



Munich Personal RePEc Archive

## **Staggered wages, inflation, and discounting**

Bonini, Patricia and Da Silva, Sergio  
Federal University of Santa Catarina

2007

Online at <http://mpra.ub.uni-muenchen.de/1979/>  
MPRA Paper No. 1979, posted 07. November 2007 / 02:08

# Staggered wages, inflation, and discounting

Patricia Bonini<sup>a</sup>, Sergio Da Silva<sup>a\*</sup>

<sup>a</sup>*Department of Economics, Federal University of Santa Catarina, 88049970 Florianopolis SC, Brazil*

## Abstract

In the literature of staggered wages (Taylor, 1979, 1980; Blanchard, 1986; Ball and Cecchetti, 1991) the discount factor is neglected in the workers' loss function. Yet discounting is to be viewed as an extra piece of micro-foundation with implications for discretionary monetary policy. We revisit the issue and show that discounting in the model of staggered wages actually lowers the time consistent steady inflation.

*Keywords:* Staggered wage model; Time consistent steady inflation; Discounting

*JEL classification:* E12; E31; E52

## 1. Introduction

In the literature of staggered wages (Taylor, 1979, 1980; Blanchard 1986; Ball and Cecchetti, 1991) the discount factor is neglected in the workers' loss function. Yet discounting is to be viewed as an extra piece of micro-foundation with implications for discretionary monetary policy. Both wage contracts in an imperfectly competitive labor market and distorting taxes on labor income generate an equilibrium wage above the socially efficient level. This inefficiency gives the monetary authority an incentive to create surprise inflation. Along the lines of Ball and Cecchetti (1991), we derive the discretionary time consistent inflation rate by considering the influence of discounting. We find that a positive money growth rate in the presence of discounting pushes the average real wage below its optimal level, shrinks the gap between the market level of output and that socially efficient, and thus diminishes the incentives for the policymaker to create surprise inflation. Thus considering discounting in the model of staggered wages actually lowers the time consistent steady inflation.

Section 2 revisits the benchmark model. Section 3 extends the model to consider discounting. Section 4 derives the equilibrium time consistent inflation rate, and then calibrates the extended model. Section 5 concludes.

---

\* Corresponding author.

Tel./Fax: +55 48 3238 1219.

E-mail address: professorsergiodasilva@gmail.com

## 2. Model

We take the nominal contract setup in which firms are on their labor demand curves (Gray, 1976; Fischer, 1977; Taylor, 1979). The perfectly symmetric firms are both price ( $p$ ) and wage ( $w$ ) takers when maximizing profits, so that aggregate output supply is

$$y_t = \frac{\alpha}{\alpha - 1} (w_t - p_t), \quad (1)$$

where all variables are in logs, and parameter  $\alpha \in (0, 1)$  is the degree of returns to scale.

Aggregate demand is given by the quantity equation with constant velocity of money, i.e.  $m_t - p_t = y_t$ . Then the aggregate price level that balances the output market is

$$p_t = \alpha w_t + (1 - \alpha) m_t, \quad (2)$$

where the money supply grows steadily at rate  $\mu$ , i.e.  $m_t = \mu t$ .

The labor market is imperfectly competitive, and there is no perfect labor mobility. The economy has two sectors, each of them with a pool of workers that are immobile thanks to their specific skills. With perfect symmetry across the sectors, individual wages match the average wage. Labor supply depends positively on the real wage, and labor contracts target  $w^*$ , which is the wage clearing the labor market in the absence of nominal rigidities. By leaving out constants one gets  $w_t^* = p_t$  in equilibrium. The contract wage  $x_t$  is renewed at time period  $t$ . Half of the labor force minimizes the losses coming from the deviations from equilibrium, i.e.

$$\sum_{t=1}^2 L_t = \rho^t (x_t - p_t)^2 \quad (3)$$

subject to  $x_t = x_{t+1}$ . This constraint means that the contracts fix a wage for two periods.

The other half of the labor force minimizes the losses  $L$  at the previous time period  $t-1$ , and fixes the wage at  $x_{t-1} = x_t$ . Parameter  $\rho \in (0, 1]$  is the discount factor. In the benchmark staggered wage model it is neglected to its borderline case  $\rho = 1$ .

A distorting tax is imposed on labor income (Ball and Cecchetti, 1991). The equilibrium pre-tax  $w^*$  is then set above the efficient level by an amount  $\hat{w}$ . The latter is a function of the tax rate and the parameter of preference over consumption and leisure entering the labor supply, and of the parameter of technology (entering the labor demand). The efficient real wage is that attained in the equilibrium with no taxes, i.e.  $w_t^E - p_t = -\hat{w}$ . It is negative because  $w^* - p = 0$  after dropping constants. As  $w^*$  is set above the efficient level, aggregate employment and output are set below their efficient level.

The monetary authority is concerned with output inefficiency (Barro and Gordon, 1983). While wage setters target  $w^* = p$ , the monetary authority cares about the loss provoked by the departure of the contract wage from  $w^E$ . This loss is  $\ell_t = \rho^t (x_t - p_t + \hat{w})^2$

for the sector renewing the contract in  $t$ . The social loss averaged across firms in the first and second period is then

$$\ell_t = \frac{1}{2} \rho (x_t - p_t + \hat{w})^2 + \frac{1}{2} \rho^2 (x_{t-1} - p_t + \hat{w})^2. \quad (4)$$

The monetary authority chooses the money each period taking the expectations of current and future money as given, so that equilibrium is discretionary. Since the time paths of wages and price are both linear functions of  $\mu$ , they are also functions of the sequence of surprises, i.e.  $\{\delta_t\} = m_t - \mu$ . Both the workers and the authority have rational expectations.

### 3. The model with discounting

Relaxing the assumption that  $\rho = 1$  in the workers' loss function (3) leads to a new rule of contract revision:

$$x_t = \frac{1}{1+\rho} p_t + \frac{\rho}{1+\rho} p_{t+1}^e. \quad (5)$$

Considering (2) and (5), a solution for  $x_t$  and  $x_{t-1}$  exists for homogeneous staggering. Its stable characteristic root ( $|\lambda| < 1$ ) is

$$\lambda = \frac{(2-\alpha)(1+\rho) - \sqrt{(2-\alpha)^2(1+\rho)^2 - 4\alpha^2\rho}}{2\alpha\rho}. \quad (6)$$

It is implied that  $\partial\lambda/\partial\rho > 0$  for  $\rho < 1$ . Thus taking discounting into account raises  $\lambda$ .

As the supply of money grows steadily and there is no surprise, the time paths of the contract wages are the first terms in the right hand side of (7) and (8):

$$x_t = \mu(t+z) + \sum_{j=0}^{\infty} (1-\lambda^{1+j}) \delta_{t-j} \quad (7)$$

and

$$x_{t-1} = \mu(t+z-1) + \sum_{j=1}^{\infty} (1-\lambda^j) \delta_{t-j}, \quad (8)$$

where

$$z = \frac{\rho(1+\lambda)[1-\lambda(\lambda\rho+1)] - \lambda(1+\rho)}{(1+\rho)(1-\lambda)(1+\lambda\rho)^2}. \quad (9)$$

The summation terms in (7) and (8) give the effect of a monetary surprise  $\delta$  on the wages. And term  $z$  summarizes the effect of discounting on the time paths of the contract wages. Neglecting the discount factor ( $\rho = 1$ ) yields  $z = 1/2$ , but  $z < 1/2$  if  $\rho < 1$ . With discounting, the non surprise time paths of aggregate real wage and output are

$$w_t - p_t = (\alpha - 1) \left( \frac{1}{2} - z \right) \mu \quad (10)$$

and

$$y_N = \alpha \left( \frac{1}{2} - z \right) \mu. \quad (11)$$

The average real wage in (10) falls short of its optimal level  $w_t^* - p_t = 0$  over the contract period because  $\mu$  is positive. And the equilibrium steady state output  $y_N$  is pushed upward; as a result, the gap between equilibrium output and its efficient level is shrunk. This diminishes the incentive for the monetary authority to create surprise inflation.

#### 4. Calibration

Now we derive the equilibrium time consistent inflation rate to show how discounting affects the time consistent steady inflation. Under rational expectations,  $\delta$  affects the monetary authority's loss function (4) through (7) and (8). The time consistent discretionary equilibrium requires zero surprise inflation each period, and this means imposing  $\sum_{t=0}^{\infty} \frac{dL(t)}{d\delta} \Big|_{\delta=0} = 0$  on the authority's problem, which leads to

$$\mu = \hat{w}d \quad (12)$$

where

$$d = \frac{2(\rho + \lambda) - \alpha(1 + \rho)(1 + \lambda)}{\rho(1 - \nu)[(2 - \alpha(1 + \lambda))] + \nu[\alpha(1 + \lambda) - 2\lambda]} \quad (13)$$

and

$$\nu = z + \alpha \left( \frac{1}{2} - z \right). \quad (14)$$

The time consistent steady inflation is then a function of  $\rho$  (and of  $\hat{w}$  and  $\alpha$ ). Ball and Cecchetti's (1991) result is a particular case arising if  $\rho = 1$  in (13), and  $\nu = z = 1/2$ .

Table 1 shows our model calibrated with selected values for  $\rho$  and  $\alpha$ , including the favorite ones in the real business cycle literature, i.e.  $\rho = .96^{25}$  and  $\alpha = .75$ . The

values of  $d$  are lower for  $\rho < 1$ , thereby showing that if discounting is considered in (3) and (4), the time consistent steady inflation  $\mu$  is reduced.

## **5. Conclusion**

We extend the standard staggered wage model to consider discounting. By deriving the novel equilibrium time consistent inflation rate, and then calibrating the resulting model, we find that discounting actually lowers the time consistent steady inflation.

Table 1. The effect of discounting in the staggered wage model

	$\alpha = .40$	$\alpha = .60$	$\alpha = .75$	$\alpha = .80$
$\rho = 1$				
$\lambda$	.127	.223	.333	.382
$z$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
$d$	1.549	1.265	1	.894
$\rho = .96^{25}$				
$\lambda$	.128	.226	.335	.384
$z$	.496	.494	.490	.487
$d$	1.541	1.258	.994	.888
$\rho = .9$				
$\lambda$	.134	.237	.351	.402
$z$	.456	.434	.412	.368
$d$	1.469	1.195	.938	.835
$\rho = .85$				
$\lambda$	.137	.243	.360	.412
$z$	.432	.399	.365	.297
$d$	1.429	1.027	.906	.698

Notes

$\rho$  = discount factor

$\alpha$  = degree of returns to scale

$\lambda$  = characteristic root

$z$  = (see equation (9))

$d$  = (see equation (13))

## References

Barro, R., Gordon, D., 1983. A positive theory of monetary policy in a natural rate model. *Journal of Monetary Economics* 91, 589–610.

Ball, L., Cecchetti, S.G., 1991. Wage indexation and discretionary monetary policy. *American Economic Review* 81, 1310–1319.

Blanchard, O., 1986. The wage-price spiral. *Quarterly Journal of Economics* 101, 543–563.

Fischer, S., 1977. Long term contracts, rational expectations and the optimal money supply rule. *Journal of Political Economy* 85, 163–190.

Gray, J., 1976. Wage indexation: A macroeconomic approach. *Journal of Monetary Economics* 3, 221–335.

Taylor, J., 1979. Staggering price setting in a macro model. *American Economic Review* 69, 108–113.

Taylor, J., 1980. Aggregate dynamics and staggered contracts. *Journal of Political Economy* 88, 1–24.