A Procurement Auction Model Under Supplier Uncertainty

Gurnani, Haresh and Ray, Tridip
Hong Kong University of Science and Technology

June 2003

A Procurement Auction Model Under Supplier Uncertainty*

Haresh Gurnani[†] Tridip Ray[‡]

This version: June 2003

Abstract

As business-to-business commerce shifts to the Internet, newer suppliers with cheaper but unreliable technologies enter the market place to win orders from firms by beating the price of their perfectly reliable (but expensive) competitors. The dilemma facing purchasing firms is the allocation of the tender across suppliers of varying supply reliability. We model the procurement problem as a sealed-bid auction where the buyer has to allocate purchases between an expensive but reliable supplier and a cheaper but unreliable supplier, and the suppliers specify prices for different proportions of the tender awarded to them. A unique feature of our model is that it allows the purchasing firm to reserve the right to change the size of the total tender awarded depending on the nature of the bids received from the suppliers. We prove that the set of Nash equilibrium outcomes coincides with the set of efficient outcomes, and for strictly convex cost functions, the outcome is unique. Further, we show that the possibility of implicit supplier collusion is strengthened in that the suppliers may structure their bids forcing the buyer to allocate the tender resulting in the worst-case (highest price) scenario for him. We also show that the Anton-Yao (A-Y, 1989) model can be interpreted as a limiting case of our model and that the efficient outcome derived in this paper is the only robust outcome in the A-Y model.

* We would like to thank Sudipto Dasgupta, Jae Hyon Nahm, Larry Qiu, Kunal Sengupta and Wen Zhou for helpful comments and suggestions on an earlier draft.

† Department of Information and Systems Management, Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong. Phone: (852) 2358-7724; Fax: (852) 2358-1946; e-mail: mnharesh@ust.hk.

‡ Department of Economics, Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong. Phone: (852) 2358-7625; Fax: (852) 2358-2084; e-mail: tridip@ust.hk.

1. Introduction

Electronic Commerce on the Internet offers an efficient way to do business by bringing together many potential producers and consumers. According to a recent article in Business Week (May 2000), B2B transactions may revolutionize the way Asian manufacturers trade with the rest of the world by allowing small and mid-sized manufacturers to find a new sphere of buyers. While the Internet provides for more opportunities for trade, the reliability and trustworthiness of suppliers may vary significantly. Newer suppliers with cheaper but unreliable technologies may enter the market place and try to win orders from firms by beating the price of their perfectly reliable (but expensive) competitors. This puts the buyers in a dilemma:

> " ... *Sure, they may save a ton of money by holding a reverse auction, but how can they be sure that the low-bidding suppliers can actually fill their orders*," (Feuerstein, October 2000).

The issue we address in this paper is whether buyers should award the full tender to reliable but expensive suppliers with whom they were doing business so far, or take a risk and award parts of the tender to the newcomers offering low prices. In particular, we consider the procurement problem for a buyer who has to allocate the purchases between an expensive but reliable supplier and a cheaper but unreliable supplier.[1] For example, a buyer in North America may have a trusted local/regional supplier, but would like to explore business opportunities with newer and cheaper (but with unproven technology) Asian suppliers who have just entered the market place.[2]

Due to uncertain production processes in certain industries (for example, semiconductor manufacturing and electronics industries), the unreliable supplier may be unable to execute the supply contract on time. For the reliable supplier, investment in better tech-

[1] Multiple sourcing is frequently observed in practice in both industrial (Treleven and Schweikhart 1988) as well as government procurements (Mayer 1987).

[2] While used or perishable goods typically lend themselves to auctions, the idea is being extended and General Electric, for instance, has a website that lists the supplies it needs, and any supplier can bid to sell them. Several success stories about such auctions have been cited by Turban (1997).

nology along with maintaining safety stock of items ensures that the supply process is not disrupted. Charging a higher price to guarantee delivery can then be justified. For buyers facing concentrated selling seasons (for example, selling toys during Christmas season), any delay in component deliveries from the supplier could result in expensive shortage situations. In some cases, the buyer may as a last resort produce critical components in-house at a high cost.

In the auction model considered in the paper, both suppliers (reliable and unreliable) submit a sealed bid that specifies prices for different proportions of the tender awarded to them. The size of the total award is a decision variable for the buyer as the existence of an unreliable supplier may induce the buyer to inflate the input order size. Conversely, the buyer may choose not to award the full tender originally announced and move some of the production to the in-house production facility.[3] Anton and Yao (1989) (henceforth A-Y) consider a similar sealed-bid procurement auction model and derive a continuum of Nash equilibrium outcomes for the case of two reliable suppliers. For each outcome, the orders are split awards, in that the total award size is fixed and the allocation decision is to determine the proportion to be awarded to each supplier. Our paper differs from A-Y paper in three ways. First, we suppose that one of the suppliers is unreliable in delivery. Second, we allow for the option of in-house production in our model. Third, and most importantly, we allow for the possibility of over-procurement (to hedge against the failure of the unreliable supplier from not delivering) and under-procurement (that is, to move part of the production in-house). As a result of this added flexibility in order allocation, we prove that the set of efficient outcomes (outcomes that minimize total production costs) coincides with the set of Nash equilibrium outcomes, and for strictly convex cost functions, the outcome is unique. Further, as compared to A-Y model, we discuss that the implicit collusive behavior of the suppliers is strengthened. For the example of quadratic production costs, we show that the buyer makes a strictly positive award to both the

[3] In the absence of an in-house production facility, the buyer may incur shortage penalty costs due to partial fulfillment of its tender requirements.

suppliers as well as to the in-house production facility. Comparative statics analysis of the effects of production costs and the reliability of supply is provided. Finally, it is shown that the A-Y model can be interpreted as a limiting case of our model and that the efficient outcome derived here is the only robust outcome in the A-Y model.

In the related literature, the problem of supply unreliability has been well studied in the context of single and multi-item random yield inventory models (Henig and Gerchak 1990, Yano and Lee 1995, Gurnani *et al.* 1996; 2000).[4] Supply unreliability is often cited as justification for why firms second source in order to hedge against the failure of one supplier from not delivering on time (Leidy 1992, Ketchpel and Garcia-Molina 1998).[5] Recently, Gurnani and Shi (2000) considered a B2B bargaining model where the buyer and the supplier have asymmetric beliefs on the level of supply unreliability. They derived a non-symmetric contract that is incentive compatible and maximizes channel profits under certain conditions.

Game theoretic treatment of various auction models are surveyed in McAfee and McMillan (1987) and Milgrom (1989), while some experimental results are reported in Kagel and Roth (1995). More specifically, in the procurement auction literature with *second sourcing*, Anton and Yao (1987) modeled the procurement problem as a two-stage process in which initial production is governed by a contract between the government and the developer. In the second stage, competition is induced by an auction in which a second source competitively bids for remaining production. The sequential sourcing process is designed to extract private cost information from the suppliers. In other papers, Anton and Yao (1989; 1992) studied the split-award auction problem with symmetric and asymmetric (that is, with private information) costs respectively. In both papers, the procurement decision for the buyer is to determine the sole sourcing or split-award equilibrium outcomes. Further, Anton and Yao discuss that while supplier diversification is

[4] In all these papers, supplier prices are assumed to be fixed, that is, they are exogenous to the model.

[5] While multiple sourcing reduces the risk of stockouts, the evidence from private and government auctions suggests that the prospect of supplier collusion leading to noncompetitive bidding increases (Meeker 1984, Boger and Liao 1988, Anton and Yao 1989).

traditionally used to promote price competition, the efficient outcome, which is one of (many possible) Nash equilibrium outcomes, exhibits strong collusive features resulting in the maximum procurement price for the buyer. In our paper, the set of efficient outcomes coincides with the set of Nash equilibrium outcomes and as such the possibility of implicit supplier collusion is strengthened.

While all the papers above assume that the number of bidders is given exogenously, in the model of Seshadri *et al.* (1991) the number of active bidders is determined endogenously – they considered a multiple source procurement problem to investigate the effect of multiple sourcing on competitive behavior *prior* to supplier selection. More recently, Seshadri (1995) analyzed the supplier selection and control problem in an integrated fashion. In his paper, two suppliers are drawn from *several* vendors through a competitive bidding process, and the winner of the bidding competition receives a large share of the contract and also benefits due to the credible commitment made by the buyer. In another paper, Klotz and Chatterjee (1995) considered a repeated procurement competition problem with production learning and entry costs (for the suppliers). Using a two-period model, they explored the effectiveness of dual sourcing in facilitating competition between the suppliers as a means of counteracting the competition-reducing effects of production learning and entry costs.

The rest of the paper is organized as follows. In the next section, we discuss the model assumptions and formulate the bidding problem for the suppliers and the procurement problem for the buyer. In section 3, we characterize the set of Nash equilibrium outcomes of the game and show that it coincides with the set of efficient outcomes. The strengthening of implicit price collusion is also established. In section 4, using the example of quadratic production costs, we do the comparative statics analysis and show that the A-Y model can be interpreted as a limiting case of our model. Finally, in section 5, we conclude and discuss future research.

2. The Model

A buyer is contemplating to procure x units of a good from two potential suppliers.[6] Supplier R is the longtime trusted and reliable supplier whereas supplier U, quite new in business having just entered the market place, has an uncertain production process leading to unreliability in delivery. Following Turnbull (1986), Leidy (1992) and Gurnani *et al.* (1996), we assume that delivery by the unreliable supplier, U, is dichotomous – either he delivers 100% of the order, or he delivers nothing at all.[7] U receives his payment only when he delivers his order. Let $\beta \in (0, 1]$ be the probability that U can fulfil his order.

In traditional procurement settings, the buyer would make allocation decisions based on fixed prices quoted by the supplier, or negotiate with the supplier for quantity-driven price discounts. With the proliferation of electronic market places, new business models have evolved and price-driven auctions have become very popular.[8] In this paper, we consider a sealed-bid auction where each supplier submits a bid that specifies prices for different proportions of x that is awarded to him. That is, if αx and γx units are awarded to suppliers U and R respectively,[9] $\alpha \in [0, 1]$ and $\gamma \in [0, 1]$, the bids are functions $P_U(\alpha)$ for supplier U and $P_R(\gamma)$ for supplier R, where $P_i : [0, 1] \rightarrow \Re$, $i = U$, R, with $P_U(0) = 0 = P_R(0)$. Unlike Anton and Yao (1989), we do not restrict α and γ to add up to unity, that is, awards are not necessarily split awards. The reason for this is very intuitive in the face of supplier unreliability. For instance, even if the full contract is awarded to supplier U (i.e., $\alpha = 1$), he might default on delivery, and, to be on the safe side, it is

[6] In our paper, we assume that two potential eligible suppliers have been identified. See Barua *et al.* (1997) for a discussion of supplier selection strategies.

[7] The 'all or nothing' nature of deliveries of the unreliable supplier can be justified as follows. When transportation costs are high, the contract may specify delivery in a single shipment to avoid the high administrative and operational costs of keeping track of part shipments. Also, for buyers facing concentrated selling seasons (for example, selling toys during Christmas season), on-time delivery is critical as late component deliveries from the suppliers become useless.

[8] The scope of our model is general and is applicable in both electronic and non-electronic auction settings.

[9] Throughout the paper we follow the notation that an order (α, γ) indicates that αx units are awarded to supplier U and γx units to supplier R.

quite natural for the buyer to order some share from the reliable supplier. Thus, $\alpha+\gamma > 1$ is a natural possibility in the presence of supplier unreliability. We also allow α and γ to add up to less than unity to capture the fact that buyers usually reserve the right not to award the full tender originally announced.[10]

Suppliers U and R face the costs of production $C_U(\alpha)$ and $C_R(\gamma)$ respectively, where $C_i : [0,1] \to \Re$, $i = U$, R, with $C_U(0) = 0 = C_R(0)$. Supplier U is unreliable because, presumably, he is using an inferior (and, of course, cheaper) technology. So we assume that $C_U(\alpha) < C_R(\alpha)$, for all $\alpha \in (0,1]$. Following Anton and Yao (1989), we also assume that the suppliers have complete information about each other's costs when they bid.

Recall that deliveries are 'all or nothing' in nature, that is, if the unreliable supplier cannot deliver his promised share on time, he does not deliver at all. If a share $\delta \in [0,1]$ of x remains undelivered, the buyer's outside option or penalty is $K(\delta)$, where $K : [0,1] \to \Re$, with $K(0) = 0$. This outside option can be given the following alternative interpretations: the buyer can produce in-house[11] at a high cost $K(\delta)$, or, alternatively, the buyer bears the penalty of nondelivery to his own customers (the opportunity cost of revenue forgone) the value of which is $K(\delta)$. Whatever be the interpretation, it is natural to assume that $K(\delta) > C_R(\delta) > C_U(\delta)$, for all $\delta \in (0,1]$.

When the buyer places any order (α, γ), the total cost to the system has three components – production costs of suppliers U and R, and the expected penalty or in-house production cost of the buyer. Given any order (α, γ), let us define the total system cost as

$$TSC(\alpha,\gamma) = C_U(\alpha) + C_R(\gamma) + \beta K\left(\{1-(\alpha+\gamma)\}^+\right) + (1-\beta)K(1-\gamma),$$

[10] If the full tender is not guaranteed to be awarded, one would expect, *ex ante*, that the suppliers would compete more aggressively in their bids.

[11] For example, in some defense procurement auctions, the government auctions production rights to private developers and also retains the government-owned production option (Dana and Spier 1994).

where $\{1-(\alpha+\gamma)\}^+ = \begin{cases} 0, & \text{if } (\alpha+\gamma) \geq 1, \\ 1-(\alpha+\gamma), & \text{if } (\alpha+\gamma) < 1. \end{cases}$

If the suppliers put the bids $P_U(\cdot)$ and $P_R(\cdot)$ and the buyer places the order $(\alpha,\, \gamma)$, supplier R earns the profit $\pi_R(\gamma) = P_R(\gamma) - C_R(\gamma)$, while supplier U's expected profit is defined by $\pi_U(\alpha) = \beta P_U(\alpha) - C_U(\alpha)$.[12] The suppliers maximize expected profits, and their participation is ensured by restricting our attention to bids for which $\pi_U(\alpha) \geq 0$ and $\pi_R(\gamma) \geq 0$.

The buyer's expected procurement cost is

$$G(\alpha,\gamma) = \beta P_U(\alpha) + P_R(\gamma) + \beta K\left(\{1-(\alpha+\gamma)\}^+\right) + (1-\beta)K(1-\gamma),$$

and the buyer places the order $(\alpha,\, \gamma)$ to minimize the expected procurement cost, that is,

$$(\alpha,\gamma) \in \underset{(\alpha,\gamma)\in[0,1]\times[0,1]}{\operatorname{argmin}} \{G(\alpha,\gamma)\}\,. \tag{1}$$

The timing of the game is as follows: the suppliers simultaneously (and non-cooperatively) submit the sealed bid functions $P_U(\cdot)$ and $P_R(\cdot)$, and then the buyer chooses the order $(\alpha,\, \gamma)$ that minimizes the expected procurement cost.

A Nash equilibrium involves a pair of bids submitted by the suppliers, $(P_U^*(\alpha), P_R^*(\gamma))$, such that the bids are mutually best responses for them. Given the equilibrium bids, if an order (α^*,γ^*) minimizes the buyer's expected procurement cost (that is, satisfies (1)), then (α^*,γ^*) is an equilibrium outcome. The equilibrium procurement price is denoted by g^*, that is, $g^* = G(\alpha^*,\gamma^*) = \beta P_U^*(\alpha^*) + P_R^*(\gamma^*) + \beta K\left(\{1-(\alpha^*+\gamma^*)\}^+\right) + (1-\beta)K(1-\gamma^*)$.

A *sole-source outcome* is an order (α,γ) such that either $\alpha = 0$ or $\gamma = 0$, whereas a *multi-source outcome* is an order (α,γ) where $0 < \alpha \leq 1$ and $0 < \gamma \leq 1$.

[12] U receives his bid price $P_U(\alpha)$ only when he delivers the promised amount αx. In case of non-delivery, he is not paid at all but has to bear the cost $C_U(\alpha)$ in either situation.

3. Nash Equilibrium, Efficiency and Implicit Price Collusion

To analyze the auction game, in this section we first characterize the Nash equilibrium bids and then use this characterization to rule out the possibility of over-ordering as an equilibrium outcome. Then, we derive the set of equilibrium outcomes and the structure of the equilibrium bids, profits and procurement prices, and demonstrate that the set of equilibrium outcomes coincides with the set of efficient outcomes. Finally, we note how implicit supplier collusion leads to the worst-case outcome for the buyer and discuss the strengthening of the implicit collusion possibility as compared to the Anton-Yao model.

3.1. Characterizing the Nash Equilibrium Bids

In this subsection we first characterize the Nash equilibrium bids and the associated outcomes.

In a sole-source award, one of the suppliers gets no order. If an arbitrary sole-source order, $(\alpha, 0)$, is awarded to supplier U, then the procurement price is $\beta P_U(\alpha) + \beta K(1-\alpha) + (1-\beta)K(1)$. Similarly, if an arbitrary sole-source order, $(0, \gamma)$, is awarded to supplier R, the procurement price becomes $P_R(\gamma) + K(1-\gamma)$. Sole-source prices play an important role in determining equilibrium bids, profits and the procurement price as shown in the following lemma.

Lemma 1. *Suppose that $(P_U^*(\alpha), P_R^*(\gamma))$ is a pair of Nash equilibrium bids, and that g^* is the corresponding equilibrium procurement price. Then there exists $\overline{\alpha} \in (0,1]$ and $\overline{\gamma} \in (0,1]$ such that $g^* = \beta P_U^*(\overline{\alpha}) + \beta K(1-\overline{\alpha}) + (1-\beta)K(1) = P_R^*(\overline{\gamma}) + K(1-\overline{\gamma})$.*

Proof. See the Appendix.

Intuitively, Lemma 1 implies that there exists at least one sole-sourcing bid for each supplier such that the buyer is indifferent to either of these sole-sourcing outcomes in equilibrium. Further, if the equilibrium allocation is a dual sourcing award, that is both

$\alpha > 0$ and $\gamma > 0$, the sole-sourcing outcomes provide an upper bound on the procurement cost for the buyer for any equilibrium outcome.

Next, we derive the deviation conditions for the suppliers which serve as necessary conditions for the Nash equilibrium outcome(s). Consider a situation where the suppliers have put the bids (P_U, P_R) and the buyer has placed the order (α, γ). Bidder U can profitably induce the buyer to switch away from the order (α, γ) if there exists another order $(\widehat{\alpha}, \widehat{\gamma})$ such that U can find a price $\widehat{p}$ at $\widehat{\alpha}$ where U's expected profit is greater $(\pi_U(\widehat{\alpha}) > \pi_U(\alpha))$ *and* the buyer faces a lower expected procurement cost $(G(\widehat{\alpha}, \widehat{\gamma}) < G(\alpha, \gamma))$, that is, $\beta\widehat{p} - C_U(\widehat{\alpha}) > \beta P_U(\alpha) - C_U(\alpha)$, *and* $\beta\widehat{p} + P_R(\widehat{\gamma}) + \beta K\left(\{1-(\widehat{\alpha}+\widehat{\gamma})\}^+\right) + (1-\beta)K(1-\widehat{\gamma}) < \beta P_U(\alpha) + P_R(\gamma) + \beta K\left(\{1-(\alpha+\gamma)\}^+\right) + (1-\beta)K(1-\gamma)$. Such a $\widehat{p}$ exists if and only if the following inequality is satisfied: $P_R(\gamma) - P_R(\widehat{\gamma}) > [C_U(\widehat{\alpha}) + \beta K\left(\{1-(\widehat{\alpha}+\widehat{\gamma})\}^+\right) + (1-\beta)K(1-\widehat{\gamma})] - [C_U(\alpha) + \beta K\left(\{1-(\alpha+\gamma)\}^+\right) + (1-\beta)K(1-\gamma)]$. After some simplifications (using $\pi_R(\gamma) = P_R(\gamma) - C_R(\gamma)$) we conclude that bidder U can profitably induce the buyer to switch away from the order (α, γ) if there exists an order $(\widehat{\alpha}, \widehat{\gamma})$ such that

$$\begin{aligned} &\pi_R(\gamma) + C_U(\alpha) + C_R(\gamma) + \beta K\left(\{1-(\alpha+\gamma)\}^+\right) + (1-\beta)K(1-\gamma) \\ &\quad > \pi_R(\widehat{\gamma}) + C_U(\widehat{\alpha}) + C_R(\widehat{\gamma}) + \beta K\left(\{1-(\widehat{\alpha}+\widehat{\gamma})\}^+\right) + (1-\beta)K(1-\widehat{\gamma}). \end{aligned} \tag{2a}$$

Similarly, bidder R can profitably induce the buyer to switch away from the order (α, γ) if there exists an order $(\widehat{\alpha}, \widehat{\gamma})$ such that

$$\begin{aligned} &\pi_U(\alpha) + C_U(\alpha) + C_R(\gamma) + \beta K\left(\{1-(\alpha+\gamma)\}^+\right) + (1-\beta)K(1-\gamma) \\ &\quad > \pi_U(\widehat{\alpha}) + C_U(\widehat{\alpha}) + C_R(\widehat{\gamma}) + \beta K\left(\{1-(\widehat{\alpha}+\widehat{\gamma})\}^+\right) + (1-\beta)K(1-\widehat{\gamma}). \end{aligned} \tag{2b}$$

To qualify to be a Nash equilibrium, a pair of bids, $(P_U^*(\cdot), P_R^*(\cdot))$, must obey Lemma 1 and the deviation conditions (2a) and (2b).

3.2. Possibility of Over-ordering

In section 2, we discussed that in the presence of supplier unreliability one cannot *a priori* rule out the possibility of over-ordering. In particular, even after awarding the full tender to the unreliable supplier, the buyer may still like to place some order to the reliable supplier just in case the unreliable supplier defaults. But, as the following lemma shows, it turns out that over-ordering cannot be an equilibrium outcome.

Lemma 2. *An order* (α, γ) *such that* $0 < \alpha \leq 1$, $0 < \gamma \leq 1$, *and* $\alpha + \gamma > 1$ *is not an equilibrium outcome if* $C_U(\cdot)$ *is strictly increasing.*

Proof. Consider the order $(\widehat{\alpha}, \gamma)$ such that $\widehat{\alpha} + \gamma = 1$. Using (2a), we find that supplier U can profitably induce the buyer to switch away from the order (α, γ) (where $\alpha + \gamma > 1$) by changing his bid at $\widehat{\alpha}$ if $C_U(\alpha) > C_U(\widehat{\alpha})$. But we have $\alpha > 1 - \gamma$, and $\widehat{\alpha} = 1 - \gamma$. Hence this condition always holds when $C_U(\cdot)$ is strictly increasing. *Q.E.D.*

The proof indicates that irrespective of the order placed with the reliable supplier (γ), the split-award outcome (that is, one with $\alpha + \gamma = 1$) always dominates the outcome with over-ordering (that is, with $\alpha + \gamma > 1$). Intuitively, since the buyer does not get any benefit from the excess supply (if it is delivered), whereas it costs more to supplier U to produce the extra amount, therefore supplier U structures his bids so as to refrain the buyer from over-ordering. This explains the somewhat surprising approach in the proof in that supplier U induces the buyer to reduce his own share of the contract.[13]

[13] The argument for ruling out over-ordering so far relies on the assumption that the buyer does not get any benefit from the excess supply, that is, there is no salvage value for excess inventory. If we allow for the possibility of salvage, Lemma 2 gets modified as follows. For a unit salvage value S, if $C_U(\alpha) > \beta S\alpha$, for all $\alpha > 0$, then (α, γ) such that $0 < \alpha \leq 1$, $0 < \gamma \leq 1$, and $\alpha + \gamma > 1$ is not an equilibrium outcome. Intuitively, the condition implies that if the salvage value is low enough such that the expected salvage revenue is less than the cheapest way to produce the items, then over-ordering will not be an equilibrium outcome. The proof is similar to that of Lemma 2 and is omitted.

3.3. Nash Equilibrium

Lemma 2 demonstrates that over-ordering cannot be an equilibrium outcome. At the same time it illustrates how we can use the general deviation conditions ((2a) and (2b)) in a particular way to rule out a potential outcome to be an equilibrium one: to rule out the possibility of (α, γ) we show the existence of another allocation $(\widehat{\alpha}, \gamma)$ such that supplier U can induce the buyer to switch away from the order (α, γ) by changing his bid at $\widehat{\alpha}$. Since the awards are not necessarily split awards in our model, that is, $\alpha + \gamma$ may not be equal to 1, we have the flexibility to switch from the order (α, γ) to $(\widehat{\alpha}, \gamma)$ without changing the allocation for supplier R. In case of Anton-Yao model, since all awards are split awards, changing the allocation for one supplier also changes the allocation for the other supplier. In this subsection, we use the deviation conditions in this particular way to narrow down the set of possible Nash equilibrium outcomes significantly.

From (2a), we find that supplier U can induce a profitable deviation away from the order (α, γ) if there exists an allocation $(\widehat{\alpha}, \gamma)$ such that

$$C_U(\alpha) + \beta K \left(\{1 - (\alpha + \gamma)\}^+\right) > C_U(\widehat{\alpha}) + \beta K \left(\{1 - (\widehat{\alpha} + \gamma)\}^+\right). \tag{3a}$$

Similarly, using (2b), we see that supplier R can induce a profitable deviation away from the order (α, γ) if there exists an allocation $(\alpha, \widehat{\gamma})$ such that

$$C_R(\gamma) + \beta K \left(\{1 - (\alpha + \gamma)\}^+\right) + (1-\beta)K(1-\gamma) > C_R(\widehat{\gamma}) + \beta K \left(\{1 - (\alpha + \widehat{\gamma})\}^+\right) + (1-\beta)K(1-\widehat{\gamma}). \tag{3b}$$

Now let us define two correspondences, $BR_U(\gamma)$ for supplier U and $BR_R(\alpha)$ for supplier R, where $BR_i : [0, 1] \to [0, 1]$, $i = U,\ R$, as follows:

$$BR_U(\gamma) = \underset{0 \leq \alpha \leq 1}{\operatorname{argmin}}\, C_U(\alpha) + \beta K \left(\{1 - (\alpha + \gamma)\}^+\right), \tag{4a}$$

$$BR_R(\alpha) = \underset{0 \leq \gamma \leq 1}{\operatorname{argmin}}\, C_R(\gamma) + \beta K \left(\{1 - (\alpha + \gamma)\}^+\right) + (1-\beta)K(1-\gamma). \tag{4b}$$

Using conditions (4a) and (4b) we can interpret these two correspondences as follows. It follows from (4a) that if the buyer places an order (α, γ) where $\alpha \notin BR_U(\gamma)$, then supplier U can profitably induce the buyer to switch away from (α, γ) by changing his bid at some $\alpha' \in BR_U(\gamma)$. Similarly, it follows from (4b) that if the buyer places an order (α, γ) where $\gamma \notin BR_R(\alpha)$, then supplier R can profitably induce the buyer to switch away from (α, γ) by changing his bid at some $\gamma' \in BR_R(\alpha)$. Note that as long as $C_U(\cdot)$, $C_R(\cdot)$ and $K(\cdot)$ are continuous, $BR_U(\gamma)$ and $BR_R(\alpha)$ are non-empty.

Now let us define the set, $N = \{(\alpha, \gamma) : \alpha \in BR_U(\gamma) \text{ and } \gamma \in BR_R(\alpha)\}$. The following lemma tells that any allocation that does not belong to this set, N, cannot be a Nash equilibrium outcome.

Lemma 3. *Suppose* $(\alpha, \gamma) \notin N$, then (α, γ) *is not an Nash equilibrium outcome.*

Proof. The proof follows immediately from the interpretations of the correspondences $BR_U(\gamma)$ and $BR_R(\alpha)$ given above. *Q.E.D.*

Lemma 3 shows that a necessary condition for (α, γ) to be an equilibrium outcome is that (α, γ) belongs to the set N. Proposition 1 says that this condition is also sufficient.

Proposition 1. (a) *Consider the set* $N = \{(\alpha, \gamma) : \alpha \in BR_U(\gamma) \text{ and } \gamma \in BR_R(\alpha)\}$. N *is the set of equilibrium outcomes of the auction game under consideration.*
(b) *Let* $(\alpha^*, \gamma^*) \in N$. *If* $(P_U^*(\alpha), P_R^*(\gamma))$ *is a pair of Nash equilibrium bids with* (α^*, γ^*) *as an equilibrium outcome, then the equilibrium procurement price* (g^*) *and supplier profits* $(\pi_U^*$ *and* $\pi_R^*)$ *satisfy the following conditions:*

$$g^* \in [\max\{TSC(\overline{\alpha}, 0),\ TSC(0, \overline{\gamma})\},\ TSC(\overline{\alpha}, 0) + TSC(0, \overline{\gamma}) - TSC(\alpha^*, \gamma^*)],^{14} \tag{5}$$

$$\pi_U^* \in [g^* - TSC(\overline{\alpha}, 0), TSC(0, \overline{\gamma}) - TSC(\alpha^*, \gamma^*)], \tag{6}$$

$$\pi_R^* \in [g^* - TSC(0, \overline{\gamma}),\ TSC(\overline{\alpha}, 0) - TSC(\alpha^*, \gamma^*)]. \tag{7}$$

[14] Note that $\overline{\alpha}$ and $\overline{\gamma}$ are as defined in Lemma 1.

The sufficiency part in the proof of Proposition 1 involves constructing bid functions for the two suppliers $(P_U^*(\cdot), P_R^*(\cdot))$ so that any $(\alpha, \gamma) \in N$ can be supported as an equilibrium outcome, that is, the bids satisfy the condition of Lemma 1 and the deviation conditions (2a) and (2b). The conditions (5), (6) and (7) are used to construct bid prices at the awards (α^*, γ^*), $(\overline{\alpha},\ 0)$ and $(0,\ \overline{\gamma})$. We discuss the outline of the proof; the details are omitted and are available from the authors.[15]

To summarize the results of this section, for any $(\alpha, \gamma) \in N$, the suppliers can structure their bids so as to support it as an equilibrium outcome. Further, in equilibrium, there exists a range of profits for the suppliers as well as a range of procurement price for the buyer. Of course, the buyer would like to pay the lowest procurement price whereas the suppliers would want to earn the maximum individual profits. This issue of conflicting objectives of the buyer and the suppliers is discussed in the next subsection.

3.4. Efficiency of Nash Equilibrium Outcomes and Strengthened Implicit Price Collusion

While one of the major justifications for multi-sourcing in procurement auctions is to promote competition, Anton and Yao (1989) establish that split-award auctions effectively present the suppliers with an invitation for implicit price collusion where the buyer ends up in paying the highest possible equilibrium procurement price. But, note that, in Anton-Yao model the buyer is restricted to split-awards only, that is, $\alpha + \gamma = 1$. In our model, we allow the buyer with the flexibility of over-procurement (to hedge against the failure of the unreliable supplier from not delivering) and under-procurement (that is, to move

[15] In order to get the lower bound on g^* and the suppliers' profits, note from Lemma 1 that the equilibrium price corresponds to the bids at the sole-sourcing outcomes $(\overline{\alpha}, 0)$ and $(0, \overline{\gamma})$. Therefore, since supplier profits are non-negative, the equilibrium price must exceed the sole-sourcing production costs of both the suppliers. The upper bounds for the suppliers' profits in (6) and (7) are determined from the deviation conditions (2a) and (2b) which ensure that (α^*, γ^*) is the equilibrium outcome and can withstand the sole-sourcing bids at $(\overline{\alpha}, 0)$ and $(0, \overline{\gamma})$ for suppliers U and R respectively. Finally, the upper bound on g^* is determined by using the definition of π_U^* and π_R^*, and using the upper bounds derived for them.

part of the production in-house). As a result of this added flexibility in order allocation, one would expect, *a priori*, that the buyer would be better of. But, to the contrary, it turns out that this added flexibility strengthens the implicit price collusion possibility between the suppliers. As an important step in establishing this, we first show that any Nash equilibrium outcome is also an efficient outcome and vice-versa.

Recall that given any order (α, γ), the total system cost is defined as $TSC(\alpha,\gamma) = C_U(\alpha) + C_R(\gamma) + \beta K\left(\{1-(\alpha+\gamma)\}^+\right) + (1-\beta)K(1-\gamma)$. An outcome (α^E, γ^E) is called *efficient* if it minimizes the total system cost, that is,

$$(\alpha^E, \gamma^E) \in \underset{(\alpha,\gamma)\in[0,1]\times[0,1]}{\operatorname{argmin}} \{TSC(\alpha,\gamma)\}. \tag{8}$$

Proposition 2. *Suppose the cost functions $C_U(\cdot)$, $C_R(\cdot)$ and $K(\cdot)$ are convex. Let $E = \left\{(\alpha,\gamma) : (\alpha,\gamma) \in \underset{(\alpha',\gamma')\in[0,1]\times[0,1]}{\operatorname{argmin}} \{TSC(\alpha',\gamma')\}\right\}$ be the set of efficient outcomes. Then $E = N$, that is, the set of efficient outcomes coincides with the set of Nash equilibrium outcomes.*

Proof. See the Appendix.

Corollary 1. *If the cost functions $C_U(\cdot)$, $C_R(\cdot)$ and $K(\cdot)$ are strictly convex, then the Nash equilibrium outcome (which is also the efficient outcome) is unique.*

Proof. When the cost functions are strictly convex, $TSC(\alpha,\gamma)$ is also strictly convex so that the efficient outcome is unique. Then it follows from Proposition 2 that the Nash equilibrium outcome is unique as well. *Q.E.D.*

Now let us turn to the issue of implicit price collusion. Recall that the equilibrium procurement price, g^*, is defined by $g^* = \beta P_U^*(\alpha^*) + P_R^*(\gamma^*) + \beta K\left(\{1-(\alpha^*+\gamma^*)\}^+\right) + (1-\beta)K(1-\gamma^*)$. On rearranging terms above, it follows that, in equilibrium, the joint profit for the suppliers is given by $\pi_U^* + \pi_R^* = g^* - TSC(\alpha^*, \gamma^*)$. Proposition 1 defines a range of equilibrium profits for the two suppliers and hence a range for the joint profit. The suppliers can collude implicitly by structuring their bids so as to achieve the maximum possible joint profit. But since (α^*, γ^*) also minimizes $TSC(\alpha, \gamma)$ (from Proposition 2), it follows from the expression for $\pi_U^* + \pi_R^*$ that coordinating the bids to attain the maximum joint profit is equivalent to forcing the buyer to pay the highest possible procurement price, $g^* = TSC(\overline{\alpha}, 0) + TSC(0, \overline{\gamma}) - TSC(\alpha^*, \gamma^*)$ (which is the upper bound of g^* in condition (5)).

Let g^{*c}, π_U^{*c} and π_R^{*c} denote the procurement price and the suppliers' profits under implicit price collusion. Then, $g^{*c} = TSC(\overline{\alpha}, 0) + TSC(0, \overline{\gamma}) - TSC(\alpha^*, \gamma^*)$. To be sustainable, the colluding bids have to be incentive compatible, that is, each supplier earns a profit under collusion that is as much as the profit from a sole-source deviation. Consider supplier U. It follows from Lemma 1 (and the definition of $\pi_U(\overline{\alpha})$) that the sole-source threat point for supplier U is $g^{*c} - TSC(\overline{\alpha}, 0)$. The incentive compatibility implies $\pi_U^{*c} \geq g^{*c} - TSC(\overline{\alpha}, 0) = TSC(0, \overline{\gamma}) - TSC(\alpha^*, \gamma^*)$. But, condition (6) says that $\pi_U^{*c} \leq TSC(0, \overline{\gamma}) - TSC(\alpha^*, \gamma^*)$. Combining the two, we get $\pi_U^{*c} = TSC(0, \overline{\gamma}) - TSC(\alpha^*, \gamma^*)$, the upper bound of π_U^* in condition (6). Similarly, for supplier R we can show that $\pi_R^{*c} = TSC(\overline{\alpha}, 0) - TSC(\alpha^*, \gamma^*)$, the upper bound of π_R^* in condition (7).

The above discussion can be summarized as follows. In the auction game under consideration, although the buyer has a flexibility in order allocation, the suppliers can (implicitly) coordinate their bids such that the buyer ends up paying the maximum (worst-case) procurement price and the bids generate the highest possible individual profit for each supplier and the maximum joint profit.

In case of Anton-Yao model, the efficient outcome is *only one* equilibrium outcome in a potentially large set of equilibrium outcomes. For each equilibrium outcome, implicit collusion leads to the highest procurement price for the buyer for *that* outcome. However, the truly worst scenario for the buyer is realized when the Nash outcome is also efficient.

In our model, the set of Nash outcomes coincides with the set of efficient outcomes and is *unique* for the case of strictly convex costs. Then, the procurement price for the buyer is the maximum possible, that is, implicit collusion *always* leads to the worst case outcome for the buyer, and hence, implicit price collusive behavior between the suppliers is strengthened.

4. An Example with Quadratic Cost Functions

In order to get closed-form solutions, in this section we assume that the cost functions take the quadratic form: $C_U(\alpha) = U\alpha^2$, $C_R(\gamma) = R\gamma^2$, and $K(\delta) = K\delta^2$, where $U < R < K$. The cost functions being strictly convex, the Nash equilibrium outcome is unique and so we can do comparative statics analysis of the effects of various costs and the reliability of delivery. Further, using this example, we illustrate that the efficient outcome is the only robust equilibrium outcome in the continuum of equilibrium outcomes in Anton and Yao (1989) model.

From Lemma 2 we know that over-ordering ($\alpha + \gamma > 1$) cannot be an equilibrium outcome. Then, we can derive the BR_i functions, $i = U$, R, as follows:

$$\begin{aligned}
BR_U(\gamma) &= \operatorname*{argmin}_{0\le\alpha\le1} C_U(\alpha) + \beta K\left(1-(\alpha+\gamma)\right) = (\frac{\beta K}{U+\beta K})(1-\gamma), \\
BR_R(\alpha) &= \operatorname*{argmin}_{0\le\gamma\le1} C_R(\gamma) + \beta K\left(1-(\alpha+\gamma)\right) + (1-\beta)K(1-\gamma) \\
&= \begin{cases} \dfrac{K}{K+R} - \left(\dfrac{\beta K}{K+R}\right)\alpha, & \text{for } \alpha \le \dfrac{R}{R+(1-\beta)K}; \\[2ex] \dfrac{(1-\beta)K}{R+(1-\beta)K}, & \text{for } \alpha > \dfrac{R}{R+(1-\beta)K}. \end{cases}
\end{aligned}$$

[**Insert Figure 1 about here**]

In Figure 1, the straight line AD represents $BR_U(\gamma)$ and the line segments GH and HF represent $BR_R(\alpha)$. The figure also illustrates the interpretation of the BR_i functions provided earlier. For instance, the order (α, γ) corresponding to point L, name it (α_L, γ_L), cannot be an equilibrium outcome because supplier U can profitably induce the buyer to switch away from (α_L, γ_L) by changing his bid at α_M which is U's share at the order corresponding to point M. Similarly, (α_M, γ_M) cannot be an equilibrium outcome because R can induce a profitable deviation away from (α_M, γ_M) by changing his bid at γ_Q which is R's share at the order corresponding to point Q.

The BR_i functions intersect at the point N resulting in the *unique* Nash equilibrium outcome (α^*, γ^*) (which is also the unique efficient outcome, as follows from Proposition 2). Using the BR_i functions we can solve for α^* and γ^* as

$$\begin{aligned} \alpha^* &= \frac{\beta K R}{(R+(1-\beta)K)(U+\beta K)+\beta K U}, \\ \gamma^* &= \frac{(1-\beta)K(U+\beta K)+\beta K U}{(R+(1-\beta)K)(U+\beta K)+\beta K U}. \end{aligned} \tag{9}$$

With quadratic cost functions, the auction game under consideration has a unique Nash equilibrium outcome in which $\alpha^* x$ units are awarded to supplier U and $\gamma^* x$ units are awarded to supplier R, where $\alpha^* + \gamma^* < 1$; the remainder, $(1 - (\alpha^* + \gamma^*))x$ units, is allocated to the in-house production facility. Interestingly, a split-award allocation ($\alpha + \gamma = 1$) is not an equilibrium in this example. The intuition for this is as follows. Since $K(\cdot)$ is quadratic, the marginal cost of producing a very small amount in-house is close to zero. But the marginal costs of the suppliers are strictly positive for any multi-sourcing split-award allocation. Thus, in the margin, it pays the buyer to devote some amount for in-house production rather than awarding the full tender to the suppliers.

4.1. Comparative Statics

We can make some interesting observations about the individual allocations to the suppliers and their total allocation by conducting some simple comparative statics exercises.

From (9) we observe, as expected, that as the reliability of supplier U decreases (that is, as β decreases), his allocation, α^*, decreases, whereas the allocation for the reliable supplier, γ^*, increases. For the total allocation, $(\alpha^* + \gamma^*)$, we observe that there exists a critical value, $\widetilde{\beta}$ (where $\widetilde{\beta} = \dfrac{1}{2} + \dfrac{R}{2K}$), such that for $\beta > \widetilde{\beta}$, the total allocation, $(\alpha^* + \gamma^*)$, increases as β decreases, whereas for $\beta < \widetilde{\beta}$, the total allocation decreases as β decreases. This can be explained as follows. Start from the situation when $\beta = 1$, that is, both the suppliers are reliable. Now, if β is slightly less than 1, the total allocation increases (in the form of higher allocation to the reliable supplier) to hedge against the failure of the unreliable supplier from not delivering on time. However, as β decreases even further, allocation for the unreliable supplier becomes riskier and the reduction in its allocation may not be offset by the increased allocation for the reliable supplier. The critical value, $\widetilde{\beta}$, determines the threshold at which the total allocation $(\alpha^* + \gamma^*)$ is maximized.

Another interesting observation is the effect of in-house production cost on supplier allocations.[16] As K increases, the total allocation to the suppliers, $(\alpha^* + \gamma^*)$, and the allocation to the reliable supplier, γ^*, increases, as expected. However, for the unreliable supplier, the allocation further depends on its variability in supply, defined as $\beta(1-\beta)$. We observe that if $\beta(1-\beta) < \dfrac{UR}{K^2}$, the allocation α^* increases in K. But if the variability in supply exceeds this threshold, allocation for the unreliable supplier becomes riskier and, therefore, α^* decreases in K.

[16] The effects of the supplier costs are as expected: as the cost of the unreliable supplier increases (that is, as U increases), the allocation to the reliable supplier increases, whereas the allocation to the unreliable supplier and the total allocation decreases; similarly, as R increases, the allocation to the unreliable supplier increases, whereas the allocation to the reliable supplier and the total allocation decreases.

4.2. Robustness of the Multiplicity of Split Award Equilibria in Anton and Yao (1989) Model

Anton and Yao (1989) derive a continuum of Nash equilibrium outcomes of which only one is efficient as defined earlier. Whereas, in our analysis, the set of equilibrium outcomes coincides with the set of efficient outcomes. It would be interesting to understand the connection between the two models.

There is no supplier unreliability in A-Y model, that is, $\beta = 1$, and the option of in-house production does not exist. But the most important difference between the two models is that there is no room for over- or under-procurement in A-Y model, the orders are always split awards, that is, $\alpha + \gamma = 1$. Since there is no scope for under-procurement, the buyer does not need to pay the penalty or to take recourse to the outside option $K(\cdot)$ as envisaged in our model. Thus, A-Y model can be thought of as a limiting case of our model in the following sense – it is the limit to our model when the penalty or the in-house production option is prohibitively high (that is, $K \to \infty$ in the quadratic cost example).

When $\beta = 1$ in our model, from (9), the equilibrium outcome becomes $\alpha^* |_{\beta=1} = \dfrac{RK}{RU + RK + KU}$, $\gamma^* |_{\beta=1} = \dfrac{KU}{RU + RK + KU}$. Taking limits as $K \to \infty$ we get $\lim\limits_{K\to\infty} \alpha^* |_{\beta=1} = \dfrac{R}{R+U}$, and $\lim\limits_{K\to\infty} \gamma^* |_{\beta=1} = \dfrac{U}{R+U}$. Note that $\lim\limits_{K\to\infty} \alpha^* |_{\beta=1} + \lim\limits_{K\to\infty} \gamma^* |_{\beta=1} = 1$. Thus, when the buyer's penalty for nondelivery is prohibitively high, he refrains from under-procurement and awards an exact split in line with the suppliers' costs of production – the low-cost supplier U receives a larger share (now he is perfectly reliable since $\beta = 1$), and the high-cost supplier R gets a lower share.

Using our notation, the set of split-award equilibrium outcomes in A-Y model (Proposition 2, pp. 544) can be described as $N = \{\alpha : C_U(\alpha) + C_R(1-\alpha) < C_U(1),\ 0 < \alpha < 1\}$. When $C_U(\cdot)$ and $C_R(\cdot)$ are quadratic, this set becomes $N = \left\{\alpha : \dfrac{1-\alpha}{1+\alpha} < \dfrac{U}{R},\ 0 < \alpha < 1\right\}$.

Continuing with quadratic $C_U(\cdot)$ and $C_R(\cdot)$, *the* efficient outcome in A-Y model, defined as, $(\alpha^E|_{A-Y}, \gamma^E|_{A-Y}) \in \underset{(\alpha,\gamma)\in[0,1]\times[0,1]}{\operatorname{argmin}} \{C_U(\alpha) + C_R(\gamma)\}$, is $(\alpha^E|_{A-Y} = \frac{R}{R+U}$, $\gamma^E|_{A-Y} = \frac{U}{R+U})$. It is easy to check that the efficient outcome belongs to N.

So, in the limit, when K is infinitely high, the equilibrium outcome in our model converges to the efficient outcome in A-Y model, and the efficient outcome is also an equilibrium outcome there. Since A-Y model can be interpreted as a limiting case of our model when K is infinitely high, our analysis demonstrates that the efficient outcome is the only robust outcome in A-Y model.

5. Conclusions and Future Research

We now summarize the main contributions of the paper, discuss the limitations, and suggest directions for future research.

The appeal of doing business on the web is clear. As more and more entrants join in the B2B marketplace, electronic hubs – or e-hubs – have proliferated offering expanded choices to buyers and suppliers. In most Net marketplaces, price-driven auctions have become the most popular matchmaking method. However, the presence of unreliable suppliers offering low-priced bids posses new challenges for buyers in creating a successful e-business strategy. An important consideration for the buyer when making allocation decisions across unreliable and reliable suppliers is the level of risk he would be willing to take. We addressed this issue in our paper by considering the procurement problem for a buyer facing bids from an unreliable but cheaper supplier and a reliable but expensive supplier. The unique feature of our model was to allow the buyer to determine the size of the total tender awarded as a function of the bids received from the suppliers. As a result of this flexibility in order allocation, the *ex-ante* expectation was that the buyer would benefit. However, for the case of convex production costs, the set of Nash equilibrium outcomes coincides with the set of efficient outcomes, and as a result, the possibility of implicit supplier collusion is strengthened and the buyer incurs the highest possible procurement cost. Using the example of quadratic costs, we performed comparative statics analysis and determined the critical threshold values of the probability and variability in delivery, which influence not only the fractional tender awarded to the suppliers but also the quantity reserved for in-house production.

The analysis in this paper assumes the case of perfect information, that is, the buyer and the suppliers have full information about each other's costs. In future research, we plan to consider the more general case of imperfect information, and analyze the effect of private cost information on the equilibrium bids and the possibility of price collusion. Second, in this paper, we focused on an "all or nothing" delivery model. In future research, it is important to study the impact of different supply models in which the unreliable supplier delivers a random fraction of the order quantity. Order allocation/diversification problems with random deliveries have been addressed in the literature (see Yano and Lee (1995) for a comprehensive review) but have assumed exogenous supplier prices in the models. Third, in many e-commerce settings – especially for the case of first-time interactions – the buyer and the unreliable supplier may have different beliefs on the nature of supply unreliability in the supply chain. How would the beliefs on supply reliability affect the buyer's relationship with its reliable suppliers? In future research, we plan to analyze this issue which would also influence the in-house production decision of the buyer.

6. Appendix

A.1. Proof of Lemma 1.

It follows from the definition of g^* that $g^* \leq \beta P_U^*(\alpha) + \beta K(1-\alpha) + (1-\beta)K(1)$, for all $\alpha \in (0,1]$, and $g^* \leq P_R^*(\gamma) + K(1-\gamma)$, for all $\gamma \in (0,1]$. We will just prove that there exists $\overline{\gamma} \in (0,1]$ such that $g^* = P_R^*(\overline{\gamma}) + K(1-\overline{\gamma})$. The proof is similar for $\overline{\alpha} \in (0,1]$.

Suppose not. Then $P_R^*(\gamma) + K(1-\gamma) > g^*$, for all $\gamma \in (0,1]$. Let $\varepsilon = \min_{\gamma \in (0,1]} P_R^*(\gamma) + K(1-\gamma) - g^*$.[17] Now consider supplier U's bid, defined by

$$P_U(\alpha) = \begin{cases} P_U^*(\alpha) + \dfrac{\varepsilon}{2}, \text{ for } \alpha \neq \alpha^*,\ \alpha \neq 0 \\ P_U^*(\alpha) + \dfrac{\varepsilon}{3}, \text{ for } \alpha = \alpha^* \\ 0, \qquad\qquad\quad \text{for } \alpha = 0, \end{cases}$$

where (α^*, γ^*) is the procurement cost minimizing order placed by the buyer in the equilibrium under consideration. But supplier U's bid is constructed such that given supplier R's equilibrium bid, $P_R^*(\gamma)$, (α^*, γ^*) remains to be the procurement cost minimizing choice for the buyer, and, at the same time, supplier U's profits are greater with $P_U(\alpha)$ than with $P_U^*(\alpha)$. This contradicts the fact that $P_U^*(\alpha)$ is a best response. *Q.E.D.*

A.2. Proof of Proposition 2.

We have demonstrated in Lemma 2 that an order (α, γ) such that $0 < \alpha \leq 1$, $0 < \gamma \leq 1$, and $\alpha + \gamma > 1$ is not a Nash equilibrium outcome. As a first step in proving the equivalence of the sets E and N we need to show that such an order is not an efficient outcome either. Compare (α, γ) such that $0 < \alpha \leq 1$, $0 < \gamma \leq 1$, and $\alpha + \gamma > 1$ with (α', γ) such that $0 < \alpha' \leq 1$, $0 < \gamma \leq 1$, and $\alpha' + \gamma = 1$. We have $TSC(\alpha, \gamma) = C_U(\alpha) + C_R(\gamma) + (1-\beta)K(1-\gamma)$ and $TSC(\alpha', \gamma) = C_U(\alpha') + C_R(\gamma) + (1-\beta)K(1-\gamma)$. Clearly, $TSC(\alpha, \gamma) > TSC(\alpha', \gamma)$

[17] If a minimum does not exist, use the infimum.

when $C_U(\cdot)$ is strictly increasing. This proves that (α, γ) such that $0 < \alpha \leq 1$, $0 < \gamma \leq 1$, and $\alpha + \gamma > 1$ is not an efficient outcome.

Now, since the possibility of over-ordering is ruled out as both Nash equilibrium and efficient outcomes, the sets E and N get reduced to the following:

$$E = \left\{ (\alpha, \gamma) : (\alpha, \gamma) \in \underset{(\alpha', \gamma') \in [0,1] \times [0,1]}{\operatorname{argmin}} C_U(\alpha') + C_R(\gamma') + \beta K\left(1 - (\alpha' + \gamma')\right) + (1 - \beta)K(1 - \gamma') \right\},$$

$$N = \left\{ (\alpha, \gamma) : \begin{array}{l} \alpha \in BR_U(\gamma) = \underset{0 \leq \alpha' \leq 1}{\operatorname{argmin}} \;\; C_U(\alpha') + \beta K\left(1 - (\alpha' + \gamma)\right) \text{ and} \\ \gamma \in BR_R(\alpha) = \underset{0 \leq \gamma' \leq 1}{\operatorname{argmin}} \;\; C_R(\gamma') + \beta K\left(1 - (\alpha + \gamma')\right) + (1 - \beta)K(1 - \gamma') \end{array} \right\}.$$

Since $C_U(\cdot)$, $C_R(\cdot)$ and $K(\cdot)$ are convex, the functions defined by: (1) $C_U(\alpha) + C_R(\gamma) + \beta K\left(1 - (\alpha + \gamma)\right) + (1 - \beta)K(1 - \gamma)$, (2) $C_U(\alpha) + \beta K\left(1 - (\alpha + \gamma)\right)$, and (3) $C_R(\gamma) + \beta K\left(1 - (\alpha + \gamma)\right) + (1 - \beta)K(1 - \gamma)$, are also jointly convex in α and γ.

First, consider the problem: $\underset{(\alpha, \gamma) \in [0,1] \times [0,1]}{\text{Minimize}} \; C_U(\alpha) + C_R(\gamma) + \beta K\left(1 - (\alpha + \gamma)\right) + (1 - \beta)K(1 - \gamma)$. Since the constraint set is compact and the objective function is continuous, solution(s) to this problem exists (exist). The solutions are then completely characterized by the following first-order conditions (with the associated complementary slackness conditions)[18]:

$$C_U'(\alpha) - \beta K'\left(1 - (\alpha + \gamma)\right) + \lambda_1 \;\; \geq \;\; 0, \tag{A.1}$$

$$C_R'(\gamma) - \beta K'\left(1 - (\alpha + \gamma)\right) - (1 - \beta)K'(1 - \gamma) + \lambda_2 \;\; \geq \;\; 0, \tag{A.2}$$

where λ_1 and λ_2 are the Lagrange multipliers corresponding to the constraints $\alpha \leq 1$ and $\gamma \leq 1$, respectively.

Next, consider the problem: $\underset{0 \leq \alpha \leq 1}{\text{Minimize}} \;\; C_U(\alpha) + \beta K\left(1 - (\alpha + \gamma)\right)$, when γ is given. Once again, the constraint set being compact and the objective function being continuous,

[18] The convexity of the objective function guarantees that the solutions to the first-order conditions are indeed minimizers, and not maximizers.

solution(s) to this problem exists (exist), and are characterized by the following first-order condition (with the associated complementary slackness conditions):

$$C_U'(\alpha) - \beta K'\left(1 - (\alpha + \gamma)\right) + \lambda_3 \geq 0, \tag{A.3}$$

where λ_3 is the Lagrange multiplier corresponding to the constraint $\alpha \leq 1$.

Finally, consider the problem: $\underset{0 \leq \gamma \leq 1}{\text{Minimize}} \quad C_R(\gamma) + \beta K\left(1 - (\alpha + \gamma)\right) + (1-\beta)K(1-\gamma)$, when α is given. The solution(s) to this problem exists (exist), and are characterized by the following first-order condition (with the associated complementary slackness conditions):

$$C_R'(\gamma) - \beta K'\left(1 - (\alpha + \gamma)\right) - (1-\beta)K'(1-\gamma) + \lambda_4 \geq 0, \tag{A.4}$$

where λ_4 is the Lagrange multiplier corresponding to the constraint $\gamma \leq 1$.

In view of the above discussion, we can redefine the sets E and N as

$$\begin{aligned} E &= \left\{(\alpha, \gamma) : (\alpha, \gamma) \text{ solves equations (A.1) and (A.2)}\right\}, \\ N &= \left\{(\alpha, \gamma) : (\alpha, \gamma) \text{ solves equations (A.3) and (A.4)}\right\}. \end{aligned}$$

Since the system of equations (A.1) and (A.2) is identical to the system of equations (A.3) and (A.4), we conclude that $E = N$. *Q.E.D.*

References

[1] Anton, J.J. and Yao, D.A. "Second Sourcing and the Experience Curve: Price Competition in Defense Procurement," *RAND Journal of Economics*, Vol. 18 (1987), pp. 57-76.

[2] Anton, J.J. and Yao, D.A. "Split Awards, Procurement, and Innovation," *RAND Journal of Economics*, Vol. 20 (1989), pp. 538-552.

[3] Anton, J.J. and Yao, D.A. "Coordination in Split Award Auctions," *The Quarterly Journal of Economics*, Vol. 107 (1992), pp. 681-707.

[4] Barua, A., Ravindran S., and Whinston, A.B. "Efficient Selection of Suppliers over the Internet," *Journal of Management Information Systems*, Vol. 13 (1997), pp. 117-137.

[5] Boger, D.C. and Liao, S.S. "Quantity-Split Strategy Under Two-Contractor Competitive Procurement Environment," Naval Postgraduate School, NPS-54-88-008, (1988).

[6] Dana, J.D. Jr. and Spier, K.E. "Designing a Private Industry: Government Auctions with Endogenous Market Structure," *Journal of Public Economics*, Vol. 53 (1994), pp. 127-147.

[7] Einhorn, B. "B2B is the Password," *Business Week*, May 22 (2000), pp. 20-21.

[8] Feuerstein, A. "Define Strategic Sourcing. Ready? Go," *Upside Today: The Tech Insider*, Oct. 6 (2000). http://www.upside.com/texis/mvm/news/news?id=39dcece10

[9] Gurnani, H., Akella, R., and Lehoczky, J. "Optimal Order Policies in Assembly Systems with Random Demand and Random Supplier Delivery," *IIE Transactions*, Vol. 28 (1996), pp. 865-878.

[10] Gurnani, H., Akella, R., and Lehoczky, J. "Supply Management in Assembly Systems with Random Yield and Random Demand," *IIE Transactions*, Vol. 32 (2000), pp. 701-714.

[11] Gurnani, H. and Shi, M. "A B2B Bargaining Model with Asymmetric Beliefs on Supplier Reliability," working paper, HKUST, (2000).

[12] Henig, M. and Gerchak, Y. "The Structure of Periodic Review Policies in the Presence of Random Yield," *Operations Research*, Vol. 38 (1990), pp. 634-643.

[13] Kagel, J.H. and Roth A.E. "The Handbook of Experimental Economics," editors J.H. Kagel and A.E. Roth, Princeton University Press, (1995).

[14] Ketchpel, S. P. and Garcia-Molina, H. "Competitive Sourcing for Internet Commerce," *Proceedings of the ICDCS'98*, Amsterdam, The Netherlands, IEEE Press, (1998).

[15] Klotz, D. E. and Chatterjee, K. "Dual Sourcing in Repeated Procurement Competitions," *Management Science*, Vol. 41 (1995), pp. 1317-1327.

[16] Leidy, M.P. "Hedging Across Suppliers Under Stochastic Input Deliveries," *Economic Notes*, Vol. 21 (1992), pp. 39-59.

[17] Mayer, A. "Military Procurements: Basic Principles and Recent Development," *George Washington Journal of International Law and Economics*, Vol. 21 (1987), pp. 165-187.

[18] McAfee, R.P. and McMillan, J. "Auctions and Bidding," *Journal of Economic Literature*, Vol. 25 (1987), pp. 699-738.

[19] Meeker, B. "Second-Source Splits: An Optimum Non-Solution," *Program Manager*, Vol. 13 (1984), pp. 2-8.

[20] Milgrom, P. "Auctions and Bidding: A Primer," *Journal of Economic Perspectives*, Vol. 3 (1989), pp. 3-22.

[21] Seshadri, S., Chatterjee, K., and Lilien, G.L. "Multiple Source Procurement Auctions," *Marketing Science*, Vol. 10 (1991), pp. 246-263.

[22] Seshadri, S. "Bidding for Contests," *Management Science*, Vol. 41 (1995), pp. 561-576.

[23] Treleven, M. and Schweikhart S.B. "A Risk / Benefit Analysis of Sourcing Strategies: Single v. Multiple Sourcing," *Journal of Operations Management*, Vol. 7 (1988), pp. 93-114.

[24] Turban, E. "Auctions and Bidding on the Internet: An Assessment," *International Journal of Electronic Markets*, Vol. 7 (1997). http://www.electronicmarkets.org

[25] Turnbull, G.K. "Theory of the Firm under the Threat of Input Supply Interruption," *Southern Economic Journal*, Vol. 53 (1986), pp. 807-817.

[26] Yano, C.A. and Lee, H.L. "Lot Sizing with Random Yields: A Review," *Operations Research*, Vol. 43 (1995), pp. 311-334.

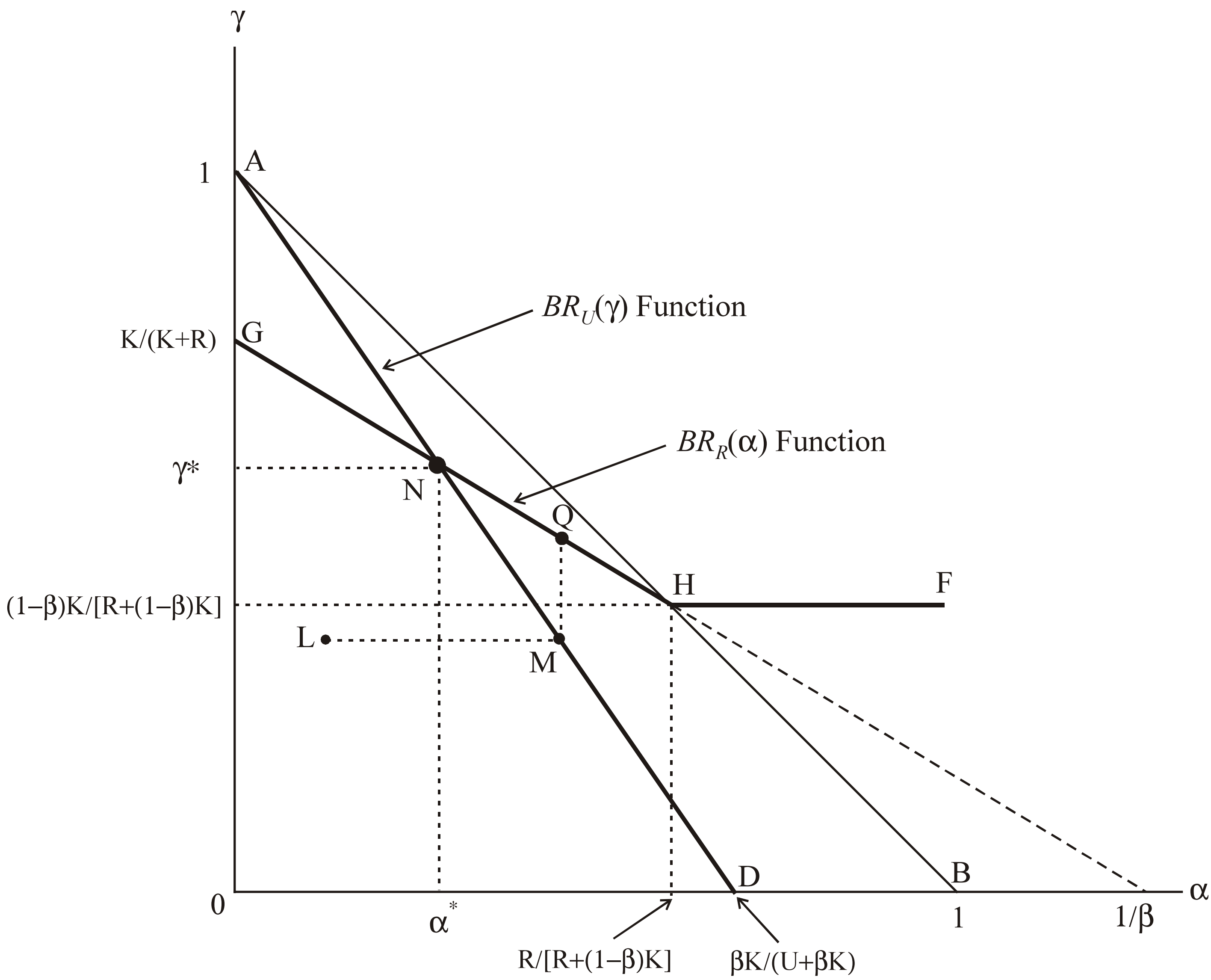

Figure 1: Nash Equilibrium Outcome with Quadratic Costs