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Competition and mergers in networks with call externalities

Concurrence et fusions entre réseaux avec externalités d'appels

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Abstract

This paper considers a model of two interconnected networks with different qualities. There are call externalities in the sense that consumers value calls they send and receive. Networks compete in two part tariffs. We show that call externalities create private incentives for each competitor to charge low access prices. This result moderates the risk of tacit collusion when competitors can freely negotiate their access charges. We also analyze the case of a merger between the two networks and give conditions under which the merger can be welfare improving.

Résumé

Cet article examine un modèle de deux réseaux de qualités différentes interconnectés. Il existe des externalités d'appels dans le sens où les consommateurs valorisent les appels qu'ils émettent et également ceux qu'ils recoivent. Les réseaux se font concurrence en tarif binôme. Nous montrons que les externalités d'appels créent des incitations privées pour chaque réseau à facturer l'accès à son réseau à un prix bas. Ce résultat modère le risque de collusion tacite lorsque les opérateurs preuvent négocier librement les prix d'accès qu'ils se facturent. Nous étudions également le cas d'une fusion entre les deux opérateurs et donnons les conditions sous lesquelles la fusion améliore le bien-être.

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1 Introduction

Analyzing telecommunications liberalization naturally leads to separate the analysis of two dynamic phases. During the transitory stage following the opening to competition, the incumbent's network represents an essential facility for new competitors. The main risk of anti-competitive practices is then concerned by the abuse of dominant position which could conduct to the new competitors' foreclosure. Access pricing in this framework has given rise to numerous papers over the last few years[1]. The telecommunication sector will enter a mature phase once competitors will have developed a sufficiently large geographic coverage allowing them to connect final users directly. This is the case in mobile telecommunications. This will arise in the fixed link network when the local loop will be effectively competitive[2]. Nevertheless, networks will continue to be interconnected in order to keep the advantages of network externalities. In this context, the main risk of anti-competitive practices will probably reside in tacit collusion. Recent literature on this issue (Armstrong (1998), Laffont, Rey, Tirole (1998a,b), Carter and Wright (1999)) shows that interconnected network providers may collude tacitly through high access charges.

New telecommunication providers will try naturally to differentiate their products in order to segment the market and to lower competition. This differentiation can have horizontal or a vertical dimension. Telecommunication services of different qualities can be offered. For example, a mobile telecommunication network can be considered of a higher quality than a fixed link network. Consumers can call and be called at any period in the day.

When consumers can adopt only one network, two kinds of calls have to be distinguished: calls between consumers of the same network ("on-net calls") and calls destined to the rival network ("off-net calls"). Off-net calls give rise to the payment of an access charge to the destined network. This distinction leads to study two regulatory questions.

First, does price discrimination between on-net and off-net calls have to be authorized by regulatory agencies ? Such price discrimination is for example in practice for calls between fixed link and mobile networks or even between same quality networks[3]. This kind of pricing's goal is to

[1] For a survey on this literature, see for example Armstrong (2002).

[2] The local loop unbundling remains very heterogeneous between countries and does not allow a perfect competition on this market today (Baranes-Gassot[2002]).

[3] This is the case for example for the *friends and family* programs.

create bandwagon effects in order for price discrimination to restore tariff-mediated network externalities despite interconnection. Indeed, the higher the number of consumers adopting a network, the higher is the probability for a call to be on-net. This reduces the total cost for consumers if on-net calls are priced lower than off-net calls. A complementary question concerns the receiver payer principle.

Second, can public authorities let competitors freely negotiate their access prices or is there a need for regulation at least in a transitory regime ? An intermediate solution consists in imposing a reciprocity principle. This rule has the advantage to be easily verifiable by regulatory agencies.

Recent literature has focussed on the possible anticompetitive use of access charges between interconnected networks. Armstrong (1998) and Laffont, Rey and Tirole (1998a) show that high access prices conduct to a 'raising each other cost effect' in a linear pricing context and when providers cannot price discriminate between on-net and off-net calls. This effect partially or totally disappears when providers can operate a price discrimination between on-net and off-net calls (Laffont et al.(1998b)) or when they compete in non-linear prices (Laffont et al. (1998a)). In those two cases, providers use one tool to compete in market shares and one tool to maximize their access revenue. The collusive power of access charges totally disappears in two part tariffs. The positive effect of higher access charges on retail profits is totally neutralized by a lower fixed fee. In a recent paper, Dessein (2002) shows that this profit neutrality result is robust to the introduction of customer heterogeneity on demand volumes, but is not when subscription is elastic. In the latter case, there exist network externalities. The 'raising each other cost' effect is reversed : lower access charges decreases usage prices which increases participation rate. This creates a virtuous circle since it enhances attractiveness for new subscribers (network externality effect) and then increases the fee that providers can charge. Firms then collude through low access charges. Gans and King (2001) show that when they compete in non linear pricing and when they can price discriminate between on-net and off-net calls, providers collude through low access charges.

In this paper, we introduce two new components. First, networks are asymmetric by their quality and compete in non linear pricing. They also choose freely the level of their access charge. Consumer's demand depends on the quality of the network they adopt. This leads to the introduction of heterogeneity on demand volumes. Second, consumers do not only value the number of calls they send but also the number they receive. Before taking his adoption decision, a consumer compares the price he pays for his communications and the prices the other pay for calling him. This is at the origin of call externalities. Little theoretical work has been realized on network competition with call externalities. Hermalin and Katz (2002) distinguish one-way and two-way calling patterns. In the former, only one party can initiate communication or exchange whereas in the latter both parties can do so. Hahn (2000), Wright (2002), Jeon et al. (2002), Kim and Lim (2001) examine retail pricing in the presence of call externalities in a one-way calling framework. Hermalin and Katz (2002) study a two-way calling model. They allow positive reception prices or the payer receiver principle (Jeon et al. (2002) and Kim and Lim (2001) also examine this principle but in a one-way calling model). Even if it can be welfare enhancing, this is not a common practice in most network competition. In this paper, we use a two-way calling model but we do not allow for such positive receive price.

The goal of this paper is twofold. First, we shed a new light on the debate over collusive concern of high access charges. We determine equilibrium pricing with call externalities. We show that providers internalize on-net calls externalities on their own network, but do not for off-net calls. However, by choosing freely his access charge, a provider can internalize off-net call externalities for its consumers. As a result, he can charge low access prices, even below marginal cost, since it increases utility of its consumers and allows him to charge higher fixed fees. We show that the (non reciprocal) access charge maximizing his profit is lower than the cost he bears for terminating calls if his network is relatively small. This corresponds to an implicit subsidy of received calls, against sent calls. However, providers lack instruments and the competitive level of access charge is too high because of a double marginalization effect. Nevertheless, this effect allows us to moderate the risks of tacit collusion when competitors can freely negotiate their access charges. Second, we bring new arguments to the debate over the optimal design market in telecommunications and more generally in the ICT industry. We show precisely in this paper that a more concentrated industry can improve welfare. Merging the two telecommunications providers solves the call externalities distortion.. We show that the merger does not affect the on-net calls price on the high quality network whereas it increases the on-net calls price on the low quality network. Off-net prices in both directions are decreased. The merger increases inter network traffic. This positive effect is a positive efficiency effect of the merger which must be put in balance with the traditional competition reduction effect. From a normative point of view, the merger does not affect consumers surplus on the low quality network whereas it improves the situation of consumers on the high quality network (and the global welfare) if they are in high enough proportion.

The paper is organized as follows. Section 2 presents the model. Section 3 is dedicated to network competition. We solve the two stage game in which providers first choose simultaneously their access charge and then compete in non linear tariffs. Section 4 analyses the merger between the two providers. We compute tariffs of the private monopoly resulting from the merger and conduce the welfare analysis. The last section offers some conclusive remarks.

2 The model

We consider two interconnected networks of different quality. The highest (lowest) quality is denoted q_1 (q_2). For example a mobile telecommunication network is considered of higher quality than a fixed link network, since it allows mobility. Fixed link networks can also be of different qualities especially in the case of national interconnected telecommunications networks or in the different components of the Internet. In a general manner, the quality of a network results from the quality of transmission infrastructures, the procedures of traffic re-routing in case of congested networks, or from more qualitative aspects as the quality of relations with consumers.

The marginal cost of a communication on network i is assumed to be constant and is decomposed in the following way : a cost for originating and terminating calls, both equal to c_i. The cost in between is normalized to zero. We assume that the fixed cost for serving a consumer is f_i for network i. Each provider has only to bear directly the costs on his network. For a communication between network j and i, firm i bears a cost c_i for terminating the call and in return bills an access price a_i to provider j. The total cost of an on-net call on network i is thus $2c_i$, while the cost of an off-net call on network i is $c_i + a_j$, with $i \neq j$. We allow $a_i \neq a_j$, which means that we do not impose a reciprocity principle.

Competitors can price discriminate between on-net and off-net calls. Each of them proposes a two part tariff where F_i is the fee for a consumer adopting network i. The unit price of an on-net call on network i is p_i whereas the one of a call originated on network i and terminated on network j is $\hat{p}_i$. Consumers only pay the calls they send[4] and are supposed to subscribe only to one network. Furthermore, we assume no information asymmetries on costs and demand conditions.

We consider a vertical differentiation model *à la* Mussa & Rosen [1978]. Consumers heterogeneity, represented by parameter θ, characterizes the willingness to pay for quality. We assume that the more consumers value the quality of communications, the higher is the utility they derive and the more is their willingness to pay for that service. This heterogeneity can also be derived from an income disparity. Consumers with a higher income are disposed to pay more for a higher telecommunication service quality. We limit the analysis to two types of consumers noted $\overline{\theta}$ and $\underline{\theta}$ ($\underline{\theta}<\overline{\theta}$). Total population is normalized to 1 and is divided exogenously in proportion α of type $\overline{\theta}$ and $(1-\alpha)$ of type $\underline{\theta}$.

The utility derived by a consumer adopting a network depends on the quality of the network, the number of calls he can send and the number of calls he receives. Given quantity of calls (sent or received) x, a consumer θ joining network i has utility $U_{\theta_i}(x) = \theta q_i W(x)$, where $W(.)$ is assumed to be non decreasing and concave.

We easily derive demand functions for a consumer θ adopting a network of quality q_i, at price p. These are solutions of $x_{\theta_i}(p) = \arg\max_{\{x\}}\{U_{\theta_i}(x)-px\}$. Therefore, indirect utility for a consumer $\bar{\theta}$ sending $x_{\overline{\theta}_i}(p)$ calls is $U_{\overline{\theta}_i}(x_{\overline{\theta}_i}(p))$ and the one for receiving $x_{\underline{\theta}_j}(p)$ calls is $U_{\overline{\theta}_i}(x_{\underline{\theta}_j}(p))$.

We have the following properties

$$\begin{array}{rclr} x_{\theta_1}(p) & > & x_{\theta_2}(p) \quad \forall\theta = \underline{\theta}, \overline{\theta} & \\ x_{\overline{\theta}_i}(p) & > & x_{\underline{\theta}_i}(p) \quad \forall i = 1,2 & (1) \end{array}$$

Proof. $x_{\overline{\theta}_i}(p)$ is given by $U'_{\overline{\theta}_i}(x_{\overline{\theta}_i}(p)) = \overline{\theta} q_i W'(x_{\overline{\theta}_i}(p)) = p$ and $x_{\underline{\theta}_i}(p)$ by $U'_{\underline{\theta}_i}(x_{\underline{\theta}_i}(p)) = \underline{\theta} q_i W'(x_{\underline{\theta}_i}(p)) = p$.

We then have $\overline{\theta} q_i W'(x_{\overline{\theta}_i}(p)) = \underline{\theta} q_i W'(x_{\underline{\theta}_i}(p))$. As $\overline{\theta} > \underline{\theta}$, this equality is possible iff $W'(x_{\overline{\theta}_i}(p)) < W'(x_{\underline{\theta}_i}(p))$. As $W'' < 0$, we derive $x_{\overline{\theta}_i}(p) > x_{\underline{\theta}_i}(p)$. ■

These two properties have an intuitive interpretation. Demand of individuals increases with respect to the quality of the good. Therefore, for a

[4]See Jeon, Laffont, Tirole (2002) or Hermalin and Katz (2002) on the receiver pays principle.

given price, a consumer θ sends more calls when he adopts a high quality network. Similarly, for a given quantity, demand increases with respect to consumer's willingness to pay. Therefore, for the same network quality, a consumer $\overline{\theta}$ sends more calls than a consumer $\underline{\theta}$.

We assume, as in most of the existing literature on networks interconnection, an isotropic or balanced calling pattern. This means that if a proportion α $(0 \leq \alpha \leq 1)$ of consumers adopt network 1 and $(1-\alpha)$ network 2, the probability for a call originated on network 1 to be terminated on the same network is α and on the network 2 is $(1-\alpha)$. Under the isotropic calling pattern assumption, if all consumers have the same demand functions and prices are the same on the two networks, then traffic flows between the two networks are the same[5].

As already pointed out by Laffont, Rey, Tirole (1998), price discrimination between on-net and off-net calls restores tariff-mediated network externalities. The utility derived from joining one of the two networks depends on the number of consumers on this network, as this determines the proportion of on-net and off-net calls. The more consumers subscribe to the same network, the higher is the proportion of on-net calls. If the price for on-net calls is lower than it is for off-net calls, then the higher the number of consumers in the same network, the more they send and receive calls. Network externalities induce a coordination problem between consumers who have to make conjectures on the others consumers' behavior. We use a rational expectation concept and at equilibrium those conjectures are correct.

In this paper, we focus only on situations where high type consumers $(\overline{\theta})$ adopt the high quality network and low type consumers $(\underline{\theta})$ adopt the low quality network. In other words, we do not examine the pooling case where both types of consumers coordinate on the same network.

Under these assumptions, if all consumers $\overline{\theta}$ coordinate on network 1, then their individual utility for sending calls is given by:

$$V^s_{\overline{\theta}_1}(p_1, \hat{p}_1) = \alpha[U_{\overline{\theta}_1}(x_{\overline{\theta}_1}(p_1)) - p_1 x_{\overline{\theta}_1}(p_1)] + (1-\alpha)[U_{\overline{\theta}_1}(x_{\overline{\theta}_1}(\hat{p}_1)) - \hat{p}_1 x_{\overline{\theta}_1}(\hat{p}_1)] \tag{2}$$

Utility for consumers $\underline{\theta}$ adopting network 2 for sending calls is:

$$V^s_{\underline{\theta}_2}(p_2, \hat{p}_2) = (1-\alpha)[U_{\underline{\theta}_2}(x_{\underline{\theta}_2}(p_2)) - p_2 x_{\underline{\theta}_2}(p_2)] + \alpha[U_{\underline{\theta}_2}(x_{\underline{\theta}_2}(\hat{p}_2)) - \hat{p}_2 x_{\underline{\theta}_2}(\hat{p}_2)]$$

[5] See Dessein (2002) for non balanced traffic models.

In order to determine the valuation for received calls, we assume that if a proportion α (respectively $1-\alpha$) of consumers adopt network 1 (resp. 2), each consumer receives $1/\alpha$ (resp. $1/(1-\alpha)$) of the total flow terminated on his network. The valuation of received calls is then given by:

$$\begin{aligned} V^r_{\overline{\theta}_1}(p_1, \hat{p}_2) &= \alpha U_{\overline{\theta}_1}(x_{\overline{\theta}_1}(p_1)) + (1-\alpha)U_{\overline{\theta}_1}(x_{\underline{\theta}_2}(\hat{p}_2)) \\ V^r_{\underline{\theta}_2}(p_2, \hat{p}_1) &= (1-\alpha)U_{\underline{\theta}_2}(x_{\underline{\theta}_2}(p_2)) + \alpha U_{\underline{\theta}_2}(x_{\overline{\theta}_1}(\hat{p}_1)) \end{aligned} \quad (3)$$

We can now write the total indirect utility for a consumer $\overline{\theta}$ or $\underline{\theta}$ depending on the network he chooses to adopt. We make at this stage the following assumption

$$\begin{aligned} V_{\theta_i}(p_i, \hat{p}_i, \hat{p}_j, F_i) &= V^s_{\theta_i}(p_i, \hat{p}_i) + V^r_{\theta_i}(p_i, \hat{p}_j) - F_i \\ &= v_{\theta_i} - F_i \end{aligned}$$

This assumption means that consumers have the same valuation for the calls they send and receive. On net and off-net calls are supposed to be non substitutable and utility is separable and additive between sent and received calls. Note also that the utility of consumer θ on network i depends not only on the prices $(p_i, \hat{p}_i)$ he pays for sending calls, but also on prices other consumers pay to call him $(p_i, \hat{p}_j)$. This last term is at the origin of call externalities.

We also need to write the indirect utility of a consumer who switches unilaterally to the other network. *Ceteris paribus*, consumer $\overline{\theta}$ who deviates on network 2 has an indirect utility

$$\begin{aligned} V_{\overline{\theta}_2}(p_2, \hat{p}_2, \hat{p}_1, F_2) &= (1-\alpha)\left[U_{\overline{\theta}_2}(x_{\overline{\theta}_2}(p_2)) + U_{\overline{\theta}_2}(x_{\underline{\theta}_2}(p_2)) - p_2 x_{\overline{\theta}_2}(p_2)\right] \\ &\quad + \alpha[U_{\overline{\theta}_2}(x_{\overline{\theta}_2}(\widehat{p}_2)) + U_{\overline{\theta}_2}(x_{\overline{\theta}_1}(\widehat{p}_1)) - \widehat{p}_2 x_{\overline{\theta}_2}(\widehat{p}_2)] - F_2 \\ &= v_{\overline{\theta}_2} - F_2 \end{aligned} \quad (4)$$

while a consumer $\underline{\theta}$ who deviates on network 1 receives an indirect utility

$$\begin{aligned} V_{\underline{\theta}_1}(p_1, \hat{p}_1, \hat{p}_2, F_1) &= \alpha[U_{\underline{\theta}_1}(x_{\underline{\theta}_1}(p_1)) + U_{\underline{\theta}_1}(x_{\overline{\theta}_1}(p_1)) - p_1 x_{\underline{\theta}_1}(p_1)] \\ &\quad + (1-\alpha)[U_{\underline{\theta}_1}(x_{\underline{\theta}_1}(\hat{p}_1)) + U_{\underline{\theta}_1}(x_{\underline{\theta}_2}(\widehat{p}_2)) - \hat{p}_1 x_{\underline{\theta}_1}(\hat{p}_1)] - F_1 \\ &= v_{\underline{\theta}_1} - F_1 \end{aligned} \quad (5)$$

At this stage, we can write the constraints which need to be verified in order to respect the allocation of each type of consumers on each network. Namely, each type of consumer has to verify incentive and individual rationality constraints. Consumers $\overline{\theta}$ adopt network 1 iff

$$\begin{aligned} V_{\overline{\theta}_1}(p_1,\hat{p}_1,\hat{p}_2,F_1) &\geq V_{\overline{\theta}_2}(p_2,\hat{p}_2,\hat{p}_1,F_2) \\ &\Leftrightarrow (v_{\overline{\theta}_1}-F_1)\geq(v_{\overline{\theta}_2}-F_2) \end{aligned} \quad \text{(IC1)}$$

and

$$(v_{\overline{\theta}_1}-F_1)\geq 0 \quad \text{(IR1)}$$

Consumer $\underline{\theta}$ adopts network 2 iff:

$$\begin{aligned} V_{\underline{\theta}_2}(p_2,\hat{p}_1,\hat{p}_2,F_2) &\geq V_{\underline{\theta}_1}(p_1,\hat{p}_2,\hat{p}_1,F_1) \\ &\Leftrightarrow v_{\underline{\theta}_2}-F_2\geq v_{\underline{\theta}_1}-F_1 \ (IC2) \end{aligned} \quad (6)$$

and

$$v_{\underline{\theta}_2}-F_2\geq 0 \quad \text{(IR2)}$$

We can now write profit on networks 1 and 2 if (6), (6), (IR1) and (IR2) hold:

$$\begin{aligned} \Pi_1(p_1,\hat{p}_1,F_1) &= \alpha^2(p_1-2c_1)x_{\overline{\theta}_1}(p_1)+\alpha(1-\alpha)(\hat{p}_1-c_1-a_2)x_{\overline{\theta}_1}(\hat{p}_1) \\ &\quad +\alpha(1-\alpha)(a_1-c_1)x_{\underline{\theta}_2}(\hat{p}_2)-f_1+F_1 \\ &= \alpha(\pi_1-f_1+F_1) \end{aligned} \quad (7)$$

$$\begin{aligned} \Pi_2(p_2,\hat{p}_2,F_2) &= (1-\alpha)^2(p_2-2c_2)x_{\underline{\theta}_2}(p_2)+\alpha(1-\alpha)(\hat{p}_2-c_2-a_1)x_{\underline{\theta}_2}(\hat{p}_2) \\ &\quad +\alpha(1-\alpha)(a_2-c_2)x_{\overline{\theta}_1}(\hat{p}_1)-f_2+F_2 \\ &= (1-\alpha)(\pi_2-f_2+F_2) \end{aligned} \quad (8)$$

3 Network Competition

In this section we examine a competitive network market where the two different providers compete in the following game. In a first stage, they choose simultaneously their access charge. In a second stage, they compete simultaneously in two part tariffs. We first derive the tariff competition equilibrium and then discuss the decentralized choices of access charges.

3.1 Tariff competition

Let us assume that provider 1 operates the high quality network and provider 2 operates the low quality network. We only focus on a duopoly equilibrium where consumers $\overline{\theta}$ adopt network 1 and consumers $\underline{\theta}$ adopt network 2. Both providers set the fee and prices for on net and off-net calls maximizing their profit. As in standard adverse selection models, they are constrained by an individual rationality constraint (IR_i) and incentive constraint an (IC_i) of consumers. Equilibrium prices $(p_i, \hat{p}_i, F_i\ )$ for provider i are solutions of the following program:

$$(p_i, \hat{p}_i, F_i) = \arg \max_{\{p_i, \hat{p}_i, F_i\}} \Pi_i(p_i, \hat{p}_i, F_i) = \alpha_i(\pi_i + F_i - f_i)$$

$$s.t.: \quad \begin{array}{ll} v_{\overline{\theta}_i} - F_i \geq 0 & (IR_i) \\ v_{\overline{\theta}_i} - F_i - v_{\overline{\theta}_j} + F_j \geq 0 & (IC_i) \end{array}$$

with $\pi_i = \alpha_i(p_i - 2c_i)x_{\theta_i}(p_i) + (1-\alpha_i)(\widehat{p}_i - (c_i + a_j))x_{\theta_i}(\widehat{p}_i) + (1-\alpha_i)(a_i - c_i)x_{\theta_j}(\widehat{p}_j)$

α_i is the proportion of consumers on network i. The resolution of this program is not standard as we are not guaranteed that incentives constraints are compatible. Both providers choose their calls prices and the fixed fee they charge to consumers. We denote $(p, \hat{p}) = (p_1, p_2, \hat{p}_1, \hat{p}_2)$. The unique duopoly equilibrium candidate for the competition in fixed fees is such that the individual rationality constraint of consumers $\underline{\theta}$ subscribed to network 2 (IR_2) and incentive constraint of consumers $\overline{\theta}$ subscribed to network 1 (IC_1) are binding. Thus for a unique duopoly equilibrium candidate, program for firm 1 writes

$$\max_{\{p_1, \hat{p}_1\}} \Pi_1(p, \hat{p}, F_1) = \alpha \left[\pi_1 + v_{\overline{\theta}_1} - \left(v_{\overline{\theta}_2} - v_{\underline{\theta}_2}\right) - f_1\right]$$

and for firm 2

$$\max_{\{p_2, \hat{p}_2\}} \Pi_2(p, \hat{p}, F_2) = (1-\alpha)\left[\pi_2 + v_{\underline{\theta}_2} - f_2\right]$$

Let us define function Ψ as:

$$v_{\overline{\theta}_1} - v_{\underline{\theta}_1} \geq v_{\overline{\theta}_2} - v_{\underline{\theta}_2} \Leftrightarrow \Psi\left(p, \hat{p}\right) \geq 0$$

The following proposition gives equilibrium prices.

Proposition 1 *(i) if* $\Psi\left(p^d, \hat{p}^d\right) \geq 0$*, there exists a unique duopoly tariff equilibrium with*

$p_1^d = c_1$, $p_2^d = c_2$, $\widehat{p}_2^d = c_2 + a_1$ *and*

$$\widehat{p}_1^d = \begin{cases} \frac{(1-\alpha)\overline{\theta}q_1}{(1-\alpha)\overline{\theta}q_1 - \alpha(\overline{\theta}-\underline{\theta})q_2}(c_1 + a_2) & if \ \frac{\alpha}{1-\alpha} < \frac{\overline{\theta}q_1}{(\overline{\theta}-\underline{\theta})q_2} \\ +\infty & otherwise \end{cases}$$

F_1^d *and* F_2^d *are given by the binding constraints : (*IR_2*) and (*IC_1*)*

(ii) Otherwise, there does not exist any equilibrium.

Proof : see appendix 1

Recall that marginal costs for on-net and off-net calls on network i are respectively $2c_i$ and $c_1 + c_2$. Unit on-net prices are exactly half of marginal cost on both networks. This comes from call externalities : each call is valued not only by the sender but also by the receiver. Providers fully internalize those externalities for on-net calls. Decreasing unit price enhances subscribers' utility and allows to charge a higher fixed fee. Recall that the valuation of (received or sent) calls depends on the type of consumers and on the quality of the network. An on-net call has the same value for the sender and for the receiver, as both are of the same type and adopt the same network. Hence, on-net calls are charged at half the marginal cost.

This is not the same for off-net call externalities. The utility surplus derived by a consumer on the opposite network from receiving off-net calls cannot be captured by the provider. This induces a distortion in usage prices. Moreover, the nature of the distortion is different on the two networks according to binding constraints. If the relative size of network 1 ($\frac{\alpha}{1-\alpha}$) is relatively small, the price of an off-net call on network 1 needs to be relatively low. Such a price limits the attractiveness of network 2. Proposition 1 indicates that the off-net calls price on network 1 is higher than the marginal cost ($c_1 + a_2$). This comes from negative externalities through off-net calls. Indeed, a diminution of off-net calls price of network 1 $\widehat{p}_1$ has two opposite effects on the incentive constraint of a consumer $\overline{\theta}$. First, it increases the number of off-net calls he sends if he stays on network 1 and then increases $v_{\overline{\theta}_1}$. Second, it increases the number of off-net calls he should receive from network 1 users if he switches to network 2 and then increases $v_{\overline{\theta}_2}$. In all cases, the latter effect increases $\widehat{p}_1$. If the relative size of network 1 is high enough $\left(\frac{\alpha}{1-\alpha} > \frac{\overline{\theta}q_1}{(\overline{\theta}-\underline{\theta})q_2}\right)$, calling consumers $\underline{\theta}$ on network 2 does not matter. A very high level of off-net calls prices on network 1 $\widehat{p}_1$ relaxes the incentive constraint and it is optimal for provider 1 to annihilate the off-net flow terminating on the other network. This is the case of connectivity breakdown, already noted by Jeon, Laffont Tirole (2002). Conversely, if the relative size of network 1 is small enough $\left(\frac{\alpha}{1-\alpha} < \frac{\overline{\theta} q_1}{(\overline{\theta}-\underline{\theta}) q_2}\right)$, the first effect has a significant weight and the provider gains to authorize outside communication for its consumers.

Finally, off-net calls price on the low quality network ($\widehat{p}_2^d$) is equal to the marginal cost $(c_2 + a_1)$. As (IC_2) is not binding, there is no such distortion coming from the incentive for consumers to switch to the competitive network. It is also interesting to note that the low quality network is never better off cutting communication with the higher quality competitive network.

3.2 Competitive access prices

In this section we solve the first stage of the game. Both providers choose simultaneously their access charge. The goal of this section is to discuss the collusive power of access charges.

In most of this literature, authors impose a reciprocity principle, i.e. $a_1 = a_2 = a$. This assumption has important implications in the comparative static analysis with respect to the access charge. Indeed, increasing the access charge does not only raise the rival's cost but also its own cost. This assumption is the key of the 'raising each other's cost' effect in Armstrong (1998) or in Laffont, Rey, Tirole (1998a,b)[6]. In our paper, we do not impose this assumption. A provider can raise its rival's cost without raising its own cost. This point is also an important regulatory issue.

In this section, we bring a new light on the risk of use of high access charges for collusive concerns. Indeed, the level of access charge will influence the level of off-net calls prices. As in the existing literature, it has an impact on the number of calls consumers will send. But the level of access charges has also an impact on the number of off-net calls consumers will receive. The latter effect acts on call externalities and we show that providers can reduce inefficiencies on call externalities when they can choose freely their access charge.

We determine the access charges maximizing providers' profit. At duopoly competitive tariff equilibrium, the profit of operator i is continue with respect to access charge and can be written as :

$$\Pi_i(a_i, a_j) = \alpha_i(\pi_i(a_i, a_j) + F_i^d(a_i, a_j) - f_i)$$

Profit depends on the access charge through two terms : the access revenue which writes $AR_i = (a_i - c_i)x_{\theta_j}(\hat{p}_j(a_i))$ and the fee F_i^d the provider can

[6]LRT (1998a) also discuss non reciprocal access charges.

charge at competitive equilibrium. We derive the optimal access charge for the two components and for both providers.

Provider 1 :

At competitive equilibrium, provider 1 binds the incentive constraint and has to give consumers an incentive rent which depends on the access charge. By differentiating F_1^d with respect to a_1 we obtain:

$$\begin{aligned}\frac{dF_1^d}{da_1} &= \frac{dv_{\overline{\theta}_1}}{da_1} - \frac{d(v_{\overline{\theta}_2} - v_{\underline{\theta}_2})}{da_1} = \left[\frac{\partial v_{\overline{\theta}_1}}{\partial \widehat{p}_2} - \frac{\partial (v_{\overline{\theta}_2} - v_{\underline{\theta}_2})}{\partial \widehat{p}_2}\right] \frac{d\widehat{p}_2^d}{da_1} \\ &= \left[(1-\alpha)\frac{\overline{\theta}q_1}{\underline{\theta}q_2}\widehat{p}_2 \frac{dx_{\underline{\theta}_2}}{d\widehat{p}_2} + \alpha(x_{\overline{\theta}_2} - x_{\underline{\theta}_2})\right] \frac{d\widehat{p}_2^d}{da_1}\end{aligned}$$

An increase in access charge to network 1 leads to two opposite effects on fee F_1: a receiving calls effect and a sending calls effect. First, a higher access charge decreases the utility of network 1 since consumers receive fewer calls from network 2 users. This is the negative receiving calls effect represented by the first term between brackets. Second, a higher access charge decreases the incentive to switch on-network 2 since consumers should pay more to send off-net calls. This is the positive sending calls effect represented by the double term between brackets. It is straightforward to show that F_1^* is first increasing and then decreasing with respect to a_1. Note that the access charge maximizing the fixed fee F_1^d is independent of c_1. In other words, the price maximizing the subscription fee for terminating a call is independent of its marginal cost.

The access revenue for provider 1 writes

$$AR_1 = (1-\alpha)(a_1 - c_1)x_{\underline{\theta}_2}\left(\widehat{p}_2^d(a_1)\right)$$

and is concave with respect to a_1. The access charge maximizing the access revenue is clearly above the marginal cost c_1. Remark that the maximization of the access revenue yields to a double marginalization problem. Note also that both access charge maximizing the subscription fee and the access revenue are independent of a_2.

Provider 2 :

At competitive equilibrium, provider 2 binds the individual rationality constraint for low type consumers. Differentiating F_2^d with respect to a_2 yields

$$\frac{dF_2^d}{da_2} = \frac{dv_{\underline{\theta}_2}}{da_2} = \frac{\partial v_{\underline{\theta}_2}}{\partial \widehat{p}_1}\frac{d\widehat{p}_1}{da_2} = \alpha\frac{\underline{\theta}q_2}{\overline{\theta}q_1}\widehat{p}_1^d \frac{dx_{\overline{\theta}_1}}{d\widehat{p}_1}\frac{d\widehat{p}_1}{da_2} < 0$$

Since only the individual rationality constraint is binding at equilibrium, provider 2 faces only a receiving calls effect. The higher its access charge, the less its consumers will receive calls from network 1's consumers, the lower is their utility and the lower is the fee he can charge. The subscription fee F_2^d is then always decreasing with its access charge a_2. As for provider 1, the access revenue is concave with respect to the access charge. The access charge maximizing the access revenue is clearly above the marginal cost. As for provider 1, it is independent of the rival's access charge.

The following proposition summarizes those results for both providers.

Proposition 2 $\left(a_1^d, a_2^d\right)$ *are equilibrium solutions of the simultaneous game of choice of access charges.*

i) Access charge to network 1 is above the marginal cost for terminating calls ($a_1^d > c_1$) *iff* $\frac{\alpha}{1-\alpha} > \overline{\alpha}$

ii) Access charge to network 2 is above the marginal cost for terminating calls ($a_2^d > c_2$) *iff* $\frac{\alpha}{1-\alpha} < \underline{\alpha}$

where $\overline{\alpha} = \frac{x_{\underline{\theta}_2}}{(x_{\overline{\theta}_2} - x_{\underline{\theta}_2})}\left[\frac{\overline{\theta}q_1}{\underline{\theta}q_2}\frac{1}{\xi_{x_{\underline{\theta}_2}}} - 1\right]$, $\underline{\alpha} = \left(\frac{\overline{\theta}q_1}{\underline{\theta}q_2} - \xi_{x_{\overline{\theta}_1}}\right)\frac{\underline{\theta}}{(\overline{\theta}-\underline{\theta})}$ *and*

$\xi_{x_{\theta_i}} = -\left.\frac{dx_{\theta_i}}{d\widehat{p}_i}\frac{\widehat{p}_i}{x_{\theta_i}}\right|_{a_i=c_i}$ *is the price elasticity of demand for consumers* θ *computed for access charge* a_i *equal to marginal cost* c_i*.*

Proof : see appendix 2

The decentralized choice of the access charge by providers leads to two opposite effects. On the one hand, it induces a double marginalization problem on off-net prices. Each provider charges a price for off-net calls : the sending provider through the usage price and the receiving provider through the access charge. This creates an incentive to increase access charges. On the other hand, as providers use two part tariffs, they can capture a part of their consumer surplus. If surplus is decreasing with respect to usage prices, this creates an incentive not to charge too high access charge. More precisely, each call sent by a customer generates a call externality for the receiver. We have shown in the previous section that each provider internalizes those call externalities for on-net calls on its own network. As a result unit prices for on-net calls are half the marginal cost because call externalities within networks are not internalized. Unit off-net calls prices do not take into account the call externality. However, those prices depend on the access charge for terminating calls. By choosing freely its access charge, a provider can influence off-net calls unit prices. In other words, choosing freely its access charge allows him to internalize off-net call externalities. He can thus reduce the distortion due to externalities. Note that there subsists a distortion since each provider faces a trade-off between maximizing the subscription fee and the access revenue. Access charge is then only one tool for many objectives. More precisely, the optimal access charge for each provider depends on the binding constraint at tariff equilibrium.

Provider 2's subscription fee only depends on the call externalities. Close to marginal cost c_2, an increase in the access charge a_2 enhances the access revenue but decreases the subscription fee provider 2 can charge. The equilibrium access charge to network 2 is above marginal cost if enough consumers choose network 2 relatively to those on the rival's network $(\alpha/(1-\alpha))$.

Provider 1's subscription fee does not only depend on call externalities but also on the incentive rent $(v_{\overline{\theta}_2} - v_{\underline{\theta}_2})$ he has to give to its consumers. When choosing its access charge a_1, provider 1 has only one instrument for three objectives : reducing the incentive rent, internalizing call externalities and maximizing its access revenue. It is easy to verify that $\frac{d(v_{\overline{\theta}_2} - v_{\underline{\theta}_2})}{d\hat{p}_2} < 0$. An increase in the access charge increases the off-net price which decreases the incentive rent. If the proportion of consumers on network 2 is high, customers on network 1 receive an important proportion of calls from network 2 (off-net calls). Provider 1 gains to charge a low access charge below the marginal cost, even if it implies an access deficit. The access deficit is compensated by higher subscriptions revenues. Remark also that the threshold $\bar{\alpha}$ is decreasing with the price elasticity of demand for low type consumers on network 2 $\xi_{x_{\underline{\theta}_2}}$. The lower $\xi_{x_{\underline{\theta}_2}}$, the more provider 1 charges an access charge below marginal cost.

Our results have important implications from a regulatory or competition policy point of view. Access charges can be used as a tool of tacit collusion by the 'raise each other's cost effect' (or the double marginalization effect). Access charges are also a tool for both providers to internalize call externalities. The latter effect counterbalances the former and contributes to moderate the risk of collusive power of access charges. It speaks for a decentralized and free determination of access charges level by providers. Note also that a regulation of access charges taking into account such call externalities effects does not seem realistic in terms of informational requirements.

As in all externalities problems, one solution to annihilate distortion is to merge parties. In the following section, we address the question of merging telecommunication operators.

4 Mergers analysis

In this section, we discuss the effects of a merger between the two networks. Traditionally, a merger leads to two opposite effects : a decrease in competition in the market and possible efficiency gains. Literature on mergers and antitrust authorities practices focus on costs reduction (see e.g. Farrell-Shapiro (1990)). Our main result here is to characterize a new potential positive effect of mergers through the reduction of distortion on call externalities. The merger between the two networks leads to firm m. This firm acts as an unregulated monopoly.

For clarity's sake, we first derive optimal prices for the merger firm under the benchmark of complete information and then we introduce asymmetric information.

4.1 Benchmark

We first consider the benchmark case of complete information. The objective of the merging entity is then to maximize the joint profits

$$\max_{p_i,\hat{p}_i,F_i} \Pi(p_i,\hat{p}_i,F_i) = \alpha(\pi_1 + F_1 - f_1) + (1-\alpha)(\pi_2 + F_2 - f_2)$$

$$s.t.: \begin{array}{ll} v_{\overline{\theta}_1} - F_1 \geq 0 & (IR_1) \\ v_{\underline{\theta}_2} - F_2 \geq 0 & (IR_2) \end{array}$$

Note that since this firm uses two part tariffs, this program is the first best solution. Both individual rationality constraints are binding at equilibrium. The solution of this program is given in the following lemma.

Lemma 3 *Under complete information, the merging entity fully internalizes call externalities. Usage prices are*

$p_1^{mc} = c_1$

$p_2^{mc} = c_2$

$\hat{p}_1^{mc} = \frac{(c_1+c_2)\overline{\theta}q_1}{\overline{\theta}q_1+\underline{\theta}q_2} < c_1 + c_2$

$\hat{p}_2^{mc} = \frac{(c_1+c_2)\underline{\theta}q_2}{\overline{\theta}q_1+\underline{\theta}q_2} < c_1 + c_2$

Proof. see appendix 3 ■

This first best benchmark allows to exhibit optimal prices which internalize perfectly all call externalities. Similarly to the competitive case, on-net calls are charged at half the marginal cost since the valuation of an on-net call is the same for the sender and for the receiver. Results are different for off-net calls which are valued differently by sender and receiver as they are of different types and adopt different network's quality. For example off-net calls from network 1 to network 2 are valued $U_{\overline{\theta}_1}(x_{\overline{\theta}_1}(\hat{p}_1))$ by the sender and $U_{\underline{\theta}_2}(x_{\overline{\theta}_1}(\hat{p}_1))$ by the receiver. The externality arising from received calls depends on both types $\overline{\theta}$ and $\underline{\theta}$ and both qualities q_1 and q_2. Note also that $\hat{p}_1 < \hat{p}_2$. This is explained by the fact that calls valuation is higher for consumers $\overline{\theta}$ on network 1 than for consumers on network 2. Then the externality is larger for calls received by high type consumers.

Note that this first best outcome could be achieved by regulating access charges in a perfect world without asymmetric information. Indeed, competitive on-net prices are optimal since providers use two-part tariff and fully internalize call externalities for on-net calls. Optimal access charges are thus implementing first best off-net prices, i.e. $\hat{p}_2^d(a_1^*) = \hat{p}_2^m$ and $\hat{p}_1^d(a_1^*) = \hat{p}_1^m$, which yields to

$$a_1^* = c_1 - \frac{(c_1+c_2)\,\overline{\theta}q_1}{\overline{\theta}q_1 + \underline{\theta}q_2} \; and \; a_2^* = c_2 - \frac{(c_1+c_2)\left[\alpha\overline{\theta} + (1-2\alpha)\,\underline{\theta}\right] q_2}{(1-\alpha)\left(\overline{\theta}q_1 + \underline{\theta}q_2\right)}$$

Optimal access charges are below marginal cost in order to internalize call externalities. This analysis constitutes only a benchmark. Introducing asymmetric information on costs or demand parameters goes beyond the goal of this paper.

4.2 Imperfect discrimination

Consider now the more realistic case where the monopoly cannot perfectly price discriminate between consumers. We show how asymmetric information affects the internalization of call externalities. In order to compare it with the competitive case, we focus only on the case of separating contracts. As the monopoly uses two part tariffs, this problem is the same as the second best program. Program of the merging entity writes

$$\max_{p_i,\widehat{p}_i,F_i} \alpha(\pi_1 + F_1 - f_1) + (1-\alpha)(\pi_2 + F_2 - f_2)$$

$$s.t.\; \begin{array}{ll} v_{\overline{\theta}_1} \geq F_1 & (IR_1) \\ v_{\underline{\theta}_2} \geq F_2 & (IR_2) \\ v_{\overline{\theta}_1} - F_1 \geq v_{\overline{\theta}_2} - F_2 & (IC_1) \\ v_{\underline{\theta}_2} - F_2 \geq v_{\underline{\theta}_1} - F_1 & (IC_2) \end{array}$$

This problem is quite similar to a classical adverse selection problem. The individual rationality constraint of the low type ($\underline{\theta}$) and the incentive constraint of the high type $(\overline{\theta})$ will be binding. However, compatibility between incentives constraints (IC_1) and (IC_2) requires the following constraint :

$$v_{\overline{\theta}_1} - v_{\underline{\theta}_1} \geq v_{\overline{\theta}_2} - v_{\underline{\theta}_2}$$

Using (IR_2) and (IC_1), fixed fees are given by

$$\begin{aligned} F_1 &= v_{\overline{\theta}_1} - (v_{\overline{\theta}_2} - v_{\underline{\theta}_2}) \\ F_2 &= v_{\underline{\theta}_2} \end{aligned}$$

The program writes

$$\max_{p_i,\widehat{p}} \alpha \left(\pi_1 + v_{\overline{\theta}_1} - (v_{\overline{\theta}_2} - v_{\underline{\theta}_2}) - f_1\right) + (1-\alpha)(\pi_2 + v_{\underline{\theta}_2} - f_2)$$
$$s.t. \qquad v_{\overline{\theta}_1} - v_{\underline{\theta}_1} \geq v_{\overline{\theta}_2} - v_{\underline{\theta}_2}$$

In order to compare the two market structures, we limit ourselves to the case where the constraint is not binding[7]. We denote λ the Lagrange multiplier of the compatibility constraint. In the following, we only develop the particular case where this constraint is not binding $(\lambda = 0)$. We refer the reader to appendix 2 for the general case.

Lemma 4 *Under incomplete information, the merger entity fully internalizes call externalities. If demand function is not too convex, prices are given by*
(i) $p_1^m = c_1$

$$p_2^m = \begin{cases} \frac{2(1-\alpha)c_2 - \alpha(x_{\overline{\theta}_2} - x_{\underline{\theta}_2})/\frac{dx_{\underline{\theta}_2}}{dp_2}}{2(1-\alpha)-\alpha(\frac{\overline{\theta}-\underline{\theta}}{\underline{\theta}})} \; if \; \frac{\alpha}{1-\alpha} < \frac{2\underline{\theta}}{(\overline{\theta}-\underline{\theta})} \\ +\infty \; otherwise \end{cases}$$

$$\hat{p}_1^m = \begin{cases} \frac{(1-\alpha)\overline{\theta}q_1}{(1-\alpha)(\overline{\theta}q_1+\underline{\theta}q_2)-\alpha(\overline{\theta}-\underline{\theta})q_2}(c_1+c_2) \; if \; \frac{\alpha}{1-\alpha} < \frac{\overline{\theta}q_1+\underline{\theta}q_2}{(\overline{\theta}-\underline{\theta})q_2} \\ +\infty \; otherwise \end{cases}$$

$$\hat{p}_2^m = \frac{\underline{\theta}q_2}{\overline{\theta}q_1+\underline{\theta}q_2}\left[(c_1+c_2) - \frac{\alpha}{1-\alpha}\left[x_{\overline{\theta}_2}(\widehat{p}_2^m) - x_{\underline{\theta}_2}(\widehat{p}_2^m)\right]/\frac{dx_{\underline{\theta}_2}(\hat{p}_2)}{d\hat{p}_2}\right]$$

(ii) F_1^m *and* F_2^m *are given by the binding constraints :* (IR_2) *and* (IC_1)

[7]The general case with a positive Lagrange multiplier would only increase technical complications without improving qualitative results.

Proof : see appendix 3.

Only the on-net calls price on the high quality network is not distorted by the informational rent. Furthermore, we find the standard result of no distortion at top only for on-net calls for high type. All other prices are then affected by the informational rent $(v_{\overline{\theta}_2} - v_{\underline{\theta}_2})$ which must be conceded to type $\overline{\theta}$.

In order to quantify the direction of this distortion, we show that :

$$\begin{aligned}
\frac{d(v_{\overline{\theta}_2} - v_{\underline{\theta}_2})}{dp_1} &= 0, \frac{d(v_{\overline{\theta}_2} - v_{\underline{\theta}_2})}{d\hat{p}_1} < 0 \\
\frac{d(v_{\overline{\theta}_2} - v_{\underline{\theta}_2})}{dp_2} &< 0, \frac{d(v_{\overline{\theta}_2} - v_{\underline{\theta}_2})}{d\hat{p}_2} < 0
\end{aligned}$$

We see that the on-net calls price on network 1 p_1 has no effect neither on utility $v_{\overline{\theta}_2}$ or on $v_{\underline{\theta}_2}$. Thus this price is not distorted by the informational rent. In contrast, off-net calls prices for calling network 2 from network 1 $\hat{p}_1$ affect both $v_{\overline{\theta}_2}$ and $v_{\underline{\theta}_2}$. More precisely, an increase in this price reduces the number of off-net calls received on network 2, thus reducing the utility of consumers adopting this network. An increase in $\hat{p}_1$ reduces the informational rent conceded to type $\overline{\theta}$. The effect is the same for p_2 and $\hat{p}_2$. The three prices are then higher than under complete information ($\hat{p}_1^m > \hat{p}_1^{mc}$, $p_2^m > p_2^{mc}$ and $\hat{p}_2^m > \hat{p}_2^{mc}$).

Note also that the profit of the merger is higher than the sum of the profit made by competitive networks. This comes from the traditional arguments of price competition relaxation and of the minimization of the incentive rent conceded to high type consumers. But it also comes from the full internalization of all call externalities This gives a new interpretation of the high bids recently observed in telecommunications mergers. This point has an important implications for competition policy.

4.3 Welfare analysis

We extend the analysis to address the question of the welfare effects of a merger between two telecommunications operators. This has been a key question and remains a relevant policy issue, in light of the recent mergers wave in the ICT industry. We compare the merger with a totally decentralized market structure where final and access prices are chosen freely by providers.

Mergers are generally analyzed by antitrust authorities under the trade-off between the competitive pressure reduction and potential efficiency gains.

The latter are traditionally evaluated in terms of cost reductions. We do not address this question in this paper. We highlight another source of efficiency gains based on call externalities and better pricing strategy.

The analysis in terms of global welfare is decomposed into two effects. The first effect of our analysis concerns the incentive rent. The merging entity optimizes the informational rent using the three prices $\hat{p}_1, \hat{p}_2, p_2$ whereas competitive provider 1 can only use one direct instrument $\hat{p}_1$ and an indirect tool a_1 which influences $\hat{p}_2$. The monopoly has then more tools to optimize the incentive rent. Moreover, even if competitor 1 can influence $\hat{p}_2$ through its access charge a_1, he has other goals. The incentive rent is only one the three objectives when he determines the optimal a_1 (the two other objectives are the access revenue and the internalization of call externalities). The second effect is the perfect internalization of all call externalities by the merging entity. This enhances the gross utility of subscribers. The following lemma compares prices in both market structures.

Lemma 5 $p_1^d = p_1^m$, $p_2^m > p_2^d$, $\hat{p}_1^m < \hat{p}_1^d\left(a_2^d\right)$ *and* $\hat{p}_2^m < \hat{p}_2^d\left(a_1^d\right)$.

Proof : see appendix 4.

The on-net price p_1 is not affected by the merger since the competitive operator fully internalizes on-net call externalities and this price does not affect the incentive rent. In contrast, the on-net price on network 2 p_2 after the merger is modified by the incentive rent let to high type consumers. This term is measured by $-\alpha \frac{\partial\left(v_{\overline{\theta}2} - v_{\underline{\theta}2}\right)}{\partial p_2} > 0$. We conclude that on-net price for network 2 is lower in the competitive market structure ($p_2^d < p_2^m$).

The comparison for off-net prices is less direct since it depends on the level of access charges. The merging entity fully internalizes call externalities which tends to reduce off-net prices. It uses the marginal cost as internal prices for charging access to the other network and uses directly the three usage prices p_1, $\hat{p}_1$, $\hat{p}_2$ to balance the incentive rent and the call externalities effect. Competitive providers also internalize call externalities if they can choose freely their access charge. But as we saw in the previous section, the level of their access charge is increased by the double marginalization effect. Access charges are then chosen at too high a level. This distortion mainly comes from a lack of instruments in the competitive market structure. Off-net prices are then higher in the competitive case ($\widehat{p}_i^d > \widehat{p}_i^m$, $i = 1, 2$).

The following proposition gives the normative implication of a merger between the two providers.

Proposition 6 *(i) Consumers' net surplus on the low quality network is not affected by the merger.*
(ii) Consumers' net surplus on the high quality network is increased iff $\frac{\alpha}{1-\alpha} > \tilde{\alpha}$
(iii) The merger between the two providers increases the global welfare iff $\frac{\alpha}{1-\alpha} > \hat{\alpha}$ *with* $0 < \hat{\alpha} < \tilde{\alpha}$.

Proof : see appendix 4.

Consumers on the high quality network send and receive more off-net calls after the merger. Their gross surplus $v_{\overline{\theta}_1}$ is then unambiguously increased. However, their net surplus is equal to their incentive rent $(v_{\overline{\theta}_2} - v_{\underline{\theta}_2})$. The merger has to two opposite effects on this incentive rent. On the one hand, off-net prices decrease and create an increase in their rent. On the other hand, the on-net price on the rival's network increases, which decreases the incentive rent. The net effect is ambiguous. We show that the former effect dominates if the proportion of consumers on the high quality network is high enough. Indeed, the incentive rent for a high type consumer depends on its traffic flow on network 2. The weight of inter networks traffic is all the more important for him since there are many consumers on network 1. Hence, the decrease of off-net prices effect dominates if $\alpha/(1-\alpha)$ is high enough.

The net surplus of consumers on the low quality network is not affected by the merger since their individual rationality constraint is binding in both market structures. However, their gross surplus $v_{\underline{\theta}_2}$ is affected since they send fewer on-net calls but send and receive more off-net calls. We can conduct the same analysis as before and show that the gross surplus is higher after the merger if $\alpha/(1-\alpha)$ is high enough. The merger decreases off-net prices and then increases inter network traffic. This counterbalances the negative effect of the increase in on-net price p_2 if the weight of external traffic is sufficiently important or if there are enough consumers on the rival's network.

Global welfare variation can be written as $\Delta W = \Delta\pi + \alpha\Delta S_1 + (1-\alpha)\Delta S_2 = \Delta\pi + \alpha\Delta S_1$ since $\Delta S_2 = 0$ (S_i denotes the surplus of consumers on network i). Profits are higher under the merger but high type consumer's surplus can be lower. Therefore, the global effect of the merger on total welfare is ambiguous. The merger increases the global welfare if consumer's surplus loss is counterbalanced by profit gains. This is the case if the weight of inter network traffic is sufficiently high or if the proportion of consumers on network 1 is high enough.

5 Conclusion

This model shows that providers internalize on-net calls externalities on their own network, but not off-net calls. However, by choosing freely its access charge, a provider internalize incoming off-net call externalities for its consumers. Providers can gain by charging low access prices below marginal cost since it improves their consumers' utility and allows them to charge higher fixed fees. However, providers lack instruments and the competitive level of access charge is too high because of a double marginalization effect. However, this result contributes to moderating the risk of high access charges for collusive concerns.

Merging the two telecommunications providers solves the call externalities distortion. We have shown that the merger does not affect the on-net calls price on the high quality network whereas it increases the on-net calls price on the low quality network. Off-net prices in both directions are lower. The merger then increases inter network traffic. It does not affect consumers' surplus on the low quality network whereas it improves the situation of consumers on the high quality network (and the global welfare) if they are in high enough proportion.

The lessons drawn from our model would be robust to changes, including the extension to a continuum of heterogeneous consumers in their demand. However, aggregating individual demand would lead to technical difficulties.

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6 Appendices

6.1 Appendix 1: proof of Proposition 1

Let us first write the following properties:

$$U'_{\overline{\theta}_i}(x_{\overline{\theta}_j}(p)) = \frac{q_i}{q_j}p \tag{9}$$

$$U'_{\overline{\theta}_i}(x_{\underline{\theta}_j}(p)) = \frac{\overline{\theta}q_i}{\underline{\theta}q_j}p \tag{10}$$

$$U'_{\underline{\theta}_i}(x_{\overline{\theta}_j}(p)) = \frac{\underline{\theta}q_i}{\overline{\theta}q_j}p \tag{11}$$

Here we solve the tariff competition game. A Nash equilibrium duopoly is a vector of prices $\left(p_1^d, \hat{p}_1^d, p_2^d, \hat{p}_2^d, F_1^d, F_2^d\right)$ for which all consumers $\overline{\theta}$ coordinate on network 1 and all consumers $\underline{\theta}$ on network 2. In other words, at equilibrium, the four incentive and individual rationality constraints $(IR1)$, $(IC1)$, $(IR2)$, $(IC2)$ have to be verified.

$$\begin{array}{ll} F_1 \leq v_{\overline{\theta}_1} & (IR_1) \\ F_2 \leq v_{\underline{\theta}_2} & (IR_2) \\ F_1 \leq F_2 + \left(v_{\overline{\theta}_1} - v_{\overline{\theta}_2}\right) & (IC_1) \\ F_1 \geq F_2 + \left(v_{\underline{\theta}_1} - v_{\underline{\theta}_2}\right) & (IC_2) \end{array}$$

- We first derive equilibrium fees F_1^d and F_2^d

At equilibrium, unit prices $\left(p^d, \hat{p}^d\right)$, (IC_1) and (IC_2) are compatible iff $\left(v_{\overline{\theta}_1} - v_{\overline{\theta}_2}\right) \geq \left(v_{\underline{\theta}_1} - v_{\underline{\theta}_2}\right)$ or $\Psi(p, \hat{p}) \geq 0$. For a given F_j, provider i would do well to charge the highest fee F_i which respects (IR_i) and (IC_i). Best responses in fees are thus given by $F_1^* = \min\left(F_2 + v_{\overline{\theta}_1} - v_{\overline{\theta}_2}, v_{\overline{\theta}_1}\right)$ and $F_2^* = \min\left(F_1 + \left(v_{\underline{\theta}_2} - v_{\underline{\theta}_1}\right), v_{\underline{\theta}_2}\right)$. It is straightforward to see that the unique equilibrium fees are given by binding $(IC1)$ and $(IR2)$.

- Now let us derive equilibrium unit prices given equilibrium fees.

Program for firm 1 writes:

$$\max_{\{p_1, \hat{p}_1\}} \Pi_1(p, \hat{p}) = \alpha\left[\pi_1 + v_{\overline{\theta}_1} - (v_{\overline{\theta}_2} - v_{\underline{\theta}_2}) - f_1\right]$$

First order conditions write :

$$\begin{aligned}\frac{\partial \Pi_1}{\partial p_1} &= \alpha^2[2U'_{\overline{\theta}_1}(x_{\overline{\theta}_1}(p_1)) - 2c_1]\frac{dx_{\overline{\theta}_1}(p_1)}{dp_1} = 0 \\ &\Leftrightarrow \; p_1^d = c_1 \qquad (12)\end{aligned}$$

$$\frac{\partial \Pi_1}{\partial \hat{p}_1} = \left\{(1-\alpha)\left[U'_{\overline{\theta}_1}(x_{\overline{\theta}_1}(\hat{p}_1)) - (c_1+a_2)\right] - \alpha\left[U'_{\overline{\theta}_2}(x_{\overline{\theta}_1}(\hat{p}_1)) - U'_{\underline{\theta}_2}(x_{\overline{\theta}_1}(\hat{p}_1))\right]\right\}\frac{dx_{\overline{\theta}_1}(\hat{p}_1)}{d\hat{p}_1} = 0$$

Using (9) and (11), this writes:

$$(1-\alpha)[\hat{p}_1 - (c_1+a_2)] - \alpha\left(\frac{q_2}{q_1}\hat{p}_1 - \frac{\underline{\theta}q_2}{\overline{\theta}q_1}\hat{p}_1\right) = 0 \qquad (13)$$

or

$$\hat{p}_1^d = \frac{(1-\alpha)\,\overline{\theta}q_1\,(c_1+a_2)}{(1-\alpha)\,\overline{\theta}q_1 - \alpha(\overline{\theta}-\underline{\theta})q_2}$$

As second order crossed derivatives are zero, second order conditions write $\frac{\partial^2\Pi_1}{\partial p_1^2} < 0$ and $\frac{\partial^2\Pi_1}{\partial \hat{p}_1^2} < 0$.

$$\begin{aligned}\frac{\partial^2 \Pi_1}{\partial p_1^2} &= 2\frac{dx_{\overline{\theta}_1}(p_1)}{dp_1} + (2p_1 - 2c_1)\frac{d^2x_{\overline{\theta}_1}(p_1)}{dp_1^2} \\ &= 2\frac{dx_{\overline{\theta}_1}(p_1)}{dp_1} < 0 \text{at equilibrium}\end{aligned}$$

$$\begin{aligned}\frac{\partial^2 \Pi_1}{\partial \hat{p}_1^2} &= (1-\alpha)\frac{dx_{\overline{\theta}_1}(\hat{p}_1)}{d\hat{p}_1} - \alpha\left(\frac{q_2}{q_1} - \frac{\underline{\theta}q_2}{\overline{\theta}q_1}\right)\frac{dx_{\overline{\theta}_1}(\hat{p}_1)}{d\hat{p}_1} \\ &\quad + \left[(1-\alpha)[\hat{p}_1 - (c_1+a_2)] - \alpha\left(\frac{q_2}{q_1}\hat{p}_1 - \frac{\underline{\theta}q_2}{\overline{\theta}q_1}\hat{p}_1\right)\right]\frac{d^2x_{\overline{\theta}_1}(\hat{p}_1)}{d\hat{p}_1^2} \\ &= (1-\alpha)\frac{dx_{\overline{\theta}_1}(\hat{p}_1)}{d\hat{p}_1} - \alpha\left(\frac{q_2}{q_1} - \frac{\underline{\theta}q_2}{\overline{\theta}q_1}\right)\frac{dx_{\overline{\theta}_1}(\hat{p}_1)}{d\hat{p}_1} \text{at equilibrium}\end{aligned}$$

$$\frac{\partial^2 \Pi_1}{\partial \hat{p}_1^2} < 0 \Leftrightarrow \frac{\alpha}{1-\alpha} < \frac{\overline{\theta}q_1}{(\overline{\theta}-\underline{\theta})q_2}$$

Program for firm 2 writes

$$\max_{\{p_2,\hat{p}_2\}} \Pi_2(p_2,\hat{p}_2,F_2) = (1-\alpha)\left[\pi_2 + v_{\underline{\theta}_2} - f_2\right]$$

First order conditions write :

$$\begin{aligned}\frac{\partial \Pi_2}{\partial p_2} &= (1-\alpha)^2[2U'_{\underline{\theta}_2}(x_{\underline{\theta}_2}(p_2)) - 2c_2]\frac{dx_{\underline{\theta}_2}(p_2)}{dp_2} = 0 \\ &\Leftrightarrow p_2^d = c_2\end{aligned}$$

$$\frac{\partial \Pi_2}{\partial \widehat{p}_2} = \alpha(1-\alpha)[U'_{\underline{\theta}_2}(x_{\underline{\theta}_2}(\widehat{p}_2)) - (a_1+c_2)]\frac{dx_{\underline{\theta}_2}(\widehat{p}_2)}{d\widehat{p}_2} = 0$$

Using (9) and (10), this writes:

$$\alpha(1-\alpha)[\widehat{p}_2 - (a_1+c_2)]\frac{dx_{\underline{\theta}_2}(\widehat{p}_2)}{d\widehat{p}_2} = 0$$

We derive

$$\hat{p}_2^d = c_2 + a_1$$

Second order conditions write

$$\begin{aligned}\frac{\partial^2 \Pi_2}{\partial p_2^2} &= 2\frac{dx_{\underline{\theta}_2}(p_2)}{dp_2} + (2p_2 - 2c_2)\frac{d^2 x_{\underline{\theta}_2}(p_2)}{dp_2^2} \\ &= 2\frac{dx_{\underline{\theta}_2}(p_2)}{dp_2} < 0 \text{at equilibrium}\end{aligned}$$

$$\begin{aligned}\frac{\partial^2 \Pi_2}{\partial \widehat{p}_2^2} &= \frac{dx_{\underline{\theta}_2}(\hat{p}_2)}{d\hat{p}_2} + (\hat{p}_2 - (c_2+a_1))\frac{d^2 x_{\underline{\theta}_2}(\hat{p}_2)}{d\hat{p}_2^2} \\ &= \frac{dx_{\underline{\theta}_2}(\hat{p}_2)}{d\hat{p}_2} < 0 \text{at equilibrium}\end{aligned}$$

- We now show that this equilibrium candidate is the unique duopoly equilibrium. Let us assume that $(p^*,\hat{p}^*)$ is a unit price equilibrium such that the four constraints $(IR1), (IC1), (IR2), (IC2)$ are compatible or $\Psi(p^*,\hat{p}^*) < 0$. We know from the previous result that the unique equilibrium fees are such that $(IC2)$ and $(IR1)$ are binding. But the best response to those equilibrium fees are $\left(p^d,\hat{p}^d\right)$. We conclude that if $\Psi(p^*,\hat{p}^*) \geq 0$, then $\left(p^d,\hat{p}^d,F_1^d,F_2^d\right)$ is the unique duopoly equilibrium. Otherwise, there does not exist any duopoly equilibrium. ■

6.2 Appendix 2 : proof of proposition 2

The global profit of operator 1 is first increasing and then decreasing with respect to a_1. Let us write the derivative for a_1 close to the marginal cost:

$$\left.\frac{\partial \Pi_1}{\partial a_1}\right|_{a_1=c_1} = (1-\alpha)x_{\underline{\theta}_2} + \alpha(x_{\overline{\theta}_2} - x_{\underline{\theta}_2}) + (1-\alpha)(c_2+c_1)\frac{\overline{\theta}q_1}{\underline{\theta}q_2}\frac{dx_{\underline{\theta}_2}}{d\widehat{p}_2}$$

Three effects can be set out. The first two are positive and respectively affect the access revenue and the incentive rent. The last is negative and represents the call externalities effect. The optimal access charge is above marginal cost iff

$$\frac{\alpha}{1-\alpha} > \frac{x_{\underline{\theta}_2}}{(x_{\overline{\theta}_2} - x_{\underline{\theta}_2})}\left[\frac{\overline{\theta}q_1}{\underline{\theta}q_2}\frac{1}{\xi_{x_{\underline{\theta}_2}}} - 1\right] = \overline{\alpha}$$

We now compute the derivative of the global profit of provider 2 close to the marginal cost c_2.

$$\begin{aligned}\left.\frac{\partial \Pi_2}{\partial a_2}\right|_{a_2=c_2} &= \alpha x_{\overline{\theta}_1} + \alpha\frac{\underline{\theta}q_2}{\overline{\theta}q_1}\widehat{p}_1\frac{dx_{\overline{\theta}_1}}{d\widehat{p}_1}\frac{d\widehat{p}_1}{da_2} > 0 \\ &\Leftrightarrow \quad \frac{\alpha}{1-\alpha} < \left(\frac{\overline{\theta}q_1}{\underline{\theta}q_2} - \xi_{x_{\overline{\theta}_1}}\right)\frac{\underline{\theta}}{(\overline{\theta}-\underline{\theta})} = \underline{\alpha}\end{aligned}$$

Note that $\left(\frac{\overline{\theta}q_1}{\underline{\theta}q_2} - \xi_{x_{\overline{\theta}_1}}\right)\frac{\underline{\theta}}{(\overline{\theta}-\underline{\theta})} < \frac{\overline{\theta}q_1}{(\overline{\theta}-\underline{\theta})q_2}$. ■

6.3 Appendix 3 : proof of lemma 3 and 4

6.3.1 Proof of lemma 3

The first best program under complete information writes

$$\max_{p_i,\hat{p}_i,F_i} \Pi(p_i,\hat{p}_i,F_i) = \alpha(\pi_1 + F_1 - f_1) + (1-\alpha)(\pi_2 + F_2 - f_2)$$

$$s.t.: \quad \begin{array}{ll} v_{\overline{\theta}_1} - F_1 \geq 0 & (IR_1) \\ v_{\underline{\theta}_2} - F_2 \geq 0 & (IR_2)\end{array}$$

Both individual rationality constraints are binding.

First order conditions write:

$$\begin{aligned}\frac{\partial \Pi^m}{\partial p_1} &= \alpha^2[2U'_{\overline{\theta}_1}(x_{\overline{\theta}_1}(p_1)) - 2c_1]\frac{dx_{\overline{\theta}_1}(p_1)}{dp_1} = 0 \\ \frac{\partial \Pi^m}{\partial p_2} &= (1-\alpha)^2[2U'_{\underline{\theta}_2}(x_{\underline{\theta}_2}(p_2)) - 2c_2]\frac{dx_{\underline{\theta}_2}(p_2)}{dp_2} = 0\end{aligned}$$

which writes using (9)

$$p_1^{mc} = c_1$$
$$p_2^{mc} = c_2$$

$$\frac{\partial \Pi^m}{\partial \hat{p}_1} = \alpha(1-\alpha)[U'_{\overline{\theta}_1}(x_{\overline{\theta}_1}(\hat{p}_1)) + U'_{\underline{\theta}_2}(x_{\overline{\theta}_1}(\hat{p}_1)) - (c_1+c_2)]\frac{dx_{\overline{\theta}_1}(\hat{p}_1)}{d\hat{p}_1} = 0$$

Using (9) and (11), this writes:

$$\alpha(1-\alpha)[\hat{p}_1 + \frac{\underline{\theta}q_2}{\overline{\theta}q_1}\hat{p}_1 - (c_1+c_2)]\frac{dx_{\overline{\theta}_1}(\hat{p}_1)}{d\hat{p}_1} = 0$$

We derive

$$\hat{p}_1^{mc} = \frac{(c_1+c_2)\overline{\theta}q_1}{\overline{\theta}q_1 + \underline{\theta}q_2} < c_1 + c_2$$

$$\frac{\partial \Pi^m}{\partial \widehat{p}_2} = \alpha(1-\alpha)[U'_{\underline{\theta}_2}(x_{\underline{\theta}_2}(\widehat{p}_2)) + U'_{\overline{\theta}_1}(x_{\underline{\theta}_2}(\widehat{p}_2)) - (c_1+c_2)]\frac{dx_{\underline{\theta}_2}(\widehat{p}_2)}{d\widehat{p}_2} = 0$$

Using (9) and (10), this writes:

$$\alpha(1-\alpha)[\widehat{p}_2 + \frac{\overline{\theta}q_1}{\underline{\theta}q_2}\widehat{p}_2 - (c_1+c_2)]\frac{dx_{\underline{\theta}_2}(\widehat{p}_2)}{d\widehat{p}_2} = 0$$

We derive

$$\hat{p}_2^{mc} = \frac{(c_1+c_2)\underline{\theta}q_2}{\overline{\theta}q_1 + \underline{\theta}q_2} < c_1 + c_2$$

As second order crossed derivatives are zero, second order conditions write $\frac{\partial^2 \Pi^m}{\partial p_i^2} < 0$ and $\frac{\partial^2 \Pi^m}{\partial \hat{p}_j^2} < 0$, $i, j = 1, 2$:

$$\frac{\partial^2 \Pi^m}{\partial p_i^2} = \alpha^2[(2p_i - 2c_i)\frac{d^2 x_{\overline{\theta}_i}(p_i)}{dp_i^2} + 2\frac{dx_{\overline{\theta}_i}(p_i)}{dp_i}] = 2\frac{dx_{\overline{\theta}_i}(p_i)}{dp_i} < 0$$

$$\begin{aligned}\frac{\partial^2\Pi^m}{\partial\widehat{p}_2^2} &= \alpha(1-\alpha)[\widehat{p}_2+\frac{\overline{\theta}q_1}{\underline{\theta}q_2}\widehat{p}_2-(c_1+c_2)]\frac{d^2x_{\underline{\theta}_2}(\widehat{p}_2)}{d\widehat{p}_2^2}+\alpha(1-\alpha)\frac{\overline{\theta}q_1+\underline{\theta}q_2}{\underline{\theta}q_2}\frac{dx_{\underline{\theta}_2}(\widehat{p}_2)}{d\widehat{p}_2}\\ &= \alpha(1-\alpha)\frac{\overline{\theta}q_1+\underline{\theta}q_2}{\underline{\theta}q_2}\frac{dx_{\underline{\theta}_2}(\widehat{p}_2)}{d\widehat{p}_2}<0\end{aligned}$$

and

$$\begin{aligned}\frac{\partial^2\Pi^m}{\partial\hat{p}_1^2} &= \alpha(1-\alpha)[\hat{p}_1+\frac{\underline{\theta}q_2}{\overline{\theta}q_1}\hat{p}_1-(c_1+c_2)]\frac{d^2x_{\overline{\theta}_1}(\hat{p}_1)}{dp_1^2}+[\frac{\overline{\theta}q_1+\underline{\theta}q_2}{\overline{\theta}q_1}]\frac{dx_{\overline{\theta}_1}(\hat{p}_1)}{dp_1}\\ &= [\frac{\overline{\theta}q_1+\underline{\theta}q_2}{\overline{\theta}q_1}]\frac{dx_{\overline{\theta}_1}(\hat{p}_1)}{d\hat{p}_1}<0\end{aligned}$$

■

6.3.2 Proof of lemma 4

The second best program under asymmetric information writes

$$\begin{aligned}&\max_{p_i,\widehat{p}_i}\alpha\left(\pi_1+v_{\overline{\theta}_1}-(v_{\overline{\theta}_2}-v_{\underline{\theta}_2})-f_1\right)+(1-\alpha)(\pi_2+v_{\underline{\theta}_2}-f_2)\\ &s.t.\quad v_{\overline{\theta}_1}-v_{\underline{\theta}_1}\geq v_{\overline{\theta}_2}-v_{\underline{\theta}_2}\end{aligned}\tag{14}$$

First order conditions write :

$$\begin{aligned}\frac{\partial\Pi^m}{\partial p_1} &= \alpha^2\left[2U'_{\overline{\theta}_1}(x_{\overline{\theta}_1}(p_1))-2c_1\right]\frac{dx_{\overline{\theta}_1}(p_1)}{dp_1}=0\\ &\Leftrightarrow\ p_1^m=c_1\end{aligned}\tag{15}$$

$$\begin{aligned}&\tfrac{\partial\Pi^m}{\partial\hat{p}_1}=\{(1-\alpha)\left[U'_{\overline{\theta}_1}(x_{\overline{\theta}_1}(\hat{p}_1))+U'_{\underline{\theta}_2}(x_{\overline{\theta}_1}(\hat{p}_1))-(c_1+c_2)\right]\\ &-\alpha\left[U'_{\overline{\theta}_2}(x_{\overline{\theta}_1}(\hat{p}_1))-U'_{\underline{\theta}_2}(x_{\overline{\theta}_1}(\hat{p}_1))\right]\}\tfrac{dx_{\overline{\theta}_1}(\hat{p}_1)}{d\hat{p}_1}=0\end{aligned}$$

Using (9) and (11), this writes:

$$(1-\alpha)[\hat{p}_1+\frac{\underline{\theta}q_2}{\overline{\theta}q_1}\hat{p}_1-(c_1+c_2)]-\alpha\left(\frac{q_2}{q_1}\hat{p}_1-\frac{\underline{\theta}q_2}{\overline{\theta}q_1}\hat{p}_1\right)=0\tag{16}$$

We derive

$$\hat{p}_1^m=\frac{(1-\alpha)\,\overline{\theta}q_1\,(c_1+c_2)}{(1-\alpha)(\overline{\theta}q_1+\underline{\theta}q_2)-\alpha(\overline{\theta}-\underline{\theta})q_2}$$

$$\frac{\partial \Pi^m}{\partial p_2} = (1-\alpha)^2[2U'_{\underline{\theta}_2}(x_{\underline{\theta}_2}(p_2)) - 2c_2]\frac{dx_{\underline{\theta}_2}(p_2)}{dp_2}$$
$$-\alpha(1-\alpha)\left[\left(U'_{\bar{\theta}_2}(x_{\bar{\theta}_2}(p_2)) - p_2\right)\frac{dx_{\bar{\theta}_2}(p_2)}{dp_2} - x_{\bar{\theta}_2}(p_2)\right]$$
$$-\alpha(1-\alpha)\left[\left(U'_{\bar{\theta}_2}(x_{\underline{\theta}_2}(p_2)) - 2U'_{\underline{\theta}_2}(x_{\underline{\theta}_2}(p_2)) + p_2\right)\right)\frac{dx_{\underline{\theta}_2}(p_2)}{dp_2} + x_{\underline{\theta}_2}(p_2)\right] = 0$$

Using (9) and (10), this writes:

$$(1-\alpha)[2p_2 - 2c_2]\frac{dx_{\underline{\theta}_2}(p_2)}{dp_2} - \alpha\left[-x_{\bar{\theta}_2}(p_2) + \left(\frac{\bar{\theta}}{\underline{\theta}}p_2 - 2p_2 + p_2\right)\frac{dx_{\underline{\theta}_2}(p_2)}{dp_2} + x_{\underline{\theta}_2}(p_2)\right] = 0$$

$$\Leftrightarrow p_2\frac{dx_{\underline{\theta}_2}(p_2)}{dp_2}\left[2(1-\alpha) - \alpha\frac{(\bar{\theta}-\underline{\theta})}{\underline{\theta}}\right] = 2c_2(1-\alpha)\frac{dx_{\underline{\theta}_2}(p_2)}{dp_2} - \alpha\left(x_{\bar{\theta}_2}(p_2) - x_{\underline{\theta}_2}(p_2)\right)$$

or

$$p_2^m = \frac{2(1-\alpha)c_2 - \alpha(x_{\bar{\theta}_2} - x_{\underline{\theta}_2})\frac{1}{\frac{dx_{\underline{\theta}_2}}{dp_2}}}{2(1-\alpha) - \alpha(\frac{\bar{\theta}-\underline{\theta}}{\underline{\theta}})}$$

$$\frac{\partial \Pi^m}{\partial \hat{p}_2} = \alpha(1-\alpha)\left[U'_{\underline{\theta}_2}(x_{\underline{\theta}_2}(\widehat{p}_2)) + U'_{\bar{\theta}_1}(x_{\underline{\theta}_2}(\widehat{p}_2)) - (c_1+c_2)\right]\frac{dx_{\underline{\theta}_2}(\widehat{p}_2)}{d\widehat{p}_2}$$
$$-\alpha^2\left[\left(U'_{\bar{\theta}_2}(x_{\bar{\theta}_2}(\widehat{p}_2)) - \widehat{p}_2\right)\frac{dx_{\bar{\theta}_2}(\widehat{p}_2)}{d\widehat{p}_2} - x_{\bar{\theta}_2}(\widehat{p}_2) - \left(U'_{\underline{\theta}_2}(x_{\underline{\theta}_2}(\widehat{p}_2)) - \widehat{p}_2\right)\frac{dx_{\underline{\theta}_2}(\widehat{p}_2)}{d\widehat{p}_2} + x_{\underline{\theta}_2}(\widehat{p}_2)\right] = 0$$

Using (9) and (10), this writes:

$$(1-\alpha)\left[\widehat{p}_2 + \frac{\bar{\theta}q_1}{\underline{\theta}q_2}\widehat{p}_2 - (c_1+c_2)\right]\frac{dx_{\underline{\theta}_2}(\widehat{p}_2)}{d\widehat{p}_2} + \alpha\left[x_{\bar{\theta}_2}(\widehat{p}_2) - x_{\underline{\theta}_2}(\widehat{p}_2)\right] = 0$$

$$\Leftrightarrow \widehat{p}_2^m(1-\alpha)\left(\frac{\bar{\theta}q_1+\underline{\theta}q_2}{\underline{\theta}q_2}\right) = (1-\alpha)(c_1+c_2) - \alpha\left[x_{\bar{\theta}_2}(\widehat{p}_2) - x_{\underline{\theta}_2}(\widehat{p}_2)\right]\frac{1}{\frac{dx_{\underline{\theta}_2}(\widehat{p}_2)}{d\widehat{p}_2}}$$

As second order crossed derivatives are zero, second order conditions write $\frac{\partial^2\Pi_1}{\partial p_i^2} < 0$ and $\frac{\partial^2\Pi_1}{\partial \widehat{p}_i^2} < 0$, $i = 1, 2$.

$$\frac{\partial^2\Pi_1}{\partial p_1^2} = 2\frac{dx_{\overline{\theta}_1}(p_1)}{dp_1} + (2p_1 - 2c_1)\frac{d^2x_{\overline{\theta}_1}(p_1)}{dp_1^2}$$
$$= 2\frac{dx_{\overline{\theta}_1}(p_1)}{dp_1} < 0 \text{using } (15)$$

$$\frac{\partial^2\Pi_1}{\partial \widehat{p}_1^2} = \left[(1-\alpha)\frac{\overline{\theta}q_1 + \underline{\theta}q_2}{\overline{\theta}q_1} - \alpha\frac{(\overline{\theta}-\underline{\theta})q_2}{\overline{\theta}q_1}\right]\frac{dx_{\overline{\theta}_1}(\hat{p}_1)}{d\hat{p}_1}$$
$$+\left[(1-\alpha)[\hat{p}_1 + \frac{\underline{\theta}q_2}{\overline{\theta}q_1}\hat{p}_1 - (c_1+c_2)] - \alpha\left(\frac{q_2}{q_1}\hat{p}_1 - \frac{\underline{\theta}q_2}{\overline{\theta}q_1}\hat{p}_1\right)\right]\frac{d^2x_{\overline{\theta}_1}(\hat{p}_1)}{d\hat{p}_1^2}$$
$$= \left[(1-\alpha)\frac{\overline{\theta}q_1 + \underline{\theta}q_2}{\overline{\theta}q_1} - \alpha\frac{(\overline{\theta}-\underline{\theta})q_2}{\overline{\theta}q_1}\right]\frac{dx_{\overline{\theta}_1}(\hat{p}_1)}{d\hat{p}_1}\text{using } (16)$$

$$\frac{\partial^2\Pi_1}{\partial \widehat{p}_1^2} < 0 \Leftrightarrow \frac{\alpha}{1-\alpha} < \frac{\overline{\theta}q_1 + \underline{\theta}q_2}{(\overline{\theta}-\underline{\theta})q_2}$$

Note that this condition is the same than the one which guarantees that $\hat{p}_1^m > 0$.

$$\frac{\partial^2\Pi^m}{\partial p_2^2} = \left[2(1-\alpha) - \alpha\frac{\overline{\theta}}{\underline{\theta}}\right]\frac{dx_{\underline{\theta}_2}(p_2)}{dp_2} + \frac{d^2x_{\underline{\theta}_2}(p_2)}{dp_2^2}\frac{\left(\alpha(x_{\underline{\theta}_2}(p_2) - x_{\overline{\theta}_2}(p_2))\right)}{dx_{\underline{\theta}_2}(p_2)/dp_2} + \alpha\frac{dx_{\overline{\theta}_2}(p_2)}{dp_2} < 0$$

which can be written close to p_2^m

$$\left(\frac{d^2x_{\underline{\theta}_2}(p_2^m)}{dp_2^2}\right) / \left(-\frac{dx_{\underline{\theta}_2}(p_2^m)}{dp_2}\right) < \frac{-\left(2(1-\alpha) - \alpha\frac{\overline{\theta}}{\underline{\theta}}\right)\frac{dx_{\underline{\theta}_2}(p_2^m)}{dp_2} - \alpha\frac{dx_{\overline{\theta}_2}(p_2^m)}{dp_2}}{\alpha(x_{\overline{\theta}_2}(p_2^m) - x_{\underline{\theta}_2}(p_2^m))} \tag{17}$$

Profit is then locally concave with respect to p_2 iff demand is not too convex.

The second order condition for $\widehat{p}_2$ writes

$$\frac{\partial^2\Pi^m}{\partial \widehat{p}_2^2} = \begin{array}{l}(1-\alpha)\left[\frac{\bar{\theta}q_1 + \underline{\theta}q_2}{\underline{\theta}q_2}\widehat{p}_2 - (c_1+c_2)\right]\frac{d^2x_{\underline{\theta}_2}(\widehat{p}_2)}{d\widehat{p}_2^2} + \left[(1-\alpha)\frac{\overline{\theta}q_1 + \underline{\theta}q_2}{\underline{\theta}q_2} - \alpha\right]\frac{dx_{\underline{\theta}_2}(\widehat{p}_2)}{d\widehat{p}_2} \\ +\alpha\frac{dx_{\overline{\theta}_2}(\widehat{p}_2)}{d\widehat{p}_2} < 0\end{array}$$

which can be rewritten at the neighboring of $\widehat{p}_2^m$

$$\left(\frac{d^2 x_{\underline{\theta}_2}(\widehat{p}_2^m)}{d\widehat{p}_2^2}\right) / \left(-\frac{dx_{\underline{\theta}_2}(\widehat{p}_2^m)}{d\widehat{p}_2}\right) < \frac{\left((1-\alpha)\frac{\overline{\theta}q_1+\underline{\theta}q_2}{\underline{\theta}q_2}-\alpha\right)\left(-\frac{dx_{\underline{\theta}_2}(\widehat{p}_2^m)}{d\widehat{p}_2}\right)-\alpha\frac{dx_{\overline{\theta}_2}(\widehat{p}_2^m)}{d\widehat{p}_2}}{\alpha(x_{\overline{\theta}_2}(\widehat{p}_2^m)-x_{\underline{\theta}_2}(\widehat{p}_2^m))} \tag{18}$$

As previously, this condition can be interpreted in terms of curvature of the demand function $x_{\underline{\theta}_2}$. Profit is locally concave iff the demand function is not too convex. Otherwise, the monopoly charges $\widehat{p}_2 = +\infty$. ■

6.4 Appendix 4. Proof of lemma and proposition 5

6.4.1 Proof of lemma 5

- $\widehat{p}_1^m < \widehat{p}_1^d\left(a_2^d\right)$

$$\begin{aligned}
\widehat{p}_1^d(a_2) &= \frac{(1-\alpha)\,\overline{\theta}q_1}{(1-\alpha)\,\overline{\theta}q_1-\alpha(\overline{\theta}-\underline{\theta})q_2}(c_1+a_2) \\
\widehat{p}_1^m &= \frac{(1-\alpha)\overline{\theta}q_1(c_1+c_2)}{(1-\alpha)(\overline{\theta}q_1+\underline{\theta}q_2)-\alpha(\overline{\theta}-\underline{\theta})q_2}
\end{aligned}$$

$$\widehat{p}_1^m < \widehat{p}_1^d\left(a_2^d\right) \Leftrightarrow a_2^d > \underline{a}_2 = c_2 - \frac{(1-\alpha)\underline{\theta}q_2(c_1+c_2)}{(1-\alpha)(\overline{\theta}q_1+\underline{\theta}q_2)-\alpha(\overline{\theta}-\underline{\theta})q_2}$$

In order to compare the position of $\underline{a}_2$ with respect to a_2^d, we compute

$$\left.\frac{\partial\Pi_2^d}{\partial a_2}\right|_{a_2=\underline{a}_2} = \alpha x_{\overline{\theta}_1} + \alpha(\underline{a}_2-c_2)\frac{dx_{\overline{\theta}_1}}{d\widehat{p}_1}\frac{d\widehat{p}_1^d}{da_2} + \alpha\frac{\underline{\theta}q_2}{\overline{\theta}q_1}\widehat{p}_1^d(\underline{a}_2)\frac{dx_{\overline{\theta}_1}}{d\widehat{p}_1}\frac{d\widehat{p}_1^d}{da_2}$$

At the point $a_2 = \underline{a}_2$, $\widehat{p}_1^d(\underline{a}_2) = \widehat{p}_1^m$. We obtain

$$\begin{aligned}
\left.\frac{\partial\Pi_2^d}{\partial a_2}\right|_{a_2=\underline{a}_2} &= \alpha x_{\overline{\theta}_1} + \alpha\left[-\frac{(1-\alpha)\underline{\theta}q_2(c_1+c_2)}{(1-\alpha)(\overline{\theta}q_1+\underline{\theta}q_2)-\alpha(\overline{\theta}-\underline{\theta})q_2}\right]\frac{dx_{\overline{\theta}_1}}{d\widehat{p}_1}\frac{d\widehat{p}_1^d}{da_2} \\
&\quad +\alpha\frac{\underline{\theta}q_2}{\overline{\theta}q_1}\frac{(1-\alpha)\overline{\theta}q_1(c_1+c_2)}{(1-\alpha)(\overline{\theta}q_1+\underline{\theta}q_2)-\alpha(\overline{\theta}-\underline{\theta})q_2}\frac{dx_{\overline{\theta}_1}}{d\widehat{p}_1}\frac{d\widehat{p}_1^d}{da_2} \\
&= \alpha x_{\overline{\theta}_1} > 0
\end{aligned}$$

We conclude that $a_2^d > \underline{a}_2$ and then $\widehat{p}_1^m < \widehat{p}_1^d$

- $\widehat{p}_2^m < \widehat{p}_2^d$

$$
\begin{aligned}
\widehat{p}_2^d(a_1) &= c_2 + a_1 \\
\hat{p}_2^m &= \frac{\underline{\theta} q_2}{\overline{\theta} q_1 + \underline{\theta} q_2} \left[(c_1 + c_2) - \frac{\alpha}{1-\alpha} \left(x_{\overline{\theta}_2}(\widehat{p}_2^m) - x_{\underline{\theta}_2}(\widehat{p}_2^m) \right) \frac{1}{\frac{dx_{\underline{\theta}_2}(\widehat{p}_2^m)}{d\widehat{p}_2}} \right]
\end{aligned}
$$

$$
\begin{aligned}
\widehat{p}_2^m &< \widehat{p}_2^d\left(a_1^d\right) \Leftrightarrow \\
a_1^d &> \underline{a}_1 = c_1 - \frac{\overline{\theta} q_1}{\overline{\theta} q_1 + \underline{\theta} q_2}(c_1 + c_2) - \frac{\alpha}{1-\alpha} \frac{\underline{\theta} q_2}{\overline{\theta} q_1 + \underline{\theta} q_2} \left[x_{\overline{\theta}_2}(\widehat{p}_2^m) - x_{\underline{\theta}_2}(\widehat{p}_2^m) \right] \frac{1}{\frac{dx_{\underline{\theta}_2}(\widehat{p}_2^m)}{d\widehat{p}_2}}
\end{aligned}
$$

In order to compare a_1^d and $\underline{a}_1$ we compute $\left.\frac{\partial \Pi_1}{\partial a_1}\right|_{a_1 = \underline{a}_1}$

$$
\begin{aligned}
\frac{\partial \Pi_1}{\partial a_1} &= (1-\alpha) x_{\underline{\theta}_2}\left(\widehat{p}_2^d(a_1)\right) + (1-\alpha)(a_1 - c_1) \frac{dx_{\underline{\theta}_2}\left(\widehat{p}_2^d\right)}{d\widehat{p}_2} \frac{d\widehat{p}_2^d}{da_1} \\
&\quad + \alpha \left[x_{\overline{\theta}_2}\left(\widehat{p}_2^d(a_1)\right) - x_{\underline{\theta}_2}\left(\widehat{p}_2^d(a_1)\right) \right] \frac{dx_{\underline{\theta}_2}\left(\widehat{p}_2^d\right)}{d\widehat{p}_2} \frac{d\widehat{p}_2^d}{da_1} + (1-\alpha) \frac{\overline{\theta} q_1}{\underline{\theta} q_2} \widehat{p}_2^d(a_1) \frac{dx_{\underline{\theta}_2}\left(\widehat{p}_2^d\right)}{d\widehat{p}_2} \frac{d\widehat{p}_2^d}{da_1}
\end{aligned}
$$

with $\frac{d\widehat{p}_2^d}{da_1} = 1$. At the point $a_1 = \underline{a}_1$, we have $\widehat{p}_2^d(\underline{a}_1) = \widehat{p}_2^m$ or $\underline{a}_1 = \widehat{p}_2^m - c_2$.

$$
\begin{aligned}
\left.\frac{\partial \Pi_1}{\partial a_1}\right|_{a_1 = \underline{a}_1} &= (1-\alpha) x_{\underline{\theta}_2}(\widehat{p}_2^m) + \alpha (x_{\overline{\theta}_2}(\widehat{p}_2^m) - x_{\underline{\theta}_2}(\widehat{p}_2^m)) \frac{dx_{\underline{\theta}_2}(\widehat{p}_2^m)}{d\widehat{p}_2} \\
&\quad + (1-\alpha) \frac{dx_{\underline{\theta}_2}(\widehat{p}_2^m)}{d\widehat{p}_2} \left(\widehat{p}_2^m - c_2 - c_1 + \frac{\overline{\theta} q_1}{\underline{\theta} q_2} \widehat{p}_2^m \right) \\
&= (1-\alpha) x_{\underline{\theta}_2}(\widehat{p}_2^m) + \alpha (x_{\overline{\theta}_2}(\widehat{p}_2^m) - x_{\underline{\theta}_2}(\widehat{p}_2^m)) \frac{dx_{\underline{\theta}_2}(\widehat{p}_2^m)}{d\widehat{p}_2} \\
&\quad + (1-\alpha) \frac{dx_{\underline{\theta}_2}(\widehat{p}_2^m)}{d\widehat{p}_2} \left(\frac{\overline{\theta} q_1 + \underline{\theta} q_2}{\underline{\theta} q_2} \widehat{p}_2^m - (c_2 + c_1) \right) \\
&= (1-\alpha) x_{\underline{\theta}_2}(\widehat{p}_2^m) + \alpha (x_{\overline{\theta}_2}(\widehat{p}_2^m) - x_{\underline{\theta}_2}(\widehat{p}_2^m)) \frac{dx_{\underline{\theta}_2}(\widehat{p}_2^m)}{d\widehat{p}_2} \\
&\quad - \alpha \left(x_{\overline{\theta}_2}(\widehat{p}_2^m) - x_{\underline{\theta}_2}(\widehat{p}_2^m) \right) \frac{dx_{\underline{\theta}_2}(\widehat{p}_2^m)}{d\widehat{p}_2} \\
&= (1-\alpha) x_{\underline{\theta}_2}(\widehat{p}_2^m) > 0
\end{aligned}
$$

We conclude that $\widehat{p}_2^m < \widehat{p}_2^d$. ■

6.4.2 Proof of proposition 6

(i) The incentive rent $v_{\overline{\theta}_2} - v_{\underline{\theta}_2}$ writes

$$v_{\overline{\theta}_2} - v_{\underline{\theta}_2} = (1-\alpha)\, A_1(p_2) + \alpha A_2(\widehat{p}_2) + \alpha A_3(\hat{p}_1)$$

with

$$\begin{aligned} A_1(p_2) &= [U_{\overline{\theta}_2}(x_{\overline{\theta}_2}(p_2)) + U_{\overline{\theta}_2}(x_{\underline{\theta}_2}(p_2)) - p_2 x_{\overline{\theta}_2}(p_2)] - [U_{\underline{\theta}_2}(x_{\underline{\theta}_2}(p_2)) - p_2 x_{\underline{\theta}_2}(p_2) + U_{\underline{\theta}_2}(x_{\underline{\theta}_2}(p_2))] \\ A_2(\widehat{p}_2) &= [U_{\overline{\theta}_2}(x_{\overline{\theta}_2}(\widehat{p}_2)) - \widehat{p}_2 x_{\overline{\theta}_2}(\widehat{p}_2)] - [U_{\underline{\theta}_2}(x_{\underline{\theta}_2}(\hat{p}_2)) - \hat{p}_2 x_{\underline{\theta}_2}(\hat{p}_2)] \\ A_3(\widehat{p}_1) &= U_{\overline{\theta}_2}(x_{\overline{\theta}_1}(\widehat{p}_1)) - U_{\underline{\theta}_2}(x_{\overline{\theta}_1}(\hat{p}_1)) \end{aligned}$$

We denote $\Delta A_j(p) = A_j(p^m) - A_j\left(p^d\right),\ j = 1,.2,3$ which is a decreasing function with respect to p.

The incentive rent is higher after the merger iff

$$\frac{\alpha}{1-\alpha} > \frac{-\Delta A_1(p_2)}{\Delta A_2(\widehat{p}_2) + \Delta A_3(\hat{p}_1)} = \tilde{\alpha}$$

Since $p_2^m > p_2^d$, $\widehat{p}_1^m < \widehat{p}_1^d$ *and* $\widehat{p}_2^m < \widehat{p}_2^d$, we have $\Delta A_1(p_2) < 0$, $\Delta A_2(\widehat{p}_2) > 0$, $\Delta A_3(\hat{p}_1) > 0$ and then $\tilde{\alpha} > 0$.

(ii)

$$\Delta W = \alpha^2 \Delta B_1(p_1) + (1-\alpha)^2 \Delta B_2(p_2) + \alpha(1-\alpha)\Delta B_3(\hat{p}_1) + \alpha(1-\alpha)\Delta B_4(\hat{p}_2)$$

with

$$\begin{aligned} B_1(p_1) &= [2U_{\overline{\theta}_1}(x_{\overline{\theta}_1}(p_1)) - 2c_1 x_{\overline{\theta}_1}(p_1)] \\ B_2(p_2) &= [2U_{\underline{\theta}_2}(x_{\underline{\theta}_2}(p_2)) - 2c_2 x_{\underline{\theta}_2}(p_2)] \\ B_3(\hat{p}_1) &= [U_{\overline{\theta}_1}(x_{\overline{\theta}_1}(\hat{p}_1)) + U_{\underline{\theta}_2}(x_{\overline{\theta}_1}(\hat{p}_1)) - (c_1 + c_2)\, x_{\overline{\theta}_1}(\hat{p}_1)] \\ B_4(\hat{p}_2) &= [U_{\underline{\theta}_2}(x_{\underline{\theta}_2}(\hat{p}_2)) + U_{\overline{\theta}_1}(x_{\underline{\theta}_2}(\hat{p}_2)) - (c_1 + c_2)\, x_{\underline{\theta}_2}(\hat{p}_2)] \\ \Delta B_j(p) &= B_j(p^m) - B_j\left(p^d\right), \quad j = 1, ..., 4 \end{aligned}$$

$\Delta B_j(p)$ is a decreasing function with respect to p.

Since $p_1^m = p_1^d$, $p_2^m > p_2^d$, $\widehat{p}_1^m < \widehat{p}_1^d$ *and* $\widehat{p}_2^m < \widehat{p}_2^d$, we have $\Delta B_1(p_1) = 0$, $\Delta B_2(p_2) < 0$, $\Delta B_3(\widehat{p}_1) > 0$, $\Delta B_4(\hat{p}_2) > 0$.

We deduce that

$$\Delta W > 0 \Leftrightarrow \frac{\alpha}{1-\alpha} > \frac{-\Delta B_2(p_2)}{\Delta B_3(\hat{p}_1) + \Delta B_4(\hat{p}_2)} = \hat{\alpha} > 0$$

Since for $\frac{\alpha}{1-\alpha} = \tilde{\alpha}$, $\Delta W > 0$ it is straightforward that $0 < \hat{\alpha} < \tilde{\alpha}$. ■