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# Can Age Discrimination Be Justified with a Lower Productivity of Older Workers?

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## Abstract

The connection between age and productivity is a widely discussed topic in the empirical literature. The present paper's aim is to contribute to the explanation of an apparent lower productivity of older individuals. If we introduce uncertainty about the future working conditions depending on present success, a decrease of productivity over the working life can be observed despite a constant a priori productivity.

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# 1 Introduction

Especially European countries are characterized by two aspects changing the age structure and the size of the population at the same time. Improvements in health care and several other reasons increase the life expectancy. Additionally, the fertility decreases. This has or may have severe consequences in all areas of life. Obviously, the pension system faces new challenges. The median voter is older. Structure and size of the health care system have to change. The demand structure in a society will adapt. And even environmental consequences can be expected.<sup>1</sup>

Theoretically, there are several impacts of an aging population on productivity. First, we have a direct impact due to influence of the population size on the number of workers and on the capital intensity in production. Second, we have two indirect effects. A longer life enforces a higher saving rate in younger years. But an older average household leads to a relatively decreased share of income spent for saving purposes.

If we concentrate on the labor market we have to state that European pension systems are in most cases organized as pay-as-you-go systems. This type of system is stressed usually by several factors. First, the number of pensioners increases directly as a consequence of the demographic changes mentioned above and as a result of regulating the labor market with the instrument of early retirement. This leads to a rejuvenation of the employees.<sup>2</sup> A problem arises: To stabilize the pension systems it is necessary to increase the age of retirement. But at the same time, older workers are discriminated in the labor market (Büsch, 2004).<sup>3</sup>

Two questions arise in this context. First: Can unjustifiable discrimination be explained? The main arguments follow two lines. The first was introduced by Becker (1957). Becker argues that persons prefer to deal with individuals of their own group defined by age, sex or race. Several commentaries have to be made concerning this theory. Firstly, this model can not explain discrimination itself. Secondly, in competitive markets discrimination should vanish in the long run. Thirdly, although this contradicts standard assumption of economic theory it is compatible with models of bounded rationality or transaction costs. The other line of arguments is associated with the model of statistical discrimination introduced by Aigner and Cain (1977).<sup>4</sup> If we assume limited information on

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<sup>1</sup>See Ono and Maeda (2001) for the last argument. Here, the impact depends on the risk aversion. If the risk aversion is small enough, aging increases the environmental quality.

<sup>2</sup>In Germany, retirement with 60 years of age after unemployment was possible until 1992. As a consequence, the retirement age decreased in OECD countries from 65 in 1965 to below 60 in 1995. At the same time we observe unemployment rates in the age group between 55 and 64 above average in Germany, whereas in the EU15, EU19 or in the OECD countries unemployment rates in this age group are always below the average. See Blöndal and Scarpetta (1999) and OECD (2008).

<sup>3</sup>We have to emphasize that the notion 'old' is not clearly defined. Tuckman and Lorge (1952) see a worker older than 45 as old, whereas Arrowsmith and McGoldrick (1996) would set the point at the age of 40 and Lee and Clemens (1985) at the age of 61. Büsch (2004) examines the interval between 27 and 53 and finds clear evidence for age discrimination.

<sup>4</sup>Arrow (1972, 1974), Phelps (1972) and Spence (1973) provided fundamentals for this

the real productivity of workers caused by difficulties in the measurement of productivity, then it could be rational to use group specific characteristics as a device. The unobservable individual attribute is replaced by the observable group-specific parameter value. Consequently, age discrimination is possible if it is assumed that the older worker has lower productivity. Additionally, measurement costs and variabilities can be different. Consequently, older worker can be discriminated if the test or the used attribute associate them *on average* with lower productivity. Again, the argument is not very convincing. This leads directly to the second question.

The second question is: Is discrimination justifiable since we have real differences in the productivity between age groups? It is a widespread stereotype that older worker are probably less productive. The arguments given in the literature are that the physical abilities decrease in general, cognitive abilities at least in some areas. Nevertheless, empirical evidence for age-dependent individual productivity is not unambiguous.<sup>5</sup> Skirbekk (2004) states that individual productivity starts to decrease with the age of 50. In a few cases, the empirical evidence that supports the so-called deficit model is derived using an investigation of the productivity or creativity in special fields. Since there is a certain agreement about decreasing physical abilities it is of interest to learn more about the change in areas where these characteristics are playing a minor role. Therefore, investigations are usually done in fields like arts and sciences. In general, Bayer and Dutton (1977) as well as Bratsberg et al. (2003) support this hypothesis. Levin and Stephan (1989, 1991) find a decline of creativity after investigating the fields of physics, earth sciences, physiology and biochemistry. Over (1982), observing the careers of psychologists, comes to the same conclusion. Weinberg and Galenson (2005) observe a decline of creativity after investigating the careers of Nobel laureates in economics. Oster and Hamermesh (1998) also chose their own colleagues as guinea pigs. Miller (1999) found a decline of creativity after following the careers of famous painters, musicians and writers - with the exception of female writers! In principle, the same result is derived by Kanazawa (2003), who also adds criminals to his sample. Korniotis and Kumar (2007) support the hypotheses after a look at the productivity of investment bankers.

In the following we want to argue that some of these results can be explained by the selection process inside the community of these groups.

## 2 The Model

The aim of this paper is to investigate the predication of a decrease of individual productivity during the professional life. We assume scientists work for three periods. Having in mind that the professional education of scientists (and other highly specialized persons) usually takes a bit more time this corresponds to approximately three decades of professional life. In the following we denote

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argument.

<sup>5</sup>See Börsch-Supan, Dützgin and Weiss (2005) for a discussion.

the decades with *I*, *II* and *III* for the first, second and third decade in the professional life of a cohort, respectively.<sup>6</sup> The productivity of a person within a decade is assumed to be constant. Two different productivity levels are possible. In general the persons have an average productivity  $l$ . But depending on individual characteristics the persons can reach an outstanding level of creativity  $h$ .<sup>7</sup> Persons of type  $\alpha$  remain on this level for the whole professional life. Persons of type  $\beta_1$  reach this level in the first two decades and have an average productivity in the third decade, whereas persons of type  $\beta_2$  start their professional life with average productivity and are characterized by high productivity in the decades *II* and *III*. And finally, persons of type  $\gamma_1$ ,  $\gamma_2$  and  $\gamma_3$  have an above-average productivity  $h$  only in the decades *I*, *II* and *III*, respectively. One could add a person of type  $\delta$  with average productivity of his or her complete professional life. But for simplicity we exclude this group completely since we entirely concentrate our interest on outstanding contributions in one's field that are also in the center of the empirical studies cited above.<sup>8</sup>

	decade		
type	<i>I</i>	<i>II</i>	<i>III</i>
$\alpha$	$h$	$h$	$h$
$\beta_1$	$h$	$h$	$l$
$\beta_2$	$l$	$h$	$h$
$\gamma_1$	$h$	$l$	$l$
$\gamma_2$	$l$	$h$	$l$
$\gamma_3$	$l$	$l$	$h$

A first look on the table suggests a peak of productivity in the second decade if we assume an equal size of each group consisting of individuals of types  $\alpha$ ,  $\beta$  and  $\gamma$ , respectively ( $N(\alpha) = N(\beta) = N(\gamma) = \frac{N}{3}$ ).<sup>9</sup> This result does not change if the group size varies between types  $\alpha$ ,  $\beta$  and  $\gamma$ . The peak remains in the second decade if there are more or less individuals of type  $\beta$  than of type  $\alpha$ . The same is true for all possible frequencies of occurrence of the basic types  $\alpha$ ,  $\beta$  and  $\gamma$ . But obviously the result changes if we have different frequencies of types  $\beta_1$ ,  $\beta_2$ ,  $\gamma_1$ ,  $\gamma_2$  and  $\gamma_3$ . A distribution of abilities  $N(\beta_1) > N(\beta_2)$  and/or  $N(\gamma_1) > N(\gamma_3)$  corresponds to a decreasing productivity with age inside a cohort. Particularly, with the given notation we can write this condition as:

$$\frac{N(\beta_1)}{N(\beta)} + \frac{N(\gamma_1)}{N(\gamma)} > \frac{N(\beta_2)}{N(\beta)} + \frac{N(\gamma_3)}{N(\gamma)}$$

If we exclude a significantly mortality risk during the working age we have to find other explanations. One reason may be the dependence of the group

<sup>6</sup>The first decade captures the age between 30 and 40, the second decade the years between 40 and 50 and the third decade the years after the 50th birthday. If the person is 60 years old he or she retires.

<sup>7</sup>Throughout the paper we assume a constant size of the population and of each group.

<sup>8</sup>In the following section we normalize  $h = 1$  and  $a = 0$ .

<sup>9</sup>If an individual of type  $\beta_1$  loses its high abilities with 45 years of age and if individuals of type  $\beta_2$  become more productive following their 45th birthday we have a constant average productivity of each cohort.

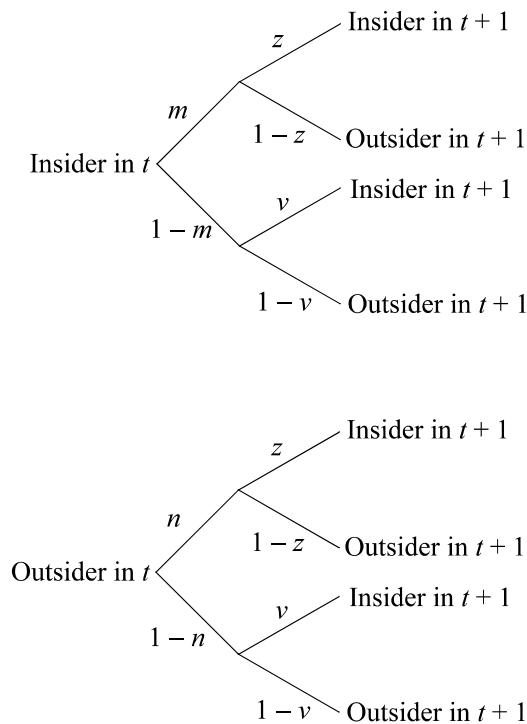


Figure 1: Probabilities and the change of the status of individuals.

affiliation at a certain point of time on the group affiliation in a previous point of time. If we assume that the belonging to a group is not exogenous but depends on endogenous variables, we have another possible explanation for decreasing average productivity over the life cycle.

For simplicity we assume that the ability to create an outstanding piece of work depends not only on individual characteristics but also on some "capital". This can be a laboratory that had to be used, a grant that is received or even the possibility to exchange ideas within the community we look at. In that case we observe a decreasing productivity in all cases where this capital is available mainly at the beginning of a career. Up to now the probability to have a smashing idea for a person of type  $\alpha$  within a certain period was equal to one. Now let us assume this is reduced to  $m$ , if the person belongs to a community, or to  $n$ , if the person is an outsider, with  $1 > m > n > 0$ . Furthermore let us add a selection mechanism. If the person was successful in period  $t$ , the probability to stay an insider is  $z$ . Without success, this probability is  $v$ . In order to make sense we have to postulate  $z > v$ . Figure (1) illustrates the change of the status of individuals.

If we assume that all persons of interest have completed their studies suc-

cessfully we can start at a point at which  $z \cdot N(\bullet)$  of each type of individuals are members of the scientific community. To simplify the notation in what follows we use capital letters for the number of individuals in each group with a superscript for the period and an asterisk for the number of the insiders.

## 2.1 "Forever Young" - Individuals of Life-long High Productivity

If we take up the cliché persons that are productive on a high level for their whole lifetime can be called "forever young".<sup>10</sup> The individuals of the  $\alpha$ -type inside the community will have success with a probability of  $m$  and can stay inside the community with a probability  $z$ . The same calculation can be made for all combinations of possibilities, ending with the number of individuals in each group at the beginning of the second decade being:

$$A^{II*} = [z \cdot (m \cdot z + (1 - m) \cdot v) + (1 - z) \cdot (n \cdot z + (1 - n) \cdot v)] \cdot A \quad (1)$$

$$A^{II} = \left[ \begin{array}{l} z \cdot (m \cdot (1 - z) + (1 - m) \cdot (1 - v)) \\ + (1 - z) \cdot (n \cdot (1 - z) + (1 - n) \cdot (1 - v)) \end{array} \right] \cdot A \quad (2)$$

If we assume that each individual makes one contribution per decade, we have an output of  $m \cdot A^{I*} + n \cdot A^I = [m \cdot z + n \cdot (1 - z)] \cdot A$  contributions from this group in this decade. We now want to calculate the output and the distribution of individuals in the second decade. At the beginning of the third decade there are the following groups of individuals of type  $\alpha$ :

$$A^{III*} = [m \cdot z + (1 - m) \cdot v] \cdot A^{II*} + [n \cdot z + (1 - n) \cdot v] \cdot A^{II} \quad (3)$$

$$A^{III} = [m \cdot (1 - z) + (1 - m) \cdot (1 - v)] \cdot A^{II*} + [n \cdot (1 - z) + (1 - n) \cdot (1 - v)] \cdot A^{II} \quad (4)$$

Output in the second and third decades is consequently:

$$m \cdot A^{II*} + n \cdot A^{II} = \left[ \begin{array}{l} m^2 \cdot z^2 + (m \cdot v + n^2 \cdot (1 - v)) \cdot z \cdot (1 - m) \\ + \left[ \begin{array}{l} (m \cdot v + n^2 \cdot (1 - v)) \cdot (1 - n) \\ + (2 \cdot m \cdot z + n \cdot (1 - z)) \cdot n \end{array} \right] \cdot (1 - z) \end{array} \right] \cdot A \quad (5)$$

$$m \cdot A^{III*} + n \cdot A^{III} = \left[ \begin{array}{l} -n^3 \cdot (v - z)^2 \cdot (z - 1) \\ + m \cdot \left( \begin{array}{l} m^2 \cdot z^3 + m \cdot v^2 \cdot (m \cdot z - 1) \\ + v \cdot (m \cdot z - 2 \cdot m^2 \cdot z^2 + 1) \end{array} \right) \\ + n^2 \cdot (v - z) \cdot \left( \begin{array}{l} m \cdot z \cdot (2 - 3 \cdot z) \\ + v \cdot (m \cdot (3 \cdot z - 2) - 1) + 1 \end{array} \right) \\ + n \cdot \left( \begin{array}{l} 1 + m \cdot z + m \cdot v^2 \cdot (m - 3 \cdot m \cdot z + 2) \\ + m^2 \cdot (z^2 - 3 \cdot z^3) \\ + v \cdot \left( \begin{array}{l} 2 \cdot m^2 \cdot z \cdot (3 \cdot z - 1) \\ - m \cdot (2 \cdot z + 1) - 1 \end{array} \right) \end{array} \right) \end{array} \right] \cdot A \quad (6)$$

<sup>10</sup>Here, we refer to Bob Dylan as an outstanding example for this group.

To find conditions for a decreasing output we can calculate  $(m \cdot A^{I*} + n \cdot A^I) - (m \cdot A^{II*} + n \cdot A^{II}) > 0$ . This leads to:

$$\begin{aligned}
0 &< \left[ \begin{array}{l} [m - m \cdot (m \cdot z + v \cdot (1 - m))] \cdot z \\ + \left[ n - \left( \begin{array}{l} m \cdot (n \cdot z + v \cdot (1 - n)) \\ + m \cdot n \cdot z + n^2 \cdot (1 - z) \end{array} \right) \right] \cdot (1 - z) \\ - [z \cdot (1 - m) + (1 - n) \cdot (1 - z)] \cdot n \cdot (1 - v) \end{array} \right] \cdot A \\
&= (m - n) \cdot (v - z) \cdot ((m - n) \cdot z + n - 1) \cdot A = \Omega_{A1} \cdot A \quad (7) \\
\Omega_{A1} &> 0 \Leftrightarrow m \cdot z < n \cdot (1 - z)
\end{aligned}$$

Obviously, a simple statement about the behavior of the system is not possible. Here, it would be interesting to know more about the output of this group over time. In principle, the group remains on the same productivity level in all three periods. Nevertheless, a short inspection of this expression reveals that a decreasing output is possible. We consider first a point with  $m = z = 1$ . Irrespective of the values of  $n$  and  $v$  this implies a certain success of the individuals in their effort if they are insiders and there is no uncertainty with respect to their status.<sup>11</sup> Evaluating the right-hand side of the expression above produces a zero; that implies a constant output.

$$\begin{aligned}
\frac{\partial \Omega_{A1}}{\partial m} &= (v - z) \cdot [2 \cdot z \cdot (m - n) + n - 1] \\
\frac{\partial \Omega_{A1}}{\partial m} \Big|_{m=z=1} &= (1 - n) \cdot (v - 1) \leq 0 \\
\frac{\partial \Omega_{A1}}{\partial n} &= (v - z) \cdot [(1 - 2 \cdot z) \cdot (m - n) - n + 1] \\
\frac{\partial \Omega_{A1}}{\partial n} \Big|_{m=z=1} &= 0 \\
\frac{\partial \Omega_{A1}}{\partial v} &= (m - n) \cdot (z \cdot (m - n) + n - 1) \\
\frac{\partial \Omega_{A1}}{\partial v} \Big|_{m=z=1} &= 0 \\
\frac{\partial \Omega_{A1}}{\partial z} &= (m - n)^2 \cdot (v - 2 \cdot z) - (m - n) \cdot (n - 1) \\
\frac{\partial \Omega_{A1}}{\partial z} \Big|_{m=z=1} &= (1 - n) \cdot [v + n \cdot (1 - v) - 1] \leq 0
\end{aligned}$$

A small deviation from the point  $m = z = 1$  results therefore in a decreasing output. If  $m, z < 1$  then a change of  $n$  and  $v$  results in a decreasing output if:

$$\begin{aligned}
0 &> [(m - n) - 2 \cdot z \cdot (m - n) - n + 1] \\
n &> \frac{1 - z \cdot m}{1 - z}
\end{aligned}$$

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<sup>11</sup>See equations (1) and (2).



But the first inequality is impossible given that  $m > n$  and  $n < 1$ . The second inequality is impossible as well since it would require values of  $n$  exceeding 1. Therefore, increasing values of  $n$  or  $v$  result in a rise of productivity and compensate partially the negative impact of decreasing values of  $m$  and  $z$ .

If we compare the second and third decade the output decreases furthermore. Here, the corresponding condition is  $(m \cdot A^{II*} + n \cdot A^{II}) - (m \cdot A^{III*} + n \cdot A^{III}) > 0$ . Similar to (7) we get:

$$\begin{aligned}\Omega_{A2} &= -(m-n)^2 \cdot (v-z)^2 \cdot ((m-n) \cdot z + n - 1) \\ \Omega_{A2} &> 0 \quad \Leftrightarrow \quad m \cdot z < n \cdot (1-z)\end{aligned}\tag{8}$$

Again we can start in a point with  $m = z = 1$  and analyze the behavior of the system:

$$\begin{aligned}\frac{\partial \Omega_{A2}}{\partial m} &= -(v-z)^2 \cdot (m-n) \cdot [3 \cdot (m-n) \cdot z + 2 \cdot n - 2] \\ \frac{\partial \Omega_{A2}}{\partial m} \Big|_{m=z=1} &= -(v-1)^2 \cdot (1-n)^2 \leq 0 \\ \frac{\partial \Omega_{A2}}{\partial n} &= -(v-z)^2 \cdot (m-n) \cdot [(m-n) \cdot (1-3 \cdot z) - 2 \cdot n + 2] \\ \frac{\partial \Omega_{A2}}{\partial n} \Big|_{m=z=1} &= 0 \\ \frac{\partial \Omega_{A2}}{\partial v} &= -2 \cdot (m-n)^2 \cdot (v-z) \cdot ((m-n) \cdot z + n - 1) \\ \frac{\partial \Omega_{A2}}{\partial v} \Big|_{m=z=1} &= 0 \\ \frac{\partial \Omega_{A2}}{\partial z} &= -(m-n)^3 \cdot (v-z)^2 \\ &\quad + 2 \cdot (m-n)^2 \cdot (v-z) \cdot ((m-n) \cdot z + n - 1) \\ \frac{\partial \Omega_{A2}}{\partial z} \Big|_{m=z=1} &= -(1-n)^3 \cdot (v-1)^2 \leq 0\end{aligned}$$

Consequently, we can apply the same arguments as we did in the the comparison of the output in the first and second decade: A small deviation from the point  $m = z = 1$  results in a decreasing output. Increasing values of  $n$  or  $v$  result in a rise of productivity and compensate partially the negative impact of decreasing values of  $m$  and  $z$ .

## 2.2 "Mid-life Crisis" - Changing Productivity Level in the Second Decade

Now we look at the individuals of the  $\beta$ -type. Here we have to distinguish between two subgroups. The first is highly productive in the first and second decades, the second in the second and third decades. Although both groups have two productive periods during their working life we will see immediately

that there is a mayor difference between them. We assume that the probabilities are equal to values used in the previous section.

The subgroup of individuals of type  $\beta_1$  starts with high a productivity. Consequently, a share of  $z$  will start as an insider in the first period. The results in the first and second decade are similar to those of the  $\alpha$ -group. However, they are unproductive in the last period. The subgroup of individuals of type  $\beta_2$  has a low productivity in the first period. Nevertheless, a share  $v$  becomes insider even in the first decade. Insiders and outsiders are now indistinguishable from  $\alpha$ -individuals and have the same probabilities for success and continuance in the group.

Subgroup  $B_1$  can be described as follows: The output in the first periods is equal to  $m \cdot B_1^{II*} + n \cdot B_1^I = [m \cdot z + n \cdot (1 - z)] \cdot B_1$ . In the second and third period, the distribution of individuals is given by:

$$B_1^{II*} = [z \cdot (m \cdot z + (1 - m) \cdot v) + (1 - z) \cdot (n \cdot z + (1 - n) \cdot v)] \cdot B_1 \quad (9)$$

$$B_1^{II} = \left[ \begin{array}{l} z \cdot (m \cdot (1 - z) + (1 - m) \cdot (1 - v)) \\ + (1 - z) \cdot (n \cdot (1 - z) + (1 - n) \cdot (1 - v)) \end{array} \right] \cdot B_1 \quad (10)$$

$$B_1^{III*} = [m \cdot z + (1 - m) \cdot v] \cdot B_1^{II*} + [n \cdot z + (1 - n) \cdot v] \cdot B_1^{II} \quad (11)$$

$$B_1^{III} = [m \cdot (1 - z) + (1 - m) \cdot (1 - v)] \cdot B_1^{II*} + [n \cdot (1 - z) + (1 - n) \cdot (1 - v)] \cdot B_1^{II} \quad (12)$$

Similar to (5) we can derive for the contribution of this subgroup in the second period:

$$m \cdot B_1^{II*} + n \cdot B_1^{II} = \left[ \begin{array}{l} m^2 \cdot z^2 + (m \cdot v + n^2 \cdot (1 - v)) \cdot z \cdot (1 - m) \\ + \left[ \begin{array}{l} (m \cdot v + n^2 \cdot (1 - v)) \cdot (1 - n) \\ + (2 \cdot m \cdot z + n \cdot (1 - z)) \cdot n \end{array} \right] \cdot (1 - z) \end{array} \right] \cdot B_1 \quad (13)$$

However, since this subgroup is "unproductive" in the third period we have:

$$m \cdot B_1^{III*} + n \cdot B_1^{III} = 0 \quad (14)$$

The change of output of this subgroup over time is obvious without a lengthy explanation. In the first two decades, the behavior is equivalent to the  $\alpha$ -group. Since the output in the third decade is zero, it decreases over the complete lapse of time.

As already mentioned, things are different with the second subgroup. Individuals in this group are unproductive in the first period. However, some of them are insiders in this period. But since they are unsuccessful they can not be distinguished from unsuccessful individuals of the  $\alpha$ -type. Therefore, with the beginning of the second period a share  $v$  of these persons become insiders. The individuals can be subdivided into insiders and outsiders according to:

$$B_2^{II*} = B_2^I = v \cdot B_2 \quad (15)$$

$$B_2^{II} = B_2^I = (1 - v) \cdot B_2 \quad (16)$$

The output of these individuals is now given by:

$$m \cdot B_2^{II*} + n \cdot B_2^{II} = (m \cdot v + n \cdot (1 - v)) \cdot B_2 \quad (17)$$

In the third period, the successful individuals of the subgroup stay inside the system with a probability of  $z$ . The unsuccessful share is again able to stay in the system with a probability of  $v$ . Consequently, we have:

$$B_2^{III*} = (z \cdot m + v \cdot (1 - m)) \cdot B_2^{II*} + (z \cdot n + v \cdot (1 - n)) \cdot B_2^{II} \quad (18)$$

$$\begin{aligned} &= [(z \cdot m + v \cdot (1 - m)) \cdot v + (z \cdot n + v \cdot (1 - n)) \cdot (1 - v)] \cdot B_2 \\ B_2^{III} &= ((1 - z) \cdot m + (1 - v) \cdot (1 - m)) \cdot B_2^{II*} \quad (19) \\ &\quad + ((1 - z) \cdot n + (1 - v) \cdot (1 - n)) \cdot B_2^{II} \\ &= \left[ \begin{array}{l} ((1 - z) \cdot m + (1 - v) \cdot (1 - m)) \cdot v \\ + ((1 - z) \cdot n + (1 - v) \cdot (1 - n)) \cdot (1 - v) \end{array} \right] \cdot B_2 \end{aligned}$$

Consequently, the output is given by:

$$m \cdot B_2^{III*} + n \cdot B_2^{III} = \left[ \begin{array}{l} ((1 - m) \cdot (1 - v) + m \cdot (1 - z)) \cdot v \\ + ((1 - n) \cdot (1 - v) + n \cdot (1 - z)) \cdot (1 - v) \end{array} \right] \cdot B_2 \quad (20)$$

The output in the third decade is bigger than in the second decade as long as:

$$\begin{aligned} 0 &< m \cdot B_2^{III*} + n \cdot B_2^{III} - m \cdot B_2^{II*} - n \cdot B_2^{II} \\ &= (m - n) \cdot (m \cdot v + n \cdot (1 - v)) \cdot (z - v) \end{aligned}$$

As expected, output of this subgroup increases as long as the probability to stay inside the community is higher after a successful period. We can now compare the output of this subgroup with the output of the  $\alpha$ -type individuals. In the second period, the output of the  $\alpha$ -group is relatively higher if:  $\frac{m \cdot A^{II*} + n \cdot A^{II}}{A} > (m \cdot v + n \cdot (1 - v))$ . This can be reduced to:

$$(m - n) \cdot (z - v) \cdot (m \cdot z + n \cdot (1 - z)) > 0$$

This inequality condition is fulfilled as long as  $m > n$  and  $z > v$ . In the third decade, the comparison reveals a higher output of the  $\alpha$ -group if:

$$\begin{aligned} 0 &< m \cdot A^{III*} + n \cdot A^{III} - m \cdot B_2^{III*} - n \cdot B_2^{III} \\ &= (m - n)^2 \cdot (v - z)^2 \cdot (m \cdot z + n \cdot (1 - z)) \end{aligned}$$

Again, the inequality condition is fulfilled. The output of the subgroup increases from zero to a positive value in the second decade and increases thereafter if the third decade approaches. But in all periods, the output per capita is below the output of individuals of type  $\alpha$ .

### 2.3 "Once in a Lifetime" - Individuals Productive for Only One Decade

Now what about the  $\gamma$ -type individuals? Similar to the  $\beta$ -individuals they start either productive with a probability of  $z$  for being insiders or unproductive with a probability of  $v$  being insiders. In later periods, productive individuals remain insider with a probability of  $z$  but become nevertheless unproductive.

For the subgroup  $G_1$  consisting of individuals of type  $\gamma_1$  we have an output similar to that of individuals of the  $\alpha$ -group and the  $\beta_1$ -subgroup. However, in the second and third decades they are unproductive. It follows directly:

$$G_1^{I*} = z \cdot G_1 \quad (21)$$

$$G_1^I = (1 - z) \cdot G_1 \quad (22)$$

$$\begin{aligned} G_1^{II*} &= (m \cdot z + (1 - m) \cdot v) \cdot G_1^{I*} + (n \cdot z + (1 - n) \cdot v) \cdot G_1^I \quad (23) \\ &= [(m \cdot z + (1 - m) \cdot v) \cdot z + (n \cdot z + (1 - n) \cdot v) \cdot (1 - z)] \cdot G_1 \end{aligned}$$

$$\begin{aligned} G_1^{II} &= (m \cdot (1 - z) + (1 - m) \cdot (1 - v)) \cdot G_1^{I*} \quad (24) \\ &\quad + (n \cdot (1 - z) + (1 - n) \cdot (1 - v)) \cdot G_1^I \\ &= \left[ \begin{array}{l} (m \cdot (1 - z) + (1 - m) \cdot (1 - v)) \cdot z \\ + (n \cdot (1 - z) + (1 - n) \cdot (1 - v)) \cdot (1 - z) \end{array} \right] \cdot G_1 \end{aligned}$$

$$G_1^{III*} = v \cdot G_1 \quad (25)$$

$$G_1^{III} = (1 - v) \cdot G_1 \quad (26)$$

The output is given by:

$$m \cdot G_1^{I*} + n \cdot G_1^I = (m \cdot z + n \cdot (1 - z)) \cdot G_1 \quad (27)$$

$$m \cdot G_1^{II*} + n \cdot G_1^{II} = 0 \quad (28)$$

$$m \cdot G_1^{III*} + n \cdot G_1^{III} = 0 \quad (29)$$

Apparently, in the first decade the output of this subgroup is equivalent to the output of subgroups  $A$  and  $B_1$ . However, in the second and third periods it drops to zero.

Subgroup  $G_2$  is unproductive in the first and third decades and highly productive in the second decade. Consequently, we can write:

$$G_2^{I*} = v \cdot G_2 \quad (30)$$

$$G_2^I = (1 - v) \cdot G_2 \quad (31)$$

$$G_2^{II*} = v \cdot G_2 \quad (32)$$

$$G_2^{II} = (1 - v) \cdot G_2 \quad (33)$$

$$\begin{aligned} G_2^{III*} &= (m \cdot z + (1 - m) \cdot v) \cdot G_1^{II*} + (n \cdot z + (1 - n) \cdot v) \cdot G_1^{II} \quad (34) \\ &= ((m \cdot z + (1 - m) \cdot v) \cdot v + (n \cdot z + (1 - n) \cdot v) \cdot (1 - v)) \cdot G_2 \end{aligned}$$

$$\begin{aligned} G_2^{III} &= (m \cdot (1 - z) + (1 - m) \cdot (1 - v)) \cdot G_2^{II*} \quad (35) \\ &\quad + (n \cdot (1 - z) + (1 - n) \cdot (1 - v)) \cdot G_2^{II} \\ &= \left[ \begin{array}{l} (m \cdot (1 - z) + (1 - m) \cdot (1 - v)) \cdot v \\ + (n \cdot (1 - z) + (1 - n) \cdot (1 - v)) \cdot (1 - v) \end{array} \right] \cdot G_2 \end{aligned}$$

The output of the subgroup is given by:

$$m \cdot G_2^{I*} + n \cdot G_2^I = 0 \quad (36)$$

$$m \cdot G_2^{II*} + n \cdot G_2^{II} = (m \cdot v + n \cdot (1 - v)) \cdot G_2 \quad (37)$$

$$m \cdot G_2^{III*} + n \cdot G_2^{III} = 0 \quad (38)$$

The change of output over time needs no further explanation. However, it should be mentioned that resulting from the relation  $z > v$  the output is relatively smaller than that of the subgroup  $G_1$  in the first decade despite an equal productivity.

To complete the picture we have to look at subgroup  $G_3$ , the group of late bloomers. The individuals in this group are relatively unproductive in the first and second decades and become productive in the last one. Therefore, we can write:

$$G_3^{I*} = v \cdot G_3 \quad (39)$$

$$G_3^I = (1 - v) \cdot G_3 \quad (40)$$

$$G_3^{II*} = v \cdot G_3 \quad (41)$$

$$G_3^{II} = (1 - v) \cdot G_3 \quad (42)$$

$$G_3^{III*} = v \cdot G_3 \quad (43)$$

$$G_3^{III} = (1 - v) \cdot G_3 \quad (44)$$

Accordingly, we can find expressions for the output:

$$m \cdot G_3^{I*} + n \cdot G_3^I = 0 \quad (45)$$

$$m \cdot G_3^{II*} + n \cdot G_3^{II} = 0 \quad (46)$$

$$m \cdot G_3^{III*} + n \cdot G_3^{III} = (m \cdot v + n \cdot (1 - v)) \cdot G_3 \quad (47)$$

The output in the third decade is equal to the output of subgroup  $G_2$  in the second decade.

### 3 "All Together Now" - The Complete Economy

We can now merge all components to get a model of the complete economy. Let  $y$  be the output per capita. It follows for the average individual:

$$\bar{y} = \frac{1}{N} \cdot \left[ \begin{array}{l} (y_A^I + y_A^{II} + y_A^{III}) \cdot A \\ + (y_{B1}^I + y_{B1}^{II} + y_{B1}^{III}) \cdot B_1 + (y_{B2}^I + y_{B2}^{II} + y_{B2}^{III}) \cdot B_2 \\ + (y_{G1}^I + y_{G1}^{II} + y_{G1}^{III}) \cdot G_1 + (y_{G2}^I + y_{G2}^{II} + y_{G2}^{III}) \cdot G_2 \\ + (y_{G3}^I + y_{G3}^{II} + y_{G3}^{III}) \cdot G_3 \end{array} \right] \quad (48)$$

To simplify the notation we use  $y_X^I + y_X^{II} + y_X^{III} = y_X$  for the total output of a subgroup and  $\frac{X_{(\bullet)}}{N} = x_{(\bullet)}$  for the weight of a subgroup as share in the total population.<sup>12</sup> Furthermore,  $Y^I$  denotes the output of the young generation,

<sup>12</sup>Note that without population growth we can skip the time indices.

$Y^{II}$  the output of the middle-aged generation and  $Y^{III}$  the output of the old generation. This results in:

$$\bar{y} = y_A \cdot a + y_{B1} \cdot b_1 + y_{B2} \cdot b_2 + y_{G1} \cdot g_1 + y_{G2} \cdot g_2 + y_{G3} \cdot g_3$$

The average output per head decreases over the life span if  $Y^I > Y^{II} > Y^{III}$ :

$$\begin{aligned} & y_A^I \cdot a + y_{B1}^I \cdot b_1 + y_{B2}^I \cdot b_2 + y_{G1}^I \cdot g_1 + y_{G2}^I \cdot g_2 + y_{G3}^I \cdot g_3 \\ \geq & y_A^{II} \cdot a + y_{B1}^{II} \cdot b_1 + y_{B2}^{II} \cdot b_2 + y_{G1}^{II} \cdot g_1 + y_{G2}^{II} \cdot g_2 + y_{G3}^{II} \cdot g_3 \\ \geq & y_A^{III} \cdot a + y_{B1}^{III} \cdot b_1 + y_{B2}^{III} \cdot b_2 + y_{G1}^{III} \cdot g_1 + y_{G2}^{III} \cdot g_2 + y_{G3}^{III} \cdot g_3 \end{aligned} \quad (49)$$

Using the results of the previous section allows us to simplify this expression:

$$\begin{aligned} y_{B2}^I &= y_{B1}^{III} = y_{G2}^I = y_{G3}^I = y_{G1}^{II} = y_{G3}^{II} = y_{G1}^{III} = y_{G2}^{III} = 0 \\ y_A^I &= y_{B1}^I = y_{G1}^I \\ y_A^{II} &= y_{B1}^{II} \\ y_{B2}^{II} &= y_{G2}^{II} \\ y_{G1}^I &> y_{G2}^{II} = y_{G3}^{III} \end{aligned}$$

We can simplify Condition (49) and write:

$$\begin{aligned} & y_A^I \cdot (a + b_1 + g_1) \\ \geq & y_A^{II} \cdot (a + b_1) + y_{B2}^{II} \cdot (b_2 + g_2) \\ \geq & y_A^{III} \cdot a + y_{B2}^{III} \cdot b_2 + y_{B2}^{II} \cdot g_3 \end{aligned} \quad (50)$$

The behavior of the system depends on the parameter values. As we did in Section 2.1 we can calculate the derivatives of the differences with respect to the parameters  $m$ ,  $n$ ,  $v$  and  $z$  and evaluate the resulting expressions at the point  $m = z = 1$ .<sup>13</sup> The probability of success of "insiders" has no clear-cut effect on the change of productivity.

$$\left. \frac{\partial (Y^I - Y^{II})}{\partial m} \right|_{m=z=1} = g_1 - (a + b_1) \cdot (1 - n) \cdot (1 - v) - (b_2 + g_2) \cdot v$$

If the subgroup of individuals of type  $\gamma_1$  is rather large, a decrease of the probability of success of insiders  $m$  results in a smaller gap in the productivity of per capita output of individuals in the first and second decades of their working life. If this subgroup is small enough, a decreasing  $m$  results in a widening of the gap.

$$\begin{aligned} \left. \frac{\partial (Y^I - Y^{II})}{\partial n} \right|_{m=z=1} &= -(b_2 + g_2) \cdot (1 - v) \leq 0 \\ \left. \frac{\partial (Y^I - Y^{II})}{\partial v} \right|_{m=z=1} &= -(b_2 + g_2) \cdot (1 - n) \leq 0 \end{aligned}$$

<sup>13</sup>See Appendix for the derivatives.

The effects of a change of the probability of success for individuals outside the community and of a change of the probability of unsuccessful insiders to stay inside the community are very similar: Both effects result in a smaller gap between the productivities of individuals in their first and second decades of their working life. The probability of successful individuals being insiders in the following period is also not unambiguous.

$$\left. \frac{\partial (Y^I - Y^{II})}{\partial z} \right|_{m=z=1} = (a + b_1 + g_1) \cdot (1 - n) - (a + b_1) \cdot (2 - n^2 \cdot (v - 1) - v + n \cdot (2 \cdot v - 3))$$

If we look at the change of productivity at the boundary of the second and third decades of working life the derivatives are even more complicated to interpret:

$$\left. \frac{\partial (Y^{II} - Y^{III})}{\partial m} \right|_{m=z=1} = -b_2 \cdot ((v - 1) \cdot (n \cdot (2 \cdot v - 1) - 2 \cdot v)) + (a + b_1) \cdot (2 + n \cdot (v - 1) - v) + v \cdot (g_2 - g_3) - a \cdot \left( \begin{array}{c} 3 \cdot (1 - v) + n^2 \cdot (v - 1)^2 + v^2 \\ + n \cdot (5 \cdot v - 2 \cdot v^2 - 3) \end{array} \right)$$

$$\left. \frac{\partial (Y^{II} - Y^{III})}{\partial n} \right|_{m=z=1} = (1 - v) \cdot \left( \begin{array}{c} (b_2 + g_2) - b_2 \cdot (n \cdot (v - 1) - v) \\ - (g_3 + b_2 \cdot (2 - n - (1 - n) \cdot v)) \end{array} \right)$$

$$\left. \frac{\partial (Y^{II} - Y^{III})}{\partial v} \right|_{m=z=1} = (1 - n) \cdot \left( \begin{array}{c} (b_2 + g_2) - b_2 \cdot (n \cdot (v - 1) - v) \\ - (g_3 + b_2 \cdot (2 - n - (1 - n) \cdot v)) \end{array} \right)$$

$$\left. \frac{\partial (Y^{II} - Y^{III})}{\partial z} \right|_{m=z=1} = b_2 \cdot (1 - n) \cdot (n \cdot (v - 1) - v) + (a + b_1) \cdot (2 - n^2 \cdot (v - 1) - v + n \cdot (2 \cdot v - 3)) - a \cdot \left( \begin{array}{c} (v - 1) \cdot (n^2 \cdot (3 \cdot v - 4) - n^3 \cdot (v - 1) - 3) \\ + n \cdot (8 \cdot v - 3 \cdot v^2 - 6) + v^2 \end{array} \right)$$

To get an idea about the likely behavior of the system we have to make an assumption about the pattern of distribution of individuals. It seems to be plausible that we have more individuals of the  $\gamma$ -type than of type  $\beta$  and more of type  $\beta$  than of the most productive  $\alpha$ -individuals. Consequently, we want to assume that:<sup>14</sup>

$$N(\alpha) = N(\beta_1) = N(\beta_2) = N(\gamma_1) = N(\gamma_2) = N(\gamma_3)$$

Starting with an extreme scenario with  $m = z = 1$  and  $n = v = 0$ , the differences are positive ( $Y^I - Y^{II} = Y^{II} - Y^{III} > 0$ ). However, the derivatives

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<sup>14</sup>This assumption implies  $\frac{N}{6}$  individuals being of type  $\alpha$ ,  $\frac{N}{3}$  of type  $\beta$  and  $\frac{N}{2}$  of type  $\gamma$ .

have different signs. Whereas a decrease of all variables increases the gap between the productivities per capita of the first and second decades, a decrease of  $m$  and  $n$  make this gap smaller but a decrease of  $v$  and  $z$  enlarges this gap. The same results can be found for  $m = z = 1$  and  $n = v = 0.1$ . In the more likely case  $m = z = 0.9$  and  $n = v = 0.1$  the differences are positive ( $Y^I - Y^{II} = Y^{II} - Y^{III} > 0$ ). Here, the derivatives with respect to  $m$  and  $z$  are positive and the derivatives with respect to  $n$  and  $v$  negative. An increase in the probabilities of success for insiders and unchanged status for successful individuals widens the gap, whereas an increase of the probabilities of success for outsiders and of unsuccessful individuals becoming insiders scale down the gap. If  $m$  and  $z$  become smaller, the difference between productivity in the first and second decades decreases and eventually turns out to be negative.<sup>15</sup> Additionally, the derivatives for the second difference with respect to  $n$  and  $v$  are positive, indicating an increasing gap with increasing probabilities compared to the second and third decades. Another extreme scenario is the perfect randomization of success and status-keeping between groups characterized by  $m = n = v = z = 0.5$ . In this case, productivity in the first decade is smaller than in the second decade, but in the second higher than in the third. This corresponds to the result for the case without uncertainty about status and success discussed above. The derivatives indicate that a change of  $v$  and  $z$  has no influence on productivity at all. However, a decrease of  $m$  and  $n$  increases the gap between the productivities in the first and second decades but decreases this gap between productivities in the second and third decades. Parameter values with  $m < n$  and  $v > z$  seem to be implausible. However, if we calculate the resulting derivatives we find a decreasing effect of increasing values of  $m$  and  $n$  and an increasing effect of increasing values of  $v$  and  $z$  for the first two periods. With respect to the second and third decades the effect of increasing values of  $m$  and  $n$  is positive but of  $v$  and  $z$  negative.<sup>16</sup> The difference between productivities depends on the parameter values; for the first two periods it can be positive or negative whereas for the last two periods it is a positive difference indicating a decrease in productivity.

## 4 Summary

In the literature, there is a great deal of discussion about the interaction between age and productivity. Especially the question of age discrimination is still unsolved. Empirical observations lead to the conclusion that productivity can decrease with age due to diminishing physical capabilities and decreasing flexibility that hinders the adaptation to new or changed requirements in one's own area of work. Contrary to these findings the increase of social competence and longtime working experience can result in increasing productivity.

The aim of the present paper is to explain an apparent lower productivity of

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<sup>15</sup>As an example we can calculate the values for  $m = z = 0.7$  and  $n = v = 0.25$ .

<sup>16</sup>As an example we used the values  $m = z = 0.25$  and  $n = v = 0.75$  as well as  $m = z = 0.2$  and  $n = v = 0.8$ .



older worker with a selection process inside the world of work. In a model with different types of workers we introduce uncertainty about the working conditions in the following decade depending on the success in the present period. Individuals with high productivity have a higher probability to enjoy good working conditions in the following period than individuals with low productivity today. For reasonable parameter values we can observe mainly patterns that allow to explain a decreasing productivity of individuals over the life cycle despite an a priori productivity that would result in a peak of productivity in the middle of lifetime. Consequently, the empirical research discussed above should be reconsidered with a special attention to the inherent selection mechanisms that give older individuals a hard time in their effort to be as productive as their younger colleagues.

Several expansions of the model are imaginable. Let us assume that productivity depends on the worker's effort. Hence, incentives play a role in explaining this effort. A temporally unrestricted employment lowers the incentives to be productive, whereas annually renewed contracts create higher incentives. However, if payment is low and work demanding, restricted contracts decrease the incentives for a high-productive individual to enter the scientific community at all. Additionally, this effort is lower if the time the individual will stay inside the scientific community becomes shorter since the return on investments in specific human capital decreases. But this is left for future research.

## 5 Appendix

The output per capita derived in Section 2 can be expressed in the following way:

$$\begin{aligned}
 y_A^I &= m \cdot z + n \cdot (1 - z) \\
 y_A^{II} &= \left[ \begin{array}{l} m^2 \cdot z^2 + (m \cdot v + n^2 \cdot (1 - v)) \cdot z \cdot (1 - m) \\ + \left[ \begin{array}{l} (m \cdot v + n^2 \cdot (1 - v)) \cdot (1 - n) \\ + (2 \cdot m \cdot z + n \cdot (1 - z)) \cdot n \end{array} \right] \cdot (1 - z) \end{array} \right] \\
 y_A^{III} &= \left[ \begin{array}{l} -n^3 \cdot (v - z)^2 \cdot (z - 1) \\ + m \cdot \left( \begin{array}{l} m^2 \cdot z^3 + m \cdot v^2 \cdot (m \cdot z - 1) \\ + v \cdot (m \cdot z - 2 \cdot m^2 \cdot z^2 + 1) \end{array} \right) \\ + n^2 \cdot (v - z) \cdot \left( \begin{array}{l} m \cdot z \cdot (2 - 3 \cdot z) \\ + v \cdot (m \cdot (3 \cdot z - 2) - 1) + 1 \end{array} \right) \\ + n \cdot \left( \begin{array}{l} 1 + m \cdot z + m \cdot v^2 \cdot (m - 3 \cdot m \cdot z + 2) \\ + m^2 \cdot (z^2 - 3 \cdot z^3) \\ + v \cdot \left( \begin{array}{l} 2 \cdot m^2 \cdot z \cdot (3 \cdot z - 1) \\ - m \cdot (2 \cdot z + 1) - 1 \end{array} \right) \end{array} \right) \end{array} \right]
 \end{aligned}$$

$$\begin{aligned}
 y_{B2}^{II} &= m \cdot v + n \cdot (1 - v) \\
 y_{B2}^{III} &= \left[ \begin{array}{l} ((1 - m) \cdot (1 - v) + m \cdot (1 - z)) \cdot v \\ + ((1 - n) \cdot (1 - v) + n \cdot (1 - z)) \cdot (1 - v) \end{array} \right]
 \end{aligned}$$

The output of the generations is now:

$$\begin{aligned}
 Y^I &= (m \cdot z + n \cdot (1 - z)) \cdot (a + b_1 + g_1) \\
 Y^{II} &= \left( \begin{array}{l} n^2 \cdot (v - z) \cdot (1 - z) + m \cdot (v + m \cdot z \cdot (z - v)) \\ + n \cdot (1 + m \cdot z \cdot (1 - 2 \cdot z) + v \cdot (2 \cdot m \cdot z - m - 1)) \end{array} \right) \cdot (a + b_1) \\
 &\quad + (m \cdot v + n \cdot (1 - v)) \cdot (b_2 + g_2) \\
 Y^{III} &= (m \cdot v + n \cdot (1 - v)) \cdot (g_3 + b_2 \cdot (1 + (m - n) \cdot (z - v))) \\
 &\quad + a \cdot \left[ \begin{array}{l} n^3 \cdot (v - z)^2 \cdot (1 - z) \\ + n^2 \cdot (v - z) \cdot \left( \begin{array}{l} 1 + m \cdot z \cdot (2 - 3 \cdot z) \\ + v \cdot (m \cdot (3 \cdot z - 2) - 1) \end{array} \right) \\ + n \cdot \left( \begin{array}{l} 1 + m \cdot z + m \cdot v^2 \cdot (2 + m - 3 \cdot m \cdot z) \\ + m^2 \cdot (z^2 - 3 \cdot z^3) \\ + v \cdot (2 \cdot m^2 \cdot z \cdot (3 \cdot z - 1) - m \cdot (2 \cdot z + 1) - 1) \end{array} \right) \\ + m \cdot \left( \begin{array}{l} m^2 \cdot z^3 + m \cdot v^2 \cdot (m \cdot z - 1) \\ + v \cdot (1 + m \cdot z - 2 \cdot m^2 \cdot z^2) \end{array} \right) \end{array} \right]
 \end{aligned}$$

The average output per capita decreases over lifetime if the following differences are positive:

$$\begin{aligned}
Y^I - Y^{II} &= (a + b_1 + g_1) \cdot (m \cdot z + n \cdot (1 - z)) \\
&\quad - (b_2 + g_2) \cdot (m \cdot v + n \cdot (1 - v)) \\
&\quad - (a + b_1) \cdot \left( \begin{array}{c} n^2 \cdot (v - z) \cdot (1 - z) \\ + m \cdot (v + m \cdot z \cdot (z - v)) \\ + n \cdot \left( \begin{array}{c} 1 + m \cdot z \cdot (1 - 2 \cdot z) \\ + v \cdot (2 \cdot m \cdot z - m - 1) \end{array} \right) \end{array} \right) \\
Y^{II} - Y^{III} &= (b_2 + g_2) \cdot (m \cdot v + n \cdot (1 - v)) \\
&\quad + (n \cdot (v - 1) - m \cdot v) \cdot (g_3 + b_2 \cdot (1 + (m - n) \cdot (z - v))) \\
&\quad + (a + b_1) \cdot \left( \begin{array}{c} -n^2 \cdot (v - z) \cdot (z - 1) + m \cdot (v + m \cdot z \cdot (z - v)) \\ + n \cdot (m \cdot ((1 - 2 \cdot z) \cdot (z - v)) - v + 1) \end{array} \right) \\
&\quad - a \cdot \left[ \begin{array}{c} m \cdot \left( \begin{array}{c} m^2 \cdot z^3 + m \cdot v^2 \cdot (m \cdot z - 1) \\ + v \cdot (1 + m \cdot z - 2 \cdot m^2 \cdot z^2) \end{array} \right) \\ - n^3 \cdot (v - z)^2 \cdot (z - 1) \\ + n^2 \cdot (v - z) \cdot \left( \begin{array}{c} 1 + m \cdot z \cdot (2 - 3 \cdot z) \\ + v \cdot (m \cdot (3 \cdot z - 2) - 1) \end{array} \right) \\ + n \cdot \left( \begin{array}{c} m \cdot \left( \begin{array}{c} m \cdot (z^2 - 3 \cdot z^3) + z \\ + v^2 \cdot (2 + m - 3 \cdot m \cdot z) \end{array} \right) \\ + v \cdot \left( \begin{array}{c} 2 \cdot m^2 \cdot z \cdot (3 \cdot z - 1) \\ - m \cdot (1 + 2 \cdot z) - 1 \end{array} \right) + 1 \end{array} \right) \end{array} \right]
\end{aligned}$$

$$\begin{aligned}
Y^I - Y^{III} &= (a + b_1 + g_1) \cdot (m \cdot v + n \cdot (1 - v)) \\
&\quad + (n \cdot (v - 1) - m \cdot v) \cdot (g_3 + b_2 \cdot (1 + (m - n) \cdot (z - v))) \\
&\quad - a \cdot \left[ \begin{array}{c} m \cdot \left( \begin{array}{c} m^2 \cdot z^3 + m \cdot v^2 \cdot (m \cdot z - 1) \\ + v \cdot (1 + m \cdot z - 2 \cdot m^2 \cdot z^2) \end{array} \right) \\ - n^3 \cdot (v - z)^2 \cdot (z - 1) \\ + n^2 \cdot (v - z) \cdot \left( \begin{array}{c} 1 + m \cdot z \cdot (2 - 3 \cdot z) \\ + v \cdot (m \cdot (3 \cdot z - 2) - 1) \end{array} \right) \\ + n \cdot \left( \begin{array}{c} m^2 \cdot (z^2 - 3 \cdot z^3) + m \cdot z \\ + m \cdot v^2 \cdot (2 + m - 3 \cdot m \cdot z) \\ + v \cdot \left( \begin{array}{c} 2 \cdot m^2 \cdot z \cdot (3 \cdot z - 1) \\ - m \cdot (1 + 2 \cdot z) - 1 \end{array} \right) + 1 \end{array} \right) \end{array} \right]
\end{aligned}$$

Now we can calculate the derivatives of the differences with respect to the

parameters  $m, n, v$  and  $z$ .

$$\begin{aligned}
\frac{\partial (Y^I - Y^{II})}{\partial m} &= -(b_2 + g_2) \cdot v + (a + b_1 + g_1) \cdot z \\
&\quad - (a + b_1) \cdot \left( \frac{z \cdot n + v \cdot (1 - n)}{+2 \cdot z \cdot (z - v) \cdot (m - n)} \right) \\
\frac{\partial (Y^I - Y^{II})}{\partial n} &= (b_2 + g_2) \cdot (v - 1) - (a + b_1 + g_1) \cdot (z - 1) \\
&\quad - (a + b_1) \cdot \left( \frac{2 \cdot (v - z) \cdot (z \cdot (m - n) + n)}{+m \cdot z \cdot (1 - v) - v + 1} \right) \\
\frac{\partial (Y^I - Y^{II})}{\partial v} &= -(m - n) \cdot ((a + b_1) \cdot (1 - (m - n) \cdot z - n) + b_2 + g_2) \\
\frac{\partial (Y^I - Y^{II})}{\partial z} &= (m - n) \cdot (g_1 + (a + b_1) \cdot (1 + (m - n) \cdot (v - 2 \cdot z) - n)) \\
\\
\frac{\partial (Y^{II} - Y^{III})}{\partial m} &= (g_2 - g_3) \cdot v - b_2 \cdot (z - v) \cdot (2 \cdot v \cdot (m - n) + n) \\
&\quad + (a + b_1) \cdot ((z - v) \cdot (2 \cdot z \cdot (m - n) + n) + v) \\
&\quad - a \cdot \left( \begin{aligned} &(m - n) \cdot \left( v^2 \cdot \left( \frac{3 \cdot z \cdot (m - n)}{+2 \cdot (n - 1)} \right) + 2 \cdot v \cdot z \right) \\ &+ \left( \frac{(m^2 + n^2) \cdot z^2 \cdot 3}{-2 \cdot z \cdot (n^2 + m \cdot n \cdot (3 \cdot z - 1))} \right) \cdot (z - 2 \cdot v) \\ &+ (1 - n) \cdot v + n \cdot z \end{aligned} \right) \\
\frac{\partial (Y^{II} - Y^{III})}{\partial n} &= b_2 \cdot ((2 \cdot v \cdot (m - n) + 2 \cdot n - m) \cdot (z - v)) + (v - 1) \cdot (g_3 - g_2) \\
&\quad + (a + b_1) \cdot \left( \frac{1 - 2 \cdot n \cdot (v - z) \cdot (z - 1)}{+m \cdot (z \cdot (1 + 2 \cdot (v - z)) - v) - v} \right) \\
&\quad - a \cdot \left( \begin{aligned} &(2 \cdot n \cdot m \cdot (3 \cdot z - 2) + 3 \cdot n^2 \cdot (1 - z)) \cdot (v - z)^2 \\ &+ m \cdot v^2 \cdot (2 + m - 3 \cdot m \cdot z) + m^2 \cdot z^2 \cdot (1 - 3 \cdot z) \\ &+ v \cdot (2 \cdot m^2 \cdot z \cdot (3 \cdot z - 1) - m \cdot (1 + 2 \cdot z) - 1) \\ &+ (v - z) \cdot (2 \cdot n - v) + m \cdot z + 1 \end{aligned} \right) \\
\frac{\partial (Y^{II} - Y^{III})}{\partial v} &= (m - n) \cdot \left( \begin{aligned} &g_2 - g_3 + b_1 \cdot (1 + n \cdot (z - 1) - m \cdot z) \\ &+ b_2 \cdot (n + (2 \cdot v - z) \cdot (m - n)) \\ &+ 2 \cdot a \cdot \left( \frac{((2 \cdot z - 1) \cdot m + n - 1) \cdot n}{+m - (m^2 + n^2) \cdot z} \right) \cdot (v - z) \end{aligned} \right) \\
\frac{\partial (Y^{II} - Y^{III})}{\partial z} &= (m - n) \cdot (b_1 - b_2) \cdot n \\
&\quad - \left( \begin{aligned} &(b_1 + b_2) \cdot v - 2 \cdot b_1 \cdot z \\ &+ a \cdot (v - z) \cdot \left( \frac{2 \cdot (1 - n)}{+(m - n) \cdot (v - 3 \cdot z)} \right) \end{aligned} \right) \cdot (m - n)^2
\end{aligned}$$

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