ECOLE POLYTECHNIQUE
CENTRE NATIONAL DE LA RECHERCHE SCIENTIFIQUE

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Décembre 2008

Cahier n° 2008-14

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Congestion pricing and long term urban form: Application to Île-de-France

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Résumé: Nous proposons un algorithme de résolution du modèle monocentrique de transport avec congestion. Nous utilisons cet algorithme afin d'explorer l'impact de différents schémas de tarification de la congestion sur la forme urbaine, et par conséquent, sur les véhicules-kilomètres (émissions de CO2) à long terme. L'application empirique concerne la région Île-de-France. Quatre régimes de tarification sont considérés : (*i*) absence de tarification, où une taxe linéaire reflète le coût d'usage du véhicule ; (*ii*) péage cordon, où les voitures payent pour passer à l'intérieur d'une zone donnée ; (*iii*) taxe linéaire optimale, proportionnelle à la distance parcourue (optimale dans la classe des taxes linéaires) ; et (*iv*) taxe optimale (optimum de premier rang). Par rapport à (*i*), la taxe optimale aboutit à des réductions de 34% et 15%, respectivement pour le rayon de la ville et la distance parcourue moyenne.

Abstract: We propose an efficient algorithm that solves the monocentric city model with traffic congestion, and use it to explore the impact of congestion pricing on urban forms and, hence on transport volume, emissions and energy consumption. The application focuses on the region Île-de-France. Four pricing policies are considered: no toll, where transport cost is equal to the vehicle operating cost, cordon toll where users pay the toll when they drive inside cordon region (location and value of the toll are both optimized) linear toll (optimal under the class of linear tolls) and optimal toll (or first-best toll). Our analysis shows that the linear toll is particularly effective in that it yields about 93\% of the welfare gain of the first-best toll. By comparison to the no-toll situation, optimal congestion pricing reduces the size of the city and the average travel distance by 34\% and 15\%, respectively.

Mots clés : Modèle monocentrique ; Calcul d'équilibre ; Tarification de la congestion ; Effets de long terme

Key Words : Monocentric model; Equilibrium computation; Transport pricing; Long term impacts

Classification JEL: R21 ; R41 ; R48

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1 Introduction

While the literature on road pricing has been abundant in the last decades, long term impact on housing and business location have not received so much attention. Recent implementation of an area-based charge in London and a few other experiments have raised concern about the overall impact on congestion, business activities and environmental conditions in the long run (cf. Santos & Fraser 2006). At the same time, the alarming levels of pollution reached in many metropolitan areas and the important increase of energy cost contribute to making the optimization of urban forms and the regulation of transport an important issue (cf. Mitchell et al. 2005).

This paper explores the impact of transport pricing on the urban form, and, hence, on transport volume, CO_2 emissions and energy consumption. We consider a monocentric model with traffic congestion where all the economic activity is located in the central business district (CBD). There are two main actors: households, whose utility is increasing with housing area, and a government that decides how much land is devoted to roads. The government collects a population tax, which is the same for all households, and a location tax that depends on where the household lives.[1] Transport congestion introduces an externality that requires public intervention for regulation.

Transport congestion was introduced in the monocentric model by Strotz (1965) and Mills (1967). In the following decade, there was growing interest in second-best allocations of land between housing and roads.[2] A synthesis of this problem may be found in Kanemoto (1980). Recently, Mun et al. (2003) have shown that second-best pricing schemes are almost as efficient as first-best pricing. Their conclusion has been confirmed by Verhoef (2005). Both models, however, are rather restrictive forms of the monocentric model. Mun et al. (2003) do not consider a variable housing area, and Verhoef (2005) assumes that the amount of land allocated to transportation is fixed. The monocentric model has been used mainly for theoretical and normative discussions, and very little for empirical applications.[3]

[1] On practical grounds, road pricing may contribute to raising funds for the transport sector (cf. De Palma et al. 2007, de Palma & Quinet 2005).

[2] Representative papers are Mills & Ferranti (1971), Solow (1972, 1973), Riley (1974), Robson (1976), Kanemoto (1977), Arnott & MacKinnon (1978), Arnott (1979), and Sullivan (1983).

[3] Empirical applications include Baum-Snow (2007), Boarnet (1994), McMillen et al. (1992) and Rouwendal & van der Straaten (2008).

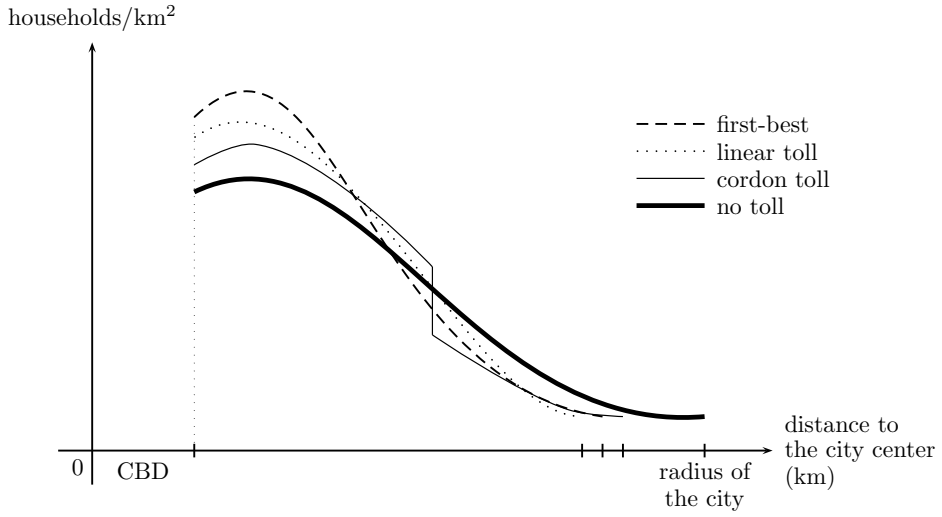

Figure 1: Impacts of congestion pricing.

We adopt the monocentric city framework using the formulation of Fujita (1989), and contribute to the literature at two stages. First, we propose a flexible and efficient algorithm to compute the optimal solution. The solution approach underlying the algorithm replaces the standard optimality conditions (cf. Fujita 1989) by a set of first-order differential equations that can be solved efficiently by standard numerical techniques.[4] The algorithm is flexible enough to be used for a number of pricing rules.

Second, we undertake an empirical application on the agglomeration of Île-de-France (IDF). In particular, we feed the model with data from IDF and find that it succeeds in adequately capturing a number of urban features. On the basis of the calibrated model, we quantify the impacts of different pricing rules: cordon, linear and first-best tolls. All policies lead to a smaller city and a reduced average trip-distance.

Figure 1 illustrates the impacts of congestion pricing on the distribution of households. Each curve reflects the distribution of households under a given regime. Road pricing motivates households to move closer to the CBD. Linear toll depends on only the travel distance, while first-best toll, which is non-linear, depends on the congestion or external cost created by the trip. A unit of trip-distance in a congested area is tolled more than the same unit

[4]The model was solved under a particular set of parameter values in Riley (1974), Robson (1976) and Kanemoto (1977), but no general solution method has been proposed.

in an uncongested area. In this monocentric geometry, congestion is higher around the CBD and it is there that the difference with linear tolls emerges. In Figure 1, the impacts of the (optimal) linear toll and first-best toll are rather similar in the outer part of the city, but they become quite distinct around the CBD.

Optimal pricing reduces the radius of the city, the average trip-distance and congestion by 34%, 15% and 13%, respectively. The optimal linear toll,[5] which we call linear toll for short, induces a comparable impact and leads to a relatively dense city. But in practice, the linear toll is equivalent to an important increase in gasoline price. Such a policy is likely to face road user opposition and has the inconvenience of depending on only the length of the trip and not on its location (origin/destination pair). For example, urban and inter-urban trips (which induce less congestion) are tolled the same way. So, under a more general transport network the linear toll will be less efficient than in the model we consider here. Efficiency is measured as the unspent part of the households' revenue, for a given level of utility.

Cordon pricing is less efficient than linear toll but still reaches an acceptable efficiency level of 62% with respect to first-best. By contrast to linear tolls, cordon tolls concern only highly congested areas and turn out to be an attractive alternative for policy makers. Indeed, similar pricing rules to cordon toll are already in use in some cities (London and Singapore, in particular), and other implementation projects are under study. From the simulation we have conducted, it appears that an optimal urban form requires both a smaller radius and a higher concentration of households around the CBD (cf. Figure 1). The first-best rules satisfy these requirements by setting the toll equal to the external cost. Linear toll is more efficient in reducing the radius of the city than in concentrating households around the CBD. In general, under the linear rule, the optimal trade-off between the two objectives requires an excessive charge on road users.

A cordon toll close to the CBD does not have a strong impact on the radius of the city. At the same time, a cordon toll away from the CBD has substantial impact on the radius of the city but does not induce any significant variation in the concentration of households inside the city. In most cases, and for data related to Île-de-France, we found that it is optimal to set the cordon toll at a distance about 21km from the city center.

Pricing reduces the size of the city but the average area occupied by house-

[5] That is, optimal among linear tolls.

holds does not decrease so much. On one hand, part of the land available for housing and transportation is lost. However, this lost area is not very large since, as the empirical observation shows, the available land for housing and transportation gets smaller as we move away from the city center. On the other hand, with congestion pricing, the surface of land allocated to roads decreases and larger areas are available for housing. Overall, both impacts have comparable magnitudes and the resulting variation in the housing area remains, in general, small.

On more general grounds, pricing congestion contributes to decreasing the level of pollution since it leads to smaller and more compact cities. Indeed, energy consumption per household decreases as urban density increases (cf. Newman & Kenworthy 1989). Since CO_2 emissions are correlated with trip-distance, congestion pricing has an appreciable environmental benefit. The set of simulations we have conducted shows that congestion pricing reduces the level of emissions by 15%, and has a comparable impact on congestion. The paper is organized as follows. In Section 2 we introduce the notation and provide the solution procedure for the land-use equilibrium. The calibration of the model to IDF is undertaken in Section 3. In Section 4 we discuss the impact of congestion pricing. We finally conclude in Section 5.

2 A general method to compute a compensated equilibrium

2.1 The basic framework

The analysis is carried out under the classical monocentric model. We adopt the formulation of Fujita (1989) and denote the model by HS_T.[6] The number of households living in the city is fixed and equal to $\overline{N}$ (closed city). The variable r denotes the distance from the center of the city. Each household makes daily trips from its location, at distance r from the center of the city, to the Central Business District (CBD) that extends to distance r_c from the center of the city. Inside the CBD, we assume that transportation is costless. The radius of the city is denoted by r_f. $N(r)$ is the number of households located further than distance r from the city center. $L(r)$ is the amount of land available for housing or transportation at r. $L_T(r)$ is the

[6]Fujita (1989) refers to the model as the Herbert-Stevens model with traffic congestion.

amount of land allocated for transportation at r. Each household consumes two goods, housing s and a composite good z, and gets a utility $U(z,s)$ where $\partial U(z,s)/\partial z > 0$ and $\partial U(z,s)/\partial s > 0$. All households have the same utility function and the same (pretax) revenue Y. The price of the composite good is normalized to 1 and the unitary price of land, or land rent, at distance r from the city center is $R(r)$. The opportunity cost of land, or the agricultural rent, is denoted by R_A. The amount of composite good necessary to achieve utility level u when the housing area is equal to s is $Z(s,u)$, which is the solution of $U(z,s) = u$ in z. Let I denote the revenue net of taxes. The household *bid rent function* $\psi(I,u)$ is given by

$$\psi(I,u) := \max_{s \geq 0} \frac{I - Z(s,u)}{s}, \tag{1}$$

where the maximum is reached at the *bid-max lot size* $S(I,u)$

$$S(I,u) := \arg\max_{s \geq 0} \frac{I - Z(s,u)}{s}. \tag{2}$$

The government is responsible for providing the transportation infrastructure, $L_T(r)$, and has the possibility of levying two kinds of taxes: a population tax that does not depend on r and is denoted by g, and a location (or congestion) tax that depends on r and is denoted by $l(r)$.

The *road occupancy* at r is defined by the ratio of the number $N(r)$ of households located further away than r from the city center to the amount $L_T(r)$ of land devoted to transport use at r. At each distance r, the transport cost depends on the road occupancy at r: $c(N(r)/L_T(r))$, where the function c is assumed to satisfy $c(w) > 0$, $c'(w) > 0$ and $c''(w) > 0$ for all $w \geq 0$. The transport cost from distance r to the CBD is

$$\tau(r) = \int_{r_c}^{r} c\left(\frac{N(x)}{L_T(x)}\right) dx. \tag{3}$$

Define the *bid rent of the transport sector* ψ_T at each distance r as the marginal benefit of land for transportation at r:

$$\psi_T\left(\frac{N(r)}{L_T(r)}\right) \quad = c'\left(\tfrac{N(r)}{L_T(r)}\right)\left(\tfrac{N(r)}{L_T(r)}\right)^2. \tag{4}$$

The bid rent $\psi_T(N(r)/L_T(r))$ represents the cumulated gain for the $N(r)$ commuters (away from r) from a unit increase of roads at r.

2.2 Solution approach

The household's problem is to maximize the utility function $U(z,s)$ over r, z and s subject to the revenue constraint $z + R(r)s = Y - g - l(r) - \tau(r)$. If we replace I in (1) by[7] $Y - g - l(r) - \tau(r)$, we obtain the household bid rent at distance r

$$\psi(Y - g - l(r) - \tau(r), u) = \max_s \frac{Y - g - l(r) - \tau(r) - Z(s,u)}{s}, \qquad (5)$$

and the corresponding bid-max lot size $S(Y - g - l(r) - \tau(r), u)$. Appendix A provides an interpretation of the HS_T model and the role played by the population tax g. Since all households are identical, it is convenient to assume that they all reach the same utility level at an optimal solution.[8] The objective of the central planner is to maximize the total surplus in the city. Let $n(r)$ denote the number of households in an annulus of unit width at r. The objective function to be maximized over (nonnegative) quantities $n(r)$, $s(r)$, $L_T(r)$ and r_f is the following total surplus $\mathscr{S}$:

$$\mathscr{S} = \int_{r_c}^{r_f} \{[Y - \tau(r) - Z(s(r),u) - R_A s(r)]n(r) - R_A L_T(r)\} dr. \qquad (6)$$

Any distribution $n(r)$ of households should satisfy the following constraints. First, the total amount of land devoted to housing and transportation must be lower than or equal to the amount of land available:

$$n(r)s(r) + L_T(r) \leq L(r) \qquad \text{for} \quad r_c \leq r \leq r_f. \qquad (7)$$

Second, the distribution of households satisfies:

$$N(r) = \int_r^{r_f} n(r) dr \qquad \text{for} \quad r_c \leq r \leq r_f. \qquad (8)$$

Finally, all households are located inside the city:

$$\overline{N} = N(r_c) = \int_{r_c}^{r_f} n(r) dr. \qquad (9)$$

[7] Indeed, $Y - g - l(r) - \tau(r)$ is the part of the income that remains for the consumption of housing (s) and the homogeneous good (z).

[8] Without this assumption, an optimal solution may imply an increasing utility as we move away from the CBD (cf. Riley 1974, Papageorgiou & Pines 1999). When all households are assumed identical such a situation may seem inconsistent and the *Mirrlees* paradigm of the "unequal treatment of equals" appears (cf. Mirrlees 1972). We avoid this discussion and consider only solutions with equal utilities among households.

Since the bid rent function $\psi(I,u)$ is continuously increasing in I, we can define $\phi(R,u)$ by

$$\phi(R,u) := I \Leftrightarrow \psi(I,u) = R. \tag{10}$$

The quantity $\phi(R,u)$ is the aftertax revenue required by a household having utility level u and willing to pay a land rent R. The optimality conditions of this problem (maximize (6) subject to constraints (7), (8) and (9)) are recalled in their standard form in Appendix A. They represent conditions for the compensated equilibrium in which the common utility u is achieved by a competitive land market with common location tax g and an optimal location tax $l(r)$. The idea of the approach we propose is to transform standard optimality conditions (Equations (22a)-(22f) in Appendix A) into a set of first-order differential equations. Brueckner (2005) proposed a similar approach but under a framework where the proportion of land devoted to roads is fixed. We have the following result.

Proposition 1. *Let $u > 0$ be a fixed utility level. The solution of the problem which consists in maximizing (6) subject to constraints (7), (8) and (9) can be computed in the following way. Solve, for all positive r_f and for $r_c \leq r \leq r_f$, the system of backward differential equations :*

$$\begin{cases} R'(r) & = -\dfrac{c'\big(\Psi_T^{-1}\big(R(r)\big)\big)\Psi_T^{-1}(R(r)) + c(\Psi_T^{-1}(R(r)))}{\dfrac{\partial \phi}{\partial R}(R(r),u)} \\ \\ N'(r) & = \dfrac{\frac{N(r)}{\Psi_T^{-1}(R(r))} - L(r)}{S(\phi(R(r),u),u)}, \end{cases} \tag{11}$$

with terminal conditions $R(r_f) = R_A$ and $N(r_f) = 0$. Then, find r_f such that $N(r_c) = \overline{N}$. From these, compute $L_T(r) = N(r)/\Psi_T^{-1}(R(r))$, $s(r) = S(\phi(R(r),u))$ and $l(r) = \int_{r_c}^{r} c'(\Psi_T^{-1}(R(r')))\Psi_T^{-1}(R(r'))dr'$ for $r_c \leq r \leq r_f$.

Proof. See Appendix B. □

This procedure assumes that $n(r) > 0$ and $L_T(r) > 0$ for all $r_c \leq r \leq r_f$. While the second condition is guaranteed at any optimal solution,[9] it is possible that households density be equal to zero at some distance r. In Appendix C we provide details on how to implement this algorithm and show how to handle the case where $n(r) = 0$ for $r > r_c$.

[9] If not, $N(r)/L_T(r)$ will be unbounded inducing a very high transportation cost.

In order to compare the optimal pricing rule with alternative policies, we relax the analytical form in the first equation of (11) by introducing the more flexible rule:

$$H(r) = \begin{cases} c'(\Psi_T^{-1}(R(r)))\Psi_T^{-1}(R(r)) & \text{(first-best)} \\ \kappa & \text{(linear toll)} \\ \xi_d \, I_{\{r_d\}}(r) & \text{(cordon toll),} \end{cases} \tag{12}$$

where κ and ξ_d are positive constants and $I_{\{r_d\}}(r)$ the function that takes value one at r_d and zero elsewhere along with replacing the first equation in (11) by $R'(r) = -(H(r) + c(\Psi_T^{-1}(R(r))))/(\partial\phi(R(r), u)/\partial R)$. Then, instead of (11), we solve the system of differential equations given by

$$\begin{cases} R'(r) & = -\dfrac{H(r) + c(\Psi_T^{-1}(R(r)))}{\dfrac{\partial\phi}{\partial R}(R(r), u)} \\ \\ N'(r) & = \dfrac{\frac{N(r)}{\Psi_T^{-1}(R(r))} - L(r)}{S(\phi(R(r), u), u)}. \end{cases} \tag{13}$$

The second pricing rule in (12) corresponds to a charge that is proportional to the length of the trip, where κ is the charge per unit of distance. Such may reflect a charge implemented as a gasoline tax. Notice that the linear toll does not depend on the origin and destination of the trip. The third pricing rule in (12) reflects cordon pricing. Each driver pays ξ_d for crossing the ring of radius r_d. Households living inside this ring do not pay the charge.

3 Calibration on Île-de-France

In this section, we calibrate the model parameters to match selected target variables related to the IDF (Île-de-France) region. The monocentric model may be criticized as being based on unrealistic assumptions. Indeed, many metropolitan regions have a polycentric structure, and many authors consider that the main effort should therefore focus on polycentric models (cf. Mieszkowski & Mills 1993, forexample). The monocentric framework, however, remains very useful for at least three reasons. First, for the case of IDF, as we discuss below, there is a high concentration of (non-industrial) activities in the CBD located inside Paris. Second, the monocentric model

is useful when we consider only part of the economic activity and the related transportation. In particular, in IDF, most economic activities with highly skilled employees are concentrated in the CBD. This issue is particularly relevant since polycentric models have not yet been used successfully. Third, given that the theory underlying the monocentric model is much more coherent and complete (many theoretical insights have already been gained), the empirical exercise can be evaluated much more accurately than if polycentric models are used. We do not intend to say that the monocentric model is superior to polycentric models, but we argue that there are many lessons we can draw from it if we remain aware of its limitations. Moreover, empirical observations still confirm the high concentration of economic activities in small areas. For the case of IDF, a recent report by Pottier et al. (2007) states that more than three million households (among a total of five million) are working in the twenty districts inside Paris. The ratio is even higher for highly skilled employees, who generally use private cars relatively frequently. Moreover, maps from AIRPARIF show a high concentration of emissions in the CBD and the region around. On the basis of these observations, we think that many urban attributes of IDF can be explored within the monocentric framework.

3.1 A specific model

The related literature has extensively considered the Cobb-Douglas utility function:[10]

$$U(z,s) = z^{\alpha}s^{\beta} \quad \text{with} \quad \alpha > 0\,, \quad \beta > 0\,. \tag{14}$$

From $U(s,z) = u$, we have the quantity of composite good

$$Z(s,u) = u^{1/\alpha}\ s^{-\beta/\alpha}, \tag{15a}$$

and the solution of (2) yields

$$S(I,u) = (\frac{\alpha+\beta}{\alpha})^{\frac{\alpha}{\beta}} u^{\frac{1}{\beta}} I^{-\frac{\alpha}{\beta}}. \tag{15b}$$

Substituting it in (1) yields the bid rent function

$$\psi(I,u) = \frac{\beta}{\alpha+\beta}(\frac{\alpha}{\alpha+\beta})^{\frac{\alpha}{\beta}} u^{-\frac{1}{\beta}} I^{\frac{\alpha+\beta}{\beta}}. \tag{15c}$$

[10] See Robson (1976), Verhoef (2005) and Kanemoto (1977).

The inverse of (15c) gives

$$\phi(R,u) = \frac{\alpha+\beta}{\alpha}\left(\frac{R\alpha}{\beta}\right)^{\frac{\beta}{\alpha+\beta}} u^{\frac{1}{\alpha+\beta}}. \tag{15d}$$

For the congestion function, we use the BPR (Bureau of Public Roads) formula (cf. Branston 1976)

$$c(\gamma) = \frac{\theta}{v_0}(1+k'\gamma^{\lambda}), \tag{16}$$

where k' is a positive constant, v_0 is the maximum travel speed and θ the households' valuation of time. This function satisfies the convexity requirement for $k' > 1$ and $\lambda > 1$. In (16) the travel cost is the sum of two terms. The first term does not depend on the road occupancy and reflects the transport cost without congestion. The second term captures the impact of congestion. Indeed, as road occupancy increases, travel speed decreases and the travel time increases. Define $k = k'\,\theta/v_0$. The impact of a marginal increase in roadoccupancy is $c'(\gamma) = k\lambda\gamma^{\lambda-1}$. Using (4) and (15) we can obtain all expressions required in the computation of (13):

$$\left\{\begin{array}{rcl} \psi_T(\gamma) & = & c'(\gamma)\gamma^2 = k\lambda\gamma^{\lambda+1} \\ \psi_T^{-1}(R) & = & (\frac{R}{k\lambda})^{\frac{1}{\lambda+1}} \\ c(\psi_T^{-1}(R)) & = & \frac{\theta}{v_0} + k^{\frac{1}{\lambda+1}}(\frac{R}{\lambda})^{\frac{\lambda}{\lambda+1}} \\ c'(\psi_T^{-1}(R))\psi_T^{-1}(R) & = & (k\lambda)^{\frac{1}{\lambda+1}} R^{\frac{\lambda}{\lambda+1}}, \end{array}\right. \tag{17}$$

where $\gamma = \psi_T^{-1}(R)$.

3.2 Base-case parameter values

We fit the above model with data from IDF.

Land available

We assume

$$L(r) = \mu(r) \times 2\pi r\,, \tag{18}$$

	Local roads	Main roads	Highways
Length (km)	14490	2752	1814
Max. speed (km/h)	50	70	110

Table 1: Road network in Île-de-France.

where $\mu(r)$ is the fraction of land devoted to housing and transportation at r.[11] Data from IDF show that the proportion of land used for housing and transportation, with respect to the total available land, decreases as we move away from the CBD. Furthermore, collective houses are more concentrated near the CBD and individual houses spread away from the city center. Collective houses are generally built on more than four levels, while individual houses are built on one or two levels. It is important to take into account this fact in order to match the observed distribution of households. We approximate $\mu(r)$ by an exponential expression, which yields

$$\mu(r) = 3.191\ e^{-8.7\times10^{-5}r} \qquad (R^2 = 0.99). \tag{19}$$

Figure 5 shows both observed values (dots) and their approximation (lines). As we move away from the CBD the fraction of land available for housing and transportation decreases substantially.

Travel speed

There are two options at least on how to compute free-flow travel speed: v_0. First, one may consider that it is constant over all the region. In this case it can be computed as the (harmonic) mean of the maximum allowed speeds over the network of three kinds of roads. The details of the network are shown in Table 1 and give a value of about 55 km/h.

A better approach is to consider that the free-flow travel speed decreases as we get closer to the CBD. This case arises because a driver inside Paris uses mainly (slow) local roads but can drive on faster roads in outer regions. To take into account the fact that the free-flow speed increases as we move away from the CBD, we approximate it as follows. At the city border a traveller mainly uses highways where the speed limit is 110 km/h. A household will

[11] Fujita & Thisse (2002) report that only 12% is used in this sense and all the remaining area is used for agriculture, protected areas, etc.

Variable	Value	Unit
r_c	3 500	m
$\overline{N}$	2 120 493	household
θ	6 900	€h^{-1}year^{-1}
R_A	48	€m^{-2} year^{-1}

Table 2: Base-case parameter values

be likely to use highways less as we get closer to the CBD. We assume[12] that to travel from the city center to the CBD, on average, 80% of the trip is made on highways, and 20% on main roads. A trip that starts closer to the CBD uses less highways but the same fraction of main roads. Instead, urban area roads (with a speed limit of 50 km/h) substitute for highways. Denoting by w_h and w_n the respective fractions of usage of highways and main roads, the average speed is the harmonic mean

$$\left(\frac{w_h}{110}+\frac{w_n}{70}+\frac{1-w_h-w_n}{50}\right)=\frac{1}{v_0},$$

or $v_0 = 3\ 850/(77-42w_h-22w_n)$. As mentioned above, w_n is fixed at 20%. Assuming a linear form of w_h and taking into account that $w_h = 0.8$ at r_f and $w_h = 0$ at r_c, we end up with the following relation between the free-flow travel speed and the distance to the city center:

$$v_0=\frac{51\ 931}{1-5.92\times10^{-6}r}.$$

Hence, the free-flow travel speed decreases from about 90km/h at distance 70km (entrance of the city) from the city center to 52km/h at distance 10km (where the maximum speed generally becomes low). This decrease is more realistic and leads to better calibration than the fixed v_0.

Households

We consider a population of drivers going to and from the city center 230 days a year,[13] and estimate costs over one year. Some parameter values are provided in Table 2. The number of households used is adjusted so that it

[12]Based on the authors judgement from a Google-Earth exploration.
[13]This is approximately: 5 days x 52 weeks - 30 days (holidays).

corresponds to the number of vehicles used for home-to-work trips. Since we consider a CBD of radius 3.5 km, and since we consider only households that make trips to the CBD, we remove half of the population located in the ring that extends from 0 to 7 kilometers. Accordingly, we consider a total population of $\overline{N} = 2\ 120\ 493$ households.

Utility function

From the Cobb-Douglas utility functions properties, we know that the ratio β/α is equal to the share of the available revenue spent on housing with respect to the share spent on the homogeneous good. Robson (1976) assumed a value of 50% and Kanemoto (1977) reduced the approximation to what seems to be a more realistic 20%. In the base-case, we consider the second value which matches recent estimations reported in INSEE (2003). Thus, we have $\alpha = 4\beta$, so that

$$U(z,s) = \left(z^4 s\right)^{\beta} . \tag{20}$$

An alternative value of β is considered for the sake of comparison.

Congestion term

The congestion function depends on the maximum speed inside the city, the value of time and parameters k' and λ in (16). Boiteux (2001) reports that the value of time in IDF in 2001 was 11.6€/h for home-to-work trips.[14] To take into account the increase since 2001, we take the value of 15€ (which corresponds to a five year growth rate at 5%). So, during a year with 230 working days and an average of two trips per day, we have $\theta = 15 \times 230 \times 2$ ($€h^{-1}\text{year}^{-1}$). Both parameters are used in the calibration of the model. As a comparative statics exercise we consider an alternative situation with a higher level of congestion and compare with the base-case.

Tolling schemes

We consider four policies:

- no toll (NT), where κ in (12) reflects vehicle operating cost;

[14] For the sake of comparison, the average value of time for work trips reported in Small & Verhoef (2007), Chapter 3, is \$9.14/h for metropolitan areas in the US in 2003.

- cordon toll (CT), where a driver pays a toll when he enters inside the ring of a given radius;
- (optimal) linear toll (LT), where κ is set to the value that maximizes the surplus in (6);
- a first-best toll (FB) that internalizes the external costs.

The "no toll" rule may be interpreted as a small tax or, better, the *vehicle operating cost* per kilometer. On the basis of a gasoline price of 1.5€ per liter, the gasoline cost per meter for an average vehicle that consumes 6 liters per 100 kilometer is 0.0207 € per meter per year. Assuming that gasoline price is half the vehicle operating cost we use $\kappa = 0.0414$ for the NT policy.[15] For the cordon toll, both the location and the value are chosen to maximize the surplus $\mathscr{S}(u)$ given in (6). For the linear toll (LT), we search for the value of κ that maximizes $\mathscr{S}(u)$. In practice, the optimization process is a tedious but straightforward task. Pricing rule NT is the reference policy, since it is close to the real situation.

Calibration

A dataset related to rings with 7km intervals is used to feed the model with data. To replicate the urban structure of IDF, we construct a loss function (denoted "*Loss*") that depends on the four parameters u, β, k and λ. The loss function is equal to the weighted sum of square errors between observed data and the output of the model. We focus on the radius of the city (r_f), the distribution of the households (pop), the travel time (tt) and the level of urban rent (rent). The expression of the loss function is

$$\begin{aligned} Loss(u,\beta,k,\lambda) = \\ \sum_{r\in 7,14,\ldots,70} \{ w_{r_f} \left(\frac{M_{r_f} - r_f}{r_f} \right)^2 + w_{\text{rent}} \left(\frac{M^r_{\text{rent}} - R(r)}{R(r)} \right)^2 + \\ w_{\text{tt}} \left(\frac{M^r_{\text{tt}} - tt(r)}{tt(r)} \right)^2 + w_{\text{pop}} \left(\frac{M^r_{\text{pop}} - pop(r)}{pop(r)} \right)^2 \}, \end{aligned} \quad (21)$$

where w_x denotes the weight of variable x and M^r_x denotes the value of x predicted by the model at r (r measured in km). The four variables are not

[15] Based on authors' judgement and data values from INSEE (2005).

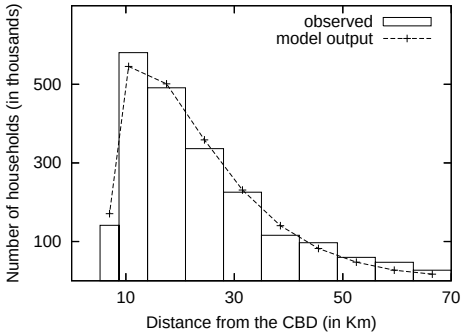

Figure 2: Distribution of households: observed and predicted ($R^2 = 0.987$).

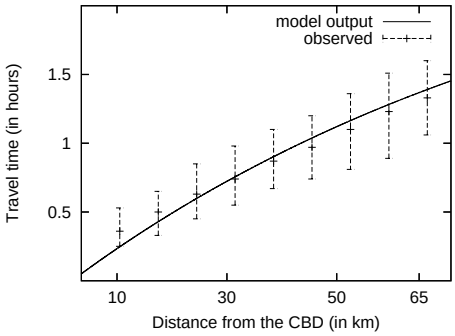

Figure 3: Travel time: observed and predicted ($R^2 = 0.97$).

measured in the same way: "rent" is the average rent between r and $r-\Delta r$ (we have used $\Delta r = 7$km), "tt" is the average travel time for households between r and $r - \Delta r$, "pop" is the number of households between r and $r - \Delta r$. The weights are set equal (and normalized to one) by default. They may be changed to focus the calibration on a given set of variables. The function $Loss(u, \beta, k, \lambda)$ reaches a unique minimum when the output of the model perfectly matches the observed values. Table 4 contains the values of target variables along ten rings as indicated in the first column. The second column contains the number of households. Values in the third column correspond to the number of vehicles used for home-to-work trips. As we are mainly interested in transport, this variable may be used instead of the number of households. The fourth column contains the travel time for the same type of trips. Rent values (based on observations from "indice notaire-INSEE" in 2007) are reported in the last column.

The model is calibrated with respect to policy LT, i.e. when households pay a tax that reflects the vehicle operating cost. The output of the model with parameter values $u = 11\,976$, $\beta = 0.2$, $\lambda = 4.02$ and $k' = 6.6 \times 10^{-12}$ fits particularly well the distribution of households and travel time. Figure 2 shows the observed distribution of households in IDF and the distribution produced by the model. The correlation is satisfactory. Figure 3 shows observed and predicted values for the travel time. The correlation between the two sets is high, even if the slope of the predicted values seems higher. The variable free-flow travel speed has been useful for refining the approximation of travel time. The only variable that does not seem to be well fitted by the

model is the land rent. This fact may be explained intuitively as follows. Under the monocentric city framework, the market rent is an exclusive result of transport costs. The attractiveness of the CBD lies in the fact that we incur lower travel time. But in reality, the attractiveness of the CBD of Paris is the result of many other attributes: a richer social life, better access to many facilities and so on. This difference is one of the limitations of the model used here.

4 Results

Simulation outputs are presented in Table 5. Table 3 contains a smaller set of the output of the base-case scenario. Under each scenario there are four pricing rules: no toll, cordon toll, linear toll and first-best toll. The base-case uses parameter values discussed above and summarized in Table 2. The first column of Table 5 provides location tax corresponding to $H(r)$ given in (12). The second column contains the radius of the city r_f. Column $\overline{s}$ corresponds to the average area occupied by a household ($\overline{s} = \int_{r_c}^{r_f} s(r)\ n(r)dr/\overline{N}$). The average (one-way) trip-distance $V_K = \int_{r_c}^{r_f} r\, n(r)dr/\overline{N}$ is given in column V_K. Column RD contains the surface of land allocated to roads. TT and TT_0 denote the average travel time and the free-flow travel time, respectively. The social cost per household is decomposed into three items (all expressed for an average household per year): C_L, the opportunity cost of land; C_T, the generalized transport cost; and C_Z, the cost of the homogeneous good. Column $\Delta\mathscr{S}$ corresponds to the impacts of pricing on the surplus of an average household per year. We now discuss the impact of each pricing rule under the base-case and then compare with two alternative scenarios.

4.1 The structure of the city

No toll

Without tolling, drivers incur only the vehicle operating cost. The urban region extends to a radius of 73km which corresponds to the actual radius of IDF. The average area occupied by a household is 84 m^2. The average length of a trip is 22km and the average duration 38 mins. The amount of taxes collected (769€) is close to annual spending on private transport in IDF. The density of households increases as we move from r_f to the CBD. It declines

	r_f	$\overline{s}$	V_K	RD	TT	$\Delta\mathscr{S}$
	City radius	Housing area	Trip length	Roads' area	Travel time	Δsurp
	km	m^2	km	$10^4\,m^2$	mins	€/y
No toll	73.423	84.236	22.075	7 539	37.7	0
Cordon toll	55.633	84.889	19.632	6 482	33.9	181
Linear toll	46.246	81.385	18.727	6 042	32.7	271
First-best	48.650	83.129	18.711	6 049	32.7	286

Table 3: The structure of the city (base-case). Cordon toll: located at 22km, value 22.5€/day; Linear: $\kappa = 0.21$/m/year

substantially near the CBD, because households living in the city center do not use their cars frequently for home-to-work trips. Vehicle emissions, in particular for CO_2, are highly correlated with the trip-distance, and a policy that reduces the latter is environmentally effective. Indeed, transportation is the leading sector in terms of CO_2 emissions in France (141 million-tons in 2005, according to ADEME). Assuming an average emission of 153 g/km (reported for 2006 by ADEME), IDF accounts for a total of 3.3 million-tons of CO_2 per year for just home-to-work trips.[16]

This situation is not optimal since congestion externality is not taken into account by road users.[17] Congestion pricing has long been advocated as the convenient tool to remove market distortions and increase welfare. We explore the impacts of three alternative policies.

Cordon toll

Drivers pay the toll when they enter a given ring. The value of the toll as well as its location are both chosen to maximize the surplus in (6). The optimal location of the cordon is at 22km from the city center and each household going inside the toll region pays 22.5€ per day. This pricing rule motivates households to locate inside the ring so they do not pay the toll. Competition

[16] Monetary values of pollution are reviewed in Zaouali & de Palma (2007).

[17] Without transport congestion (externality) the unregulated equilibrium is optimal.

for land inside the cordon raises land rent near the CBD. The land rent curve shifts upwards near the cordon location (cf. Figure 6). A similar jump appears in the distribution of households as shown in Figure 7.

In quantitative terms, this policy reduces the radius of the city and the average length of a trip by 27% and 13%, respectively. The average area occupied by a household slightly increases by 1% because the land allocated to roads is smaller. Congestion decreases by 11.5%. The gain in surplus results from the decrease in the opportunity cost of land (C_L) and transport cost (C_T). The consumption of the homogeneous good increases, but overall the surplus increases by 181€ per household per year.

Notice that the housing area increases slightly under CT, despite the important decrease in the radius of the city. Indeed, the decrease in the radius of the city induces a relatively smaller decrease in the available land: from (18) and (19), the available land for housing and transportation is relatively small when r is large. The decrease in the radius of the city therefore does not have a large impact (in relative terms) on the total amount of land available for housing and transportation. At the same time, the amount of land allocated to roads decreases at all distances from the city center. Overall, the resulting variation in the housing area remains almost the same.

The (optimal) linear toll

The linear toll requires that each household pay 210€ per kilometer (of daily trips) per year. We obtain a particularly small city with a radius reduced by 40% in comparison with the no toll situation. The trip-distance decreases by 17% which may be seen as a decrease in CO_2 emissions. The corresponding city is characterized by a reduction in transport cost and opportunity cost of land as well as an increase in the consumption of the composite good. The main weakness of the linear toll is that it significantly reduces the area occupied by households significantly: the average housing area decreases by 4.4% which is relatively higher than the variation under cordon and first-best tolls.

The linear toll reaches a good efficiency level in comparison with the optimal toll. In our simulations, however, we found that it is equivalent to a large increase in the gasoline price (about 12€/liter). Hence, the policy is likely to face strong opposition from road users.

First-best toll

The optimal toll leads, as expected, to a compact and dense city. The radius of the city and trip-distance decrease by 34% and 15%, respectively. The consumption of the homogeneous good increases, but the opportunity cost of land and transport costs are reduced. Travel time decreases by 13%. The decrease in the radius and trip-distance remain, however, slightly lower than under the linear toll. Policy FB is particularly effective in concentrating households around the CBD (cf. Figure 7).[18]

Optimal congestion pricing increases the welfare by 286€ per household per year. Cordon and linear tolls get 62% and 93% of this gain, respectively. The amount of the toll collected is relatively higher in comparison with all the other pricing schemes. With the optimal toll, the government budget is balanced (cf. condition (23)) in the sense that total taxes are equal to the cost of land used for transportation. Since the other pricing schemes provide lower revenues, the government must find alternative funding schemes.

4.2 Higher congestion

When k in the congestion function (16) increases (Scenario 1 in Table 5), congestion costs increase, and an efficient urban form corresponds to a further concentration around the CBD. The radius of the city increases under NT and decreases under FB. CT and LT induce a small increase. The higher congestion is followed by an increase in the land rent around the city center, motivating households to locate further away from the CBD. This incentive is higher than the opposing one induced by the (private) travel cost. Appropriate tolling makes the second incentive higher. There is more land allocated to roads.

The transport cost and expenses on the homogeneous good increase under the four regimes, while the land cost decreases only under FB. It is clear that an increase in congestion has a negative impact on welfare. The intuition for this result is straightforward (notice the decrease in housing area given by $\overline{s}$). CO_2 emissions and other pollutants related to fuel consumption vary in the same direction as VK: a higher congestion is followed by a higher pollution under all regimes except FB.

[18] A set of simulations we do not report here confirms this fact under a larger set of parameter values.

4.3 Alternative preferences

As the preference for housing increases (Scenario 2 in Table 5), the city radius r_f increases under the four regimes. The variations with respect to the base-case are 25%, 28%, 15% and 13%, respectively for NT, CT, LT, and FB. At the same time, the increase in the average housing area is relatively small for NT in comparison to LT and FB. This situation is brought about when congestion is unpriced and too much land is devoted to roads, leaving smaller areas for housing.

From the output in Table 5 we can see that the larger city leads to higher travel time, higher consumption of the homogeneous good and higher CO_2 emissions. This result requires a higher compensation for the households and yields a lower level of surplus.

Under NT the increase in $\overline{s}$ is relatively small in comparison with the increase in r_f. Indeed, with unpriced congestion the expansion of the city leads to an over-investment in roads. CT and FB yield a higher area per household because a smaller area is devoted to roads.

5 Conclusion

This paper examined the impacts of congestion pricing on the urban form. Our analysis reveals the importance of tolling on household decisions and consequently on the urban structure of the city. As a solution of the optimality conditions we derive, among other variables, the households distribution and the amount of land allocated to transportation over the city. Our analysis concludes that convenient congestion pricing leads to more efficient urban forms. The increase in welfare results from the reduced travel cost and the better allocation of land between housing and roads. In monetary terms, first-best toll yields a welfare increase estimated at 606 M€,[19] Accounting for environmental impact, the welfare gain of congestion pricing will be greater. Using an alternative empirical approach, Daniel & Bekka (2000) estimate that congestion pricing leads to a 10% reduction of emissions. We found that vehicle-kilometer (and so the related emissions) can be reduced by 16%. The difference is due to the fact that we integrate the long-term impacts on housing. De Palma & Lindsey (2006) obtain higher but comparable results.

[19]From the base case in Table 5, we have 286€ as impact on the surplus. Aggregating over the total population yields the value of 606M€.

They take into account a more general set of trips (not only home-to-work) and other sources of externalities (noise, accidents, etc.).

The linear toll reaches a good efficiency level in comparison to the first-best scheme, but its implementation is equivalent to an important increase in the vehicle operating cost. In practice, the cordon toll represents a potential alternative. Indeed, it induces a satisfactory increase in the housholds' surplus and encounters lower opposition from road users, as revelaed by true experience in recent years.

The model we have considered does not intend to perfectly reproduce housing and transportation in IDF. The monocentric model has well known limitations and there is a number of issues relevant to the region IDF that we have not discussed. In particular, there are multiple (smaller) business centers outside the CBD, and many working trips do not concern the CBD. We have assumed that all households have the same revenue, the same preferences and make only a daily home-to-work trip.[20] One further limitation in this model is that the attractiveness of the CBD is limited to savings in transport costs. This assumption, which is acceptable in simplified contexts, is not reasonable for agglomerations such as Paris where other facts such as the richer social life play an important role.

It is not easy to deal with all these facts at the same time, but the theory of the polycentric city is not yet sufficiently coherent and complete to represent a better alternative. Indeed, polycentric models do not refer to a precise model, but rather to a class of models. It would be useful to develop an analysis based on polycentric models that overcomes the weaknesses of the monocentric model. Still, this approach would require an identification of the limitations, both theoretical and empirical, of the monocentric model. Some of these limitations are direct extensions of the monocentric city model, and we plan to address these issues in future research in which we also plan to add the multi-cordon toll scheme. Indeed, the solution approach adopted here can be adapted to cordon pricing. At the same time, our conclusions about the impact of congestion pricing on the urban form and the levels of emissions should extend to more complicated frameworks.

[20] Brueckner & Selod (2006) discusses the optimal choice of transport systems.

Acknowledgements

We are very grateful to IAURIF for data availability, to Hakim Ouaras, Nathalie Picard, Dany Nguyen and Néjia Zaouali for the data they have collected on Île-de-France. The paper has benefited from comments of Robin Lindsey and participants at the Third International Conference on Funding Transportation Infrastructure, June 19-20, Paris (France). André de Palma acknowledges financial support from "Project Carbone: Gestion du transport et de la mobilité dans le cadre du changement climatique (ref 07 MT S063)" and Institut Universitaire de France. Moez Kilani acknowledges financial support from region Île-de-France (R2DS) under the TARIFU contract and from École des Ponts ParisTech.

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A Interpretation of the HS_T model

The necessary and sufficient conditions for a solution to the HS_T model, formulated in Section 2 above, are (7)-(9) and the following (22a)-(22f):[21]

$$R(r) = \begin{cases} \max(\psi(Y-g-l(r)-\tau(r),u), \psi_T(\frac{N(r)}{L_T(r)})) & r_c \le r \le r_f \\ R_A & r \ge r_f, \end{cases} \tag{22a}$$

$$R(r) = \psi(Y-g-l(r)-\tau(r),u) \qquad \text{if} \quad n(r) > 0, \tag{22b}$$

$$R(r) = \psi_T\left(\frac{N(r)}{L_T(r)}\right) \qquad \text{if} \quad L_T(r) > 0, \tag{22c}$$

$$s(r) = S(Y-g-l(r)-\tau(r),u), \qquad r_c \le r \le r_f, \tag{22d}$$

$$n(r) = \frac{L(r)-L_T(r)}{S(Y-g-l(r)-\tau(r),u)}, \qquad r_c \le r \le r_f, \tag{22e}$$

$$l(r) = \int_{r_c}^{r} c'\left(\frac{N(x)}{L_T(x)}\right)\frac{N(x)}{L_T(x)}dx, \qquad r_c \le r \le r_f. \tag{22f}$$

Equations (22a), (22b) and (22c) state that each piece of land should be allocated to the highest bidder. It follows that if both $n(r)$ and $L_T(r)$ are (strictly) positive, then the households' bid rent is equal to the bid rent of the transport sector. Furthermore, at the outside boundary of the city (at r_f) the bid rent function is equal to the agricultural rent R_A. Condition (22d) ensures that each household is choosing its bid-max lot size to maximize its utility (from (1) and (2)). Equation (22e) implies that constraint (7) is binding at the optimum, i.e. all the available land within the city is used either for housing or transportation. The location tax in (22f) reflects external costs induced by each household. It can be shown that under this congestion pricing the optimal solution yields

$$\int_{r_c}^{r_f} R(r)L_T(r)dr = \int_{r_c}^{r_f} l(r)n(r)dr. \tag{23}$$

meaning that the cost of transforming (agricultural) land to roads is just equal to the total amount of congestion tolls collected. The government budget is balanced in this sense.

[21] See Fujita (1989).

Transportation introduces externalities in the monocentric model and efficient solutions can no longer be obtained without public intervention.[22] In particular, the location tax given in (22f) is devised so that households internalize the external costs they impose on other road users.[23] The efficient allocation can then be decentralized through a compensated equilibrium (given by (22)), where the government chooses g, $l(r)$ and $L_T(r)$. The decentralization is a consequence of the fact that the solution to any compensated equilibrium can be obtained as a solution to the HS_T model and vice versa. The government can reach any target utility level by imposing adequate population taxes. The government decides on the taxes to collect and the amount of land to allocate to roads at each distance. Let $\text{HS}_T(u)$ refer to the Herbert-Stevens model with traffic congestion when the target utility is equal to u.[24] The following result (adapted from Fujita (1989)) states the relation between the HS_T model and competitive equilibria.

Proposition 2. *$(R(r), n(r), s(r), L_T(r), r_f, g^*, l(r))$ is a solution to the $HS_T(\overline{u})$ if and only if it is a compensated equilibrium under target utility $\overline{u}$.*

The total surplus in (6) may be written as

$$\mathscr{S} = \int_{r_c}^{r_f} \{ \left(\frac{Y - g - l(r) - Z(s(r), u)}{s(r)} - R_A \right) s(r) n(r) \\ - L_T(r) R_A + g n(r) + l(r) n(r) \} \, dr. \quad (24)$$

Using (5), (22b) and (23), it becomes

$$\mathscr{S} = \underbrace{\int_{r_c}^{r_f} (R(r) - R_A) L(r) dr}_{\mathbf{TDR}} + g \, \overline{N}, \quad (25)$$

where **TDR** stands for *total differential rent*. To illustrate the solution for varying utility levels, let us write $\mathscr{S}$, **TDR** and g as a function of u. We have from (24) and (25):

$$\mathscr{S}(u) = \mathbf{TDR}(u) + N \; g(u). \quad (26)$$

[22] In the absence of transportation externalities the competitive solution without government intervention is efficient.

[23] The impact of unpriced congestion is discussed in Arnott (1979) and Arnott & MacKinnon (1978).

[24] We discuss how the values of u (and/or g) are chosen (and what it reflects).

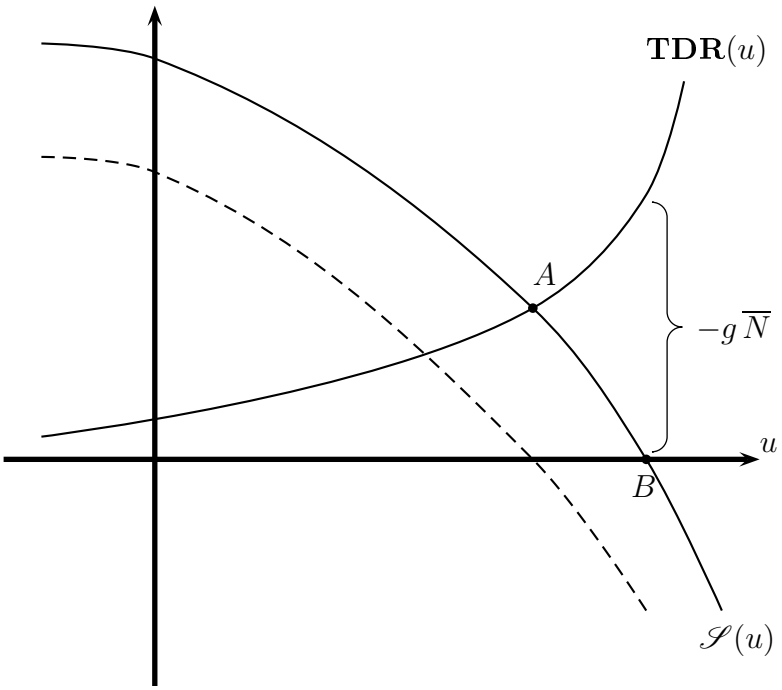

Figure 4: HS_T model and compensated equilibria.

The function $\mathscr{S}$ has the following properties:[25] $\mathscr{S}(u)$ is continuously increasing in u, and $\lim_{u\to-\infty}\mathscr{S}(u) = \overline{N}\,(Y-\tau(r_c))$ and $\lim_{u\to+\infty}\mathscr{S}(u) = -\infty$. The function g has the following properties: $g(u)$ is continuously decreasing in u, $\lim_{u\to-\infty} g(u) = Y - \tau(r_c)$ and $\lim_{u\to+\infty} g(u) = -\infty$.

Figure 4, which is adapted from Fujita (1989), is useful for understanding the relationship between the solution to the HS_T and compensated equilibria. The surplus related to the first-best optimum is given by curve $\mathscr{S}(u)$. When the tolling scheme is not optimal, we necessarily obtain a lower level of surplus for any utility level. Under a non-optimal congestion pricing, curve $\mathscr{S}(u)$ therefore moves downwards as the dashed curve. The total differential rent can either be redistributed to the households or to an absentee land owner. In the latter case, the households revenue is just Y. From Proposition 2 it is clear that point A corresponds to the solution of the competitive equilibrium or to the compensated equilibrium with target utility u^*. This solution is

[25]These properties are obtained as an extension to the model without transportation externalities (see Fujita 1989, page 74).

obtained under optimal congestion pricing, so if we set $l(r)$ to a different level we obtain a lower level of surplus. To reach the utility level at point B the households must receive a total subsidy equal to $\mathbf{TDR}(u)/\overline{N}$. Such is the situation where the total differential rent is redistributed to city citizens. The same is the solution to a competitive equilibrium with an absentee land owner but where the revenue Y is replaced by $Y + \mathbf{TDR}(u)/\overline{N}$. The case where only part of the rent is redistributed is an intermediate case between the two extremes.

In this sense, g may be interpreted as a control variable that indicates how much of the total differential rate is redistributed to city residents. The HS_T model can be seen from another perspective. If the utility level is given, the population tax should be designed so that condition (9) is met. That is, the population in the city remains equal to $\overline{N}$. Indeed, g appears in the solution as a multiplier for this condition (See Fujita 1989, page68).

In many papers, (Kanemoto 1977, Robson 1976, Pines & Sadka 1985, inter alios) the problem has been formulated as utility maximization under the revenue constraint. This result may be obtained from the HS_T model by finding the highest level of utility given the budget constraint

$$\overline{N}\,Y \geq \int_{r_c}^{r_f} \{[\tau(r) + Z(s(r), u) + R_A s(r)]n(r) + R_A L_T(r)\}dr$$

is satisfied, i.e. total revenue is higher than total costs. However, notice that this constraint is just $\mathscr{S}(u) \geq 0$ which in Figure 4 coincides with point B.

B Proof of Proposition 1

Let us denote the road occupancy at r by $\Gamma(r)$, i.e.

$$\Gamma(r) := \frac{N(r)}{L_T(r)} \tag{27}$$

Replacing (8) and (27) by an equality between differentials with appro-

priate boundary conditions, we may easily write all equations (7)-(9) as

$$\left\{\begin{array}{rcll} N'(r) & = & -n(r) & \\ L_T(r) & = & N'(r)s(r) + L(r) & \\ \Gamma(r) & = & \dfrac{N(r)}{L_T(r)} & \text{for} \quad r_c \le r \le r_f, \\ T'(r) & = & c(\Gamma(r)) & \\ s(r) & = & S(Y - g - l(r) - \tau(r), u) & \end{array}\right. \tag{28}$$

with boundary conditions

$$\left\{\begin{array}{rcl} \tau(r_c) & = & 0 \\ l(r_c) & = & 0 \\ N(r_c) & = & \overline{N} \\ N(r_f) & = & 0\,. \end{array}\right. \tag{29}$$

Now, let us examine equations (22a)- (22c) involving $R(r)$. Recall that we have assumed $n(r) > 0$ and $L_T(r) > 0$ for $r_c \le r < r_f$. Thus, the three equations (22a)- (22c) are equivalent to

$$\left\{\begin{array}{rcl} R(r) & = & R_A \quad \text{for} \quad r \ge r_f \\ R(r) & = & \psi_T(\Gamma(r)) \quad \text{for} \quad r_c \le r < r_f \\ \psi_T(\Gamma(r)) & = & \psi(Y - g - l(r) - \tau(r), u) \quad \text{for} \quad r_c \le r < r_f\,. \end{array}\right. \tag{30}$$

Notice that by continuity at r_f the first two equations imply that

$$\psi_T(\Gamma(r_f)) = R_A\,.$$

Now, let us consider the third equation. Notice that

$$\begin{array}{rcll} & & \psi(Y - g - l(r) - \tau(r), u) = R(r) & \\ \Leftrightarrow & & Y - g - l(r) - \tau(r) = \phi(R(r), u) & \text{by (10)} \\ \Leftrightarrow & & l(r) + \tau(r) = Y - g - \phi(R(r), u) & \\ \Leftrightarrow & \left\{\begin{array}{rcl} l'(r) + \tau'(r) & = & -\dfrac{\partial \phi}{\partial R}(R(r), u))R'(r) \\ \psi(Y - g, u) & = & R(r_c) \quad \text{by (29)} \end{array}\right. & \end{array}$$

Recollecting all the results above gives the following equivalent formulation of conditions (22a)-(22e):

$$\left\{\begin{array}{rcl} R'(r) & = & -\dfrac{l'(r)+c(\Gamma(r))}{\dfrac{\partial\phi}{\partial R}(R(r),u)} \quad \text{for} \quad r_c \le r < r_f \\ R(r_f) & = & R_A \\ N'(r) & = & \dfrac{\frac{N(r)}{\Gamma(r)} - L(r)}{S(\phi(R(r),u),u)} \\ N(r_c) & = & \overline{N} \\ N(r_f) & = & 0 \\ L_T(r) & = & \dfrac{N(r)}{\Gamma(r)} \\ g & = & Y - \phi(\psi_T(\Gamma(r_c)),u) \\ s(r) & = & S(\phi(R(r),u),u) \\ R(r) & = & \left\{\begin{array}{l} \psi_T(\Gamma(r)) \quad \text{for} \quad r_c \le r \le r_f \\ R_A \quad \text{for} \quad r \ge r_f \end{array}\right. \end{array}\right. \tag{31}$$

We end up by replacing $\Gamma(r)$ by $\psi_T^{-1}(R(r))$ in the two differential equations.

C Numerical implementation

An NSP software was developed to solve (11).

- a first function computes an approximate solution $R_{r_f}(r)$ and $N_{r_f}(r)$ of the double backward differential equation (11) over the interval $[r_c; r_f]$,

rewritten here:

$$\begin{cases} R'(r) = -\dfrac{H(\psi_T^{-1}(R(r))) + c(\psi_T^{-1}(R(r)))}{\partial_R \phi(R(r), u)} \\ N'(r) = \dfrac{N(r)/\psi_T^{-1}(R(r)) - L(r)}{S(\phi(R(r), u), u)} \\ \text{with final conditions } R(r_f) = R_A,\ N(r_f) = 0 \end{cases}$$

In order to cautiously solve the equations above, the following numerical procedure was used:

1. **Initialization**. Set $n = 0$, $r^0 = r_f$, $R^0 = R_A$, $N^0 = 0$
2. **while** $r^n > r_c$ **do**
   (a) compute $R'(r^n)$ and $N'(r^n)$
   (b) compute $\delta r^n = \min\left[r^n - r_c, \max\left(\delta r_{min}, \min(\delta r_{max}, \frac{\varepsilon R_A}{|R'(r^n)|}, \frac{\varepsilon \overline{N}}{|N'(r^n)|})\right)\right]$; note that three parameters are used: the maximum and minimum admissible values for δr^n, δr_{min} and δr_{max}, and a fraction ε limiting the progress of the numerical integration.
   (c) $n \to n + 1$
   (d) $r^{n+1} := r^n - \delta r^n$, $R^{n+1} = R^n - \delta r^n R'(r^n)$, $N^{n+1} = N^n - \delta r^n N'(r^n)$
3. **Conclusion**. Since $r^n = r_c$, set $R_{r_f}(r_c) = R^n$ and $N_{r_f}(r_c) = N^n$

- a second function searches and finds r_f (using dichotomy) such that $N_{r_f}(r_c) = \overline{N}$. The algorithm is the following:

1. **Initialization**. Set $r_1 = r_c$, $r_2 = 2r_c$, $r_3 = 3r_c$; compute $N_j = N_{r_j}(r_c) - \overline{N}$, for $j \in 1, 2, 3$.
2. **while** $\frac{|N_1|}{\overline{N}} > 10^{-6}$ **and** $\frac{|r_3 - r_1|}{r_r 2} > 10^{-5}$**, do**
   (a) if $N_1 N_2 < 0$ (the solution lies in $[r_1; r_2]$) then
      i. set $r_3 = r_2$, $N_3 = N_2$, $r_2 = (r_1 + r_2)/2$
      ii. compute $N_2 = N_{r_2}(r_c) - \overline{N}$
   (b) else if $N_2 N_3 < 0$ (the solution lies in $[r_2; r_3]$) then
      i. set $r_1 = r_2$, $N_1 = N_2$, $r_2 = (r_2 + r_3)/2$
      ii. compute $N_2 = N_{r_2}(r_c) - \overline{N}$

(c) else the solution does not lie in $[r_1; r_3]$) then

i. set $r_1 = r_2$, $N_1 = N_2$, $r_2 = r_3$, $N_2 = N_3$, $r_3 = 1.1\ r_3$

ii. compute $N_3 = N_{r_3}(r_c) - \overline{N}$

There is a further detail that should be taken into account in the iterations. Since $N'(r)$ is always negative, from the second line in (11) we have:

$$\psi_T^{-1}(R(r)) \geq \frac{N(r)}{L(r)}.$$

Hence, $\psi_T^{-1}(R)$ in (15) is replaced by

$$\psi_T^{-1}(R(r)) = \max\left\{\frac{N(r)}{L(r)}, (\frac{R(r)}{k\lambda})^{\frac{1}{1+\lambda}}\right\}.$$

This change is important when the area just next to the CBD border is exclusively allocated for transportation.

D Notation

Variable	description
r	distance to the city center $[L]$
r_c	radius of the district center $[L]$
r_f	radius of the urban fringe $[L]$
$c(\gamma)$	marginal transport cost $[€L^{-1}]$ as function of road occupancy
g	population tax per household $[€]$
$l(r)$	location tax per household at r $[€]$
$N(r)$	number of households located further away than r from the city center
$\overline{N}$	total households in the city
Y	annual income $[€]$
$n(r)$	lineic density of households at r $[L^{-1}]$
$R(r)$	rent at r per unit of area $[€L^{-2}]$
R_A	opportunity cost of land $[€L^{-2}]$
$s(r)$	housing area per agent at r $[L^2]$
$L(r)$	total amount of land available at r $[L^1]$
$L_T(r)$	amount of land devoted to transport use at r $[L^1]$

Ring (km)	Households	Veh. used for home-to-work trips	travel time (hours)	Land rent (€/m*m/year)
3.5-7	674 832	140 956		102.39
7-14	1 260 076	580 538	0.37	82.03
14-21	771 020	491 077	0.50	74.07
21-28	438 363	336 195	0.63	70.22
28-35	252 913	225 534	0.74	69.41
35-42	130 083	115 719	0.87	68.64
42-49	109 050	96 744	0.97	71.14
49-56	72 312	59 604	1.10	68.83
56-63	54 568	47 144	1.23	72.31
63-70	32 559	26 983	1.33	69.34
total		2 120 493		

Table 4: Data on IDF for ten rings.

	tx	r_f	$\overline{s}$	VK	RD	TT	TT_0	C_L	C_T	C_Z	$\Delta\mathscr{S}$
	Total tax	City radius	Housing area	Trip length	Roads' area	Travel time	Free-flow TT	Land cost	Transp. cost	Composite good cost	Δsurp
	€/y	km	m^2	km	$10^4\,m^2$	mins	mins	€/y	€/y	€/y	€/y
Base-case.											Cordon toll: located at 21.998km, value 22.457€/day; Linear: $\kappa = 0.21$/m/year
No toll	769	73.423	84.236	22.075	7 539	37.68	21.00	5 750	3 469	41 671	0
Cordon toll	2 410	55.633	84.889	19.633	6 482	33.90	18.24	5 499	3 081	42 147	181
Linear toll	3 541	46.246	81.385	18.727	6 042	32.70	17.22	5 185	2 960	42 479	271
First-best	7 172	48.650	83.129	18.711	6 049	33.70	17.22	5 360	3 021	42 222	286
Scenario 1. k' : 6.6 → 10.0											Cordon toll: located at 21.496km, value 26.065€/day; Linear: $\kappa = 0.24$/m/year
No toll	778	75.241	81.339	22.280	8 196	40	21	5 760	3 673	42 057	0
Cordon toll	2 534	55.493	82.629	19.579	6 949	36	18	5 539	3 285	42 445	236
Linear toll	3 605	44.935	78.635	18.521	6 394	35	17	5 222	3 157	42 765	347
First-best	8 141	47.788	80.726	18.556	6 430	35	17	5 331	3 163	42 629	368
Scenario 2. $\alpha + \beta = 1$ and β : 0.20 → 0.21											Cordon toll: located at 22.200km, value 24.783€/day; Linear: $\kappa = 0.22$/m/year
No toll	790	89.677	85.224	22.583	7 571	40	22	5 805	3 667	44 907	0
Cordon toll	2 625	68.474	87.307	20.522	6 722	37	19	5 716	3 369	45 135	173
Linear toll	3 530	52.415	84.401	19.545	6 261	36	18	5 469	3 253	45 385	273
First-best	8 037	55.953	86.104	19.494	6 253	35	18	5 549	3 246	45 293	292

Table 5: Summary statistics under different pricing regimes and parameter values.

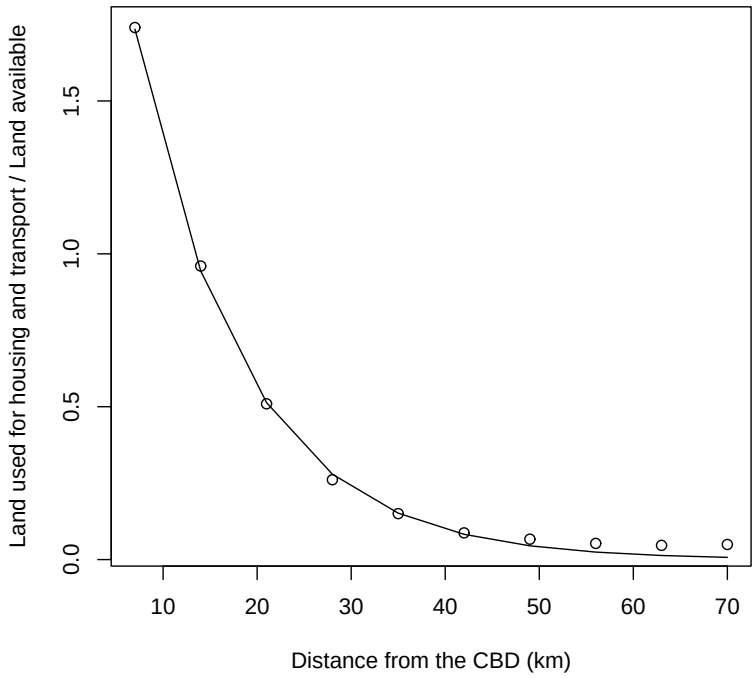

Figure 5: The fraction of land available for transportation and housing.

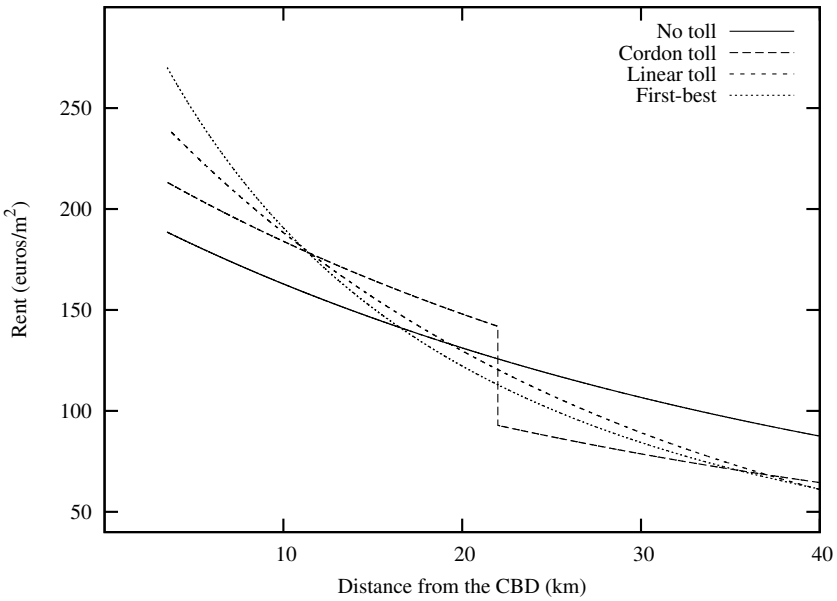

Figure 6: Land rent (base-case).

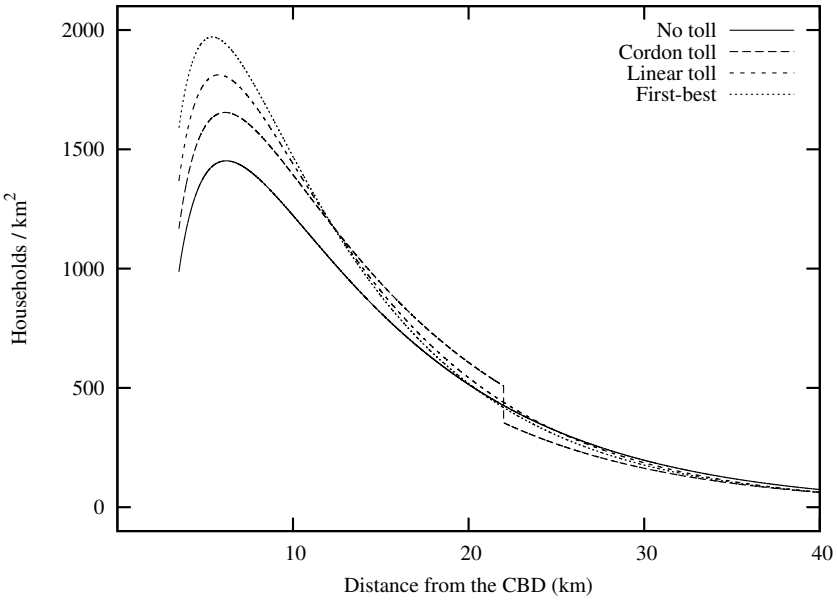

Figure 7: Distribution of households (base-case).

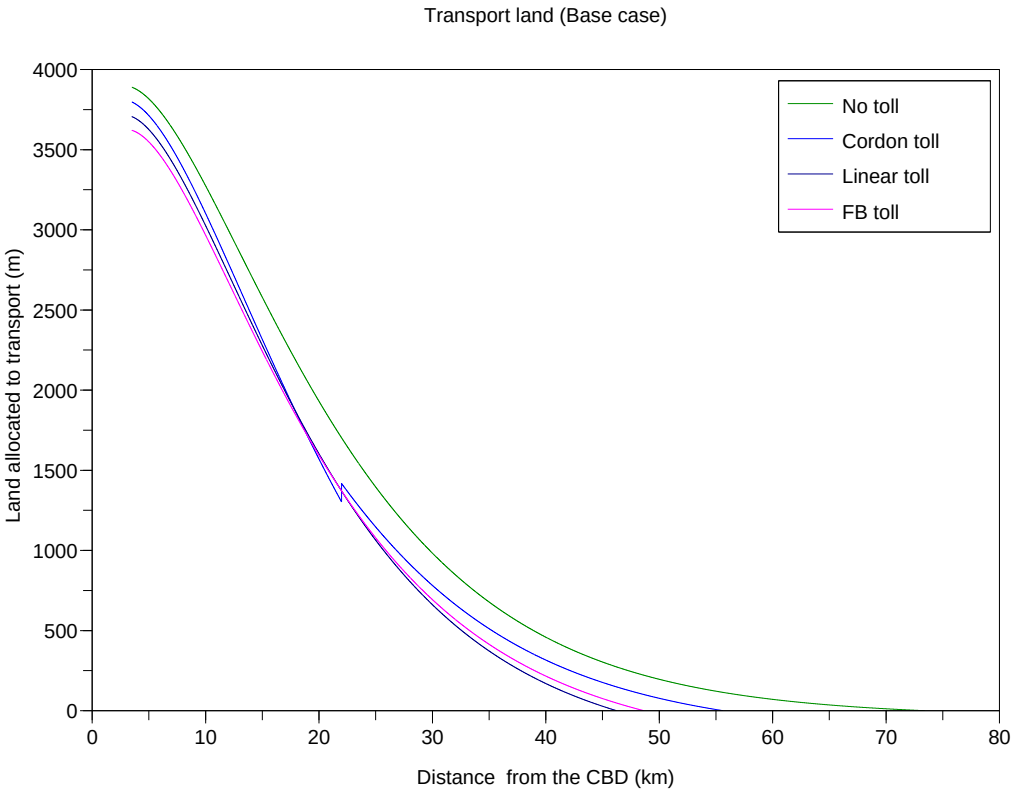

Figure 8: Land allocated to transport (base-case).