iq_j}(x, q_j(\theta_j))dx \geq 0. \tag{10}$$

Equations (9) and (10) implicitly define the firms' best-responses, denoted by $R_i(q_j(\theta_i))$. With respect to the complete information benchmark, where the best-responses were such that marginal utility equals marginal cost for each product, asymmetric information implies that the best-responses curves are shifted downwards. Everything happens here as if firm i anticipates that the consumer's valuation for product j is given by $\overline{\theta}_j$; since quality levels are increasing in the consumer's valuations, firm i anticipates too high a quality level purchased by the consumer for product j; products being substitutes, firm i's quality level tends to decrease w.r.t. the full information benchmark. However, there is a feedback effect to consider in order to fully apprehend the equilibrium pattern of quality levels. Suppose indeed that θ_j is close enough to $\overline{\theta}_j$; then firm i's quality level is not much distorted; suppose simultaneously that θ_i is close to $\underline{\theta}_i$; then firm j's quality level is much distorted; since products are substitutes, firm i over-supplies and firm j under-supplies quality at equilibrium. The pattern of distortion is represented graphically in Figure 2.

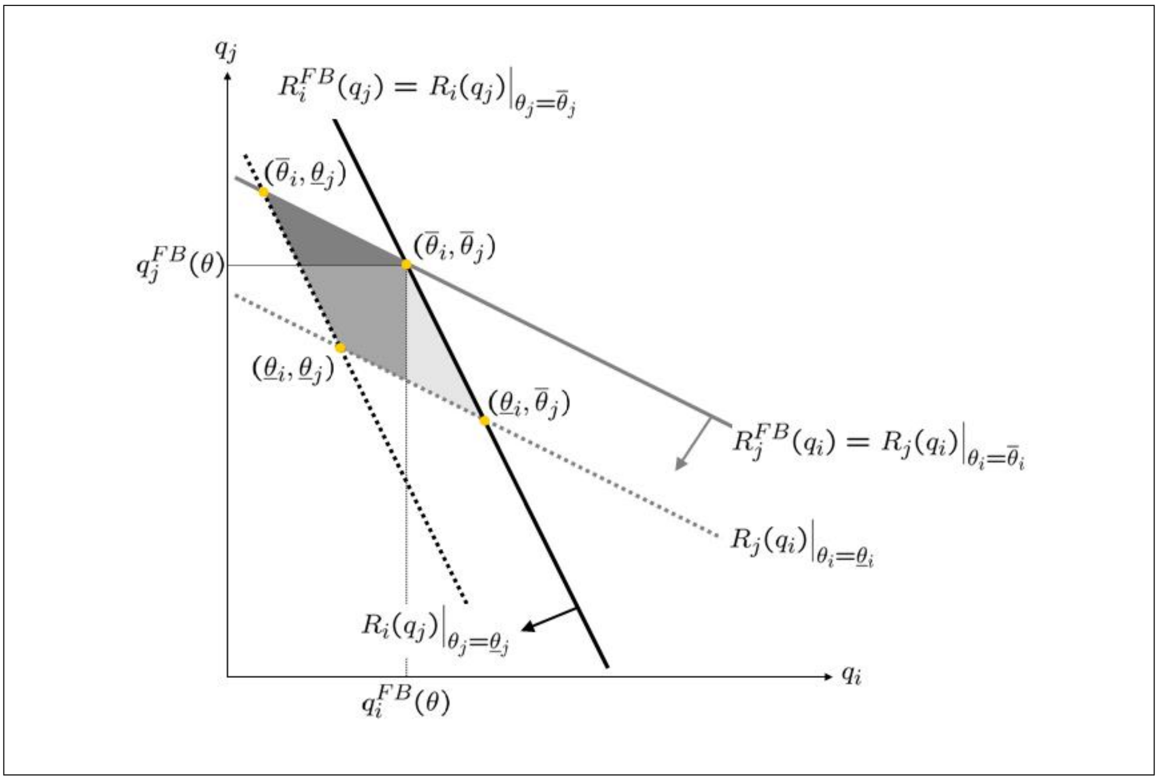

Figure 2: Best-responses and the set of quality levels in the PDP equilibrium with the possibility of exclusive shopping.

Summarizing, we get the following proposition.

Proposition 1. *Under delegated common agency, there exists a unique partially-discriminatory* *pricing equilibrium, with indifferent types $\theta_j^* = \overline{\theta}_j$ and $\theta_i^* = \overline{\theta}_i$. The rent of a consumer is increasing in his valuation for each product, while his net rent is decreasing. With respect to the first-best case, there is either over-supply of quality for one good and under-supply of quality for the other good, or under-supply of quality for both goods. Only the indifferent types get the first-best qualities.*

Intrinsic common agency. In this setting competition is less severe since the consumer's outside opportunities are now null. How is the analysis affected?

As previously, consider firm i and assume that the consumer's rent behaves monotonically with respect to θ_j. Then, we can define the indifferent type θ_j^* such that the participation constraint is exactly binding, or

$$U(\theta_i, \theta_j^*) = -\tilde{p}_i(\theta_i) - \tilde{p}_j(\theta_j^*) + u(q_i(\theta_i), q_j(\theta_j^*)) = 0.$$

It is immediate that the first-order condition that characterizes firm i's optimal quality level is still given by (5). The reason is that the previous case (delegated common agency) differs from the current situation only in the outside opportunity, which was independent of q_i in the delegated common agency case. Therefore, if the indifferent types under intrinsic common agency are the same as under delegated agency, then both settings will yield the same quality levels and will differ only in the allocation of total surplus between the consumers and the firms. As we shall soon see, the indifferent types are different in the current setting, which yields different equilibria from those obtained under delegated common agency.

Remember that, using net prices introduced earlier, the consumer's rent could be expressed as

$$U(\theta) = -\tilde{p}_i(\theta_i) - \tilde{p}_j(\theta_j) + u(q_i(\theta_i), q_j(\theta_j))$$

Taking the partial derivative with respect to θ_j yields

$$\frac{\partial}{\partial \theta_j} U(\theta) = -\tilde{p}_j'(\theta_j) + u_{q_j}(q_i(\theta_i), q_j(\theta_j)) q_j'(\theta_j),$$

which can no longer be signed a priori: *firm i's ranking over the different (unknown) types of consumer is endogenous as it depends now directly on firm j's offer to the consumer.*

We now determine the indifferent types. Suppose that firm i expects firm j's offer to be such that consumer with larger valuations for product j earn smaller rents, or $\frac{\partial}{\partial \theta_j} U(\theta) \leq 0$. Firm i's offer is then such that $\tilde{p}_i(\theta_i) = -\tilde{p}_j(\overline{\theta}_j) + u(q_i(\theta_i), q_j(\overline{\theta}_j))$. The consumer's rent is then equal to

$$\begin{aligned} U(\theta) &= [\tilde{p}_j(\overline{\theta}_j) - u(q_i(\theta_i), q_j(\overline{\theta}_j))] - [\tilde{p}_j(\theta_j) - u(q_i(\theta_i), q_j(\theta_j))], \\ &= \int_{\theta_j}^{\bar{\theta}_j} [\tilde{p}_{j\theta_j}(z) - u_{q_j}(q_i(\theta_i), q_j(x)) q_j'(x)] dx. \end{aligned}$$

Partial derivation with respect to θ_i yields

$$\frac{\partial}{\partial\theta_i}U(\theta) = \int_{\theta_j}^{\overline{\theta}_j} -u_{q_iq_j}(q_i(\theta_i), q_j(x))q_j'(x)q_i'(\theta_i)dx,$$

which is positive. We have therefore proven that if the indifferent type for firm i is $\overline{\theta}_j$, then the indifferent type for firm j is $\underline{\theta}_i$. In a similar way, it can be proven that if the indifferent type for firm i is $\underline{\theta}_j$, then the indifferent type for firm j is $\overline{\theta}_i$. Therefore, the indifferent types are either $\{\underline{\theta}_i, \overline{\theta}_j\}$ or $\{\overline{\theta}_i, \underline{\theta}_j\}$: multiple equilibria emerge, each equilibrium leading to rent and quality levels which differ from those obtained in the delegated common agency setting.

As in any model of competition, the contractual offer proposed by one competitor to the consumer affects the behavior of the latter in its relationship with the rival firm. With asymmetrically informed firms, there is an additional effect though. Indeed, each firm being informed on the piece of information unknown to its rival, the offer of, say, firm i will also affect how firm j ranks the different types of buyer: the relevant unknown information for firm j is not only θ_i but also the information transmitted in firm's proposal. Preferences of the consumer for the quality of the various products become endogenous. This is best observed in the intrinsic common agency setting in which only the sum of prices is defined at equilibrium; in that case, if firm j offers a contract such that consumers with larger valuations for its product earns larger rents, then firm i offers in turn a contract with the reverse property, namely that consumers with larger inherent valuations for product i obtain smaller rent. Under delegated common agency, the participation constraints pin down uniquely the prices offered by the firms, leading to a unique equilibrium; in that case, the need to prevent exclusive shopping forces a given firm to give up larger rents to consumers with larger valuations for its product.

We conclude this case with an analysis of the equilibrium quality levels. Adapting the previous analysis, we observe that best-responses are shifted in opposite directions: at equilibrium, there is now over-supply of quality for one good and under-supply for the other product. The equilibrium quality levels and rent profiles in that case are represented in Figure 3 and Figure 4 respectively.

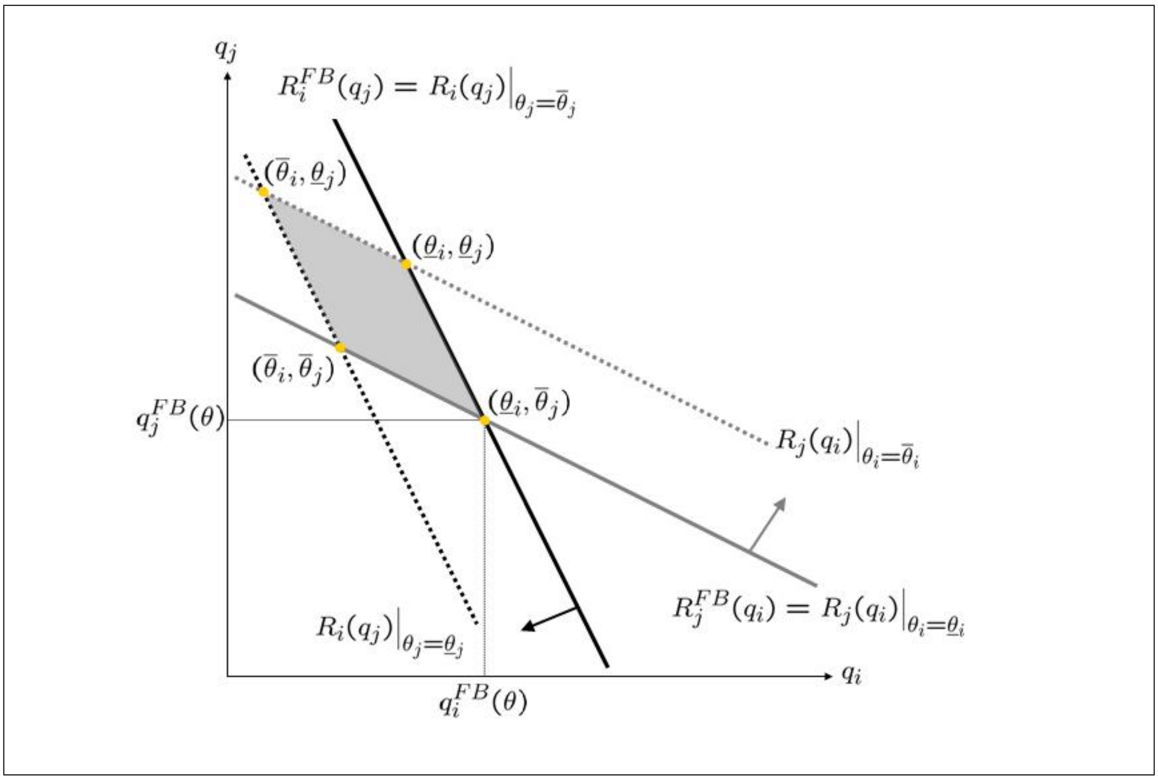

Figure 3: Best-responses and the set of quality levels in the PDP equilibrium without the possibility of exclusive shopping (indifferent types $\{\underline{\theta}_i, \overline{\theta}_j\}$).

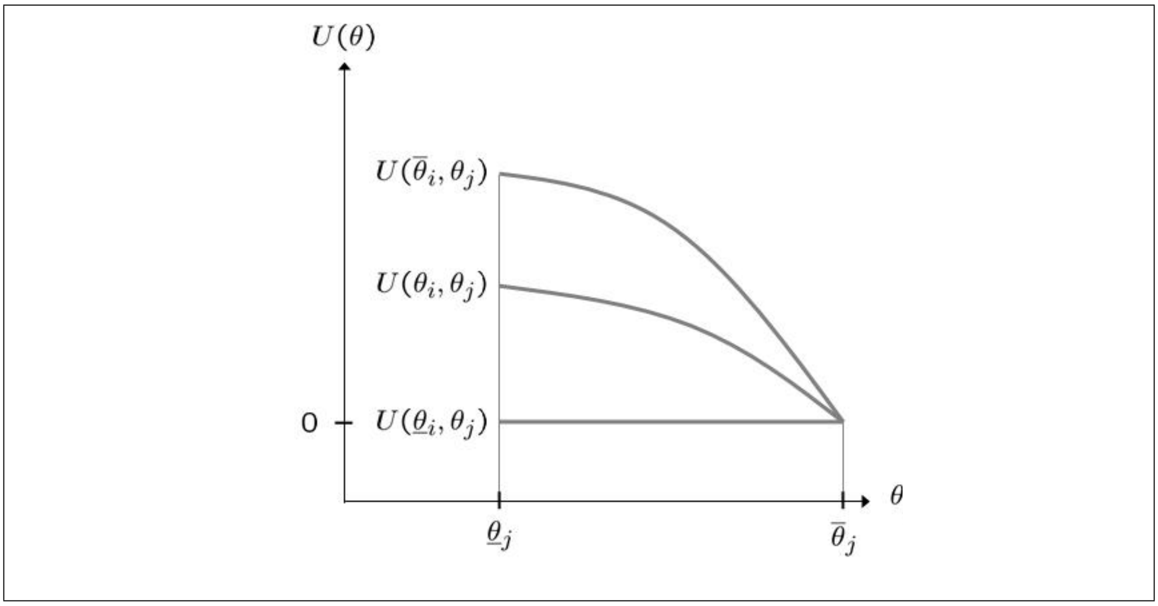

Figure 4: Rent profiles in the PDP equilibrium without the possibility of exclusive shopping (indifferent types $\{\underline{\theta}_i, \overline{\theta}_j\}$).

Summarizing, we obtain the following proposition.

Proposition 2. *Under intrinsic common agency, there exist two symmetric partially-discriminating equilibria: one with indifferent types $\{\underline{\theta}_i, \overline{\theta}_j\}$, the other with indifferent types $\{\overline{\theta}_i, \underline{\theta}_j\}$. In both equilibria:*

(i) the consumer's rent is increasing in the valuation for one good and decreasing in the valuation for the other good.

(ii) with respect to the first best, there is over-supply of quality for one good and under-supply for the other; only the indifferent types get the first-best qualities.

In these equilibria, consumers who are loyal to a brand, that is, those who value much more one brand than the other, earn either the largest rents, or the smallest rents depending on which brand they fancy. Indifferent consumers, that is those who equally like both brands are the only ones whose rent do not vary across the two equilibria Hence, brand loyalty does not pay off for all consumers. Only consumers who are loyal to one of the brands get the highest rent[22]. We discuss these features more extensively in the next section.

Consumers participation: From Common to Exclusive Agency. So far, we assumed implicitly that firms serve all consumers, i.e., there is full participation at equilibrium since for all types, the equilibrium rent is larger than the corresponding outside opportunities. However, one could find circumstances under which a firm prefers not to serve some potential customers in order to extract more surplus from the remaining participating consumers. In terms of our model, in the delegated common agency case, this would mean that firm i sets $\theta_j^* < \overline{\theta}_j$.

From firm i's perspective, the more 'costly' consumers are those who have large valuations for the rival firm's good since firm i must give them up sufficiently large rents to ensure that they will not shop exclusively with the rival. High-valuations customers who put too strong an implicit threat on both firms end up consuming only their most preferred good only: for those consumers, $U(\theta_1, \theta_2) \leq U_i^{out}(\theta_j)$ for $i, j = 1, 2$ and they choose brand i iff $U_j^{out}(\theta_i) \geq U_i^{out}(\theta_j)$.

Figure 5 illustrates the consumers' choices under delegated common agency when both firms non-cooperatively adopt such a strategy.[23]

[22]Our result support a popular wisdom, which believes that it is sometimes good to buy from the rival even when his brand is less appreciated. This is supposed to maintain a pressure on the firms, who may otherwise "abuse" consumers' loyalty.

[23]Whether it is optimal requires to put more structure on the model (in particular on the utility function and the distributions of heterogeneity).

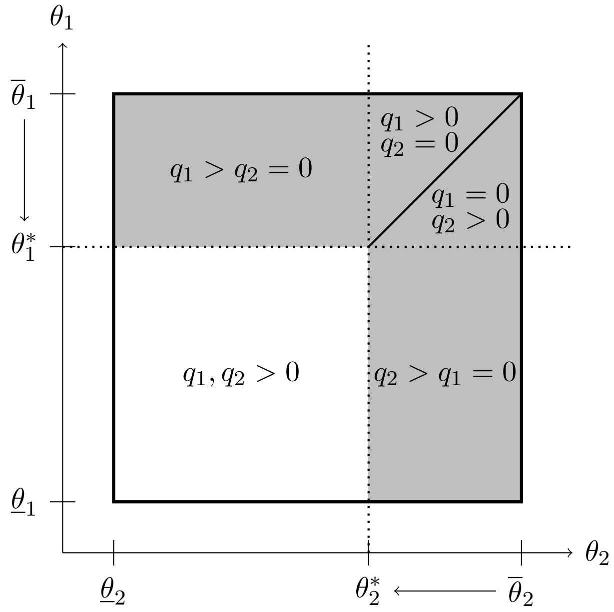

Figure 5: Participation decisions when consumers have the possibility of exclusive shopping (delegated common agency).

It is worth noting that our model can therefore easily accommodate situations in which some consumers buy both goods (as in the common agency framework) whereas others decide to buy only one of the available brand (as in an exclusive agency framework).

Notice that the pattern of consumers' decisions depends again importantly on the possibility to shop exclusively with one firm or not. Indeed, had we considered the intrinsic common agency case, we would have observed the pattern of participation describe in the Figure 6.

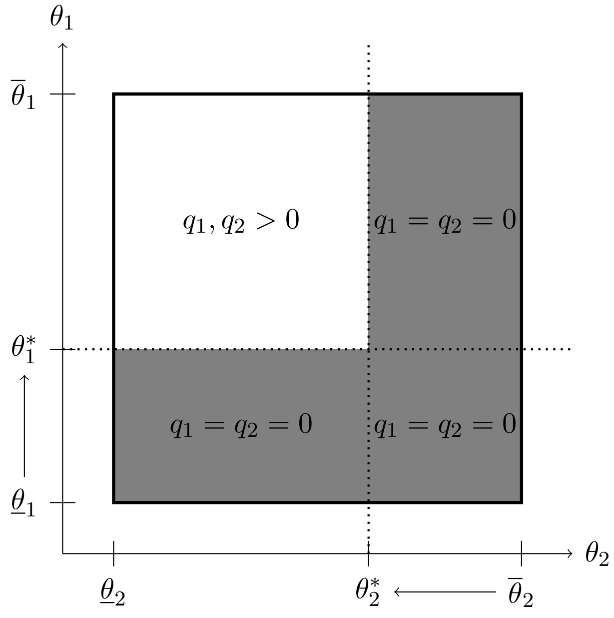

Figure 6: Participation decisions when consumers must buy both products (intrinsic common agency).

4 Competition in Fully-Discriminatory Pricing

As regards the previous section, one may object that firms might try to better tailor their offers to the different characteristics of the consumers. We now address this issue by analyzing competition in nonlinear prices. Under incomplete information, firm i can achieve such a second-degree price discrimination by facing the consumer with a price schedule $p_i(\theta_i, q_i)$, defined for all positive q_i, such that different types of a consumer choose different quality levels. In that context, the firms' pricing policy consists in a mix of first-degree price discrimination along the known dimension of heterogeneity and second-degree price discrimination along the unknown dimension of the consumer's preference: firms fully discriminate the different types of buyers.

For a given price schedule offered by firm j, there is no loss of generality in applying the Revelation Principle[24] to determine firm i's optimal price and quality levels. However, the consumer's behavior and therefore his incentive vis-à-vis firm i depend on the price schedule offered by the rival firm j. Hence, we define $\hat{U}_i(\theta_j, q_i)$ the indirect utility function which gives the maximal gain of the consumer with type $\theta = (\theta_i, \theta_j)$ for a given consumption level q_i when that buyer chooses optimally his the quality level for product j as follows[25]

$$\hat{U}_i(\theta, q_i) = \max_{q_j} \{-p_j(\theta_j, q_j) + U^g(\theta, q_i, q_j)\},$$

$$\hat{q}_j(\theta, q_i) = \arg\max_{q_j} \{-p_j(\theta_j, q_j) + U^g(\theta, q_i, q_j)\}.$$

[24] See Green and Laffont (1977) or Myerson (1981) for instance.
[25] See Martimort and Stole (2002) for the detailed methodology.

$\hat{q}_j(\theta, q_i)$ highlights the linkage between the consumer's choices of qualities: a change in the quality of one good affects the consumer's marginal utility for the other good. When designing its offer for the consumer, each firm accounts for the impact of its rival's offer on the choice of quality levels by the consumer. We assume that $\hat{q}_j(\theta_j, q_i)$ is defined through the following first-order condition as follows[26]

$$-P_{jq_j}(\theta_j, \hat{q}_j) + U^g_{q_j}(\theta, q_i, \hat{q}_j) = 0. \tag{11}$$

Now, from the viewpoint of firm i, everything happens as if it were facing a buyer with rent given by

$$U(\theta) \equiv \max_{q_i} \left\{ -p_i(\theta_i, q_i) + \hat{U}_i(\theta_j, q_i) \right\}.$$

We can now apply the standard methodology[27] to find the conditions for (local) incentive compatibility from the viewpoint of firm i. These conditions are, as usual, expressed in terms of rent-quality pairs instead of price-quality pairs:

$$\frac{\partial U}{\partial \theta_j}(\theta) = \frac{\partial \hat{U}_i}{\partial \theta_j}(\theta_j, q_i(\theta)) = -P_{j\theta_j}(\theta_j, \hat{q}_j) + U^g_{\theta_j}(\theta_j, \hat{q}_j), \tag{$FOIC_i$}$$

$$\frac{\partial^2 \hat{U}_i}{\partial q_i \partial \theta_j}(\theta_j, q_i(\theta)) \times \frac{\partial q_i}{\partial \theta_j}(\theta) \geq 0. \tag{$SOIC_i$}$$

From $(FOIC_i)$, we observe that firm i can obtain the revelation of the unknown information only in an indirect way: the quality of its product q_i does not affect directly the buyer's rent. As in the partially-discriminatory pricing case, $(FOIC_i)$ also shows that one cannot sign a priori the derivative of the consumer's rent w.r.t. to firm i's unknown information θ_j since this derivative depends on the endogenous price schedule offered by firm j.

The Spence-Mirrlees condition is now endogenous and must be assumed to ensure that the problem remains well-behaved: $\forall(\theta_j, q_i)$, $\frac{\partial^2 \hat{U}_i}{\partial q_i \partial \theta_j}(\theta_j, q_i) < 0$. Provided that this condition is satisfied, local incentive compatibility of an allocation ensures that it is also globally incentive compatible and the local second-order condition for incentive compatibility reduces to a standard monotonicity constraint on the quality profile of good i.

If the consumer decides to buy only the rival's product, then the corresponding quality and rent are given by

$$U_i^{out}(\theta_j) = \max_{q_j} \{-p_j(\theta_j, q_j) + U^g(\theta, q_i = 0, q_j)\} = \hat{q}_j(\theta, 0),$$
$$q_j^{out}(\theta_j) = \arg\max_{q_j} \{-p_j(\theta_j, q_j) + U^g(\theta, q_i = 0, q_j)\} = \hat{U}_j(\theta, 0).$$

The next lemma turns out to be useful.

Lemma 2. *When qualities are substitutes and firms compete in price schedules, both outside opportunities are positive.*

Proof. See Ivaldi and Martimort (1994). □

[26] This requires to check that the consumer's program is concave at equilibrium.
[27] See, e.g., Guesnerie and Laffont (1984).

The intuition is similar to the one underlying Lemma 1.

We now show that the indifferent types are the same in the FDP and the PDP equilibria; this, however, implies different kinds of distortions of the quality profiles.

Equilibrium quality profiles given the indifferent types. Consider that the rent profile is strictly monotonic in the adverse selection parameters. Then, from the viewpoint of firm i there exists an indifferent type $\theta_j^* \in \{\underline{\theta}_i, \overline{\theta}_i\}$ such that the participation constraint is binding, or

$$U(\theta_i, \theta_j^*) = \begin{cases} U_i^{out}(\theta_j^*) & \text{under delegated common agency,} \\ 0 & \text{under intrinsic common agency.} \end{cases}$$

Denote by $\tilde{\theta}_j$ the boundary of Θ_j different from θ_j^*. For further references, we note that the consumer's rent can be rewritten as follows

$$U(\theta) = \begin{cases} U_i^{out}(\theta_j^*) + \int_{\theta_j^*}^{\theta_j} \frac{\partial \hat{U}^i}{\partial \theta_j}(y, q_i(\theta_i, y))dy & \text{under delegated common agency,} \\ \int_{\theta_j^*}^{\theta_j} \frac{\partial \hat{U}^i}{\partial \theta_j}(y, q_i(\theta_i, y))dy & \text{under intrinsic common agency.} \end{cases}$$

Leaving aside the second-order condition for incentive compatibility, the Hamiltonian associated to firm i's maximization problem is

$$H_i = f_j(\theta_j)\left\{\hat{U}^i(\theta, q_i(\theta)) - C_i(q_i(\theta)) - U(\theta)\right\} + \mu_j(\theta_j)\frac{\partial \hat{U}^i}{\partial \theta_j}(\theta_j, q_i(\theta)),$$

where $\mu_j(\theta_j)$ is the co-state variable. Since there is no transversality condition in $\theta_j = \tilde{\theta}_j$, we have $\mu_j(\tilde{\theta}_j) = 0$. The Maximum Principle implies that $-\frac{\partial H_i}{\partial U} = \dot{\mu}_j(\theta_j)$. Therefore, we obtain that $\mu_j(\theta_j) = \int_{\tilde{\theta}_j}^{\theta_j} f_j(x)dx$. Then, optimizing w.r.t. the control variable leads to (this requires that the Hamiltonian be concave in q_i)

$$\hat{U}^i_{q_i}(\theta, q_i(\theta)) - C_{iq_i}(q_i(\theta)) = -\frac{\int_{\tilde{\theta}_j}^{\theta_j} f_j(x)dx}{f_j(\theta_j)} \frac{\partial^2 \hat{U}_i}{\partial q_i \partial \theta_j}(\theta_j, q_i(\theta)). \tag{12}$$

At equilibrium we must have $q_j(\theta) = \hat{q}_j(\theta, q_i(\theta))$. Hence, (11) becomes

$$-P_{jq_j}(\theta_j, q_j(\theta)) + U^g_{q_j}(\theta, q_i(\theta), q_j(\theta)) = 0. \tag{13}$$

Equation (13) can be differentiated w.r.t. θ_i and θ_j to further simplify (12); straightforward manipulations yield

$$U^g_{q_i}(\theta, q_i, q_j) - C_{iq_i}(q_i) = \frac{\int_{\tilde{\theta}_j}^{\theta_j} f_j(x)dx}{f_j(\theta_j)} \frac{\frac{\partial q_i}{\partial \theta_j}\frac{\partial q_j}{\partial \theta_i} - \frac{\partial q_i}{\partial \theta_i}\frac{\partial q_j}{\partial \theta_j}}{\frac{\partial q_i}{\partial \theta_i}} u_{q_i q_j}(q_i, q_j).$$

Equilibrium quality profiles are characterized by a set of nonlinear partial differential equations. Unfortunately, we were not able to study the behavior of the optimal quality profiles in this general context; this, in turns, prevents from showing that the necessary conditions stated previously are also sufficient.[28]

As in the PDP equilibrium case, we show next that the nature of the distortions on the best-responses, and hence on the equilibrium quality profiles, created by asymmetric information strongly depends on the indifferent types.

The indifferent types under delegated common agency. Remember that the buyer's outside opportunity w.r.t. firm i is defined by $U_i^{out}(\theta_j) \equiv \max_{q_j} \{-p_j(\theta_j, q_j) + U^g(\theta, 0, q_j)\} = \hat{U}_i(\theta_j, 0)$. Therefore, the derivative of the net rent is given by

$$\frac{\partial}{\partial \theta_j} \left[U(\theta) - U_i^{out}(\theta_j) \right] = \int_0^{q_i(\theta)} \frac{\partial^2 \hat{U}_i}{\partial \theta_j \partial q_i}(\theta_j, x) dx < 0.$$

This implies that the indifferent types are $\{\overline{\theta}_i, \overline{\theta}_j\}$, as in the PDP equilibrium. As a consequence, (12) shows that firm i tends to reduce its quality for a given quality offered by firm j; at equilibrium, as in the PDP equilibrium, there could be either under-provision of quality for both firms, or under-provision for one firm and over-provision for the other.

The indifferent types under intrinsic common agency. Using the corresponding writing of the firm's rent, it is immediate to show that if $\frac{\partial U}{\partial \theta_j}(\theta) > 0$ then it must be the case that $\frac{\partial U}{\partial \theta_i}(\theta) < 0$: the indifferent types are either $\{\underline{\theta}_i, \overline{\theta}_j\}$ or $\{\overline{\theta}_i, \underline{\theta}_j\}$. Consequently, (12) shows that at equilibrium one firm over-provides and the other one under-provides quality.

Discussion. As shown in the previous analysis, the indifferent types in the FDP equilibria are the same as the indifferent types in the corresponding PDP equilibria. The best-responses in a FDP equilibrium are therefore shifted in the same direction than in the corresponding PDP equilibrium. However, in terms of the distortions that arise at equilibrium, there is a difference between the fully-discriminatory and partially-discriminatory pricing cases. Indeed, in a FDP equilibrium, there is no distortion on the quality levels for a consumer with type $(\tilde{\theta}_i, \tilde{\theta}_j)$, whereas in the PDP equilibrium, there is no distortion on the quality levels for a consumer with type (θ_i^*, θ_j^*). Consequently, the pattern of distortions on the equilibrium quality levels in the FDP case is 'opposite' to the one obtained in the PDP case. To illustrate this point, let us consider two examples in the delegated common agency case:

- *Buyers with large valuations for both goods.* When θ_i and θ_j are high, then, as regards the full information benchmark, there will be weak distortions on the quality levels in the PDP case, but large downward distortions in the FDP case. A reverse conclusion in the case the buyer has low valuations for both products.

- *Buyers with a large valuation for one product and a low valuation for the other good.* Consider for instance that θ_i is low and θ_j is large. In the PDP equilibrium, the quality of good i is distorted upwards whereas the quality of product j is distorted downwards; in the corresponding FDP equilibrium, the reverse holds.

[28]Indeed, we have to show that (i) the firms' problem are concave, (ii) the consumer's problem $\max_{\{q_1,q_2\}}\{-p_1 - p_2 + U^g\}$ is concave so that (11) is valid, (iii) that the Spence-Mirrlees condition is met at equilibrium and (iv) that the second-order condition for incentive compatibility is also satisfied at equilibrium. In Section 5, we study a quadratic-uniform specification of our model in which all these conditions are indeed satisfied at equilibrium.

Summarizing, we get

Proposition 3. *Under fully discriminatory pricing, the indifferent types are the same as under partially discriminatory pricing. This, however, does not hold for quality distortions; under FDP, "no distortion" occurs only for the type at the opposite bound of the indifferent type. Hence, the pattern of distortions is 'opposite' to the ones obtained under PDP.*

Now we shall try to explain certain market features. Indeed, our results under intrinsic common agency may help to explain apparently 'odd' features in the market for software viewers. Recall (from the introduction) the example from Martimort and Stole (2003b) about hardware and software, which perfectly fits the setting of common agency with substitute goods. Our asymmetric equilibria under intrinsic common agency seem to be descriptive of these markets. In the computer markets, firms price discriminate by offering products (the main difference being about the processor) at different prices. The same is true of the market for softwares, in which firms also price discriminate through different versions of a software (from basic to one offering many features). Hence, in both markets, there is price discrimination.

In the market for softwares, producers commonly offer basic versions of their products for free while more sophisticated versions are highly priced. For instance, Apple offers its Quick Time Player for free and charges \$29.99 for its Quick Time Pro. Real Networks offers its Real Player for free and charges \$49.99 for its Real Player Plus. Hence, high valuation buyers pay an *infinitely* higher percentage of the price paid by low valuation buyers. On the other hand, a DELL desktop computer would feature a price of \$599 for a Celeron processor, and \$699 for a (faster) Pentium processor;[29] in this market, a high valuation buyer pays *only 17 percent* of the price paid by the low valuation buyer. Hence, it *seems* that, while high valuation buyers of hardware get higher rent than their low valuation peers, the opposite occurs in the software market.[30] This is precisely a distinctive feature of the equilibria that we obtain under intrinsic common agency.

Shapiro and Varian (1998) have argued that this pricing behavior in the software market, which they refer to as 'versioning', is intended to avoid competition in a market with intense competition, high fixed costs and marginal costs close to zero: without versioning, competition would drive prices to zero, making it impossible for firms to recoup their fixed costs. There would be nothing to add to Shapiro and Varian's argument if the market for software viewers was highly competitive. However, Apple, Real Networks and Microsoft (with its free Media Player) enjoy some market power (as a matter of fact, versioning consists in offering different price-version pairs). If it is the case that low-profile users earn higher rents, then this way of making business by extracting surplus *mainly* from the high-profile users would stand in contrast with the standard theory of second-degree price discrimination. Therefore, we believe that our model might be useful in empirical works if standard models perform poorly when confronted with the data.

As another example, within the car industry our model may explain the evidence that price markups are much lower in base models than in models with more options (for more,

[29]Prices on October 18 2005, for the Dimension 3000 series, at www.dell.com.

[30]See Wang (2003). Of course, one cannot conclude that this is strictly the case, because consumers' valuations remain unobservable. However, by giving basic software for free to low-profile buyers, while charging a high price for a slightly more sophisticated one, it seems more likely, or at least possible, that the low-profile users are getting more surplus that the high-profile ones.

see Verboven, 1999). In the same paper, a citation from Louis Philips concludes that "one has the impression that extra options are overpriced, to extract the highest possible price from those who want fancy tires or extra horsepower". Philips seems to suggest that consumers with preference for the basic models may actually get a higher surplus than those who prefer the sophisticated versions.

Finally, from a more theory-oriented perspective, Proposition 3 has an important implication for the theory of duopoly competition that uses the common agency framework. Indeed, in standard common agency models, equilibrium quality allocations are unaffected by the setting (delegated or intrinsic) even under incomplete information.[31] In other words, whether the consumer has the extra option or not to accept one firm's offer only does not change the equilibrium quality he consumes; only the distribution of the rent between consumers and firms is affected. In our model, the fact that competition in nonlinear prices yields different indifferent types in the intrinsic and the delegated case implies that:

Corollary 1. *With asymmetrically informed firms, equilibrium qualities are affected by the setting (delegated or intrinsic).*

In the next section we compare these equilibria for a particular specification of the model in which adverse selection parameters are uniformly distributed (thereby neutralizing potential bias in the comparison if, say, the joint distribution puts 'more weight' on higher valuations consumers). Under this specification, all the sufficient conditions (for the consumer's and the firms' optimization problems) are satisfied at equilibrium.

5 Comparisons of Pricing Policies

In this section, we put more structure on our model in order to obtain explicit solutions of the quality profiles in the FDP equilibrium. We use a quadratic specification of the consumer's gross utility function

$$U^g = (a+\theta_1)q_1 - \frac{1}{2}q_1^2 + (a+\theta_2)q_2 - \frac{1}{2}q_2^2 - \lambda q_1 q_2$$

with $\lambda \in [1,0]$[32]. Cost functions are assumed to be quadratic and identical

$$C_i(q_i) = \frac{1}{2}q_i^2, \quad i = 1,2$$

and the adverse selection parameters are assumed to be uniformly distributed on their respective support. We restrict attention to the delegated common agency case.

Computing the equilibrium quality and price levels is straightforward in a PDP equilibrium. However, it becomes more involved in a FDP equilibrium; in that case, we focus attention on solutions of the nonlinear partial differential equations which are linear in

[31] This is true as long as full participation is assumed; see Martimort and Stole (2003b).

[32] See Ivaldi and Martimort (1994) or Miravete and Röller (2003, 2004) for theoretical and applied analysis using a similar specification for the utility function.

the adverse selection parameters. The determination of the price schedules is not inherently difficult but requires to perform cumbersome computations that will not be detailed here.[33]

To make sensible comparisons, we compute the expected welfare, firm's profit and consumer's rent, where the expectation is taken over both adverse selection parameters. Before presenting our results, let us emphasize that the PDP and the FDP equilibrium perfectly coincide when goods are independent from the consumer's viewpoint (i.e., when $U^g_{q_1q_2} = 0$). Indeed, in that case firm i cannot affect the consumer's incentive to reveal θ_j since the choices of q_i and q_j are independent. Then we obtain the following results.

Result 1. *Under delegated common agency,*

- *from the firms' viewpoint, partially discriminatory pricing dominates fully discriminatory pricing;*
- *the consumer's rent is larger under fully discriminatory pricing than under partially discriminatory pricing.*

Proof. See Web Appendix □

To trace out fully the origin of this result, it is worth mentioning that in both equilibria the profit of each firm (gross of the quality cost) can be decomposed into two parts. The first (positive) part corresponds to the standard surplus extracted from the consumer (modulo the information rent given up to that consumer for incentive reason) and the second (negative) part corresponds to the outside opportunity of consumers which has to be given up by the firm.

Fully discriminatory pricing enable firms to finely tailor their offers to the characteristics of the buyer. Hence, discriminatory pricing performs better in extracting surplus from the different types of consumer. However, discriminatory pricing also triggers an intense competition between firms, which provides the consumer with higher outside opportunities, thereby leading to higher equilibrium rents. As said earlier, the 'threat' to consume only one good is credible and a uniform pricing strategy turns out to be preferred by firms. Although it may appear at first sight inefficient, the decision not to tailor offers to all the consumer's characteristics allows the duopolists to somehow refrain themselves from competing: *Strategic ignorance of consumer's taste for the rival's brand softens competition.*

Finally, Result 1 is also in the spirit of those obtained in the literature on price discrimination under complete information (e.g., Thisse and Vives, 1988, Holmes, 1989, and Corts, 1998, among others). In these papers, uniform pricing may dominate third-degree price discrimination. Our result extends this argument to settings with both first- and second-degree price discrimination by arguing that partial discrimination dominates full discrimination when the consumer can threaten each firm not to consume its good.

Besides, Result 1 has also some empirical implications. Indeed, it argues, first, that the 'nature' of asymmetric information (i.e., whether firms are symmetrically or asymmetrically un-informed) is important and should be tested and, second, that attention should

[33] These computations are available at the following urls: `http://center.uvt.nl/staff/diaw/` or `http://ceco.polytechnique.fr/home/pouyet`.

not be restricted exclusively to nonlinear tariffs since 'simpler contracts' might be used by firms.

For instance, Miravete and Röller (2003, 2004) consider competition in the cellular telephone industry; their structural model follows the lines of Ivaldi and Martimort (1994) and therefore is identical to our model except that firms are un-informed about both private information parameters; however, they also consider that firms have a priori different knowledge about the consumer's characteristics and argue that one possible reason is that a firm has a better knowledge than its rival about some of the consumer's characteristics. Ivaldi and Martimort (1994) focus on the French energy distribution market; analyzing the data, they argue that asymmetric information is multi-dimensional but that "there is not too much asymmetric information" (p. 96) and that there is evidence of discrimination between groups of buyers; moreover, they argue that an energy supplier usually observes the technology used by a client and therefore might be informed on the client's willingness-to-pay for its energy. This suggests that in these contexts it would be relevant to test the nature of incomplete information.

Moreover, these papers focus mainly on nonlinear price schedules. Our analysis also suggests that simpler contracts can emerge as a strategic response in a competitive environment when consumers have the option to shop exclusively with one competitors at equilibrium. This may explain why fully nonlinear tariffs are seldom observed in practice.[34]

6 Conclusion

In this paper, we have developed a model which allows to explain situations in which competing firms price-discriminate, yet sometimes provide lower valuations consumers with higher utility levels. Our model also offers a rich environment that incorporates both direct and indirect price discrimination and that could serve as a building block for more applied work.

Some interesting questions have been left aside. For instance, we have assumed that firms can credibly commit to a partially-discriminatory pricing policy. It would be worth investigating if firms manage to reach their preferred outcome when they can choose (prior to the competition stage) either to discriminate or not. In the same vein, if firms are able to credibly disclose their information to their competitors prior the competition stage, it would be interesting to study whether they decide to share their information of if some prisoner's dilemma occurs at equilibrium.

Recent research in mechanism design has started to examine the impact of information sharing between principals. In a setting with one informed and one uninformed principals, Bond and Gresik (1998) show that the possibility for the informed principal to communicate his information to the uninformed principal leads to no effective information transmission at equilibrium.[35] In a sequential common agency context, Calzolari and

[34] Miravete (2004) considers a monopoly under incomplete information and argues that the firm should use 'simple' mechanisms since the gains to offer fully discriminatory contracts are negligible. Our model shows that a qualitatively similar result holds in a competitive environment under certain conditions, but for different reasons.

[35] More precisely, they show that the most efficient equilibrium when principals can communicate is equivalent to a pooling equilibrium that emerges in the game with no direct communication between principals.

Pavan (2004) study the possibility for the first-mover principal to disclose the information learned from contracting with the agent to the last-mover principal and show that, under certain conditions, information transmission between principals occurs at equilibrium. Coming back to our setting of asymmetrically un-informed principals, it would be worth investigating the possibility for one principal to extract the piece of information commonly known by the rival principal and the agent through, for instance, 'Maskinian' mechanisms.[36]

These extensions are left for future research.

A Appendix

A.1 Proof of Lemma 1

We adapt the proof of Ivaldi and Martimort (1994) to the PDP equilibrium.

First, consider that both outside opportunities strictly negative at equilibrium. Then we have $-p_i(\theta_i)+u_i(\theta_i, q_i(\theta_i))+u(0, q_i(\theta_i)) < 0$ for $i = 1, 2$, implying that $-p_1(\theta_1)-p_2(\theta_2)+u_1(\theta_1, q_1(\theta_1))+u_2(\theta_2, q_2(\theta_2))+u(0, q_2(\theta_2))+u(q_1(\theta_1), 0) < 0$. With demand substitutes, the following inequality holds: $u(q_1(\theta_1), 0) + u(0, q_2(\theta_2)) > u(0, 0) + u(q_1(\theta_1), q_2(\theta_2))$, implying in turn that the equilibrium rent of the consumer must be strictly negative, a contradiction.

Second, consider that $U_2^{out}(\theta_1) > 0$. Since the net rent is decreasing in θ_1, we have $U(\overline{\theta}_1, \theta_2) = U_2^{out}(\overline{\theta}_1)$, or $p_2(\theta_2) = u_2(\theta_2, q_2(\theta_2)) + u(q_1(\overline{\theta}_1), q_2(\theta_2)) - u(q_1(\overline{\theta}_1), 0)$. Then, simple manipulations show that $U_1^{out}(\theta_2) = -\int_0^{q_1(\overline{\theta}_1)} \int_0^{q_2(\theta_2)} u_{q_1 q_2}(x, y)dydx > 0$.

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[36] See Maskin (1979).

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