Can information asymmetry cause agglomeration?

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30. January 2009

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January 2009

Abstract

The modern literature on city formation and development, for example the New Economic Geography literature, has studied the agglomeration of agents in size or mass. We investigate agglomeration in sorting or by type of worker, that implies agglomeration in size when worker populations

*The authors are very grateful to the Department of Economics and Finance at City University of Hong Kong for funding that facilitated their collaboration. Daisuke Oyama and the participants at the Tokyo Workshop on Spatial Economics made comments that substantially improved the paper. We thank Sukkoo Kim for help with data, and Tilman Borgers, Guido Cataife, Tom Holmes, Bob Hunt, Tomoya Mori, Gianmarco Ottaviano, Ross Starr, Will Strange, Matt Turner, and Ping Wang (Boss) for helpful comments, but we retain responsibility for any errors.

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differ by type. This kind of agglomeration can be driven by asymmetric information in the labor market, specifically when firms do not know if a particular worker is of high or low skill. In a model with two types and two regions, workers of different skill levels are offered separating contracts in equilibrium. When mobile low skill worker population rises or there is technological change that favors high skilled workers, integration of both types of workers in the same region at equilibrium becomes unstable, whereas sorting of worker types into different regions in equilibrium remains stable. The instability of integrated equilibria results from firms, in the region to which workers are perturbed, offering attractive contracts to low skill workers when there is a mixture of workers in the region of origin.
Keywords: Adverse Selection, Agglomeration.
JEL Codes: R12, D82, R13.

1 Introduction

What are the driving forces behind the formation and growth of cities? This question has vexed urban economists for many years. Informal explanations have been offered, but formal models of the important and ubiquitous phenomenon have proved elusive. The answers to this question have important policy implications, since the various models could feature equilibrium allocations that are efficient, or second best, or worse. Thus, it is important to know which model is prevalent in each case, so that appropriate corrective policy, if needed, can be applied. For these reasons, it is important to have both a variety of models as well as testable hypotheses to distinguish among them. It is unlikely that one model, such as the one presented below or that of the New Economic Geography (see the early work of Abdel-Rahman, 1988, 1990; Fujita, 1988; Abdel-Rahman and Fujita, 1990; and the modern model development of Krugman, 1991, and Fujita *et al* 1999), will explain the economics of all cities in all time periods. For example, Ellison and Glaeser (1997, 1999) find that at least half the explanation for agglomeration lies in natural advantages of a location. Natural advantages are important factors at the historical initiation of a city, but market factors are what keep cities where they are and help them to grow after the initial natural advantages diminish.[1]

[1]Mining towns have natural advantages when natural resources are discovered, but might vacate with resource exhaustion.

It is generally difficult to construct equilibrium models of agglomeration. Studies of the formation and growth of cities are subject to Starrett's Spatial Impossibility Theorem (see Mills, 1967; Starrett, 1978; Fujita, 1986; and Fujita and Thisse, 2002, chapter 2.3), namely that a model featuring a closed economy with no relocation cost, location independent preferences and production, and perfect and complete markets everywhere has no equilibrium where any commodity is transported. An implication is that there is no agglomeration of agents in equilibrium.[2] Various models, such as those used in the New Economic Geography or regional science more generally, employ delicate combinations of agglomerative and repulsive forces to avoid the Theorem (by violating at least one of its assumptions) and to generate equilibria with cities and agglomeration.

The modern literature on cities has a focus on agglomeration in size. Hints about the sources of a broader kind of agglomeration can be found in data and empirical work. For example, Berry and Glaeser (2005) find that levels of human capital in cities have been diverging over time. In other words, more skilled and less skilled workers are agglomerating separately.[3] Combes *et al* (2008) find strong evidence that wage disparities between French cities are driven by sorting by skill. What is the explanation for agglomeration with sorting? Observations about two other phenomena can help address this question. U.S. Department of Commerce (1975) data show that over the long term, labor has moved out of agriculture and into other industries, thus freeing low skill workers from ties to land and allowing them to become mobile. Second, rising income and wage inequality have been attributed to skill-biased technological change (see for example Acemoglu, 1999; Berman *et al*, 1994; and Caselli, 1999). The purpose of our work is to provide a model that is consistent with all of these phenomena. We show that *asymmetric information in the labor market drives agglomeration of workers sorted by skill.*[4] When mobile low skilled worker population rises or there is technological change that favors higher skilled workers, integration of worker types in the same location at equilibrium becomes unstable, while sorting of worker types into different

[2] Or, alternatively, there is no equilibrium at all.

[3] One common definition of agglomeration can be found at http://www.thefreedictionary.com/agglomeration: "bunch, clump, cluster, clustering - a grouping of a number of similar things; 'a bunch of trees'; 'a cluster of admirers'" Of course, we would like to add "a cluster of workers with the same skill level."

[4] For our purposes, skills could be represented by human capital, as is standard in the literature, or by social skills, as in Blum *et al* (2006).

locations in equilibrium remains stable. Therefore, our model suggests that *increased mobility of low skill labor and skill biased technological change causes the geographic sorting of workers by skill.*

The basic elements of models explaining agglomeration of any kind can be stripped down to a two region framework, where there is no presumed asymmetry among regions. A geographically symmetric equilibrium is present, where the economic activity at every location looks like that at any other location. Models that succeed in generating agglomeration feature (another) stable asymmetric equilibrium where economic agents separate into two locations: one with large population and one with small population; see Krugman (1991). These ingredients are insufficient to explain sorted agglomeration, since the population shares of each type can be the same in both regions.[5]

Consider a separating equilibrium in adverse selection problems when there is asymmetric information in the market. In a separating equilibrium, agents reveal their types, and different types are separated by their actions. Can this separation by selection be one of the driving forces of sorting and thus agglomeration? We present a model that features classical asymmetric information in the labor market resulting in adverse selection. A stable equilibrium in this model has sorted agglomeration of agents.

We use a competitive contracting framework where there are large numbers of both firms and workers; each worker can work for only one firm, and each firm can employ at most one worker. There are two locations and two types of workers, high ability and low ability. The total populations of the two types of workers are fixed exogenously. The high type dislikes work more than the low type; this conforms to the commonly used single crossing property. Firms have the same technology for production, regardless of location, of a single consumption commodity that depends on the skill level of the worker employed. They know the overall distribution of types, but the type of a particular worker is private information to that worker. The firms compete with both potential entrants and firms in the other region. A firm offers a labor contract that specifies a lump sum wage based on hours worked; the latter is an indicator of type. We show that no pooling equilibrium, where both types of workers receive the same contract, exists. Location is irrelevant to equilibrium, so any distribution of types across

[5]Models with only one type of producer and consumer have no hope of explaining *sorted* agglomeration.

regions is an equilibrium.

Our stability analysis performs a perturbation test on equilibria as follows. A small fraction of workers is pushed from one region to the other. New firms enter into the region where workers arrive and offer new contracts. New firms in the region of origin enter, not knowing the types of the perturbed workers, and make counteroffers. The perturbed workers decide whether to accept these new contracts in their new region or return to their region of origin and work under the terms of the new contracts there. To return, there is a small moving cost for high skilled workers, none for low skilled workers. If for all perturbations, there is a subgame perfect Nash equilibrium where no workers want to return, then the equilibrium is unstable. Otherwise, it is stable.

We assume that firms in one region cannot observe worker behavior, in particular labor supply or type, in another region.[6] So if a worker is perturbed from a region that has both types of workers in equilibrium, the firms in the new region cannot infer her type with certainty. Neither can the entering firms in the region of origin if the workers return. All of these firms can only use their beliefs, and these beliefs are based on the equilibrium proportions of types in the region of origin. Therefore, at an equilibrium where types are sorted, for example all the low types reside in region 1 whereas all the high types reside in region 2, then the type of a perturbed worker can be inferred by all since the region of origin is known and there is only one type of worker in that region in equilibrium. This is called a *sorted equilibrium*. This certainty about worker type can be exploited by firms, and can render such sorted equilibria stable. In contrast, there can also be *integrated equilibria* where both types cohabit at least one region. Depending on parameters, an integrated equilibrium can either be stable or unstable.

Both the total populations of the two regions and the numbers of workers

[6] Our work is a distant relative of the important paper of Fang (2001). The major differences include the following. First, our consumer choice, namely equilibrium choice of region, is not costly, whereas Fang's is costly. Second, Fang has a noisy signal, a test value where noise is essential, whereas the second consumer choice in our model, labor supply, is not noisy. (In Fang's model, if there were no noise in the test score, there would be no reason for the culture signal.) Third, Fang uses Bayesian Nash equilibrium, whereas we employ a stability concept natural in the spatial setting. The defining notion of region in our model is that worker labor supply is only observable to active firms in the region where they work in equilibrium. In Fang's model, the analogous notion would be that worker test scores are only observable among the equilibrium group of workers to which they belong (this is not the assumption of that model). Due to these differences, the results from the models are qualitatively different.

of each type inhabiting the regions will, in general, differ in a stable separating equilibrium, sorted or integrated. Stable integrated equilibria exist only when the proportion of mobile low skill workers in total population is small and the technological advantage in productivity of the high skill workers is small. As the exogenous parameter reflecting productivity of the high ability workers increases, perhaps due to skill biased technological progress, or as more low skill workers become mobile, integrated equilibria become unstable and there is a transition to the stable sorted equilibria.[7] The intuition behind the mechanism is that as the productivity of the high skilled rises or as the proportion of low skill rises, it is easier for firms to offer pooling contracts at *equilibrium proportions* that dominate the equilibrium (first best) contracts for the low types (see Figure 4 below). Thus, the low types can use their disguise (as an unknown type) to upset the stability of integrated equilibrium. The sorted equilibria are special, since types are known with certainty, so there can be no disguise.

In this contracting environment, the assumption of the Impossibility Theorem that is violated is the assumption of perfect and complete (labor) markets everywhere. It is our hope that our work prompts further investigation of the importance of information asymmetry in the urban context.[8]

Other models induce the sorting kind of agglomeration in different ways, though their primary purpose might not be to explain agglomeration. For instance, Konishi (2008) is a fine example of sorting driven by local public goods in the Tiebout tradition; he also considers adverse selection in that context. Mori and Turrini (2005) is a fine example in the New Economic Geography tradition. Agent heterogeneity in and of itself is insufficient to drive sorting or agglomeration, as the Starrett theorem certainly allows heterogeneity. So the main driving force of agglomeration in any of these models cannot be heterogeneity.

We proceed to explain the reasons underlying a few of our modeling choices.

[7] This transition is similar to the comparative static transition in the New Economic Geography models due to population growth or a transport cost decrease; see Krugman (1991). However, unlike the models of the New Economic Geography, our model does not use transport cost or product differentiation. Instead, it features asymmetric information, adverse selection, and a rather standard competitive contracting environment. It is analytically solvable, in contrast with the models of the New Economic Geography (aside from those employing quasi-linear utility).

[8] DeCoster and Strange (1993) provide an interesting model featuring asymmetric information and agglomeration. However, the underlying driving force for agglomeration is the presence of exogenous natural advantages of certain locations.

A natural competitor to our model is a model of perfect competition that has a localization externality between firms within a region. This externality could, for example, be represented by a Cobb-Douglas firm production function where output is dependent on private inputs as well as the aggregate quantity of labor of one or both types employed in the region. If each type is complementary to only firms employing workers of the same type in the same region, then separation of types is a natural feature of equilibrium. We wish to make three points about this alternative. First, in such an alternative model with or without land, the agglomeration of all workers in one location is also a stable equilibrium. Second, in the alternative model with land, the benefits from the localization externality are likely to be completely capitalized into land rents and thus passed on to the landowners, provided that land supply is inelastic in each region. This would yield a large set of stable equilibria, with arbitrary population distributions. Third, our model is based on microfoundations, whereas the alternative is not, but the alternative model makes assumptions that in a not very subtle manner yield the outcome.

One alternative to the competitive contracting environment that we have chosen is a purely competitive market framework, assuming that there are many participants on both sides of the labor market. However, asymmetric information in the form of adverse selection causes a breakdown of the competitive market for standard reasons. The low skill workers are the "lemons" in the labor market. Nevertheless, there are many agents on both sides of the labor market, so we use a competitive contracting environment. When we examine stability, there is another reason to consider a contracting model: there are few consumers (an arbitrarily small measure) and many firms in the labor market.

The opposite of a competitive approach would be to assume that there is only one firm in each region, and thus there is a monopoly. We expect that our results extend to this framework as well, though the assumption that there is a monopoly in each region does not seem as reasonable empirically as the competitive assumption. If monopolies were observed in regions, one would probably want to employ a large fixed cost rather than a decreasing returns production technology. Another alternative is to use monopolistic competition or oligopoly for the labor market, but these have the same drawbacks as the monopoly assumption and add further complication to the model. After all, we are trying to explain how asymmetric information can cause agglomeration in the simplest framework possible.

We do not attempt to explain the dynamics of agglomeration in this paper. Rather, as in the New Economic Geography literature, we perform an extended comparative static exercise to show how critical exogenous parameters affect the characteristics of a stable equilibrium.

The paper proceeds as follows. In section 2, we introduce the model and notation. In section 3, we analyze separating equilibrium, show that there are no pooling equilibria, and examine the stability properties of sorted as well as integrated separating equilibria. A general discussion of the numerical results, with a focus on the comparative statics in productivity of the high ability workers and in the share of the population of high productivity workers in the total population of mobile workers, is found in section 4. Section 5 provides conclusions and directions for future research. Section 6, the appendix, contains all proofs.

2 The model

2.1 Notation

There are two regions in this economy indexed by $j = 1, 2$. There are two types of mobile workers in the economy, indexed by $i = H, L$. Each worker is endowed with one unit of labor. Workers supply labor to firms and earn a lump-sum wage. Workers are differentiated by their ability (high type and low type). Their populations are denoted by $N_H, N_L \in \mathbb{R}_{++}$. A labor contract $(w, l) \in \mathbb{R}_+ \times [0, 1]$ between a firm and a worker specifies a wage and a quantity of labor. Since workers can only decide whether to take the offer or not, but cannot choose a quantity of labor not offered in a contract, there is no loss of generality in using a lump-sum wage.

If a type i worker accepts the offer (w, l) from a firm, her utility is

$$u_i(w, l) = w - \theta_i l.$$

Parameters $\theta_H, \theta_L \in (0, \infty)$, where $\theta_H > \theta_L > 0$, denote the marginal disutility of labor of the two types. For a given utility level, $\frac{dw}{dl}|_{u_i} = \theta_i$. A larger θ_i means a higher disutility from work and that wage or consumption has relatively lower value. This is the single crossing property used in models of asymmetric information, for instance in the vast literatures on optimal income taxation, industrial organization, health insurance, and education economics. As in these literatures,

we presume that $\theta_H > \theta_L$ rather than the opposite for the simple reason that the opportunity cost of time is higher for the high type workers.[9]

There are a large number of potential firms in both regions that will hire these two types of workers. For the convenience of analysis, we assume that firms are small and one firm hires at most one worker.[10] Firms have access to two types of decreasing returns to scale technologies. The high type technology requires high type labor while the low type technology requires low type labor. Each of the firms commit to a production technology upon entering the market. If the firm faces uncertainty about the type of labor they might hire, the firm may adopt a mixed technology by choosing a probability mix of the high and low type technologies. That is, the firm can play a mixed strategy over technologies. The output of a firm is given by two cases: if a firm adopts the high type technology and employs l units of type H labor, its production function is

$$f^H(l) = \alpha l^\beta.$$

where $\alpha > 1$ and $0 < \beta < 1$. If a firm adopts the low type technology and employs l units of type L labor, the production function is

$$f^L(l) = l^\beta.$$

Parameter α represents the technological advantage of the high skill workers over the low skill workers. Type H workers are of higher productivity and are lazier (due to a higher disutility of labor). Take the produced consumption commodity as numéraire. For a firm that hires with contract (w, l), we discuss its profit function in two cases:

[9] We do find that under the interesting circumstances when worker types are reversed as $\theta_L > \theta_H$, a pooling equilibrium may exist. In this case, workers can accept a pooling contract at a corner solution $(w, 1)$: A deviating contract that attracts type H but not type L would be outside the bound of $l = 1$. Any contract with $l < 1$ that attracts type H will also attract type L. A deviating contract that attracts type L but not type H can be ruled out under proper parameters where the type L indifference curve passing through $(w, 1)$ does not intersect the type L production function at labor supply less than or equal to 1. If we assume $\theta_L > \theta_H$, we again obtain separating equilibria as the outcome; the analogous pictures and algebra yield a contract structure where the low skill type is at a tangency whereas the high skill type might not be at a tangency. A second reason we do not use this version of the model is that it predicts that the high skill wage rate (computed as an average over hours worked) will be lower than the low skill wage rate.

[10] It is easy to relax this assumption, but at the cost of more notation.

(1) When a firm knows with certainty the types of the workers, its profit function takes the form

$$\pi^H(w,l) = f^H(l) - w$$

if it hires a type H worker with contract (w,l), and its profit function takes the form

$$\pi^L(w,l) = f^L(l) - w$$

if it hires a type L worker with contract (w,l).

(2) When a firm does not know the types of the workers, given free mobility of workers, it can infer the probability of hiring a particular type based on the exogenously given proportion of types in the economy. The probability of hiring a type H worker is $\frac{N_H}{N_H+N_L}$. So firms can adopt a mixed production function and have expected profit function

$$\bar{\pi}(w,l) = \frac{N_H}{N_H+N_L} f^H(l) + \frac{N_L}{N_H+N_L} f^L(l) - w.$$

Firms maximize expected profits over contract offers. Facing potential entrants, firms will earn zero expected profit in equilibrium.

2.2 Equilibrium

In equilibrium, workers choose the most preferred contract terms among all offers. This gives us incentive compatibility conditions. In addition, all accepted contracts must give nonnegative utility to workers. These are voluntary participation conditions.[11] Firms maximize profits, while taking workers' actions into account, by choosing among contracts that satisfy incentive compatibility and voluntary participation conditions. This is a sequential game where firms move first with contract offers and workers choose the best contracts.

The definition of equilibrium is formalized in a general way, allowing as many contract terms offered in the market as the number of firms. The actual number of contracts in the market in any particular equilibrium will be very small as we will see below. Finally, there is free entry in both regions; therefore, equilibrium expected profit is zero.

With free mobility of workers and free entry of firms, location or region is irrelevant to the equilibrium concept. It becomes quite relevant when studying

[11] For example, as an outside option they could work in agriculture.

stability, since firms cannot observe worker behavior, in particular labor supply, in the other region.

Let μ denote Lebesgue measure on $\mathbb{R}$ and let $M \subseteq [0, \infty)$ denote the (Lebesgue measurable) set of firms that enter the market; note that in equilibrium the measure of M is total worker population. All statements about firms should be taken as almost sure (in other words, except possibly for a set of agents of measure zero) with respect to Lebesgue measure in firms or consumers, appropriate to the context. As is standard in measure theory, we denote by "a.s." the term "almost surely." A **contract structure** is a set of active firms and a triple of measurable functions, $(M, \hat{w}, \hat{l}, \hat{d})$, where $\hat{w} : M \to \mathbb{R}_+$, $\hat{l} : M \to [0, 1]$, and $\hat{d} : M \to [0, 1]^4$. Here M is the set of active firms, $(\hat{w}(k), \hat{l}(k))$ is the contract offered by firm k, and $\hat{d}(k)$ specifies the region in which the firm enters and the type of technology and labor it employs. Specifically, $\hat{d}(k) = (1, 0, 0, 0)$ means that firm k enters in region 1 and employs the high type technology with the high skill type of labor, $\hat{d}(k) = (0, 1, 0, 0)$ means that firm k enters in region 1 and employs the low type technology with the low skill type of labor, $\hat{d}(k) = (0, 0, 1, 0)$ means that firm k enters in region 2 and employs the high type technology with the high skill type of labor, whereas $\hat{d}(k) = (0, 0, 0, 1)$ means that firm k enters in region 2 and employs the low type technology with the low skill type of labor. Since technology choice is tied with labor types, we do not use extra notation for technology.

Let $k \in M$, a firm that has entered the labor market. For ease of notation, we denote $\hat{w}^k = \hat{w}(k)$ and $\hat{l}^k = \hat{l}(k)$. Let $\mathcal{C}$ be the collection of all contract structures.

Next we define the profit of a firm under a contract structure, and subject to incentive compatibility. Fix a contract structure $(M, \hat{w}, \hat{l}, \hat{d})$. For expositional purposes, it is best to do this using several cases, with our discussion embedded. Call the expected profit function of firm $k \in M$: $\Pi^k(M, \hat{w}, \hat{l})$. Define the firms offering contracts that are incentive compatible for the high type as $IC_H = \left\{k' \in M \mid u_H\left(\hat{w}^{k'}, \hat{l}^{k'}\right) \geq u_H\left(\hat{w}^k, \hat{l}^k\right) \text{ almost surely for } k \in M\right\}$. Analogously, define the firms offering contracts that are incentive compatible for the low type as $IC_L = \left\{k' \in M \mid u_L\left(\hat{w}^{k'}, \hat{l}^{k'}\right) \geq u_L\left(\hat{w}^k, \hat{l}^k\right) \text{ almost surely for } k \in M\right\}$. It is possible that either or both of these sets is empty. In equilibrium, they will not be empty. Define the set of firms offering contracts satisfying voluntary participation

(VP) conditions as follows:[12]

$$\begin{aligned} VP_H &= \left\{k' \in M \mid u_H\left(w^{k'}, l^{k'}\right) \geq 0\right\}, \\ VP_L &= \left\{k' \in M \mid u_L\left(w^{k'}, l^{k'}\right) \geq 0\right\}. \end{aligned}$$

In contrast with standard mechanism design, here there are competing firms or principals, so we must specify profits, and thus which workers are attracted to firms, before defining equilibrium. If there were only one firm, then the distribution of workers could be an equilibrium selection rather than a piece of the definition of firm profit.

In essence, the next step before we can define equilibrium is to define the profit of a firm for any profile of strategies (contracts offered) by all firms. This is a rather technical exercise. Then we can define equilibrium using this profit function, since we will then know profits of each firm under unilateral deviations.

Embedded in the exercise of defining profit for a firm is a set of beliefs, one for each firm, about the type of worker they will attract given the profile of strategies of all firms. The appendix contains a complete and formal definition of a **consistent** contract structure, namely that when firms calculate profits given the contracts offered by other firms, they account for both the incentive compatibility constraints and the voluntary participation constraints in calculating the type of worker they will attract, and thus the profit they expect to generate from production.

Let n_H^1 and n_L^1 denote the number of type H and type L workers in region 1, and let n_H^2 and n_L^2 denote the number of the two types of workers in region 2. Notice that we use superscripts to denote regions and subscripts to denote labor types.

An equilibrium subject to incentive compatibility is defined as the following.

Definition. An **equilibrium** is a consistent contract structure and a population distribution $\left\{(M, \hat{w}, \hat{l}, \hat{d}), n_H^1, n_H^2, n_L^1, n_L^2\right\} \in \mathcal{C} \times \mathbb{R}_+^4$ such that:

(i) Almost surely for firms $k \in M$, they maximize expected profit:

$$\Pi^k(M, \hat{w}, \hat{l}) \geq \Pi^k(M, \hat{w}', \hat{l}')$$

[12] It will turn out (see Proposition 1) that the voluntary participation constraints never bind in equilibrium, so these can be removed if desired.

for all consistent contract structures $(M, \hat{w}', \hat{l}', \hat{d}') \in \mathcal{C}$ such that $\hat{w}'(k') = \hat{w}(k')$, $\hat{l}'(k') = \hat{l}(k')$ a.s. $k' \in M$.

(ii) Firms earn zero expected profit due to free entry:[13] almost surely for firms $k \in M$

$$\Pi^k(M, \hat{w}, \hat{l}) = 0$$

(iii) Population distribution is feasible:

$$\int_M \hat{d}(k) d\mu(k) = \begin{pmatrix} n_H^1 \\ n_L^1 \\ n_H^2 \\ n_L^2 \end{pmatrix}$$

$$\begin{aligned} n_H^1 + n_H^2 &= N_H \\ n_L^1 + n_L^2 &= N_L \end{aligned}$$

Condition (i) simply says that given the contract choices by other firms, any firm is choosing a contract that maximizes expected profit.[14] Condition (ii) says that due to free entry, in equilibrium any firm's profit must be zero. Condition (iii) features a Lebesgue integral, and says that in equilibrium, in each region and for each type of worker, the number of firms that are active is equal to the number of workers, and that the sum across regions of the number of workers of each type is equal to the exogenously given total populations.

There are many possible patterns of equilibria; potentially there can be continua of them. For example, each region may have only one type of worker (sorted) or a mixture of both types of workers (integrated). Firms may offer different contracts to different types (separation) or they may offer the same contract to both

[13] It would be possible to derive this at equilibrium from the free entry condition. In that case, one would have the firms as $[0, \infty)$, with the inactive firms using contract $(0, 0)$. Then at equilibrium, if profit were positive for any firm, another would enter and replicate its contract and location, contradicting positive profit in equilibrium.

[14] Notice that the profit function of firm k, Π^k, is independent of firm behavior, in particular firm deviations, on a set of measure zero. Hence, when firm k deviates from strategy $(\hat{w}(k), \hat{l}(k))$ to strategy $(\hat{w}'(k), \hat{l}'(k))$, but all other firms k' retain the original contract strategy $(\hat{w}(k'), \hat{l}(k'))$, this new contract structure is the same as the old one up to a set of measure zero, measurable, consistent, and yields a mathematically convenient way to represent deviations to check that the strategy profile is a Nash equilibrium.

types (pooling). We rule out unstable equilibria by a stability notion that operates by perturbing the populations between the two regions.[15]

In the following sections, we will examine two patterns of equilibria: the separating equilibrium where there is only one type of worker in each region, called sorted, and equilibria where both types are present in at least one region, called integrated. Of the latter class of equilibria, the pooling equilibria where the same contract is offered to both types is of interest. Various kinds of equilibria will exist for various exogenous parameter values.

2.3 Stability analysis

We conduct the following stability analysis on equilibria:[16]

1. Disturb the equilibrium by moving an arbitrarily small fraction of workers from a region to the other. We consider a game played from this point on by the perturbed workers and firms that might enter either region. Its extensive form and justification are as follows. Given that the number of consumers moved is arbitrarily small, and the number of firms that are potential entrants in the market is assumed to be large, even if there were no information asymmetry, the consumers are at an advantage relative to the firms. Therefore, facing competition, firms that enter will earn zero profit.

2. Firms do not observe workers' labor supply in the region of origin, but they do know the equilibrium distribution of workers by type. Each entering firm makes a contract offer based on this information:

2.1 If worker types are identified at the region of origin, firms will offer the first best contract for that type. In this case, consumers have no informational advantage over firms, but they do have an advantage in that there are few consumers and many potential firms. Thus, firms will choose profit maximizing production plans given that they know each worker's type, but will compete until profits are zero. This will turn out to be a special case of (2.2), in the circumstance where firms know with certainty workers' types.

[15] It is possible that in equilibrium, one region is empty, in other words it has no workers or firms. This situation could be eliminated as an equilibrium by adding land to the model.

[16] The New Economic Geography literature also relies on notions of stability for equilibrium selection, but in contrast the stability concepts employed there tend to be very complicated and driven by computation; see Fujita and Mori (1997). Krugman (1991) pioneered the use of stability in two region models whereas Fujita *et al* (1999) develop it more fully.

2.2 If worker types are not identified at the region of origin, meaning that there is a mixture of both types of workers in that region, risk-neutral firms make contract offers that maximize expected profit. In this case, workers have advantages over firms both in numbers and in information. An entering firm will make an offer before observing labor supply or type, based on population proportions of types in the region of origin at equilibrium.

3. Firms back in the region of origin, from where the perturbed workers have been displaced, can make (zero expected profit) counteroffers to the perturbed workers to return home. Again, they only know the equilibrium distribution of the perturbed worker types. The high type workers face a small moving cost for the return, the low types face no moving cost.[17] (The motivation for this assumption is that high types feature location-specific capital that comes into play for perturbations and moves, whereas low types do not. For example, low types might be involved in manufacturing, whereas high types are involved in research that uses teams or labs.) The firms in the region to which workers have been pushed are aware that the firms in the region of origin might make counteroffers.

4. The equilibrium is unstable if for all small perturbations, there is a subgame perfect Nash equilibrium where no workers return home. Otherwise the equilibrium is called stable.

5. There is a small continuity issue in the case of sorted equilibria, in that there are only workers of one type in each region, so equilibria with a very small population of the other type in each region could have different stability properties than the sorted equilibrium. Thus, we examine stability of equilibria with small populations of the other type in the region, if any are close by, and attribute their stability properties to the limiting equilibrium as population becomes completely sorted.

Loosely speaking, the motivation for this notion of stability is that for every objection to the equilibrium, in the form of a perturbation to a new region and corresponding new offers, there is a counterobjection in that some agents will return home. If no agents return home, then there is an objection without counterobjection, so the equilibrium is unstable. In the end, this will reduce to a comparison of the first best contract for the low types to a pooling contract,

[17] There could also be a moving cost associated with the initial perturbation of workers, but since this perturbation is an involuntary move, it makes no difference to the analysis.

where the equilibrium is stable if the pooling contract is not better for the low types.

2.4 Signalling versus Screening

The difference between signalling and screening models is in the order of moves of the game. For example, a worker might signal their ability by choosing a costly signal, education, moving before the firm that hires them. Screening models have the firm moving first, for example by presenting a menu of contracts for the worker to choose from.

We wish to emphasize that here we employ a variant of a screening model but with many competing firms. Choice of location by consumers is *not* a signal, since it has no impact on equilibrium. The *aggregate* location choices of consumers do have an impact on stability of the equilibrium.

Both signalling and screening models are used in the literature. Screening models have been used in literatures on optimal income taxation, procurement, and insurance. Signalling models are often used in labor economics. However, there is also a large literature, both theoretical and empirical, that employs screening in the labor economics context; see, for example, Landeras and Perez de Villarreal (2005).

For the purposes of our analysis, and in particular the notion of stability that we employ, there must be some residual uncertainty, conditional on the signal, about a worker's ability. This allows for screening after the signal. There are several reasons this might occur in the real world. The most obvious one is that education, a natural signal, is not a perfect indicator of ability, perhaps because it is noisy. It also might not be a good indicator of social skills, that Blum *et al* (2006) find to be important as a component of unobserved ability. Moreover, education generally is used for human capital accumulation as well as signalling. It would be hard for a worker to choose a scalar, such as education, to optimize both the signal and the quantity of human capital accumulation; likely it optimizes a convex combination of the two. Finally, it is possible that at the equilibrium of the signalling game, the two types end up pooled. In any of these cases, there is residual uncertainty conditional on the signal, and that will lead to wage dispersion in screening equilibrium conditional on the signal.

3 Characterization of Equilibrium

3.1 Existence and Uniqueness of Separating Equilibrium

There are two possible types of equilibria: *separating* equilibrium, where worker types are revealed by their contract choices, and *pooling* equilibrium, where both types of workers choose the same contract. We provide in this section a complete characterization of equilibrium contracts. We present a few properties, namely necessary conditions, of the equilibrium contracts first. We say that a constraint, such as IC_L, IC_H, VP_L, or VP_H **binds** if and only if it holds with equality for a set of firms of positive Lebesgue measure. This standard terminology means that the solution to the unconstrained optimization problem of a firm is not the same as the one with the constraint imposed.

Proposition 1. *The following hold in equilibrium:*

(i) VP_H and VP_L do not bind.

(ii) There is only one contract for each type of worker across locations and firms.

(iii) If IC_L (respectively, IC_H) does not bind, f^H (respectively f^L) is tangent to a type H (respectively type L) indifference curve at the equilibrium contract.

(iv) In a separating equilibrium, IC_H does not bind.

Proof. See the Appendix.

For the next proposition, we require a couple of definitions to reduce notation. Define

$$\widetilde{l} = \left[\frac{\theta_H}{\beta(\frac{N_L + \alpha N_H}{N_L + N_H})} \right]^{\frac{1}{\beta - 1}}$$

so that $\widetilde{l}$ is the best the high type can do with the production function mixed between the high and low types at the economy-wide proportions. Let $\hat{l}^*$ be the solution to

$$\theta_L \hat{l}^* - \alpha \left(\hat{l}^* \right)^{\beta} + \left(\frac{\beta}{\theta_L} \right)^{\frac{\beta}{1-\beta}} - \theta_L \left(\frac{\beta}{\theta_L} \right)^{\frac{1}{1-\beta}} = 0.$$

Proposition 2.

(i) When $\alpha\beta\left(\hat{l}^*\right)^{\beta-1} \leq \theta_H$*, so* IC_L *does not bind, a separating equilibrium exists and type L workers receive contract* $\left(\hat{w}_L, \hat{l}_L\right) = \left(\left(\frac{\beta}{\theta_L}\right)^{\frac{\beta}{1-\beta}}, \left(\frac{\beta}{\theta_L}\right)^{\frac{1}{1-\beta}}\right)$*, whereas type H workers receive contract* $\left(\hat{w}_H, \hat{l}_H\right) = \left(\alpha\left(\frac{\alpha\beta}{\theta_H}\right)^{\frac{\beta}{1-\beta}}, \left(\frac{\alpha\beta}{\theta_H}\right)^{\frac{1}{1-\beta}}\right)$.

(ii) When $\alpha\beta\left(\hat{l}^*\right)^{\beta-1} > \theta_H$*, so* IC_L *binds, a separating equilibrium exists if and only if*

$$\hat{w}_L - \theta_L\left(\hat{l}_L - \hat{l}^*\right) - \theta_H\hat{l}^* \geq \widetilde{l} \cdot \left[\frac{N_L + \alpha N_H}{N_H + N_L} - \theta_H\right]$$

At such an equilibrium, type L workers receive contract $\left(\hat{w}_L, \hat{l}_L\right) = \left(\left(\frac{\beta}{\theta_L}\right)^{\frac{\beta}{1-\beta}}, \left(\frac{\beta}{\theta_L}\right)^{\frac{1}{1-\beta}}\right)$*, whereas type H workers receive contract* $\left(\hat{w}'_H, \hat{l}^*\right) = \left(\hat{w}_L - \theta_L\left(\hat{l}_L - \hat{l}^*\right), \hat{l}^*\right)$.

(iii) Workers reveal their types in equilibrium. In other words, there is no pooling equilibrium.

Proof. See the Appendix.

Please refer to Figure 1 for a graphical depiction of the equilibrium contracts with nonbinding IC_L, and Figure 2 for the case when IC_L binds. In these pictures, the horizontal axis represents labor supply whereas the vertical axis represents wage, output or numéraire. Only the separating equilibrium where firms can distinguish worker types exist.

Figure 3 illustrates why a pooling equilibrium cannot exist. Define $t = N_H/\left(N_H + N_L\right)$. The difficulty with a pooling equilibrium lies in the firms' ability to propose a deviating separating contract that attracts only the more productive (type H) workers. It is always possible to profit from deviating to a separating contract with type H workers. Such contracts are represented by the shaded area in Figure 3.

The reason a separating equilibrium might not exist when the incentive constraint IC_L binds is that a pooling contract can dominate the high type contract for the high type utility when the incentive constraint binds. In Figure 2, this pooling contract might be in the area between the linear indifference curve for the high type at the separating contract and the high type production function. This

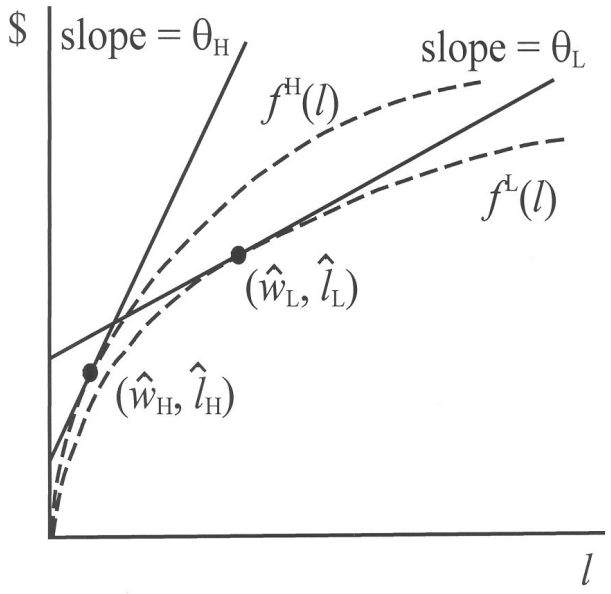

Figure 1: Separating equilibrium, IC_L not binding

pooling contract can, in turn, itself be dominated as in Figure 3.

To this point in the analysis, location is irrelevant. Any distribution of the types between the two regions can be an equilibrium, provided that all contracts are separating.

Worker types are identified by their contract choices in the market. Types will not be pooled together at the same contract. Yet in a spatial setting, there is another kind of integration. Worker types can be integrated in a region or they can be sorted between two regions. In the next subsection we distinguish by their stability properties these two types of separating equilibria.

3.2 Stability properties of separating equilibria

Suppose a separating equilibrium has population distribution $(\hat{n}_H^1, \hat{n}_L^1, \hat{n}_H^2, \hat{n}_L^2)$. Let $s^j = \widehat{n}_H^j / \left(\widehat{n}_H^j + \widehat{n}_L^j\right)$ be the high type share of region j. There are two kinds of separating equilibria: a *sorted separating* equilibrium has only one type of worker in each region, i.e., $s^j \in \{1, 0\}$, whereas an *integrated separating* equilibrium has at least one region containing a mixture of the two types, i.e., $0 < s^1 < 1$ or $0 < s^2 < 1$. We examine their stability as follows. Before stating the precise result, it is useful to present a preliminary argument.

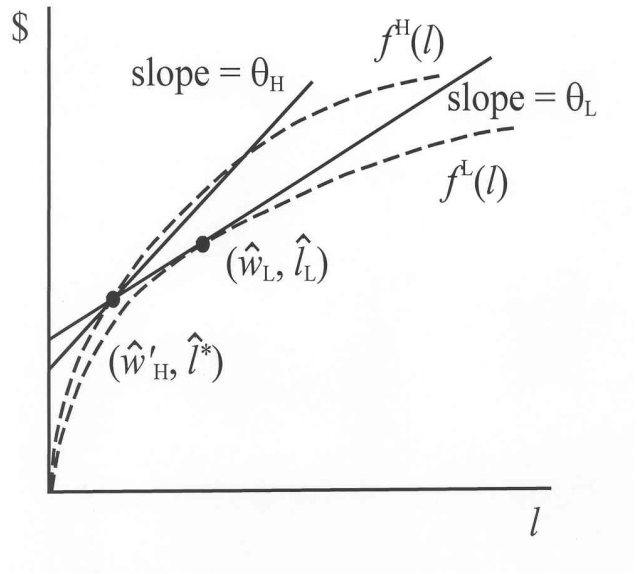

Figure 2: Separating equilibrium, IC_L binding

Claim. *An integrated separating equilibrium is stable if and only if the pooling contract at equilibrium proportions for each region is not better for the low type than the first best contract for the low type.*

This means that stability analysis of an integrated separating equilibrium boils down to examining the first best contract for the low type and the two pooling contracts at equilibrium proportions, one for each region. That examination is found in the next proposition.

The proof of the claim is rather brief, but informative, so we give it here. Recall first from the definition of stability that firms will be indifferent among the contracts they offer, since such contracts all yield zero expected profit. Due to the small moving cost, the firms in the region of origin know that any offer they make to high types will be rejected, since it can be dominated by an offer by firms in the new region, who move first. Thus, stability is completely determined by the low types. Suppose first that the pooling contract dominates the first best contract for the low types. Then if the firms in the new region offer only the pooling contract, and firms in the region of origin are left with only the first best contract for the low types (since the high types will never return), we have a

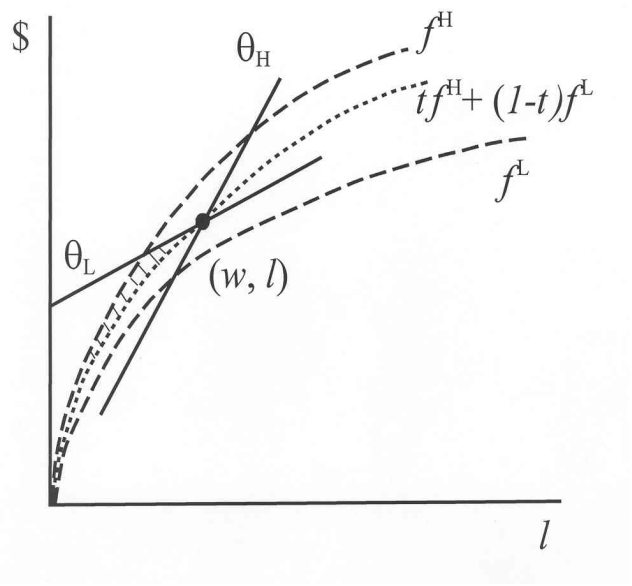

Figure 3: Nonexistence of pooling equilibrium

subgame perfect Nash equilibrium in which nobody moves back to the region of origin, so the equilibrium is unstable. If the first best contract for the low types is at least as good as the pooling contract, then the only subgame perfect Nash equilibrium has the low types always offered the first best contract for them, and they move back (assuming that if they are indifferent, they move back).

Proposition 3.

(i) A sorted separating equilibrium with nonbinding IC_L is always stable.

(ii) A sorted separating equilibrium with binding IC_L is always stable.

(iii) An integrated separating equilibrium with either a binding or a nonbinding IC_L is stable if and only if

$$\left(s^j\alpha+1-s^j\right)\left(\frac{\left(s^j\alpha+1-s^j\right)\beta}{s^j\theta_H+\left(1-s^j\right)\theta_L}\right)^{\frac{\beta}{1-\beta}}-\theta_L\left(\frac{\left(s^j\alpha+1-s^j\right)\beta}{s^j\theta_H+\left(1-s^j\right)\theta_L}\right)^{\frac{1}{1-\beta}}$$
$$-\left(\frac{\beta}{\theta_L}\right)^{\frac{\beta}{1-\beta}}+\theta_L\left(\frac{\beta}{\theta_L}\right)^{\frac{1}{1-\beta}}\leq 0,\ j=1,2.$$

(iv) Fixing other parameters except for α, there are critical values $1>\bar{s}(\alpha)>0$ and $\bar{\alpha}(s^j)>1$ such that any integrated separating equilibrium with regional high type shares $s^1,s^2>0$ is unstable: a) if $\min\left[s^1,s^2\right]<\bar{s}(\alpha)$, b) if and only if

$\alpha > \min\left[(\bar{\alpha}(s^1), \bar{\alpha}(s^2)\right]$.

Proof. See the Appendix.

The key intuitions and implications of this result are as follows. For the purpose of simulations, we shall focus on the case of a nonbinding IC_L. As detailed above, the issue of instability of equilibrium reduces to the question of whether the pooling contract at equilibrium proportions of population for at least one of the two regions is better for the low type than the first best contract. An entering firm makes an offer to a perturbed worker before observing labor supply or type, based on population proportions of types in the region of origin at equilibrium. For a pooling contract, risk neutrality on the part of firms leads to their use of average production functions and average disutility of labor. Thus, an entering firm can offer a pooling contract that maximizes expected profit under the average slope of the high and low type production functions and the average disutility of labor θ^*, where the average is taken according to the population of types in the region of origin at equilibrium. Furthermore, competition drives firms' profit to zero. Stability analysis employs disequilibrium behavior of firms and workers, in contrast with the equilibrium contracting behavior studied in section 3.1.

The sorted separating equilibrium is always immune to a perturbation of workers since all agents are fully informed, so contracts are first best. Firms will not offer a better contract to attract perturbed workers, and the low types will always return to the region of origin. In contrast, when a mixture of workers of different types is moved to another region, a firm entering in the destination region will offer a contract based on its expectations. This may give the low type a contract better than the equilibrium contract, for the following reason. Consider an integrated separating equilibrium. When the share of the high type in total mobile population is large so that the deviating contract is very close to the high type equilibrium contract, it is not attractive to the low type. This is because the high type equilibrium contract is not attractive to the low type by condition IC_L. So an integrated separating equilibrium can be stable. When the share of high type in total population is small enough, an entering firm will offer a more attractive contract (w^*, l^*) to the low type. This renders the integrated separating equilibria unstable (see the illustration in Figure 4). For any given high type productivity α,

there is a critical regional high type share $\bar{s}(\alpha)$ such that, for any smaller shares, integration of types is unstable.

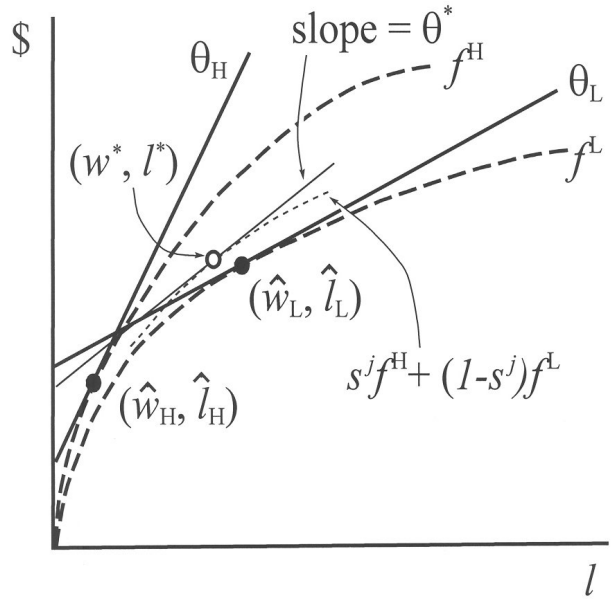

Figure 4: Unstable integration of types

Higher α creates a larger difference between the productivity of the two types. This allows entering firms to offer a more attractive contract to the low type when integrated with the high type. Thus, when the productivity of the high type is relatively low, integrated separating equilibria can be stable. But when this productivity is relatively high, they will be unstable. For any fixed regional high type share s, there is a critical value of high type productivity, $\bar{\alpha}(s)$, such that a larger α means integration of types is unstable. We will illustrate numerically these critical values next in Section 4.[18]

4 Simulation results

In this section, we illustrate with numerical examples the qualitative effects of two key parameters, the technological advantage of high type workers, α, and the

[18] In our model (without land), whether or not the fully agglomerated equilibrium with all agents in one region (and thus integrated) is stable or not depends on the parameters.

share of high type workers in the mobile population, t, on stable equilibria. The equilibrium contract of the low type is fixed by their preferences and production function. The following parameter values are used in the computations: $\beta = 0.4$, $\theta_H = 1.2$, $\theta_L = 0.45$.

4.1 Equilibrium

As α takes a higher value, the high type production function becomes higher and more concave. It pushes both the high type equilibrium wage and labor quantity up. As a result, a high type worker enjoys a higher utility level. Table 1 shows a set of numerical results. Parameter α takes values from 1.1 to 1.2. VP and IC conditions are satisfied in this range. IC_L binds for $\alpha \geq 1.201$. To keep this section simple, we only consider $\alpha < 1.201$.

α	1	1.02	1.04	1.06	1.08	1.1	1.12	1.14	1.16	1.18	1.2
w_H	0.481	0.497	0.513	0.530	0.547	0.564	0.581	0.598	0.616	0.633	0.651
l_H	0.160	0.166	0.171	0.177	0.182	0.188	0.194	0.199	0.205	0.211	0.217
u_H	0.288	0.298	0.308	0.318	0.328	0.338	0.348	0.359	0.369	0.380	0.391

Table 1

See Figure 5 for a graphical representation. Parameter α is graphed on the horizontal axis, whereas the values of the inequality constraints are on the vertical axis. Nonnegative values mean that the constraints are satisfied. VP: thin, IC: thick, type H: dotted, type L: dashed. When α exceeds 1.201, IC_L binds, implying that the low skill type will be indifferent between the equilibrium contracts for low and high skill types.

4.2 Stability

The share of the high type in total mobile population in a region and the technological advantage of high type workers both affect the stability of integrated separating equilibria. As we have seen in Proposition 3, there are critical values of the high type share and the technological advantage parameter for which integrated separating equilibria are not stable. We analyze the parameter range where all integrated separating equilibria are unstable for all possible regional high type shares. It is convenient to use the highest of the two regional high

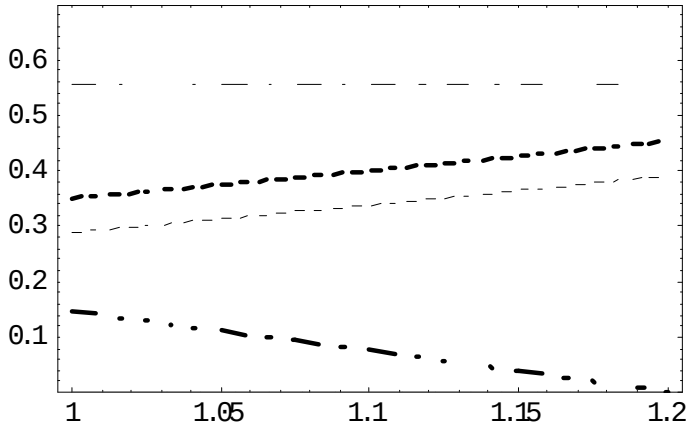

Figure 5: IC and VP conditions as a function of α

type shares, and compare the highest high type share to the critical share. Recall that $t = N_H/(N_H + N_L)$ is the economy-wide share of the total high type mobile population in the total mobile population. Actually, this share t is the highest value of the minimum high skill share of the two regional shares:

$$t = \max_{n_H^1+n_H^2=N_H, n_L^1+n_L^2=N_L} \min[\frac{n_H^1}{n_H^1+n_L^1}, \frac{n_H^2}{n_H^2+n_L^2}].$$

If one region has a high type share higher than t, the other region must have a share lower than t. Thus, the task is reduced to finding the critical high type share in total population $\bar{t}$ such that, if $t < \bar{t}$, then any integrated separating equilibrium is unstable.

In order to discuss comparative statics with respect to t, it is convenient to introduce a large population of immobile workers of the low skill type in the background. These immobile workers can engage in agriculture, tied to land. Their presence allows us to discuss in a simple way how equilibrium changes when more low skill workers become mobile by switching to work in manufacturing, in particular resulting in a decrease in t.

For a fixed t share, there is a critical value $\bar{\alpha}$ such that any integrated equilibrium is unstable for $\alpha > \bar{\alpha}$. Pairs of critical $(\bar{\alpha}, \bar{t})$ constitute a critical curve that separates the parameter space into two parts. For α values above or t values below the critical curve, no equilibrium with integration of types is stable. The benefit of a deviating pooling contract for the low type (utility from a deviating pooling contract minus utility from the equilibrium or first best contract) is presented in

Figure 6 (a positive value means integration-of-types is unstable).

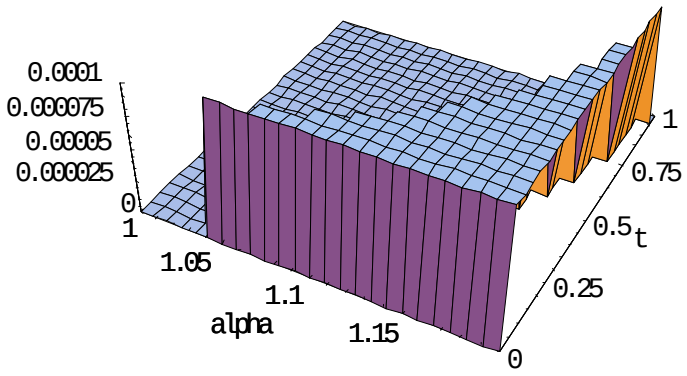

Figure 6: Unstable range for type integration

A set of critical values $(\bar{\alpha}, \bar{t})$ is reported in Table 2.

$\bar{\alpha}$	1	1.02	1.04	1.06	1.08	1.1	1.12	1.14	1.16	1.18	1.2
$\bar{t}$	0	0.023	0.050	0.081	0.117	0.162	0.218	0.290	0.392	0.555	0.944

Table 2

With immobile low skill workers in the background (say, working in agriculture), as more are released to become mobile, t shrinks and eventually only sorted separating equilibria are stable. When the technological advantage of the high type α increases, the integrated separating equilibria eventually become unstable, whereas the sorted separating equilibria remain stable.

4.3 Transition of Stable Equilibria

We illustrate the transition of stable equilibria using a concentration index $\sigma = s^1 - s^2 \in [-1, 1]$. The index σ represents the degree to which type H workers are concentrated in region 1 relative to region 2. When $\sigma = -1$, all type H workers are in region 2. When $\sigma = 0$, both regions have the same share of type H. When $\sigma = 1$, all type H workers are in region 1. A larger σ means a higher share of

type H workers in region 1 relative to the share in region 2. The variable σ is endogenous.

We can now evaluate σ at equilibrium for changing parameters α and t.[19] When t is fixed and the technological advantage parameter α increases from 1, integration of types is stable for middle ranges of σ. In Figure 7, the shaded area represents the values of pairs of the productivity parameter α and the concentration index σ such that the associated integrated separating equilibrium is stable. This range is diminishing as α is larger. When α passes the critical $\bar{\alpha}(t)$, any equi-

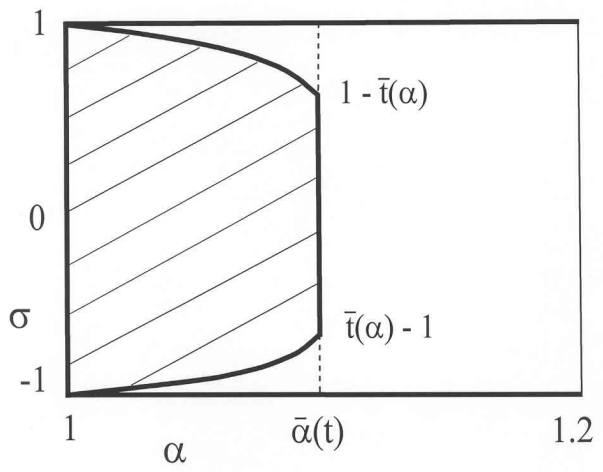

Figure 7: Transition over α for fixed t

librium with integration of types is unstable. Then only the sorted separating equilibrium is stable, and we have agglomeration. Thus, *sorted agglomeration can be caused by increased productivity of high skill workers.* For example, if $t = 0.29$, the critical $\bar{\alpha} = 1.14$.

When α is fixed and the high type share in total population t varies, we represent equilibria in Figure 8. The shaded area represents pairs of high type share t and concentration index σ such that the associated integrated separating equilibrium is stable. For a large high type share t, integration of types is stable

[19] Let $N^1 = n_L^1 + n_H^1$ and $N^2 = n_L^2 + n_H^2$. For a given high type share in total population t, any regional share combinations (s^1, s^2) can be supported as a separating equilibrium (sorted or integrated) if the following condition is satisfied: $s^1 N^1 + s^2 N^2 = t\left(N^1 + N^2\right)$. Among this continuum of equilibria, those with $(s^1, s^2) \geq (\bar{t}, \bar{t})$ are stable.

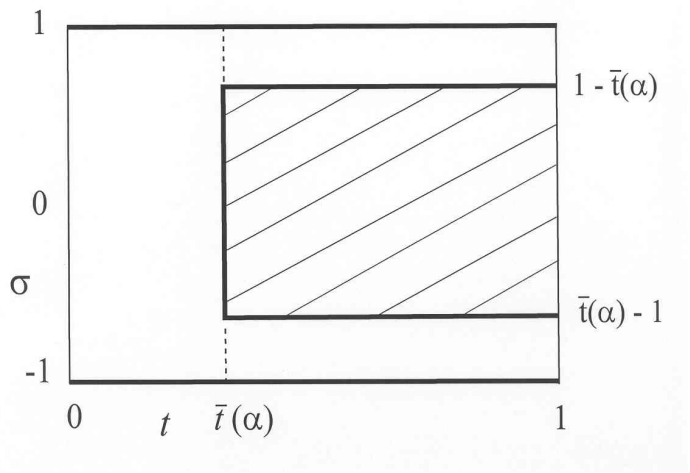

Figure 8: Transition over t for fixed α

for intermediate values of σ. As more low types become mobile, t decreases and for $t \leq \bar{t}(\alpha)$, no equilibrium with integration of types is stable. Then only the sorted separating equilibrium is stable, and once again we have *sorted agglomeration caused by mobility of more low skill workers.*

5 Conclusion

In this article, we examine whether adverse selection in a labor market with asymmetric information can be a factor that generates agglomeration. Agglomeration is defined in a broad sense as a stable but unequal population distribution between regions. If this is a consequence of sorting agents by type, then we call this "sorted agglomeration". We find that separation of workers by contract type is sustained as the only equilibrium outcome. There are different contracts for different types of workers in equilibrium. Workers of different types can be integrated in their equilibrium locations. When there is a large share of high type mobile workers in the total mobile population, integration of types is stable. An integrated, stable equilibrium features a similar mixture of workers in each region. When more low type workers are released from their immobility, integration of types becomes unstable. Empirically, this represents a shift of low skill workers from agriculture, where they are tied to land, to manufacturing, where they are free to move. Calculations of the authors from U.S. Department of Commerce

(1975, p. D 11-23) show that the percentage of the total labor force *not* in agriculture in the U.S. rose from 52% in 1870 to 96% in 1970. With a small proportion of high type mobile workers in the total mobile population, integration of types is unstable. Any stable equilibrium has the large population of low type mobile workers in one region and high type mobile workers separated in the other region. Thus, the increase in the number of low skill workers from 1870 to 1970 can help explain agglomeration during this period. The technological advantage of high skill workers is also a key factor in the stability of integrated equilibria. If the productivity of the high type increases, integration of types becomes less stable. Given the same share of high type in total mobile workers, a larger technological productivity advantage of high skill workers results in the agglomeration of workers by type. This skill biased technological change is consistent with evidence of more recent sorted agglomeration, for example the Berry and Glaeser (2005) work on human capital differences between cities or the Combes *et al* (2008) work on wage dispersion across cities. It is also consistent with a general increase in average human capital, provided that greater human capital for the high types is causing their productivity to rise. So, *given asymmetric information in the labor market, either increased mobility of low skill workers or increased productivity of high skill workers can result in separate agglomeration of workers by type.*

Extensions of the model include the following. First, land markets can be added and the functional form assumptions can be generalized. We expect similar results. In its current form, the transition to agglomeration is abrupt, as in early models of the New Economic Geography. We expect that, analogous to those models, the addition of land or amenities to our sparse model could smooth the transition. Our functional forms were chosen so that the model is easy to solve analytically. The cost of other functional forms would be more complex calculations; the cost of general functional forms could be no method to solve the model analytically.

The model could be extended to include more regions and more types of consumers (in particular, a continuum of types). More generally, heterogeneity of firms could be added. If firm types were common knowledge, then the results would likely be straightforward and similar. But if firm types were private information, that would complicate the model substantially, since there would be two sided uncertainty in the labor market.

Extensions involving multiple periods and dynamic information revelation are

possible but are likely difficult. In communities with small populations, our model might not be relevant because the type of a particular worker could be easily observable.

Further questions to be addressed include welfare properties of equilibrium allocations and testable implications. Evidently, in the case of nonbinding incentive constraints, the equilibrium will be first best, but when an incentive constraint binds, the equilibrium will generally be second best. In the latter case, subsidies to low skill workers (say, conditional on acceptance of a low skill contract) have the potential to loosen the incentive constraint, improving welfare (independent of any equity effects). We have presented some comparative statics that might serve as testable implications. In particular, it is evident from our pictures that high skill workers receive a higher average wage than low skill workers, so one can look for increasing wage dispersion for cities in a country over time, or larger wage dispersion for cities in developed countries in contrast with cities in developing countries.[20]

6 Appendix: Formal Definition of a Consistent Contract Structure and Proofs

Definition of a Consistent Contract Structure:

Let $\backslash$ denote set subtraction. A contract structure $(M, \hat{w}, \hat{l}, \hat{d})$ is called **consistent** if $\hat{d}$ satisfies the following rules. Fix $k \in M$. (i) If the contract offered by firm k does not give either type as much utility as that offered by another firm or the outside option, then it attracts no workers and profits are zero: if $k \in M \backslash [(IC_H \cap VP_H) \cup (IC_L \cap VP_L)]$ then $\Pi^k(M, \hat{w}, \hat{l}) = 0$. In this case, $\hat{d}(k) = (0, 0, 0, 0)$. (ii) If firm k offers a contract that is taken by only one type of worker, then that firm knows with certainty the type it attracts: if $k \in (VP_H \cap IC_H) \backslash (IC_L \cap VP_L)$, then $\Pi^k(M, \hat{w}, \hat{l}) = \pi^H\left(\hat{w}^k, \hat{l}^k\right)$ and $\hat{d}(k) = (1, 0, 0, 0)$ (if the firm is in region 1) or $\hat{d}(k) = (0, 0, 1, 0)$ (if the firm is in region 2); if $k \in (VP_L \cap IC_L) \backslash (IC_H \cap VP_H)$, then $\Pi^k(M, \hat{w}, \hat{l}) = \pi^L\left(\hat{w}^k, \hat{l}^k\right)$ and $\hat{d}(k) = (0, 1, 0, 0)$ (if the firm is in region 1) or $\hat{d}(k) = (0, 0, 0, 1)$ (if the firm is in

[20] A more direct approach, suggested by Bob Hunt, is to examine the extent of geographic localization of information about worker/consumers, for example in the form of credit bureaus; see Hunt (2005).

region 2). (iii) Consider the case where the contract offered by the firm optimizes the utility of both types of workers given the contract structure. Then it is possible for a firm to attract any profile of workers, leading to a profit correspondence. To avoid unnecessary complications, and as is standard in the literature on mechanism design, we select a profile. It is a discontinuous selection, but again we will guess and verify equilibrium, so its continuity properties are not important. (iii.a) If $k \in IC_H \cap VP_H \cap IC_L \cap VP_L$ and $\mu([IC_H \cap VP_H)]\backslash[IC_L \cap VP_L]) = 0$ and $\mu([IC_L \cap VP_L]\backslash[IC_H \cap VP_H]) = 0$, then $\Pi^k(M, \hat{w}, \hat{l}) = \frac{N_H}{N_H+N_L} f^H\left(\hat{l}^k\right) + \frac{N_L}{N_H+N_L} f^L\left(\hat{l}^k\right) - \hat{w}^k$ and $\hat{d}(k) = (\frac{N_H}{N_H+N_L}, \frac{N_L}{N_H+N_L}, 0, 0)$ (if the firm is in region 1) or $\hat{d}(k) = (0, 0, \frac{N_H}{N_H+N_L}, \frac{N_L}{N_H+N_L})$ (if the firm is in region 2). That is, when the firms offering contracts that optimize utility for the high and low types are the same, then a firm in this set can expect the economy-wide distribution of workers. (iii.b) If $k \in IC_H \cap VP_H \cap IC_L \cap VP_L$ and $\mu([IC_H \cap VP_H)]\backslash[IC_L \cap VP_L]) = 0$ and $\mu([IC_L \cap VP_L]\backslash[IC_H \cap VP_H]) > 0$, then $\Pi^k(M, \hat{w}, \hat{l}) = \pi^H\left(\hat{w}^k, \hat{l}^k\right)$ and set $\hat{d}(k) = (1, 0, 0, 0)$ if the firm is in region 1 or $\hat{d}(k) = (0, 0, 1, 0)$ if the firm is in region 2. That is, if a firm offers a contract that optimizes utility for both types of workers, but contracts are offered by other firms that are as good for the low type but not as good for the high type, then the firm expects only high types. Similarly, if $k \in IC_H \cap VP_H \cap IC_L \cap VP_L$ and $\mu([IC_H \cap VP_H)]\backslash[IC_L \cap VP_L]) > 0$ and $\mu([IC_L \cap VP_L]\backslash[IC_H \cap VP_H]) = 0$, then $\Pi^k(M, \hat{w}, \hat{l}) = \pi^L\left(\hat{w}^k, \hat{l}^k\right)$ and $\hat{d}(k) = (0, 1, 0, 0)$ (if the firm is in region 1) or $\hat{d}(k) = (0, 0, 0, 1)$ (if the firm is in region 2). (iii.c) Finally, if a firm offers a contract that optimizes utility for both types of workers but other firms offer contracts that are as good for only the low type, whereas yet other firms offer contracts that are as good only for the high type, then the firm expects to get the economy-wide mixture of workers: if $k \in IC_H \cap VP_H \cap IC_L \cap VP_L$ and $\mu([IC_H \cap VP_H)]\backslash[IC_L \cap VP_L]) > 0$ and $\mu([IC_L \cap VP_L]\backslash[IC_H \cap VP_H]) > 0$, then $\Pi^k(M, \hat{w}, \hat{l}) = \frac{N_H}{N_H+N_L} f^H\left(\hat{l}^k\right) + \frac{N_L}{N_H+N_L} f^L\left(\hat{l}^k\right) - \hat{w}^k$ and $\hat{d}(k) = (\frac{N_H}{N_H+N_L}, \frac{N_L}{N_H+N_L}, 0, 0)$ (if the firm is in region 1) or $\hat{d}(k) = (0, 0, \frac{N_H}{N_H+N_L}, \frac{N_L}{N_H+N_L})$ (if the firm is in region 2).[21]

Proposition 1. *The following hold in equilibrium:*

(i) VP_H and VP_L do not bind.

[21] Since such stategies are never profitable, we could also assume that the firm attracts no workers in this case.

(ii) There is only one contract for each type of worker across locations and firms.

(iii) If IC_L (respectively, IC_H) does not bind, f^H (respectively f^L) is tangent to a type H (respectively type L) indifference curve at the equilibrium contract.

(iv) In a separating equilibrium, IC_H does not bind.

Proof. (i) Suppose a type H worker accepts an equilibrium contract $(w', l') \neq (0, 0)$ with $u_H(w', l') = 0$. Thus, $w' = \theta_H l'$. By zero profit, $f^H(l') = w'$ and by concavity $\frac{d}{dl} f^H(l') < \theta_H$. We can find a new contract (w'', l'') by reducing the labor supply required by $\varepsilon < 0$, so that $l'' = l' - \varepsilon$ and $w'' = w' - \theta_H \varepsilon$. This new contract gives the worker the same utility (implying IC_H) but increases the firm's profit. Note that (w', l') satisfies IC_L and $u_L(w'', l'') = w' - \theta_L l' + \theta_L \varepsilon - \theta_H \varepsilon < u_L(w', l')$, so (w'', l'') satisfies IC_L. These arguments apply to type L as well.

(ii) We prove this for the two types separately. First, suppose there are two distinct contracts (w_1, l_1) and (w_2, l_2) offered to type H in equilibrium, and $u_H(w_1, l_1) = u_H(w_2, l_2)$. There are three possibilities: both firms are certain about worker types and use f^H, both firms are uncertain about worker types and use the expected production function $\frac{N_H}{N_H+N_L} f^H(\cdot) + \frac{N_L}{N_H+N_L} f^L(\cdot)$, or one firm uses f^H and the other uses the expected production function.

Case 1: two firms use f^H. By zero profit, $f^H(w_1, l_1) = f^H(w_2, l_2) = 0$. There is a new contract $((w_1 + w_2)/2, (l_1 + l_2)/2)$ that is indifferent for type H (implying IC_H) and yields more profit. The new contract satisfies IC_L since both (w_1, l_1) and (w_2, l_2) satisfy IC_L.

Case 2 can be argued the same way as Case 1.

Case 3: One firm uses f^H and the other uses the expected production function. Using zero profit, it must be that $f^H(l_1) = w_1$, $\frac{N_H}{N_H+N_L} f^H(l_2) + \frac{N_L}{N_H+N_L} f^L(l_2) = w_2$ and $f^H(l_2) > w_2$. There is a new contract $(w_1 + \theta_H \varepsilon, l_1 + \varepsilon)$ for small $\varepsilon > 0$ such that it is indifferent for type H (implying IC_H), attracts only type H workers, and increases profit. It also satisfies IC_L.

Second, suppose there are two distinct contracts (w_1, l_1) and (w_2, l_2) accepted by type L in equilibrium. There are three possibilities: both firms use f^L, both firms use the expected production function $\frac{N_H}{N_H+N_L} f^H(\cdot) + \frac{N_L}{N_H+N_L} f^L(\cdot)$, or one firm uses f^L and the other uses the expected production function.

Cases 1 and 2 can be argued in the same way as Case 1 for type H.

Case 3: Using zero profit, it must be that $\frac{N_H}{N_H+N_L} f^H(l_1) + \frac{N_L}{N_H+N_L} f^L(l_1) = w_1$,

$f^H(l_2) = w_2$ and thus $\frac{N_H}{N_H+N_L} f^H(l_2) + \frac{N_L}{N_H+N_L} f^L(l_2) > w_2$. There is a new contract $(w_1 + \theta_H\varepsilon, l_1 + \varepsilon)$ for small $\varepsilon > 0$ such that it is indifferent for type L (implying IC_L) and increases profit. It also satisfies IC_H since both (w_1, l_1) and (w_2, l_2) satisfy IC_H.

(iii) Suppose type H workers accept an equilibrium contract (w', l') (this is unique by property (ii) and nonzero by property (i)) and IC_L does not bind. There is a small $\varepsilon > 0$ such that contracts $(w' + \theta_H\varepsilon, l' + \varepsilon)$ and $(w' - \theta_H\varepsilon, l' - \varepsilon)$ violate none of the VP or IC conditions. If $\frac{d}{dl} f^H(l') > \theta_H$, the firm can profitably deviate to contract $(w' + \theta_H\varepsilon, l' + \varepsilon)$. If $\frac{d}{dl} f^H(l') < \theta_H$, the firm can profitably deviate to $(w' - \theta_H\varepsilon, l' - \varepsilon)$. So, $\frac{d}{dl} f^H(l') = \theta_H$. The tangency condition can be proved in the same way for type L workers and IC_H.

(iv) Due to the single crossing property and property (ii), given that IC_L binds, IC_H also binds in and only in a pooling equilibrium. Since we are considering only separating equilibrium, IC_L does not bind. Suppose we have a separating equilibrium with contract (w_1, l_1) for type H and (w_2, l_2) for type L and IC_L does not bind. Therefore, a firm hiring a type H worker has the tangency condition: $f^H(w_1, l_1) = \theta_H$ (by property (iii)). By the concavity of the production functions, $u_H(w_1, l_1) \geq u_H\left(f^H(l), l\right) > u_H\left(f^L(l), l\right)$ for all $l > 0$. This means IC_H cannot bind since $w_2 = f^L(l_2)$ by the zero profit condition. ∎

Proposition 2.

(i) When $\alpha\beta\left(\hat{l}^*\right)^{\beta-1} \leq \theta_H$, *so* IC_L *does not bind, a separating equilibrium exists and type L workers receive contract* $\left(\hat{w}_L, \hat{l}_L\right) = \left(\left(\frac{\beta}{\theta_L}\right)^{\frac{\beta}{1-\beta}}, \left(\frac{\beta}{\theta_L}\right)^{\frac{1}{1-\beta}}\right)$, *whereas type H workers receive contract* $\left(\hat{w}_H, \hat{l}_H\right) = \left(\alpha\left(\frac{\alpha\beta}{\theta_H}\right)^{\frac{\beta}{1-\beta}}, \left(\frac{\alpha\beta}{\theta_H}\right)^{\frac{1}{1-\beta}}\right)$.

(ii) When $\alpha\beta\left(\hat{l}^*\right)^{\beta-1} > \theta_H$, *so* IC_L *binds, a separating equilibrium exists if and only if*

$$\hat{w}_L - \theta_L\left(\hat{l}_L - \hat{l}^*\right) - \theta_H\hat{l}^* \geq \widetilde{l} \cdot \left[\frac{N_L + \alpha N_H}{N_H + N_L} - \theta_H\right]$$

At such an equilibrium, type L workers receive contract $\left(\hat{w}_L, \hat{l}_L\right) = \left(\left(\frac{\beta}{\theta_L}\right)^{\frac{\beta}{1-\beta}}, \left(\frac{\beta}{\theta_L}\right)^{\frac{1}{1-\beta}}\right)$, *whereas type H workers receive contract* $\left(\hat{w}'_H, \hat{l}^*\right) = \left(\hat{w}_L - \theta_L\left(\hat{l}_L - \hat{l}^*\right), \hat{l}^*\right)$.

(iii) Workers reveal their types in equilibrium. In other words, there is no

pooling equilibrium.

Proof. (i) and (ii) We construct the unique separating equilibrium contracts by utilizing results in Proposition 1. First, since IC_H does not bind, the low type is always offered a contract at a tangency. Suppose a firm hires a type L worker with contract (w_L, l_L). By zero profit, $(l_L)^\beta - w_L = 0$ and by tangency of the type L production function and the type L indifference curve, $\frac{d}{dl} f^L(l_L) = \beta (l_L)^{\beta-1} = \theta_L$. The equilibrium contract is

$$\hat{l}_L = \left(\frac{\beta}{\theta_L}\right)^{\frac{1}{1-\beta}}, \hat{w}_L = \left(\frac{\beta}{\theta_L}\right)^{\frac{\beta}{1-\beta}}.$$

By the concavity of f^L, $w_L - \theta_L l_L > 0$ and VP_L is satisfied.

Second, since $u_L\left(\hat{w}_L, \hat{l}_L\right) > 0$ and $f^H\left(\hat{l}_L\right) > f^L\left(\hat{l}_L\right) = \hat{w}_L$, this particular indifference curve passing through $\left(\hat{w}_L, \hat{l}_L\right)$ intersects f^H at a point $\left(\hat{w}^*, \hat{l}^*\right)$ such that $\hat{l}^* < \hat{l}_L$. This $\hat{l}^*$ can be solved from zero profit:

$$f^H\left(\hat{l}^*\right) = \hat{w}_L - \theta_L\left(\hat{l}_L - \hat{l}^*\right),$$

or

$$\theta_L \hat{l}^* - \alpha\left(\hat{l}^*\right)^\beta + \left(\frac{\beta}{\theta_L}\right)^{\frac{\beta}{1-\beta}} - \theta_L\left(\frac{\beta}{\theta_L}\right)^{\frac{1}{1-\beta}} = 0.$$

For part (i), if $\frac{d}{dl} f^H\left(\hat{l}^*\right) \leq \theta_H$, or

$$\alpha\beta\left(\hat{l}^*\right)^{\beta-1} \leq \theta_H,$$

then type H can achieve a higher payoff than $\left(\hat{w}^*, \hat{l}^*\right)$ at a contract $\left(\hat{w}_H, \hat{l}_H\right)$ such that $\hat{l}_H \leq \hat{l}$. This is solved from zero profit, $\alpha (l_H)^\beta - w_H = 0$, and the tangency of the type H production function and the type H indifference curve: $\frac{d}{dl} f^H(l_H) = \alpha\beta (l_H)^{\beta-1} = \theta_H$. Therefore,

$$\hat{l}_H = \left(\frac{\alpha\beta}{\theta_H}\right)^{\frac{1}{1-\beta}}, \hat{w}_H = \alpha\left(\frac{\alpha\beta}{\theta_H}\right)^{\frac{\beta}{1-\beta}}.$$

By the concavity of production functions, VP_H, IC_H and IC_L are all satisfied.

For part (ii), if $\frac{d}{dl} f^H\left(\hat{l}^*\right) > \theta_H$, then $\left(\hat{w}^*, \hat{l}^*\right)$ is the highest payoff type H can get under zero profit and IC_L, since IC_L binds. Note that VP_H is satisfied

by concavity whereas IC_H is satisfied due to the slope difference, $\theta_H > \theta_L$, of indifference curves.

To this point of the proof, we have used necessary conditions for equilibrium contracts to solve for them. To prove formally that these are equilibrium contracts, we must show that there are no independent, profitable firm deviations. For part (i), this can easily be seen, for example, using Figure 1. These are first best contracts. Any alternative contract offered by a firm will yield negative profit, or will violate a production constraint. For part (ii), we must ensure that there is no pooling contract that will give higher utility to the high type than the proposed separating contract. Calculations yield the weak inequality given in part (ii).

(iii) Suppose there is a nontrivial pooling contract $(w, l) \neq (0, 0)$ in the market that both types of workers accept with a high type share $\frac{N_H}{N_H+N_L}$. If a firm can offer a different contract arbitrarily close to (w, l) that attracts type H workers but not type L workers, it can use production function f^H instead of the average of two production functions. This brings more profit since the increase in production is a discontinuous jump. A contract $(w - \theta_L\varepsilon, l - \varepsilon)$ for small $\varepsilon > 0$ is that kind of deviating contract. A type H worker is indifferent between the deviating contract and (w, l), while a type L worker prefers (w, l), since $u_L(w - \theta_H\varepsilon, l - \varepsilon) = w - \theta_L l + (\theta_L - \theta_H)\varepsilon < u_L(w, l)$. ∎

Proposition 3.

(i) A sorted separating equilibrium with nonbinding IC_L is always stable.

(ii) A sorted separating equilibrium with binding IC_L is always stable.

(iii) An integrated separating equilibrium with either a binding or a nonbinding IC_L is stable if and only if

$$\left(s^j\alpha + 1 - s^j\right)\left(\frac{\left(s^j\alpha + 1 - s^j\right)\beta}{s^j\theta_H + \left(1 - s^j\right)\theta_L}\right)^{\frac{\beta}{1-\beta}} - \theta_L\left(\frac{\left(s^j\alpha + 1 - s^j\right)\beta}{s^j\theta_H + \left(1 - s^j\right)\theta_L}\right)^{\frac{1}{1-\beta}} - \left(\frac{\beta}{\theta_L}\right)^{\frac{\beta}{1-\beta}} + \theta_L\left(\frac{\beta}{\theta_L}\right)^{\frac{1}{1-\beta}} \leq 0,\ j = 1, 2.$$

(iv) Fixing other parameters except for α, there are critical values $1 > \bar{s}(\alpha) > 0$ and $\bar{\alpha}(s^j) > 1$ such that any integrated separating equilibrium with regional high type shares $s^1, s^2 > 0$ is unstable: a) if $\min[s^1, s^2] < \bar{s}(\alpha)$, b) if and only if $\alpha > \min[(\bar{\alpha}(s^1), \bar{\alpha}(s^2)]$.

Proof.

(i) Sorted separating equilibrium with nonbinding IC_L:

When the two types of workers are sorted by location in a separating equilibrium, firms know for sure the type of a worker coming from a particular region. All agents are fully informed. Thus, when a worker moves to another region, an entering firm offers a first best contract that yields zero profit. This contract turns out to be that same first best contract that the worker receives in equilibrium. This is where part 5 of the stability notion comes into play. Although the high types will not move back to their region of origin due to the moving cost, any low types will (assuming that if they are indifferent, they move back). This yields a stable equilibrium, as the limit of stable integrated equilibria that tend to the sorted equilibrium.

(ii) Sorted separating equilibrium with binding IC_L:

In equilibrium, the contract for the high type is second best. When a high type worker is perturbed, an entering firm offers the first best contract, and the worker stays in the new region. However, the same argument as in (i) applies for low skill workers and equilibria that are integrated but close to sorted. See condition 5 of section 2.3.

(iii) Integrated separating equilibrium with a nonbinding IC_L:

Using the claim, we must simply compare, for the low types, the pooling contract to the first best contract.

The stability of an integrated separating equilibrium depends on the composition of its worker populations, since the composition determines the pooling contracts (one each for workers perturbed from the two regions). When a small measure of workers is moved from region j to the other region, the firms hiring the perturbed workers have expected output

$$s^j f^H(l) + \left(1 - s^j\right) f^L(l).$$

Since the workers cannot be distinguished, firms will pay a uniform wage rate to all workers that equals the expected disutility of labor

$$\theta^* = s^j \theta_H + \left(1 - s^j\right) \theta_L.$$

Profit maximization determines the quantity of labor hired l^*:

$$\frac{d}{dl}\left(s^j f^H\left(l^*\right)+\left(1-s^j\right) f^L\left(l^*\right)\right)=\theta^*.$$

This means

$$l^*=\left(\frac{\left(s^j\alpha+\left(1-s^j\right)\right)\beta}{\theta^*}\right)^{\frac{1}{1-\beta}}.$$

By competition, the firm will offer a total wage w^* at zero profit.

$$\begin{aligned} w^* &= \left(s^j\alpha+1-s^j\right)\left(l^*\right)^\beta, \\ &= \left(s^j\alpha+1-s^j\right)\left(\frac{\left(s^j\alpha+\left(1-s^j\right)\right)\beta}{s^j\theta_H+\left(1-s^j\right)\theta_L}\right)^{\frac{\beta}{1-\beta}}. \end{aligned}$$

Type L workers will prefer it over the first best contract if

$$w^*-\theta_L l^*>\hat{w}_L-\theta_L\hat{l}_L, \text{ or}$$

$$\begin{aligned} &\left(s^j\alpha+1-s^j\right)\left(\frac{\left(s^j\alpha+1-s^j\right)\beta}{s^j\theta_H+\left(1-s^j\right)\theta_L}\right)^{\frac{\beta}{1-\beta}}-\theta_L\left(\frac{\left(s^j\alpha+1-s^j\right)\beta}{s^j\theta_H+\left(1-s^j\right)\theta_L}\right)^{\frac{1}{1-\beta}} \\ &>\left(\frac{\beta}{\theta_L}\right)^{\frac{\beta}{1-\beta}}-\theta_L\left(\frac{\beta}{\theta_L}\right)^{\frac{1}{1-\beta}}. \end{aligned}$$

Integrated separating equilibrium with a binding IC_L:

Exactly the same calculations work when IC_L binds, since the behavior of the high type is irrelevant. Hence, the equilibrium is unstable if and only if the low type workers want to stay in their new region, thus rendering the behavior of high types irrelevant, and reducing the problem to the same one as with a nonbinding IC_L.

(iv) Let

$$\delta\left(\alpha,s\right)=\left(s\alpha+1-s\right)\left(\frac{\left(s\alpha+1-s\right)\beta}{s\theta_H+\left(1-s\right)\theta_L}\right)^{\frac{\beta}{1-\beta}}-\theta_L\left(\frac{\left(s\alpha+1-s\right)\beta}{s\theta_H+\left(1-s\right)\theta_L}\right)^{\frac{1}{1-\beta}}$$

denote the utility level of a low type worker from a deviating contract and s is the high type share of the original region. Therefore, the contract is attractive if

$$\delta\left(\alpha,s\right)>\delta\left(\alpha,0\right).$$

First, let $\gamma(\alpha, s) = \frac{(s\alpha+1-s)\beta}{s\theta_H+(1-s)\theta_L}$.

$$\begin{aligned}\frac{\partial \delta(\alpha, s)}{\partial s} &= (\alpha-1)(\gamma(\alpha, s))^{\frac{\beta}{1-\beta}} + (s\alpha+1-s)\frac{\beta}{1-\beta}(\gamma(\alpha, s))^{\frac{2\beta-1}{1-\beta}}\frac{\partial \gamma(\alpha, s)}{\partial s} \\ &\quad -\frac{\theta_L}{1-\beta}(\gamma(\alpha, s))^{\frac{\beta}{1-\beta}}\frac{\partial \gamma(\alpha, s)}{\partial s},\end{aligned}$$

where

$$\frac{\partial \gamma(\alpha, s)}{\partial s} = \beta\frac{(\alpha-1)(s\theta_H+(1-s)\theta_L)-(s\alpha+1-s)(\theta_H-\theta_L)}{(s\theta_H+(1-s)\theta_L)^2}.$$

We have

$$\begin{aligned}\gamma(\alpha, 0) &= \frac{\beta}{\theta_L}, \\ \frac{\partial \gamma(\alpha, s)}{\partial s} &\mid_{s=0} = \beta\frac{(\alpha-1)\theta_L-(\theta_H-\theta_L)}{(\theta_L)^2}.\end{aligned}$$

This means

$$\begin{aligned}\frac{\partial \delta(\alpha, s)}{\partial s} &\mid_{s=0} = (\alpha-1)\left(\frac{\beta}{\theta_L}\right)^{\frac{\beta}{1-\beta}} + \frac{\beta}{1-\beta}\left(\frac{\beta}{\theta_L}\right)^{\frac{2\beta-1}{1-\beta}}\beta\frac{(\alpha-1)\theta_L-(\theta_H-\theta_L)}{(\theta_L)^2} \\ &\quad -\frac{\theta_L}{1-\beta}\left(\frac{\beta}{\theta_L}\right)^{\frac{\beta}{1-\beta}}\beta\frac{(\alpha-1)\theta_L-(\theta_H-\theta_L)}{(\theta_L)^2}, \\ &= (\alpha-1)\left(\frac{\beta}{\theta_L}\right)^{\frac{\beta}{1-\beta}} > 0.\end{aligned}$$

Therefore, $\delta(\alpha, s) > \delta(\alpha, 0)$ for all s close enough to 0. An integrated equilibrium is unstable if s^1 or s^2 is small enough.

Second, since

$$\frac{\partial \gamma(\alpha, s)}{\partial \alpha} = \frac{s\beta}{s\theta_H+(1-s)\theta_L},$$

$$\begin{aligned}\frac{\partial \delta(\alpha, s)}{\partial \alpha} &= s(\gamma(\alpha, s))^{\frac{\beta}{1-\beta}} + (s\alpha+1-s)\frac{\beta}{1-\beta}(\gamma(\alpha, s))^{\frac{2\beta-1}{1-\beta}}\frac{d\gamma(\alpha, s)}{d\alpha} \\ &\quad -\frac{\theta_L}{1-\beta}(\gamma(\alpha, s))^{\frac{\beta}{1-\beta}}\frac{d\gamma(\alpha, s)}{d\alpha} \\ &= (\gamma(\alpha, s))^{\frac{\beta}{1-\beta}}\left[s + \frac{s\beta}{1-\beta} - \frac{\theta_L s\beta}{(1-\beta)(s\theta_H+(1-s)\theta_L)}\right], \\ &= (\gamma(\alpha, s))^{\frac{\beta}{1-\beta}}\frac{s}{1-\beta}\left[1 - \frac{\theta_L\beta}{s\theta_H+(1-s)\theta_L}\right].\end{aligned}$$

We have, for all $s > 0$,

$$\frac{\partial \delta(\alpha, s)}{\partial \alpha} > 0$$

because $\beta < 1$ and $\frac{\theta_L}{s\theta_H + (1-s)\theta_L} < 1$. Moreover, for fixed $s > 0$, $\frac{\partial \delta(\alpha,s)}{\partial \alpha}$ is increasing in α, and thus is bounded away from zero. Notice, however, that

$$\frac{\partial \delta(\alpha, s)}{\partial \alpha} \mid_{s=0} = 0$$

We conclude that $\delta(\alpha, s) > \delta(\alpha, 0)$ if and only if α is large enough. ∎

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