Modelling extreme financial returns of global equity markets

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Abstract

Extreme asset price movements appear to be more pronounced recently and have major consequences for an economy's financial stability and monetary policies. This paper investigates the extreme behaviour of equity market returns and quantifies the probabilities of these losses. Taking fourteen major equity markets the study is able to ascertain similarities and divergences in the tail returns from around the world. To do so, it applies extreme value theory to equity indices representing American, Asian and European markets. The paper finds that all markets tail realisations are adequately modelled with the fat-tailed Fréchet distribution. Furthermore tail realisations associated with the downside of a distribution are greater than those associated with the upside, and extreme returns for Asian markets are usually larger than their European and American counterparts.

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Modelling extreme financial returns of global equity markets

1. Introduction

Extreme asset price movements appear to be more pronounced recently and have major consequences for an economy's financial stability and monetary policies. This paper investigates the extreme behaviour of equity market returns and quantifies the probabilities of these losses. Taking fourteen major equity markets the study examines similarities and divergences in the tail returns in equity markets from around the world. By their very nature, the estimation of extreme returns is highly dependent on accurate modelling of rare events, and the paper models extreme equity returns using extreme value theory.

The paper provides predictions of the frequency and severity of extreme returns for a comprehensive set of American, European and Asian markets. Using daily returns from 1985 through 2000 the study is able to incorporate the effects of major financial crises such as the 1987 crash, the Asian crises and the recent technology bubble by illustrating and analysing the extent of market movements across all markets. Whilst previous studies have examined tail behaviour under a number of different headings including portfolio allocation (Jansen et al, 2000), risk management (Cotter, 2001), methodological issues (Quintos et al, 2001) and for different assets such as currencies (Cotter, 2004), this is the first study to comprehensively examine tail returns across the main global markets and to identify similarities and divergences in the recent decades.[1]

[1] By way of contrast, Pownall and Koedijk (1999) examine Asian markets by focusing on the Asia 50 index rather than separating these out across countries.

Extreme price movements are found during periods of manias and crashes (Kindleberger, 2000), adequately describing the scenario of the 30% fall in US equities over a week during the 1987 crash.[2] This paper assumes that an extreme return occurs if market movements exceed some predetermined threshold value on either side of a probability distribution of equity returns. Specifically the paper calculates ex post unconditional tail probabilities for global equity markets separately and uses these to determine the frequency of occurrence of large price movements.[3] These measures are underpinned by an analysis of the unconditional distributions of American, Asian, and European equity markets.

The remainder of the paper proceeds as follows. In section 2, the estimation procedures are presented with a brief synopsis of the theoretical underpinnings of extreme value theory. Section 3 provides a description of the markets indices chosen for analysis and their time varying dynamics. Section 4 presents the empirics detailing unconditional extreme value estimates. Finally, a summary of the paper and some conclusions are given in section 5.

2. Theory and Estimation Methods

We begin by providing a short synopsis of the salient features of extreme value theory as it applies to modelling extreme financial returns (for comprehensive

[2] Identifying whether market crashes occur or not is a controversial issue. For instance two excellent treatises by Garber (1990) and Kindleberger (2000) disagree on whether actual events such as Tulip mania in the 17th century constitute an asset price bubble. Although it is hard to have a clear-cut answer on whether equity prices reflect economic fundamentals at any moment in time, major deviations result in asset prices being prone to exhibiting major corrections associated with market crashes. Asset price bubbles are driven by a breakdown in the agency relationship and in the case of equity markets where institutional investors do not face the full consequence of crises arising from their investment decisions (Allen and Gale, 2000).

[3] The approach has also been used in a multivariate setting examining extreme spillovers between markets (Hartmann et al, 2004) and estimating extreme correlations for bull and bear markets (Longin and Solnik, 2001).

details see Embrechts et al, 1997). The theoretical framework distinguishes three types of unconditional asymptotic distributions that models tail realisations, the Gumbel, the Weibull and the one of concern to this study, the fat-tailed Fréchet distribution. The fat-tailed property has been documented for the extreme returns of many financial time series, such as index returns (Cotter, 2004), single equities (Danielsson and de Vries, 2000), foreign exchange (Huisman et al, 2001) and derivatives (Cotter, 2001). The property indicates the propensity for financial time series to exhibit upside and downside returns of very large magnitude relative to the normal distribution for given probability levels. The fat-tailed property causes a relatively slow decay for convergence towards the limit, vis-à-vis the normal distribution.

Begin by assuming that a random variable, such as financial returns, is independent and identically distributed (iid) and belonging to the true unknown cumulative probability density function F(r).[4] Taking the full distribution, returns are defined as the equity index's first difference of daily logarithmic price, $r_t = \ln(p_t) - \ln(p_{t-1})$, measuring daily price movements. To examine extreme tail returns only, let (M_n) be the maxima of n random variables placed in ascending order such that $M_n = \max\{R_1, R_2,..., R_n\}$, and the (tail) probability that the maximum value exceeds a certain price change, r, [5]

[4] The successful modelling of financial returns using GARCH specifications in the literature that replicates serial correlation clearly invalidates the iid assumption. However, this assumption is relaxed as de Haan et al (1989) examine less restrictive processes more akin with index returns. In these cases only the assumption of stationarity is required. This convention is generally followed in the financial literature as it is in this paper.

[5] Extreme value theory is usually detailed for upper order statistics where the random variable is placed in ascending order that focuses on the maxima of upper tail values. The remainder of the paper follows this convention. This study also examines empirically the lower order statistics where the random variable is placed in descending order that focuses on the minima of lower tail values. In this case, the minima of the random variable is $\text{Min}\{R_1, R_2,..., R_n\} = -\text{Max}\{-R_1, -R_2,..., -R_n\}$.

$$P\{M_n > r\} = P\{R_1 > r, \ldots, R_n > r\} \quad = 1 - F^n(r) \qquad (1)$$

This represents the tail probability.

Whilst the exact distribution is allowed to be unknown, asymptotically it behaves like a fat-tailed distribution.

$$1 - F^n(r) \approx ar^{-\alpha} \qquad (2)$$

where the scaling constant is given by a and α is the tail index, for $\alpha > 0$, and for $r \to \propto$. Leadbetter et al (1983) outline the theoretically convergence of fat-tailed distributions to the fat-tailed extreme value distribution, and the asymptotic convergence of extreme financial returns to the Fréchet distribution is in Longin (1996). The Fréchet extreme value distribution unifies fat-tailed distributions to have tail equivalence and allows for unbounded moments:

$$\lim_{t \to \infty} \frac{1 - F(tr)}{1 - F(t)} = r^{-\alpha} \qquad (3)$$

Asymptotically it allows the tail to vary with $-\alpha$, which follows a power law. The consequence of the power law is that the fat-tailed returns decline at a slow rate in comparison to other distributional shapes. These alternative distributions can be divided into three separate groups depending on the value of the tail index α. A commonly assumed class of distributions used for financial returns includes the set of thin tailed densities, and most notably amongst these, the normal or lognormal distributions. This classification of densities includes the normal and exponential distributions and these belong to the Gumbel distribution, having a characteristic of tails decaying exponentially. In contrast, the classification of a Weibull distribution ($\alpha < 0$) includes the uniform example where the tail is bounded by having a finite right end point and is a short tailed distribution. Of primary concern to the analysis of fat-tailed distributions is the Fréchet classification, and examples of this type generated here are the Cauchy, Student-t, and sum-stable distributions. This important classification of distributions that accurately models extreme index returns incorporates tail values that decay by a power function.

By unifying fat-tailed distributions by having tail equivalence it implies distributions such as Student-t and sum-stable distributions exhibit identical limiting tail behaviour. The tail index, α, measures the degree of tail thickness and the number of bounded moments. For example, a tail index of two implies that the first two moments, the mean and variance, exist whereas financial studies have cited value between 2 and 4 suggesting that not all the first four moments of the price changes are always finite (Loretan and Phillips, 1994). The tail index has also been used to distinguish between different distributions with for instance, α interpreted as representing the degrees of freedom of a Student-t distribution and equals the characteristic exponent of the sum-stable distribution for $\alpha < 2$.

Given the asymptotic relationship of the random variable to the fat-tailed distribution, non-parametric tail estimation takes place giving separate upside and downside tail probabilities. The tail probability estimator is obtained from taking a second order expansion of $F^n(r)$ as $r \rightarrow \propto$, avoiding all higher order terms in the expansion r, and rearranging to incorporate sample estimates. The tail probabilities focus on extreme price movements only and determine the probability of various price movements, p:

$$p \quad = \quad M/n(R_{M+1}/r)^{\alpha} \tag{4}$$

Here we are estimating the probability of exceeding a certain very large threshold, r, given the ratio of number of tail values to sample size and the tail index estimate. This probability estimate is used to examine various quantiles across the spectrum of indexes analysed. Also the measure can be extended to provide out-of-sample risk estimates at low probability values.

The statistical features of the tail probability estimator are equivalent to those of the tail estimator that assumes that returns belong to the fat-tailed Fréchet distribution. The widely used Hill (1975) moment estimator is used to determine tail quantiles and probabilities. The Hill estimator represents a maximum likelihood estimator of m order statistics:

$$\gamma = 1/\alpha = (1/m) \sum [\log r_{(n+1-i)} - \log r_{(n-m)}] \qquad \text{for } i = 1....m \qquad (5)$$

To compare these estimators the tail stability test of Loretan and Phillips (1994) is applied to examine whether the tail probabilities vary according to the index chosen assuming that the underlying data is iid with Fréchet type tail behaviour:

$$V(\gamma^x - \gamma^y) = [\gamma^x - \gamma^y]^2/[\gamma^{x2}/m^x + \gamma^{y2}/m^y]^{1/2} \qquad (6)$$

for γ^x (γ^y) are used to denote different equity indexes x and y. Here the tail stability test determines the extent by which indexes x and y deviate from each other ($V(\gamma^x - \gamma^y)$). Furthermore this statistic is rearranged to examine whether probabilities across the tails of the distribution are distinguishable. Thus tail behaviour can be examined for constancy across indexes and across the upside and downside of any return distribution. If both hypotheses of constancy holds it suggests that tail returns are first similar across indexes, and second, similar across the distribution of an index itself. Loretan and Phillips (1994) show that this moments based test statistic is asymptotically normally distributed.

We now illustrate how extreme value theory models tail returns in the context of extreme financial returns. Under extreme value theory tail returns are modelled separately from the full probability distribution function. Each set of tail returns can be approximated by an extreme value distribution. Tail returns are analysed separately for lower and upper distribution values respectively represented by M_n giving the maxima (minima) from a sample of size n. The tail probability measuring the likelihood of experiencing an extreme negative return exceeding some predetermined quantile for n returns is:

$$P_{downside} = P\{M_n < r_{long}\} = \quad c \qquad (7)$$

r_{long} represents the predetermined quantile for a long trading position and c is the tail probability given by $F^n(r)$.

Likewise, using the same framework the tail probability measuring the likelihood of experiencing an extreme positive return exceeding some predetermined quantile for n returns is:

$$P_{upside} = P\{M_n > r_{short}\} = \quad b \qquad (8)$$

r_{short} represents the predetermined quantile on a short trading position and b is the unknown tail probability given by $1 - F^n(r)$.

Notwithstanding that the main analysis examines the unconditional distribution of returns, an introductory discussion of the conditional distribution of index returns gives us an understanding of relationship between the magnitude of returns and their occurrence during periods of high and low levels of volatility. A description of the time varying dynamics is provided from fitting a GARCH (1, 1) model to the returns series (Bollerslev, 1986). This allows for modelling of serial dependency that exists in financial returns and provide a description of the conditional environment.

$$h_t^2 = \alpha_o + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{q} \beta_j h_{t-j}^2 \qquad (9)$$

Here volatility is time varying and modelled adaptively on past squared values of the disturbance term and past values of the conditional volatility process.

3. Data Description

Daily logarithmic returns from a broad spectrum of equity market indices representing major markets from around the world are analysed for the period between January 1 1985 and December 31 2000. The data include three US, five European and six Asian equity market indices allowing for an extensive investigation of extreme returns for major financial markets across a wide geographical spread. The indices chosen and their abbreviations are NASDAQ Composite (US NASDAQ), S&P500 Composite (US S&P500), Dow Jones Industrials (US DOW JONES), FTSE All Share (UK), DAX100 DS-Calculated (GERMANY), France DS-Calculated (FRANCE), Italy DS-Calculated (ITALY), Amsterdam EOE (HOLLAND), Nikkei 225 Stock Average (JAPAN), Hang Seng (HONG KONG), Singapore Straits Times (SINGAPORE), Bangkok S.E.T. (THAILAND), Jakartha SE Composite (INDONESIA), and Kuala Lumpar Composite (MALAYSIA).

A box plot of the observed distribution is presented in figure 1 for S&P500 returns indicating the large magnitude of extreme values located in the tails of the distribution. Characteristics of the returns series are provided by the summary statistics in table 1. Mean values indicate on average positive returns, and standard deviation values suggest daily volatility around 1%, although in general the Asian series exhibit higher values pointing to their high level of inherent risk. These indices also indicate the largest interquartile range with the largest individual return occurring for the Hong Kong index with a daily loss in excess of 40%. Clearly all series are non-normal given the results for the Jarque-Bera test statistic. This lack of normality is reflected in the excess skewness and more importantly for this study, the excess kurtosis that is evident for all series analysed. This implies that any attempt to model these returns using a normal distribution would clearly underestimate the tail densities and thus fails to adequately predict the likelihood of extreme events.

INSERT FIGURE 1 HERE

INSERT TABLE 1 HERE

To provide a description of the conditional environment and time varying volatility an AR (1) – GARCH (1, 1) model is fitted to the daily returns series. Conditional volatility is obtained allowing for an examination of the pattern and magnitude of return fluctuations in periods of tranquillity and turbulence. The model's parameters and associated probability values using Bollerslev and Wooldridge's (1992) robust standard errors are presented in table 2. In addition the commonly noted fat-tailed characteristic of financial returns is accounted for, by modelling the error terms with Student-t distributions. Generally, the conditional volatility models are similar in their attributes indicating that past return volatility impacts on current volatility as is typical of a GARCH type process. Generally the autoregressive term is significant in the conditional mean equation and both ARCH and GARCH effects are documented in the conditional volatility equation.

INSERT TABLE 2 HERE

The models are chosen using the AIC and BIC selection criteria. The post fitting diagnostics suggest that the models appear well specified. The standardised residual series are white noise in all cases except for the UK, Singapore and Indonesian indexes as can be seen from the Ljung-Box test results. Furthermore, the serial correlation associated with financial returns series is removed after fitting the GARCH model as the standardised residuals for the squares, generally satisfy the null of no fourth order linear dependence of the Ljung-Box $Q^2(12)$ tests. The only exception is the Hong Kong index.

Using the GARCH model provides some information on the dynamics of the indexes returns. A time series plot of the returns and volatility series is presented in Figure 2 for the US NASDAQ index although the same conclusions can be garnished from the other indexes. The largest price movements are associated with the October 1987 equity market crash, although there has been a general increase in price movements and associated volatility towards the end of the sample period. Notably we see the extent to which financial markets can go from periods of tranquillity where market movements are reasonably stable to periods of turbulence featuring instability.

The periods of instability result in high levels of volatility and may be a result of large positive or large negative price movements or a combination of both. In the returns plot we see the large return reversals around the October 1987 crash that each individually would be included in any analysis of equity market crises.[6] Finally whilst the GARCH model provides a good description of the time-varying dynamics in general across the full distribution it suffers from event risk in trying to model tail returns (Longin, 2000). This is unsurprising and should occur for adaptive time varying processes in general that try and model the full distribution of returns. Turning to the tail modelling we now examine the estimates of extreme value theory that concentrates on tail values only thereby minimising event risk and provides estimates based on the unconditional features of the equity index returns.

INSERT FIGURE 2 HERE

4. Extreme value findings

We have already seen similarities and divergences across the full distribution of index returns and we now turn our attention to the extremes. To begin, the Hill tail estimates are calculated and compared across the distribution of each index. Next extreme value quantiles for very low probabilities are discussed. Tests of stability in tail behaviour between the respective markets are then outlined. Finally, comparative tail probability estimates are scrutinized for the fourteen major equity markets.

Hill tail estimates and associated quantiles for major equity markets are given in table 3 and are now discussed. We begin with the number of tail returns used to

[6] Generally the movements for the US index were negative around the 1987 crash with extreme negative returns being recorded on October 17 (-12.026%) and October 22 (-14.022%). Within this period extreme returns associated with a boom were also recorded for example October 21 (7.083%).

generate the tail probability estimates. Alternative approaches in determining the optimal number of tail returns, m, analysed are available (Danielsson et al, 2001) and the approach adopted here follows the bootstrap procedure that minimizes the asymptotic mean squared error of the Hill estimator described by Hall (1990).[7] The initial starting value chosen for the procedure is $n^{0.6}$. To overcome possible biases in the number of tail returns chosen and the inferred tail probabilities, Hill estimates for a large range of tail returns are calculated and plotted as Hill plots. Inferences on extreme upside and downside returns imply that the constancy of the tail index value is paramount. Tail constancy is examined and confirmed for the equity indices by the Hill plots with an illustration given for the Hong Kong index in figure 3.[8] Here constant estimates are obtained for a large spectrum around the number of tail values chosen. The maximum number of values required in tail index estimation occurs for the US NASDAQ and US DOW JONES with m = 155, although all estimation uses less than 5% of each indexes returns (5% = 209 returns from a sample size of 4174 returns).

INSERT TABLE 3 HERE

INSERT FIGURE 3 HERE

Turning now to the Hill values themselves, point estimates in table 3 for tail index values are generally between 2 and 4 within small confidence intervals. An exception to this is the Indonesian index with an estimate of 1.81 based on the

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[7] Formally this involves:
$m_0(n) := \arg_m \min \text{Asy } E(\gamma_m(m) - \gamma)^2$

[8] A single example is given for conciseness and the remaining plots are available on request. Embrechts et al (1997) note the tail estimates can vary considerably for very low and large numbers of tail values and suggest using the Hill plot to identify regions of stability for statistical inference purposes. Due to areas of large divergence of tail estimates Embrechts et al (1997) suggested the term 'hill horror plot' as a way of describing and identifying the phenomenon of instability in the Hill estimates.

upper tail of its distribution.[9] In general, the lower the tail index estimates, the fatter the density mass of the tail. This rule recognizes the impact of the very large spikes occurring for some of the Asian series, especially the Indonesian series, and also the US NASDAQ. The implication is of a propensity for greater price movements for these series in accordance with the noted maximum and minimum values. In contrast the European and American indexes have higher tail estimates indicating relatively thinner tails. An exception in this generalization is the Asian Japanese index with a Hill estimate of 3.41 for the lower tail of the distribution. This only indicates that the Nikkei 225, although from an Asian market, represents a reasonably stable market and behaves similarly to the European and American indexes. Similar conclusions are made for the well-diversified old economy index, the S&P500, which has a relatively thinner tail with higher Hill estimates compared to the well-diversified new economy index, the NASDAQ. Surprisingly, the thinnest tail documented is for the Italian index and one would not normally consider it as being the safest market.[10]

Using the Hill estimates inferences are also made regarding the number of defined moments of an empirical distribution. Previous studies usually indicate a Hill estimate of between 2 and 4 suggesting that the first two moments, the mean and variance are defined but this is not necessarily so for higher moments such as kurtosis (Loretan and Phillips, 1994). Thus for a second moment to exist, tail index values should not be less than 2, which is a hypothesis that is not rejected

[9] This estimate shows the impact of regulatory change where a deregulation of the exchange resulted in a single days return in excess of 40% in Indonesia at the end of 1988.

[10] It is important to point out that a Hill tail index indicates the relative risk of the tail value relative to the starting point of the tail. Thus, whilst the Asian markets are the most risky from analysing across the full distribution of returns, it does not necessarily imply that they will have the largest tail estimates as it is a comparison between each tail return and the threshold tail return that the Hill index describes.

for any index (with the exception of Indonesia), thereby supporting the existence of a second moment.[11] Turning to the fourth moment, kurtosis is defined if tail index estimates are greater than 4. This hypothesis, although never supported for the point estimates themselves is generally rejected with the exception of incidences for UK, French, Dutch, Italian and Japanese markets respectively. Overall the results are in line with previous studies where there is ambiguity whether a fourth moment is defined for financial series.

Extreme equity market returns quantiles are also presented in table 3 and are now discussed. These extremal equity index values provide evidence on the severity and timing of extreme financial returns. The quantiles represent estimated extremal returns based on various probabilities, for example, $Q_{1/n}$ occurring once over the sample period of 16 years and for $Q_{1/2n}$ occurring once over the sample period of 32 years. The low probability levels for in-sample quantiles occur 0.024% of the time whereas for out-of-sample quantiles occur 0.012% of the time given the sample size (n = 4174 returns) chosen for analysis. The evidence supports the view that (with the exception of Japan) the Asian markets exhibit a greater propensity for extreme returns. For instance, there is a 1 in 4714 chance that an upside return of 38.38% would occur for the Indonesian index that exhibits the largest extreme returns whereas the corresponding loss for the UK index is 6%. Also in terms of geographical location, the European market with the largest extreme values is the Amsterdam index from Holland. For the US the NASDAQ provides an exception to the relatively stable American indexes and may be driven by the uncertainty associated with the technology sector.

[11] The critical value for these one-tail tests is 1.64.

As well as comparing extreme quantiles for different geographical areas we can also determine the variation across upside and downside extreme returns. Some of the Asian contracts including Indonesia, Japan and Malaysia have larger extreme returns associated with the upside distribution than the downside. In contrast, the outcomes for the less risky European and American markets suggest that extreme positive returns are smaller than negative outcomes. For example, the UK index exhibits an extremal return of 6.00% at the probability $Q_{1/n}$ from analyzing the upper tail in comparison to 9.75% for the lower tail. Comparison of upside and downside extreme returns are further applied to out-of-sample estimates, where similar conclusions are garnished for the tails of the distribution.

It is interesting to formally determine the extent to which tail behaviour deviates across markets and trading positions. Using the stability test discussed in Loretan and Phillips (1994) estimates are presented in table 4 determining the extent to which the Hill tail estimates of each index deviates from each other for each trading position. Overall the vast majority of tail estimates for the indexes analysed are similar in magnitude with very few statistically significant estimates. This is particularly pronounced in examining the downside tail statistics with all but 13 cases from 91 have similar sized tail values for a critical value of 1.96. Thus the fat-tailed behaviour associated with extreme financial returns is not just prone to affecting certain geographical markets but impacts equity markets per se. Deviations that do occur tend to be from the relatively thin-tailed Japanese and the relatively fat-tailed Indonesian index. This implies the extreme tail return behaviour is reasonably similar across all equity markets and is generally homogeneous across American, European and to a lesser degree, Asian markets.

INSERT TABLE 4 HERE

Turning to comparing tail behaviour of the markets for the upside returns, similar conclusions are inferred from examining stability across indexes (although there is a greater degree of divergence with 35 significant test statistics). Distinctions occur for different indexes benchmarked against the most fat-tailed and thin-tailed tail estimate. Once more the Indonesian index represents the relatively fat-tailed asset and is statistically divergent from all other indexes. Now the relatively thin-tailed asset is the Italian index with 7 statistically significant diverging tails. Otherwise, European, American and Asian markets exhibit reasonably similar tail behaviour for upper tail returns.

An interesting extension for the Hill index values is to provide tail probability estimates and these are presented in table 5 and are now discussed. These provide information on the probability of these indexes incurring price movements that reach certain thresholds such as 10%. The tail probabilities implicitly feed directly from the Hill index values. Tail probability estimates are provided for three thresholds and for extreme negative and positive returns on an annualised basis. The predetermined extreme losses chosen allow for a thorough investigation into the propensity for any of the market to experience daily losses of a very large magnitude. Overall a clear distinction can be made from examining the likelihood of experiencing various extreme returns for the markets in the different geographical regions. The tail probability values for the Asian indexes dwarf their American and European counterparts. Exceptions are the relatively low values for the well-developed Japanese index and the relatively high values for the technology dominated US NASDAQ.

INSERT TABLE 5 HERE

To illustrate taking the Malaysian index the estimate of 0.3440 suggests a very high probability of occurrence of –10%. These extreme returns would occur once every (k = 1/p: 1/0.3340) 3 years approximately whereas in contrast, the occurrence for the UK index is much less estimated at every (1/0.0589) 20 years approximately. These may appear to be rare events but two important points remain. First the events occur with some frequency for example every 20 years might suggest 2 times in the average lifespan of a professional investor, and second, and of more importance is the size of these rare events occurring at daily frequency which have disastrous conclusions for a range of economic agents. The most risky European and Asian markets for large extreme returns are Holland and Indonesia respectively whereas the safest markets are the UK and Japan. For America the NASDAQ index of equities represents the riskiest in terms of tail behaviour in contrast to the relatively safe S&P500. Similar probability findings hold across the Asian, American and European bourses at the different thresholds. Thus, these estimates represent a greater propensity for equity market crashes occurring in Asian with respect to other international markets.

More obvious distinctions can be made with the tail probability estimates using upside and downside values using upper and lower tails of distributions than that of the Hill estimates. Here we can compare the probabilities of extreme outcomes for lower and upper tails of a distribution of returns and the length of time waiting for their occurrence. Whilst the earlier analysis suggests that only 2 of the indexes for the UK and Italian markets have statistically diverging Hill estimates, the tail probability estimates for the upper and lower tail estimates show the extent of movements resulting from extreme market returns. Tail probabilities associated with the lower tail of returns are much higher than for the upper tail return with few exceptions. We can now conclude that the likelihood of an equity market crash is greater than that of a boom with the odd exception of the Japanese and Indonesian indexes. For instance taking the French market as an illustration, the propensity of a price movement of –10% (10%) is 0.0695 (0.0237) implying occurrences of once every 13 (40) years approximately.[12] Furthermore these distinctions become more pronounced as you move to more extreme returns such as –30% (30%).

5. Summary and Conclusions

This paper examines the prediction of the frequency and severity of extreme market returns for a range of global equity indices. The emphasis is on the statistical calculation of extreme price movements and their associated consequences using extreme value theory. Extreme price movements appear to be more pronounced recently with the 1987 crash, the Asian crises and the recent technology bubble and these events drive financial instability. This paper comprehensively investigates the extreme behaviour of equity market returns and quantifies the probabilities of these losses. Taking fourteen major world equity markets using American, European and Asian indexes the study is able to

[12] The average waiting period is $k = 1/p$.

ascertain similarities and divergences in the extreme tail returns from around the world – which is an issue not explored in previous studies.

Given previous findings of financial returns being fat-tailed, the limiting Fréchet distribution is applied to extreme upside and downside returns. The paper then reports a number of interesting findings. First, the analysis confirms the non-normality of equity market returns, and in particular the leptokurtosis indicative of fat-tailed distributions. Relatedly, the returns series exhibit positive tail indices implying that the limiting extreme value distributions are characterized by a (fat-tailed) Fréchet distribution. Second, the extreme returns associated with lower tails are generally higher in absolute terms than those of the upper tails; this implies that large negative movements are more severe in magnitude than large positive movements. Third, with the exception of Japan, the extreme returns of the Asian market indices are higher than their US and European counterparts suggesting that the frequency and severity of extreme returns on these markets are greater than those on Western markets.

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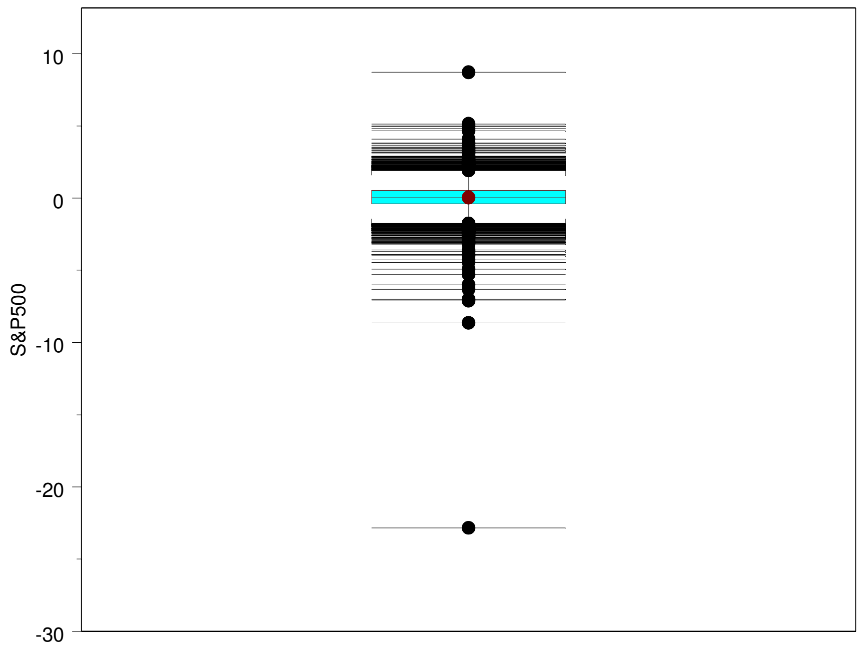

Figure 1. Box plot of US S&P500 returns.
Notes: The plot shows the median of the S&P500 returns surround by whiskers that spans 1.5 times interquartile range of returns, and values beyond this are outliers.

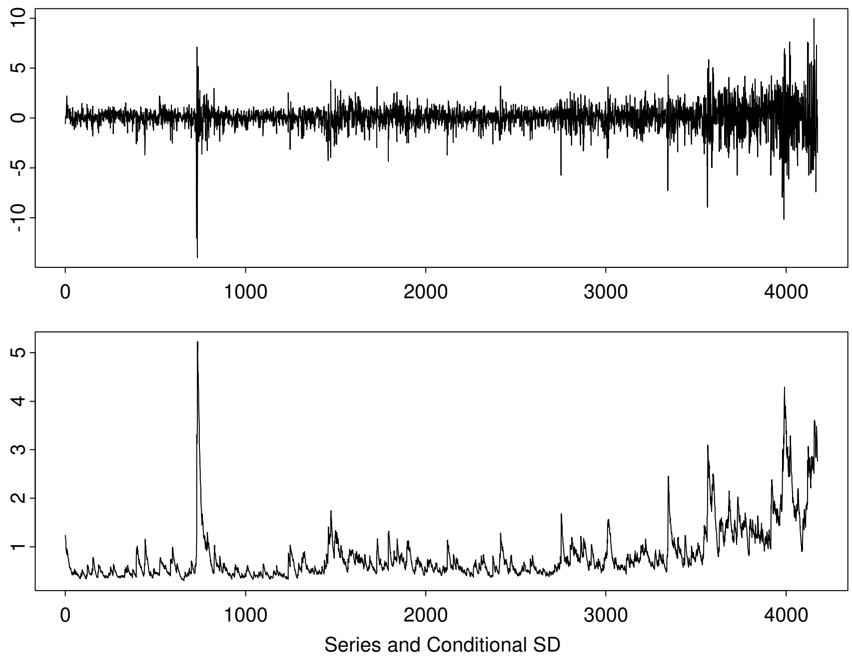

Figure 2. Time series plot of returns series (upper box) and conditional standard deviation series (lower box) of US NASDAQ index.

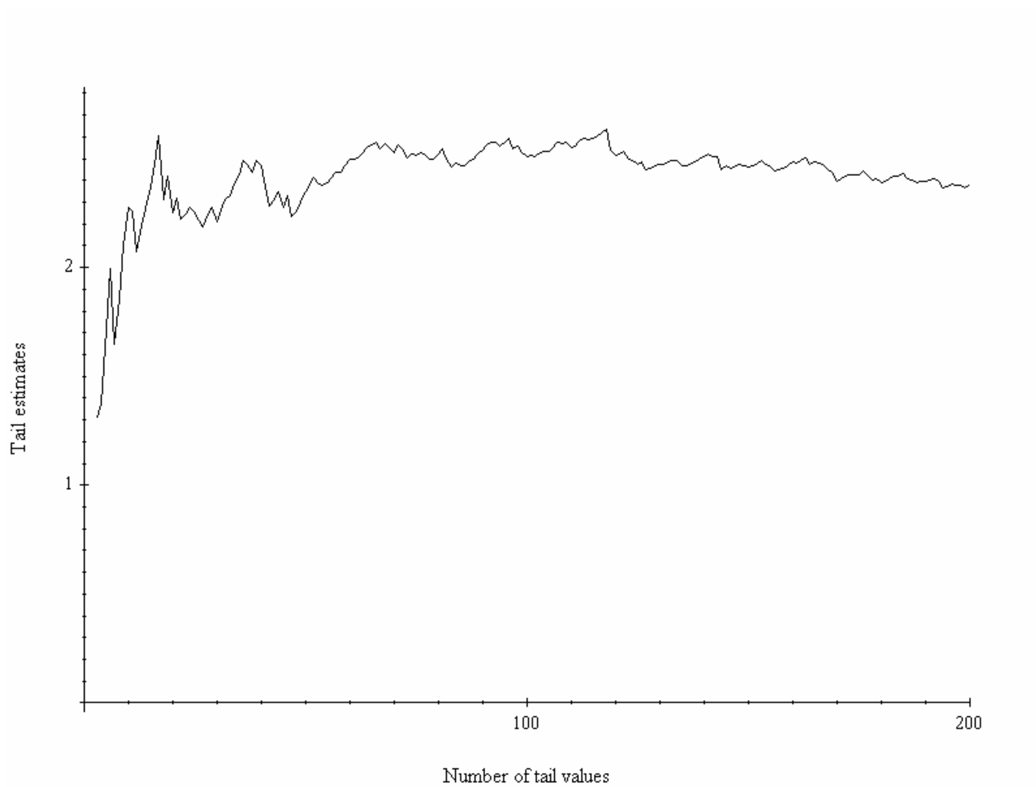

Figure 3. Hill plot for upper tail returns of Hong Kong index

Notes: the Hill estimates are reasonably constant using between 70 and 200 tail values and diverge considerably from using a very small number of tail returns.

Table 1. Summary statistics for daily returns series

Index	Mean	Std D	Min	Max	Iq R	Skew	Kurt	J-B
American								
NASDAQ	0.055	1.26	-14.00	9.96	0.98	-0.90	17.00	34643
S&P500	0.050	1.03	-22.83	8.71	0.92	-2.98	67.26	724413
DOW JONES	0.052	1.06	-25.64	9.67	0.97	-3.74	92.32	1397123
European								
UK	0.039	0.87	-11.91	5.70	0.95	-1.31	20.32	53380
GERMANY	0.041	1.22	-13.46	6.79	1.25	-0.78	11.69	13547
FRANCE	0.055	1.09	-9.89	7.97	1.13	-0.61	9.87	8476
ITALY	0.050	1.30	-8.44	8.40	1.34	-0.20	6.65	2344
HOLLAND	0.048	1.17	-12.78	11.18	1.09	-0.57	14.96	25106
Asian								
JAPAN	0.004	1.34	-16.14	12.43	1.24	-0.17	13.01	17453
HONG KONG	0.061	1.79	-40.54	17.25	1.49	-3.56	81.66	1084852
SINGAPORE	0.027	1.45	-29.19	15.48	1.22	-2.11	57.05	510688
THAILAND	0.015	1.71	-10.03	11.35	1.37	0.12	8.95	6177
INDONESIA	0.044	1.68	-22.53	40.31	0.72	3.97	110.03	2003473
MALAYSIA	0.019	1.71	-24.15	20.82	1.30	-0.26	34.60	173703

Notes: The mean, min and max values represent the average, lowest and highest returns respectively. The interquartile range (IqR) gives the spread between the 75^{th} and 25^{th} percentiles and Std D represents the standard deviation. These statistics are presented in percentages. The skewness (Skew) statistic is a measure of distribution asymmetry with symmetric returns having a value of zero. The kurtosis (Kurt) statistic measures the shape of a distribution vis-à-vis a normal distribution with a gaussian density function having a value of 3. Normality is formally examined with the Jarque-Bera (J-B) test with a critical value of 3.84. All skewness, kurtosis and normality coefficients are significant at the 5 percent level.

Table 2. GARCH (1, 1) model for daily returns series

	AR	α_0	α_1	β_1	AIC	BIC	Q(12)	$Q^2(12)$	t parameter
American									
NASDAQ	0.157	0.008	0.072	0.879	10748.500	10786.520	8.2950*	15.5700*	5.046
	[0.000]	[0.000]	[0.000]	[0.000]			[-0.762]	[-0.212	[-0.397]
S&P500	0.0151*	0.004	0.029	0.944	10288.240	10326.260	17.8700*	14.0400*	4.794
	[-0.161]	[0.000]	[0.000]	[0.000]			[-0.120]	[-0.298	[-0.346]
DOW JONES	0.0067*	0.006	0.029	0.940	10479.750	10517.770	14.6200*	10.6500*	4.745
	[-0.327]	[0.000]	[0.000]	[0.000]			[-0.263]	[-0.559	[-0.329]
European									
UK	0.098	0.011	0.059	0.904	9418.450	9456.470	22.250	8.0330*	9.038
	[0.000]	[0.000]	[0.000]	[0.000]			[-0.035]	[-0.783	[-0.688]
GERMANY	0.040	0.015	0.064	0.894	12154.230	12192.250	14.8000*	2.4060*	6.260
	[-0.007]	[0.000]	[0.000]	[0.000]			[-0.252]	[-0.999	[-0.428]
FRANCE	0.099	0.020	0.064	0.889	11472.930	11510.950	14.7900*	6.8190*	7.306
	[0.000]	[0.000]	[0.000]	[0.000]			[-0.253]	[-0.869	[-0.649]
ITALY	0.127	0.026	0.065	0.884	13026.720	13064.740	19.8500*	7.8360*	6.291
	[0.000]	[0.000]	[0.000]	[0.000]			[-0.070]	[-0.798	[-0.510]
HOLLAND	0.0116*	0.011	0.052	0.909	11385.860	11423.880	20.4400*	3.5180*	5.879
	[-0.227]	[0.000]	[0.000]	[0.000]			[-0.059]	[-0.991	[-0.405]
Asian									
JAPAN	-0.0015*	0.008	0.061	0.899	12635.580	12673.600	14.6300*	5.5860*	5.024
	[-0.462]	[0.000]	[0.000]	[0.000]			[-0.262]	[-0.936	[-0.363]
HONG KONG	0.067	0.039	0.060	0.870	14288.460	14326.480	20.5800*	262.000	84.602
	[0.000]	[0.000]	[0.000]	[0.000]			[-0.057]	[0.000	[-0.299]
SINGAPORE	0.206	0.054	0.116	0.756	12290.140	12328.160	24.070	6.2580*	4.740
	[0.000]	[0.000]	[0.000]	[0.000]			[-0.020]	[-0.903	[-0.287]
THAILAND	0.157	0.005	0.093	0.861	14021.050	14059.070	89.2600*	11.3200*	4.465
	[0.000]	[-0.002]	[0.000]	[0.000]			[0.000]	[-0.501	[-0.363]
INDONESIA	0.217	0.001	0.131	0.708	9412.660	9450.680	67.700	1.1370*	82.152
	[0.000]	[0.000]	[0.000]	[0.000]			[0.000]	[-1.000	[-0.101]
MALAYSIA	0.170	0.034	0.095	0.800	13227.100	13265.120	27.290	2.3640*	3.834
	[0.000]	[0.000]	[0.000]	[0.000]			[-0.007]	[-0.999	[-0.230]

Notes: An AR1-GARCH (1, 1) specification is fit to the daily returns series assuming a conditional t-distribution. The associated p-values are in parentheses based on Bollerslev-Wooldridge standard errors. Q(12) is a Ljung-Box test on the squared residuals. $Q^2(12)$ is a Ljung-Box test on the squared standardised residuals. * denotes insignificance at the 5% level. Feasability of the models is based on Akaike's (AIC) and Schwarz's (BIC) selection criteria. The estimated parameters from and the associated standard errors are in [].

Table 3. Hill estimates for daily returns series and associated quantiles

	Lower Tail				Upper Tail			
	m^-	$\gamma-$	$Q^-_{1/n}$	$Q^-_{1/2n}$	m^+	$\gamma+$	$Q^+_{1/n}$	$Q^+_{1/2n}$
American								
NASDAQ	121	2.56	16.39	21.49	155	2.61	14.12	18.42
		[2.07, 3.11]				[2.23, 3.16]		
S&P500	131	3.02	9.22	11.62	143	3.31	7.85	9.69
		[2.54, 3.64]				[2.91, 4.00]		
DOW JONES	121	2.56	9.88	12.55	155	2.61	7.17	8.72
		[2.13, 3.07]				[2.19, 3.07]		
European								
UK	132	2.61	9.75	12.72	143	3.50	6.00	7.31
		[2.18, 3.06]				[2.99, 4.01]		
GERMANY	131	2.91	12.39	15.73	145	3.09	10.26	12.84
		[2.45, 3.31]				[2.71, 3.75]		
FRANCE	130	3.01	10.35	13.03	139	3.56	7.63	9.28
		[2.52, 3.52]				[3.01, 4.12]		
ITALY	134	2.97	12.31	15.55	137	3.86	8.78	10.49
		[2.51, 3.43]				[3,28, 4.51]		
HOLLAND	132	2.50	14.97	19.74	148	2.78	11.30	14.50
		[2.15, 2.95]				[2.37, 4.37]		
Asian								
JAPAN	132	3.41	10.86	13.30	140	3.04	12.35	15.51
		[2.90, 4.21]				[2.60, 3.63]		
HONG KONG	132	2.49	21.37	28.24	143	2.85	16.17	20.63
		[2.08, 3.02]				[2.45, 3.31]		
SINGAPORE	141	2.43	17.56	23.36	142	2.66	14.92	19.36
		[2.04, 2.89]				[2.31, 3.15]		
THAILAND	133	2.64	19.99	26.00	139	2.67	20.40	26.45
		[2.34, 3.10]				[2.25, 3.09]		
INDONESIA	121	2.12	24.51	33.99	145	1.81	38.38	56.26
		[1.87, 2.52]				[1.58, 2.13]		
MALAYSIA	137	2.59	19.19	25.07	143	2.39	21.33	28.50
		[2.26, 3.18]				[2.09, 2.75]		

Notes: Hill tail estimates, γ, are calculated for each equity index based on Hall's (1990) bootstrap method to determine the optimal number of tail values, m. This approach minimises the asymptotic mean squared error of the Hill tail estimates. 95% confidence intervals for the tail estimates are given in []. For the given sample size n = 4174, percentage return quantiles, Q, are presented for in-sample, $Q_{1/n}$, and out-of-sample, $Q_{1/2n}$, probabilities. With a sample size of 4174 returns, the low probability levels for in-sample quantiles occur 0.024% of the time whereas for out-of-sample quantiles occur 0.012% of the time. The analysis is completed separately for lower and upper tails.

Table 4. Tail stability tests for daily returns series

Lower Tail

	S&P500	DOW JONES	UK	GERMANY	FRANCE	ITALY	HOLLAND	JAPAN	HONG KONG	SINGAPORE	THAILAND	INDONESIA	MALAYSIA
NASDAQ	-1.31	0.00	-0.15	-1.02	-1.28	-1.18	0.19	-2.25	0.22	0.42	-0.25	1.46	-0.09
S&P500		1.31	1.18	0.30	0.03	0.14	1.52	-0.98	1.55	1.77	1.09	2.75	1.25
DOW JONES			-0.15	-1.02	-1.28	-1.18	0.19	-2.25	0.22	0.42	-0.25	1.46	-0.09
UK				-0.88	-1.15	-1.05	0.35	-2.14	0.38	0.59	-0.09	1.64	0.06
GERMANY					-0.27	-0.17	1.23	-1.28	1.26	1.47	0.79	2.48	0.95
FRANCE						0.11	1.49	-1.01	1.52	1.74	1.06	2.72	1.22
ITALY							1.40	-1.12	1.43	1.65	0.96	2.65	1.12
HOLLAND								-2.47	0.03	0.23	-0.44	1.31	-0.29
JAPAN									2.50	2.72	2.05	3.65	2.21
HONG KONG										0.20	-0.48	1.28	-0.32
SINGAPORE											-0.68	1.10	-0.53
THAILAND												1.74	0.16
INDONESIA													-1.60

Upper Tail

	S&P500	DOW JONES	UK	GERMANY	FRANCE	ITALY	HOLLAND	JAPAN	HONG KONG	SINGAPORE	THAILAND	INDONESIA	MALAYSIA
NASDAQ	-2.02	0.00	-2.47	-1.45	-2.58	-3.20	-0.55	-1.30	-0.76	-0.76	-0.19	3.10	0.76
S&P500		2.02	-0.47	0.58	-0.61	-1.28	1.48	0.71	1.26	1.26	1.79	4.76	2.69
DOW JONES			-2.47	-1.45	-2.58	-3.20	-0.55	-1.30	-0.76	-0.76	-0.19	3.10	0.76
UK				1.05	-0.14	-0.82	1.94	1.18	1.72	1.72	2.24	5.14	3.13
GERMANY					-1.19	-1.84	0.90	0.14	0.69	0.69	1.23	4.30	2.15
FRANCE						-0.67	2.06	1.31	1.85	1.85	2.36	5.19	3.23
ITALY							2.69	1.96	2.48	2.48	2.97	5.66	3.81
HOLLAND								-0.76	-0.21	-0.21	0.34	3.55	1.28
JAPAN									0.54	0.54	1.08	4.13	2.00
HONG KONG										0.00	0.55	3.69	1.48
SINGAPORE											0.55	3.69	1.48
THAILAND												3.16	0.93
INDONESIA													-2.32

Notes: Tail stability tests for each index are calculated using Loretan and Phillips (1994) procedure described in the text. The critical value is 1.96.

Table 5: Tail probability estimates for daily returns series

	Lower Tail			Upper Tail		
	-10%	**-20%**	**-30%**	**10%**	**20%**	**30%**
American						
NASDAQ	0.2194	0.0372	0.0132	0.1542	0.0253	0.0088
S&P500	0.0472	0.0058	0.0017	0.0284	0.0029	0.0007
DOW JONES	0.1079	0.0183	0.0065	0.1004	0.0164	0.0057
European						
UK	0.0589	0.0096	0.0033	0.0106	0.0009	0.0002
GERMANY	0.1168	0.0155	0.0048	0.0681	0.0080	0.0023
FRANCE	0.0695	0.0086	0.0025	0.0237	0.0020	0.0005
ITALY	0.1157	0.0148	0.0044	0.0374	0.0026	0.0005
HOLLAND	0.1711	0.0302	0.0110	0.0913	0.0133	0.0043
Asian						
JAPAN	0.0836	0.0079	0.0020	0.1213	0.0147	0.0043
HONG KONG	0.4113	0.0732	0.0267	0.2530	0.0351	0.0110
SINGAPORE	0.2461	0.0457	0.0171	0.1812	0.0287	0.0098
THAILAND	0.4182	0.0671	0.0230	0.4257	0.0669	0.0227
INDONESIA	0.4249	0.0978	0.0414	0.7163	0.2043	0.0981
MALAYSIA	0.3440	0.0571	0.0200	0.3816	0.0728	0.0276

Notes: The values in this table represent the likelihood of extreme returns, for example -10%, on an annualised basis. These extreme returns would occur once every k (k = 1/p) years.