A general refutation of Okishio's theorem and a proof of the falling rate of profit

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ABSTRACT

This is the first published general refutation of the Okishio theorem. An earlier refutation based on a specific example was published by Kliman and McGlone in 1988.

Okishio's theorem, published in 1961, asserts that if real wages stay constant, the rate of profit necessarily rises in consequence of any cost-reducing technical change. It proves this within a simultaneous equation (general equlibrium) framework.

This paper establishes that this proposition is false within a differential equation (temporal) approach. In such a framework the denominator of the rate of profit rises continuously, regardless of whether or not there is technical change, unless capitalist consumption exceeds profit, as occurs in a slump.

Okishio himself asserts that his theorem is 'contrary to Marx's Gesetz des Tendentiellen Falls der Profitrate' – contrary to Marx's law of the tendency of the rate of profit to fall. This assertion is, within the literature, universally taken to be the substantive content of the 'Okishio Theorem'. Thus, if Marx's approach to value is in fact temporal, and not simultaneist, this assertion by Okishio is false, since it applies not to Marx's own theory, but to the interpretation of that theory subsequently attributed to Marx by a specific school of thought represented principally by Bortkiewicz, Sweezy, Morishima, Seton, and Steedman.

The subsequent accumulation of hermeneutic evidence strongly supports the thesis that Marx's theory is temporalist and not simultaneist.

Since the Okishio theorem makes the general assertion that the rate of profit must necessarily rise if there are cost-saving technical changes, and since Kliman and McGlone demonstrate a particular case in which cost-saving technical change leads to a fall in the profit rate, the Kliman-McGlone paper is the first published refutation of the Okishio theorem. The present paper is a generalisation of this refutation which establishes the precise conditions under which the profit rate rise or falls, and establishes the general result that the profit rate necessarily falls as a consequence of capitalist accumulation with a constant real wage, until and unless accumulation ceases in value terms.

Consequently the mathematical findings set out in this paper, refute the Okishio Theorem.

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1 INTRODUCTION

It is almost universally believed that Professor N. Okishio's (1961) justly-celebrated theorem disproves Marx's analysis of the tendency of the profit rate to fall, by showing that if real wages stay constant the rate of profit rises as a result of productivity-enhancing technical change. All attempted refutations accept Okishio's approach which like many others, notably the Surplus Approach school, is based a system of simultaneous linear equalities.

This paper is a complete departure from the simultaneous method, which we believe was alien to Marx, theoretically deficient, and utterly unrealistic. Applying the method of a growing body of writers convinced that simultaneous systems cannot represent the actual formation of values or prices,[1] it provides a fully general refutation rooted in a rigorous differential equation formalism. We show that the denominator of the rate of profit falls continuously unless capitalist consumption exceeds profit, as occurs in a slump. The resultant fall in the profit rate can be permanently offset only in such circumstances. Since this accurately reflects observed reality, we conclude that simultaneous systems – including neoclassical general equilibrium – cannot represent a capitalist economy, and that the premises of Okishio's calculation are false.

2 GENERAL EQUILIBRIUM MARXISM

The best-known progenitor of the simultaneous presentation of Marx's theory, from Bortkiewicz (1906, 1907) first appeared in Max Weber and Werner Sombart's *Archiv für Sozialwissenschaft und Sozialpolitik*.

Bortkiewicz's explicit debt to Walras, the founder of modern mathematical general equilibrium theory, was the fruit of a lifelong correspondence with him (see Gattei 1982) which he began in 1887 at the age of nineteen with the following words (Jaffé 1965 Vol II:230)

> Your writings, sir, have awakened in me a lively interest in the application of mathematics to political economy, and has pointed out to me the road to travel in my researches into the methodology of economic science.

His famous article offers the following, revealing remark:

> Alfred Marshall said once of Ricardo: 'He does not state clearly, and in some cases he perhaps did not fully and clearly perceive how, in the problem of normal value, the various elements govern one another *mutually*, not *successively*, in a long chain of causation.' This description applies even more to Marx...[who] held firmly to the view that the elements concerned must be regarded as a kind of causal chain, in which each link is determined, in its composition and its magnitude, only by the preceding links...Modern economics is beginning to free itself gradually from the successivist prejudice, the chief merit being due to the mathematical school led by Leon Walras [2]

The mathematics used to formalise Marx's theory are thus the explicit fruit of the doctrine now known as neoclassical general equilibrium theory. The lynchpin of the construction is the rejection of Marx's 'successivist prejudice' in favour of the Walrasian, simultaneist assumption that the prices and values of outputs at the end of a period must be equal to the prices and values of inputs at the beginning of the *same* period, a logically inconceivable assertion. By the same token it fully accounts for the transformation of flows into stocks and vice versa, an essential feasibility condition absent from all simultaneous treatments.

The emerging non-equilibrium alternative is based on two principal tenets:

(i) Time is sequential, not simultaneous. Production and exchange are both represented by difference equations, and in the limit, differential equations.

(ii) The value transferred by consumed means of production to the value of outputs is given by the value of the money paid for them; variable capital is correspondingly given by the value of the money wage.

To arrive at a complete alternative a third aspect is dealt with in this paper: the effect of capital stocks, or historically-inherited value, and their effect on the determination of value. Neglect of this issue is the basis of the errors introduced by Okishio's formalism.

3 NOTATION

Mathematical notation is not neutral. Our principle is that the same symbol always stands for the same commodity in the same capital, while value is distinguished from use value, and stocks from flows, by varying the type or by additional symbols. This emphasises the unity of the commodity form. It also makes it easier to use the same letters as Marx, whose English translators tend to use C for everything and V for everything else.

Every commodity has two aspects: use value and value. Value (and price) magnitudes will be represented with a £ sign in front unless the context is unambiguous.[3]

The basic symbols are matrices C, W, X and B, and vectors V, L, λ and p:

- C^i_j constant capital employed: quantity of commodity *j* in capital *i* (whose value is hence £C^i_j)
- V^i variable capital (labour power) employed by capital *i*, in hours
- £L^i value-creating capacity of V^i, (value-product) in pounds.
- X^i_j produced output of commodity *j* in capital *i*
- W^i_j quantity of commodity *j* in the purchases from wages of workers in capital *i*
- B^i_j quantity of commodity *j* in sector *i*
- λ^j value of a unit of commodity *j* measured in pounds
- p^j price of a unit of commodity *j* measured in pounds

Columns represent commodities and rows represent capitals or sectors. This corresponds to Marx's usage rather than the Leontief tradition which shows sectors as columns.[4]

There may be more than one producer of the same commodity so C^i_j may not in general be square. We use a *reduced form* (Freeman 1991) of C in which each column is a sector producing a distinct commodity.[5] X is therefore diagonal.

Workers' consumption is represented by a matrix (W) rather than a vector, so wages may differ from sector to sector though of course they may be the same.

To distinguish rows from columns we use the convention that superscripts vary over columns and subscripts over rows. Thus:

£$C^{farmers}$ is a column vector giving the farmers' constant capital, and so on.

£C_m is a column vector in which £C^i_m is the value of money held in sector *i*.

The important matrix K gives the distribution of the total stocks of all commodities in the economy except labour power. A problem of signs arises. It is conventional, and anything else would be obscurantist, to represent the *consumption* of C, W and B as positive. But consumption actually diminishes a stock and, strictly, should be represented as negative.[6] The stock of a commodity is then *minus* the sum (or integral) of consumption flows. The stock of C is thus represented by –C, just as assets on a balance sheet appear as a debit, something owing to the owner. In writing down the relation between K and other stocks this cannot be avoided and we write

$$K = X - C - W - B$$

Row and column sums are represented by a subscript or superscript Σ thus:

£K^{Σ} is a row vector giving the value of each commodity in the economy

£K_{Σ} is a column vector giving the value of capital stock in each sector

The diagonal matrix formed from K^{Σ} is called $\hat{K}$, so that $\hat{K}^{i}_{j}= 0$ when $i \neq j$ and $\hat{K}^{i}_{i}$ is the quantity of commodity *i* in existence. $\hat{C}$, $\hat{W}$ and so on are similarly defined.

Note $\hat{X}$=X.

4 MARKET PRICES AND THE VALUE OF MONEY

All simultaneous presentations fall back on the Ricardian notion of price as the ratio at which goods are bartered for each other. This eliminates money and excises the real dynamics of the capitalist economy. It is possible only on the basis of the 'simplifying assumption' that rates of profit actually equalise, that is, that commodities sell for their prices of production instead of their market prices. This assumption is in fact a postulate. Without it, there is no simultaneous solution.

We differ in three decisive respects. First, we deal with sale at arbitrary market prices, without assuming the rate of profit is everywhere equal. Of course, the special case in which profit rates equalise is covered by this fully general assumption. Second we deal with sale at *money* prices, as does Marx and a capitalist economy. Third, more subtly, the market price of a commodity is the *value* of the money for which it exchanges. A fully dynamic treatment must recognise that a given quantity of money will represent different magnitudes of value at different times, for example as a result of price inflation. Changes in the value of money must be corrected for, and this is indeed one of the principal reasons an independent measure of the magnitude of value – labour hours – is necessary to decipher the underlying movement of the economy. No-one seriously claims that, if price inflation raises capital stock from £1000 to £2000, a profit of £1,000 has been made. Value must be measured in terms which abstract from changes in price that do not depend on the production of new goods (See for example Elson, Ramos, Rodriguez, Freeman 1995).

All money prices are therefore henceforth expressed in terms of the value which any given quantity of money represents in exchange at any given time.The value of 1£ is then the total value of all goods in circulation, divided by their total money-price. From now on we assume, as did Marx throughout Volume III of *Capital*, a constant value of money.[7]

This calculation differs from that known as the New Approach in which the value of money is determined in relation to the net product. In our view this requires it to be deduced from prior assumptions about production and technology, since without knowing the structure of production we cannot say what the net product is. Our derivation of the price-value relation (Freeman 1995) proceeds from the exchange relations introduced by Marx in the first part of Volume I of *Capital* and is independent of any particular assumptions about production or reproduction. We argue that *all* goods serving as commodities (that is, goods to which the market attaches a price, whether or not they are sold) take part in circulation. The price-value relation therefore applies to the total commodity-stock K.

Marx's first equality, the equality of total price and value, follows tautologically from this definition; it applies to the total stock of capital.

5 THE SEQUENTIAL APPROACH

If we assume (which Marx did not) that all goods are turned over exactly once, then X=K, the starting point of the Bortkiewicz approach. This can be used to illustrate some of the ideas of our approach which may be unfamiliar to newer readers and to show its relation to the simultaneous approach.

Our starting point is a *time-dependent* relation between values in one period and values in the next, given by the standard assumptions of value theory: the value of outputs at time $t+1$ is equal to the value of constant capital consumed in period $[t,t+1]$ plus the value product of this period

$$X\lambda^{t+1} = C^t\lambda^t + £L^t \tag{1}$$

or more simply

$$£X^{t+1} = £C^t + £L^t \tag{2}$$

This provides a difference equation for λ (as throughout this article, we omit the time superscript where it can be unambiguously supplied, so that $£L^t$ is simply written £L):

$$\lambda^{t+1} = X^{-1}C\lambda^t + X^{-1}£L \tag{3}$$

Given the initial value λ_0 this has a positive determinate solution at all times provided inputs, hours worked and gross outputs are positive. It is difficult to conceive how this could be violated.

The simultaneous solution is a special case of this equation in which it is assumed that λ does not change:

$$\lambda = X^{-1}C\lambda + X^{-1}£L \tag{4}$$

In terms of the normal procedures of dynamic analysis this is the *particular solution* of the general dynamic equation. It is illegitimate, and would be unacceptable in any other science, to regard a succession of particular solutions brought about by changes in the parameters C, X and £L as a description of the evolution of the state of the system (3), though this is the procedure portrayed as Marxist for ninety years. It implies λ has two distinct magnitudes at the same time; its equilibrium magnitude at the end of one period and its new equilibrium magnitude at the beginning of the next.

5.1 The value transfer vector

The total market price $K^{\Sigma}p$ and total value $£K^{\Sigma}{}_{\Sigma}$ of all goods in society are equal by definition. The price and value of any subset of these goods are not in general equal. The formation of p therefore transfers value between the holders of commodity stocks. This is independent of how much is actually traded. If Nick Leeson pays $1000,000,000 for futures which fall to $100,000,000 then he has instantly lost $900,000,000 whether or not he sells them. We need to relate such value transfers to the price-value differences which induce them.

Consider the vector $£K^{\Sigma}$ each of whose elements represents the value of the total stock of one particular commodity, and the vector

$$\hat{K}p$$

each of whose elements represents the price of the total stock of one particular commodity. Their difference is a (time-varying) vector of value-price transfers which we shall term £E:

$$£E = \hat{K}p - £K^{\Sigma}$$

Clearly the elements of £E sum to zero by definition. Corresponding to this vector £E is a unique vector of *unit* value transfers we shall term *e*.

$$e = £E\hat{K}^{-1}$$

This lets us calculate the value-price difference of any given collection of commodities. In particular, it allows us to calculate the difference between the value and the price of the output of any given period:

$$X^t e^{t+1} = X^t p^{t+1} - £X^t \qquad (5)$$

5.2 The second equality

Eqation (5) exhibits the value-price relation as a relation of *transition*; it allows us to separate any changes in value which have resulted from production, and have hence been embodied in $£X^t$ during period [*t*, *t* +1], from those which result from circulation at the end of this period, and which cause prices at *t*+1 to diverge from values at *t*+1. Were it not for the effects of supply, demand and the movement of capital, we would have

$$e^{t+1}X^t = 0.$$

In this case the unit values expressed in $£X^t$ would become the unit values of the inputs to the next period, $£C^{t+1}$ and $£V^{t+1}$. The Bortkiewicz tradition claims that Marx 'forgot to transform inputs', that is, asserts that the value transferred to the product by C^{t+1} is $£C^t$, and that the value $£V^{t+1}$ appropriated by workers is $£V^t$.

A growing body of 'non-dualist' writers (Wolff-Callari-Roberts, Moseley, Ramos and Rodriguez) have established, though working with simultaneous equations, that Marx's conception was a different one; that the value transferred to the product by C is equal to the price of the elements of C, namely *p*C, and that he similarly perceived variable capital as *p*V. He did not need to transform inputs in Volume III since he had already done it in Volume I.

Space does not permit repetition of the clear textual evidence that this was indeed Marx's view which these authors and others have provided. It is, however, almost undeniable. The 'error' of which Marx has stood accused for ninety years does not exist.

Marx's idea is however easier to understand in the framework properly his own; namely, that of a succession of periods of production and circulation. From this point of view, the issue is this: the value of the elements of C, when they serve as inputs, is determined not by production alone but by production *followed by* circulation. The time superscript completely clarifies this. The issue is the difference between $£X^t$, values determined by the previous period of production alone, and $£C^{t+1}$, a subset of these same goods whose values have been modified by circulation, transferring value both between the elements of C and between consumers and producers.

This fully clarifies Marx's second equality in the simple case where all capital turns over equally in a given period. Define surplus value £S like Marx as the difference between £L (the value added by workers) and Vp_L (the price of consumed variable capital). The difference on the other hand between market price and cost price is capitalist profit, a row vector we call £Π. Hence (the subscript *t* being omitted where unambiguous)

$$£S = £L - £V = £L - Vp_L$$

$$£\Pi^{t+1} = £X^{t+1} - £C - £V = £L - £V + £E^{t+1}$$

$$= \pounds S + \pounds E^{t+1} \qquad (6)$$

Whatever the time average of £Π, each actual sale will deviate from it. Nevertheless, just as a general law regulating exchange (the first equality) applies to all market prices, a second general law regulates profits. Summing (6) across sectors gives

$$\pounds\Pi^{\Sigma} = \pounds L^{\Sigma} - \pounds V^{\Sigma} + E^{\Sigma t+1}$$

But $E^{\Sigma t+1}$ is 0; therefore

$$\pounds\Pi^{\Sigma} = \pounds L^{\Sigma} - \pounds V^{\Sigma} = \pounds S^{\Sigma},$$

Marx's 'second equality'. Being established for the general case where profits are not equal, this is certainly true for the special case where they do, that is where market prices equal prices of production.

6 CAPITAL

The assumption that all capital turns over equally in a given period is both false and fatal. Every attempt to abandon this 'simplification' has come to grief. This is for two reasons. First, the confusion clarified above; and second because of the parallel failure to integrate the relation between stocks and flows into the dynamics of value and price.

The fundamental question is: what happens when the value of a pre-existing stock of capital is modified as a result of the operation of the price system? We have just shown that newly-produced goods leaving one period of production transfer to the next period of production a value which is different from the value with which they left the last period. The question then arises: what happens to the value of goods which were not produced in the immediately preceding period but which were preserved in the form of stocks?

If, a stock of some commodity – say computers – is tied up in production then this both affects, and is affected by, their current price. As they become cheaper all capitals containing them depreciate. This depreciation, however, is a money sum which the capitalists have to find. If a capitalist buys a computer for £3,000 and it is now worth £1,000, then not even Berlusconi can simply write off the difference. The value of a *capital* is what the owner paid at the time of purchase; according to all simultaneous systems it is what other people pay for it now.

The capital gains and losses cannot simply be written off, and herein lies the principal fault of all simultaneous treatments, above all Okishio's. The error in these treatments, is the conception that a new technology is instantly and costlessly adopted. This is absurd. Actually as a new technology is introduced, an *average* price emerges which transfers value from the owners of the old technology to the producers of the new. The cheapening of old capital is balanced by transfers of value to new producers. As long as there are 386 and 486 computers in the world, Pentium computers sell at a higher price and possess a higher value than they would in the ideal simultaneous world in which the stock of old computers is costlessly wiped out overnight.

This is the basis for the systematic immiseration of three-quarters of the world's humans. As a *theory* – which does not necessarily apply to the economists who use it – the notion of costless technical change is one of the most profoundly apologetic of modern economics: if it were true there would be no third world and Eastern Europe would be rich.

Similar points have been recognised (see for example Alberro and Persky) but they have to be rigorously incorporated into the theory to provide a proper mathematical foundation for what Marx was really saying. This we now proceed to do.

The accounting concept of 'depreciation' contains two elements, as Marx discussed at some length; an element of genuine wear and tear, and an element of 'moral' depreciation or loss in value purely due to improvements in technology.

There are thus transfers of value resulting from price variations not just between *current* outputs but *historically-produced* outputs. It is these price-value transfers which must be systematically accounted for to formalise Marx's account of both transformation and of the rate of profit correctly.

6.1 The price-value relation with stocks

Marx analysed reproduction, exactly as Bortkiewicz disparagingly remarks, as a succession of periods of production and circulation. This perfectly rigorous analytical distinction corresponds to the mathematical operation of partial differentiation. The total change in the value of any stock over any period of time is the sum of two distinct partial effects; the change resulting from production and consumption, that is the labour process, and the change resulting from the operation of the price mechanism and from trade, that is circulation.

In any given period we use the symbol Δ_P to mean the change in the magnitude of a stock induced by production (in which from now on we include consumption), and Δ_C to mean the change induced by circulation. In the continuous case these become partial derivatives induced by either circulation or production/consumption. The total change Δ is just the sum of the two.

In passing from the discrete to the continuous case we shall represent

$$\lim_{\Delta t \to 0}\left(\frac{\Delta_C C}{\Delta t}\right) = \left.\frac{\partial_C C}{\partial t}\right|_{Circulation} \quad \text{by } C_C\text{, and so on.}$$

C_P, defined analogously, is what Marx calls the turnover of constant capital, V_p the turnover of variable capital, X_P is output per unit time, and so on. Thus

$$C' = C_C + C_p,$$

$$£C' = £C_C + £C_p,$$

and so on. One subtle point is that $£L_P$, the *rate of value generation*, is the the total new value added by labour per unit time, or the monetary expression of hours worked per unit time. Note further that the true relation between prices p and values λ is now more easily expressible as a change in the *same* qualitative entity over time. In the transition from p^t to p^{t+1} there are two distinct mutations given by Δ_P and Δ_C.

$$p^{t+1} = p^t + \Delta_P p^t + \Delta_C p^t$$

Thus λ^t, shorn of the mystery with which generations have shrouded it, is another name for $p^t + \Delta_P p^t$, the partial derivative of price with respect to production, and the celebrated transformation of values into prices is the partial derivative of price with respect to circulation, given by $p^{t+1} = \lambda^t + \Delta_C p^t$.[8]

Circulation cannot create or destroy use value. It can only redistribute it. Therefore

$$\Delta_C K^{\Sigma}$$

is identically zero. Equally, however (and for this reason) it cannot create or destroy exchange value. Hence also

$$\Delta_C £K^{\Sigma} = 0 \qquad \text{(the 'first equality')}$$

However circulation can transfer value between capitals, so that in general

$$\Delta_C(Kp - £K) = \Delta £E \neq 0$$

This means that profit is distinguished from surplus value not just by the difference between the price and value of current outputs, but by the value transferred from one capital to another through the price mechanism. If, for example, I hold stocks of oil worth £1m which rise to £1,500,000 then £500,000 is transferred to me from other capitalists in the system even if I produce or consume nothing; this is just as much a profit, albeit a speculative one, as if I had just produced the oil yesterday.

This calls for a generalisation of the definition of profit, and a corresponding generalisation of the definition of surplus value. We begin by accounting rigorously for the values created and transferred in production with stocks of capital.

6.2 The value accounting identity

Assume for simplicity that workers consume all wage goods in the current period. Consumed variable capital V is therefore always equal in price and hence value to the price of consumed wage goods Wp consumed during the same period.[9]

During production each stock decreases except X, because production creates new use values $\Delta_P X$. A portion of K^t survives intact to subsequent periods and preserves the value it has inherited. K^{t+1}, the total goods now in circulation, are equal to this portion plus X^t. It follows that this intact portion has magnitude

$$K^{t+1} - \Delta_P X^t$$

or

$$K^t + \Delta_P K^t - \Delta_P X^t \quad (7)$$

(Another way of deriving the same result is to say that this intact portion is equal to K^t less consumption of C, V, W and B). This preserves the value it possessed when production began, and contributes this to the total supply of value in society as if it had just been produced. This component of new value is equal to

$$(\hat{K}^t + \Delta_P \hat{K}^t - \Delta_P X^t)\, p^t$$

Production creates new goods whose value comprises two components, namely the value transmitted by the consumed constant capital $\Delta_P C^t$ and the value added by labour power $\Delta £L^t$. The total value in the economy following production is therefore the sum of preserved and new values,

$$(\hat{K}^t + \Delta_P \hat{K}^t - \Delta_P X^t)\, p^t + \Delta_P C^t\, p^t + \Delta_P £L^t$$

On this basis, new unit values are formed. These are a social average, equal to the total value of each commodity divided by the total use value of the same commodity. Representing new unit values as $p+\Delta_P p$, the total value of all stocks in circulation is also given by

$$\hat{K}^{t+1}(p+\Delta_P p)$$

that is

$$(\hat{K} + \Delta_P \hat{K})(p+\Delta_P p)$$

where we drop the superscript t where unambiguous.

hence $$(\hat{K}+ \Delta_P \hat{K})(p+\Delta_P p) = (\hat{K}+ \Delta_P \hat{K} - \Delta_P X)\, p + \Delta_P C\, p + \Delta_P £L$$

Expanding and simplifying yields

$$\hat{K}\Delta_P p + \Delta_P \hat{K}\Delta_P p = -\Delta_P X p + \Delta_P C p + \Delta_P £L$$

that is

$$\hat{K}\Delta_P p + \Delta_P X\, p = \Delta_P C p + \Delta_P £L + o(2) \quad (8)$$

We now divide through by Δt and pass to the limit as $\Delta t \rightarrow 0$ This gives the *value accounting identity*

$$\hat{K} p_P + X_P p = C_P p + £L_P \quad (9)$$

or, in slightly more familiar form

$$(X_P - C_P)\, p = £L_P - \hat{K}^t p_P \quad (10)$$

This should be compared with the value equation when all stocks are considered to turn over during the period of production, which can be written:

$$(X_P - C_P) p = £L_P$$

The difference is the term

$$\hat{K}p_P,$$

the *moral depreciation term*, representing capital gains and losses.

Suppose now that in circulation goods sell, not at prices equal to values $\lambda(= p+\Delta_P p)$ but at new prices $p+\Delta p$ $(= p+\Delta_P p+\Delta_C p)$ where in general $\Delta p \neq \Delta\lambda$. The term $\Delta_C p$ representing value transfers in circulation is just Δe and we can write

$$p+\Delta p = p+\Delta_P p +\Delta e.$$

The same reasoning as above now yields the *price accounting identity*,

$$\hat{K}p' + X_P p = C_P p + £L_P + £E_C \tag{11}$$

identical to (9) except for the new term $£E_C$, value transfers induced by circulation.

Equations (9) and (11) are the basic dynamic relations of price and value. They can be rearranged to show how new value is created and redistributed in the economy thus:

$$\hat{K}p_P + (X - C)_P\, p = £L_P \tag{12}$$

that is, *new value enters the economy at the rate* $£L_P$, and

$$\hat{K}p' + (X - C)_P\, p = £L_P + £E_C \tag{13}$$

showing how this new value is redistributed through by transfer vector $£E_C$.

6.3 Surplus value and profit with fixed capital

The capitalists begin production with stocks K – W, that is, everything except wage goods, and variable capital V whose value is Wp. Their gross value is therefore

$$(K - W)\, p + Wp = Kp$$

At the end of production they have used up $\Delta_P C$ and $\Delta_P V$ and created new use values X_P.[10] They therefore own stocks equal to

$$K + \Delta_P X - \Delta_P C$$

and have also used up V_P of their variable capital. Their new worth is equal to the new price of their stocks

$$(K + \Delta_P X - \Delta_P C)(p+\Delta_P p) - \Delta_P V$$

and assuming that the value of variable capital is equal to the current price of wage goods, this is equal to

$$(K + \Delta_P X - \Delta_P C - \Delta_P W)(p+\Delta_P p)$$

Gross wealth including current consumption is therefore

$$(\Delta_P K + \Delta_P X - \Delta_P C - \Delta_P W)(p+\Delta_P p)$$

Subtracting current gross wealth from initial gross wealth gives net surplus value:

$$(K + \Delta_P X - \Delta_P C - \Delta_P W)(p+\Delta_P p) - Kp$$
$$K\Delta_P p + \Delta_P X\, p - (\Delta_P C + \Delta_P W)\, p + o(2)$$

But the value equation (8) established that

$$\hat{K}\Delta_P p + \Delta_P X\, p = \Delta_P C\, p + \Delta_P £L + o(2)$$

Substituting for $p\Delta_P X$, and proceeding to the limit, yields the rate at which surplus value is produced or the *rate of surplus value generation*

$$£S_P = £L_P - £V + (K - \hat{K})p_P \tag{14}$$

This is the value-product of labour power $£L_P$, less variable capital $£V_P$, plus a *redistribution* term

$$(K - \hat{K})p_P.$$

This latter reflects the result of the competitive struggle between capitals through depreciation. All capitals whose value has risen have appropriated surplus value from all capitals whose value has fallen through depreciation. The rate of *profit generation* is given similarly by

$$£\Pi_P = £L_P - £V_P + (K - \hat{K})p_P + £E_C \qquad (15)$$

that is, the rate of surplus value generation plus the transfer vector $£E_C$. Since $£E^{\Sigma}{}_C$ is zero and $K^{\Sigma}=\hat{K}^{\Sigma}$ we have

$$£\Pi^{\Sigma}{}_P = £S^{\Sigma}{}_P,$$

Marx's second equality..Lastly the equations of price and profit yield a simple relation connecting price and profit on a sectoral basis

$$£\Pi_P = (X - C)_P\, p - £V_P + Kp' \qquad (16)$$

6.4 The stock accounting identity

Our final aim is to produce the fundamental equation of accumulation, governing the value of the total invested capital £K over time. We must first account rigorously for the transformations of *use values* resulting from production and circulation. Since we cannot assume market clearing we must account for the differences between produced and purchased commodities, that is, the relation between stocks and flows. This we term the general time-dependent stock accounting identity; combining it with the time-dependent value and price accounting identities yields the equation of accumulation governing £K′.

Circulation, as we have discussed, alters the distribution of stocks but not their total quantity. There is no automatic way to predict the proportions of these exchanges. We know only the relation between them given by the definition:

$$K = X - C - W - B \qquad (17)$$

The same applies to any changes of stock levels, so that

$$\Delta K = \Delta(X - C - W - B) \qquad (18)$$

This is likewise true for any isolated source of change, so that

$$\Delta_C K = \Delta_C(X - C - W - B) \qquad (19)$$

Now, K may change in circulation through a redistribution of commodities, but circulation can neither create nor destroy use values. It follows that the row sum of $\Delta_C K$ is zero.

Therefore summing (19) across rows – capitals – produces a fundamental statement, a sort of Kirchoff's Law of circulation, which any commodity economy must obey:

$$\Delta_C(X - C - W - B)^{\Sigma} = 0 \qquad (20)$$

In consequence the quantity ΔK, changes in K over the whole of reproduction, can *only* be due to production (in which, recall, we include private consumption).

Therefore (recalling that $\Delta = \Delta_P + \Delta_C$)

$$\Delta K^{\Sigma} = \Delta_P(X - C - W - B)^{\Sigma} \qquad (21)$$

That is, the change in the total stock of each commodity over an entire cycle of production and circulation reduces to the change due to production alone. We term this the *fundamental stock accounting identity*. It is the most general statement we can make. If any magnitudes in it are specified in more detail – for example by a production function or a theory of consumer demand – then we have a particular model of the economy, which whatever its special properties, must obey the equation (21)

6.5 Capitalist accumulation

The wealth of society falls into two main portions: the wage fund *W* owned by workers and everything else, owned by capitalists. This latter is capital; it consists of those commodities which, broadly speaking, enter into the equalisation of profit rates. In this we include the wealth of collectors, speculators, hoarders and rentiers; in short every form of wealth which acts as a receptacle for surplus value and which, as a component in a portfolio of wealth, may be exchanged for other commodities in pursuit of a higher rate of growth of real value, that is, profit. Neglecting variable capital this is given by $K - W$.

However capital also seeks a return on variable capital along with all other advances of money. The value of the capital seeking a share of surplus value is therefore simply the scalar quantity

$$\mathrm{K}^{\Sigma}p = £\mathrm{K}^{\Sigma}$$

The total rate of accumulation of society, is the rate at which this magnitude grows. This is therefore

$$£\mathrm{K}^{\Sigma\prime} = (\mathrm{K}^{\Sigma}p)' = \mathrm{K}p' + \mathrm{K}'p$$

the sum of two quantities, one the result of the accumulation and capitalist consumption of use-values and the other the result of price and value changes. The second term is given by the equation of value production which can be simplified to:

$$\mathrm{K}^{\Sigma}p' + \mathrm{X}^{\Sigma}{}_{\mathrm{P}}p = \mathrm{C}^{\Sigma}{}_{\mathrm{P}}p + £\mathrm{L}^{\Sigma}{}_{\mathrm{P}}$$

hence

$$£\mathrm{K}^{\Sigma\prime} = £\mathrm{L}^{\Sigma}{}_{\mathrm{P}} + (\mathrm{K}^{\Sigma} - \mathrm{X}^{\Sigma} + \mathrm{C}^{\Sigma})_{\mathrm{P}}p$$

However, the stock accounting identity tells us

$$\mathrm{K}^{\Sigma}{}_{\mathrm{P}} = (\mathrm{X}^{\Sigma} - \mathrm{C}^{\Sigma} - \mathrm{B}^{\Sigma} - \mathrm{W}^{\Sigma})_{\mathrm{P}}$$

Thus the rate of growth of capital, summed over society, is therefore

$$£\mathrm{K}^{\Sigma\prime} = £\mathrm{L}^{\Sigma}{}_{\mathrm{P}} - £\mathrm{B}^{\Sigma}{}_{\mathrm{P}} - £\mathrm{W}^{\Sigma}{}_{\mathrm{P}}$$

$$£\mathrm{K}^{\Sigma\prime} = £\mathrm{S}^{\Sigma}{}_{\mathrm{P}} - £\mathrm{B}^{\Sigma}{}_{\mathrm{P}} \qquad (22)$$

The only way this can be negative is if the bourgeoisie disinvest in value terms. This may be achieved either through direct diversion of use values to capitalist consumption through arms or other unproductive expenditure; or it may be, as takes place in a slump, because investment slackens off to simple replacement and so the stock of capital depreciates towards its new equilibrium value.

6.6 The general law governing the rate of profit

We are now in a position to state the general law governing the variation of the rate of profit. Since we have made no special assumptions concerning wage rates, supply and demand, capitalist behaviour or the structure of production, this law is absolutely general and must therefore apply in all special cases.

The general or average rate of profit is given by the ratio between $£\mathrm{S}^{\Sigma}{}_{\mathrm{P}}$, the rate at which profit is generated, and $£\mathrm{K}^{\Sigma}{}_{\Sigma}$, the volume in value terms of capital seeking a return on investment. For simplicity here we use S and K for these two terms.

The rate at which the profit rate changes is then

$$r' = \frac{\mathrm{d}}{\mathrm{d}t}\frac{£\mathrm{S}}{£\mathrm{K}} = \frac{£\mathrm{K}£\mathrm{S}' - £\mathrm{S}£\mathrm{K}'}{£\mathrm{K}^2} = \frac{£\mathrm{S} - r£\mathrm{K}'}{£\mathrm{K}}$$

But we can substitute from the numerator for £K′ using equation (22), to give

$$r' = \frac{£\mathrm{S} - r\,(£\mathrm{S} - £\mathrm{B})}{£\mathrm{K}} = \left(\frac{£\mathrm{L}^{\Sigma}{}_{\mathrm{P}} - £\mathrm{V}^{\Sigma}{}_{\mathrm{P}} - r£\mathrm{I}^{\Sigma}{}_{\mathrm{P}}}{£\mathrm{K}}\right)$$

where $\mathrm{I}^{\Sigma}{}_{\mathrm{P}}$ is the rate of investment, that is, surplus value less capitalist consumption. We can now formulate precisely the conditions for this to be a positive magnitude (rising profit rate) or a negative magnitude(falling profit rate). First, if $£\mathrm{L}^{\Sigma}{}_{\mathrm{P}}$ and $£\mathrm{V}^{\Sigma}{}_{\mathrm{P}}$ are constant (constant rate of value creation and wage in value terms), then the rate of profit must fall unless the capitalists disinvest in value terms, that is, unless $£\mathrm{I}^{\Sigma}{}_{\mathrm{P}}$, the rate of investment, is negative. Thus (the law as such) investment produces a continuously falling profit rate.

Second, this can be offset (countervailing tendencies) by raising £$L^{\Sigma}{}_{P}$ – making the workers work harder or employing more of them – or by decreasing £$V^{\Sigma}{}_{P}$, the share of national product which they consume in value terms. However there are *absolute limits to either*. $L^{\Sigma}{}_{P}$ here is a social total. Differences between less or more skilled labour average out, and therefore it is in a fixed ratio to hours worked. And V cannot be decreased below zero or the workers die.

We thus find – an astonishing and salutory result – that after a hundred years of nit-picking at Marx's original statement of the general law of the falling rate of profit, that this law is not merely valid, but scientifically and rigorously exact.

7 REFERENCES

Note: space does not permit the comprehensive bibliography given in Freeman and Carchedi (1995); we confine ourselves to what is cited in the text and the direct precursors of the ideas it presents or criticises.

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8 NOTES

[1] See the references in this article to Ernst, Langston, Carchedi, Freeman, Andrews, Naples, Kliman and McGlone, and Giussani and Walker's 1988 survey.

[2] Bortkiewicz (1906:23-24). I am indebted to Michele Naples for pointing out this passage.

[3] This leads to a pedantic but necessary distinction: One pound’s worth of value will be represented as £1 but one pound coin or note itself will be represented 1£.

[4] Schefold (1980) is an exception in using this convention.

[5] This does not exclude joint production. It means that joint products have been allocated from the sector of origin to the main sector producing them, using the standard procedure employed by input-ouput statisticians to convert the 'make' matrix into the 'commodity' matrix.

[6] I am in debt to Bruce Roberts for drawing my attention to this problem in a very patient reading of a first draft of a section of this paper. It seems a rather strong illustration of the scant attention economics has paid to the stock-flow relation that this dilemma is not recognised.

[7] This assumption can be relaxed; space does not permit this here. See Freeman (1995).

[8] To those who exclaim in shock or glee that this removes the connection between labour and value we reply: look at the equation of production. Labour is the substance of price as well as value. The shock, and the glee, result from the misconceived idea that price and value are qualitatively different things, instead of different stages of the same thing.

[9] This can be corrected to allow for *secondary exploitation,* transfers of value to and from consumer durables, but we shall omit this correction here.

[10] $\Delta_P B$, bourgeois consumption, is part of profits and should not be deducted before these are calculated.