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Economic theory tells that a command system allocates resources poorly because of the impossibility of economic calculation. Therefore, once prices are freed and start to operate at quasi-equilibrium (market-clearing) levels, the hidden inefficiencies come into the open and a massive resource reallocation would have to take place. More precisely, the issue refers to the possible and probable intensity of resource reallocation in view of constraints like the balance between exit and entry in the labour market, the size of the budget deficit and the means for its non-inflationary financing, social and political stability, etc. This paper makes an attempt to conceptualise the emergence of strain emerges an economic system when relative prices change dramatically, and explores what can be implications for stabilisation policy. The start is made with the closed economy, after which the open economy case is looked at, and a possible formalised expression of strain is suggested, The distributional struggle, as a consequence of resource reallocation, is highlighted. Some modelling and empirical analysis help in substantiating the main thesis. it is contented that the line of reasoning espoused herein can help in developing an economic explanation of shocks in economic systems.

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ECONOMETRIC INFERENCE INTO THE MACROECONOMIC DYNAMICS OF EAST EUROPEAN ECONOMIES

STRAIN AND THE INFLATION -UNEMPLOYMENT RELATIONSHIP

- a conceptual and empirical investigation -

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- Abstract -

Economic theory tells that a command system allocates resources poorly because of the impossibility of economic calculation. Therefore, once prices are freed and start to operate at quasi-equilibrium (market-clearing) levels, the hidden inefficiencies come into the open and a massive resource reallocation would have to take place. More precisely, the issue refers to the possible and probable intensity of resource reallocation in view of constraints like the balance between exit and entry in the labour market, the size of the budget deficit and the means for its non-inflationary financing, social and political stability, etc. This paper makes an attempt to conceptualise the emergence of strain emerges an economic system when relative prices change dramatically, and explores what can be implications for stabilisation policy. The start is made with the closed economy, after which the open economy case is looked at, and a possible formalised expression of strain is suggested, The distributional struggle, as a consequence of resource reallocation, is highlighted. Some modelling and empirical analysis help in substantiating the main thesis. it is contented that the line of reasoning espoused herein can help in developing an economic explanation of shocks in economic systems.

Daniel Daianu and Lucian-Liviu Albu*)

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1. Introduction

The train of thought of this paper is that more attention needs to be paid to an essential issue of the transformation of post-command economies: i.e., the magnitude of required resource reallocation. For, a fundamental tenet in economic theory - which was confirmed by reality - is that a command system allocates resources poorly because of the impossibility of economic calculation [1]. Therefore, once prices are freed and start to operate at quasi-equilibrium (market-clearing) levels, the hidden inefficiences come into the open and a massive resource reallocation would have to take place - from low onto high productivity areas. More precisely, the issue refers to the possible and probable intensity of resource reallocation in view of constraints suchlike: the balance between exit and entry in the labor market, the size of the budget deficit and the means for its non-inflationary financing, social and political stability, etc.

A hypothesis used in several studies [2] is that the magnitude of the process of resource reallocation - the imbalance between exit and entry - brings about tremendous strain in the system. Flemming (1992), Aghion and Blanchard (1993), Sachs and Woo (1993), Gavin (1993) are among those who captured analytically implications of the magnitude of required resource reallocation in a post-command economy. Flemming, for example, focuses on the very shock caused by the brutal changes in relative prices which - as it is argued - would ask for temporary subsidies for the declining sectors. These analyses should be contrasted with Mussa (1982), who considered adjustment in a frictionless environment. It can be submitted that when the expansion of the private sector is slow, the foreign support is insufficient, the external (negative) shocks are

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powerful, and the underground economy is not effective enough in absorbing the labor shedded by the official economy, the strain in the system can lead to its growing destabilization - in spite of possibly vigorous efforts at macroeconomic stabilization. Strain should not leave decision-makers insensitive to how they evaluate macroeconomic linkages and work out the policy-mix.

By relying on a previous study (Daianu, forthcoming) this paper makes an attempt to conceptualize strain and to analyse it empirically.

2. The closed economy case

The relevance of the closed economy framework could be questioned. We would argue that, apart from a purely theoretical interest and the help it provides in scrutinzing the open economy model, its features fit the case of a very large economy – like the Russian economy, though even this economy suffered the impact of the collapse of Eastern markets.

Let us take the simplified case of a two commodity economy (Figure 1). The initial production combination, (a1, b1), still reflects the central planners' preferences; the latter are indicated by the price line P1. Were consumers sovereign, the production combination would be (a2, b2) and the price line denoting equilibrium (market-clearing) prices would be P2.Took resource reallocation place without friction - with no imbalance between exit and entry - there would be no strain in the system; the shift from (a1, b1) to (a2, b2) would take place along the production possibilities curve.

In a real economy friction is unavoidable. Furthermore, the imbalance between exit and entry can be considerable, and it can cause the production combination A to be substantially inside the production possibilities curve - the fall of the output of (a) is not accompanied by a corresponding growth of the output of (b). This means a significant reduction of aggregate utility - from I1 to I2 - if the expansion of the unofficial economy does not offset it. Over time the production combination would have to come ever closer to (a2, b2). This process is shown by the thick arrow in Figure 1. The magnitude of the required resource reallocation can be illustrated by the ratio:

$$J = \frac{p_{a}^{*} |q_{a}^{*} - q_{a}| + p_{b}^{*} |q_{b}^{*} - q_{b}|}{p_{a}^{*} q_{a}^{*} + p_{b}^{*} q_{b}^{*}}$$
(1)

where (p*) and (q*) refer to equilibrium values, whereas (p) and (q) correspond to the current (distorted) resource allocation. J can be viewed as a measure of aggregate disequilibrium (in the system) as against the vector of equilibrium prices and quantities [3]. The general form of (1) is:

$$\mathbf{J} = \frac{\sum \mathbf{p}_{i}^{*} |\mathbf{q}_{i}^{*} - \mathbf{q}_{i}|}{\sum \mathbf{p}_{i}^{*} \mathbf{q}_{i}^{*}}$$
(2)

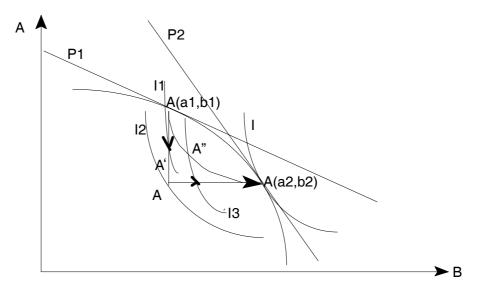


Figure 1: reallocation in a closed economy

The size of the above ratio measures the *strain* within the system and reflects the magnitude of aggregate disequilibrium.

It can be assumed that the possible level of unemployment is related to the degree of *strain* in the system: the higher is *strain* (resource misallocation) the higher is the unemployment that would be brought about by the required resource reallocation. This is a major reason which lies behind the temptation to tolerate high inflation rates as a way to diffuse the tension within a system. Thus, in the social cost function $Z(U, \pi) = (U - U^n) + \beta \pi^2$ the weighting parameter (β) is lower the more pressuring is the unemployment level, or when inflation emerges as the only way for diffusing tension in the system. Thus let us consider the Phillips curve relationship, U=Un - $\alpha(\pi - \pi^e)$, where π^e is the expected inflation

rate. The social cost function can be written as $Z(U, \pi)=(U^n - \alpha(\pi - \pi^e) - U^n) + \beta \pi^2$, or $Z(U,\pi) = -\alpha(\pi - \pi^e) + \beta \pi^2$. It can be seen that Z is inversely linked with unexpected inflation, and directly linked with effective inflation. Minimization of the social loss function implies: $Z'(p) = -\alpha + 2\beta\pi = 0$. Therefore, the optimum level of inflation is $\pi = \alpha/2\beta$. The smaller is (β) - i.e., the more disturbing is the unemployment level - the higher would be the optimal inflation rate. U^n denotes the natural unemployment rate when resource misallocation would have been, basically, dealt with. Therefore, U^n is to be distinguished from the NAIRU (the non-accelerating inflation rate of unemployment) under circumstances when there is an intense pressure to reallocate resources; (π) is the inflation rate.

It can be argued that the above mentioned trade off is less valid the more distabilizing is the very level of inflation - like in the case of a hyperinflation that can lead to an implosion of production. In the latter case, the fight against the highly destabilizing inflation should be the top policy priority. Therefore, analysis needs to consider the dynamics of inflationary expectations and, further, of money velocity.

Let us consider the following aspect. It is beyond contention that U^J , the unemployment that would be involved by an immediate and total resource redeployment, would not be tolerated by the system. Therefore, an optimal level of unemployment, U^* , can be imagined - a level that reflects the various constraints connected with the budget, the concern not to fuel inflationary expectations, the need not to impair the dynamic of the private sector in particular, and of the economy, in general, etc [4]. This optimal level implies a resource transfer via explicit and implicit subsidies. The transfer appears as a combination of non-inflationary and inflationary financing. The major inference is that by presuming a limit to the volume of available non-inflationary financing - either through deliberate money creation, or through the growth of temporary quasi-inside money (inter-enterprise arrears [5]). It follows that the expected inflation, π^e depends on *strain*: $\pi^e = \pi^e$ (J). Figure 2 shows how *strain* (J) affects the tradeoff between inflation and unemployment; a higher J leads to an outward shift of the curve. In Figure 2 J₁ < J₂ < J₃.

Are there any phenomena which can alleviate *strain*? Yes, and some of the most important are: inter-enterprise arrears [6], monopoly pricing, explicit and implicit subsidies,

spillover effects [7], the elimination of negative value-added activities, *learning*, and last, but not least, the efficiency reserves of producers (who operate within their production possibilities curves themselves).

In figure 1 A' denotes the action of the mentioned phenomena, whereas A" indicates the expansion of the production of (b) as well.

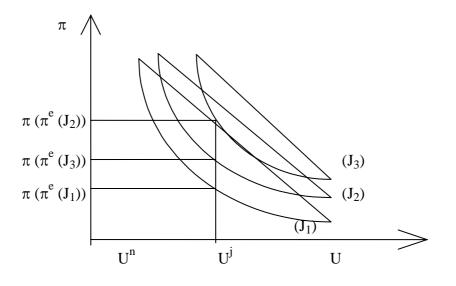


Figure 2: how strain affects expected inflation

The more numerous are those who would lose their jobs because of the needed resource reallocation the more intense would be the opposition against it, against restructuring. Paradoxically, but not surprisingly, *strain* and, relatedly, the acutely felt need to reduce it, can induce a logic of motion of the system that is liable to perpetuate flaws of the old mode of resource allocation.

3. The open economy

There are several notable differences as compared to the closed economy model. The main one is the existence of comparative advantages which, supposedly, orient - together with consumers'preferences - the allocation of resources. As it is known, under a complete specialization of the economy comparative advantages - alone - would determine the structure of production, while consumption would be determined by the structure of demand

and by foreign trade. But no real economy evinces complete specialization; either comparative advantages are not fully transparent, or total specialization is not possible (and desirable) - at least, because of the existence of non-tradeables. Besides, sometimes it pays to think in terms of dynamic comparative advantages [8]. It is noteworthy that in the case of post-command economies the structure of demand overlaps considerably with the structure of production suggested by comparative advantages - in the sense of the needed expansion of "hard" goods. This is shown in Figure 3 where the optimal production indicates the opportunities for specialization (A_3). For the sake of simplification Figure 3 overlooks non-tradeables.

A second major difference is that domestic prices reflect the open character of the economy - the international exchanges. Since the economy is assumed to be a price-taker, world relative prices shape domestic relative prices. In Figure 3 this is shown by the world price line P^{W} . It is obvious that the best production combination is A₃, which, through international trade would bring about the highest aggregate welfare; I₃ is higher than I₂ (that does make use of foreign trade), and both are superior to I.

Another difference is that significant demand - and supply-related external shocks [9] lead to a compression (be it temporary) of the production possibilities curve - the broken curve in Figure 3. This compression amplifies both the reallocation problem and the related distributional struggle issue.

As in the case of the closed economy, intense friction and the magnitude of aggregate disequilibrium make it such that resource reallocation does not occur along the production possibilities curve; at the new prices P, and were the inefficient activities done away with immediately, A_1 would be considerably within the production possibilities curve. Taking into account the functional opening of the economy *strain* can be illustrated by a modified ratio:

$$\mathbf{J} = \frac{\sum \mathbf{p}_{i}^{w} |\mathbf{q}_{i}^{w} - \mathbf{q}_{i}|}{\sum \mathbf{p}_{i}^{w} \mathbf{q}_{i}^{w}}$$
(3)

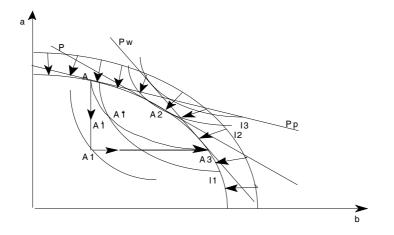


Figure 3: resource reallocation in an open economy

As in the closed economy case, similar phenomena can alleviate the strain in the system. Additionally, there are three phenomena specific to the open economy. One is the possibility to export strain. The recent years provide a remarkable case-study in this respect. Thus, after reunification, Germany exported strain to her neighbours, which led to the facto desmantling of the Exchange Rate Mechanism and other consequences. This possibility is more likely to be present the more active (and sizeable) is an economy in the world space, and the more it operates as a price-maker. For example, after the dramatic rise in the oil price in the seventies, the major industrialized countries experienced a lot of *strain* under the guise of stagflation. But, since these countries (the USA, in particular) operate as pricemakers in the world economy, a partial transfer (diffusion) of this strain took place via increases of the prices of capital goods. Moreover, the USA, due to their role as provider of the main world currency, were able to pay their way out by simply printing money - by exporting inflation. Exporting strain can take the form of protectionism as well. Another phenomenon refers to how comparative advantages have to be viewed in a world of global sourcing and procurement. By this is meant the non-negligeable chances for activities which, presumeably, would have to be discarded, to be saved by their getting into a worldwide network of interconnected operations under the aegis of global companies. And finally, possible positive external shocks need to be taken into account.

The path suggesting the probable dynamic of resource reallocation in the open economy is indicated by the thick arrow in Figure 3.

4. The distributional conflict.

Another way of portraying *strain* is to focus on the scope of the required process of overall income (wages) readjustment which should fit the new market-clearing prices. Under market equilibrium conditions wages equal the marginal productivity of labor: $w_i = q'(n)=dq$ $(n_i)/dn_i$. For the declining and substantially overstaffed sectors the equilibrium wage is fairly low - even below zero for negative value-added activities. The reverse is the situation for the sectors enjoying comparative advantages, or for which domestic demand is very high.

The modified form of J' that builds on wages is:

$$\mathbf{J}' = \frac{\sum \mathbf{n}_i |\mathbf{w}_i^* - \mathbf{w}_i|}{\sum \mathbf{n}_i \mathbf{w}_i} \tag{4}$$

where n denotes labor in sector (i), and w_i^* and w_i refer to equilibrium and actual wage, respectively, for the sector (i). Σ n_i =N, where N refers to all labor resources. For the inefficient, subsidized (explicitly, or implicitly) sectors actual wage exceeds the marginal productivity of labor: $w_i > dq_i / dn_i$. The higher is J', i.e., the higher is strain, the more fierce would be the distributional struggle [10].

It is realistic to presume that the change of the regime of functioning of the economy (the revolution) has led to high expectations concerning steadily increasing incomes (wages) all across the social spectrum. Therefore J' can be written as:

$$\mathbf{J}' = \frac{\sum \mathbf{n}_i |\mathbf{w}_i^e - \mathbf{w}_i^*|}{\sum \mathbf{n}_i \mathbf{w}_i^*}$$
(5)

where w_i denotes the expected income (wage) of agent (i) following the change of the regime of functioning of the economy. It is very likely that the expectations adjusted J' is higher than the level that overlooks them.

One should consider a factor that mitigates the intensity of *strain* when expectations are factored into the analysis. In the old system there were informal channels of income redistribution; most of redistribution was caused by shortages (which forced people to pay premiums for getting hold of goods in short supply), but also by perks and privileges enjoyed by the party and the bureaucracy nomenklatura. The closer those channels brought the initial

income distribution to that outlined by the required resource reallocation (by the equilibrium prices) the more significant the above mentioned mitigation is.

The difference between equilibrium and actual wages reflects the resource transfer (subsidies) practiced by the system; the higher is this difference, the stronger will be the forces that oppose change. When the actual (subsidized) wage is significantly more attractive than the unemployment benefit and the labor market opportunities the resistence to quit job is heavy; therefore when:

$$w_i + (w_i^* - w_i) > r g d + (1-r) w_j$$
 (6)

where ($w_i^* - w_i$) measures the total subsidy, (ρ) is the likelyhood of becoming unemployed, (d) is the unemployment benefit, (γ) \in (0,1) is a coefficient that corrects the utility of the unemployment benefit by a *psychological cost*, and (1 - ρ) refers to the likelyhood of finding a job in a viable sector (j).The psychological cost can help explain the permanence of what was previously seen as temporary unemployment - the **hysteresis** phenomenon.

The distribution of income between labor and the other factors of production (capital) has an impact on the formation of coalitions of interests. In the potentially expanding sectors wages should be lower than the marginal labor productivity: w < q'(n) - this would suggest the room for labor reallocation until w=q'(n); the equalization would take place by a decreasing marginal labor productivity. Nonetheless, to the extent workers behave as quasi-owners and wages equalize the marginal labor productivity, or even the average labor productivity [11] (for tradeunions set the wage rates) a bizarre situation comes up: the workers in the potentially expanding sectors, too, would oppose resource reallocation should the latter be seen as affecting the wage level which is equal to the marginal labor productivity. Here one meets a *wage illusion* since the workers in the efficient sectors do not realize that it is them who make up the source for the subsidies granted to the inefficient sectors - the inflation tax included. They do not realize, too, that following an adequate restructuring of the economy, the dynamics of subsidies (their diminishing) would more than offset the tendency for the marginal labor productivity to be reduced because of new labor resources entering the expanding sectors. But this realization would require the workers in the efficient sectors to optimize by comparing what they gain because of a diminished resource transfer - to the subsidized sectors - with the effect of a lower marginal labor productivity owing to labor reallocation.

Critical elements for judging the prospects for resource reallocation is the unemployment benefit (d) and the size of wages in the expanding sectors (w_j) . The lower are these the more difficult is reallocation. In this respect Blanchard and Aghion talk about an optimal speed of restructuring, which should be evaluated in conjunction with the state of the budget deficit and, relatedly, the impact on the development of the private sector - for the latter finances, partially, the budget deficit. Sachs and Woo underline, implicitly, the distributional struggle issue when they stress the requirement of providing financial incentives to the workers in the subsidized sectors in order to intensify job search and, consequently, to enhance labor mobility. To this end they stress the importance of foreign aid as a the means for breaking a vicious circle of macroeconomic stabilization.

For the post-command (transforming) systems, the *distributional struggle*, which is related to the required resource reallocation, appears as a structural origin of *strain* and as a structuralist-type explanation of inflation. Though it is aimed at the Latin-American experience mostly, Williamson's remark is quite relevant in this respect: "One can look beyond the Patinkinian vision of the economy as a Walrasian system plus a money demand equation, and ask whether it may be necessary to incorporate one important idea in the postwar history of inflation theory...This idea is the notion of inflation as the result of inconsistent claims for real income shares" (1994, pp.68-69). He adds: "...stabilization is not only a matter of fiscal probity plus a nominal anchor plus credibility: it may also demand a choice between social confrontation and achievement of a social consensus" (Ibid., p.72).

It can be submitted that in the case of post-command economies the *distributional conflict* gets a dimension which goes beyond Williamson's meaning; this dimension is given by "the speed and, particularly, by the scope and magnitude of income redistribution entailed by the required resource reallocation". In the countries Williamson [12] focuses on the distributional struggle takes place against the background of a pattern of income distribution which reflects a, relatively, stable allocation of resources. Therefore, individuals' expectations are relatively fulfilled. Differently, in the post-command economies a massive resource reallocation is underway, which affects considerably and brutally income distribution. Additionally, many individuals' expectations are profoundly unfulfilled. The frustration caused by unfulfilled expectations is magnified by exogenous shocks, which have led to a compression of production [13] under circumstances when individuals are used to a certain pattern and level of consumption.

5. Strain and inflation-unemployment relationship: modeling and empirical investigation

5.1. An attempt to explain new empirical trends

Empirical data demonstrate, against the background of business cycles, some major changes of trends in Western countries in the last 25 years. Among these we can mention:

- an impressive decrease in inflation followed by a continuing growth of unemployment;

- a general diminution of the growth rate of production (GDP);

- a diminution of the extension area of points within all the three plans.

An important question is if a smaller area of main macroeconomic indicators' variation represents greater economic stability and, relatedly, less strain.

In any case the evolution is from a period where the main symptom of strain was inflation toward one where unemployment became the principal symptom and factor of strain.

On the other hand, in Eastern European countries there was an opposite situation after 1989; open inflation rose rapidly in the region whereas unemployment did also rise but at a smaller pace. Probably, the long-term trends will be similar to that registered in Western countries. The most important question is how long this transition period will be. In a such context to estimate strain and to evaluate the costs of its diminution becomes an important topic for research.

A possible way to assess strain is as the distribution of the actual points around an average spatial distance (from the origin of axes of co-ordinates). Another way is to calculate the dispersion degree related to one average value. The formulas used to estimate the three-dimensional distance for the two methods are:

- Method I

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$$MdoX := \frac{\sum_{t} \sqrt{\left(u\%X_{t}\right)^{2} + \left(p\%X_{t}\right)^{2} + \left(y\%X_{t}\right)^{2}}}{n}$$
(7)

- Method II

$$dX := \frac{\sum_{t} \sqrt{\left(u\%X_{t} - MuX\right)^{2} + \left(p\%X_{t} - MpX\right)^{2} + \left(y\%X_{t} - MyX\right)^{2}}}{n}$$
(8)

where X represents country; t - years; n - number of years within the period; u% - unemployment rate; p% - inflation rate; y% - growth rate of GDP; and

$$MuX := \frac{\frac{1}{t}u\%X_{t}}{n} \qquad MpX := \frac{\frac{1}{t}p\%X_{t}}{n} \qquad MyX := \frac{\frac{1}{t}y\%X_{t}}{n}$$

The following table presents synthetically the results of the two methods. From this table and by using the first method one can group the countries by the average value of global distance:

- under 10.0 Germany and Netherlands;
- between 10.0-13.0 Denmark, Belgium, and France;
- between 13.0-16.0 United Kingdom and Italy;
- over 17.0 Ireland, Greece, Portugal, and Spain.

A similar classification results by using the second method.

A preliminary conclusion from this type of analyse is that strain is probably smaller when an economy is within one of the first groups.

This approach gives us only a general preliminary base to investigate strain in an economic system. Therefore a deeper analysis is necessary. Also, the empirical data suggest to us the possibility of the existance of some persistent trends and long-run attractors. In Appendix 1 the trends in selected Western countries are shown on the same graph in every case of the three plans. Considering empirical data we can also draw generically the main trends within the three R2 spaces (u-p is unemployment-inflation plan; y-p is production-inflation plan; and u-y represents the unemployment-production plan). For Western economies the generic trends are shown in Figures 4a, 4b, and 4c and

Method I	Method II	
MdoG = 7.214 MdoNL = 9.665 MdoD = 10.821 MdoB = 10.939 MdoF = 11.09 MdoUK = 13.049 MdoI = 14.512 MdoIR = 17.408 MdoP = 17.671 MdoGR = 17.852	dG = 3.26 dNL = 4.395 dB = 4.566 dD = 4.913 dF = 4.974 dI = 5.95 dUK = 5.96 dGR = 6.853 dP = 7.593 dIR = 7.822	
MdoS = 18.814	dS = 8.996	

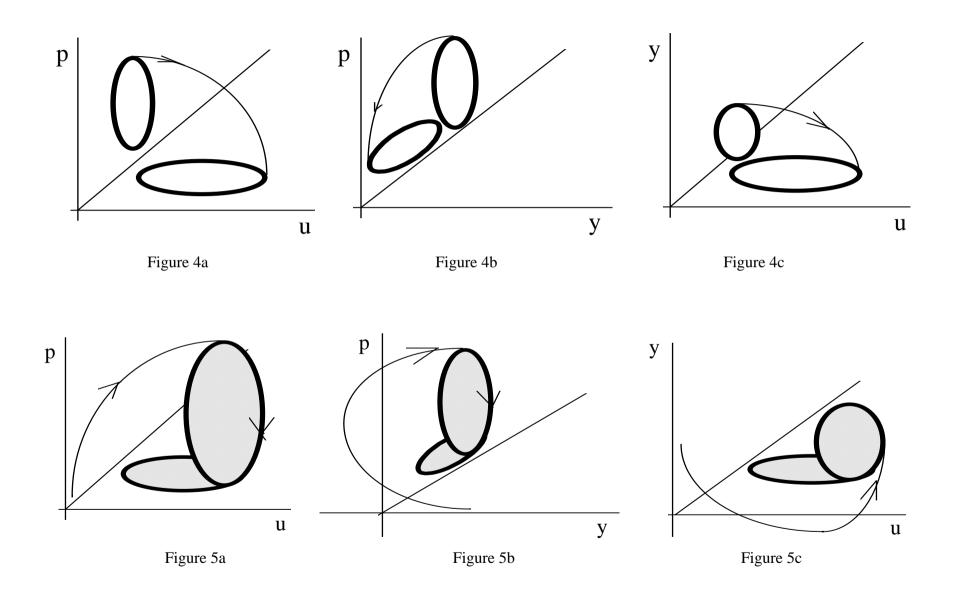
for Eastern economies they are shown in Figures 5a, 5b, and 5c. On the graphs the arrows represent the long-run trends or "time arrow".

The main idea from these figures is that the long-run dynamic seems to demonstrate the existence of some attractors. In our cases these are represented in the two-dimensional spaces by the ellipses.

We can remark several differences between Eastern economies and Western economies. Thus, the main problem in Eastern countries is transition from the first type of attractors to the second type. For instance, within the unemployment-inflation plan the transition implies a move from the ellipse having its long axe oriented along the inflation axe to the ellipse having its long axis along the unemployment axis. Also, we can remark on the graph an important bifurcation point represented by the extreme-right point of intersection between the two attractors (see also Albu and Ungureanu, 1994). From this point there are two paths: one on the first ellipse and another on the second ellipse. The main problem is to escape from the first ellipse-attractor because the economic system has an important quantity of inertial forces and remnant memory (hysteresis phenomenon).

We can also interpret the "confirmation" of the Phillips curve for the 1950s and 1960s and then its "death" decreed by economists. Possibly the change is due to a transition toward the mentioned attractor within the unemployment-inflation relationship. Also the dynamics demonstrated by many Western economies during last two decades seem to confirm the so-called Stop-Go Cycle. The governments, by first promoting and then stopping expansionary macro policies - and then repeating the process - have moved the economy through a series of clockwise "loops" in the inflation-versus-unemployment graph. Generally, the steps illustrate typical stop-go cycles, alternating periods of expansionary and contractionary demand management that produce a succession of accelerating inflation, inflationary recession, disinflation, and finally, reflation (Kohler, 1992). Moreover, what is more important is that both inflation and unemployment move within a strictly closed region of the unemployment-inflation. plan. Until the year 1990 the closed region seems to be similar to a circle but then it looks like an ellipse having its long axe along the unemployment axe. The last trend would be attributed to the taking into account the natural rate of unemployment within macroeconomic policies. Probably by taking into account a "natural rate of inflation" within macroeconomics policies would remove the attractor toward one circle-attractor.

During the last period inflation and unemployment seem to be the two forms of manifestation of a same economic process. Moreover, they are interchangeable (completely interchangeable in the case of a circle-attractor; also we note that by relating them to the natural unemployment the two variables can become completely interchangeable in the case of an ellipse-attractor). This is not the case in economies where there is a very large unemployment rate and/or a higher inflation rate.



5.2. A simple discrete-continuous model to estimate state parameters for unemployment-inflation relationship

During the periods where the two variables are interchangeable we think the Okun's discomfort index becomes relevant opposite to other periods. Here, we consider the opportunity to present a methodology in order to capture some estimators of the system's stability and implicitly of its contained strain. Varying with the values of some essential parameters a large menu of dynamics including chaotic dynamics can be simulated. A main conclusion regarding macroeconomic policy derives from the model: any economic policy having impact on one variable represents a "falsification" induced in the system and it will generate a compensatory reaction of the other variable from the joint plane. Implicitly the impact will manifest later on the other plans. When the impact does not push the parameters over some threshold-values the dynamic of system is stable. But when the essential parameters will be moved over these threshold-values the emergence of chaos becomes possible.

Firstly, we consider the following relation:

$$\mathbf{S} = \mathbf{u} + \mathbf{p} \tag{9}$$

where u is unemployment rate and p is the inflation rate. Also, we note their product by following expression:

$$\mathbf{P} = \mathbf{u} \cdot \mathbf{p} \tag{10}$$

We write the shares of u and respectively p in S by x1 and respectively x2 as following:

$$x1 = u / S \quad \text{and} \quad x2 = p / S \tag{11}$$

and their product as:

$$PP = (u / S)^{-} (p / S)$$
(12)

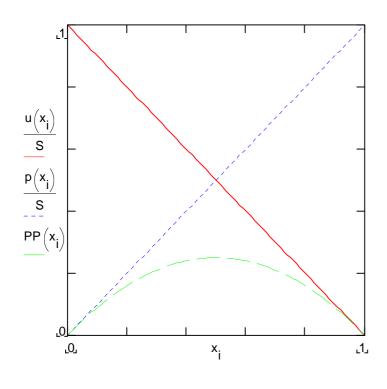
Now, we consider the generalised variable x representing shares of S. In Figure 6 is presented the complete map, where both functions of x-variable - the unemployment rate (u) and respectively the inflation rate (p) - are supposed to be linear curves. The model can be expressed by the following system of equations:

$$\mathbf{p}(\mathbf{x}) = \mathbf{x}^{\top} \mathbf{S} \tag{13}$$

$$\mathbf{u}(\mathbf{x}) = \mathbf{S} - \mathbf{p}(\mathbf{x}) \tag{14}$$

From this it results the following expression of PP:

 $PP(x) = x^{+}(1 - x)$





In this part of paper we call some results derived from the Catastrophe Theory or, more generally, from the Structural Stability Theory [14]. In order to elaborate a continuous-time model we consider that the main equation of our model can be derived from an existing potential function V(x;m):

$$dV / dx = f(x; m) = 0$$
 (16)

where x is the rapid variable of the system and m - the slow or control variable (both are, implicitly, functions of time). In our case, we chose the following potential function:

$$V(x;m) = (-x^{3})/3 + (x^{2})/2 - m x$$
(17)

to which it corresponds the following equation of potential surface:

$$-x^{2} + x - m = 0$$
(18)

Comparing this with the relation of PP already obtained it results that in terms of our model the slow variable m can be estimated by the following expression:

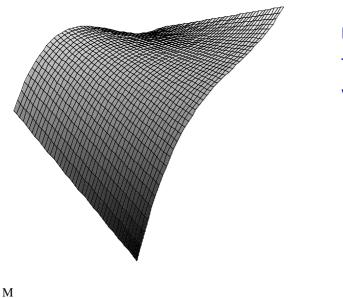
$$\mathbf{m} = \mathbf{P} / (\mathbf{S}^2) \tag{19}$$

(15)

Considering the analysis of the graph of function V, it results some threshold values of the parameter m. Therefore, for x having values among 0 and 1, there are the following cases:

when 0 < m < 3 / 16, V has 3 real roots (0 and other two separated); when m = 3 / 16, V has 2 real roots (0 and an other double root); when 3 / 16 < m < 1, V has only 0 as real root.

The graph of function V is shown in Figure 7.



Rotation 50 (0 to 360) Tilt 10 (0 to 360) Vertical Scale 100 (1 to 100 magnifications)





A very important threshold value of parameter m is 1/4, where the maximum, minimum, and inflexion points are confounded. Other important conclusions concerning evolution and stability of the system are:

- there are two equilibrium curves on the potential surface - a stable equilibrium curve (C1) and an unstable equilibrium curve (C2);

- for values of x smaller than (C2) the trajectories are attracted to (C1) (the long-run effect);

- for value of x greater than (C2) the system is strong attracted in an intense troubled zone;

- a rich menu of alternatives can be deduced by investigation of the function V map moreover this can offer some larger possibilities of statistical data interpretation.

A decisive problem represents the estimation of m. If this were quantified we would adopt desired alternatives by knowing their consequences. The resulting decisive importance of quantifying m strongly contradicts the authors who consider unemployment-inflation relationship having insignificant or neutral influence on the entire economic system. Consequently it must be ignored or both its factors must be strongly repressed simultaneously. When m equals 1/4 the system there is in an absolute equilibrium state and u = p. However, the remaining question is how can one solve better the problem of stability within zone (0; 0.25). For this we shall try to adopt a discrete model as an alternative.

Considering parameter \mathbf{a} as an essential parameter that assures the dynamics of the system in case of the discrete model, the equation will be:

$$\mathbf{x}_{t} = \mathbf{a}^{T} \mathbf{x}_{t-1}^{T} (1 - \mathbf{x}_{t-1})$$
(20)

This equation (which represents a canonical form of May's equation) possesses a wide range of dynamic behaviour, which is well known in the specialised literature. Limiting our attention to the initial conditions of the dynamics of x included in the interval [0; 1], the following "windows" of **a** were identified: 0 < a < 1, x moves monotonously towards the stationary solution x=0; 1 < a < 2, x moves monotonously towards the stationary solution x=0; 1 < a < 2, x moves with a flattened oscillatory movement to the stationary solution x=(a-1)/a; 3 < a < 4, x demonstrates a complex of permanent oscillations. For a = 3.57, one can observe an infinite number of fixed points with different periodicity and an infinite number of different periodical cycles; there also exist innumerable combinations of initial conditions from which completely **a**-periodical, although bounded, trajectories begin; it is from this threshold that the chaotically region begins. For a > 4, the model explodes.

Introducing the values of parameter \mathbf{a} derived from the discrete-time analysis, we obtained the following system of relations:

$$m(x) = -x^{2} + x$$
 (21)

$$a(x) = 1/(1-x)$$
 (22)

Now, eliminating x between the two equations, the relation among the two parameters can be written as:

$$m = (a-1)/a^2$$
(23)

and the graphical representation is shown in Figure 8. On this graph, we can see three remarkable points of parameter **a**: for **a** lower than 1, the parameter **m** becomes negative (that is the value 1 represents the lower limit of parameter **a** in case of our model); for parameter **a** equals value 2, parameter **m** has a maximum value (that is, the well-known value 1/4); for parameter **a** equals value 3, the function of **m** has an inflexion point. But from the previous section we know that for parameter **a** having the value greater than 3 the way to chaos is open. Thus, now we can affirm that for our model solely condition m < 1/4 is not sufficient. Moreover, it must be doubled by this last restriction on the scale of parameter **a**.

Having this new information a deeper analysis on the continuous map of the potential function V demonstrates that for the value m = 2/9 the maximums' branch (curve (C2)) has an inflexion point on the potential surface. This is the image of inflexion point of function m(a) in the potential function representation (in our graphical representation this inflexion point on V surface does not appear too clear). Only introducing parameter **a** in analysis permitted its discovery. So, a rise of unemployment within S in the proximity or over 2/3 value provokes firstly multiple-cycles and then emergence of a chaotic behaviour of the system. A similar situation occurs when the share of inflation within S equals the value 2/3.

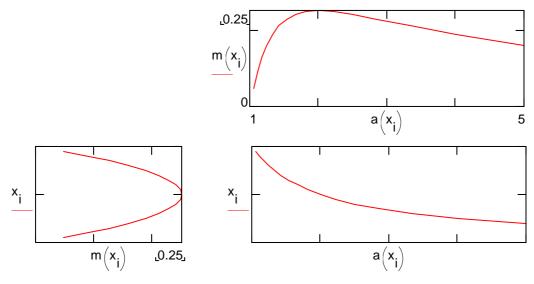


Figure 8

To illustrate the correspondence between u, p, m and \mathbf{a} we present in Appendix 2 a simple simulation-model. Also, it encloses a graphical representation of the transition to chaos function with \mathbf{a} -parameter variance.

Certainly, the analysis based on the simple model can be refined further more by considering new parameters. Unfortunately, this development introduces complicated deformations in the model, new non-linearities, and surely the model must be fundamentally changed. But the solution will be also to analyse the relations and variables grouped separately by considering simultaneously only two or, under special conditions, three of them. In terms of the models derived from our existing economic theory this is the manner of restriction [15]. Our results may be considered as an interlocution in the chaos/chance debate. In fact, the non-linear estimate is one that is confirmed in several fields. But the irregularities are explained by the stochastic, not the deterministic component, inasmuch as the control parameter reaches either the chaotic or the oscillatory bifurcation.

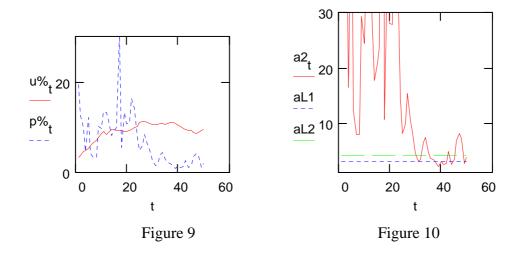
In Appendix 3 are presented the estimates of parameter **a** (only the largest values "a2" that are significant for our model) for the unemployment-inflation plan in case of selected Western economies. The figures represent the estimate values for the period 1970-1995. On graphic representations the interval that signifies the transition from stability to chaos is delimited by the values aL1=3 and respectively aL2=4.

To estimate more accurately the parameters a deeper analysis by subperiods will be necessary but this will be the goal of an other special study.

However, we can conclude that when the estimate value of **a** is within chaotic zone the strain in economic system is larger. As we stressed the symptoms of strain are unemployment and inflation. In the Western countries there was a specific transition during the last 25 years: unemployment replaced inflation as the main symptom of strain. Moreover, it would seem that such long periods of experiencing high unemployment may have infused a larger acceptance by the system of relative large unemployment. This must be viewed in a direct connection with a continuous development of social security programmes in Western countries. But as it was pointed out at the beginning of this section, such an inference needs to be buttressed by judging the evolution of other variables such as: the dynamic of budget deficits and their sustainability, number of strikes, etc.

In Eastern countries it seems that the acceptance of large unemployment is smaller at least during the present transition period. Moreover, in these countries the development of social security programmes is only in an incipient phase. It should be underlined, nonetheless, that an increasing underground sector developed as a valve for absorption of a large part from open unemployment.

For Romania, the trends for unemployment and respectively for inflation are presented in Figure 9, where monthly data cover the period 1990-1995. Also, in Figure 10 is presented the estimated evolution of parameter a for the unemployment-inflation plan. Here the monthly inflation rate was replaced by a yearly equivalent rate. Trends can be observed that seem to be toward the long-run trends in other European countries. However, a special research would be necessary to evaluate the distance from the situation existing in other economies and from the minimum levels requested by the integration within European Union.



5.3. Estimation of fractal dimension

Another methodology to estimate trends in economic series and to appreciate their type is that based on fractal dimension. In our study we chose the method called Hurst exponent. According to statistical mechanics, the Hurst exponent (H) should equal 0.5 if the series is a random walk. In other words, the range of cumulative deviation should increase with the square root of time. When H differed from 0.5, the observations were not independent. Each of these carried a "memory" of all the events that preceded it. This is not a short-term memory, which is often called Markovian. This memory is different: it is long-term. More recent events had a grater impact that distant events, but there was still residual influence. On a broader scale, a system that exhibits Hurst statistics is the result of a long stream of interconnected events. Time is very important. Inclusion of a time arrow is not possible in standard econometrics, which supposes series are invariant with respect to time [16].

There are three distinct intervals for the Hurst exponent: (1) H = 0.50, (2) 0 < H < 0.50, and (3) 0.50 < H < 1.00. First case denotes a random series. Events are random and uncorrelated. The present does not influence the future. Its probability density function can be normal curve. The standard statistics assume that nature follows the normal distribution, but H is typically greater than 0.5 for numerous series. The second type of system is an antipersistent, or ergodic, series. If the system has been up in the previous period, it is more likely to be down in the next period. In the third case we

have a persistent, or trend-reinforcing, series. If the series has been up (down) in the last period, then the chances are that it will continue to be positive (negative) in the next period. The closer H is to 0.5, the noisier it will be, and the less defined its trends will be. Persistent series are fractional brownian motion, or biased random walks. The strength of the bias depends on how far H is above 0.50. Persistent time series are plentiful in nature, as are probably many economic time series. Persistent time series are fractal because they can also be described as fractional brownian motion. The Hurst exponent describes the likelihood that two consecutive events are likely to occur. Because each point is not equally likely (as it is in a random walk), the fractal dimension of the probability distribution is not 2; it is a number between 1 and 2. Mandelbrot (1972) has shown that the inverse of H is the fractal dimension. Note that a random walk is truly 2-dimensional and would fill up a plane.

To estimate H and fractal dimension for the plan u-p and for global surface (u-py), we propose the following dynamic series:

$$doup_{t} := \sqrt{\left(u\%_{t}\right)^{2} + \left(p\%_{t}\right)^{2}}$$
(24)

$$do_{t} := \sqrt{\left(u\%_{t}\right)^{2} + \left(p\%_{t}\right)^{2} + \left(y\%_{t}\right)^{2}}$$
(25)

In Appendix 4 we present an own methodology of calculus used by us to estimate Hurst exponent and some applications.

6. Final remarks

The working hypothesis of this paper relies on the exceptional magnitude of the required resource reallocation in the former command economies; it aims at emphasising the extraordinary *strain*, these systems are undergoing. Ignoring this *strain*, would be equivalent to accepting a nonsensical proposition - that the command system was capable of allocating resources satisfactorily, eventually. One needs to highlight also another factor that enhances *strain*: the change of the regime of functioning of the economy and, relatedly, the scarcity of organisational and institutional capital which explain high systemic fragility and vulnerability.

It can be argued that the degree of *strain*, in post-command economies is the main impediment for the achievement of quick and durable macroeconomic stabilisation.

The assessment of realistic policy choices needs to consider various constraints: the size of the budget deficit and the available non-inflationary means for its financing, the concern not to fuel inflationary expectations, the impact of restructuring on the dynamics of the private sector, the level of external aid (financing), the social consensus regarding the speed of restructuring, the privatisation policy, etc.

Aside from the attempt to decipher *strain*, and subject it to empirical analysis this paper has several main messages. Firstly, it does underline that what can be done quickly should be done accordingly, delays can bring about very damaging detours and can create a hard to escape from "path-dependency". Secondly, it cautions against unavoidable tradeoffs among policy goals and commends the need to understand what is possible and probable to achieve bearing in mind the complexity of transformation and the *strain*, in the system. From this perspective it is argued that durable macroeconomic stabilisation and bringing inflation to a one level digit takes time and depends on the evolving institutional body of the post-command economies and on the pace of restructuring (resource reallocation). Thirdly, it is contended that the reasoning proposed herein can be applied to any socio-economic aggregate (economy) undergoing heavy shocks (external and internal) and, in which, consequently, an intense *strain* emerges. Finally, as the experience of Western countries indicates, *strain* can emerge incrementally in advanced economies as well unless adjustment mechanisms work effectively enough.

The dynamics and trends in the world economy, which *strain* increasingly national economies and can enhance their vulnerability, warrant further research on this topic.

ENDNOTES:

[1] Apart from the suppression, or divertion, of the entreprenurial spirit, which - as best indicated by the Austrian School (Schumpeter, von Mises, Kirszner, Rothbard) - is vital for the dynamics of an economy.

[2] See Daianu (1994a, 1994b) forthcoming.

[3] See also Portes (1986).

[4] Such constraints are highlighted by Aghion and Blanchard, or Sachs and Woo, for example.

[5] Inter-enterprise arrears, as a sympton of strain, endogenize partially the money supply dynamics and emasculate monetary policy.

[6] Daianu (1994b).

[7] When supply does not react sufficiently fast consumers may persist in using certain substitues for a while.

[8] We think, mainly, of segments of highly qualified labor.

[9] Such like the collapse of eastern European markets and the worsening of the terms of trade of most post-command economies in their trade with Russia.

[10] Daianu (1994a, pp.202-203).

[11] When wages consume the whole product nothing is left for capital renewal or enlargement (for investment) - which is a paradoxical situation bearing in mind the potential for expansion of those sectors. An even worse case is when there is **asset stripping**, when wages are higher than the average labor productivity.

[12] Williamson glosses on what he considers to be a standard thinking on macroeconomic stabilization - well exemplified by the book edited by Bruno, Fischer, Helpman, Liviatan and Meridor.

[13] In the ratio (4) the denominator can be written as $(1-\phi) \Sigma n_i \times w_i$, $\phi \in [0,1)$, where $(1-\phi)$ illustrates the degree of reduction of the production possibilities curve (and, implicitly, of incomes) because of exogenuous shocks. Clearly, J' increases and the *strain* in the system as well.

[14] In this version of the paper we omitted the theoretical argumentation. However, here we present only a brief incursion in the recent literature and some possible connections with the subject of underground economy development. So, in the last years there has been an increasing preoccupation of mathematical economists for

studying dynamic economic systems in non-linearity and discontinuity conditions. Although the disputes between the theorists representing the two great theories (the catastrophe theory, as a special case of bifurcation theory, whose emphasis is on discontinuity in the large; and the chaos theory, and its close relative fractal geometry, whose emphasis is on discontinuity in the small) continued, there are signs of a reconciliation within a metatheoretical sketch of the General Theory of Economic Discontinuities through some works of syntheses. Considering new mathematical developments (Grandmont, 1988), an increasing number of studies investigates nonlinearities in models of economic dynamics (Day, 1994; Vilder De, 1995; etc.). Some of the most important problems which have been dealt with recently are: endogenous business cycles and competitive market dynamics; overlapping generations models; speculative bubbles and crashes; optimal accumulation in two-sector models; imperfect financial intermediation; multiplier-accelerator models; cobweb model; dynamics of the perfect foresight and rational expectations; dynamics of the adaptive expectations; discrete time model of monetary dynamics. A discrete-continuous model relatively similar to that used in this paper was used to estimate the size and dynamic of underground economy (Albu, 1995).

[15] The mentioned conclusions may be result from our limited possibilities to model, from the limits of our present science and our existing mathematical apparatus, from our own power of understanding reality (the scientists from other disciplines as physics, for instance, had already the power to affirm frankly their own limits in investigating and understanding, but some economists have yet reticence to affirm this). Maybe, the main preoccupation of research would be only to register the evidence of facts, even when they refuse to come into our existent models. Also the investigation of some economic time series or constructing some models including up to 4 or even 5 slow parameters, can be developed by using the recent results of Chaos Theory and its close Fractal Geometry and respectively Catastrophe Theory. But frequently the explanations will be beyond those accepted by standard economic theory. Consequently, many of these studies will be contested by the partisans of standard economics.

[16] For details see Peters (1991).

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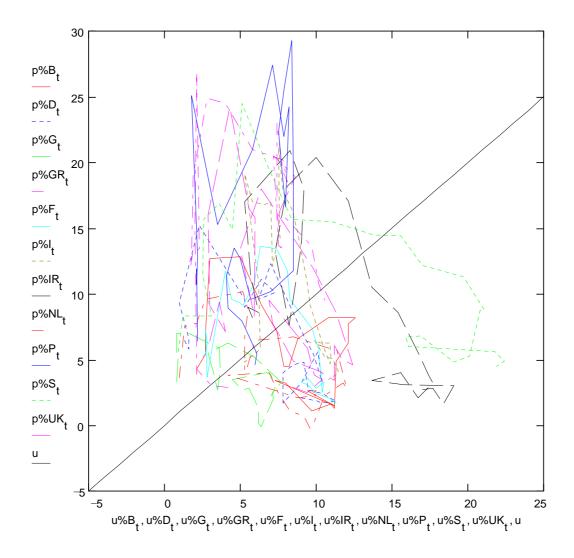
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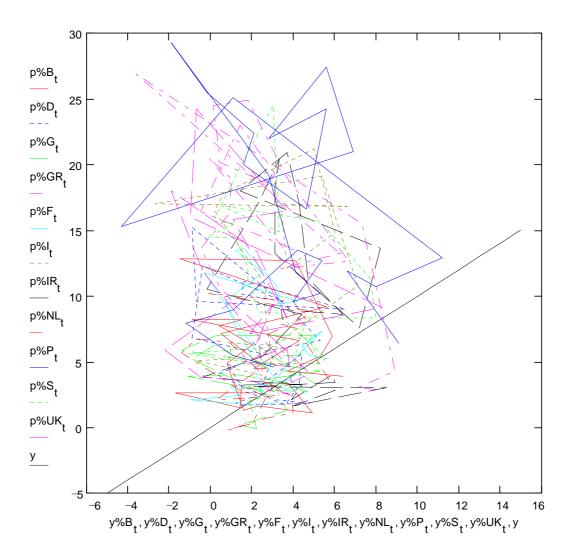
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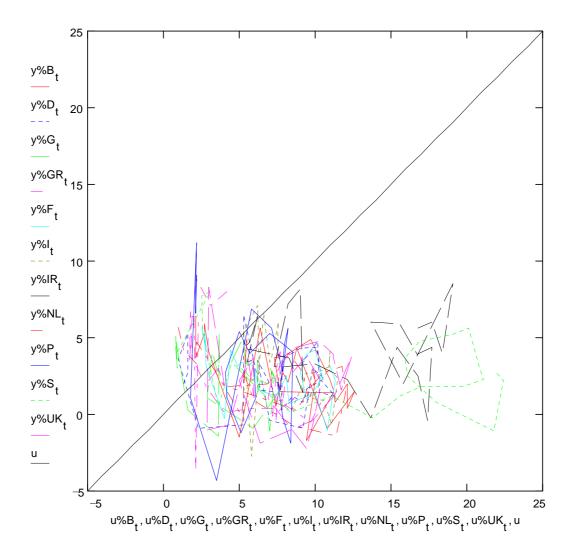
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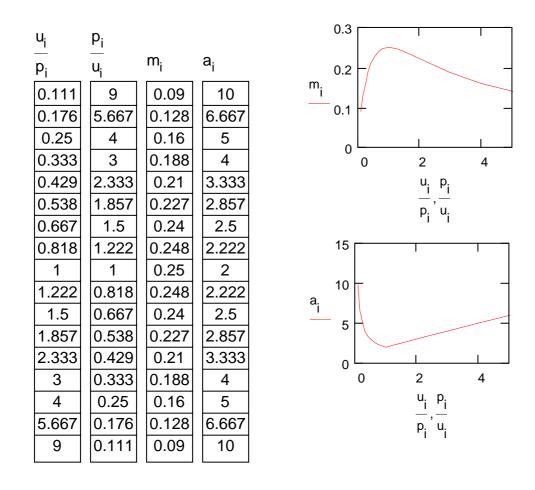


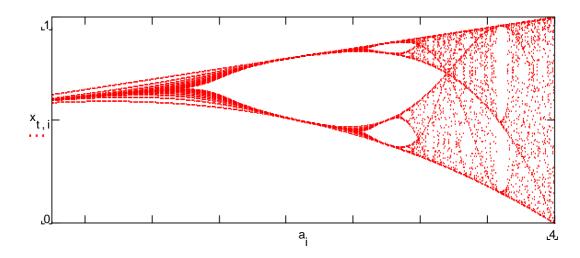




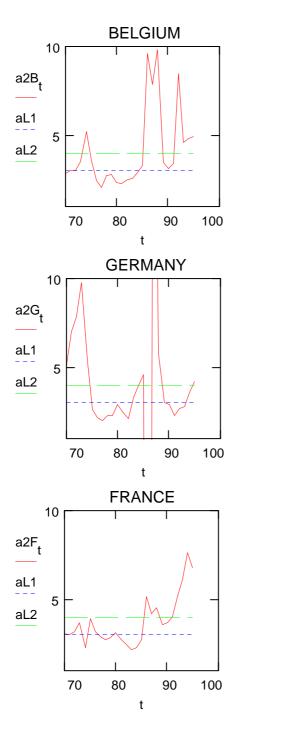


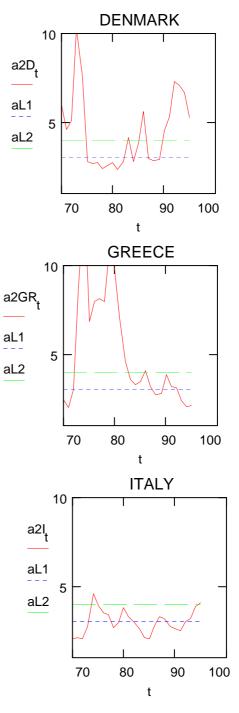


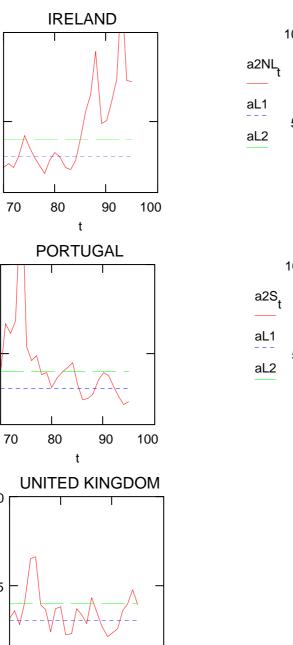












10

5

10

5

10

5

70

80

t

90

100

a2UK_t

aL1

aL2

a2P_t

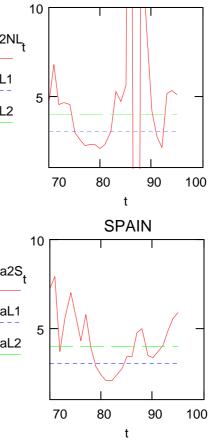
aL1

aL2

a2IR_t

aL1

aL2



NEDERLANDS

Appendix 4

First, we considered an existing time series, t, with i observations:

$$Y_{t,N} := \begin{bmatrix} t \\ \sum_{i=1}^{t} (x_i - M_N) \end{bmatrix}$$

where Y is cumulative deviation over N periods; x - value of variable (named doup, do, doyp, and douy); M - average x over N periods.

Then, after some laborious transformation algorithms, the following relationship (formulated by Hurst) permitted to us to estimate H:

$$\mathbf{R} / \mathbf{S} = (\mathbf{b} \cdot \mathbf{N})^{\mathbf{H}}$$

where R represents range of Y (R=Max(Y) - Min(Y)); S - standard deviation of the observations; N - number of observations; b - a constant.

In fact we followed the steps:

s1 :=
$$x_1$$
 s2 := s1 + x_2 s3 := s2 + x_3 . . .
X1 := x_1

$$X2 := \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \qquad \begin{array}{l} n := 2 \quad t := 1..2 \\ Y2 := \begin{pmatrix} s1 - mean(X2) \\ s2 - 2 \cdot mean(X2) \end{pmatrix} \qquad \begin{array}{l} Z2 := \frac{max(Y2) - min(Y2)}{stdev(X2)} \end{array}$$

n := 3 t := 1..3
X3 :=
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$
 Y3 := $\begin{pmatrix} s1 - mean(X3) \\ s2 - 2 \cdot mean(X3) \\ s3 - 3 \cdot mean(X3) \end{pmatrix}$ Z3 := $\frac{max(Y3) - min(Y3)}{stdev(X3)}$

.

n := 2..N

 $z_2 := Z2$ $z_3 := Z3$ $z_4 := Z4$. . .

$$\begin{split} H &:= \frac{(N-1) \cdot \sum_{n} \left(\log(n) \cdot \log\left(z_{n}\right) \right) - \left(\sum_{n} \log(n) \cdot \sum_{n} \log\left(z_{n}\right) \right)}{(N-1) \cdot \sum_{n} \log(n)^{2} - \left[\sum_{n} \left(\log(n) \right) \right]^{2}} \\ b &:= \frac{\left(\sum_{n} \log(n)^{2} \right) \cdot \left(\sum_{n} \log\left(z_{n}\right) \right) - \left[\sum_{n} \log(n) \cdot \sum_{n} \left(\log(n) \cdot \log\left(z_{n}\right) \right) \right]}{(N-1) \cdot \sum_{n} \log(n)^{2} - \left[\sum_{n} \left(\log(n) \right) \right]^{2}} \end{split}$$

By applying the methodology on data relating to the selected Western countries resulted the estimate values of H for the period 1970-1995 which are presented in the following table:

	Hdoup	Hdo
Belgium	0.861	0.841
Denmark	0.889	0.897
France	0.901	0.906
Germany	0.773	0.677
Greece	0.823	0.812
Italy	0.943	0.938
Ireland	0.739	0.747
Netherlands	0.792	0.754
Portugal	0.978	0.966
Spain	0.909	0.907
United Kingdom	0.885	0.856

We can observe that by value of H the hierarchy is some different from the other methods used in this chapter. So, aside of Germany and Netherlands, which are the two countries that demonstrate less strain conforming with precedent methods, moreover there is Ireland. The other western countries seem to demonstrate more strain as a general characterisation of the period 1970-1995 as consequence of some strong persistence time series.

As a comparison for Romania the calculated value of Hdoup was about 0.895 for the period November 1993-December 1995, where the monthly inflation rate was replaced by a yearly equivalent rate.