Which Method for Pricing Weather Derivatives ?

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ABSTRACT

Since the introduction of the first weather derivative in the United-States in 1997, a significant number of work was directed towards the pricing of this product and the modelling of the daily average temperature which characterizes most of the traded weather instruments. The weather derivatives were created to enable companies to hedge against climate risks. They respond more to a need to cover seasonal variations which may cause loss of profits for companies than to a coverage need in property damage. Despite the abundance of work on the topic, no consensus has emerged so far about the methodology for evaluating weather derivatives. The major problems of these instruments are on one hand, they are based on an meteorological index that is not traded on financial market which does not allow the use of traditional pricing methods and on the other hand, it is difficult to get round this obstacle by susbtituting the underlying for a linked exchanged security since the weather index is weakly correlated with prices of other financial assets. To further the question of evaluation, we propose in this paper to, firstly, shed light on the difficulties of implementing the three major pricing approaches suggested in the literature for the weather derivatives (actuarial, arbitrage-free and consumption-based methods) and, secondly, to compute the prices of a weather contract by the three methodologies for comparison.

Keywords : weather derivatives, arbitrage-free pricing method, actuarial pricing approach, consumption-based pricing model, risk-neutral distribution, market price of risk, finite difference method, Monte-Carlo simulations.

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1. Introduction

Introduced in 1997 by two energy companies, weather derivatives know certain success. In 1999, the Chicago Mercantile Exchange (CME) dedicates them an electronic platform of exchange (Globex). The London International Financial Futures and Options Exchange (LIFFE) follows the step in July 2001 by creating I-Wex and by launching on the market three contracts based on temperature for London, Paris and Berlin as well as in January 2002, a range of indices called NextWeather in association with Meteo France and following the merger with Euronext. It is the deregulation of the energy sector in the United States in 1996 that by pulling the competition is at the origin of the phenomenon.

A weather derivative is a financial contract whose incomes depend on the evolution of an underlying meteorological index. It was created to enable companies to protect themselves against risks related to climate. The meteorological index can be calculated, for instance, from measurements of temperature, precipitation or humidity. At the moment, the most actively traded contracts are on the temperature and more specifically on cooling degree day (CDD) which counts the daily average temperature above 65 ° F when it comes to the summer and on heating degree day (HDD) which counts the daily average temperature being below 65 ° F during the winter period. The daily average temperature is the average of the maximum and minimum temperature of the day. The temperature-based contracts have a great success because they meet the coverage needs of the energy companies which currently represent the main actors on the climate market.

In spite of the interest aroused by the weather derivatives, their development is not so rapid and significant as we had hoped. Several reasons can be expressed for that : departure of the market of the main actors such as Enron, Aquila and El Paso which has lowered the number of transactions, participants are too limited to energy companies which does not promote the liquidity of the market and also, distrust of investors for the weather products that they still deem too risky. But the main hindrance to climate market expansion is the difficulty in evaluating weather derivatives which has for consequence the high price of the contracts (the seller tends to fix a high bonus to compensate for the difficult exercise of evaluation).

The obstacles to the pricing of the weather derivatives are several kinds. The non-negotiability on the market of the meteorological index compromises the use of the traditional techniques of evaluation such as the method of Black and Scholes (1973). The current practice dealing with this kind of problem would be to replace the weather index by an exchanged security whose price is intimately linked to the index value. But in the case of the weather derivatives, this solution is hardly feasible as the level of the underlying is weakly correlated with the prices of the other financial assets. In addition, the exchange of certain weather contracts (options on weather index) primarily on the over-the-counter market does not facilitate the price transparency and does not authorize the calibration of the pricing models with regard to the information supplied by the market. There are, however, weather contracts of futures type which are traded on the regulated market and which are the subject of a quotation. At the moment, it is not possible to use most of the quoted prices of these contracts as the majority of them still lacks liquidity.

Despite the abundance of the led work, no consensus appeared, until now, as for the method to be adopted to price the weather derivatives. To shed light on the choice of a method, we propose in this article, first, to assess the advantages and disadvantages of the three main valuation methods that were suggested in the literature (actuarial, arbitrage-free and consumption-based methods). We will also draw up the difficulties of implementing each of these approaches in order to provide answers. Lastly, we will calculate the prices of a weather contract by means of the three methodologies to answer the following question which, hitherto, had not yet been addressed : do the three main techniques of valuation of the weather derivatives lead to divergent results ?

The paper is organized as follows : section 2 presents the three pricing methods proposed in the literature to deal with the weather derivatives, Section 3 describes the implementation of these methods and compares the obtained prices and section 4 concludes the paper.

2. Review of the pricing methods

The valuation of a derivative is to determine its value at the present moment or at any time before the maturity date of the contract, knowing that it will provide a variable payoff to the buyer at some point in the future. The amount of the payoff depends on the evolution of the underlying asset. The pricing task is particularly complex in the case of the weather derivatives since the meteorological index is not traded on the financial market. The arbitrage-free method which is traditionally used to price derivatives is questionable for weather derivatives and other approaches, such as the actuarial method and the consumption-based model, have been suggested for valuing weather derivatives. We begin by presenting the arbitrage-free method in order to understand to what extent the use of this methodology is compromised when the underlying is not an traded asset on the financial market. We will see that several kinds of difficulties are encountered when evaluating the weather derivatives by this approach. Proposals will be made to solve these problems. We will then present the actuarial method and we will discuss the consumption-based pricing model.

2.1 Arbitrage-free pricing method

Mainly, derivatives are made up of options and futures contracts. The pricing of an option is to calculate the premium paid by the purchaser at the time of the arrangement made with the seller while determining the value of a futures contract refers to the calculation of the strike price. It is customary to evaluate products from the arbitrage-free method. For example, on a market without friction and operating continuously, the value of an option giving a payment at the maturity date is obtained by creating a self-financing portfolio composed of the quantities of the underlying and of a riskless asset which will duplicate the results of the option at maturity. To prohibit any arbitrage opportunity (i.e. the possibility of making profits without risk), the price of the option at the present moment must be equal to the initial cost of the duplication portfolio since they provide both the same income at the expiration date. Cox, Ross and Rubinstein (1979) show that from the equality between the value of the portfolio and the payment of the option at the maturity date and that downgrading in time, the price of the option at the present moment is determined in a unique way and corresponds to the computation of the expected payment of the option (at maturity) discounted by the risk-free rate and defined using the so-called risk-neutral probabilities of the underlying. The limit of the formula for the option price of Cox, Ross and Rubinstein (1979) when the number of periods becomes infinitely large, coincides with the formula in continuous time of Black-Scholes (1973).

The weather options can not be evaluated by this method because it is not possible to create the portfolio of duplication with the meteorological index since the latter is not traded on the financial market. To solve this problem, Geman (1999) has suggested to substitute a contract on energy for the meteorological index in the portfolio by stressing the dependence of the energy price with the climate. However, Brix, Jewson and Ziehmann (2002) point out, for example, that the price of gas is more closely linked to the demand than to the temperature. They propose instead to use the weather futures contracts whose prices are, in their view, highly correlated with the underlying of the weather options which we want to value. For these authors, there remains a major obstacle for their use in the immediate future : these contracts are not yet sufficiently liquid, which makes it impossible, at the moment, the construction of a portfolio of perfect duplication.

But it is possible to consider other strategies that that of the duplication of the payoff of the option. We can choose the quantities of the securities in the portfolio, for example, so as to maximize the expected utility of the agent or in order to reduce the variance of the difference between the value of the portfolio and the result of the option at the end of the period. Authors such as Frittelli (2000) have shown that maximizing the expected utility with an utility function of exponential type gave rise to the calculation of the price of the option as being the conditional expectation of the payment of the option, discounted by the riskless rate and defined with the risk-neutral distribution of the underlying asset called "minimal entropy martingale measure" because it presents the particularity to minimize the relative entropy or distance of Kullback-Leibler of a probability with regard to the other one defined *a priori*. In the case of reducing the variance, the conditional expectation of the option price is calculated by using a risk-neutral distribution of the underlying called "variance optimal martingale measure" (Heath, Platen and Schweizer (2001)). It was shown that in an incomplete market (when the payoff of the option is not reachable by a self-financing portfolio), there was a multitude of arbitrage-free prices for the contingent claim (Karatzas and Kou (1996)). On the other hand, the price was unique when the market was complete because there was only a single measure of risk-neutral probability, the other measures becoming confused with that stemming from the strategy of perfect duplication.

These two strategies outlined above are perfectly feasible to treat the problem of liquidity of the weather futures contracts when creating a self-financing portfolio. The question is to know which one to choose. But the problem lies not only in this choice to be made, it also concerns the difficulty to implement these approaches. To obtain the price of a contingent claim from one of these strategies, it is advisable to calculate the expectation of its terminal payment by means of the risk-neutral distribution corresponding to the chosen strategy. There are two ways to calculate the expectation under one of these probability measures : either by extracting a risk-neutral distribution from the quoted prices or by solving the partial differential equation with a terminal condition whose the conditional expectation of the contingent claim price is the unique solution. We begin by exposing the first way to end then by the description of the second one.

2.1.1 Inference of a risk-neutral probability

The price at the moment t of the weather call option expiring at t_m on the HDD index for the cold period (November to April) is defined by:

$$C(t,I_t^H) = \delta\, e^{-r(t_m-t)}\, E^Q[\, \max(I_{t_m}^H - K,0) \mid F_t\,] = \delta\, e^{-r(t_m-t)} \int_K^{\infty} (x - K)\, f_Q(x)\, dx \qquad (1)$$

where $I_{t_m}^H = \sum_{j=1}^{n} \max(65 - T_j,0)$ represents the HDD[1] index on n days of the contract period, $T_j = \dfrac{T_j^{min} + T_j^{max}}{2}$ is the daily average temperature, δ corresponds to the value attributed to one degree-day, r is the risk-free interest rate, K denotes the strike price, $E^Q[..]$ stands for the conditional expectation under a risk-neutral probability called Q and $f_Q(x)$ is a risk-neutral density of the variable $I_{t_m}^H$.

In the case of the weather derivatives, there are several possible risk-neutral distributions. To calculate the price of the weather option by the arbitrage-free method, it is necessary to choose a risk-neutral distribution. The choice can be made by extracting the distribution from the quoted prices of the option. The problem here is that there are no quotations for the weather option because it is negotiated only on the OTC market. To solve this problem, it is possible to use the prices of the weather futures contracts traded on the CME instead of those of the weather option. For contracts based on the same meteorological index, the risk-neutral distribution must be the same. Indeed, the theoretical value at time t of the weather futures on the HDD index expiring at t_m is given by:

$$F(t,I_t^H) = \delta\, E^Q[\, I_{t_m}^H \mid F_t\,] = \delta \int_0^{\infty} x\; f_Q(x)\, dx. \qquad (2)$$

The inference of a risk-neutral distribution from the quotations will be made by searching the values f_Q so as to reduce the distances between the price given by the pricing formula (2) and the observed price on the market. This research involves solving the following optimization problem[2] :

$$\underset{f_Q}{\text{Min}} \sum_{t=1}^{M} \left(F(t,I_t^H) - F_t^m\right)^2 \qquad (3)$$

where F_t^m refers to the observed price of the weather futures contract at date t.

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(1) During the warm period (May to October), it is the CDD index that is used : $I_{t_m}^C = \sum_{j=1}^{n} \max(T_j - 65,0)$.

(2) The objective function has in fact a more complex expression. It has to contain a regularization term because the number of unknowns f_Q is superior to the number of equations to be solved. We recommend using the optimization program of Jackwerth and Rubinstein (1996) because it has the advantage of not requiring a closed-form expression for the theoretical price of the contracts because it is difficult or impossible to reduce the price formula of the weather derivatives to an analytical expression.

The theoretical value $F(t,I_t^H)$ is calculated by approximating the conditional expectation by Monte-Carlo simulations because it is difficult or impossible to reduce the price formula (2) to an analytical expression. It then generates N paths for the daily average temperature over a period of time. For each of these paths, we construct the HDD index and we assign it a risk-neutral probability. The N risk-neutral probabilities are estimated by solving the optimization program (3). Once obtained the optimal values of f_Q , we can then use them to value the weather futures contracts and options based on the same index. To obtain a "fair" price for the weather derivatives, the inference will be materialized from the quotations of the liquid contracts. At the moment, only weather futures contracts of Chicago, Cincinnati and New York can be considered as liquid (VanderMarck (2003) and Jewson (2004)).

2.1.2 Resolution of a partial differential equation

Pirrong and Jermakyan (2001) have suggested to calculate the arbitrage-free prices of weather options by inducing the market prices of risk from the quotations of the weather futures. The market price of risk is the difference between the expected rate of return of the underlying and the riskless interest rate, reported to the quantity of risk measured by the volatility. In incomplete market, there are as many market prices of risk as risk-neutral distributions because the market price of risk depends on the portfolio strategy adopted by the agent to reduce the risk and, at the same time, on the risk-neutral distribution associated with the strategy. The market prices of risk, noted λ_t,, that we seek to induce from the quotations, minimize the following objective function [(3)] :

$$\underset{\lambda_t}{\mathrm{Min}} \sum_{t=1}^{M} \left(F(t,T_t,I_t^H) - F_t^m\right)^2 \tag{4}$$

where the theoretical price $F(t,T_t,I_t^H)$ of the weather futures is solution of the following partial differential equation (under a risk-neutral probability Q) :

$$\frac{\partial F}{\partial t} + \left(\frac{dT_t^m}{dt} + \alpha(T_t^m - T_t) - \lambda_t \sigma_t\right)\frac{\partial F}{\partial T} + \max(65-T_t,0)\frac{\partial F}{\partial I^H} + \frac{1}{2}\sigma_t^2 \frac{\partial^2 F}{\partial T^2} = 0 \tag{5}$$

with the terminal condition $F(t_m,T_{t_m}, I_{t_m}^H) = I_{t_m}^H$.

[(3)] The objective function requires, in principle, a regularization term because there are more unknowns λ_t than equations. Indeed, market prices are quoted only during the working days while λ_t must be computed continuously and particularly when one uses the finite difference to solve numerically the partial differential equation that verifies the theoretical price $F(t,T_t,I_t^H)$. But the addition of the regularization term risks to increase considerably the computation time in the presence of the partial differential equation in two dimensions to be solved. To tacle this problem, we assume that prices during non-working days remain identical to the last quoted price so we can have as many unknowns as equations. This solution is feasible because the weather contract prices are not volatile.

The behaviour of the daily average temperature T_t is modeled by using the Ornstein-Uhlenbeck process (under the real probability):

$$dT_t = \left[\frac{dT^m_t}{dt} + \alpha(T^m_t - T_t) \right] dt + \sigma_t \, dW_t \qquad (6)$$

where α represents the speed of mean-reversion, σ_t is the volatility of the temperature, $dW_t = \tilde{\varepsilon}_t \sqrt{dt}$ with $\tilde{\varepsilon}_t \sim\!> N(0,1)$,

$$T^m_t = \alpha_0 + \alpha_1 \, t + \sum_{k=1}^{K} \alpha_{c,k} \cos(k\omega t) + \alpha_{s,k} \sin(k\omega t) \quad \text{et} \quad \omega = \frac{2\pi}{365}. \qquad (7)$$

The resolution of the partial differential equation can be led numerically by means of the finite difference method which will be described in section 3.

It is possible to try to calculate the market prices of risk [(4)] instead of trying to extract them from quotations. But this possibility is very expensive in computing time because we must solve numerically two PDEs, one to determine the weather contract price and the other to find the market price of risk λ_t.

In conclusion of this section, we can say that the arbitrage-free pricing method is applicable only when quotations are available for the weather contracts in order to extract a risk-neutral distribution or to infer market prices of risk. In addition, the inference of a risk-neutral distribution and market prices of risk requires the liquidity of the quoted weather contracts. Consequently, this method of valuation is applicable at the moment only to the weather contracts on the temperature which are the most traded on the market. It is possible to circumvent these problems by trying to calculate instead of extracting the market prices of

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[(4)] Within the framework of a model where an asset X endowed with the stochastic volatility Y follows this process:

$$dX_t = X_t \left(\mu(t,Y_t) \, dt + Y_t \, dW^1_t \right) \quad \text{with} \quad dY_t = a(t,Y_t) \, dt + b(t,Y_t) \, dW^2_t,$$

where $\rho \in [-1,1]$ represents the correlation coefficient of the brownian motions W^1_t et W^2_t, $dW^2_t = \rho \, dW^1_t + \sqrt{1-\rho^2} \, dW^\perp_t$ and W^1_t et $W^\perp_t$ are independent, Heath, Platen et Schweizer (2001) show that the price $C(t,X_t,Y_t)$ of the option on the asset X from the portfolio strategy of minimization of the variance of the residual risk is solution of the following partial differential equation :

$$\frac{\partial C}{\partial t} + \left[a(t,Y_t) - b(t,Y_t) \tilde{\lambda}_t \right] \frac{\partial C}{\partial Y} + \frac{1}{2} X^2_t Y^2_t \frac{\partial^2 C}{\partial X^2} + \frac{1}{2} b^2(t,Y_t) \frac{\partial^2 C}{\partial Y^2} = 0$$

with

$$\tilde{\lambda}_t = b(t,Y_t) \frac{\partial J}{\partial Y} \quad \text{and} \quad J(t,Y_t) = -\log E\left[\exp\left(\int_t^T \left(\frac{\mu(s,Y_s)}{Y_s} \right)^2 ds \right) \right]$$

where $\tilde{\lambda}_t$ is the market price of risk associated with the probability called "variance optimal martingale measure" and J is the solution of the following partial differential equation :

$$\frac{\partial J}{\partial t} + a \frac{\partial J}{\partial Y} + \frac{1}{2} b^2 \frac{\partial^2 J}{\partial Y^2} - \frac{1}{2} b^2 \left(\frac{\partial J}{\partial Y} \right)^2 + \left(\frac{\mu}{Y} \right)^2 = 0.$$

risk associated with the risk-neutral probability chosen to price the weather derivatives. However, it is still necessary to have quotations for the traded asset replacing the meteorological index in the self-financing portfolio in order to estimate the parameters of the price process of the susbtitution asset and this as well for the strategy of maximization of the expected utility as for the strategy of minimization of the variance. Remember that it is not possible to include the weather index in the self-financing portfolio since it is not traded on the market. Faced with the difficulty of establishing the arbitrage-free method, authors such as Brix, Jewson and Ziehmann (2002) and Augros and Moréno (2002) have proposed to value the weather derivatives by the actuarial method. The main advantages of this method are its simplicity of implementation and its application to any type of weather derivative that is liquid or not, that it possesses or not quotations and that it has or not a substitution asset.

2.2 Actuarial pricing method

The actuarial method evaluates the weather derivatives as being the conditional expectation of the future payment of these products, defined under the real probability of the underlying asset and to which is added a discounted compensation for the risk supported by the seller of the contract. The actuarial prices of the weather call option and futures on the HDD index at time t are respectively expressed as follows:

$$C^A(t,T_t,I^H_t) = \delta\, e^{-r(t_m-t)} \left(E[\,\text{payoff} \mid \mathcal{F}_t\,] + \kappa\, \sigma_{payoff}\right) \tag{8}$$

and

$$F^A(t,T_t,I^H_t) = \delta \left(E[\,I^H_{t_m} \mid \mathcal{F}_t\,] + \kappa\, \sigma_{I^H}\right) \tag{9}$$

where payoff = $\max(I^H_{t_m} - K, 0)$, K represents the strike price, δ is the tick size, r corresponds to the risk-free interest rate, t_m refers to the maturity date of the contracts, $E[\ldots \mid \mathcal{F}_t]$ designates the conditional expectation operator under the real probability, $\kappa\, \sigma_{payoff}$ and $\kappa\, \sigma_{I^H}$ stand for the risk premiums and σ_{payoff} and σ_{I^H} denote respectively the volatility of the payoff and the volatility of the HDD index.

This method is based on the law of large numbers which clarifies that by repeating a large number of time an experience, in a independent way, we obtain a more and more reliable estimate of the true value of the expectation of the observed phenomenon. The expectation under the real probability can be computed in two ways, either by using historical data on temperature, or using the technique of Monte-Carlo simulation. The first approach called "burn analysis" is to accumulate over a year degree days, then determine the payoff of the derivative for this year, then repeat this operation for other years. The expectation of the weather derivative price then corresponds to the average annual payoffs. The second approach is based on the use of a model for the daily average temperature to generate a set of paths. For each of these paths, we construct the HDD index. The HDD index so obtained is used to calculate the payoff. The expectation of the weather derivative price is then equivalent to the average of the payoffs from all the generated paths. By disregarding here opportunities of arbitrage, the implementation of this method puts no technical difficulty because there is no need to consider a martingale measure in particular. In addition, the authors (Brix, Jewson and Ziehmann (2002), Jewson (2004), Augros and Moréno (2002), Platen and West (2005)) little mention the estimation of the risk premium which is assumed to be zero or is equal to an arbitrary value. Platen and West (2005) argue that the risk premium tends to decrease when the insurance companies are in competition.

Cao and Wei (1998, 2004) show, on the contrary, that the risk premium is significant in the case of the weather derivatives by using the general equilibrium model of Lucas (1978). For example, in the case of the weather options, the risk premium would occupy 11.61% of the price for a risk aversion parameter corresponding to -40. To understand this result, we are going to present the model of Cao and Wei.

2.3 Consumption-based pricing method

The model of Cao and Wei (1998, 2004) is based on the general equilibrium model of Lucas (1978) which considers an economy in which a representative agent chooses the quantities of consumption, risky and riskless securities so as to maximize the expectation of the intertemporal utility while respecting a budget constraint. The problem faced by the agent is written in the following manner:

$$\underset{C_\tau,\, Q_\tau,\, Q_\tau^0}{\text{Max}} \quad E\left[\sum_{\tau=t}^{\infty} \beta^{\tau-t} U(C_\tau) \mid F_t\right], \quad 0<\beta<1 \tag{10}$$

under the constraint :

$$C_\tau + P_\tau Q_\tau + P_\tau^O Q_\tau^O \leq (P_\tau + D_\tau) Q_{\tau\,-M} + P_\tau^O Q_{\tau\,-M}^O + L_\tau \tag{11}$$

where U (…) expresses the utility function of the individual, this one is increasing and concave what is translated by the conditions U ' > 0 and U " < 0, C_τ corresponds to the actual consumption of the individual at the date τ, , P_τ et P_τ^O define respectively the price of the risky asset and the price of the riskless asset, D_τ means dividends, Q_τ et Q_τ^O give the quantities of the risky and riskless assets, , $Q_{\tau\,-M}$ et $Q_{\tau\text{-}M}^O$ correspond to the quantities held for M periods, L_τ represents the agent's salary and β coresponds to a discount factor which reflects the preference of the agent for the present : when β goes to 0, the individual prefers the present consumption what is less the case when β converge to 1.

The optimal quantities C, Q, Q^0 meet the conditions of the first order of the optimization problem stipulated by the equations (10) and (11). The expression P et P^0 of the asset prices are derived from the equations of the first order (called the Euler conditions). Cao and Wei use these relations to price the weather derivatives. For a power utility function [(5)], the authors show that the prices of the HDD weather derivatives at the time t are written as:

$$C^{cons}(t,T_t,I_t^H) = \delta\, E\left[\beta^{(t_m-t)} \frac{U'(C_{t_m})}{U'(C_t)} \max(I_{t_m}^H - K,0) \mid F_t\right] = \delta\, E\left[\beta^{(t_m-t)} \left(\frac{C_{t_m}}{C_t}\right)^{-\varphi} \max(I_{t_m}^H - K,0) \mid F_t\right] \tag{12}$$

for a call option,

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(5) Cao and Wei (2004) employ the utility function $U(C_t) = \frac{C_t^{1+\varphi}}{1+\varphi}$ where $\varphi < 0$ defines the constant coefficient of risk aversion. The problem with this expression is that the utility is negative when $\varphi < 0$. To be strict, we choose to use the following expression of the utility : $U(C_t) = \frac{C_t^{1-\varphi} - 1}{1-\varphi}$ with $\varphi \geq 0$.

$$F^{cons}(t,T_t,I_t^H) = \delta\, E\left[\, \beta^{(t_m - t)} \frac{U'(C_{t_m})}{U'(C_t)}\; I_{t_m}^H \;\middle|\; \mathcal{F}_t \right] \;=\; \delta\, E\left[\, \beta^{(t_m - t)} \left(\frac{C_{t_m}}{C_t}\right)^{-\varphi} I_{t_m}^H \;\middle|\; \mathcal{F}_t \right] \qquad (13)$$

for a futures contract

where $\beta^{(t_m-t)} \frac{U'(C_{t_m})}{U'(C_t)}$ represents the intertemporal marginal rate of substitution of the consumption.

Given the difficulty of determining an exact expression for the conditional expectation, Cao and Wei use Monte Carlo simulations to calculate the expectation.

These prices can be reduced to those of the arbitrage-free method if one considers that the intertemporal marginal rate of substitution coincides with the stochastic discount factor which designates a random variable that can only take positive values (Campbell, Lo and MacKinlay (1997)). The existence of this factor implies the absence of arbitrage opportunities on the market. It is unique only when the market is complete. As we assumed that the utility function was increasing, the intertemporal marginal rate of substitution is therefore positive. We can then write :

$$M_{t,t_m} = \beta^{(t_m - t)} \left(\frac{C_{t_m}}{C_t}\right)^{-\varphi} \qquad (14)$$

where M_{t,t_m} refers to the stochastic discount factor.

Besides, Cox, Ingersoll and Ross (1985) demonstrate that :

$$\beta^{(t_m - t)} \frac{U'(W_{t_m}^e)}{U'(W_t^e)} = e^{-r(t_m - t)}\, \zeta_{t_m} \qquad (15)$$

where W_t^e corresponds to the wealth of the agent at the moment t, we have $W_t^e = S_t\ \forall t$ if we suppose that the agent invests all his wealth in the purchase of the risky asset S at every date t,

$$\zeta_{t_m} = \frac{f_Q(S_{t_m})}{f_P(S_{t_m})}\,. \qquad (16)$$

By using these results, we can rewrite the equations (12) and (13) in the following way:

$$C^{cons}(t,T_t,I_t^H) = \delta\, E[\, M_{t,t_m} \max(I_{t_m}^H - K,0) \mid \mathcal{F}_t\,] \qquad (17)$$

$$= \delta \int_0^{\infty} \beta^{(t_m - t)} \left(\frac{C_{t_m}}{C_t}\right)^{-\varphi} \max(I_{t_m}^H - K,0)\; f_P(I_{t_m}^H)\, dI_{t_m}^H \qquad (18)$$

$$= \delta \int_0^{\infty} e^{-r(t_m - t)}\, \zeta_{t_m} \max(I_{t_m}^H - K,0)\; f_P(I_{t_m}^H)\, dI_{t_m}^H \qquad (19)$$

$$= \delta \int_0^{\infty} e^{-r(t_m - t)} \max(I_{t_m}^H - K,0)\, f_Q(I_{t_m}^H)\, dI_{t_m}^H \qquad (20)$$

$$= \delta\, e^{-r(t_m-t)}\, E^Q[\max(I^H_{t_m} - K,0) \mid \mathcal{F}_t] \tag{21}$$

$$\text{et} \quad F^{cons}(t,T_t,I^H_t) = \delta\, E^Q[I^H_{t_m} \mid \mathcal{F}_t] \ \text{ with } r = 0. \tag{22}$$

When the coefficient φ is zero, we observe that prices are identical to those of the actuarial method. This situation occurs when the consumption is independent of the level of the meteorological index because the seller of the contract is, in this case, neutral towards the risk. The risk premium is therefore zero.

Cao and Wei (2004) show that the total consumption of goods and services in the USA is significantly correlated with the mean of the temperatures of Atlanta, Chicago, Dallas, New York and Philadelphia. The authors determine the significance of the relationship between the consumption and the temperature by considering the following equation:

$$\ln C_t = a_0 + a_1 \ln C_{t-1} + \mu_t \tag{23}$$

where $a_1 \leq 1$, $\mu_t = \sqrt{1-\rho^2}\, \nu_t + \rho\, \varepsilon_t + \eta_1\varepsilon_{t-1} + \ldots + \eta_m\varepsilon_{t-m}$, ν_t is a white noise which is independent of the disturbance ε_t of the temperature, ρ means the correlation coefficient between consumption and temperature and $\eta_1,\ldots, \eta_m$ represent the parameters of past impacts of the temperature on consumption. The disturbances $\varepsilon_t, \varepsilon_{t-1}, \ldots$ are approximated by using residuals from the estimated process of the daily average temperature. The result obtained by Cao and Wei (2004) implies that the constant coefficient of risk aversion can not be equal to zero in the case of the weather derivatives and that, consequently, the actuarial method is not acceptable for valuing the weather derivatives. Following this result, Cao and Wei (2004) determine the proportion occupied by the risk premium in the value of the weather options and futures contracts by comparing prices when the correlation coefficient between the temperature and consumption is assumed to be zero and non-zero. They indicate that the gap between the two prices represents the risk premium because the price is broken down as follows (in the case of the futures contract):

$$\hat{F}^{cons}(t,T_t,I^H_t) = \frac{F^{cons}(t,T_t,I^H_t)}{E[M_{t,t_m} \mid \mathcal{F}_t]} = \delta\, \frac{E[M_{t,t_m} I^H_{t_m} \mid \mathcal{F}_t]}{E[M_{t,t_m} \mid \mathcal{F}_t]} = \delta \left(E[I^H_{t_m} \mid \mathcal{F}_t] + \frac{\mathrm{cov}(I^H_{t_m}, M_{t,t_m})}{E[M_{t,t_m} \mid \mathcal{F}_t]} \right) \tag{24}$$

when there is a dependence between the consumption and the temperature, otherwise we have:

$$\hat{F}^{cons}(t,T_t,I^H_t) = \delta\, E[I^H_{t_m} \mid \mathcal{F}_t]. \tag{25}$$

Cao and Wei have calculated the weather derivatives prices by postulating values stemming from the theory for the risk aversion coefficient due to the lack of liquidity of the weather derivatives. For liquid weather contracts such as Chicago, Cincinnati and Chicago ones, it is possible to estimate the risk aversion parameter and the discount factor from the quotations by using the generalized method of moments (GMM) or the simulated method of moments (SMM). The basic idea of the generalized method of moments is to approximate the conditional expectation in the expression of the prices by the average of sampling (empirical moment) and to estimate the risk aversion coefficient and the discount factor so that the averages of sampling are as close to zero. The SMM approach is different from the GMM method in the sense that we look for the values of β and φ such as the estimated moments from the Monte-Carlo simulations are the closest to the observed prices :

$$\frac{1}{N}\sum_{t=1}^{N}\left[\beta^{(t_m-t)}\left(\frac{C_{t_m}}{C_t}\right)^{-\varphi} I^H_{t_m}\right] \approx \tilde{F}^m_t \tag{26}$$

where $\tilde{F}^m_t$ is the quoted price on the market and N is the number of simulations for the variables $\left(\frac{C_{t_m}}{C_t}\right)$ and $I^H_{t_m}$.

It is possible to use either of these methods to estimate the parameters of the weather derivatives price. But it was shown that the simulated method of moments engendered estimates giving rise to prices closer to those from estimates provided by the generalized method of moments (Hamisultane (2007)). For this reason, we shall privilege this method here. Because it is necessary to estimate the risk aversion coefficient and the discount factor from the quotations to obtain prices close to the reality, the consumption-based pricing model does not allow to evaluate any type of weather contracts. Only the liquid contracts being the subject of a quotation or having a substitution asset can be estimated from this model. It thus presents the same drawback as that of the arbitrage-free pricing method. The detailed review of the pricing methods has shown that despite the different theoretical basis, the prices from the three approaches are connected between them. We will see if the three 3 methodologies provide prices close or not between them on the empirical plan.

3. Comparison of the computed prices of a weather derivative

We have at our disposal the price sample of the weather futures of Cincinnati expiring in March 2005 that includes 23 observations of the working days for the period 01/03/2005-31/03/2005 in order to conduct the calibration of the arbitrage-free and consumption-based models. Afterwards, we calculate the prices of the weather derivatives of Cincinnati for the out-sample period 05/03/2006-31/03/2006 during which we have quotations from the site of the Chicago Mercantile Exchange (CME), which will be compared to the estimated prices. We shall calculate here only the prices of the weather futures but all the techniques which we shall use can be applied to the weather options. We describe in this section the implementation of the three pricing methods. As for all the pricing methods, the modelling of the behaviour of the daily average temperature is essential, we start by treating this point.

3.1 Modelling the dynamics of the daily average temperature

Several processes have been suggested in the literature to model the dynamics of the daily average temperature for which there is a steady movement often formalised by a sine function, a mean-reversion, an autoregressive pattern, an upward trend due to the global warming and a seasonal volatility. These processes can be ordered in two broad categories: continuous time processes and discrete time processes. The first ones were developed in part to facilitate the handling of stochastic calculus and the latter ones to reflect with greater precision the complex dynamics of the daily average temperature. In calculating the prices, we hold a continuous time process and a discrete time process : the first one to calculate the prices by the arbitrage-free method whose main findings were made in continuous time and, the second to calculate the prices by the actuarial and consumption-based methods whose work has been done mostly in discrete time. Regarding the continuous time process, our decision was focused on the Ornstein-Uhlenbeck process with a seasonal non-parametric model for the volatility (model of Benth and Šaltytė-Benth (2005)), whose results appear in Table 1. While for the discrete time process, we retain a high order autoregressive process also equipped with the seasonal non-parametric model for the variance (see results in Table 2). These formulations were selected by comparing the models proposed by Cao Wei (1998, 2004), Roustant (2002), Campbell and Diebold (2005) and Benth and Šaltytė-Benth (2005)) on the basis of the information criteria (Akaike and Schwarz). To estimate the parameters of both processes, we used data of the daily average temperature of Cincinnati which are available on the site of the CME for the period 01/01/1993-12/31/2005. The seasonal non-parametric model for the volatility is determined by first combining the squared residuals at time t per year and then by taking the average of these data over the years for each date t giving rise to 365 averages which constitute the series of volatility for any given year. In view of the results in Tables 1 and 2, we note that the coefficients of both processes are all significantly different from zero (all t-statistics in absolute value are above 1.96 at 5% level of risk) , which validates the chosen formulations for these two processes.

Table 1 : Estimation of the parameters of the Ornstein-Uhlenbeck process with a seasonal non-parametric model for the volatility :

$$dT_t = \left[\frac{dT_t^m}{dt} + \alpha(T_t^m - T_t) \right] dt + \sigma_t dW_t$$

$$T_t^m = \alpha_0 + \alpha_1 t + \sum_{k=1}^{K} \alpha_{c,k} \cos(k\omega t) + \alpha_{s,k} \sin(k\omega t) \quad , \quad \omega = \frac{2\pi}{365},$$

$$\sigma_t^2 = \eta \overline{\hat{\varepsilon}^2}_{(365\times1)} \otimes \ell_{(Y\times1)} \quad ,$$

$\hat{\varepsilon}^2$ represents the squared residuals of the estimated Ornstein-Uhlenbeck process,

ℓ is a vector of size $Y\times1$ where Y corresponds to the number of years.

	Estimation	**t-statistic**
	$\mathbf{T_t^m}$	
α_0	53.26	217.45
α_1	0.0003	3.45
$\alpha_{c,1}$	-21.22	-122.76
$\alpha_{s,1}$	-7.05	-40.72
$\alpha_{c,2}$	-1.07	-6.20
$\alpha_{s,2}$	0.32	1.83
	Speed of mean-reversion	
α	0.27	29.20
	$\boldsymbol{\sigma_t^2}$	
η	1	48.62

Table 2 : Estimation of the parameters of the autoregressive process with a seasonal non-parametric model for the volatility :

$$T_t = T_t^m + \sum_{i=1}^{p} \rho_i (T_{t-i} - T_{t-i}^m) + \varepsilon_t ,$$

$$T_t^m = \alpha_0 + \alpha_1 t + \sum_{k=1}^{K} \alpha_{c,k} \cos(k\omega t) + \alpha_{s,k} \sin(k\omega t) \quad , \quad \omega = \frac{2\pi}{365} ,$$

$$\varepsilon_t = \sigma_t \, \tilde{\varepsilon}_t \ , \ \ \tilde{\varepsilon}_t \sim\text{>iid}(0,1),$$

$$\sigma_t^2 = \eta \, \overline{\hat{\varepsilon}^2}_{(365\times1)} \otimes \ell_{(Y\times1)} \quad ,$$

$\hat{\varepsilon}^2$ represents the squared residuals where $\hat{\varepsilon}_t = T_t - \hat{T}_t^m - \sum_{i=1}^{p} \hat{\rho}_i (T_{t-i} - \hat{T}_{t-i}^m)$,

ℓ is a vector of size $Y\times1$ where Y corresponds to the number of years.

	Estimation	**t-statistic**
	$\mathbf{T_t^m}$	
$\boldsymbol{\alpha_0}$	53.26	217.45
$\boldsymbol{\alpha_1}$	0.0003	3.45
$\boldsymbol{\alpha_{c,1}}$	-21.22	-122.76
$\boldsymbol{\alpha_{s,1}}$	-7.05	-40.72
$\boldsymbol{\alpha_{c,2}}$	-1.07	-6.20
$\boldsymbol{\alpha_{s,2}}$	0.32	1.83
	Autoregressive part	
$\boldsymbol{\rho_1}$	0.89	67.13
$\boldsymbol{\rho_2}$	-0.28	-16.53
$\boldsymbol{\rho_3}$	0.10	7.82
	$\boldsymbol{\sigma_t^2}$	
η	1.01	48.43

3.2 Implementation of the arbitrage-free method

We infer in this part elements contributing to the price formation by the arbitrage-free method such as the risk-neutral distribution and the market price of risk.

Extraction of a risk-neutral distribution

To extract a risk-neutral distribution from the quotations, it is necessary to resort to the Monte-Carlo simulations to approximate the conditional expectation of the theoretical price of the weather derivatives. The Monte-Carlo simulations are realized by means of the antithetic variates technique which allows to reduce the number of random trials by associating with each trial x_j its opposite $-x_j$. The expectation of the average of the functions $g(x_j)$ and $g(-x_j)$ is calculated to approximate the true value of $E[g(x)]$. In our case, we generate 1000 random numbers for the antithetic pairs $\{x_j,-x_j\}$ representing the disturbances of the daily average temperature over the period 03/01/2005-03/31/2005. For each path of the temperature, we compute the heating degree-day and we accumulate them over the considered period of time to construct the indexes $I^{H,\nu1}_{t_m,i}$ et $I^{H,\nu2}_{t_m,i}$ for i = 1,…, 1000 with ν1 and ν2 indicating the first and second antithetic variate. The HDD index is then given by :

$$I^{H}_{t_m,i} = \frac{1}{2}\left(I^{H,\nu1}_{t_m,i} + I^{H,\nu2}_{t_m,i}\right). \tag{27}$$

The theoretical price of the weather futures at time t is obtained by associating to each of the indexes $I^{H}_{t_m,i}$, a risk-neutral probability noted $f_{Q,i}$ for which we are trying to estimate, i.e. :

$$F(t,T_t,I^H_t) \approx \left(\sum_{i=1}^{N} I^{H}_{t_m,i}\, f_{Q,i}\right)_t . \tag{28}$$

For a contract in date t> 0 on M days and starting in 0, the price is calculated by generating the paths of the temperature for dates ranging from t to M and by taking for the dates prior to t (0 to t-1) the observed values of the temperature. The inference of a risk-neutral distribution is performed by solving the optimization problem given by Eq. (3).

Extraction of a market price of risk

The inference of a market price of risk requires the resolution of the partial differential equation given by Eq. (5) by a numerical method such as the finite difference method which consists in defining at first an uniform grid. For example here, the mesh is made by $N \times M \times G$ points where N, M and G correspond respectively to the number of points for the variables t, T_t and I^H_t. Secondly, it consists in replacing each derivative in the equation by a discrete operator (usually truncated Taylor series) and in gathering the terms in order to highlight a resolution scheme (explicit, implicit, Crank-Nicolson,…). By means of the resolution scheme, we determine the discrete values of $F(t,T_t,I^H_t)$ for each node of the grid. To avoid the oscillation of the solutions, we choose to use the implicit scheme which appears to be more effective than the Crank-Nicolson method (see Harris (2003)). The implicit scheme replaces the time derivative by a forward difference and the space derivative by a second-order central difference. By combining the terms between them, we end in the following equation which is to be solved for each point on the grid :

$$F^g_{i+1,j} = \alpha_j\, F^g_{i,j-1} + \beta_j\, F^g_{i,j} + \zeta_j\, F^g_{i,j+1} \tag{29}$$

where $F^g_{i,j} = F(i\Delta t, j\Delta T, g\Delta I^H)$ with $i = 0, 1,\ldots,N$, $j = 0, 1,\ldots,M$ and $g = 0, 1,\ldots,G$,

$$\alpha_{i,j} = \frac{1}{2}\Delta t\left(\frac{1}{j\Delta T}\Lambda_{i,j} - \sigma^2_{i-1}\frac{1}{(j\Delta T)^2}\right) , \tag{30}$$

$$\beta_{i,j} = 1 + \sigma^2_{i-1}\frac{\Delta t}{(j\Delta T)^2}, \tag{31}$$

$$\zeta_{i,j} = \frac{1}{2}\Delta t\left(-\frac{1}{j\Delta T}\Lambda_{i,j} - \sigma_{i-1}^2\frac{1}{(j\Delta T)^2}\right) \quad (32)$$

and

$$\Lambda_{i,j} = \frac{\Delta T_i^m}{\Delta t} + \alpha(T_{i-1}^m - j\Delta T) - \lambda_{i-1}\,\sigma_{i-1} \quad (33)$$

where σ_i corresponds to the seasonal non-parametric process for the volatility.

The terminal condition at expiration (i.e. $F(t_m,T_{t_m},I_{t_m}^H)$) and the boundary conditions (i.e. $F(t,0,I_t^{H,max})$ and $F(t,T_t^{max},0)$) are respectively :

$$F_{N,j}^g = g\Delta I^H \quad \forall\, j, \forall\, g\,, \quad (34)$$

$$F_{i,0}^g = G\Delta I^H \quad \forall\, i, \forall\, g \quad (35)$$

et

$$F_{i,M}^g = 0 \quad \forall\, i, \forall\, g. \quad (36)$$

The condition $F_{i,0}^g = G\Delta I^H$ is explained by the fact that when the temperature reaches 0 ° F (or any lowest level) at a date t, it is highly probable that it has maintained this low level until the date t and it maintains it until the date of maturity. At the expiration of the contract, the cumulative index reaches then its highest level which is $I_t^{H,max}$. For $F_{i,M}^g = 0$, the reasoning is similar. When the temperature reaches a very high level at a time t (its maximal value), there is a strong chance that the value of the cumulative index is zero at the maturity date. In Eq.(29), We have three unknown values ($F_{i,j-1}^g$, $F_{i,j}^g$, $F_{i,j+1}^g$) linked to one known value $F_{i+1,j}^g$. The value $F_{i+1,j}^g$ is given by the terminal condition at expiration. To solve Eq. (29), we must go backward in time (from i=N-1, …, 0).

We choose a mesh of size 200 × 200 × 31. We attribute respectively 0 and 100 to the minimal and maximal value of the daily average temperature with a spacing of 0.5 between points as well as 0 and 1000 to the lower and upper limits of the HDD index with a step of 5 and finally 0 and 31 to the limits of the time variable with a distance of 1 between points. The value 31 is the total number of days for the period of the contract expiring in March. Prices will be calculated for t ranging from 0 to 30 with the implicit scheme defined by equations (381) to (384). To avoid the oscillation of the solutions due to the lack of the diffusion term $\frac{\partial^2 F}{\partial (I^H)^2}$ in the partial differential equation given by Eq.(5), we use the approach adopted by Dewynne and Wilmott (1995) who is to consider that the index value remains constant between the dates of observation, what is verified in our case because the accumulation of degree-days is made only after the observation of the minimal and maximal temperature of the day and thus to solve an equation of dimension one between the dates. The component $\frac{\partial F}{\partial I^H}$ for which there is no corresponding diffusion term disappears then. Given that prices cannot suffer discontinuities when the index value is modified by the accumulation of degree-days, we must ensure that the following condition is met between the dates (Wilmott P., Dewynne J.N. and Howison P. (2000)) :

$$F(t_i^-, T_i^-, I_i^{H,-}) = F(t_i^+, T_i^+, I_i^{H,+}) \tag{37}$$

where t_i^- corresponds at the moment just before date i and t_i^+ the moment, just after date i, for which the value of the index is modified, $I_i^{H,-}$ represents the HDD index value until moment t_i^- and T_i^- refers to the temperature at the date t_i^-. We obtain the prices $F(t_i^+, T_i^+, I_i^{H,+})$ at the end of the resolution of the equation of dimension one. These prices were calculated by means of the values of the index appearing on the grid. To determine the price $F(t_i^-, T_i^-, I_i^{H,-})$ from the obtained prices $F(t_i^+, T_i^+, I_i^{H,+})$ when the quantity $I_i^{H,+} = I_{i-1}^{H,+} = I_{i-1}^{H,-} + \max(65\text{-}T_{i-1}^-, 0)$ does not coincide with the values of the index appearing on the grid, we resort to the linear interpolation.

3.3 Implementation of the actuarial method

To aproximate the conditional expectation in the weather futures theoretical price, we use the Monte-Carlo simulations and more specifically the antithetic variates technique. To do this, we generate 5000 random numbers for the antithetic pairs $\{x_j, -x_j\}$. The number of replications can be more important here than for the extraction of a risk-neutral distribution because they will not be used in an optimization problem. The more the number of replications is important in an optimization problem and the more the computation time to find the optimum is long. As previously, for each random trial, we calculate the degree-day and we accumulate them over the given period of time to obtain the indexes $I_{t_m,i}^{H,v1}$ et $I_{t_m,i}^{H,v2}$ for i = 1, …, 5000 with ν1 et ν2 representing the first and second antithetic variate. The actuarial price at date t of the weather futures contract on the HDD index is given by :

$$F(t, T_t, I_t^H) = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{2} \left(I_{t_m,i}^{H,v1} + I_{t_m,i}^{H,v2} \right)_t \quad \forall\, t \tag{38}$$

where N=5000 simulations.

3.3 Implementation of the consumption-based method

With the aim of estimating the constant coefficient of risk aversion by means of the simulated method of moments, we use the consumption of non-durable goods in the United States. The consumption of non-durable goods and services is generally employed in the empirical work. Because we see a greater correlation of the temperature with the consumption of non-durable goods than with the consumption of non-durable goods and services, we chose to work with the first data. The consumption data from households come from the site of the Federal Bank of St. Louis. They are in constant prices, seasonally adjusted, of a monthly frequency and extend over the period from 01/01/1993 to 31/12/2005. To obtain the per capita consumption, we divide the aggregate by the total population. Given that we want to calculate the price for each day of the weather contract, we use the linear interpolation to transform the monthly consumption in daily consumption. We estimate the parameters of the consumption process by using data on the period 01/01/1993-31/12/2005 (see Table 3). To approximate the conditional expectation of the prices by the Monte-Carlo simulations, we run 2000 paths for the temperature and the consumption. There is here more moments than parameters to be estimated. There is therefore no single solution checking all equations given by Eq. (26). In the presence of a large number of moments, the optimization algorithm can converge with difficulty towards a solution. The convergence will depend on the choice of the matrix of weights in the objective function to minimize. For reasons of efficiency, we choose to build the matrix of weights using the HAC matrix (Heteroskedasticity and Autocorrelation Covariance) of Newey and West (1994). Results are presented in Table 4.

Table 3 : Estimation of the parameters of the consumption process over the period 01/01/1993 – 31/12/2005

	Estimation	**t-statistic**
$\mathbf{a_0}$	-0.0017	-5.26
$\mathbf{a_1}$	1.0002	26903.27
$\boldsymbol{\rho}$	6.34×10^{-7}	1.22
$\boldsymbol{\sigma_v}$	0.0002	

$\ln C_t = a_0 + a_1 \ln C_{t-1} + \rho\, \varepsilon_t + \sqrt{1-\rho^2}\, \nu_t$, $\hat{\sigma}_v = \dfrac{\hat{\sigma}_e}{\sqrt{1-\hat{\rho}^2}}$

where $\hat{\sigma}_e$ corresponds to the estimated standard deviation of e_t

$e_t = \ln C_t - (\hat{a}_0 + \hat{a}_1 \ln C_{t-1} + \hat{\rho}\, \hat{\varepsilon}_t)$

Table 4 : Estimation of the parameters of the consumption-based pricing model by the simulated method of moments with the quotations of the Cincinnati weather futures over the period 01/03/2005 – 31/03/2005

	$\boldsymbol{\beta}$	$\boldsymbol{\varphi}$
Estimations	1.0078 (900.86)	49.9974 (3.38)
Bandwidth of Newey and West (1994)	15	
J-statistic	124.37	
Probability	0.00	

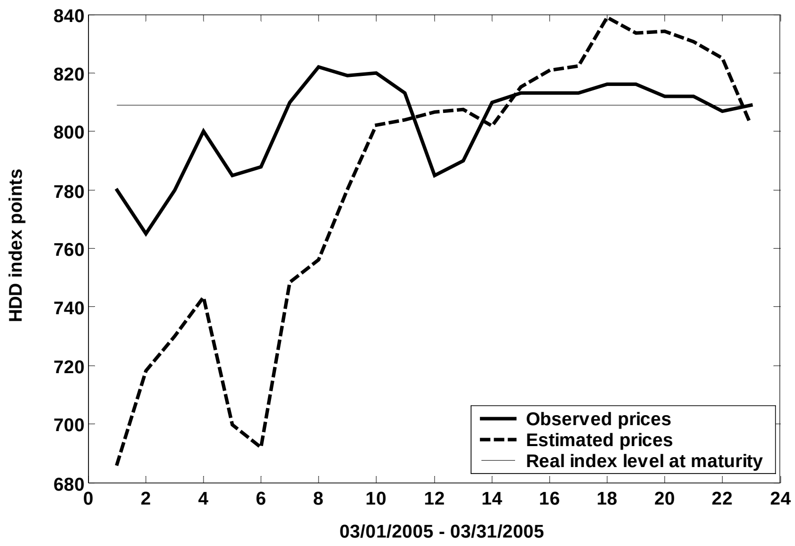
The figure in brackets indicates the t-statistic.

In view of the results, the consumption of non-durable goods is not correlated with the temperature as the t-statistic associated with the estimated value $\hat{\rho}$ of the correlation coefficient is less than 1.96 at 5% level of risk. Even using monthly data on consumption and aggregating the observations of temperature, we reach again the same conclusion. This finding which is contrary to the correlation observed by Cao and Wei (2004), is explained by the fact that Cao and Wei have examined the dependence between the average temperature of the five U.S. cities and the total consumption of goods and services in the United-States while we have considered only the temperature of cities taken individually and we have compared it with the consumption of non-durable goods of the entire U.S. territory. Using the equation of consumption with a very low correlation coefficient to generate trajectories of the aggregate has for consequence a large aversion parameter beyond 10 to match the estimated prices to the observed ones. But the theory states that the reasonable values of the aversion coefficient is between 0 and 10. Many empirical work (Mehra and Prescott (1985), Grossman, Melino and Shiller (1987), Mankiw and Zeldes (1991)) has also highlighted the aberrant results when the consumption is weakly correlated with the prices of the financial assets. We notice that the probability (< 5%) attached to the J-statistic does not indicate a perfect adequacy between the estimated prices and the quoted prices. We can attribute this result to the implementation in two stages of the simulated method of moments which is less effective than the iterative implementation but which is simpler to set up.

3.4 Results

The joint graphic representation of the quoted prices, the actuarial prices and the real level of the index at maturity during March 2005 shows that the quotations are, in general, closer to the effective realization of the index than the actuarial prices (see Figure 1). This observation encourages us to calibrate the pricing models with regard to the quotations.

Figure 1 : Observed prices and estimated actuarial prices (expressed in HDD index points and for working days) of the Cincinnati weather futures contract over the period 03/01/2005 – 03/31/2005



Further to the calibration, the differences between the estimated price and the real value of the index appear, in general, much less important in the mid-period of the contract for the prices supplied by the arbitrage-free method (concerning the inferred market prices of risk) and by the consumption-based approach than for those emanating from the actuarial method (see Figures 2, 3 and 4). On the other hand, we notice that the inferred risk-neutral distribution has generated estimated prices identical to those of the actuarial method (see Figure 5). This failure can be explained in part by the arbitrary choice of the values of departure necessary for the starting up of the algorithm of optimization. Given that it is difficult to have an idea of the values to be attributed to the risk-neutral probability at the beginning of the optimization program, we have allocated them the frequencies 1/N with N corresponding to the number of simulations for the weather variable. Although the arbitrage-free method concerning the inference of the market prices of risk has led to lower differences between the estimated prices and the observed ones in the mid-period of the contract in comparison with those of the actuarial method, it has also engendered a greater volatility of the estimated prices. This is due to the large number of inferred parameters that governs the behaviour of the estimated prices (there are as many market prices of risk than the estimated prices) and also to the high variability of the market prices of risk (see Figure 6). The consumption-based pricing method which has also brought satisfactory results in the mid-period, did not present this disadvantage because of the restricted number of parameters to be calibrated (limited to the discount factor and the risk aversion coefficient). Following these remarks, the consumption-based model appears to be more appropriate to evaluate the weather derivatives than the absence-free method for the mid-period of the contract. For the beginning and the end of the period, the actuarial method provides prices that are less distant from the real level of the index than those from the consumption-based model.

Figure 2 : Observed prices and predicted prices (expressed in HDD index points and for working days) of the Cincinnati weather futures from the actuarial pricing method over the period 03/01/2006 – 03/31/2006

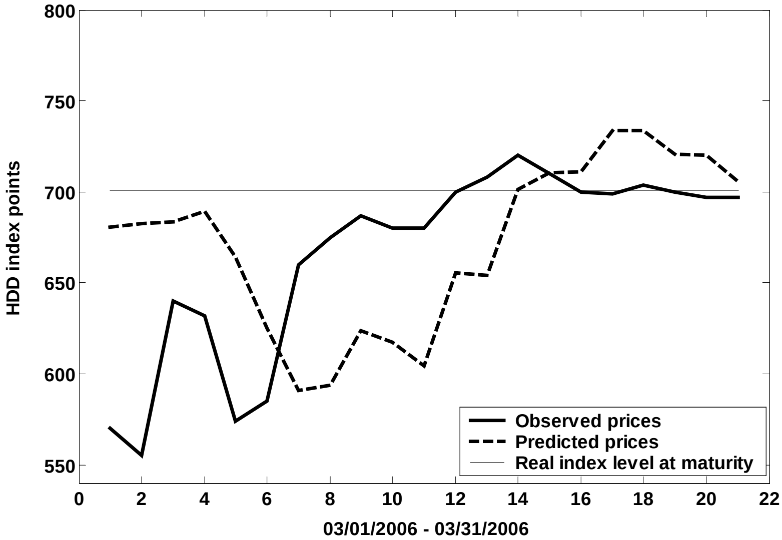


Figure 3 : Observed prices and predicted prices (expressed in HDD index points and for working days) of the Cincinnati weather futures from the arbitrage-free pricing method by using inferred market prices of risk

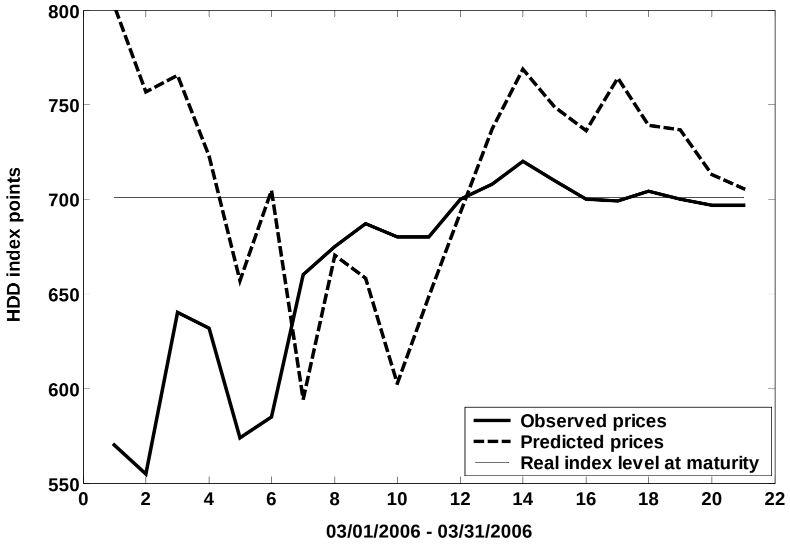


Figure 4 : Observed prices and predicted prices (expressed in HDD index points and for working days) of the Cincinnati weather futures from the consumption-based pricing method

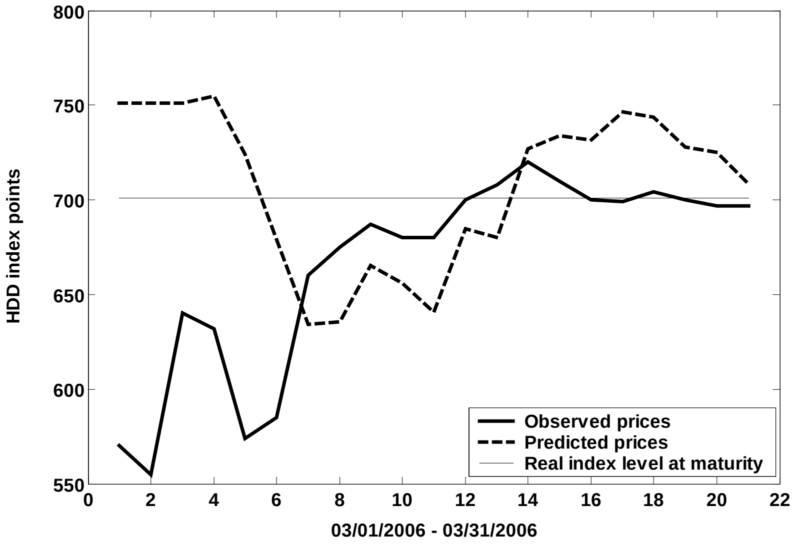


Figure 5 : Observed prices and predicted prices (expressed in HDD index points and for working days) of the Cincinnati weather futures from the arbitrage-free pricing method by using inferred risk-neutral distribution

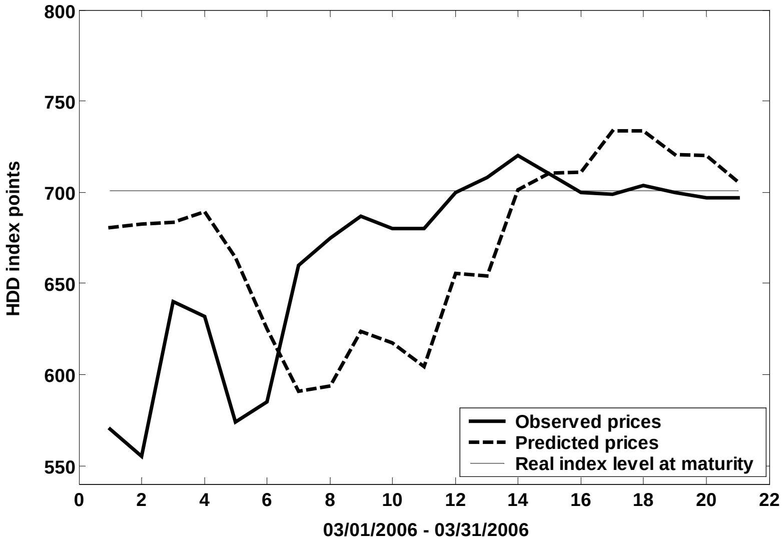
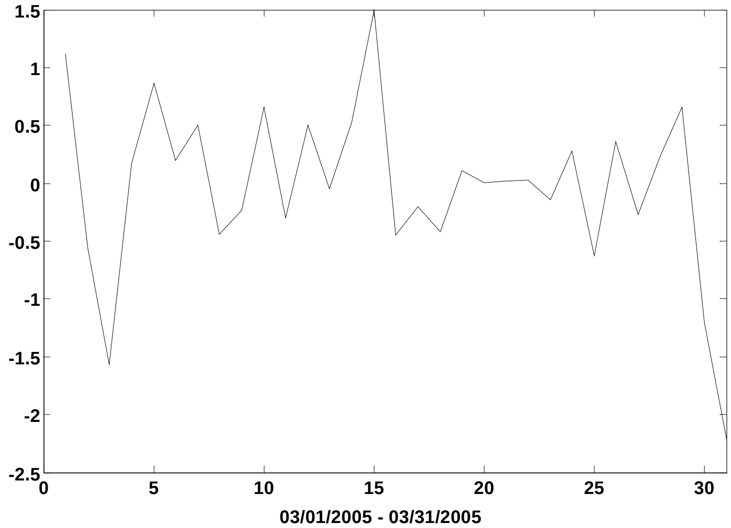
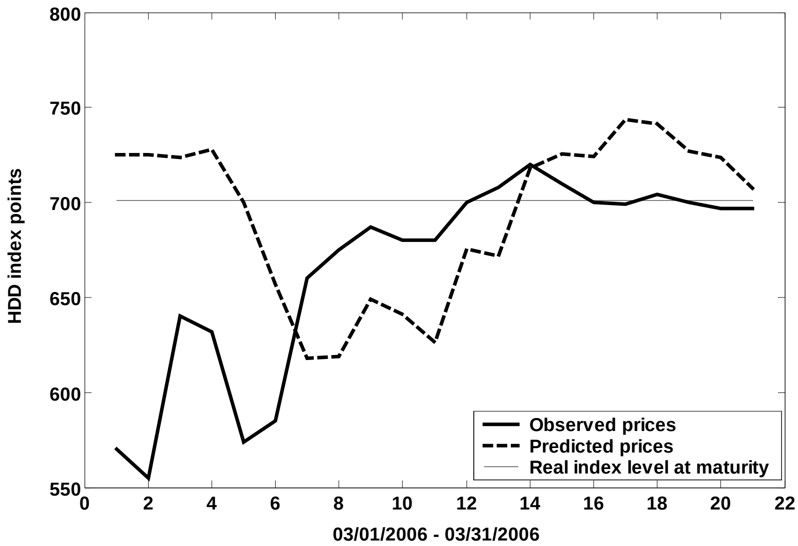


Figure 6 : Inferred market prices of risk from quotations of the Cincinnati weather futures expiring in March 2005



In case we would not possess the quotations relative to the weather contract for which we want to price by calibration, the alternative could be to use the quoted prices of the weather contract whose underlying is strongly correlated with the degree-day index of the derivative that we intend to value. To find out if this solution is feasible, we have calculated the prices of the Cincinnati weather futures by calibrating the pricing models with regard to the quotations of the New York weather futures whose the daily average temperature is closely linked to that of Cincinnati (the correlation coefficient is equal to 0.98). The results of the calibration of the consumption-based model appear in Figure 7.

Figure 7 : Observed prices and predicted prices (expressed in HDD index points and for working days) of the Cincinnati weather futures from the consumption-based pricing method (coefficients estimated with the quotations of the New York weather futures : $\beta = 1.004$ and $\varphi = 19.950$)



The similarity found here with the previous results leads us to conclude that the undertaken initiative constitutes a solution to the problem of unavailability of the price data for a contract that we seek to value.

4. Conclusion

The comparison, on the theoretical plan, of the pricing methods brought us to conclude that links united methodologies in spite of their different basis. The presentation of the consumption-based pricing model revealed us an additional tool to determine the arbitrage-free prices. It is to calculate the intertemporal marginal rate of substitution of the consumption or stochastic discount factor. On the other hand, the actuarial approach determines the weather derivative prices by using the historical probabilities of the underlying and not the risk-neutral ones. We have seen through the general equilibrium model that this approach meant admitting that the risk premium is non significant. But Cao and Wei (2004) have shown that the compensation for the risk was significant in the case of the weather derivatives. The use of the actuarial method is so questionable for valuing the weather contracts. On the empirical plan, the joint graphic representations of the quoted prices, the estimated prices and the real level reached by the degree-day index at the expiration date, have shown that the prices from the three methodologies are divergent and more precisely that the prices stemming from the arbitrage-free method (concerning the inference of the market prices of the risk) and from the consumption-based approach were closer to the actual level of the index than the prices resulting from the actuarial method in the mid-period of the contract. Although the prices from the inference of the market prices of risk gave satisfaction, they are nonetheless very volatile. The consumption-based pricing method has emerged as the most appropriate for evaluating the weather derivatives. However, this method has a drawback: it requires quotations to estimate the risk-aversion coefficient and the discount factor. We have shown that we could address the lack of price data by using the quotations of a liquid weather contract similar to one that we tried to price and for which we did not possess quotations. Nevertheless, despite the criticisms levelled against the actuarial method, it was observed that it gave good prices for the beginning and the end of the contract period. A combination of the actuarial and consumption-based methods can be then considered for evaluating the weather derivatives.

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