Active firms in horizontal mergers and cartel stability *

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Abstract. In this paper, we study the optimal number of active firms in a coalition and in a merger. We consider two kinds of game : a merger game and a coalition game, both in the context of price competition with horizontal product differentiation. These are two-stage games. The first stage consists of determining the number of active firms; the second stage is price competition between active firms. Firms belonging to the same owner or to the same coalition play cooperatively between themselves but face competition between other firms.

We show that when there is no competitive pressure (i.e. no outside firm) then only merged equilibria can occur in the merger case. In the coalition case we obtain a similar result in which the number of active firms in the second stage is less than the initial number of firms.

Moreover we show that if competitive pressure is high enough then the initial number of firms in the industry is the same as the number of active firms in the last stage for each kind of game.

Keywords: Mergers, Coalitions, Product differentiation

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1 Introduction

In their seminal paper (1983), Salant, Switzer and Reynolds (SSR) [8] have shown that a merger in a standard Cournot framework with linear demand and linear costs is not profitable unless a large majority of the relevant firms are involved in the merger (80 %). This is so because outsiders benefit more than the firms participating in the merger, the "insiders". Since production costs are linear, any coalition of firms is indifferent to how total production is divided between the members of the coalition, so that every coalition of firms behaves as if it were a single firm.

Perry and Porter (1985) [7] but also Farrell and Shapiro (1990) [3] have challenged the view that a merged firm is no larger than any of the constituent firms. These papers introduce the existence of some crucial assets that are in limited supply in order to capture the notion that some firms are larger than others in a homogeneous product industry. This assumption implies rising marginal cost of output production and, consequently, internal cost savings from mergers could make a merger profitable.

Deneckere and Davidson (1985) [2] have found an opposite result in the case of price competition with differentiated products so that a merger is always beneficial for the insiders.

Kamien and Zang (1990) [6] have explored the possibility of endogenous monopolization of a homogeneous product Cournot oligopoly through one firm's acquisition of the others. They adopt two different approaches; first, an analysis of a centralized game : an owner who has acquired several firms behaves as one entity (as in SSR, 1983). Secondly, they explore the possibility for an owner, possessing several firms, to choose the optimal number of active firms, each of them competing between themselves, this being a decentralized game. More precisely, in this kind of game, they emphasize that an owner, possessing several firms, chooses to operate more than one firm. They show that, for the two kinds of game, monopolization can only occur in industries composed ex-ante of a small number of firms. Moreover, in the centralized game (SSR context (1983)) with a large number of firms, merged equilibria (that means the number of active firms is fewer than the initial number of firms) are non-existent. This confirms and strengthens the SSR's results (1983).

More recent literature takes into account strategic delegation (Gonzalez-Maestre and Lopez-Cunat (2001) [4] or Ziss (2001) [9]) to study merger profitability. What differs from the decentralized game in Kamien and Zang

(1990) [6] is the two types of competition : in production and also in the remuneration of managers. Delegation increases competition between entities inside the firm. Consequently, the incentives to merge and the profitability of merger, under delegation, are considerably increased in stark contrast to the position where there is no delegation. Ziss (2001) [9] argues that a merger will result in the merged entity operating only one firm.

Commitment through delegation may be limited by the possible renegotiation of the delegation contract in the absence of a strong enforcing institutional setting avoiding false disclosure and private renegotiation. Precommitment effects seem to rest on the crucial assumption that contracts, once publicly disclosed, cannot be secretly renegotiated. This is at odds with reality: whether legally enforceable or of a more implicit nature, actual contracts can almost always be renegotiated if both parties agree (Caillaud, Jullien and Picard, 1995 [1]). In the same way, in the decentralized game of Kamien and Zang (1990) [6], internal competition is not rational because if contracts were renegotiable ex-post, firms may act cooperatively. In our model, we consider that firms belonging to the same owner play in a cooperative way.

Our purpose is twofold; first, we analyse the incentives to merge in the context of price competition with horizontal product differentiation. We suppose a two stage game. In the first stage, an owner possessing several firms chooses the number of active firms. The second stage is price competition between active firms.

The number of active firms plays a major rôle : since products are horizontally differentiated, demand increases with the number of active firms. This means that a merger can gain market share, but equilibrium price is lower. Active firms create internal competition but reinforce competition with other firms at the same time.

We show that if the market structure of the industry is duopoly then only merged equilibria can occur. This means that only equilibria in which an owner of several firms chooses to let less active firms than he owns exist. Moreover, if the competitive pressure is high enough then merged equilibria can not occur in this game.

Second, we analyse cartel stability in a static case. As previously we consider a two-stage game. In the first stage, each coalition has to decide the number of its active firms but contrary to the merger game, non active firms are not closed, they receive the same profits as active firms. In the second stage only active firms make price competition.

We show that total cartelization of the industry is an equilibrium of this game, which is in contrast with the results obtained by Kamien and Zang (1990) [6].

A major difference exists between these two games. In the merger game, one owner can possess several firms; whereas in the second game, several firms can belong to a same coalition but each firm in a coalition belongs to one owner.

This article is organized as follows. In section 2 we describe the basic model. Characterizations of equilibria are provided in section 3. Concluding remarks follow in section 4. Proofs of results appear in the appendix.

2 The model

We consider the following utility function derived from Häckner (2000) [5]:

$$U(\mathbf{q}, I) = \sum_{i=1}^{n} q_i - \frac{1}{2}\left[\sum_{i=1}^{n} q_i^2 + 2\gamma \sum_{i \neq j}(q_i q_j)\right] + I \tag{1}$$

The parameter $\gamma \in [0,1]$ is a measure of the substitutability between products. Utility is quadratic in the consumption of the n horizontally differentiated products and linear in the consumption of other goods: I, which price is normalized to one.

The demand function is given by:

$$q_i(p_i, p_j, n) = \frac{1}{1+\gamma(n-1)}\left[1 - \frac{1+\gamma(n-2)}{1-\gamma}p_i + \frac{\gamma}{1-\gamma}\sum_{j \neq i} p_j\right] \tag{2}$$

We assume that entry into the industry is difficult and that each producer operates at a constant and identical marginal and average cost (c). Without loss of generality, we assume that $c = 0$. All the relevant variables and strategies available to the firms are common knowledge.

We posit an initial industry consisting of 16 identical and independent firms.

Let us now turn to the formal description of our two games.

MERGER GAME

- *Stage 1: Number of active firms.*

  Let K_j be the number of firms owned by a merged entity M_j (firms belonging to a same owner have been previously purchased by this owner) and Z, the number of outside firms which are firms not belonging to a merged entity M_j.
  Each owner decides the number of his firms which are active.
  Let k_j ($0 \leq k_j \leq K_j$) be the number of active firms within M_j.
  A SPNE (Subgame Perfect Nash Equilibrium) in the merger game is said to be merged if the number of firms operated by all owners is fewer than the initial number of firms .

- *Stage 2: Price competition.*

  Firms belonging to the same owner act cooperatively amongst one another but face competition with each other. The active firms in the merged entities (M_j) and the outside firms compete in price.

COALITION GAME

- *Stage 1 : Number of active firms.*

  Each coalition C_j owning K_j independent firms decides the number of its active firms, denoted by k_j. Contrary to the merger stage, non active firms (they do not compete at the competition stage) are not closed. For example those firms can receive an allowance from the active firms. We assume that firms, active or passive, receive the same individual profit which is the total profit of the coalition shared in a equal way between the members of the coalition. Also, let Z be the number of firms not belonging to a coalition.

- *Stage 2 : Price competition.*

  As in the merger case, active firms in coalition and the outside firms compete in price. Firms belonging to the same coalition act cooperatively amongst one another but face competition with each other.

We then characterize pure strategy SPNE of these two games.

3 Analysis of equilibria

This section characterizes the set of equilibria depending on the kind of game in question : merger or coalition.

3.1 Merger case

We analyse successively the case in which two owners have previously bought some firms : the owners M_1 and M_2, owning respectively K_1 and K_2 firms, and after the situation where only one owner has previously bought firms.

3.1.1 Two mergers

In this section, we consider the case in which two owners, denoted by M_1 and M_2 own K_1 and K_2 firms ($K_i \geq 1, \forall i = 1, 2$). Each of them operates k_1 and k_2 units respectively. There are Z outside firms.
M_1, M_2 and the outside firms simultaneously choose the product price for each of their firm whilst seeking to maximize their profit.

lemma 1. *Equilibrium prices of the two mergers (p_1^* and p_2^*) and the outsiders (p^*) is given by the following three functions :*

$$\begin{cases} p_1^* = \frac{(1-\gamma)(2+2(Z-1)\gamma+2\gamma k_1+\gamma k_2)(2+(2Z-3)\gamma+2\gamma(k_1+k_2))}{A} \\ p_2^* = \frac{(1-\gamma)(2+(2Z-3)\gamma+2\gamma(k_1+k_2))(2+2(Z-1)\gamma+\gamma(k_1+2k_2))}{A} \\ p^* = \frac{(1-\gamma)(2+2(Z-1)\gamma+2\gamma k_1+\gamma k_2)(2+2(Z-1)\gamma+\gamma(k_1+2k_2))}{A} \end{cases}$$

with

$A = 2\gamma^2 k_1^2(4 - 4\gamma + 3Z\gamma + 3\gamma k_2) + 2(1 + (Z-1)\gamma + \gamma k_2)(2(2 + (Z-3)\gamma)(1 + (Z-1)\gamma) + \gamma(4 - 4\gamma + 3Z\gamma)k_2) + \gamma k_1(2(8 + 5(Z-2)\gamma)(1 + (Z-1)\gamma) + \gamma k_2(22 - 25\gamma + 17Z\gamma + 6\gamma k_2))$.

We check that $A > 0$.

Proof. See Appendix A. □

Equilibrium profit of the merged entity M_1 is given by:

$$\pi^{M_1} = \frac{1}{(1 + \gamma(k_1 + k_2 + Z - 1))A^2}(1-\gamma)k_1(1 + (Z-1)\gamma + \gamma k_2)$$
$$(2 + 2(Z-1)\gamma + 2\gamma k_1 + \gamma k_2)^2(2 + (2Z-3)\gamma + 2\gamma(k_1 + k_2))^2 \quad (3)$$

The expression for merger M_2 is symmetrical.

We now determine if an owner of several firms will choose to close some of them or to keep all of them active.

proposition 1.

- *If $Z \geq 2$ then merged equilibria can not occur in this game.*
- *If $Z < 2$ the game can result in merged equilibria depending on product substitutability. Merged equilibria can occur when there is little product differentiation.*

Proof. See Appendix B. □

The existence of merged equilibria depends on the number of outsiders. Precisely, the presence of outsiders increases competitive pressure, so that when the number of outsiders is high enough ($Z \geq 2$), an owner of several firms will not close some of them to maintain market power.

In the remainder of this paper, we assume $\gamma = 0.9$ in order to consider all the different cases (merged or unmerged equilibria).

The objective now is to analyse if the number of firms owned by each owner influences the number of their active firms.

The reaction function of the merger M_1 is defined as $k_1^*(k_2)$.

lemma 2. *The reaction function of the merger M_1 is given by :*

$$\boldsymbol{k_1^*(k_2, Z)} = \begin{cases} K_1 \;\; if \;\; Z > 1, \;\; \forall k_2 \\ f(k_2) < K_1 \;\; if \;\; Z = 1 \;\; and \;\; k_2 \geq 8 \\ K_1 \;\; if \;\; Z = 1 \;\; and \;\; k_2 < 8 \\ g(k_2) \;\; if \;\; Z = 0 \end{cases}$$

Proof of this is obtained by numerical simulation. Appendix C discloses exact values of functions f and g as well as values of profit functions.

The reaction function of the merger K_2 is a symmetrical function of $k_1^*(k_2)$.

We observe that $k_1^*(k_2)$ is a decreasing function, so k_1 and k_2 are strategic substitutes.

lemma 3.

- *If there is not competitive pressure ($Z = 0$), then the owner of several firms will choose to close some of its firms if the other owner keeps all his firms active.*
- *It competitive pressure is strong enough ($Z > 1$) then neither of the two owners will close firms.*
- *If there is only one outside firm, one owner will choose to close some of his firms only if the other owner lets all his firms continue to be active.*

proposition 2. *If the market structure of the industry is a duopoly then only merged equilibria can occur.*

3.1.2 One merger

In this section, we interject a second course in which only the owner M_1 has previously bought some firms and can close some of his firms. The game is solved as previously.

We obtain the following proposition :

proposition 3. *If the market structure of the industry is complete or one of partial monopolization then no merged equilibria can occur.*

So whatever the number of outsiders is, only unmerged equilibria occur.

3.2 Coalition case

In this section, we analyse the equilibria of the game considering successively the case of two and one coalitions.

3.2.1 Two coalitions

We create two coalitions in this game and we test for stability. The two coalitions and the outsiders choose their price simultaneously in order to maximize their profit. This is exactly the same as in the merger case. It follows that Lemma 1, Proposition 1, Lemma 2 and Lemma 3 are satisified for the coalition game as well.

Now we have to determine equilibria structures for the industry in question. For this we test for internal and external stability for each possible market structure.

Recall that external stability requires that firms inside do not find it desirable to exit and internal stability that firms outside do not find it desirable to enter.

proposition 4. *Only two coalition structures are stable :*
$K_1^* = 7, K_2^* = 7, Z = 2$[1]
$\{K_1 > 0, K_2 > 0\}/K_1^* + K_2^* = 16$

Proof. See Appendix D. □

The second structure corresponds to the case of maximal concentration in the industry which is in contrast with the results obtained by Kamien and Zang (1990) [6]. In this case ($Z = 0$), one of the two coalitions will allow all its firms to continue to be active whereas the other will allow fewer firms that he owns to remain active. [2]

proposition 5. *Maximal concentration in the industry is an equilibrium of this game and corresponds to the case of one coalition which allows all its firms to be active whereas the other not.*

3.2.2 One coalition

In this section we analyse the case in which only one coalition is present in the industry. The game is solved as previously.

lemma 4. *Equilibrium prices of the coalition (p_1^*) and the outsiders (p^*) are given by :*

$$\begin{cases} p_1^* = \frac{(1-\gamma)(2+2(Z-1)\gamma+2\gamma k_1)(2+(2Z-3)\gamma+2\gamma k_1)}{A'} \\ p^* = \frac{(1-\gamma)(2+2(Z-1)\gamma+2\gamma k_1)(2+2(Z-1)\gamma+\gamma k_1)}{A'} \end{cases}$$

with

$A' = 2\gamma^2 k_1^2(4-4\gamma+3Z\gamma) + 2(1+(Z-1)\gamma)(2(2+(Z-3)\gamma)(1+(Z-1)\gamma)) + \gamma k_1(2(8+5(Z-2)\gamma)(1+(Z-1)\gamma))$. [3]

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[1] In this case $\pi^{out}(7,7,2) > \frac{\pi^{C_1}}{7}(K_1 = 7, K_2 = 7, Z = 2) = \frac{\pi^{C_2}}{7}(K_1 = 7, K_2 = 7, Z = 2)$ so coalition is not profitable for the insiders.

[2] Proof : See lemma 2

[3] We check that $A' > 0$.

The coalition will allow all its firms to be active whatever the number of outsiders (proof is obtained by setting $k_2 = 0$ in lemma 2).

proposition 6. *There are two equilibria in this game :*
$K_1^* = 16, Z = 0$
$K_1^* = 6, Z = 10$

So maximum concentration is again an equilibrium of this game (all the firms belong to the coalition).

4 Concluding remarks

In this paper, we have studied the optimal number of active firms in a coalition and a merger. We have considered two kinds of game : a merger game and a coalition game, both in the context of price competition with horizontal product differentiation. These are two-stage games. The first stage consists of determining the number of active firms. The second stage is price competition between active firms. We assume that firms belonging to the same owner or to the same coalition play cooperatively between themselves but face competition between each other firm.

We show that when there is no competitive pressure (i.e. no outside firm) then only merged equilibria can occur in the merger case. In the coalition case we obtain a similar result in which the number of active firms in the second stage is less than the initial number of firms.

Moreover, we show that when the competitive pressure is high enough ($Z \geq 2$) then the initial number of firms in the industry is the same as the number of active firms in the last stage for each kind of game.

Appendix A : Proof of lemma 1

Firms belonging to the same owner play in a cooperative way so the maximization program of the owner M_1 with k_1 active firms is :

$$\max_{p_1^{M_1}, p_2^{M_1}, \dots, p_{k_1}^{M_1}} (\pi^{M_1}) \tag{4}$$

where

$$\pi^{M_1} = \sum_{i=1}^{k_1} (p_i^{M_1} - c) \frac{1}{1 + \gamma(k_1 + k_2 + Z - 1)} \times \tag{5}$$

$$\left[1 - \frac{1 + \gamma(k_1 + k_2 + Z - 2)}{1 - \gamma} p_i^{M_1} + \frac{\gamma}{1 - \gamma} \left(\sum_{i \neq j} p_j^{M_1} + \sum_{j \in M_2} p_j + \sum_{j \in out} p_j \right) \right] \tag{6}$$

We stipulate $j \in out$ the firms which are outsiders.

We obtain k_1 First Order Conditions (FOC) which are symmetrical, so $p_i^{M_1} = p^{M_1}, \forall i \in M_1$. After simplifications, we obtain the best response functions:

$$p^{M_1}(p^{-M_1}) = \frac{1 - \gamma + \gamma \sum\limits_{i \notin M_1} (p_i)}{2 \left[1 + \gamma(k_2 + Z - 1) \right]} + \frac{c}{2} \tag{7}$$

The best-reply function is symmetrical for the merger M_2.

The maximization program of an outside firm is:

$$\max_{p_i} (p_i - c) \frac{1}{1 + \gamma \left(\sum\limits_{i=1}^{2} k_i + Z - 1 \right)} * \tag{8}$$

$$\left[1 - \frac{1 + \gamma \left(\sum\limits_{i=1}^{2} k_i + Z - 2 \right)}{1 - \gamma} p_i + \frac{\gamma}{1 - \gamma} \left(\sum_{j \in M_1} p_j + \sum_{j \in M_2} p_j + \sum_{\left\{ \substack{j \neq i \\ j \in out} \right.} p_j \right) \right] \tag{9}$$

As before, prices of outsiders are equal, we then replace, p_i^* by p^{out} for all $i \notin (M_1, M_2)$. We obtain :

$$p^{out}(p^{-out}) = \frac{1-\gamma+\gamma(\sum_{j\in M_1} p_j + \sum_{j\in M_2} p_j + c[1+\gamma(\sum_{i=1}^{2} k_i + Z - 2)]}{2[1+\gamma(\sum_{i=1}^{2} k_i + Z - 2)] - \gamma(Z-1)} \tag{10}$$

In order to simplify, we replace

$$(p^{M_1}(p^{-M_1}), p^{M_2}(p^{-M_2}), p^{out}(p^{-out})) \tag{11}$$

by

$$(p_1, p_2, p). \tag{12}$$

The intersection of best response functions yields to :

$$\begin{cases} p_1^* = \frac{(1-\gamma)+\gamma(k_2p_2+Zp)}{2[1+\gamma(k_2+Z-1)]} \\ p_2^* = \frac{(1-\gamma)+\gamma(k_1p_1+Zp)}{2[1+\gamma(k_1+Z-1)]} \\ p^* = \frac{(1-\gamma)+\gamma(k_1p_1+k_2p_2)}{2[1+\gamma(k_1+k_2+Z-2)]-\gamma(Z-1)} \end{cases}$$

Appendix B : Proof of proposition 1

Numerical simulation gives the number of active firms in the merger M_2 in order having $k_1^* < K_1$.

Z=	γ =0.5 $k2 \geq$	γ =0.9 $k2 \geq$
0	11	1
1	17	8
2	23	14
3	30	20
4	36	26
5	42	33

This table can be read in this way :"for Z=3 and $\gamma = 0.9$, k_2 must be higher than 20 to π^{M_1} have an interior maximum ($k_1^* < K_1$)". Note that for γ =0.1, k_2 must be very high for merged entity M_1 have a maximum.

Appendix C : Proof of lemma 2

Table 1: Reaction functions of the two mergers for $Z = 1$

K_1	K_2	k_1	k_2	π^{M_1}	π^{M_2}
1	14	1	14	0.004226	0.024267
2	13	2	13	0.004656	0.016495
3	12	2.19	12	0.004818	0.015507
4	11	2.36	11	0.005013	0.014701
5	10	2.61	10	0.005255	0.013668
6	9	3.02	9	0.005562	0.012277
7	8	3.8	8	0.005965	0.010335
8	7	8	3.8	0.010335	0.005965
9	6	9	3.02	0.012277	0.005562
10	5	10	2.61	0.013668	0.005255
11	4	11	2.36	0.014701	0.005013
12	3	12	2.19	0.015507	0.004818
13	2	13	2	0.016495	0.004656
14	1	14	1	0.024267	0.004226

Table 2: Reaction functions of the two mergers for $Z = 0$

K_1	K_2	k_1	k_2	k_1	k_2
1	15	$\oslash$	$\oslash$	0.156969	15
2	14	2	0.247657	0.157632	14
3	13	3	0.203337	0.158403	13
4	12	4	0.186461	0.159311	12
5	11	5	0.177512	0.160394	11
6	10	6	0.171958	0.161711	10
7	9	7	0.168174	0.163346	9
8	8	8	0.165428	0.165428	8
9	7	9	0.163346	0.168174	7
10	6	10	0.161711	0.171958	6
11	5	11	0.160394	0.177512	5
12	4	12	0.159311	0.186461	4
13	3	13	0.158403	0.203337	3
14	2	14	0.157632	0.247657	2
15	1	15	0.156969	$\oslash$	$\oslash$

For $Z = 0$ and $K_i > 1, \forall i = 1, 2$, two cases are possible for each structure (K_1,K_2).

Table 3: Best-response functions ($k_1^*(K_1,Z)$)

K_1	Z=0	Z=1	Z=2	Z=3	Z=4	Z=5	Z=6	Z=7	Z=8	Z=9	Z=10	Z=11	Z=12	Z=13	Z=14
1	0.156969	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	(0.157632;2)	2	2	2	2	2	2	2	2	2	2	2	2	2	.
3	(0.158403;3)	2.19	3	3	3	3	3	3	3	3	3	3	3	.	.
4	(0.159311;4)	2.36	4	4	4	4	4	4	4	4	4	4	.	.	.
5	(0.160394;5)	2.61	5	5	5	5	5	5	5	5	5	.	.	.	.
6	(0.161711;6)	3.02	6	6	6	6	6	6	6	6	.	.	.	.	.
7	(0.163346;7)	3.8	7	7	7	7	7	7	7	.	.	.	.	.	.
8	(0.165428;8)	8	8	8	8	8	8	8	.	.	.	.	.	.	.
9	(0.168174;9)	9	9	9	9	9	9	.	.	.	.	.	.	.	.
10	(0.171958;10)	10	10	10	10	10	.	.	.	.	.	.	.	.	.
11	(0.177512;11)	11	11	11	11	.	.	.	.	.	.	.	.	.	.
12	(0.186461;12)	12	12	12	.	.	.	.	.	.	.	.	.	.	.
13	(0.203337;13)	13	13	.	.	.	.	.	.	.	.	.	.	.	.
14	(0.247657;14)	14	.	.	.	.	.	.	.	.	.	.	.	.	.
15	15	.	.	.	.	.	.	.	.	.	.	.	.	.	.

Table 4: Profit of the merger M_1 function of the number of outside firms (Z) and the number of firms owned by the merger (K_1)(where for $Z = 0$, the first number dictates the profit of the merger M_1 when $k_1^* < K_1$ and $k_2^* = K_2$ and for the second number it is the inverse.)

K_1	Z=0	Z=1	Z=2	Z=3	Z=4	Z=5	Z=6	Z=7	Z=8	Z=9	Z=10	Z=11	Z=12	Z=13	Z=14
1	241.36	42.26	22.9277	15.3169	11.4897	9.27698	7.43159	6.94726	6.30024	5.84244	5.51693	5.28856	5.13482	5.04097	4.99723
2	(242.078,1168.3)	46.56	31.0011	23.2071	18.7127	15.811	13.9909	12.6817	11.7559	11.098	10.6365	10.326	10.1364	10.0481	.
3	(242.909,1329.68)	48.18	35.5813	28.6077	24.2304	21.3159	19.3009	17.8781	16.8679	16.1599	15.6837	15.3931	15.2578	.	.
4	(243.881,1405.44)	39.3371	33.2096	29.1471	29.1471	26.3468	24.3739	22.9754	21.9966	21.3386	20.9374	20.7506	.	.	.
5	(245.034,1449.74)	52.55	43.1892	37.7761	34.0615	31.4529	29.6083	28.3193	27.4541	26.9268	26.6814	.	.	.	.
6	(246.423,1478.86)	55.62	47.6051	42.7738	39.3964	37.0162	35.3569	34.2451	33.5683	33.2535	.	.	.	.	.
7	(248.13,1499.47)	59.65	52.9569	48.6037	45.5497	43.428	42.0094	41.1473	40.7466	.	.	.	.	.	.
8	(250.277,1514.84)	103.35	59.674	55.7287	53.51	1814.50	0784.49	5664.	.	.	.	.	.	.	.
9	(253.06,1526.73)	122.77	68.3702	64.7999	62.4307	60.9978	60.336	.	.	.	.	.	.	.	.
10	(256.81,1536.22)	136.68	80.0361	76.859	74.9451	74.0599	.	.	.	.	.	.	.	.	.
11	(262.137,1543.96)	147.01	96.4303	93.7582	92.5262	.	.	.	.	.	.	.	.	.	.
12	(270.303,1550.39)	155.07	121.038	1192.03	.	.	.	.	.	.	.	.	.	.	.
13	(284.404,1555.82)	164.95	161.899	.	.	.	.	.	.	.	.	.	.	.	.
14	(314.653,1560.48)	242.67	.	.	.	.	.	.	.	.	.	.	.	.	.
15	1564.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.

Appendix D : Proof of proposition 4

A cartel is stable if it satisfies a property of both internal and external stability.

Internal stability

Internal stability implies that no cooperating firm in a coalition finds it desirable to become independant or to rejoin the other coalition. So internal stability for $C_i, \forall i = 1, 2$, is given by :

$$\frac{1}{K_i}\pi^{C_i}(K_i, K_j, Z) > \begin{cases} \pi^{out}(K_i - 1, K_j, Z + 1) \\ and \\ \frac{1}{K_j+1}\pi^{C_j}(K_i - 1, K_j + 1, Z) \end{cases}$$

External stability

External stability implies that no independent firm finds it desirable to join a cartel. So external stability is given by :

$$\pi^{out}(K_1, K_2, Z) > \begin{cases} \frac{1}{K_1+1}\pi^{C_1}(K_1 + 1, K_2, Z - 1) \\ and \\ \frac{1}{K_2+1}\pi^{C_2}(K_1, K_2 + 1, Z - 1) \end{cases}$$

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