Scenarios, probability and possible futures

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Abstract

This paper provides an introduction to the mathematical theory of possibility, and examines how this tool can contribute to the analysis of far distant futures. The degree of mathematical possibility of a future is a number between O and 1. It quantifies the extend to which a future event is implausible or surprising, without implying that it has to happen somehow. Intuitively, a degree of possibility can be seen as the upper bound of a range of admissible probability levels which goes all the way down to zero. Thus, the proposition 'The possibility of X is $\Pi(X)$' can be read as 'The probability of X is not greater than $\Pi(X)$'.

Possibility levels offers a measure to quantify the degree of unlikelihood of far distant futures. It offers an alternative between forecasts and scenarios, which are both problematic. Long range planning using forecasts with precise probabilities is problematic because it tends to suggests a false degree of precision. Using scenarios without any quantified uncertainty levels is problematic because it may lead to unjustified attention to the extreme scenarios.

This paper further deals with the question of extreme cases. It examines how experts should build a set of two to four well contrasted and precisely described futures that summarizes in a simple way their knowledge. Like scenario makers, these experts face multiple objectives: they have to anchor their analysis in credible expertise; depict though-provoking possible futures; but not so provocative as to be dismissed out-of-hand. The first objective can be achieved by describing a future of possibility level 1. The second and third objective, however, balance each other. We find that a satisfying balance can be achieved by selecting extreme cases that do not rule out equiprobability. For example, if there are three cases, the possibility level of extremes should be about 1/3.

Keywords: Futures, futurible, scenarios, possibility, imprecise probabilities, uncertainty, fuzzy logic

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1 Introduction

This paper is written for the long range planners who have not been exposed to the theory of possibility, and are puzzled by requests for 'probabilities of scenarios'. Recognizing that probability theory is more often than not irrelevant to quantifying the degree of uncertainty about distant futures, the paper argues that this need could generally be satisfied with quantitative possibility levels.

The paper is organized as follows. Section 2 reviews the problem with probabilities in Futures Studies. It is generally agreed that there should be a distinction between *Scenarios* and *Forecasts*. Both are detailed descriptions of a system's future, with the conventional difference that a set of scenarios is presented without quantifying any degree of confidence or likelihood, while forecasts are assigned a probability distribution. We argue that making this distinction does not really solve the recurring controversy about whether and how to quantify uncertainty for far distant futures.

Section 3 discusses the meaning of the sentence "The possibility level of future f is π", where π is a number between 0 and 1. It calls *Futurible* a future which is assigned a possibility level, and shows that the concept is missing from previous uses of fuzzy modeling in Futures Studies, which were more interested in dynamic system modeling and experts elicitation.

Section 4 deals with the question "At which level of possibility should the futuribles be selected?". We examine from a normative point of view how the experts should build a set of two to four well contrasted and precisely described futuribles that summarizes in a simple way complex information. We argue that the set should contain at least one future at possibility level 1 and that if it contains extreme scenarios, their possibility should be $1/n$.

2 Uncertainty in Future Studies

2.1 The sempiternal demand for probabilities

The Intergovernmental Panel on Climate Change [IPCC, 2000] elaborated long-term greenhouse gas emissions scenarios to help assess the urgency of action pertaining to climatic change. One important purpose of these scenarios is to drive global ocean-atmosphere general circulation models. This is why they had to be quantitative, and not only specify the political framework, but also tabulate precise time series of future emissions disaggregated by world region. Even if some combinations of values which lead to high emissions, such as high per capita income growth and high population growth, appear less likely than other combinations, IPCC did not report any likelihood considerations. All the scenarios were considered 'equally sound'.

Using these scenarios led IPCC to report a range of global warming over the next century from 1.4 to 5.8 °C. This turned out to be controversial, as it dramatically revised the top-range value which was previously 3.5 °C, a revision that was not driven by much new scientific knowledge. This figure of 5.8 °C appears to result from the combination of the new high emissions scenario with the unchanged high parameter of climate sensitivity. That led to controversies illustrating a recurring debate between the makers and the users of scenarios:

- Schneider [2001] and others argued that the absence of any probability assignment would lead to confusion, as users select arbitrary scenarios or assume equiprobability. As a remedy, there has been a steady flow of literature estimating the Probability Density Function of global warming in 2100. Reilly et al. [2001] estimated that the 90% confidence limits were 1.1 to 4.5 °C, while Wigley and Raper [2001] found 1.7 to 4.9 °C for the same 1990 to 2100 warming.
- Grübler and Nakićenović [2001] and others argued that good scientific arguments preclude determining the probability of occurrence or the likelihood of events that far in the future. According to the IPCC report's lead authors, no method of assigning probabilities to a 100-year climate forecast was sufficiently widely accepted and documented to pass the review process. They underlined the difficulty of assigning reliable probabilities to socioeconomic trends in the latter half of the 21st century, the difficulty of obtaining consensus range for quantiles like climate sensitivity, and the possibility of a nonlinear geophysical response.

This IPCC case is only one recent manifestation of a deeper problem. The need to quantify the level of truth of each scenario is a source of permanent tension in Futures Studies. Decision-makers often demand quantification of uncertainty, but consultants in strategic scenario planning are reluctant to probabilize. As reported in Best [1991], Adam Kahane from Shell explicits well the problem with probabilities:

> We don't assign probabilities to our scenarios, for several reasons. First, we intentionally write several scenarios that are more or less equally plausible, so that none is dismissed out of hand. Second, by definition, any given scenario has only an infinitesimal probability of being right because so many variations are possible. Third, the reason to be hesitant about all scenario quantification — not probabilities, economic growth rate or whatever — is that there is a very strong tendency for people to grab onto the numbers and ignore the more important conceptual or structural messages.

P. Wack concludes more sanguinely on probability:

> But I have a strong feeling that it will be poisonous and will contaminate the logic of scenarios.

To which David Kline, supervisor of the gas forecasting and model development unit of the California Energy Commission, respectfully disagreed and added:

> Trying to bridge just this kind of gap [. . .] represents one of the most important intellectual and practical endeavors of our time.

2.2 Probability is not of the essence

Many specialists of the Bayesian school argue that probabilities can and should be used because they allow to model any subjective judgment of plausibility degree. Yet the persistence of the controversy on scenarios versus forecasts clearly shows that this Bayesian 'probability applies to everything' approach is not convincing for many specialists. Besides the arbitrariness of deciding which theory is interesting, there are indeed many useful results and technological applications of the fuzzy logic, for example in process control and expert elicitation.

One way to analyze that controversy on 'The probability of scenarios' is to frame the activity of Futures Studies as a situation of communication from analysts A to business people B. Scenarios and Forecasts are two different communication tools to make B aware of ideas that A regards as actually or potentially valid. Whether scenarios should be probabilized is just a question of terminology, in this paper the convention is *to define* scenarios as futures without probabilities. The interesting question is to know when to use a given tool.

In order to discuss in which circumstances scenarios may be more appropriate than forecasts, consider the following taxonomy of ignorance situations. These are only some of those discussed in Smithson [1988, Fig. 1.1] :

Surprise: A pattern which is discrepant from everything one has been exposed to before. Like many physiological stimuli, surprise has opposite effects at low and high doses. A small discrepancy is likely to recruit attention: novelty is necessary for learning. But a large discrepancy leads to pure and simple ignorance: a situation that does not fit into any existing schemata cannot be recognized.

Taboos: Things that should not be known, or even asked about.

Strategic uncertainty: All kinds of disinformation, lies and fallacies that are at the core of interactions when people and organizations compete with each others.

Randomness: The prototype of randomness is to draw one card from a well mixed deck so that all outcomes can be considered symmetrically. Some repetitive events such as the toss of a dice prove to be in reasonably close agreement with the mathematical concept of independently repeated random events [Savage, 1954, p. 3]. Probability is the well known historical approach to modeling randomness.

Vagueness: A characteristic of natural language, the prototype being the statement that someone is 'Tall'. Fuzzy theory was elaborated to process information given in natural language.

Terminology is conventional and inevitably arbitrary to some extent. There are probably as many definitions of the word 'Risk' as there are risk analysts. While the categorization above relates to the interesting question 'What is Risk

?', it is not an attempt towards a comprehensive taxonomy. The point is only that there are many kinds of ignorance to deal with. To each kind of ignorance corresponds a class of methods, mathematical or not, which are regarded as more or less appropriate by a given community of practice. Depending on the kind of ignorance A and/or B wants to reduce, different communication tools will be appropriate.

One of the main purposes of scenarios is to prevent surprises by training the decision-makers' recognition ability. Surprise possibly leads to very inadequate decisions. For example, at 07:02 December 7, 1941 the radar operators J. Lockard and G. Elliot tracked a very large reflexion 136 miles north of their position in Oahu. This was the first use of radar in warfare by U.S. Military Forces, and the untrained lieutenant on duty dismissed their report. One hour later, 183 Japanese airplanes were attacking Pearl Harbor. Extending the field of perception gives a competitive advantage by allowing the decision-makers to respond faster to their changing environment. This explains why it makes sense to present the scenarios as easy-to-remember narrative storylines: stories in which situations are rich in concrete details and create emotions are likely to be more efficient at making a long lasting human impression.

Futures studies are also used to explore strategic uncertainties, for example by playing war games in military planning. Game theory can be regarded as the most appropriate mathematical method to model this kind of situations. Arguably, it is more fundamental than probability to describe the workings of some social systems such as the economy.

The study of taboos in Anthropology and Sociology rarely uses probabilities or anything quantitative, preferring more narrative methods.

Why is the idea that probabilities can be used to quantify degrees of likelihood in all kinds of situations so common? This paper is written on the premise that it is not because the majority of people are strict Bayesians, but because they are simply unaware of the alternatives, and common sense prescribes to stick to a traditional theory that works well enough. That could explain :

- Unawareness about other kinds of ignorance. To some extent the privileged focus on probabilities can be related to the huge practical success of Shannon [1948]'s mathematical theory of communication. However, that approach does not relate to any human dimension at all:

  > Semantics aspects of the communication are irrelevant to the engineering problem.

  The above shows that social sciences are interested in other issues than fighting white noise.

- Unawareness that there are tools to deal with ignorance beyond randomness. The next section presents a few of those.

Unawareness tends to cause exclusion of non-probabilistic kinds of ignorance from the field of scientific inquiry. But negotiating on the meaning of 'scientific' and on the perimeter of the system studied would leave little doubt that the subject matter of Futures Studies involves social systems, therefore has a need to describe situations of ignorance beyond randomness. This paper proposes to do so by using possibility theory.

2.3 Fuzzy modeling and scenarios

The algebra of possibilities can not be separated from the fuzzy theory, and there is a significant body of literature on the use of fuzzy modeling to study the evolution of dynamic systems. Two specific applications to scenario building will be discussed, based respectively on fuzzy cognitive maps and on fuzzy system dynamics.

Misani [1997] proposes to use fuzzy cognitive maps to build scenarios. A fuzzy cognitive map is an oriented graph like a classical influence diagram, where the strength of causal relationships between uncertain variables is quantified. The specificity of fuzzy cognitive maps is that each variable x_i is represented by an activation level in [0, 1], and the intensity of each causal link is represented by a number $l_{i,j}$ in [-1, 1]. Such a map can be compared to a Bayesian network stripped of all its probabilistic machinery. It is very much like an associative neural network, with the difference that the topology of the cognitive map reflects knowledge about the working of the system, while the neural network's topology is given a priori.

These maps are used like neural networks, by iteration. The iterating rule is that the activation level x_i of a node i is determined by the activation level of its parents $pa(i)$ and by the strength of the link. Formally, $x_i(t+1)$ is a function of $\sum_{j \in pa(i)} x_j(t) l_{i,j}$. Iterations are performed until a stable state is found, and the trajectory represents a qualitative simulation of the system dynamics. Different initial configurations may reveal different final stable patterns, each of which can be used to write a consistent scenario.

Canarelli [1997] proposes a conceptually similar approach, altough more flexible. The knowledge base has two parts. The first part represents the system dynamics. It is a collection of linguistic rules such as 'if the standard of living is high, then car ownership is high'. The second part represents linguistic values of the variables. It is a collection of possibility distributions defining to which extent 'the standard of living in the area of study is high' given the numerical value of the underlying variables (for example income).

That approach is similar to fuzzy cognitive maps in that the evolution of the system is computed by iterating from one date to the next. The additional flexibility comes from the fuzzy inference unit, which is more sophisticated than simply a neuron-like linear combination of the inputs. An essential difference of the scenario building method proposed by Canarelli is that it looks only at the transitory dynamics, not at the final pattern. Thus the initial state is fixed to whatever is observed in reality, and the dynamics is governed by exogenously specified policy variables. Each scenario corresponds to a set of specifications for the policy variables.

Both these methods allow for rapid model development based on expert's knowledge. As practical tools for approximate reasoning, they potentially offer interesting ways to build futuribles. Historically, the kind of mathematics used above has been of interest for two distinct epistemic communities. The applications of fuzzy modeling presented above can be seen as deriving from Zadeh [1978]'s ideas about 'level of confidence' in fuzzy theory, which has spread in a community led by engineers and computer scientists. The other community is characterized by social scientists, who would trace possibility to Shackle [1953]'s notion of 'potential surprise' and claim a much older interest on rationality and decision-making in Economics. This article rather belongs to the latter school. While applying the methods presented in section 4 below could benefit from advances in fuzzy modeling, to understand them better it is necessary to review mathematical and philosophical foundations of what is a level of possibility.

3 Degrees of possibility: the mathematical theory

This section discusses the definition of the degree of possibility of a future. The first part 3.1 gives a taste of the basic elements and interpretations of possibility theory. Next, 3.2 presents de Finetti [1937] interpretation defining possibility levels using gambling odds. Finally, 3.3 formalizes this view of possibility using the concept of upper probability and precaution.

The discussion mostly draws upon Dubois and Prade [1998], especially section 5.6 on possibility as an upper bound on probability, and on Walley [1991, sec. 3.8]'s canonical correspondences between probability sets and preference relations. It is argued that the relation 'future f is more probable than future g' is a partial order, so that even though the possibility $\pi(f)$ of future f is 1 and $\pi(g) = 0.6$, it can not be said that 'f is more probable than g'.

3.1 Possibility 101

Let Ω denote the universe of discourse, that is the set of all futures that can be described. Let x be a variable ranging on Ω (for example x is the global warming at the end of the century, in °C). Denote $\pi_x(f)$ the possibility level that $x = f$. Dubois and Prade [1998] explain that:

> A possibility distribution represents a state of knowledge distinguishing what is plausible from what is less plausible, what is the normal course of things from what is not, what is surprising from what is expected. The function π_x represents a flexible restriction of the values of x with the following conventions:
>
> $\pi_x(f) = 0$ means that $x = f$ is impossible.
>
> $\pi_x(f) = 1$ means that $x = f$ is totally possible (=plausible).
>
> Distinct values may simultaneously have a degree of possibility equal to 1. Flexibility is modelled by letting $\pi_x(f)$ between 0 and 1. [...] Clearly, if Ω is the complete range of x, then at least one of

the elements of Ω should be fully possible as a value of x, so that $\exists f,\ \pi_x(f) = 1$ (normalization).

This paper considers only normalized possibility distribution: we assume that Ω is an exhaustive description of the futures. This explicitly excludes situations of surprises, which is the main reason why we stay away from the word Scenario. Since the word Forecast is also reserved for the probabilistic context, this paper uses the word Futurible. It is a contraction of *futur possible*, a neologism that has been used for more than 40 years by the French school of Prospective [Mousli and Roëls, 1995].

Possibility theory is similar to probability in the sense that it uses numerical set functions to measure the degree of confidence assigned to a subset $S \subset \Omega$, and that the measure is based on a distribution defined on the elements $f \in \Omega$. The possibility measure Π describes to what extent the information encoded by π_x is consistent with the statement 'the value of x will be in S'. The possibility measure is defined from the distribution using:

$$\Pi(S) = \max_{f \in S} \pi_x(f) \tag{1}$$

In a professional setting, the rule can be understood as follows: when the analyst presents a set of futuribles, for example $S = \{f, g, h\}$, the convincing power of the set is assessed by looking only at the most plausible element regardless of the number of futuribles in S.

Formally, the possibility measure is normalized by $\Pi(\Omega) = 1$ and $\Pi(\oslash) = 0$ (The glyph $\oslash$ denotes the empty subset). It has the property that for any pair of subsets S, T :

$$\Pi(S \cup T) = \max\left(\Pi(S)\,,\, \Pi(T)\right) \tag{2}$$

The possibility of the union is indeed the possibility of the most plausible future in $S \cup T$, that is $\Pi(S \cup T) = \max_{f \in S \cup T} \pi(f)$. If one considers that S and T are theories predicting the future, this property states that the degree of plausibility of the theory 'the future will be in S OR T' is the degree of the most plausible prediction of the two.

In possibility theory the max operator plays the role of addition in probability theory. Recall that whenever P is a probability measure, $P(S)+P(\overline{S}) = 1$ for any event S and its complement $\overline{S}$. With possibility the corresponding equality is $\max(\Pi(S), \Pi(\overline{S})) = 1$. Whenever Π is a possibility measure, the dual function defined by $N(S) = 1 - \Pi(\overline{S})$ is called a necessity measure.

That distinction between possibility and necessity does not appear with probabilities, but is key to any theory used to quantify the plausibility of precisely described futures, which fall into the $\Pi(S) \leq 1$ and $N(S) = 0$ case. Only non-probabilistic approaches allows to assign a non-zero possibility while recognizing at the same time that the necessity is infinitesimal. This solves the problem of the infinitesimal probability of scenarios (mentioned in A. Kahane's quote, section 2.1) : when S contains just a few precisely described futuribles and Ω is large, then the probability $P(S)$ can only be zero, since all the probability weight goes to $P(\overline{S})$.

3.2 A meaning for possibility levels

Within scientists interested in decision-making, there have been controversies on the foundations of statistics [Savage, 1954]. These controversies pertain as much to possibility theory as to probability theory. Different interpretations of uncertainty take different observable variables as fundamental. Three points of view should be mentioned:

- An objective approach can be used to derive possibilities from data. For example, one could use a database of model simulations to draw a possibilistic histogram of the CO_2 concentration in 2100.
- A subjective approach can be used to derive possibilities from expert's verbal statements, or with the P300 Event-Related Potential, an electric measure of mental activity the amplitude of which increases with unpredictable, unlikely, or highly significant stimuli.
- A logicist approach can be used to understand why the theory of possibility is paradoxical. If S is an event, $N(S) = \Pi(S) = 1$ means that S is true (or will certainly happen), $N(S) = \Pi(S) = 0$ means that S is false (or impossible), but ($\Pi(S) = 1$ and $N(S) = 0$) means neither true or false, simply no information. This goes beyond Aristotelian logic, more specifically against the axiom of the excluded middle. While reasoning without this axiom is unusual, the 'constructive mathematics' school demonstrated that most interesting mathematical theorems can be demonstrated without this axiom [Bishop, 1967].

In this paper we rely on a kind of subjective interpretation, De Finetti's point of view, for reasons that will be explained below. Other approaches, which might fit better the models reviewed in 2.3, are equally consistent but there is no space to discuss the pros and the cons of the alternative interpretations. These controversies never prevented the practical use of probability, so hopefully they will not prevent the use of possibility either.

To start with an informal introduction to possibility, consider a race between my horse and yours. One can define 'The possibility of my horse winning the race is 0.3' as a shortcut for the imprecise piece of information 'The probability of my horse winning the race lies in the [0, 0.3] interval'. More generally, the proposition 'The possibility of S is $\Pi(S)$' can be read as 'The probability of S is not greater than $\Pi(S)$'.

That explanation of possibility is not a satisfying fundamental definition, since it depends on what probability means. Following de Finetti [1937]'s ideas, we define 'The possibility of my horse winning the race is Π' as synonymous with the following gambling rules:

- Don't bet that my horse will win the race.

- Bet that it will loose the race if and only if that pays more than $\frac{\Pi}{1-\Pi} : 1$

Note that there are odds for which no bet would be accepted at all. The intuition behind these rules is an attitude of rational precaution. Implicitly, it is assumed that only gambles with positive expected value are desirable, and that any probability level in $[0, \Pi]$ is equally admissible. Since I admit that the probability of my horse winning can be zero, betting on it would not be safe. On the other hand, the net expected gain of a bet on your horse at rate $r : 1$ is $(1 - p) \times r - p \times 1$, which is known to be larger than $(1 - \Pi) \times r - \Pi$. So if $r > \frac{\Pi}{1-\Pi}$, the net expected gain is positive and it makes sense to desire such a gamble.

This approach is operational since it can in principle be reversed to model a person's beliefs from that person's observed gamble behavior. It fits well with the widespread idea that markets transmit information. However this point needs more discussion. In economic terms, consider a contingent financial asset that is worthless if event S happens, but pays off 100 euros otherwise. Observing an expert who bought that asset for 50 euros (but passed over at 51) only reveals that: the expert believes that the probability (and therefore the possibility) of S happening is not greater than 0.5.

To conclude more precisely, one needs either additional assumptions or additional observations. An additional observation of the behavior with respect to the complementary asset (the one that pays off 100 euros only if S happens) is especially illuminating. The beliefs of an expert that would not buy it event at 0.01 euro could be represented with a possibility level. This contrasts with an expert who believes that the probability of S is precisely 0.5, and would buy at 49.99 euros.

Another way to get more observations is to involve more experts. During the last decade, predictive markets have been set up to trade this kind of contingent assets with the explicit goal to reveal what informed people think on politics, sports or security issues (See for example University of Iowa or intrade.com.) The possibilistic case corresponds to the opening of the markets, when only one side is making offers. Actively traded contracts define a probability all the more precise than the bid-ask spread is small.

There are two reasons that make the De Finetti's approach especially relevant for futuribles. First, in the context of Futures Studies there is no need for additional observations of the kind discussed above. This is because there is a natural additional assumption: the degree of necessity of any precisely described future is zero. So while the infinitesimal probability of scenarios is a problem with probabilistic methods, it is actually an useful assumption with a possibilistic framework. Second, De Finetti's interpretation naturally allows to define possibility as upper probability, as discussed next.

3.3 Possibility as upper probability

We now go back to the formal theory of possibilities. This section gives a formal definition of the notions of admissible probability and desirable gambles introduced above. Finally, it defines when a futurible is more probable than another.

Imagine for example that there are n different theories about the future, each suggesting a different probability distribution p_i. To treat these theories equally, it is natural to consider the linear pool of all probability distributions $\{p = \sum \alpha_i p_i$, for all the the $\alpha_i \geq 0$ such that $\sum \alpha_i = 1\}$. In one dimension, this set would be an interval such as [0, 0.3]. More generally, the set can be represented as a solid polyhedron defined by its vertexes p_i. In the sequel, we are interested in sets of admissible probabilities, which are closed convex sets of probabilities.

A convex solid can also be defined by the planes containing its faces. Say that a measure of probability p is *admissible* in front of the measure of possibility π when $p(S) \leq \pi(S)$ for any event S. This defines a set of all admissible probability measures in front of π, denoted $\mathcal{C}_\pi$ in the sequel. That set is never empty: since the normalization axiom implies that there exists f such that $\pi(f) = 1$, the probability stating that f is certain ($p(f) = 1$) is admissible.

$$\mathcal{C}_\pi = \{p \text{ such that: for any } S \subset \Omega,\ p(S) \leq \Pi(S)\} \tag{3}$$

Given a set of admissible probability measures $\mathcal{C}$, one can turn the definition around and consider the maximum of $p(S)$ for all p in $\mathcal{C}$. This will be called S's plausibility level and denoted $\overline{p}(S)$.

$$\overline{p}(S) = \max_{p \in \mathcal{C}} p(S) \tag{4}$$

Finally, Walley [1991, sec. 3.8]'s theorems allow to define two mathematical structures that are naturally related to any set of admissible probabilities $\mathcal{C}$:

The partial order relation defined by: a future f is *more probable* than g if and only if $p(f) > p(g)$ for all p in $\mathcal{C}$.

A desirable gamble is a function ranging on Ω with positive expected value for any admissible probability.

The notion of desirable gambles offers the link with de Finetti's approach. There is a geometric duality: the larger the set of admissible probabilities, the smaller the set of desirable gambles.

The partial order associated with a possibility distribution is the partial order of the set of admissible probabilities implied by this distribution.

Table 1 displays a numeric example of the above.

4 Application: guidelines for assessing far distant futures

After having discussed the meaning of a single futurible's possibility level, we now turn to the problem "at which level of possibility should the futuribles be

S	$\{a\}$	$\{b\}$	$\{c\}$	$\{a,b\}$	$\{b,c\}$	$\{c,a\}$	$\{a,b,c\}$
$\Pi(S)$	0.4	1	0.2	1	1	0.4	1
$N(S)$	0	0.6	0	0.8	0.6	0	1
$p_1(S)$	0	1	0	1	1	0	1
$p_2(S)$	0.2	0.6	0.2	0.8	0.8	0.4	1

Table 1: Example with $\Omega = \{a, b, c\}$: a measure of possibility Π, the corresponding measure of necessity N and two admissible measures of probability p_1 and p_2. Future b is more probable than c (for any p admissible, $p(c) \leq 0.2$ and $p(b) \geq 0.6$) but future a does not compare with c.

selected ?". To answer this question, we model a situation of communication from A (the analyst) to B (the businesspeople, or the decision-maker) with the following assumptions.

The analyst's information will be represented by a set of admissible probabilities $\mathcal{C}$ on an universe of discourse Ω. The analyst can only communicate a summary (S, π), where $S \subset \Omega$ contains n=2 to 4 futures and π is their possibility distribution. We assume there is a real-valued objective function $J(f)$ that defines the performance of a future f with respect to the businesspeople's goals. Finally, what makes the problem interesting is the assumption that both Ω and $\mathcal{C}$ are very large. In other words, the set of futures that can be described is infinite, and the information given by $\mathcal{C}$ is very imprecise.

We study how to choose S and π. First we show that this problem has more dimensions than the classical problem of transmitting a signal over a limited bandwidth channel. There are three goals that the analysts would like to reach simultaneously: no future should be dismissed out of hand as less probable than another, the set of futuribles should be plausible, and it should explore a wide range of outcomes. We show that these three principles interact to suggest that the extreme futuribles should be selected at possibility level $1/n$ exactly.

4.1 The problem

Given (Ω, $\mathcal{C}$, J), we proceed in two steps to define an interesting (S, π). First, given S, how to choose π ? Assume that the analyst's priority is to not imply undesirable policies. Denoting $\mathcal{D}_A$ the set of desirable gambles for A, this assumption means that $\mathcal{D}_B \subset \mathcal{D}_A$. By duality, this means that $\mathcal{C}_\pi$ includes the restriction of $\mathcal{C}$ to S. In other words, the possibility level $\pi(f)$ presented to the businesspeople must not be inferior to the maximum probability of f admissible to the analyst. Since there is no reason to restrict $\mathcal{D}_B$ further:

$$\pi(f) = \max_{p \in \mathcal{C}} p(f) = \overline{p}(f) \tag{5}$$

Second, how to choose the focal futuribles S ?

From a classical signal theory point of view, one would seek to minimize the unavoidable information loss occurring during the communication. The difficulty is that imprecise probabilities explicitly recognize that ignorance (and therefore information) is not unidimensional, as discussed in 2.2.

To intuitively explain why it would be difficult to determine S by simply minimizing information loss using a single real-valued measure of uncertainty, consider the classical Entropy definition of information. Klir [1999] noticed that a good measure of the informational content of $\mathcal{C}$ is the maximum of the entropy reached by all admissible probability distributions $p \in \mathcal{C}$. Geometrically, this value measures a distance between the set $\mathcal{C}$ and the point corresponding to equiprobability. But this does not inform about the size of $\mathcal{C}$, since no single index could summarize both the size and the position of a geometrical figure at the same time. Generalized entropy cannot discriminate between void information and equiprobability. Actually, as soon as equiprobability is part of $\mathcal{C}$, generalized entropy is maximized.

Instead of defining S by optimization, I propose a toolbox of three mathematical principles to complement the many existing informal techniques of Futures Studies. Three criteria could guide the choice of focal futuribles. First, none should be preferred to another. Second, the set of futuribles should be maximally plausible. Third, that set of futuribles should span the largest possible interval with respect to the decision-maker's variables of interest.

4.2 Incomparability

In line with section 3.3, a future f is said to be more probable than g if and only if $p(f) > p(g)$ for all p in $\mathcal{C}$. It is not always possible to say that any one of two futures is more probable than the other. If $\mathcal{C}$ is large enough, it might contain two probability distributions p_1 and p_2 such that on the one hand $p_1(f) > p_1(g)$ but on the other hand $p_2(f) < p_2(g)$. Incomparability is a good thing here because in a set of futuribles, none should be dismissed out of hand as less probable than another. It suggests to choose S such that no future in it is more probable than any other.

A sufficient condition for this is that S maximizes generalized entropy. If equiprobability on S is in the interior of $\mathcal{C}$, then no element of S is more probable than another. This suggests:

Rule 1 (No preference) *Choose S such that equiprobability on S belongs to the interior of $\mathcal{C}$.*

This sufficient condition is not necessary, consider for example an urn with 20 red balls and 70 'black or yellow' balls. But it is justified in practice by the following considerations. When S is presented without information about the relative likelihood of the different scenarios, a Bayesian decision-maker is likely to use a so-called uninformative, that is to say uniform, probability distribution. One advantage of this method is that at least the decision-maker is not led to use a wrong (inadmissible) probability distribution.

4.3 Plausibility

The second principle focuses on getting the most likely futures on board. To this end, consider equation 4 defining S's plausibility level $\overline{p}(S)$ as the maximum of $p(S)$ for all p in $\mathcal{C}$. That plausibility should be maximized:

Rule 2 (Maximum plausibility) *Choose S to maximize $\overline{p}(S)$.*

When $\mathcal{C}$ is defined by a possibility distribution, the way to satisfy this condition is to include one (or more) futurible of possibility 1, which means ideally that it should be so realistic that nobody would bet against it at any rate. One practical justification for this principle could be that when executives hire a team of external consultants to write about the future, the qualification of the team's report is assessed by how well the baseline displays knowledge of the conventional thinking in the industry.

Note that the rule "include at least one possibility 1 futurible" is not equivalent to "Include one Business-as-Usual scenario". It depends on how $(\Omega, \mathcal{C})$ is defined. It is only when $(\Omega, \mathcal{C})$ is so precise that there is only one f^* such that $\overline{p}(f^*) = 1$ that the rule implies a Business-as-Usual scenario. It is then necessary and sufficient to include f^* in S to maximize the plausibility of the scenario set. But introducing a Business-as-Usual future may not be desirable because it would become an 'official future' exclusive point of focus, thus defeating the very purpose of any kind of uncertainty analysis. There are several technical ways to make sure that $\mathcal{C}$ is large enough to avoid this.

One way is to derive $\mathcal{C}$ from possibility distributions with flat tops, where more than one future can have possibility 1.

Another way is to use constraint-based methods. With those, $\mathcal{C}$ derives from a possibility distribution that can only take the values 0 or 1: any future that can be described is either viable or impossible. The admissible probabilities can be defined simply as those which give a zero probability to impossible futures. With constraint-based methods, the first and second principles simply rule out impossible futures.

A third way to avoid a central future could be to resort to more sophisticated theories to define $\mathcal{C}$, such as Dempster-Shafer theory of evidence. In this case, the maximum plausibility principle would imply to account for all the dissonant points of views.

Having an even number of futuribles is within the scope of the methods presented in this paper. The choice to include or not a Business-as-Usual future is left with the analyst, depending on the institutional context as much as on the precision of available information.

4.4 Contrast

The third desirable property of a good set of futuribles is that it should be well contrasted to represent the range of possible futures. Contrast is defined relative to the performance criteria J for the system being analyzed. For business futures

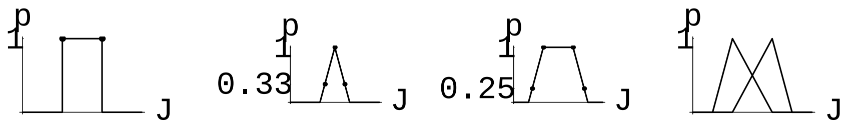

Figure 1: Where to choose the futures on the possibility distribution curve. In each of the four panels, the curve shows the possibility distribution (the π vertical axis) of the variable of interest J (for example, global warming in 2100). There are three principles: 1/ To ensure that no future is less probable than another, choose only points above the $\pi = 1/n$ possibility level, where n is the number of futures. 2/ To ensure credibility, choose at lease one point at level 1. 3/ To provide constrast, choose points as apart as possible. These principles play out differently depending upon the shape of the possibility distribution and the number of futures to be determined, as illustrated.

J may be profits, for public policy social welfare, for environmental economics the pollution level.

The range $[\inf_{f \in S} J(f)\ , \max_{f \in S} J(f)]$ is an indicator of diversity that should be maximized:

Rule 3 (Diversity) *Choose S to maximize the range of the variable of interest.*

This rule suggests to choose two extreme futuribles that respectively minimize and maximize J. This principle may seem ambiguous since it is not always possible to compare two intervals $[J(f_1), J(f_2)]$ and $[J(f_3), J(f_4)]$ with respect to inclusion. However, it is possible to mix and match futuribles, and consider the least performing of $J(f_1), J(f_3)$ and the most performing of $J(f_2), J(f_4)$.

An important case is when a future is described by a n-uple $(x_1, \ldots, x_n)$, each argument x_i being a numerical parameter, and J is monotonous in each parameter. In this practical case, there is a 'worst' and a 'best' future. The former has all the parameters minimizing J, the later maximizing it.

In the common case when extreme futures are those with the smallest possibility (bell-shaped distribution), this rule tends to push the futuribles apart. But it is limited by the principle of no preference. The possibility of each element in a set of n futures should be greater than $1/n$. So while these futures are extreme, their possibility remains at a significant level.

4.5 Illustration

In summary, Figure 1 illustrates how the principles of incomparability, plausibility, and contrast help determine futuribles. The four graphs correspond to three classes of methods which can be used to define a set of admissible probabilities : constraints, possibility distribution (with a pointy or a flat top) or finally evidence theory. The first, second and third graphs show the distribution of the performance criteria J according to the analyst's information, defined as the maximum of the possibility of all futures in Ω that reach J. That is: $\pi_J(u) = \max_{J(f)=u} \pi(f)$.

The first graph, left, represents the case in which constraints are used to sort out viable futures from the impossible or unacceptable ones. The set of viable futures is a segment at $\pi = 1$, and the two focal futuribles are the dots at the ends of this segment.

The second graph depicts a setting where a Business-as-Usual storyline is presented to the decision-maker, as the possibility distribution has a single maximum. The set of three futuribles is displayed with dots. The second principle suggests to choose the least surprising futurible, at possibility level 1. The first and third principle interact to suggest to choose the other two futuribles at possibility level 1/3.

The third graph illustrates how to choose four futures using a trapezoidal possibility function so that there is no Business-as-Usual. In this case the extreme futuribles can be chosen at possibility level 0.25, since there are four futures.

Finally the fourth graph, right, is more sophisticated. The vertical axis is no more a possibility scale, but means to represent plausibility levels in Dempster-Shafer theory of evidence. The two different triangles represent two different but internally consistent points of view about the future. Maximizing plausibility implies to choose one futurible at the top of each triangle, in order to represent each point of view.

5 Discussion and concluding remarks

The paper suggests a novel approach for the integrated assessment of far distant futures. In this approach, possibility distributions are used to define a small set of two to four *futuribles* which has to satisfy three properties: Plausibility (the set should be maximally realistic), Incomparability (no futurible should be more probable than another), and Contrast (it should describe a maximal range for the variable of interest). It is shown that plausibility is achieved if at least one futurible is totally possible. Contrast and incomparability balance each other. We find that a satisfying balance can be achieved by selecting extreme cases that do not rule out equiprobability. For example, if there are three cases, the possibility level of extremes should be about 1/3.

In the toolbox of methods for long range analysis, the approach discussed here lies between probabilistic forecasting and narrative scenario making. There are several limitations to this approach:

- This is a formal way to summarize rich information into a simpler form in which it can be passed on easily to policy makers. Mathematical methods may have harmful effects because people have a tendency to discount the extreme cases and grab onto numbers, ignoring the more important structural messages. Also, the view that information flows one way from the analyst to the decision makers discounts the interactive dimension of the analysis. Scenario making is better for these two aspects.

- Possibility theory is not as well developped as probability theory. It follows that it is less well known, and there is much less commercial software to deal with it. With respect to decision making, as discussed above, De Finetti's subjective point of view on statistics provides an epistemic interpretation of possibility. It allows to define which gambles are desirable, whatever the meaning of a gamble (policy option or investment opportunity). However, possibility theory does not easily allow to compute optimal decisions by expected value maximisation. Probabilistic forecasting is better at this.

- This method is relevant only for the part of the problem which can be represented formally by a possibility distribution. With respect to the other dimensions of ignorance such as vagueness or strategic uncertainty, other mathematical tools are needed (respectively fuzzy theory and game theory).

This research was motivated by the IPCC report about the possible global warming over the next century: +5.8 °C, without probability or likelyhood considerations. When it comes to long term environmental issues, there is no accepted method to assign a probability to any precisely described future, and that number would be infinitesimal anyway. This is a problem for [IPCC, 2000] scenarios, because although all of them are equally valid the one leading to 5.8 °C of global warming in 2100 is more extreme in an unspecified way.

We found that the concept of degrees of possibility can be a useful addition to the toolbox of methods for assessing far distant futures. It allows to quantify the unlikelihood of future events, without giving a false probabilistic precision. This explains why bounds on probabilities have found their way in the current IPCC guidelines for expressing risk and uncertainty. Further empirical work is needed to assess a degree of possibility for the +5.8 °C warming by 2100 figure.

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