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ON DECOMPOSING NET FINAL VALUES: EVA, SVA AND SHADOW PROJECT

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Abstract. A residual-income model, named *Systemic Value Added* (SVA), is proposed for decision-making purposes, based on a *systemic* approach introduced in Magni (2000, 2003, 2004). The model translates the notion of residual income (excess profit) giving formal expression to a counterfactual alternative available to the decision maker. Relations with other residual income models are studied, among which Stewart's *Economic Value Added*. The index here introduced differs from EVA in that it rests on a different interpretation of the notion of excess profit and is formally connected with the EVA model by means of a *shadow* project. The SVA is formally and conceptually threefold, in that it is economic, financial, accounting-flavoured. Some results are offered, providing sufficient and necessary conditions for decomposing Net Final Values. Relations between a project's SVA and its *shadow* project's EVA are shown, all results of Pressacco and Stucchi (1997) are proved by making use of the systemic approach and the *shadow* counterparts of those results are also shown.

Keywords and Phrases: decomposition, excess profit, residual income, systemic, shadow project, EVA, SVA, Net Final Value.

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Introduction

The Net Final Value is a well-known tool for projects' economic analysis, capital budgeting, and in general, business and financial decision-making. The problem of decomposing Net Final Values has gained in recent years a renewed interest in both American and European literature, since such a decomposition gives voice, in a formal sense, to an all-pervasive notion in economics: excess profit, also known as residual income. Several decomposition models have been recently developed so that the notion of excess profit seems to be open to multiple formalizations. The importance of this subject is both theoretical and practical. As for business and financial decision-making, the concept of excess profit may be used for analysing periodic performance of a project (or a firm), for valuing firms, for measuring shareholders value creation, for assessing the performance of managers or business units, as a reference parameter for executive compensation, and as a tool of corporate governance. From a mathematical perspective, relations among the models may be searched for and formal results may give interesting insights and suggest further ideas for both decision-making and theoretical modelling. From a cognitive point of view, the concept of excess profit may be seen as the formal translation of a counterfactual conditional, as it is given by the difference between a (factual) profit and a (counterfactual) profit that could have been realized if an alternative course of action had been undertaken. Consequently, multiple translations imply multiple conceptualizations of the same notion. From an epistemological point of view, the possible existence of multiple formal translations may address the problem of conventionalism in scientific research (in the sense of Poincaré, 1902).

In particular, I shall focus on the contributions of Stewart (1991), Peccati (1987, 1991, 1992), Pressacco and Stucchi (1997), Magni (2000, 2003, 2004). Stewart proposes the Economic Value Added (EVA) which has attracted increasing attention among academics and professionals (O'Byrne, 1999), is presented in any recent finance text-book (e.g. Brealey and Myers, 2000), is used for business and corporate finance applications (Damodaran, 1999, Fernández, 2002), and even its critics recognize that "the introduction of EVA ... can rightly be regarded as one of the most significant management innovations of the past decade" (Biddle, Bowen and Wallace, 1999, p.78); Peccati develops the concept of periodic Net Final Value (NFV) of a project[1] while Pressacco and Stucchi generalize Peccati's model in the sense of Teichroew, Robichek and Montalbano (1965a, 1965b) by introducing a two-valued rate for the project balance; Magni proposes an index named Systemic Value Added (SVA) which decomposes the Net Final Value (NFV) of a cash-flow stream by treating the investor's wealth as a dynamic system (whence the name of the index).

The former three models share a common perspective (they are all, so to say, NFV-flavoured) whereas Magni's model is mathematically and cognitively different. To put it in a nutshell, the NFV-based models employ, implicitly or explicitly, the following line of reasoning, which is typically financial: Let w_{s-1} and x be the capital invested in a project at the beginning of period s and its rate of return respectively. Suppose the evaluator could invest the capital in an alternative course of action whose rate of return is equal to i; this implies that the project's return is xw_{s-1} and the investor foregoes the counterfactual profit iw_{s-1}. The difference $xw_{s-1} - iw_{s-1}$ between the two alternative profits represents the excess profit for period s. As for the SVA model, the line of argument stems from the fact that wealth is regarded as a dynamic system and profit is seen as the difference between wealth at time s and wealth at time $s-1$ (i.e. between consecutive states of the dynamic system). Let E_s be the investor's wealth at time s if she undertakes the project at time 0 (thus E_s incorporates the capital w_{s-1}), let E^s be the investor's wealth if the alternative course of action is selected at time 0. If she invests in the

[1]Peccati decomposes both the Net Present Value and the Net Final Value. I will be concerned only with the latter, as the two notions are equivalent (see *infra*).

project, the (factual) income in period s is $E_s - E_{s-1}$, if she instead invests money at the alternative rate i her (counterfactual) income is $E^s - E^{s-1}$. The difference between the two is the excess profit of period s. It will be shown that this kind of reasoning takes account of the fact that if the investor had selected the counterfactual course of action her wealth would have increased periodically at the (counterfactual) rate of return i. So the SVA model is such that the rate i is applied not to w_{s-1} but to a different (counterfactual) capital: That capital which the evaluator foregoes by selecting the factual project.

Mathematically, it is shown that strict relations hold between the two perspectives: The NFV-based models and the SVA model are formally connected with a *shadow project* whose EVA coincides with the original project's SVA. This striking result allows us to interpret the SVA model as an EVA model (and vice versa) so that the two models are, from this point of view, two sides of the same medal: Each project has a shadow project and is itself a shadow project of some other project. This is the reason why we can see the SVA as an EVA or the EVA as a SVA. In this sense the SVA model suggests an interpretation of excess profit such that a project's excess profit is obtained by means of computation of an Economic Value Added, not referred to the project itself, but to its shadow. The shadow project, so essential in connecting the notion of EVA and SVA, has also a striking economic meaning in changing the framing of the decision-making process. The original decision process is such that the two alternatives are: (i) to invest in the project or (ii) not to invest in the project (the null alternative). The notion of shadow project enables us to reframe the decision process as: (i) to invest in the project or (ii) to invest in the shadow project. Letting P denote the project and $\overline{P}$ its shadow, we can say that the systemic outlook removes the null alternative, and the decision as to whether or not invest in P is replaced by the decision as to whether undertake P or $\overline{P}$.

Generalizations of both the EVA model and the SVA model are provided in the sense of Teichroew, Robichek and Montalbano, *op.cit.*, and relations with Pressacco and Stucchi's model are provided. All results of the latter authors are re-demonstrated making use of the systemic approach. In particular, Pressacco and Stucchi prove a fundamental theorem on the generalization of Peccati's model resting on a rule of factorization of particular bivariate polynomials. A systemic approach enables one to prove the same theorem with no need of such a rule (to be precise, a more general theorem is proved, which implies Pressacco and Stucchi's theorem); as a matter of fact, all proofs given throughout the paper only rely on the economic concept of Systemic Value Added and do not depend on formal properties of polynomials. Also, Pressacco and Stucchi's assumptions are relaxed in order to provide more general results. Morevoer, it will be shown that the introduction of the notion of shadow project, alongside the adoption of a systemic outlook, allows one to restate parallel results with no need of capitalization factors. Indeed, the SVA perspective does not rest on (explicit) capitalization process, it just relies on computation of initial capital invested and net profit.

Finally, we will discover that the SVA model has a distinctive threefold quality: It is, at one time, an economic, financial, and accounting(-flavoured) measure. It is economic for it individuates excess profit, it is financial for it considers cash values and may be inferred by means of financial arguments, it is accounting-based in that it derives from standard accounting equations, dismisses capitalization and focuses on net worth. This surprising fact sheds lights on theoretical and decisional aspects: As for the theory, we have the opportunity of understanding how these three disciplines may combine to shape a fundamental concept in economics, as well as the chance of discovering fruitful formal and conceptual relationships among them; as for decision-making, we find a model that makes use of concepts retrieved from disciplines that are considered not suited for decision-making (accounting) and incompatible one another (e.g. economics and accounting, finance and accounting).[2] The SVA model picks from these three areas the most fruitful lines of reasoning and combine them to reach a fecund solution of the decision-making process.

The paper is structured as follows. The first section is a review of the existing models. In the second section the concepts of EVA and SVA are generalized, the concept of shadow project is introduced and connections between the models are shown. The third section presents, among others, all results proved by Pressacco and Stucchi, restating them and redemonstrating them with the use of the systemic approach. The fourth section provides the systemic counterparts of the results of the previous section, where the shadow EVA replaces the EVA and capitalization factors are discharged. Some conclusive comments focus on possible future researches.

1. A critical review of the existing models

Consider a project P whose initial outlay is $a_0 \in \mathbb{R}^+$, with subsequent periodic cash flows $a_s \in \mathbb{R}$ at time s=1,2,. . . ,n. Suppose that the evaluator currently invests her wealth in an asset C whose rate of return is i. She is faced with the alternative of

(i) withdrawing the sum a_0 from asset C and investing it in project P, or
(ii) keeping the sum invested at the rate i.

Then, the rate i is the so-called opportunity cost of capital. Let E_0 be the initial net worth,[3] $E_0 \in \mathbb{R}$. The Net Final Value (NFV) of project P is given by the difference between alternative final net worths. Denote with E_n and E^n the evaluator's net worth at time n relative to case (i) and case (ii) respectively; horizon n is assumed finite and fixed throughout the paper. We have

$$\begin{aligned}\mathrm{NFV}(i) = E_n - E^n &= (E_0 - a_0)(1+i)^n + \sum_{s=1}^{n} a_s(1+i)^{n-s} - E_0(1+i)^n \\ &= -a_0(1+i)^n + \sum_{s=1}^{n} a_s(1+i)^{n-s}. \qquad (1)\end{aligned}$$

Eq. (1) presupposes that C is an account where the cash flows released by project P are reinvested in (if positive) or withdrawn from (if negative). The Net Present Value (NPV) is NPV(i)=NFV(i)$(1+i)^{-n}$. The *outstanding capital* or *project balance* w_s at the rate x is defined as

$$\begin{aligned} w_0 &:= a_0 \\ w_s &:= w_{s-1}(1+x) - a_s \qquad s = 1, \dots, n \end{aligned}$$

where x denotes an internal rate of return for P, that is NPV(x)=0. We also have w_n=NFV(x)=0.

To decompose the NFV of project P, Peccati uses the following argument: At the outset of each period s the investor invests in a (fictitious) one-period project, whose initial outlay is $-w_{s-1}$. At the end of the period, she will receive the sum a_s along with the value w_s. Denoting with G_s the Net Final Value of this one-period project we have

$$G_s = -w_{s-1}(1+i)^{n-(s-1)} + (w_s + a_s)(1+i)^{n-s} = w_{s-1}(x-i)(1+i)^{n-s}. \qquad (2)$$

[2]It is important to stress that the SVA perspective does not use accounting as such, it just employs the way of representing economic facts typical of accountancy: This is actually very satisfying from a diachronic point of view (see Remark 4.1).

[3]The term "net worth" is to be intended as a synonym of wealth.

G_s is the quota of the project's NFV generated in period s. Using the project balance equation, it is easy to verify that summing for s we have $\sum_{s=1}^{n} G_s = \text{NFV}$. Peccati then extends his model and assumes that the investment is partly financed by a loan contract consisting of an initial receipt $f_0 \in \mathbb{R}^+$ and subsequent cash flows $f_s \in \mathbb{R}$ at time $s=1,\ldots,n$. The *outstanding debt* or *debt balance* at the debt rate δ is defined as

$$\begin{aligned} D_0 &:= f_0 \\ D_s &:= D_{s-1}(1+\delta) - f_s \qquad s = 1, \ldots, n. \end{aligned}$$

Using the same argument as before, modified so as to take debt into account, we have

$$G_s = \big(w_{s-1}(x-i) - D_{s-1}(\delta - i)\big)(1+i)^{n-s}. \tag{3}$$

Summing for s we find back the NFV.

Pressacco and Stucchi (henceforth P&S) extend the first version of Peccati's model by allowing for two pairs of rates (i_P, i_N) and (x_P, x_N) in the sense we now show.

We will make use of the following

Definition 1. The balance of asset C is

$$\begin{aligned} C_0 &:= -a_0 \\ C_s &:= C_{s-1}(1 + i(C_{s-1})) + a_s \qquad s = 1, \ldots, n \end{aligned} \tag{4a}$$

where

$$i(C_{s-1}) = \begin{cases} i_P & \text{if } C_{s-1} > 0, \\ i_N & \text{if } C_{s-1} < 0, \end{cases}$$

with $i_P \neq i_N$.[4]

Definition 2. The outstanding capital of project P is

$$\begin{aligned} w_0 &:= a_0 \\ w_s &:= w_{s-1}(1 + x(w_{s-1})) - a_s \qquad s = 1, \ldots, n \end{aligned} \tag{4b}$$

with

$$x(w_{s-1}) = \begin{cases} x_P & \text{if } w_{s-1} > 0, \\ x_N & \text{if } w_{s-1} < 0, \end{cases}$$

so that $w_n=0$ (in other terms, (x_P, x_N) is an internal pair).[5] It is also worth noting that compounding w_{s-1} we have $w_{s-1}(1 + x(w_{s-1})) = w_s + a_s$.

Therefore, P&S generalize Peccati's model only under a particular perspective. In fact, they assume $D_s=0$ for all s whereas Peccati allows for $D_s \neq 0$; conversely, they handle reinvestment and external

[4] P stands for "positive", N for "negative". In this paper the notational conventions and the presentation of P&S's model differ considerably from P&S's exposition. My exposition is consistent with the systemic outlook that shall be developed later.

[5] I shall never define the value of a rate when its argument is zero, so we can pick whatever value according to our needs.

financing by introducing the pair (i_P, i_N) where i_N acts just whenever the value of C is negative (Peccati's model can be seen as assuming $i_P{=}i_N{=}i$).[6]

As one can note, the assumption $C_0{=}{-}a_0$ is equivalent to the assumption $E_0{=}0$ in Peccati's model, and the entire model is tied to this assumption.[7] The project's NFV is then

$$\text{NFV} = E_n - E^n = -a_0(1+i(C))^{0,n} + \sum_{s=1}^{n} a_s(1+i(C))^{s,n} \tag{5}$$

with

$$(1+i(C))^{s,n} := \prod_{k=s+1}^{n}(1+i(C_{k-1})),\ s<n, \qquad (1+i(C))^{n,n} := 1,$$

so that $(1{+}i(C))^{s,m+1}{:=}(1{+}i(C))^{s,m}(1{+}i(C_m))$ for $m{<}n$. The main result of P&S can be summarized as follows:

P&S Theorem. *Assume $C_0{=}{-}a_0$. Peccati's model can be generalized in a two-rate capitalization of periodic shares so that*

$$G_s = w_{s-1}(x_P - i_N)(1+i(C))^{s,n} \qquad \textit{or} \qquad G_s = w_{s-1}(x_N - i_P)(1+i(C))^{s,n}$$

if and only if

$$x(w_{s-1}) = x_P \qquad \textit{iff} \qquad i(C_{s-1}) = i_N.$$

In such a case, we have

$$\text{NFV} = \sum_{s:w_{s-1}>0}^{n} w_{s-1}(x_P - i_N)(1+i(C))^{s,n} + \sum_{s:w_{s-1}<0}^{n} w_{s-1}(x_N - i_P)(1+i(C))^{s,n}.$$

Stewart's model is well-known in the literature. He introduced his measure for corporate use, but it may be used for whatever project we can think of. There are three versions of the EVA model (see Fernández, *op.cit.*, chapters 13 and 14). The first one takes the point of view of an investor holding both stocks and bonds of the firm (in the same ratio as the firm's debt/equity ratio). Stewart suggests us to calculate the firm's total cost of capital, given by the product of the Weighted Cost of Capital (WACC) and the total capital's book value (TC_b) invested at the outset of period s; then the total cost of capital is subtracted from the Net Operating Profit After Taxes (NOPAT). Notationally, we have, for period s,

$$\text{EVA}_s = \text{NOPAT} - \text{WACC} * \text{TC}_b. \tag{6a}$$

[6] P&S take as a starting point the idea of Teichroew, Robichek and Montalbano (henceforth TRM) of a project balance depending on two rates. Notwithstanding, TRM rest on the Net Present Value rule, as they assume that unlimited funds are available to the investor and can be employed by the investor at the same rate ϱ: with our notations, this means $i_P{=}i_N{=}\varrho$, so that account C evolves according to the recurrence equation

$$C_s = C_{s-1}(1+\varrho) + a_s.$$

P&S's treatment is such that they do not merely allow for an internal pair (x_P, x_N), but generalize further on and introduce an external pair (i_P, i_N). Under these assumptions, the NPV rule cannot be applied any more and the choice between two or more alternative courses of action must be based on the net final values.

[7] I will henceforth use the two assumptions interchangeably.

Taking the point of view of an investor who holds only stocks, the EVA is

$$\text{EVA}_s = \text{PAT} - k_e * \text{E}_b \tag{6b}$$

where PAT is the profit after taxes, E_b is the equity's book value and k_e is the (opportunity) cost of equity (the counterfactual rate of return). Eqs. (6a) and (6b) mix accounting values (NOPAT, TC_b, E_b) with market values (k_e, WACC); a third version rests only on market values:

$$\text{EVA}_s = \big(\text{TSR} - k_e\big)\text{E} \tag{6c}$$

where TSR is the Total Shareholder Return and E is the market value of equity (all values in (6) obviously refer to period s).

Stewart's EVA model and Peccati's decomposition model are cognate:[8] In fact, applying the Economic Value Added concept to project P, we have TSR=$\frac{xw_{s-1}-\delta D_{s-1}}{w_{s-1}-D_{s-1}}$, E=$w_{s-1}-D_{s-1}$, k_e=i, so that

$$w_{s-1}(x-i) - D_{s-1}(\delta - i) = \big(\frac{xw_{s-1} - \delta D_{s-1}}{w_{s-1} - D_{s-1}} - i\big)(w_{s-1} - D_{s-1}) = \text{EVA}_s. \tag{7}$$

The relation between (6) and (3) is then given by

$$G_s = \big(w_{s-1}(x-i) - D_{s-1}(\delta - i)\big)(1+i)^{n-s} = \text{EVA}_s(1+i)^{n-s}. \tag{8}$$

Consequently, P&S's model, as a generalization of Peccati's model, can be viewed as a formal extension of Stewart's model in the case $x(w_{s-1})$=x_P, $i(C_{s-1})$=i_N, and in the case $x(w_{s-1})$=x_N, $i(C_{s-1})$=i_P respectively.

Magni's SVA model is a different decomposition model, based on the notion of *system*. The investor's net worth is seen as a (financial) dynamic system structured in various accounts, which are periodically activated to consider withdrawals and reinvestments of cash flows. He assumes, like P&S, that the balances are functions of a two-valued rate, but generalizes allowing for whatever $E_0 \in \mathbb{R}$. The financial system presents a different structure according to the course of action selected. We can conveniently depict it by means of a double-entry sheet where sources and uses of funds are pointed out. If alternative (i) is followed then we have, at time s,

$$\begin{array}{r|l} \underline{\text{Uses}} & \underline{\text{Sources}} \\ C_s & E_s \\ w_s & \end{array} \tag{9a}$$

for s, s=$0, 1, \ldots, n$, where C_s, w_s are the balances of asset C and project P respectively, and E_s is the investor's wealth. The structure evolves diachronically according to the recurrence equations (4a) (where the initial condition is replaced by the more general C_0=E_0-a_0, $E_0 \in \mathbb{R}$), (4b), and (4c) here added:

$$E_s = C_s + w_s = E_{s-1} + i(C_{s-1})C_{s-1} + x(w_{s-1})w_{s-1}. \tag{4c}$$

If alternative (ii) is instead selected, we have, at time s,

[8] Actually, all these models have been forerun by Edwards and Bell (1961, chapter 2, Appendix B).

$$\begin{array}{c|c} \underline{\text{Uses}} & \underline{\text{Sources}} \\ C^s & E^s \end{array} \tag{9b}$$

for s, $s{=}0,1,\ldots,n$, where C^s and E^s denote the values of asset C and net worth respectively. The financial system is then de-structured, so to say, and C^s coincides with E^s for all s. The rate of interest for account C will be obviously i_P or i_N depending on the sign of C^s. We may describe these facts with the recurrence equation governing the evolution of the system:

$$\begin{aligned} &E^0 = C^0 = E_0 \\ &E^s = C^s = C^{s-1}(1+i(C^{s-1})) = E^{s-1}(1+i(E^{s-1})) \end{aligned} \tag{10}$$

with

$$i(C^{s-1}) = \begin{cases} i_P & \text{if } C^{s-1} > 0, \\ i_N & \text{if } C^{s-1} < 0. \end{cases}$$

Thanks to (10), we can also write $i(C^{s-1}){=}i(E_0)$ for all $s{\geq}\ 1$. Also, equation (10) implies that if $C^0 \lesseqgtr 0$ then $C^s \lesseqgtr 0$ for all s (equivalently for E^s, since $E^s{:=}C^s$). Under this systemic perspective, the excess profit for period s is given by the difference between what the investor would earn in period s if she chooses alternative (i) at time 0 and what she would earn should she decide to keep on investing at the rate i, i.e. to choose alternative (ii). This is formally translated into a difference between net profits relative to the two courses of action. The net profit *sub* (i) is

$$E_s - E_{s-1} = i(C_{s-1})C_{s-1} + x(w_{s-1})w_{s-1}, \tag{11a}$$

whereas for (ii) we have

$$E^s - E^{s-1} = i(C^{s-1})C^{s-1}. \tag{11b}$$

Eq. (11a) informs us that if the investor undertakes project P her profit will be given by the return on the capital invested in the project (equal to $x(w_{s-1})w_{s-1}$) added to the interest gained on asset C (equal to $i(C_{s-1})C_{s-1}$). Eq. (11b) informs us that the net profit for (ii) is just the return on asset C (equal to $i(C^{s-1})C^{s-1}$). The excess profit for each period s, here named *Systemic Value Added* (SVA_s), is then

$$\begin{aligned} \text{SVA}_s &= \big(E_s - E_{s-1}\big) - \big(E^s - E^{s-1}\big) \\ &= x(w_{s-1})w_{s-1} + i(C_{s-1})C_{s-1} - i(C^{s-1})C^{s-1}. \end{aligned} \tag{12}$$

Summing for s we have the overall *Systemic Value Added* (SVA) of project P. The latter coincides with the Net Final Value of P:

$$\text{SVA} = \sum_{s=1}^{n} \text{SVA}_s = \sum_{s=1}^{n} \big(E_s - E_{s-1}\big) - \big(E^s - E^{s-1}\big) = E_n - E^n = \text{NFV}. \tag{13}$$

Further, we have

$$\begin{aligned} \text{SVA} &= \text{NFV} = E_n - E^n \\ &= E_0\bigg(\big(1+i(C)\big)^{0,n} - \big(1+i(E_0)\big)^n\bigg) - a_0\big(1+i(C)\big)^{0,n} + \sum_{s=1}^{n} a_s\big(1+i(C)\big)^{s,n} \end{aligned} \tag{14}$$

since

$$
\begin{aligned}
E_n &= (E_0 - a_0)(1+i(C))^{0,n} + \sum_{s=1}^{n} a_s(1+i(C))^{s,n} \\
E^n &= E_0(1+i(C^0))^n = E_0(1+i(E_0))^n
\end{aligned}
$$

Note that picking E_0=0 (i.e. C_0=$-a_0$) we get to (5) as in P&S's model.

2. EVA, SVA and $\overline{\text{EVA}}$

In this section, after introducing some definitions and proving some preliminary results, I generalize the concept of EVA inserting it in a world *à la* P&S, where we know that the pairs (i_P, i_N), (x_P, x_N) replace the rate i and x respectively. The notion of shadow project is introduced, whose balance offers interesting insights. On the basis of this notion a fundamental theorem is then proved that enables us to appreciate the formal connection between the NFV-based models and the SVA model: The SVA$_s$ of a project coincides with the EVA$_s$ of its shadow project, so that the project's NFV is just the sum of such (noncapitalized) EVAs. Relations between SVA and EVA are stated and we will see that the shadow project is conceptually flexible: It may be seen as representing two different dynamic systems (financially, two different accounts), and the SVAs may be interpreted as the net cash flows of a levered project $\overline{P}$, financed by project P.

Definition 3. A pair (i_P, i_N) is said to be a twin-pair if for all s, $i(C^s)$=$i(C_s)$.

The above definition states that if account C has the same sign in each period in both the factual and counterfactual mode, then the interest rate applied is the same in both modes. As particular cases, the following definition covers those instances where account C does not change sign over time.

Definition 4. A pair (i_P, i_N) is said to be an i_P-twin-pair if it is a twin-pair and $i(C_s)$=i_P. A pair (i_P, i_N) is said to be an i_N-twin-pair if it is a twin-pair and $i(C_s)$=i_N.

The following fundamental definition introduces the concept of shadow project, as a function of both project P and account C (in both modes). This definition will enable us to find a striking connection between EVA and SVA and to prove all results of P&S as well as the systemic counterparts of those very results (see Section 4).

Definition 5. A project $\overline{P}$ is said to be the *shadow project* of P (or the *shadow* of P) if it consists of the sequence of cash flows

$$(-\overline{a}_0, \overline{a}_1, \ldots, \overline{a}_n)$$

available at time 0,1,. . . ,n respectively, such that

$$
\begin{aligned}
\overline{a}_0 &= a_0 \\
\overline{a}_s &= (w_{s-1} - w_s) + 2x(w_{s-1})w_{s-1} + i(C_{s-1})C_{s-1} - i(C^{s-1})C^{s-1} \qquad s = 1, 2, \ldots, n
\end{aligned}
$$

The notion of *parallel* pairs will also be useful. To this end, let us have the following notations:

$$\overline{w}_s := C^s - C_s \qquad \text{and} \qquad \overline{x}(\overline{w}_{s-1}) := \begin{cases} \overline{x}_P & \text{if } \overline{w}_{s-1} > 0, \\ \overline{x}_N & \text{if } \overline{w}_{s-1} < 0, \end{cases}$$

where

$$\overline{x}_P := x_P \frac{w_{s-1}}{\overline{w}_{s-1}} \qquad \text{and} \qquad \overline{x}_N := x_N \frac{w_{s-1}}{\overline{w}_{s-1}}.$$

Then we have the following

Definition 6. *The shadow pair* $(\overline{x}_P, \overline{x}_N)$ *and the internal pair* (x_P, x_N) *are said to be parallel if, for all* s*,*

$$x(w_{s-1}) = x_P \qquad \textit{iff} \qquad \overline{x}(\overline{w}_{s-1}) = \overline{x}_P.$$

Definition 6 boils down to saying that the shadow pair and the internal pair are parallel if their outstanding capital w_s and $\overline{w}_s$ have the same sign for all s. I will often use the notion of parallel pairs and twin-pair throughout the paper.[9] I will also make use of the following straightforward result, based on Definition 6: We have

$$x(w_{s-1})w_{s-1} = \overline{x}(\overline{w}_{s-1})\overline{w}_{s-1} \tag{\#}$$

if and only if the shadow pair and the internal pair are parallel (which means $x(w_{s-1})w_{s-1}{=}x_P w_{s-1}$ and $\overline{x}(\overline{w}_{s-1})\overline{w}_{s-1}{=}\overline{x}_P\overline{w}_{s-1}$, or $x(w_{s-1})w_{s-1}{=}x_N w_{s-1}$ and $\overline{x}(\overline{w}_{s-1})\overline{w}_{s-1}{=}\overline{x}_N\overline{w}_{s-1}$).

The following Definition associates the classical notion of Soper (1959) project with the correspondent rate of return:

Definition 7. P is said to be a Soper project if for all s $x(w_{s-1}){=}x_P$. $\overline{P}$ is said to be a Soper project if for all s $\overline{x}(\overline{w}_{s-1}){=}\overline{x}_P$.

For the sake of convenience I shall label some propositions occurring frequently in the paper with the following notations:

(Par):= the internal pair (x_P, x_N) and the shadow pair $(\overline{x}_P, \overline{x}_N)$ are parallel

(SP):= P is a Soper project

(S$\overline{P}$):= $\overline{P}$ is a Soper project

(Twin):= (i_P, i_N) is a twin-pair

(i_P-Twin):= (i_P, i_N) is an i_P-twin-pair

(i_N-Twin):= (i_P, i_N) is an i_N-twin-pair

In the sequel, I shall assume $x_P{\neq}x_N$ and $i_P{\neq}i_N$ unless otherwise specified.

Let us now assume that a decision maker invests in a levered project consisting of the shadow project financed by a debt whose cash flows are a_0 at time 0 and $-a_s$ at time $s{\geq}\,1$. We have then the following

Lemma 2.1. *If* (Twin)*, then* $\overline{a}_s - a_s = \mathrm{SVA}_s$ *for all* s *and*

$$\sum_{s=1}^{n}\overline{a}_s = -a_0\big(1+i(C)\big)^{0,n} + \sum_{s=1}^{n}a_s\big(1+i(C)\big)^{s,n} + \sum_{s=1}^{n}a_s.$$

Proof: Use Definition 5, (4b), (12), (13), and the fact that (Twin) implies (5). (Q.E.D.)

The above result reveals that the net cash flows of the levered project are just the SVA_s and that the sum of the shadow project's cash flows coincide with the sum of project P's NFV and the sum of the project P's cash flows or, equivalently, the NFV of the levered project calculated at a zero rate coincides with the NFV of project P calculated at the rate $i(C_{s-1})$ (in other terms, the shadow project is that project that enables us to overlook capitalization).

[9] It is worth reminding that particular cases of twin-pair and parallel pairs occur if $i_P{=}i_N$ and $x_P{=}x_N$, which is frequent in practice.

Lemma 2.2. *We have*

$$\overline{w}_s = \overline{w}_{s-1}(1 + i(C_{s-1})) - a_s \qquad s = 1, \ldots, n \tag{$\otimes$}$$

if and only if (Twin).

Proof: Assume (Twin). We have

$$\begin{aligned}\overline{w}_s &= C^s - C_s \\ &= C^{s-1}(1 + i(C^{s-1})) - \big(C_{s-1}(1 + i(C_{s-1})) + a_s\big) \\ &= [\text{for (Twin)}] = \big(C^{s-1} - C_{s-1}\big)(1 + i(C_{s-1})) - a_s \\ &= \overline{w}_{s-1}(1 + i(C_{s-1})) - a_s.\end{aligned}$$

Assume now ($\otimes$). On the one hand, we have $\overline{w}_s{:=}C^s{-}C_s$. On the other hand, by assumption, $\overline{w}_s{=}\overline{w}_{s-1}(1 + i(C_{s-1})){-}a_s$. The result is obtained by developing both right-hand sides and subtracting the two equations. (Q.E.D.)

Lemma 2.3. *We have*

$$\overline{w}_s = \overline{w}_{s-1}(1 + \overline{x}(\overline{w}_{s-1})) - \overline{a}_s \qquad s = 1, \ldots, n \tag{$\otimes\otimes$}$$

if and only if (Par).

Proof: Assume (Par). Then

$$\begin{aligned}\overline{w}_{s-1}(1 + \overline{x}(\overline{w}_{s-1})) - \overline{a}_s &= [\text{for (Par), by (\#), by definition of } \overline{w}_{s-1}] = C^{s-1} - C_{s-1} + x(w_{s-1})w_{s-1} - \overline{a}_s \\ &= [\text{by Lemma 2.1}] = C^{s-1} - C_{s-1} + x(w_{s-1})w_{s-1} - a_s - \mathrm{SVA}_s \\ &= [\text{by (12)}] = C^{s-1}(1 + i(C^{s-1})) - C_{s-1}(1 + i(C_{s-1})) - a_s \\ &= C^s - C_s \\ &= \overline{w}_s.\end{aligned}$$

Assume now $\otimes\otimes$. We have then

$$\begin{aligned}\overline{w}_{s-1} + \overline{x}(\overline{w}_{s-1})\overline{w}_{s-1} - \overline{a}_s &= \overline{w}_s \\ &= C^s - C_s \\ &= C^{s-1}(1 + i(C^{s-1})) - C_{s-1}(1 + i(C_{s-1})) - a_s \\ &= [\text{by (12)}] = C^{s-1} - C_{s-1} + x(w_{s-1})w_{s-1} - a_s - \mathrm{SVA}_s \\ &= [\text{by Lemma 2.1}] = C^{s-1} - C_{s-1} + x(w_{s-1})w_{s-1} - \overline{a}_s \\ &= \overline{w}_{s-1} + x(w_{s-1})w_{s-1} - \overline{a}_s\end{aligned}$$

whence

$$\overline{x}(\overline{w}_{s-1})\overline{w}_{s-1} = x(w_{s-1})w_{s-1}$$

which implies (Par). (Q.E.D.)

Remark 2.1: Lemma 2.2 implies that if (Twin), then

$$\overline{w}_s = a_0(1+i(C))^{0,s} - \sum_{k=1}^{s} a_k(1+i(C))^{k,s} \qquad s=1,\dots,n.$$

Also, (4b) implies

$$w_s = a_0(1+x(w))^{0,s} - \sum_{k=1}^{s} a_k(1+x(w))^{k,s} \qquad s=1,\dots,n,$$

where

$$(1+x(w))^{k,s} := \prod_{h=k+1}^{s}(1+x(w_{h-1})) \qquad k<s \qquad \text{and} \qquad (1+x(w))^{s,s} := 1.$$

Then $\overline{w}_s$ is just w_s where we substitute $i(C_{s-1})$ for $x(w_{s-1})$. In other terms, $\otimes$ is equivalent to (4b): They define a very closed recurrence (the same except for the rate). Hence, both imply (from a formal perspective) the same solution. From this point of view, $\overline{w}_{s-1}$ may be interpreted as the project balance of P under the assumption that the rate of return is $i(C_{s-1})$. Equivalently, it is the NFV of project P (changed in sign) if the latter is truncated at time s. But, by definition, $\overline{w}_s$ is the counterfactual sum that the investor forgoes selecting the factual alternative (i) (she could have held C^s but she only holds C_s). Consequently, the truncated NFV of P coincides, in absolute value, with the capital foregone by the investor. The latter is a *lost capital* and the interest term $i(C_{s-1})\overline{w}_s$ is the lost profit, so that the SVA may be rewritten

$$x(w_{s-1})w_s - i(C_{s-1})\overline{w}_s.$$

In other words, the SVA could be named *lost-capital residual income.*

Owing to Lemma 2.3, $\overline{w}_{s-1}$ may also be interpreted as the outstanding balance of the shadow project $\overline{P}$ at the rate $\overline{x}(\overline{w}_{s-1})$ (the parallelism condition is stringent: If parallelism does not hold, then the sign of w_{s-1} differs from the sign of $\overline{w}_{s-1}$ and the economic significance of $\overline{x}(\overline{w}_{s-1})$ is befogged). Formally, $\otimes$ and $\otimes\otimes$ represent the same dynamic system seen with different eyes. Note that $\otimes\otimes$ depends on a rate defined as the ratio of w_{s-1} to $\overline{w}_{s-1}$, that is the ratio of the (factual) capital invested in the period to the (counterfactual) capital foregone by the investor (the lost capital). That is, the rate of return $\overline{x}(\overline{w}_{s-1})$ is given by project P's internal rate of return weighted by the proportion of the factual capital on the counterfactual capital. Lemmas 2.2 and 2.3 tell us that, under suitable conditions, the capital foregone by the decision maker increases periodically at the rate $\overline{x}(\overline{w}_{s-1})$ while decreases by the sum $\overline{a}_s$, and, equivalently, increases at the rate $i(C_{s-1})$ and decreases by the sum a_s. This twofold quality lies at the ground of the fundamental Theorem 2.1 (see *infra*).

To complete the parallelism between P and $\overline{P}$ let us give the following definitions:

Definition 8. If (Twin), the Economic Value Added of P is

$$\text{EVA}_s := w_{s-1}(x(w_{s-1}) - i(C_{s-1})) \tag{15a}$$

Definition 9. If (Twin) and (Par), the Economic Value Added of $\overline{P}$ (or *shadow* EVA) is the product

$$\overline{\text{EVA}}_s := \overline{w}_{s-1}(\overline{x}(w_{s-1}) - i(C_{s-1})), \tag{15b}$$

These Definitions are based on the following way of reasoning: At the beginning of each period, an investor can invest the capital w_{s-1} ($\overline{w}_{s-1}$ for $\overline{P}$) either at the rate $x(w_{s-1})$ ($\overline{x}(\overline{w}_{s-1})$ for $\overline{P}$) or at the rate $i(C_{s-1})$ (the same for $\overline{P}$). Accepting the first alternative her profit will be $x(w_{s-1})w_{s-1}$ ($\overline{x}(\overline{w}_{s-1})\overline{w}_{s-1}$ for $\overline{P}$); the other course of action will leave her with $i(C_{s-1})w_{s-1}$ ($i(C_{s-1})\overline{w}_{s-1}$ for $\overline{P}$). The residual income is then given by the difference between the two, whence we obtain (15).

Remark 2.2: The definition of EVA in (15a) is unambiguous if (Twin), and the same is true for the definition of $\overline{\text{EVA}}$ in (15b). What if (Twin) does not hold? We may wonder whether one should use $i(C_{s-1})$ or $i(C^{s-1})$. But, at a closer glance, we may posit that (15a) and (15b) are still meaningful: An EVA-minded reasoner is, as seen, inclined to employ a factual capital with a counterfactual rate, so we may well stipulate that the decision maker selects the factual value C_{s-1} of account C as the argument of the counterfactual rate. If one accepts this, Definitions 8 and 9 hold with no need of the (Twin) assumption.[10]

This brings about the following fundamental

Theorem 2.1. *If* (Twin) *and* (Par)*, then the Systemic Value Added coincides with the Economic Value Added of the shadow project, that is*

$$\text{SVA}_s = \overline{\text{EVA}}_s \qquad \textit{for every } s. \tag{16a}$$

In this case we have

$$\begin{aligned}\text{SVA} = \text{NFV} = &\sum_{s:\overline{w}_{s-1}>0, C_{s-1}<0}^{n} \overline{w}_{s-1}(\overline{x}_P - i_N) + \sum_{s:\overline{w}_{s-1}<0, C_{s-1}>0}^{n} \overline{w}_{s-1}(\overline{x}_N - i_P)\\ &+ \sum_{s:\overline{w}_{s-1}>0, C_{s-1}>0}^{n} \overline{w}_{s-1}(\overline{x}_P - i_P) + \sum_{s:\overline{w}_{s-1}<0, C_{s-1}<0}^{n} \overline{w}_{s-1}(\overline{x}_N - i_N).\end{aligned} \tag{16b}$$

Proof: Using Lemma 2.2 and Lemma 2.3 we have

$$\overline{w}_{s-1}(1 + \overline{x}(\overline{w}_{s-1})) - \overline{a}_s = \overline{w}_{s-1}(1 + i(C_{s-1})) - a_s$$

which implies

$$\overline{x}(\overline{w}_{s-1})\overline{w}_{s-1} - \text{SVA}_s = i(C_{s-1})\overline{w}_{s-1}$$

whence

$$\text{SVA}_s = \overline{w}_{s-1}\big(\overline{x}(\overline{w}_{s-1}) - i(C_{s-1})\big) = [\text{by (Definition 9)}] = \overline{\text{EVA}}_s$$

(Q.E.D.)

Remark 2.3: According to Theorem 2.1 the Systemic Value Added model we have obtained by means of a systemic argument resembles Stewart's decomposition: We just have to use the concept of Economic Value Added and decompose the shadow of P. Thus, the SVA model can be interpreted as a derivation of the EVA model. Likewise, the EVA model itself can be seen as a derivation of the SVA model: P is the shadow project of some other project P' and then the EVA_s of P coincides with the Systemic Value Added of P'. Thanks to Theorem 2.1 we may affirm that the SVA has a distinctively financial

[10] Conversely, the (Par) condition seems crucial in Definition 9 otherwise $\overline{w}_{s-1}$ is not economically interpretable as the outstanding capital of $\overline{P}$ at the rate $\overline{x}(\overline{w}_{s-1})$.

feature (not merely because it is concerned with cash values, but also) because it may be interpreted as an Economic Value Added, which is the result of a typically financial way of reasoning.

Lemma 2.4. *If* (Twin), *then*

$$\mathrm{SVA}_1 = \mathrm{EVA}_1 \tag{18a}$$

and

$$\mathrm{SVA}_s = \mathrm{EVA}_s + i(C_{s-1}) \sum_{k=1}^{s-1} \mathrm{EVA}_k (1+i(C))^{k,(s-1)} \qquad \textit{for every } s > 1. \tag{18b}$$

Proof: We have

$$\begin{aligned}
\mathrm{SVA}_1 &= [\text{by (12)}] = x(w_0)w_0 + i(C_0)C_0 - i(C^0)C^0 \\
&= [\text{for (Twin)}] = x(w_0)w_0 - i(C_0)\overline{w}_0 \\
&= [\text{for } w_0{=}\overline{w}_0] = w_0\big(x(w_0) - i(C_0)\big) \\
&= [\text{by (15a)}] = \mathrm{EVA}_1.
\end{aligned}$$

If $s>1$, we get

$$\begin{aligned}
\big(C^{s-1} - C_{s-1}\big) &= [\text{for (Twin)}] = w_0(1+i(C))^{0,(s-1)} - \sum_{k=1}^{s-1} a_k (1+i(C))^{k,(s-1)} \\
&= w_0(1+i(C))^{0,(s-1)} - \sum_{k=1}^{s-1} \big(w_{k-1}(1+x(w_{k-1})) - w_k\big)(1+i(C))^{k,(s-1)} \\
&= [\text{rearranging terms}] = w_{s-1} - \sum_{k=1}^{s-1} w_{k-1}\big(x(w_{k-1}) - i(C_{k-1})\big)(1+i(C))^{k,(s-1)}.
\end{aligned} \tag{19}$$

We have then

$$\begin{aligned}
\mathrm{SVA}_s &= [\text{for (Twin)}] = x(w_{s-1})w_{s-1} - i(C_{s-1})\big(C^{s-1} - C_{s-1}\big) \\
&= [\text{by (19)}] = w_{s-1}(x(w_{s-1}) - i(C_{s-1})) + i(C_{s-1}) \sum_{k=1}^{s-1} w_{k-1}\big(x(w_{k-1}) - i(C_{k-1})\big)(1+i(C))^{k,(s-1)} \\
&= [\text{by (15a)}] = \mathrm{EVA}_s + i(C_{s-1}) \sum_{k=1}^{s-1} \mathrm{EVA}_k (1+i(C))^{k,(s-1)}.
\end{aligned}$$

(Q.E.D.)

Theorem 2.2. *If* (Twin) , *then*

$$\sum_{k=1}^{s} \mathrm{SVA}_k = \sum_{k=1}^{s} \mathrm{EVA}_k (1+i(C))^{k,s} \qquad \textit{for every } s \geq 1. \tag{20}$$

Proof: Using induction, we have, for s=1, SVA_1=EVA_1 (Lemma 2.4). Suppose (20) holds for s=m. Then,

$$\sum_{k=1}^{m+1} \mathrm{SVA}_k = \sum_{k=1}^{m} \mathrm{SVA}_k + \mathrm{SVA}_{m+1}$$

$$= [\text{by Lemma 2.4}] = \sum_{k=1}^{m} \mathrm{SVA}_k + \mathrm{EVA}_{m+1} + i(C_m) \sum_{k=1}^{m} \mathrm{EVA}_k (1+i(C))^{k,m}$$

$$= [\text{by ind. hyp.}] = \sum_{k=1}^{m} \mathrm{EVA}_k (1+i(C))^{k,m} + \mathrm{EVA}_{m+1}(1+i(C))^{m+1,m+1} +$$

$$+ i(C_m) \sum_{k=1}^{m} \mathrm{EVA}_k (1+i(C))^{k,m}$$

$$= \sum_{k=1}^{m+1} \mathrm{EVA}_k (1+i(C))^{k,m+1} \qquad \text{(Q.E.D.)}$$

Remark 2.4: Theorem 2.1, Theorem 2.2, Lemma 2.1 and Lemma 2.4 not only provide us with fundamental relations between the SVA_s of the project and the EVA_s of the shadow project. They also give us invaluable insights about the relations existing between the SVA_s of the project and the SVA_s of its shadow project, as well as between the EVA_s of the project and the EVA_s of the shadow project. Let P_p be a project and let P_{p+1} be its shadow project (then P_{p-1} denotes a project such that P_p is its shadow project). Denote with SVA_s^p and EVA_s^p the Systemic Value Added and the Economic Value Added of P_p respectively. We have, by Theorem 2.1, Theorem 2.2 and Lemma 2.4:

$$\mathrm{SVA}_s^p = \mathrm{EVA}_s^{p+1}$$

$$\mathrm{SVA}_s^{p-1} = \mathrm{EVA}_s^p$$

$$\mathrm{SVA}_s^p = \mathrm{SVA}_s^{p-1} + i(C_{s-1}) \sum_{k=1}^{s-1} \mathrm{SVA}_k^{p-1} (1+i(C))^{k,(s-1)}$$

$$\mathrm{EVA}_s^{p+1} = \mathrm{EVA}_s^p + i(C_{s-1}) \sum_{k=1}^{s-1} \mathrm{EVA}_k^p (1+i(C))^{k,(s-1)}$$

$$\mathrm{SVA} = \sum_{s=1}^{n} \mathrm{SVA}_s^p = \sum_{s=1}^{n} \mathrm{EVA}_s^{p+1} = \sum_{s=1}^{n} \mathrm{EVA}_s^p (1+i(C))^{s,n}.$$

3. The EVA Theorems

In this section I provide some results on the decomposition of a NFV which include, among others, all results obtained by P&S (though stated in our systemic parlance), but I have a different outlook so that the proofs do not rest on formal properties of polynomials (as those of P&S) but on the just introduced concept of Systemic Value Added.

Reminding that $C_0{=}-a_0$ is equivalent to $E_0{=}0$ we have the following

Proposition 3.1. *If for all s C_s and C^s are both nonnegative or both nonpositive, then* (Twin).

Proof: From Definition 3 (and pointing out that $i(0)$ can be defined *ad libitum*). (Q.E.D.)

Proposition 3.2. *If $E_0{=}0$, then* (Twin) *and $C_s{=}-\overline{w}_s$ for all s.*

Proof: We have $C^s{=}0$ for all s (cf. equation (10)) and $-C_s{=}C^s - C_s{=}\overline{w}_s$ for all s. Further, $C^s{=}0$ for all s implies that, for all s, C_s and C^s are both nonnegative or both nonpositive, whence (i_P, i_N) is a twin-pair (Proposition 3.1). (Q.E.D.)

Proposition 3.3. *If $E_0{=}0$, then* NFV*$=E_n=C_n$.*

Proof: If $E_0{=}0$, we have $E^n{=}0$ (cf. equation (10)), and reminding that $w_n{=}0$ we obtain

$$\begin{aligned}\text{NFV} &= E_n - E^n \\ &= E_n \\ &= C_n + w_n = C_n.\end{aligned}$$

(Q.E.D.)

Theorem 3.1. *Assume* (Twin). *Then Peccati's model can be generalized in a two-rate capitalization of periodic shares G_s so that*

$$G_s = \text{EVA}_s\big(1+i(C)\big)^{s,n}. \tag{21a}$$

In this case, we have

$$\sum_{s=1}^{n} G_s = \sum_{s=1}^{n} \text{EVA}_s(1+i(C))^{s,n} = \text{NFV}$$

or, more explicitly,

$$\begin{aligned}\text{NFV} = &\sum_{s:w_{s-1}>0,C_{s-1}<0}^{n} w_{s-1}(x_P - i_N)(1+i(C))^{s,n} + \sum_{s:w_{s-1}<0,C_{s-1}>0}^{n} w_{s-1}(x_N - i_P)(1+i(C))^{s,n} \\ &+ \sum_{s:w_{s-1}>0,C_{s-1}>0}^{n} w_{s-1}(x_P - i_P)(1+i(C))^{s,n} + \sum_{s:w_{s-1}<0,C_{s-1}<0}^{n} w_{s-1}(x_N - i_N)(1+i(C))^{s,n}\end{aligned} \tag{21b}$$

Proof: Applying Peccati's argument (with equation (2)) and introducing the $(w_s + a_s)$ term from (4b) we have

$$\begin{aligned}G_s &= -w_{s-1}(1+i(C))^{(s-1),n} + (w_s + a_s)(1+i(C))^{s,n} \\ &= w_{s-1}(x(w_{s-1}) - i(C_{s-1}))(1+i(C))^{s,n} \\ &= [\text{by (15a)}] = \text{EVA}_s(1+i(C))^{s,n}\end{aligned}$$

and

$$\begin{aligned}\sum_{s=1}^{n} G_s &= \sum_{s=1}^{n} \text{EVA}_s(1+i(C))^{s,n} \\ &= [\text{by Theorem 2.2}] = \sum_{s=1}^{n} \text{SVA}_s \\ &= [\text{by (13)}] = \text{SVA} = \text{NFV}.\end{aligned}$$

(Q.E.D.)

Corollary 3.1. *If* $C_0{=}{-}a_0$ *then* (21) *holds.*

Proof: From Proposition 3.2 and Theorem 3.1. (Q.E.D.)

Lemma 3.1. *If* $E_0{=}0$*, then, for all* s*,*

$$\overline{x}(\overline{w}_{s\text{–}1}) = \overline{x}_P \qquad \text{iff} \qquad i(C_{s\text{–}1}) = i_N. \tag{22}$$

Proof: If $E_0{=}0$ then $\overline{w}_s{=}{-}C_s$ for all s (Proposition 3.2). Then, for all s,

$$\overline{x}(\overline{w}_{s\text{–}1}) = \overline{x}_P$$

if and only if

$$0 \leq \overline{w}_{s\text{–}1} = -C_{s\text{–}1}$$

if and only if

$$i(C_{s\text{–}1}) = i_N.$$

(Q.E.D.)

Proposition 3.4. *If* $C_0{=}{-}a_0$ *and* (Par)*, then* $w_s{\geq}0$ *implies* $C_s{\leq}0$*. Likewise,* $w_s{\leq}0$ *implies* $C_s{\geq}0$*.*

Proof: The first hypothesis implies (22) (Lemma 3.1). Eq. (22), (Par) and $w_s{\geq}0$ imply $i(C_s){=}i_N$, that is $C_s{\leq}0$. The second part is analogous. (Q.E.D.)

Corollary 3.2. *If* $C_0{=}{-}a_0$, (Par) *and* $E_s{>}0$ *for some* s*, then*

$$-C_s < w_s \leq 0 < E_s \leq C_s \qquad \textit{or} \qquad -w_s < C_s \leq 0 < E_s \leq w_s.$$

Proof: $E_s{>}0$ implies $w_s{>}{-}C_s$. Then, if $w_s{\leq}0$ we have $-C_s{<}w_s{\leq}0{<}E_s{=}w_s{+}C_s{\leq}C_s$; if $w_s{>}0$ we have $C_s{\leq}0$ (Proposition 3.4) so that $-w_s{<}C_s{\leq}\ 0{<}E_s{=}w_s{+}C_s{\leq}w_s$. (Q.E.D.)

Letting $\text{EVA}_{P,N}{:=}w_{s\text{–}1}(x_P - i_N)$ and $\text{EVA}_{N,P}{:=}w_{s\text{–}1}(x_N - i_P)$ we have the following

Theorem 3.2. *Assume* $E_0{=}0$*. Peccati's model can be generalized in a two-rate capitalization of periodic shares so that*

$$G_s = \text{EVA}_{P,N}^{\pi}\text{EVA}_{N,P}^{1-\pi}(1 + i(C))^{s,n} \tag{23a}$$

(with π *being a boolean variable*[11]*) if and only if* (Par)*. In this case, we have*

$$\begin{aligned}\text{NFV} &= \sum_{s=1}^{n} G_s \\ &= \sum_{s:w_{s\text{–}1}>0}^{n} w_{s\text{–}1}(x_P - i_N)(1 + i(C))^{s,n} + \sum_{s:w_{s\text{–}1}<0}^{n} w_{s\text{–}1}(x_N - i_P)(1 + i(C))^{s,n}\end{aligned} \tag{23b}$$

Proof: $E_0{=}0$ implies (22) (Lemma 3.1) and (Twin) (Proposition 3.2). (Twin) implies (21) (Theorem 3.1).

[11] That is, either $G_s = \text{EVA}_{P,N}(1 + i(C))^{s,n}$ or $G_s = \text{EVA}_{N,P}(1 + i(C))^{s,n}$.

Suppose first (Par). (22) and (Par) imply

$$x(w_{s-1}) = x_P \qquad \text{iff} \qquad i(C_{s-1}) = i_N.$$

The latter and (21) imply (23). Suppose now that (23) holds. Then

$$x(w_{s-1}) = x_P \qquad \text{iff} \qquad i(C_{s-1}) = i_N.$$

The latter and (22) imply (Par). (Q.E.D.)

Remark 3.1: It is worthwhile noting that Theorem 3.2 implies P&S Theorem. The latter is proved by the authors by means of a rule on the factorization of particular bivariate polynomials (see P&S, p.177, Rule 6.1). As we see, there is no need of such a rule. The proof here given rests on the economic concept of Systemic Value Added and does not depend on formal properties of polynomials (as any other proof in the paper), deriving from the more general result in Theorem 3.1.

The following Theorem mirrors Proposition 6.1 of P&S (p.179):

Theorem 3.3. *If* $C_0{=}{-}a_0$*,* (i_N-Twin) *and* (SP)*, then*

$$\text{NFV} = \sum_{s=1}^{n} w_{s-1}(x_P - i_N)(1 + i_N)^{n-s}. \tag{24}$$

Proof: (i_N-Twin) implies (Twin). (Twin) implies $\sum_{s=1}^{n} \text{SVA}_s{=}\sum_{s=1}^{n} \text{EVA}_s(1 + i(C))^{s,n}$ (Theorem 2.2). (SP) and (i_N-Twin) imply $\text{EVA}_s{=}w_{s-1}(x_P - i_N)$ and $(1+i(C))^{s,n}{=}(1+i_N)^{n-s}$. Then, (24) holds, since $\text{NFV}{=}E_n - E^n{=}\sum_{s=1}^{n} \text{SVA}_s$. (Q.E.D.)

Note that the above proof does not make use of the first assumption, so we can relax it and state the following more general:

Theorem 3.4. *If* (i_N-Twin) *and* (SP)*, then*

$$\text{NFV} = \sum_{s=1}^{n} w_{s-1}(x_P - i_N)(1 + i_N)^{n-s}.$$

Proposition 3.5. If $C_0{=}{-}a_0$ and (Par), then

$$E_n = C_n = \sum_{s:w_{s-1}>0}^{n} w_{s-1}(x_P - i_N)(1 + i(C))^{s,n} + \sum_{s:w_{s-1}<0}^{n} w_{s-1}(x_N - i_P)(1 + i(C))^{s,n} \tag{25}$$

Proof: Use Proposition 3.3 and Theorem 3.2. (Q.E.D.)

The following Proposition partially extends the results in (25) so as to be valid for every s (but note that we cannot generalize the coincidence of net worth and value of account C).

Proposition 3.6. *If* $C_0{=}{-}a_0$ *and* (Par)*, then*

$$E_s = \sum_{k:w_{k-1}>0}^{s} w_{k-1}(x_P - i_N)(1 + i(C))^{k,s} + \sum_{k:w_{k-1}<0}^{s} w_{k-1}(x_N - i_P)(1 + i(C))^{k,s} \tag{26}$$

where $1 \le k \le s$*.*

Proof: $C_0{=}-a_0$ implies $E^s{=}0$ for all s. We have then

$$\mathrm{SVA}_s = (E_s - E_{s\text{–}1})$$

so that

$$E_s = E_0 + \sum_{k=1}^{s} \mathrm{SVA}_k = \sum_{k=1}^{s} \mathrm{SVA}_k. \tag{27}$$

$E_0{=}0$ implies (Twin) (Proposition 3.2), which in turn implies (20) (Theorem 2.2). $E_0{=}0$ implies (22) (Lemma 3.1). (22) and (Par) imply

$$x(w_{s\text{–}1}) = x_P \qquad \text{iff} \qquad i(C_{s\text{–}1}) = i_N.$$

The latter and (20) imply

$$\sum_{k=1}^{s} \mathrm{SVA}_k = \sum_{k=1}^{s} \mathrm{EVA}_{P,N}^{\pi} \mathrm{EVA}_{N,P}^{1-\pi} (1 + i(C))^{k,s}$$

(with π being a boolean variable), whence, using (27), we get to (26). (Q.E.D.)

Proposition 3.7. *If* $C_0{=}-a_0$*, then*

$$E_s = E_{s\text{–}1}(1 + i(C_{s\text{–}1})) + \mathrm{EVA}_s. \tag{28}$$

Proof: We have

$$\begin{aligned}
E_s &= E_{s\text{–}1} + \mathrm{SVA}_s = E_{s\text{–}1} + x(w_{s\text{–}1})w_{s\text{–}1} + i(C_{s\text{–}1})C_{s\text{–}1} - i(C^{s\text{–}1})C^{s\text{–}1} \\
&= E_{s\text{–}1} + x(w_{s\text{–}1})w_{s\text{–}1} + i(C_{s\text{–}1})C_{s\text{–}1} \\
&= E_{s\text{–}1} + i(C_{s\text{–}1})(C_{s\text{–}1} + w_{s\text{–}1}) - i(C_{s\text{–}1})w_{s\text{–}1} + x(w_{s\text{–}1})w_{s\text{–}1} \\
&= [\text{by (4c), first equality}] {=} E_{s\text{–}1}(1 + i(C_{s\text{–}1})) + w_{s\text{–}1}(x(w_{s\text{–}1}) - i(C_{s\text{–}1})) \\
&= E_{s\text{–}1}(1 + i(C_{s\text{–}1})) + \mathrm{EVA}_s
\end{aligned} \tag{29}$$

(the last equality follows since Proposition 3.2 is verified and allows one to use Definition 8 and its eq. (15a)). (Q.E.D.)

Adding a condition of parallelism we can further specify eq. (28):

Corollary 3.3. *If* $C_0{=}-a_0$ *and* (Par)*, then*

$$E_s = E_{s\text{–}1}(1 + i(C_{s\text{–}1})) + \mathrm{EVA}_{P,N}^{\pi} \mathrm{EVA}_{N,P}^{1-\pi} \tag{30}$$

where π *is a boolean variable.*

Proof: $C_0{=}-a_0$ implies (28) (Proposition 3.7) and (22) (Lemma 3.1). Eq. (22), (Par) and (28) imply (30). (Q.E.D.)

Corollary 3.4. *If* $C_0{=}-a_0$*, we have both* $E_s{=}E_{s\text{–}1}{+}\mathrm{SVA}_s$ *and* $\mathrm{SVA}_s{=}\mathrm{EVA}_s + i(C_{s\text{–}1})E_{s\text{–}1}$.

Proof: Straightforward from the proof of Proposition 3.7. (Q.E.D.)

Remark 3.2: Corollary 3.4 informs us that when $E_0=0$ the SVA_s is the profit, and the difference between SVA_s and EVA_s is given by the interest gained on the initial net worth E_{s-1}. This Corollary enables us to appreciate the distinctive accounting quality of the SVA model in that the end-of-period net worth is given by the sum of the initial net worth and the net profit (which in this case coincides with SVA_s). Proposition 3.7 provides us with an equation according to which the sum E_{s-1} must be compounded at the rate $i(C_{s-1})$ and the EVA_s must be added to it in order to obtain the end-of-period wealth. The latter relation is such that we can see E_s as the value of an account E providing us with the periodic value of the whole invested capital. Actually, such an investment is *The Investment* pre-eminently, where the investor invests E_{s-1} at the rate $i(C_{s-1})$ and at the end of period the EVA_s is payed into account E (see (28)). Here different perspectives are at work: One reminds accounting, measuring the profit and summing to it the initial capital invested (initial wealth+profit), the other one is NFV-based, measuring the differential gain and summing to it the compounded initial wealth (compounded wealth+excess profit). This aspect is actually a salient feature of the two models: The EVA model is grounded on a financial reasoning according to which the entire wealth is an investment whose rate of return is $i(C_{s-1})$ and whose cash flows are just the EVAs. Conversely, the SVA model satisfies a sound accrual reasoning which presupposes that the end-of-period wealth is given by the initial wealth plus the profit of the period.

4. The shadow Theorems

In the previous section I have focused on the concept of EVA, but all results have been proved by means of a systemic approach. Among others, all results already proved by P&S have been restated in our systemic parlance and redemonstrated with the only aid of the notion of SVA, that is using sound economic reasoning and deduction. In this section I will focus on the concept of shadow EVA: The new results offered are the companions of those of the previous section in the sense that by employing the concept of shadow project one obtains specular conclusions while escaping the use of capitalization factors. This amounts to saying that this section's results enable us to stress that the concept of excess profit (as given by the SVA) may be seen, at the same time, both as a financial-based measure and as an accounting-based measure: It is a financial-based measure in that it depends on the cash-flow stream of the project and because it may be derived by means of an EVA-based reasoning (which is typically financial); it is an accounting-flavoured measure to the extent that issues of capitalization are ruled out and standard accounting equations (expressed in cash values) are used. Yet, the index is purely (micro)economic, in that it expresses profit in excess of normal profit. In a sense, the traditional idea that accounting-based indexes have nothing to do with financial ones is here disconfirmed, and the threefold essence of the shadow EVA (i.e. the SVA) appears to be a crucial feature of the concept of excess profit, once we adopt a systemic reasoning.

Proposition 4.1. *If* $C_0=-a_0$ *and* (Par)*, then (16) holds.*
Proof: From Proposition 3.2 and Theorem 2.1. (Q.E.D.)

Using the above Proposition we may prove the counterpart of Theorem 3.2, which is a qualification of Theorem 2.1:

Theorem 4.1. *Assume* $E_0=0$*. Then*

$$\mathrm{SVA}_s = \overline{\mathrm{EVA}}^{\pi}_{P,N}\overline{\mathrm{EVA}}^{1-\pi}_{N,P} \tag{31a}$$

if and only if (Par) *(with obvious meaning of the symbols and* π *being a boolean variable). In this case,* *we have*

$$\begin{aligned}\text{NFV} &= \text{SVA}\\ &= \sum_{s:\overline{w}_{s-1}>0}^{n} \overline{w}_{s-1}(\overline{x}_P - i_N) + \sum_{s:\overline{w}_{s-1}<0}^{n} \overline{w}_{s-1}(\overline{x}_N - i_P)\end{aligned} \tag{31b}$$

Proof: E_0=0 implies (22) (Lemma 3.1). Suppose first (Par). E_0=0 and (Par) imply (16) (Proposition 4.1). Eqs. (16) and (22) imply (31). Suppose now that (31) holds. Then (16) holds *a fortiori*. Hence,

$$\begin{aligned}\overline{w}_{s-1}(\overline{x}(\overline{w}_{s-1}) - i(C_{s-1})) &= \overline{\text{EVA}}_s\\ &= \text{SVA}_s\\ &= [\text{by (12)}] = x(w_{s-1})w_{s-1} + i(C_{s-1})C_{s-1} - i(C^{s-1})C^{s-1}\\ &= [\text{for (Twin)}] = x(w_{s-1})w_{s-1} - i(C_{s-1})\overline{w}_{s-1}\end{aligned}$$

whence

$$\overline{x}(\overline{w}_{s-1})\overline{w}_{s-1} = x(w_{s-1})w_{s-1}$$

which implies (Par). (Q.E.D.)

Lemma 4.1. *If both* (SP) *and* (S$\overline{P}$), *then* (Par). *In particular,* $x(w_{s-1})=x_P$ and $\overline{x}(\overline{w}_{s-1})=\overline{x}_P$.

Proof: Use Definitions 6 and 7. (Q.E.D.)

Now we state the counterpart of Theorem 3.3. The latter requires P to be a Soper project. But in the systemic approach we are provided with two projects, project P and its shadow $\overline{P}$. What about $\overline{P}$ in order to reach a decomposition analogous to (24)? For $\overline{P}$ to be worth of being named "shadow" of P, we expect it to adhere to project P's features. In fact, we have the following:

Theorem 4.2. *If* C_0=$-a_0$, (i_N-Twin), (SP) *and* (S$\overline{P}$), *then*

$$\text{NFV} = \sum_{s=1}^{n} \overline{w}_{s-1}(\overline{x}_P - i_N). \tag{32}$$

Proof: (SP) and (S$\overline{P}$) imply (Par) (Lemma 4.1). C_0=$-a_0$ and (Par) imply (31) (Theorem 4.1). (31) and (i_N-Twin) imply (32). (Q.E.D.)

We can relax the first assumption as the proof can be reshaped as follows:

Proof:. (i_N-Twin) implies (Twin). (SP) and (S$\overline{P}$) imply (Par), with $\overline{x}(\overline{w}_{s-1})$=$\overline{x}_P$ (Lemma 4.1). (Par) and (Twin) imply (16) (Theorem 2.1). $\overline{x}(\overline{w}_{s-1})$=$\overline{x}_P$, (16) and ($i_N$-Twin) imply (32). (Q.E.D.)

We have then proved:

Theorem 4.3. *If* (i_N-Twin), (SP) *and* (S$\overline{P}$), *then*

$$\text{NFV} = \sum_{s=1}^{n} \overline{w}_{s-1}(\overline{x}_P - i_N)$$

which is the counterpart of Theorem 3.4. The companion of Proposition 3.5 is the following:

Proposition 4.2. *If* $C_0 = -a_0$ *and* (Par), *then*

$$E_n = C_n = \sum_{s:\overline{w}_{s-1}>0}^{n} \overline{w}_{s-1}(\overline{x}_P - i_N) + \sum_{s:\overline{w}_{s-1}<0}^{n} \overline{w}_{s-1}(\overline{x}_N - i_P) \tag{33}$$

Proof: Use Proposition 3.3 and Theorem 4.1. (Q.E.D.)

A partial generalization of the above Proposition is found in the following one, which is the counterpart of Proposition 3.6:

Proposition 4.3. *If* $C_0 = -a_0$ *and* (Par), *then*

$$E_s = \sum_{k:\overline{w}_{k-1}>0}^{s} \overline{w}_{k-1}(\overline{x}_P - i_N) + \sum_{k:\overline{w}_{k-1}<0}^{s} \overline{w}_{k-1}(\overline{x}_N - i_P) \tag{34}$$

where $1 \leq k \leq s$.

Proof: $C_0 = -a_0$ implies $E^s = 0$ for all s. We have then

$$\mathrm{SVA}_s = (E_s - E_{s-1})$$

so that

$$E_s = E_0 + \sum_{k=1}^{s} \mathrm{SVA}_k = \sum_{k=1}^{s} \mathrm{SVA}_k.$$

$C_0 = -a_0$ and (Par) imply

$$\mathrm{SVA}_k = \overline{w}_{k-1}(\overline{x}_P - i_N)^{\pi}\overline{w}_{k-1}(\overline{x}_N - i_P)^{1-\pi}$$

(Theorem 4.1). We have then

$$E_s = \sum_{k=1}^{s} \mathrm{SVA}_k = \sum_{k:\overline{w}_{k-1}>0}^{s} \overline{w}_{k-1}(\overline{x}_P - i_N) + \sum_{k:\overline{w}_{k-1}<0}^{s} \overline{w}_{k-1}(\overline{x}_N - i_P)$$

with $1 \leq k \leq s$. (Q.E.D.)

The counterpart of Corollary 3.3 is:

Proposition 4.4. *If* $C_0 = -a_0$ *and* (Par), *then*

$$E_s = E_{s-1} + \overline{\mathrm{EVA}}^{\pi}_{P,N}\overline{\mathrm{EVA}}^{1-\pi}_{N,P} \tag{35}$$

where π *is a boolean variable.*

Proof: We have $E^s = 0$ for all s so that

$$E_s = E_{s-1} + \mathrm{SVA}_s.$$

$C_0 = -a_0$ and (Par) imply (31a) (Theorem 4.1), so that

$$E_s = E_{s-1} + \mathrm{SVA}_s = E_{s-1} + \overline{\mathrm{EVA}}^{\pi}_{P,N}\overline{\mathrm{EVA}}^{1-\pi}_{N,P}.$$

(Q.E.D.)

As you see, in the SVA model you just have to sum the initial period net worth to project $\overline{P}$'s EVA$_s$, whereas in the NFV-based models you have to compound the net worth and then sum it to project P's EVA. As for Corollary 3.2, in the SVA model it becomes:

Proposition 4.5. *If* $C_0{=}{-}a_0$, (Par) *and* $E_s{>}0$ *for some* s, *then*

$$\overline{w}_s < w_s \le 0 < E_s \le -\overline{w}_s \qquad or \qquad -w_s < -\overline{w}_s \le 0 < E_s \le w_s.$$

Proof: As we know, $C_0{=}{-}a_0$ implies $C_s{=}{-}\overline{w}_s$. The conclusion follows from Corollary 3.2. (Q.E.D.)

It is worthwhile noting that in case of zero net worth, account C acts as the shadow project, as the following Proposition shows:

Proposition 4.6. *If* $E_0{=}0$, *then* $C{=}\overline{P}$ *and* $E_s{=}w_s - \overline{w}_s$ *so that*

$$\begin{array}{r|l} \underline{\text{Uses}} & \underline{\text{Sources}} \\ w_s & \overline{w}_s \\ & E_s \end{array} \tag{36}$$

Proof: Obvious, since $C_s{=}{-}\overline{w}_s$. (Q.E.D.)

Remark 4.1: Let us focus on the intriguing concept of shadow project and assume, for notational convenience, $i(\cdot){=}i$, $x(\cdot),{=}x$, $\overline{x}(\cdot){=}\overline{x}$. The outstanding balance $\overline{w}_{s-1}$ is the sum the evaluator will forego, at the beginning of period s, if she undertakes P at time 0. In the latter case, she will invest the sum w_{s-1} at the rate x. Therefore, she renounces to the return $i\overline{w}_{s-1}$ in order to receive the return xw_{s-1}, which can be written as $\overline{x}\,\overline{w}_{s-1}$. The difference is the excess profit. Thus, economically, the shadow project may be interpreted as a course of action alternative to project P. In our systemic perspective alternatives (i) and (ii) may then be replaced by the two following alternatives:

(I) undertaking project P

(II) undertaking project $\overline{P}$.

The shadow project accomplishes a shift in the conceptualization of the decision-making process. The financial systems corresponding to (I) and (II) are

$$\begin{array}{r|l} \multicolumn{2}{c}{(I)} \\ \underline{\text{Uses}} & \underline{\text{Sources}} \\ C_{s-1} & E_{s-1} \\ w_{s-1} & \end{array} \qquad\qquad \begin{array}{r|l} \multicolumn{2}{c}{(II)} \\ \underline{\text{Uses}} & \underline{\text{Sources}} \\ C_{s-1} & E^{s-1} \\ \overline{w}_{s-1} & \end{array}$$

$$\begin{array}{r|l} \multicolumn{2}{c}{(I)} \\ \underline{\text{Uses}} & \underline{\text{Sources}} \\ C_s = C_{s-1}(1+i)+a_s & E_s \\ w_s = w_{s-1}(1+x)-a_s & \end{array} \qquad \begin{array}{r|l} \multicolumn{2}{c}{(II)} \\ \underline{\text{Uses}} & \underline{\text{Sources}} \\ C_s = C_{s-1}(1+i)+a_s & E^s \\ \overline{w}_s = \overline{w}_{s-1}(1+\overline{x})-\overline{a}_s & \end{array} \tag{37}$$

for time $s-1$ and s respectively. The decision maker must select the preferred alternative; (I) ensures a profit equal to $E_s - E_{s-1} = xw_{s-1} + iC_{s-1}$ whereas (II) offers a profit of $E^s - E^{s-1} = i\overline{w}_{s-1} + iC_{s-1}$ (note that sheet (II) in (37) is just (9b) in a different form). But C_s is shared by both courses of action, so that, ruling out the shared return iC_{s-1}, the return xw_{s-1} is compared to the return $i\overline{w}_{s-1}$. Stewart and Peccati, as well as P&S, implicitly replace $\overline{w}_{s-1}$ by w_{s-1} in (II) so that $E_s = E^s$ and SVA_s boils down to EVA_s (that is $xw_{s-1} - i\overline{w}_{s-1}$ turns to $xw_{s-1} - iw_{s-1}$). Such a replacement is, from a cognitive point of view, rich of implications, in that the counterfactual reasoning of the evalutor changes from a wealth-oriented outlook (focus on wealth's evolution) to a project-oriented outlook (focus on project's evolution).

From a systemic point of view, the replacement of $\overline{w}_{s-1}$ with w_{s-1} brings about some problems. Actually, if we substitute $\overline{w}_{s-1}$ for w_{s-1} for all s, we have, for s^* fixed,

$$E^{s^*-1} = C_{s^*-1} + w_{s^*-1} \tag{38a}$$

$$E^{s^*} = C_{s^*} + w_{s^*}; \tag{38b}$$

but (38a) implies

$$E^{s^*} = C_{s^*-1}(1+i) + w_{s^*-1}(1+i) \tag{38c}$$

since (II) implies that the net worth is invested at the rate i. Eqs. (38b) and (38c) are incompatible since

$$w_{s^*-1}(1+x) \neq w_{s^*-1}(1+i).$$

This whimsical result is followed by the ambiguous idea of compounding the EVAs to obtain the NFV. As we have seen, the latter can be seen as the sum of uncompounded SVAs or, alternatively, as the sum of compounded EVAs. In a sense, the SVA enables us to overlook capitalization. This is an interesting result, as it seems contrary to basic financial calculus. Further, if we sum the net profits we obtain the difference $E_n - E_0$, which is, financially speaking, the total interest gained on the net worth invested at time 0. Note also that the NFV can be seen as the sum of uncompounded shadow EVAs. We could then call the SVA model a "shadow EVA model". With the plain EVA model we have n amounts each of which referring to time s, so they must be compounded with the factor $(1+i)^{n-s}$. This seems to distort the process of imputation: $(1+i)^{n-s}$ collects interest that is generated in periods subsequent to period s. Should we regard it as belonging to the excess profit of period s? This seems to be the idea of Peccati, according to whom the s-th quota of the NFV is G_s, which refers to time n. So then, is EVA_s or $\text{EVA}_s(1+i)^{n-s}$ to be ascribed to period s? In the latter case, we impute interest that is generated in other periods. In the former case, we have n excess profits whose sum do not lead to the overall residual income (NFV): The sum of the parts does not coincide with the whole. The SVA model does not have such drawbacks. It accomplishes a perfect partition, for the sum of excess profit generates, as one expects, the overall residual income.[12]

Final comments

This paper has several goals: First of all, it aims at showing that Stewart's model, Peccati's model, Pressacco and Stucchi's model bear strong relations one another from a formal point of view; secondly,

[12] I do not state here that the EVA model is incorrect and that the SVA model is correct. The inconsistency I have shown is such only because we are in a systemic-diachronic outlook, so the evolution of the financial system is relevant. Further, the adoption of either method is, in my opinion, a matter of convention. The index the decision maker has to use depends on the information she wishes to obtain, that is on the notion of excess profit she is inclined to adopt.

it generalizes the concept of EVA by including it in a TRM framework where two-valued rates are used. Thirdly, an alternative model is offered based on a different interpretation of the notion of excess profit, first introduced in Magni (2000) and developed in Magni (2003, 2004). Following such an interpretation, which I call *systemic* or *lost-capital* (see Remark 2.1), some results on the decomposition of a NFV are shown, including all results obtained by P&S. Such results are generalized by relaxing the assumptions (for example the P&S's stringent assumption E_0=0 is replaced allowing $E_0 \in \mathbb{R}$). Fourthly, all proofs makes no use of formal properties of polynomials but are grounded on economic reasoning and deduction. Fifthly, the concept of shadow project is introduced, which enables us to partition a NFV through a systemic outlook, though using a NFV-based argument (i.e. a financial argument). Furthermore, the index we obtain does not explicitly rest on capitalization and therefore seems to formally trespass the basic rules of financial calculus (but these rules are being satisfied implicitly). Each result has its own shadow counterpart so that decomposition can be illustrated by focusing on the shadow project. Moreover, the idea of a shadow project gives us the opportunity to see the SVA model as an EVA model, where we compute the shadow project's EVA to decompose a project's NFV. Actually, the SVA model seems to be more satisfying from the point of view of the financial system's evolution and from the point of view of a correct decomposition. As for the latter, the EVA model provides us with shares whose sum does not offer the whole, as we would instead expect; as for the former, the EVA model shows some inconsistencies, which I have not analysed and which deserve further investigation. The SVA model solves these problems by neutralizing capitalization and offering indexes whose crude sum gives the whole, while from an evolutionary perspective the financial system is correctly grasped by the formal idea of dynamic system, conveniently depicted as a sequence of double-entry sheets, which record the value of the accounts at each time.

In the SVA model, as well as in P&S's model, there are some conventional elements that are worth pointing out. As we know from TRM, *op.cit.*, there are infinite internal pair (x_P, x_N) so that w_n=0: Which one is the pair to be selected for decomposing the Net Final Value in order to achieve a *correct* excess profit? P&S do not say anything about it. In my opinion the choice is conventional, only in some simple cases being straightforward (if the project is a Soper project then we have a unique internal rate of return x_P). If $C_0 = -a_0$ we could rely on the fact that

$$\text{NFV}(x_P, x_N) = w_n = -a_0(1 + x(w))^{0,n} + \sum_{s=1}^{n} a_s(1 + x(w))^{s,n} = 0.$$

We know that the NFV implicitly defines x_P as function of x_N and vice versa. We can then pick alternatively $x_P := i$ or $x_N := i$ so that

$$x_N = x_N(x_P) = x_N(i) \qquad \text{or} \qquad x_P = x_P(x_N) = x_P(i).$$

We have then

$$\text{NFV}(x_P, x_N) = \text{NFV}(i, x_N(i)) = 0 \qquad \text{or} \qquad \text{NFV}(x_P, x_N) = \text{NFV}(x_P(i), i) = 0. \tag{39}$$

The decision maker must choose one of the two above so that EVA_s, SVA_s and $\overline{\text{EVA}}_s$ will be univocally determined. The choice is not immediate and future researches could be devoted to the problem of selecting the most significant one from an economic point of view. Also, if we assume $i_P \neq i_N$, as we have done in this paper, there arise other problems: unless (i_N-twin) or (i_P-twin), there exist some periods in which $i(C_s)=i_P$ and some other periods in which $i(C_s)=i_N$. Then the evaluator does not know which is the one to be chosen in (39). Also, the idea of assuming a unique market rate i is *economically* different from our assumption of a pair (i_P, i_N). In the latter case we are assuming that funds can be borrowed at a rate i_N differing from the reinvestment rate i_P. To be precise, we are assuming that account C is a sort of current account where different rates apply depending on the sign of C, whereas TRM rest on the assumption of a unique opportunity cost of capital (obviously, if $i_P{=}i_N$ we get back to TRM's model). It is also worthwhile noting that if (i_P, i_N) is not a twin-pair, the analysis of TRM cannot be applied, since

$$\text{NFV}(x_P, x_N) \neq w_n$$

so that the concept of Net Final Value does not coincide with the concept of project balance at time n. There arises the problem of defining what an internal pair is: Is it a pair such that NFV=0 or is it a pair such that $w_n{=}0$? Theoretically, it can be interesting to investigate the behavior of the NFV in relation to w_n when $E_0{\neq}0$ and try to provide some rules in order to select the most significant pair (x_P, x_N), so that the meaning of EVA_s and SVA_s is economically transparent. However, the selection is natural if (x_P, x_N) is fixed *a priori*, which occurs whenever the project is connected to an account w (e.g. for a financial agreement) where cash flows are invested in or withdrawn from: The value of such an account is obviously w_s. In such a case, when a_s is positive, w reduces by the sum a_s while C increases by the same sum; when a_s is negative, w raises by the sum a_s and C decreases by the same sum. The decomposition is then straightforward as the four rates to be used are fixed *a priori* and univocally determined for each period by the sign of the two accounts. Operationally, if we adopt Stewart's point of view many such problems can be overlooked. According to an EVA approach, investors forecast the value of the capital invested w_s and the periodic rate of return for period s: x_s. No problem of existence or uniqueness of rate of return arises. So doing, we simply have $\text{EVA}_s = w_{s\text{–}1}(x_s - i)$ or, with debt, $\text{EVA}_s = w_{s\text{–}1}(x_s - i) + D_{s\text{–}1}(i - \delta_s)$ where δ_s is the cost of debt referred to period s; the rate i is sometimes taken as variable over time, so that i is replaced by i_s. As for the SVA_s we have $\text{SVA}_s = x_s w_{s\text{–}1} - i_s(C^s - C_s)$ or, with debt, $\text{SVA}_s = x_s w_{s\text{–}1} - \delta_s D_{s\text{–}1} - i_s(C^s - C_s)$.

The SVA model gives invaluable theoretical insights in terms of interdisciplinary research. One of these is that it provides an economic measure based on both financial and accounting reasoning. Contrary to what is usually stated, economics, finance, accountancy are reconciled just in the fundamental notion of excess profit. The systemic approach, the SVA model and the concept of shadow project give rise to a conceptual intersection between such disciplines so that we are left with a sound interdisciplinary measure: It is economic in the sense that the (micro)economic marshallian concept of excess profit is individuated; it is financial not simply because cash values are considered but also because a typically financial reasoning may be used to derive it; it is accounting-based in that standard accounting equations are used to derive it, capitalization is discharged as in accountancy, and attention is focused on the pre-eminent investment of the investor's net worth.

So this paper provides a framework which changes the NFV-based formula

$$\text{NFV} = -a_0 + \sum_{s=1}^{n} a_s(1 + i(C))^{s,n}$$

based on cash flows, into Stewart-Peccati's formula

$$\text{NFV} = \sum_{s=1}^{n} \text{EVA}_s\big(1 + i(C)\big)^{s,n}$$

based on the concept of excess profit. Hence, we dismiss capitalization and offer the systemic formula

$$\text{NFV} = \sum_{s=1}^{n} \text{SVA}_s$$

which is based on differential net profits. The latter can be in turn rewritten in terms of Economic Value Added by means of the shadow project, so that

$$\text{NFV} = \sum_{s=1}^{n} \overline{\text{EVA}}_s.$$

Both theoretical and operational developments can be investigated in future researches. From a theoretical point of view, more relations among SVA, EVA, $\overline{\text{EVA}}$ can be investigated, as well as connections between the Net Final Value of P and the Net Final Value of $\overline{P}$, and the concept of internal pair should be clarified. Further, the conceptual difference between the EVA model and the SVA model should hopefully attract attention: The notion of excess profit seems non-univocal, at least two interpretations can be proposed. Are other interpretations possible? Aren't they mere conventions (in the sense of Poincaré *op.cit.*)? And if they are, can we say they are not *arbitrary* conventions? With two different approaches at disposal of scholars and practitioners, the fundamental question should arise about the cognitive and epistemological grounds of both perspectives. In computing the excess profit, the decision maker should now answer a preliminary question: What kind of information do I expect to draw from the notion of excess profit? Only after having answered this basic question she may give up the model not in tune with her needs; if choice is made according to tradition or to chance she may be deemed as a reasoner willing to elude the problem (with the consequence that her choice is being based on nonrational motivations). Such issues are strictly connected with the problem of how one should undo a given scenario and translate it in formal terms (i.e. translate the counterfactual alternative course of action in mathematical language).[13] Cognitive psychology, currently so involved in the study of counterfactuals as a cognitive tool for decision-making (see, among others, Kahnemann and Tversky, 1982; Kahnemann and Miller, 1986; McConnell *et al.*, Lundberg and Frost, 1992; Roese, 2000) may be concerned in the development of the subject: Actually, if more than one formal translation is possible, which is the natural one? Which is the most plausible for decision makers? And what cognitive differences are there between the two approaches? Do the two approaches really translate the same counterfactual in a different way or do they formalize different counterfactual conditionals? The latter question unearthes a cornucopia of implications: As a matter of fact, the EVA-minded reasoner may be viewed as willing to obtain an answer to the following question:

> *What would my profit be if I invested in the alternative course of action?*

But she may also be viewed as willing an answer to the following different question:

> *What would the profit of a generic investor be if he held my capital w_{s-1} and invested it in the alternative course of action?*

This striking fact unveils that the EVA may be conceptualized in at least two ways (uncovering an intrinsic ambiguity?) that deserve thorough investigation.

[13] Buchanan (1969), uses the terms "might be" and "might have beens" referring to the opportunity cost (which is intrinsic in the notion of excess profit).

Moreover, one may think of an axiomatization of the notion here studied by resting on nice formal properties, economically significant, that an excess profit should satisfy. Such properties could be, for example: Additivity (the sum of the parts must equal the whole), symmetry (the index must be frame-independent, that is it should not change in absolute value if the description of the decision process is changed), time coherence (the evolution of the investor's wealth must be correctly represented), and the counterfactual operator should be a multiplicative homeomorphism (the undoing of the factual alternative must be such that the alternative course of action is genuinely counterfactual).[14]

From an operational point of view, rules should be given to forecast the correct SVA_s and thus to compute the correct path for the financial dynamic system (i.e. to draw up a correct sequence of double-entry sheets). Future researches could be addressed to extending the results by allowing for many C-type accounts and/or a portfolio of projects and/or multiple loan contracts (see Magni, 2003). Practical applications may highlight the divergent answers provided by EVA and SVA, both in terms of values and signs (for some hints on the possible answers, see Ghiselli Ricci and Magni, 2006).

As a final remark, it is important to underline that the SVA here presented presented enjoys an aggregation property which is useful for forecasting a project's (firm's) value: we have

$$\text{NPV}=\frac{1}{(1+i)^n}\sum_{s=1}^{n}\text{SVA}_s$$

Any permutation of the vector (SVA_1, SVA_2, … ,SVA_n) leads to the same NPV, which means that forecasting errors due to timing are nullified: one does not have to worry about timing, i.e. one does not have to forecast each and every SVA_s, but only needs to estimate the grand total $\sum_{s=1}^{n}\text{SVA}_s$. This may lead to invaluable insights for the theoretical notion of residual income as well as for real-life application (see also Magni, 2009).

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[14] It may be proved that such properties are satisfied by the Systemic Value Added and not by the Economic Value Added.

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