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Productive Efficiency and Contestable Markets

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Résumé:

Abstract: This paper provides a new game theoretic model consistent with the premises of contestable markets. Two firms repeatedly compete for a natural monopoly position. The limit price of the incumbent is disciplined by a hit and run strategy of the entrant. In this model, contrarily to the well known Maskin and Tirole model (1988): i) productive efficiency is encouraged, the more efficient firm gets a higher rent as an incumbent than the one the less efficient firm would, ii) rent dissipation does not necessarily prevails, even in the case of equally efficient firms. This opens the way to a reassessment of the merits of contestable markets.

Mots clés :

Key Words : contestable markets, productive efficiency, limit pricing, Markov strategies.

Classification JEL: C73, D43, L10

1 Introduction[1]

This paper investigates if and how competition for a natural monopoly position leads to productive efficiency and price discipline. This subject first appeared in the limit pricing literature (Gaskins, 1971). Renewed attention occurred with contestable markets (Baumol, Panzar and Willig, 1982). It is an important antitrust issue in the cases of concentrated market structures.

Such situations may be modelled as repeated entry games. A proper selection process has to be introduced to eliminate collusive equilibria and to focus the attention on the more competitive ones. The main contribution in this area has been made by Maskin and Tirole (1988), the selection process is based on a Markov hypothesis.

However, Maskin and Tirole did not explicitly address the productive efficiency issue, since their paper only concerns equally efficient firms. When applied to firms with different productive efficiencies, the Markov approach behaves strangely (Lahmandi, Ponssard and Sevy, 1996). To see this, observe that to deter entry a firm has to limit its instantaneous profit and, the less efficient it is, the more severe the limitation, since this limitation is based on the profit function of the other firm and not on its own. In a finite horizon game, if the game is long enough, entry deterrence induces the less efficient firm to be unprofitable, thus to give up its incumbency position (Gromb, Ponssard and Sevy, 1997). The Markov approach, because of its circularity in an infinite horizon, is such that an inefficient firm may remain a permanent incumbent, and productive efficiency is not encouraged.

This paper proposes a new approach directly based on the premises of contestable markets: firms differ in productive efficiencies, there is an incumbent and the entrant uses a hit and run strategy. It is proved that productive efficiency is encouraged. However, it will also be proved that rent dissipation may not necessarily prevail, it depends on the economic characteristics of the natural monopoly situation. A taxonomy is proposed to revisit the merits of contestable markets in the light of this model .

The paper is organized as follows. Section 2 reviews the Markov approach, the associated literature and provides the motivation for the new model. The infinitely repeated entry game under analysis is introduced in section 3. The subset of perfect Nash equilibria to be selected as its solution is precisely defined in section 4. The main properties of the selected equilibria are studied in section 5. Section 6 discusses their economic properties. The concluding section discusses limitations and further research.

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2 Review of the literature and motivation

Potential competition is ordinarily analyzed under the following general framework:

- the firms maximize their discounted profit over a long period of time, possibly infinite;

- time is discrete and divided into stages;

- the economic context is a natural monopoly; within a stage, if more than one firm are active, *i.e.* produce, the stage profits of at most one active firm may be strictly positive due to high fixed costs; in this context, it is enough to consider competition among two firms;

- competition takes the form of short run commitments; this embeds two ideas: first, at any point of time, a firm can only commit to a short period and, second, during that short period, the other firm can react; this also means that the discount rate between stages is close to zero;

- the two firms may have different characteristics, one being strong and the other weak, for instance the fixed costs may differ; when firms have identical characteristics the are said to be symmetric;

- the proposed solution should be a subset of perfect Nash equilibria (which are expected to be many);

- the solution should be tested with regards to economic properties such as rent dissipation and selection (productive efficiency).

This general framework involves designing a game form, which depends on the underlying situation featuring capacity, quantity, price, renewal policy competition ..., and a solution concept that should be general enough to apply to all relevant game forms.

This section reviews the previous works on this subject, points out the difficulties encountered so far and provides a preview of the results that will follow in the paper.

The discussion is carried out using an illustrative example. Price is the decision variable and noted $p \in [0,1]$. The pure monopoly profit is $v(p) = p(1-p) - f$. The fixed cost f is incurred at each stage but only in case the firm is active on the market. It is not a set up cost incurred once for all at the beginning of the game. Firms differ only in their fixed cost with $f_1 \leq f_2$, so that firm 1 is more efficient than firm 2.

Prices are set simultaneously. For a given price p_j of firm j the demand function of firm i is kinked using a positive constant switching cost denoted s. It is the pure monopoly demand $1 - p_i$ on the range $[0, p_j - s]$, it is zero on the range $[p_j + s, 1]$ and it decreases linearly on the range $[p_j - s, p_j + s]$. It is easily checked that as long as $p_j \geq 3s$ the best response of firm i is either $p_i(p_j) = p_j - s$, in which case it is active, or $p_i(p_j) \geq p_j + s$, in which case it is inactive. It is assumed all along that the condition $p_j \geq 3s$ holds for both firms on the relevant range of analysis: the fixed costs are relatively high and the price competition is tough.

Average cost pricing is characterized as the lower respective prices such that $v_i(p_i) = 0$. Theses prices are denoted p_1^{ac} and p_2^{ac}. Observe that $p_1^{ac} \leq p_2^{ac}$.

2.1 Static competition and selection

In a one stage game, any price $p_1 \in [p_1^{ac}, p_2^{ac} + s]$ is such that the best response of firm 2 is to be inactive so that firm 1 static limit price is $p_2^{ac} + s$. Denote $p_1^{\max}$ this limit price. Similarly denote $p_2^{\max}$ the static limit price of firm2. As long as $p_2^{ac} \leq p_2^{\max}$ both $(p_1^{\max}, p_1^{\max} + s)$ and $(p_2^{\max}, p_2^{\max} + s)$ are pure strategy Nash equilibria.

Eventually, if the difference in efficiency between the two firms is high enough, $p_2^{ac} > p_1^{ac} + s$. Entry of the less efficient firm is blockaded. Static competition induces a selection process. Suppose it does not. The key question addressed in this paper concerns the existence and nature of a dynamic selection process in an infinitely repeated game.

Note that this perspective is different from the dilemma between strategic entry barrier versus accommodation ordinarily discussed in one shot Stackelberg games (Dixit, 1980).

2.2 The Maskin and Tirole approach

In their 1988 paper, Maskin and Tirole analyze dynamic competition with large fixed costs in an infinite horizon. The proposed solution is based on a Markov approach in which the state variable is the move to which the other player is currently committed. Though the example discussed in that paper refers to quantity competition, as mentioned by the authors, the approach applies as well to the case of price competition. Note also that an alternating move situation taken from Cyert and DeGroot (1970) is used and not as here a simultaneous move case. This is a minor detail and what follows certainly does not depend on this peculiar feature of the game form. More importantly, Maskin and Tirole only consider the case $f_1 = f_2$.

Let the discount factor between stages be denoted δ. Let p^* be the unique solution (with $p^* < 1/2$, $1/2$ is the unrestricted monopoly price) to the equation in p

$$v(p - s) + \delta v(p)/(1 - \delta) = 0$$

The Markov equilibrium strategies when δ is close to 1 are as follows:

- along the equilibrium path only one firm, say firm i, is active and it selects $p_i = p^*$, it remains active all the time

- the reaction function of the outsider, say firm $j \neq i$, is such that if $p_i > p^*$ then $p_j = Min(p^* - s, p^*)$ but if $p_i \leq p^*$ then $p_j = p^* + s$.

This solution has many good economic properties. The exercise of monopoly power by the incumbent is disciplined by potential competition. The equation that defines the equilibrium path is easy to interpret: the first term may be seen as an entry cost that has to be recovered by future streams of revenues as an active firm, while preventing further re-entry. From an economic standpoint it is reminiscent of the limit pricing literature (Gaskins, 1971, Kamien and Schwartz, 1971, and Pyatt, 1971). This equation is known as the key recursive equation of dynamic entry games (Wilson, 1992). Eaton and Lipsey (1980) and Farrell

(1986) had already derived similar equations through intuitive arguments based on rational expectations. The Markov assumption provides a rigorous game analysis.

As can be seen from this equation, it must be that $v(p^*) \to 0$ as $\delta \to 1$, which means that, as the length of the short term commitment decreases, the instantaneous profit goes to zero. This is the celebrated rent dissipation property (Fudenberg and Tirole, 1987) which confirms the heuristic stories of Grossman (1981) and Baumol, Panzar and Willig (1982) on limit properties of contestable markets.

However, the theory of contestable markets is mostly concerned about productive efficiency, i.e. the selection of the most efficient firm through the competitive process. The Markov approach is inadequate to discuss this issue. Lahmandi et al. (1996) applied this approach to the case of asymmetric firms. The equilibrium now depends on the two prices p_1^* and p_2^* (with $p_i^* \leq 1/2$) which solve a system of two equations:

$$v_1(p_2 - s) + \delta v_1(p_1)/(1-\delta) = 0$$
$$v_2(p_1 - s) + \delta v_2(p_2)/(1-\delta) = 0$$

The Markov equilibrium strategies are as follows:

- along the equilibrium path only one firm, say firm i, is active and it selects $p_i = p_i^*$, it remains active all the time

- the reaction function of the outsider, say firm $j \neq i$, is such that if $p_i > p_i^*$ then $p_j = Min(p_i^* - s, p_j^*)$ but if $p_i \leq p_i^*$ then $p_j = p_i^* + s$.

In other words, the solution is almost not affected by the introduction of efficiency differences among the two firms. Surprisingly, rent dissipation still prevails as $\delta \to 1$. But the most striking feature of this approach is that the less efficient firm may indefinitely remain as a permanent incumbent and that, for δ close to 1, its incumbency rent is greater than the one of the more efficient one. This is easy to prove analytically. Using the system of equations that defines p_1^* and p_2^* one gets:

$\delta(v_2(p_2^*) - v_1(p_1^*)) = (1-\delta)(v_1(p_2^* - s) - v_2(p_1^* - s))$
$= (1-\delta)((p_2^* - p_1^*)(1 + 2s - (p_2^* + p_1^*)) + f_2 - f_1)$

which is strictly positive since $p_2^* > p_1^*$ (because for δ close to 1, p_i^* is close to p_i^{ac} and $p_2^{ac} > p_1^{ac}$) and $p_i^* < 1/2$.

This result can be illustrated numerically:

$s = .02$ and $\delta = .9$	efficient firm $i = 1$	inefficient firm $i = 2$
f_i	.15	.152
p_i^{ac}	.184	.187
$p_i^{\max}$	.207	.204
p_i^*	.185	.189
$v_i(p_i^*)$	.001	.002

This result is in contradiction with economic intuition: if the more efficient firm cannot obtain the incumbency position from the competitive process, it should at least be able to profitably buy it from the less efficient firm.

It is also in contradiction with a finite horizon approach. In Gromb et al. (1997), it is proved that, however small the difference in efficiency, as long as the discount factor is close to 1, selection always prevails in a long enough game. Indeed, the entry preventing strategy of the less efficient firm is recursively determined by the cumulative profit the more efficient firm would make as an incumbent. The cumulative profit of the less efficient firm eventually decreases below zero. At that stage, the less efficient firm is blokaded from entry as in section 2.1.

The Markov assumption creates a circularity in the infinite horizon game which is not consistent with a finite horizon approach.

2.3 The approach followed in this paper

The key idea is to assume that the less efficient firm may not be able to deter entry for ever. It plays an explicit hit and run strategy. This can be illustrated through the same numerical example, except that the discount factor is taken as $\delta = 1$ to make things more readable.

The proposed equilibrium is detailed in the following table

$s = .02$ and $\delta = 1$	$i = 1$			$i = 2$		
time dependance	stationary	$t = 4$	$t = 3$	$t = 2$	$t = 1$	$t = 0$
p_i^t	.193	.204	.195	.186	.178	.170
$v_i(p_i^t)$	*.0057*	.010	.005	.000	-.006	-.011
$v_j(p_i^t - s)$	**-.009**	*.000*	*-.0057*	*-.0114*	*-.0171*	
$\Sigma_{t=1}^{t=4}(v_2(p_2^t))$		.010	.015	.015	**.009**	-.002

The more efficient firm (firm 1) uses the stationary strategy $p_1 = .193$. If that firm were to set a slightly higher price, say p_1', the less efficient firm (firm 2) would "hit", i.e. move in setting its price at $p_1' - s$ and then "run" for four stages setting its price successively at .178, .186, .195 and finally at .204 (its static limit price). Since $p_1' > .193$ it must be that $v_2(p_1' - s) > -.009$, but $\Sigma_{t=1}^{t=4}(v_2(p_2^t)) = .009$, this hit and run strategy is profitable.

Now, once the less efficient strategy starts running, the best response of the more efficient firm is to wait until it goes away. Indeed, along the path, for all t, one gets $v_1(p_2^t - s) = -(4-t)v_1(.193) = -.0057(4-t)$. Moreover, firm 1 knows that firm 2 is running away along a four stage path since firm 2 cannot profitably deter entry along a 5 stage path ($\Sigma_{t=0}^{t=4}(v_2(p_2^t)) = -.002$).

This paper establishes the basic properties of this equilibrium and explores its most direct economic properties. It will be proved that either selection prevails or, if it does not, that the more efficient firm can profitably buy the incumbency position from the less efficient one. An interesting taxonomy of natural monopoly situations will emerge.

3 The repeated entry game Γ_∞^δ

The game Γ_∞^δ involves two players, player 1 and player 2. Γ_∞^δ is the infinite repetition of a stage game G. The players only use pure strategies. They maximize the sum of their discounted payoffs using δ as the discount factor $(0 < \delta < 1)$. The attention is focused on δ close to one.

3.1 The stage game G

Let $i \in \{1,2\}$ be anyone of the two players and $j \in \{1,2\}, j \neq i$ be the other one.

A strategy for player i in G is a real number $x_i \in I_i$ where $I_i =]-\infty, a_i]$ with a_i to be defined later on. The profit functions are denoted $\pi_i(x_i, x_j)$.

Assumption 1 *(natural monopoly):* It is assumed that π_i be increasing in x_j and that if $\pi_i > 0$ then $\pi_j \leq 0$.

If $\pi_j(x_i, x_j) = 0$ and if for all $x'_j \geq x_j$, $\pi_j(x_i, x'_j) = 0$ player j is said not to be active. Otherwise player j is said to be active.

For any x_i define $x_j^+(x_i)$ as the minimal x_j for which player j is not active that is:

$$x_j^+(x_i) = Min\{x_j \,|\text{for all } x'_j \geq x_j, \pi_j(x_i, x'_j) = 0\}$$

Denote $v_i(x_i)$ the pure monopoly profit function for player i. It writes:

$$v_i(x_i) = \pi_i(x_i, x_j^+(x_i))$$

The move associated to average cost pricing is denoted x_i^{ac}. It is such that $v_i(x_i) = 0$.

For any x_i define $x_j^-(x_i)$ as the best entry move of player j that is:

$$x_j^-(x_i) = Arg\{Max_{x_j < x_j^+(x_i)} \pi_j(x_i, x_j)\}$$

Denote $C_j(x_i)$ the entry cost function for player j. It writes:

$$C_j(x_i) = -\pi_j(x_i, x_j^-(x_i))$$

The one stage limit strategy is denoted $x_i^{\max}$. It is such that $C_j(x_i) = 0$.

The functions $x_j^+(.)$ and $x_j^-(.)$ are assumed to be well defined and strictly increasing.

Assumption 2 *(Nash equilibria of G):* Any pair of strategies $(x_i, x_j^+(x_i))$ such that $x_i^{ac} \leq x_i \leq x_i^{\max}$ is a Nash equilibria of the game G.

This implies that, for $i \in \{1,2\}$, $x_i^{ac} \leq x_i^{\max}$.

Assumption 3 *(monotonicity of the payoff functions on the range of analysis):* The functions $v_i(.)$ are strictly increasing and the functions $C_j(.)$ are strictly decreasing.

This assumption simplifies the analysis in the sense that border conditions need not be considered.

Assumption 4 *(technical assumption):* For mathematical convenience it will be further assumed that the functions v_i and C_i have derivatives and that these derivatives are uniformly bounded away from zero and from infinity.

The intervals I_i may now be precisely defined as $I_i =]-\infty, x_i^+(x_j^{\max})]$. Note that assumption 3 implies that $x_i^+(x_j^{\max})$ is lower than the unrestricted monopoly price of player i.

These assumptions generalize the simultaneous price competition model used in Ponssard (1991) and the one used in section 2 to cases such that $s > p_i/3$. In particular it may be that the best entry move generates an unprofitable duopoly situation rather than a monopoly one .

4 The selected set of perfect equilibria

The game Γ_∞^δ is a standard repeated game, the folk theorem applies and its set of perfect Nash equilibria is extremely large. The selection process introduced in section 2 is now used to focus on competitive equilibria reminiscent of contestable markets.

The two players are arbitrarily distinguished as a long term player, denoted as player L, and a short term player, denoted as player S. At this point no assumption is made regarding the relative efficiency of the two players.

Given an integer n, when they exist, define the real number $y_L \in I_L$ and the sequence $(y_S^t)_{t=0}^{t=n}$ in I_S that solve the following system to be denoted Φ_δ^n :

$$\begin{array}{lll}
\text{for } t \in \{0,1,2...n\} & C_L(y_S^t) = v_L(y_L)\delta(1-\delta^{n-t})/(1-\delta) & (1)\\
& C_S(y_L) = \Sigma_{t=1}^{t=n}\delta^t v_S(y_S^t) & (2)\\
& v_L(y_L) \geq 0 & (3)\\
\text{for } t \in \{1,2...n\} & \Sigma_{t'=t}^{t'=n}\delta^{t'-t} v_S(y_S^{t'}) \geq 0 & (4)\\
& \Sigma_{t'=0}^{t'=n}\delta^{t'} v_S(y_S^{t'}) < 0 & (5)
\end{array}$$

The fact that the functions $C_i(.)$ be strictly decreasing ensures the following lemma.

Lemma 1 *If it exists, the sequence $(y_S^t)_{t=0}^{t=n}$ is strictly increasing in t.*

Let H be the set of strategy pairs in G defined as

$$H = \{(y_L, x_S^+(y_L)), (y_S^t, x_L^+(y_S^t))_{t=1}^{t=n}\}.$$

To simplify notations, a pair h of H is identified by its first element that is, the active player and his move. Then

$$H = \{y_L, (y_S^t)_{t=1}^{t=n}\}.$$

The selection process will basically define two equilibrium paths which consist of sequences $(h^k)_{k=1}^{k=\infty}$ of pairs in H. On the one hand, an equilibrium path in which the long term player uses the stationary strategy y_L so that $h^k = y_L$

for all k. On the other hand, an equilibrium path in which the short term player uses a hit and run strategy $(y_S^t)_{t=1}^{t=n}$ so that $h^k = y_S^k$ for all $k = 1$ to $k = n$ and $h^k = y_L$ for all $k > n$. The is made precise by the following definition.

Definition 2 The selection process *defines the paths in the game* Γ_∞^δ *which consists of sequences* $(h^k)_{k=1}^{k=\infty}$ *in* H *such that for any pair* h *equal to some* h^k, *if* $h^+ = h^{k+1}$, *the pair* h^+ *is such that:*

if $h = y_L$

if $x_L = y_L$	*for all* $x_S \geq x_S^+(y_L)$		$h^+ = y_L$
	for all $x_S < x_S^+(y_L)$		$h^+ = y_S^1$
if $x_S = x_S^+(y_L)$	*for all* $x_L > y_L$		$h^+ = y_S^1$
	for all $x_L \leq y_L$		$h^+ = y_L$
if $x_L \neq y_L$	*and* $x_S \neq x_S^+(y_L)$		$h^+ = y_L$

if $h = y_S^t$

if $x_S = y_S^t$	*for all* $x_L \geq x_L^+(y_S^t)$	*for all* $t < n$	$h^+ = y_S^{t+1}$
		if $t = n$	$h^+ = y_L$
	for all $x_L < x_L^+(y_S^t)$		$h^+ = y_L$
if $x_L = x_L^+(y_S^t)$	*for all* $x_S > y_S^t$		$h^+ = y_L$
	for all $x_S \leq y_S^t$	*for all* $t < n$	$h^+ = y_S^{t+1}$
		if $t = n$	$h^+ = y_L$
if $x_S \neq y_S^t$	*and* $x_L \neq x_L^+(y_S^t)$	*for all* $t < n$	$h^+ = y_S^{t+1}$
		if $t = n$	$h^+ = y_L$

The sequences selected by this process may start with any pair in H but observe that $h^k = y_L$ for all $k > n$, player L is active indefinitely. For $t \in \{1, ..n\}$ denote H_S^t the sequence which starts with $h^1 = y_S^t$ and H_L the sequence in which $h^k = y_L$ for all k. After at most n stages, how the game started is irrelevant, for all $h^1, h^{1+n} = y_L$.

Suppose the selected sequence is H_L. If player L is too "greedy "$(x_L > y_L)$ or if player S is too "tough" $(x_S < x_S^+(y_L))$ player S becomes the active player at the next stage that is, $h^{k+1} = y_S^1$ and the game goes on as in H_S^1.

The is also true on the initial part of any path H_S^t, interchanging the roles of the players with the game going on as in H_L.

Denote by $V_L(.)$ and $V_S(.)$ the respective discounted payoffs of the two players as a function of the selected path. It is easily seen that:

$$\begin{array}{rl} & V_L(H_L) = v_L(y_L)/(1-\delta) \\ \text{for } t \in \{1, 2...n\} & V_L(H_S^t) = \delta^{n-t+1} V_L(H_L) \\ & V_S(H_L) = 0 \\ \text{for } t \in \{1, 2...n\} & V_S(H_S^t) = \Sigma_{t'=t}^{t'=n} \delta^{t'-t} v_S(y_S^{t'}) \end{array}$$

Conditions (3) and (4) of Φ_δ^n ensures that these payoffs are non negative.

Lemma 3 *Any strategy pair generated by the selection process is a perfect Nash equilibrium of* Γ_∞^δ.

Proof. Consider a path H_S^t with $h^1 = y_S^t, V_L(H_S^t) = \delta^{n-t+1}V_L(H_L)$ and $V_S(H_S^t) = \sum_{t'=t}^{t'=n} \delta^{t'-t} v_S(y_S^{t'})$.

Consider a deviation from player L. If $x_L > x_L^+(y_S^t)$ he would get $\pi_L(y_S^t, x_L) + \delta V_L(H_S^{t+1}) = 0 + \delta V_L(H_S^{t+1}) = V_L(H_S^t)$. If $x_L < x_L^+(y_S^t)$ he would get $\pi_L(y_S^t, x_L) + \delta V_L(H_L)$. Start with $t < n$, one may write:

$$\begin{aligned} \delta V_L(H_L) &= \delta v_L(y_L)/(1-\delta) \\ &= \delta(1 + .. + \delta^{n-t-1} + \delta^{n-t}...)v_L(y_L) \\ &= \delta(1-\delta^{n-t})v_L(y_L)/(1-\delta) + \delta^{n-t+1}v_L(y_L)/(1-\delta) \end{aligned}$$

Using condition (1) of Φ_δ^n and the explicit expression of $V_L(H_S^t)$, it follows that $\delta V_L(H_L) = C_L(y_S^t) + V_L(H_S^t)$. Since $x_L < x_L^+(y_S^t)$ player L is active and his payoff is less or equal to the one associated to his best entry move so that $\pi_L(y_S^t, x_L) \leq \pi_L(y_S^t, x_L^-(y_S^t)) = -C_L(y_S^t)$. Hence $\pi_L(y_S^t, x_L) + \delta V_L(H_L) \leq V_L(H_S^t)$.

If $t = n$, since $\pi_L(y_S^n, x_L) \leq \pi_L(y_S^n, x_L^-(y_S^n)) = -C_L(y_S^n) = 0$, one gets $\pi_L(y_S^n, x_L) + \delta V_L(H_L) \leq V_L(H_S^n) = \delta V_L(H_L)$.

Consider a deviation from player S. If $x_S > y_S^t$ he would get $\pi_S(x_S, x_L^+(y_S^t)) + \delta V_S(H_L) = \pi_S(x_S, x_L^+(y_S^t)) + 0$. It should be that this is less than $V_S(H_S^t)$. This is certainly true if $\pi_S(x_S, x_L^+(y_S^t)) < 0$. Otherwise, by assumption 2, y_S^t is a best response to $x_L^+(y_S^t)$ in G, so that $0 \leq \pi_S(x_S, x_L^+(y_S^t)) \leq \pi_S(y_S^t, x_L^+(y_S^t))$. Since $V_L(H_S^t) \geq 0$ and since the sequence $(y_S^{t'})_{t'=t}^{t'=n}$ is strictly increasing this implies $\pi_S(x_S, x_L^+(y_S^t)) \leq V_L(H_S^t)$.

If $x_S < y_S^t$, with the notational convention $V_S(H_S^{n+1}) = V_S(H_L) = 0$, he would get $\pi_S(x_S, x_L^+(y_S^t)) + \delta V_S(H_S^{t+1}) = v_S(x_S) + \delta V_S(H_S^{t+1})$. Since $v_S(.)$ is strictly increasing, $v_S(x_S) + \delta V_S(H_S^{t+1}) < v_S(y_S^t) + \delta V_S(H_S^{t+1}) = V_S(H_S^t)$.

Similar arguments hold with the other paths. ■

An equilibrium obtained through the selected process is denoted a $SPE(n)$ which stands for a "selected perfect equilibrium for a given n".

5 The main properties of the selected set of perfect equilibria and a further refinement

Some preliminary comments are in order.

Our attention is focused on the case δ close to 1. It will be convenient to work on the system Φ_1^n which is the limit of Φ_δ^n as δ goes to 1. It is easily seen that the system Φ_1^n is

$$\begin{array}{lll} \text{for } t \in \{0,1,2...n\} & -C_L(y_S^t) + (n-t)v_L(y_L) = 0 & (1') \\ & -C_S(y_L) + \Sigma_{t=1}^{t=n} v_S(y_S^t) = 0 & (2') \\ & v_L(y_L) \geq 0 & (3') \\ \text{for } t \in \{1,2...n\} & \Sigma_{t'=t}^{t'=n} v_S(y_S^{t'}) \geq 0 & (4') \\ & \Sigma_{t'=0}^{t'=n} v_S(y_S^{t'}) < 0 & (5') \end{array}$$

It may be useful to develop some intuition about the solution of system Φ_1^n to follow the mathematical construction. Start with some x_L so that $v_L(x_L) > 0$.

Condition (1') generates a strictly increasing sequence $(x_S^t)_{t=t'}^{t=n}$ backwards from an initial $x_S^n = x_S^{\max}$. The expression $\Sigma_{t=t'}^{t=n} v_S(x_S^t)$ is bell shaped and there is a $t' = t^*$ such that condition (2') almost holds (the value of n has to be scaled down to $n - t^*$). A small change in x_L results in a big change in $\Sigma_{t=1}^{t=n} v_S(x_S^t)$ so that condition (2') may be fixed. Condition (4') will hold by construction but condition (5') may not. If it does, theorem 4 below proves that the solution is unique. If it does not, Φ_1^n has no solution. In this construction, the smaller $v_L(y_L)$, the closer y_L to x_L^{ac}, the larger the value of n and the greater $C_S(y_L)$. This gives theorem 5. If Φ_1^n has a solution whatever the value of n, as n goes to infinity, the limit of $v_L(y_L)$ goes to zero. Suppose this is the case and suppose that $C_L(x) = 1 - x$, then $y_S^{t+1} - y_S^t = v_L(y_L)$ so that conditions (4') and (5') may be seen as $\int_{y_S^0}^{y_S^n} v_S(x)dx$ and $\int_{y_S^1}^{y_S^n} v_S(x)dx$ and so y_S^0 and y_S^1 converges to x^* solution of $\int_{x^*}^1 v_S(x)dx = 0$. This condition is restated in general terms in theorem 6.

A technical difficulty arises regarding the fact that the limit solution of Φ_δ^n may not be the solution of Φ_1^n. Theorem 4 proves that if Φ_1^n has a solution it is necessarily unique. The same arguments would apply to Φ_δ^n when δ is close to 1. Because Φ_δ^n is continuous in δ, its unique solution when δ goes to 1 converges to the unique solution of Φ_1^n. The properties of the solution of Φ_1^n may then be used to infer the properties of the solution of Φ_δ^n.

To avoid ambiguity, the respective solutions of Φ_δ^n and Φ_1^n, when they exist, are denoted as $y_L(n,\delta)$ and $(y_S^t(n,\delta))_{t=0}^{t=n}$, on the one hand, and $y_L(n)$ and $(y_S^t(n))_{t=0}^{t=n}$, on the other hand.

Theorem 4 *Φ_1^n admits at most one solution for large enough n.*

Proof of this theorem is in the appendix.

Theorem 5 *If there exists a solution respectively in Φ_δ^m and in Φ_δ^n with $m > n$ then $y_L(m,\delta) \leq y_L(n,\delta)$.*

Proof. Suppose $y_L(m,\delta) > y_L(n,\delta)$ then $v_L(y_L(m,\delta)) > v_L(y_L(n,\delta))$. Since C_L is strictly decreasing this implies for all $t \in \{0, 1, 2...n\}$:

$$y_S^{m-t}(m,\delta) < y_S^{n-t}(n,\delta)$$

so that

$$\Sigma_{t=n}^{t=0} \delta^{n-t} v_S(y_S^{m-t}(m,\delta)) < \Sigma_{t=n}^{t=0} \delta^{n-t} v_S(y_S^{n-t}(n,\delta)).$$

For $t \in \{n+1, ..m\}$ we still have $y_S^{m-t}(m,\delta) < y_S^0(n,\delta)$ and, because of (5) we also certainly have $v_S(y_S^0(n,\delta)) < 0$ then

$$\Sigma_{t=m-1}^{t=0} \delta^{m-1-t} v_S(y_S^{m-t}(m,\delta)) \leq \Sigma_{t=n}^{t=0} \delta^{n-t} v_S(y_S^{m-t}(m,\delta)).$$

Then

$$\Sigma_{t=m-1}^{t=0} \delta^{m-1-t} v_S(y_S^{m-t}(m,\delta)) < \Sigma_{t=n}^{t=0} \delta^{n-t} v_S(y_S^{n-t}(n,\delta)).$$

By construction the left hand side should be greater or equal to zero while the right hand side should be strictly negative thus a contradiction. ■

Theorem 6 *If $\delta < 1, \exists n_\delta$ such that $\forall n > n_\delta$ the system Φ_δ^n has no solution. If $\delta = 1$ and if the system Φ_1^n has a solution for all values of n then:*

$$\lim_{n\to\infty} v_L(y_L(n)) = 0,$$

$$\lim_{n\to\infty} \Sigma_1^n v_S(y_S^t(n)) = C_S(x_L^{ac}),$$

$$\lim_{n\to\infty} y_S^1(n) = x^*,$$

in which x^* is uniquely defined as:

$$\int_{x^*}^{x_s^{\max}} v_S(x)\frac{dC_L}{dx}(x)dx = 0.$$

Proof of this theorem is in the appendix.

Theorem 7 *If there is a solution in Φ_1^n for arbitrarily large values of n it is necessary that:*

$$v_S(x^*) + C_S(x_L^{ac}) \leq 0.$$

If

$$v_S(x^*) + C_S(x_L^{ac}) < 0$$

there is a solution in Φ_1^n for arbitrarily large values of n.

Proof. Consider the first part. Using (5'), for all n we have $v_S(y_S^0(n)) + C_S(y_L(n)) < 0$ so that at the limit we certainly have $v_S(x^*) + C_S(x_L^{ac}) \leq 0$.

As for the second part, theorem 5 proves in fact that in the construction of theorem 3 for n large enough $\hat{y}_S^1$ converges to x^* as $\hat{y}_L$ goes to x_L^{ac}. Since $v_S(x^*) + C_S(x_L^{ac}) < 0$ it must be that (5') will be satisfied and a solution is obtained. ■

Theorem 8 *Suppose $v_S(x^*) + C_S(x_L^{ac}) < 0$, then $\lim_{\delta\to 1} v_L(y_L(n_\delta, \delta)) = 0$.*

Proof. Since $v_S(x^*)+C_S(x_L^{ac}) < 0$, Φ_1^n has a solution for arbitrarily large n, this proves that $\lim_{\delta\to 1} n_\delta = \infty$. But $\lim_{n\to\infty} v_L(y_L(n)) = 0$. Since $\lim_{\delta\to 1} n_\delta = \infty$, whatever n the solutions of Φ_1^n and of Φ_δ^n exist and can be made arbitrarily close so that $\lim_{\delta\to 1} v_L(y_L(n_\delta, \delta) = 0$. ■

Definition 9 *The selection process is further refined so that* the solution *of Γ_∞^δ is taken as the $SPE(n_\delta)$ in which n_δ in the maximal n for which Φ_δ^n has a solution.*

The motivation to select this $SPE(n_\delta)$ among the other $SPE(n)$'s comes from theorem 5, which may be interpreted in two ways. Compare the strategies in $SPE(n_\delta)$ and in any $SPE(n)$. Firstly, player S maximizes his total rent over his incumbency time $(\Sigma_{t'=1}^{t'=n} v_S(y_S^{t'}(n)))$. Secondly, the long term incumbent L deters the most aggressive entry from player S. In the concluding section this particular selection is further discussed. The economic properties of $SPE(n_\delta)$ when δ goes to 1 are discussed in the next section. In particular, it is important to give an economic content to theorem 8.

6 Economic properties of the solution

The exploration proceeds in two steps. Firstly, a complete study of the special case in which the functions v and C are linear, which is easy because the solution may be derived analytically. Secondly, a discussion of these results in relationship with the literature on contestable markets.

6.1 A benchmark: the linear case

A simple linear model may be seen as a first cut in future work:

$$\begin{array}{cc} v_1(x_1) = \lambda x_1 & C_1(x_2) = 1 - x_2 \\ v_2(x_2) = \lambda x_2 - \Delta f & C_2(x_1) = 1 + \Delta f - x_1 \end{array}$$

In this setting, firm 1 is more efficient than firm 2 if and only if firm $\Delta f \geq 0$. Average cost pricing and static limit moves are respectively:

$$\begin{array}{rc} x_1^{ac} = & 0 \\ x_1^{\max} = & 1 + \Delta f \\ x_2^{ac} = & \Delta f / \lambda \\ x_2^{\max} = & 1 \end{array}$$

To satisfy assumption 3 it must be that:

$$-1 \leq \Delta f \leq \lambda$$

This game may be played either with player L as firm 1 or as firm 2. In both cases the system Φ_1^n may be solved analytically. Index by 1 or 2 its solution.

Proposition 10 *In the linear case, the selected equilibrium is such that:*
with firm 1 as player L and firm 2 as player S

$$\begin{array}{cc} \text{if } \Delta f < -1 & \text{entry of firm 1 is blockaded} \\ \text{if } -1 < \Delta f \leq (\lambda - 1)/2 & y_{L1} = x_1^{ac} = 0 \\ \text{if } (\lambda - 1)/2 < \Delta f < \lambda & y_{L1} = 1 - \lambda + 2\Delta f \\ \text{if } \lambda < \Delta f & y_{L1} = x_1^{\max} = 1 + \Delta f \end{array}$$

with firm 2 as player L and firm 1 as player S

$$\begin{array}{cc} \text{if } \Delta f < -1 & y_{L2} = x_2^{\max} = 1 \\ \text{if } -1 < \Delta f \leq \lambda(1-\lambda)/(1+\lambda^2) & y_{L1} = 1 - \lambda(1 + \Delta f) \\ \text{if } \lambda(1-\lambda)/(1+\lambda^2) < \Delta f \leq \lambda & y_{L2} = x_2^{ac} = \Delta f / \lambda \\ \text{if } \lambda < \Delta f & \text{entry of firm 2 is blockaded} \end{array}$$

Proof. Consider the first part.

By construction $\int_{x^*}^{x_2^{\max}} v_S(x) \frac{dC_L}{dx}(x) dx = \int_{x^*}^{x_2^{\max}} v_2(x) \frac{dC_1}{dx}(x) dx = \int_{x^*}^{1} (\lambda x_2 - \Delta f) dx$. It follows that $x^* = 2\Delta f / \lambda - 1$.

To obtain rent dissipation, it must be that $v_S(x^*) + C_S(x_L^{ac}) = v_2(x^*) + C_2(x_1^{ac}) < 0$. That is, $\Delta f < (\lambda - 1)/2$.

If there is no rent dissipation, using (1') one gets $y_S^t = 1 - (n-t)\lambda y_L$, so that $\Sigma_{t=1}^{t=n} y_S^t = n - \lambda y_L n(n-1)/2$. Condition (2') writes $-C_S(y_L) + \Sigma_{t=1}^{t=n} v_S(y_S^t) = 0$ which gives $-(1 - y_L) - \Delta f - n\Delta f + \lambda n - \lambda^2 y_L n(n-1)/2 = 0$.

The maximal duration n_δ when $\delta \to 1$ is obtained using (5'). Denote $n_1 = \lim_{\delta\to 1} n_\delta$. Assuming away integer problems, n_1 solves $\Sigma_{t=0}^{t=n_1} v_S(y_S^t) = 0$. But $\Sigma_{t=0}^{t=n} v_S(y_S^t) = -(n+1)\Delta f + \lambda(n+1) - \lambda^2 y_L n(n+1)/2$, so that $n_1 = 2(\lambda - \Delta f)/\lambda^2 y_L$.

Substituting n by $2(\lambda - \Delta f)/\lambda^2 y_L$ in the expression of y_L gets $y_L = 1 - \lambda + 2\Delta f$.

Hint : first use the fact that $\Sigma_{t=0}^{t=n} v_S(y_S^t) = 0$ to eliminate Δf in
$-(1 - y_L) - \Delta f - n\Delta f + \lambda n - \lambda^2 y_L n(n-1)/2 = 0$
to get $1 + \lambda = y_L(1 + \lambda^2 n)$ and then substitute n by $2(\lambda - \Delta f)/\lambda^2 y_L$.

The proof of the second part is obtained through similar calculations

Corollary 11 *The incumbency rents increases as the difference in efficiency between the two firms increases.*

■

Proof. A diagram may to used to visualize this result (see figure 1). It depicts y_{L1} and y_{L2} as a function of Δf. The respective average costs are also drawn so that the incumbency rent may be read graphically as $\lambda(y_{Li} - x_i^{ac})$. The diagram is drawn for $\lambda = 1/2$. The incumbency rents are of course identical for $\Delta f = 0$. Then $\lambda(y_{L1} - x_1^{ac})$ is seen to increase with Δf while $\lambda(y_{L2} - x_2^{ac})$ is seen to decrease. Note also the continuity of y_{L1} and y_{L2} as entry becomes blockaded. It is easily seen that this remains true for all values of λ. ■

ftbpFU327.25pt245.9375pt0ptThe incumbency rents in the linear caseFigure

The impact of productive efficiency can be analyzed. Start with $\lambda = 1$, $y_{L2}(-\Delta f) = y_{L1}(\Delta f) = 2\Delta f$. An innovation that gives a competitive advantage in average cost of Δf generates an instantaneous rent not of Δf but of $2\Delta f$! The factor 2 may be seen as a "Shumpeterian multiplier". This multiplier varies with λ, for instance it increases as the switching cost increases (to increase s amounts to decrease λ). This is to be contrasted with the MT approach in which there is no rent what ever the competitive advantage.

A taxonomy of natural monopoly situations is introduced to discuss further how productive efficiency works in this model.

Definition 12 *A natural monopoly is said to be a situation of:*

- under-competition *if a less efficient incumbent can deter entry forever and make stationary positive profits;*
- selection *if a less efficient incumbent is not able to deter entry forever and make stationary positive profits, but a more efficient incumbent may;*
- excess-competition *if a more efficient incumbent were to choose to deter entry forever, it would have to dissipate all of its profits.*

These results of proposition 10 may be combined to depict the taxonomy in a single diagram (figure 2).

ftbpFU363.375pt273pt0ptThe taxonomy of natural monopolyFigure 2

6.2 Discussion

As will be shown shortly, examples such as the one introduced in section 2 are suggestive of $\lambda \leq 1$. It corresponds to tough price competition.

To be a little bit more general than in section 2, let $R(p)$ be the revenue function with $R' > 0$ (at least in the relevant range for p) and $R'' < 0$.The fixed cost is denoted f. The symmetric entry game is thus defined with:

$$v(p) = R(p) - f$$
$$C(p) = -(R(p-s) - f\ )$$

Proposition 13 *In the simple price competition game, there is no rent dissipation if the switching cost s is small enough but there is as the switching cost s goes to zero.*

Proof. For small s, we certainly have $p^{\max}$ close to p^{ac}. Linear approximations of the v and C functions may be used. According to proposition 10 the ratio $v(p^{\max})/C(p^{ac})$ relative to 1 characterizes the situation. Write $v(p^{\max})/C(p^{ac}) = -(v(p^{\max}) - v(p^{ac})/(p^{\max} - p^{ac}))/(C(p^{ac}) - C(p^{\max}))/(p^{ac} - p^{\max}))$ so that $v(p^{\max})/C(p^{ac})$ is close to $-v'(p^{ac})/C'(p^{ac}) = R'(p^{ac})/R'(p^{ac} - \varepsilon) < 1$ since $R'' < 0$. ■

To obtain illustrations suggestive of $\lambda \geq 1$, entry should not be so tough, it should generate an (unprofitable) duopoly at that stage. The Eaton and Lipsey model of durable capital as an entry barrier (1980) provides such an illustration (see Gromb et al., 1997, section 4.3 for more details).

With these illustrations in mind, come back to the literature on contestable markets. The standard game theoretic idea models contestable markets through short term commitments. Rent dissipation prevails as the discount factor goes to 1 (section 2.2). With this new approach it is not enough, it must be that the switching cost, which provides an incumbency advantage, also goes to zero. This view is in line with Weizman (1983): if contestable markets mean average cost pricing, at the limit case, they correspond to a frictionless situation.

But this model also allows a discussion which is in line with critics according to whom the intensity of competition in case of entry should play a role in the argument. This point is seen as crucial by Dasgupta and Stiglitz (1986), its impact has been formalized in a two stage game in Henry (1988), Sutton (p. 35, 1991) provided ample empirical evidence that "a very sharp fall in price suffices to deter entry and maintain a monopoly outcome". This corresponds to the fact that it makes a clear difference whether $\lambda < 1$ or $\lambda > 1$. In the first case, competition is tough and there is no rent dissipation, in the second case, competition is soft and rent dissipation prevails.

Consider now the productive efficiency issue. In this model, productive efficiency is favored either directly through selection or at least through the fact that, if the less efficient firm may remain as a permanent incumbent, its incumbency rent is lower than the one of the more efficient one. The competitive process works well for large differences in efficiency. Otherwise, in the zone of undercompetition, firms should be encouraged to trade assets rather than engaged into price competition. In the zone excess competition, firms should only invest on significant breakthroughs.

It seems worthwhile to carry on this discussion, not through the simple linear case, but through a direct analysis of more substantial economic models. This will be done in a companion paper.

7 Concluding comments

The approach proposed in this paper requires some further theoretical work. In this last section some open questions are pointed out.

Question 1: The results should be extended to other game forms. The extension to the Stackelberg model with endogenous leadership introduced in Gromb, Ponssard and Sevy (1997) is straightforward, the interested reader will note that assumption 2 in this model and assumption D in GPS may be dropped, as long as for $i \in \{1, 2\}$, $x_i^{ac} \leq x_i^{\max}$ is preserved to make the game interesting. The extension to the alternate move model used by Maskin and Tirole (1988) requires more work.

Question 2: A more formal selection process would certainly be helpful. A starting point may be in extending the forward induction approach introduced in Ponssard (1991) to the case of infinitely repeated games.

Question 3: The fact that a player plays a stationary strategy when he is active gives him a tremendous advantage. This explains why a weak player may stay as a permanent incumbent with the MT approach. This advantage is not completely wiped out in our approach (player L enjoys it). It would be interesting to define an approach in which it is. An approach which might be worth investigating would be one in which there is a preliminary stage where each player decides how long he could stay, say n_1 for player 1 and n_2 for player 2. The choices n_1 and n_2 are then revealed and a repeated game $\Gamma_\infty^{n_1,n_2}$ is played in which each player can only use SPE strategies according to the number of stages announced at the preliminary stage. It is suspected that the equivalent of theorem 6 holds (*i.e.*, given n_i the best response $n_j(n_i)$ is the highest n_j for which the entry game $\Gamma_\infty^{n_1,n_2}$ has an equilibrium). If this were indeed the case, the taxonomy of competitive situations introduced in section 6 would lie on a stronger ground namely:

- *selection*: only the strong player would select to stay infinitely (the limit equilibria in $\Gamma_\infty^{n_1,n_2}$ with respect to large values of (n_1, n_2) would have $n_1^* = \infty, n_2^* < \infty$, where player 1 is the strong player) ;

- *under-competition*: either player could select to stay infinitely but if one does, the other would not wish to, the preliminary game would be similar to a battle of the sexes game (there would be two limit equilibria $\Gamma_\infty^{n_1,n_2}$ with $n_1^* = \infty, n_2^* < \infty$ and $n_1^* < \infty, n_2^* = \infty$) ;

- *excess-competition*: either player would select to stay infinitely whatever the other one does, the preliminary game would be similar to a prisoner dilemma game (formally $\Gamma_\infty^{n_1,n_2}$ would have no limit equilibrium with respect to large values of (n_1, n_2), the best response $n_1(n_2)$ being ∞ and vice versa, while both equilibrium payoffs in a game $\Gamma_{\infty}^{n_1,n_2}$ would decrease as (n_1, n_2) increases).

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9 Appendix : Proofs

Proof. [2] (Theorem 4) The proof runs as follows. Firstly prove that conditions (1'-2'-3'-4') of Φ_1^n have a unique solution. Secondly, check whether condition (5') is satisfied: if it is, the unique solution of Φ_1^n is obtained; if it is not, Φ_1^n has no solution.

To prove the first part, for all $x_L \in [x_L^{ac}, x_L^{\max}]$, define the function $W(x_L) = C_S(x_L) - \Sigma_1^n v_S(x_S^t)$ in which the sequence (x_S^t) is derived from x_L through (1') that is,

$$-C_L(x_S^t) + (n-t)v_L(x_L) = 0 \qquad \text{for } t \in \{0, 1, 2...n\}$$

then, show that $W(x_L)$ is negative (step 1) then positive (step 2) and that its derivative is strictly positive (step 3) so that there is a unique solution to the equation $W(x_L) = 0$.

Step 1: if $x_L = x_L^{ac}$ then $W(x_L) < 0$

In that case $x_S^t = C_L^{-1}(0)$ for all t so that $W(x_L^{ac}) = C_S(x_L^{ac}) - nv_S(x_S^{\max})$ by assumption $v_S(x_S^{\max}) > 0$ so that for n large enough $W(x_L^{ac}) < 0$.

Step 2: if $x_L = x_L^{\max}$ then $W(x_L) > 0$

Since C_L is strictly decreasing, the sequence (x_S^t) is a strictly increasing sequence bounded by $x_S^{\max}$. Since v_S is strictly increasing this implies that $\Sigma_1^n v_S(x_S^t)$ is certainly negative for n large enough so that $W(x_L^{\max}) = -\Sigma_1^n v_S(x_S^t)$ is certainly positive.

Step 3: $\frac{dW}{dx_L} > 0$

We have

$$\frac{dW}{dx_L} = \frac{dC_S}{dx_L} - \Sigma_{t=1}^{t=n}(\frac{dv_S}{dx_S^t} \cdot \frac{dx_S^t}{dx_L})$$

Using (1') we get:

$$\frac{dx_S^t}{dx_L} = (n-t)\frac{dv_L}{dx_L} / \frac{dC_L}{dx_S^t}$$

[2] I am indebted to Rida Laraki for providing the argument for this proof.

By substitution it follows that:

$$\frac{dW}{dx_L} = \frac{dC_S}{dx_L} - \frac{dv_L}{dx_L}\Sigma_{t=1}^{t=n}((n-t)\frac{dv_S}{dx_S^t}/\frac{dC_L}{dx_S^t})$$

By assumption $-\frac{dv_S}{dx_S^t}/\frac{dC_L}{dx_S^t}$ is uniformly bounded away from zero by ε so that

$$\frac{dW}{dx_L} \geq \frac{dC_S}{dx_L} + \frac{dv_L}{dx_L}\frac{n(n-1)}{2}\varepsilon$$

Since $\frac{dv_L}{dx_L}$ is bounded away from zero and since $\frac{dC_S}{dx_L}$ is bounded away from $-\infty$ we certainly have $\frac{dW}{dx_L} > 0$ for n large enough.

Hence for a given n large enough there is a unique solution to $W(x_L) = 0$ that is, to (2'). This solution is in $]x_L^{ac}, x_L^{\max}[$ so that (3') is also satisfied. Denote $\hat{y}_L$ this solution and $(\hat{y}_S^t)$ for $t \in \{0, 1, 2...n\}$ the associated sequence obtained through (1'). Observe that (4') is satisfied as well : since v_S is increasing the function $\Sigma_{t'=t}^{t'=n} v_S(\hat{y}_S^{t'})$ is bell shaped with respect to t so for all t we have:

$$\Sigma_{t'=t}^{t'=n} v_S(\hat{y}_S^{t'}) \geq Min(\Sigma_1^n v_S(\hat{y}_S^t), v_S(\hat{y}_S^n)) = Min(C_S(\hat{y}_L), v_S(x_S^{\max}) > 0$$

because $\hat{y}_L < x_L^{\max}$ implies $C_S(\hat{y}_L) > 0$ and $v(x_L^{\max}) > 0$ by construction.

It is now a simple matter to check whether (5') holds or not. If it does a complete solution to Φ_1^n is obtained, if it does not there cannot be a solution for that value of n since conditions (1') through (4') have a unique solution. ■

Proof. (Theorem 6)

Suppose $v_L(y_L(n,\delta)) \geq \varepsilon > 0$ for all n, then using (1) the sequence $(y_S^t(n,\delta))_{t=0}^{t=n}$ is a strictly increasing sequence defined backwards from $y_S^n(n,\delta) = x_S^{\max}$. Consequently, for an arbitrarily large number of items in this sequence $v_S(y_S^t(n,\delta)) < 0$ whereas for a finite number of them, $v_S(y_S^t(n,\delta)) \geq 0$. It follows that (4) cannot hold. Hence either there cannot be a solution for arbitrarily large n or, if there is one, $\lim_{n\to\infty} v_L(y_L(n,\delta)) = 0$.

Suppose $\lim_{n\to\infty} v_L(y_L(n,\delta)) = 0$, it is now proved that, if $\delta < 1$, we have a contradiction. Indeed, since $v_L(y_L(n,\delta))$ is close to zero, using (1) it is seen that the whole sequence $(y_S^t(n,\delta))_{t=0}^{t=n}$ can be made arbitrarily close to $x_S^{\max}$. By assumption $x_S^{ac} < x_S^{\max}$, so that (5) cannot hold. This completes the first part of the theorem. Furthermore we proved that if Φ_1^n has a solution for arbitrarily large n, $\lim_{n\to\infty} v_L(y_L(n) = 0$.

Combining this result with (2') we get $\lim_{n\to\infty} \Sigma_1^n v_S(y_S^t(n)) = C_S(x_L^{ac})$.

Consider now the last point. First of all, given that $\frac{dC_L}{dx}$ is bounded away from infinity and from zero and that $v_S(x)$ is bounded away from zero, there exists a unique $x^* < x_S^{ac}$ such that

$$\int_{x^*}^{x_S^{ac}} v_S(x)\frac{dC_L}{dx}(x)dx = 0$$

For all $x \leq x_S^{ac}$ define $F(x) = \int_x^{x_S^{ac}} v_S(u) \frac{dC_L}{du}(u)du$, the function F is such that $F(x) > 0$ iff $x < x^*$.

We now show the convergence of $y_S^1(n)$ to x^*.

Using (1') and (2') we get :

$$\begin{aligned} C_S(y_L(n))v_L(y_L(n)) &= \Sigma_{t=1}^{t=n} v_S(y_S^t(n))v_L(y_L(n)) \\ &= \Sigma_{t=1}^{t=n} v_S(y_S^t(n)) \left[C_L\left(y_S^{t-1}(n)\right) - C_L\left(\ y_S^t(n)\right)\right] \end{aligned}$$

When $v_L(y_L(n)$ is small this non negative expression is close to $F(y_S^1(n))$.

To see this, make the change of variable from x_S to $u = C_L(x_S)$. As t goes from 1 to n, x_S increases from $y_S^1(n)$ to $y_S^n(n)$ and u from $u^1(n) = C_L\left(y_S^1(n)\right)$ to $u^n(n) = C_L(y_S^n(n)) = 0$ but $u^{t-1}(n) - u^t(n)$ remains t independent and equals $v_L(y_L(n))$, let $\Delta u(n) = v_L(y_L(n))$.

We may then write

$$v_L(y_L(n))\Sigma_{t=1}^{t=n} v_S(y_S^t(n)) = \Sigma_{t=1}^{t=n} v_S(C_L^{-1}(u^t(n)))\Delta u(n)$$

For large values of n we have

$$\Sigma_{t=1}^{t=n} v_S(C_L^{-1}(u^t(n)))\Delta u(n) \approx \int_{u^1(n)}^{0} v_S(C_L^{-1}(u))du = \int_{y_S^1(n)}^{x_S^{ac}} v_S(x)\frac{dC_L}{dx}(x)dx.$$

This proves that $y_S^1(n)$ cannot be far below x^*. Using (2') and (5') for the two sequences n and $n+1$, it is clear that $y_S^1(n+1)$ and $y_S^1(n)$ cannot be far apart either. More precisely:

$$\left|y_S^1(n+1) - y_S^1(n)\right| \leq -Min(\frac{dC_L}{dx}(y_S^1(n)), \frac{dC_L}{dx}(y_S^0(n)))v_L(y_L(n))$$

Since $\frac{dC_L}{dx}$ is bounded away from infinity, $\lim_{n\to\infty} \left|y_S^1(n+1) - y_S^1(n)\right| = 0$, this is enough to prove that $y_S^1(n)$ converges to some limit and this limit can only be x^* since

$\lim_{n\to\infty} C_S(y_L(n))v_L(y_L(n)) = \lim_{n\to\infty} C_S(y_L(n)) \lim_{n\to\infty} v_L(y_L(n)) = C_S(0).0 = 0.$ ■