Analysis of dependencies in low frequency financial data sets

Ardia, David
Swiss Federal Institute of Technology, Zurich

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Abstract

This empirical study proposes a dependency analysis of monthly financial time series. We use the overlapping technique and non-parametric correlation in order to increase both accuracy and consistency. Copulas are used to test extreme co-movements between financial securities. Our results indicate that even in a low-frequency framework, the common practice of assuming independence over time should be taken with caution due to the presence of GARCH effects. In addition, extreme co-movements are observed across securities, especially for interest rates.

Author:	**D. Ardia**
Supervisor:	**U. A. Müller**
Co-supervisor:	**A. J. McNeil**

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1 Introduction

Until now, many studies have investigated linear and non-linear dependency in financial data sets, at weekly, daily and intradaily frequencies.

At the univariate level, it is now well established that returns contain little serial correlation, in agreement with the efficient market theory[1]. However, the absence of autocorrelation does not imply independence over time. In effect, from the series of absolute value of daily log-returns, we can clearly see the observation of Mandelbrot [24] and Fama [15] that large absolute returns are more likely than small absolute returns to be followed by a large absolute return. Ding, Granger and Engle [11] observe this effect for up to 10 years. This stylized fact is referred to as the *volatility clustering* and argues against serial independence in financial time series. To reproduce such an empirical behaviour, practitioners have developed a lot of econometric models. ARCH [14] and GARCH [3] models have been seeds of a flourishing literature since the 90's.

In a multivariate framework, it makes sense to observe cross-dependencies since assets may have similar risk exposure. In general, the linear correlation is used as a measure of dependence between financial instruments. This measure lies at the heart of traditional asset pricing models like the Capital Asset Pricing Model (CAPM) or the Arbitrage Pricing Theory (APT) and is founded on the assumption of multivariate normally distributed observations. However, in a non-Gaussian world, this measure can be misleading and does not completely describe dependencies [13, 12]. The fashionable copula methodology has given new insight on dependencies across data and has allowed to shed light on extreme co-movements previously hidden by usual measures [4, 26, 6].

Most of empirical studies on time- or cross-dependencies are based on daily observations. Surprisingly, when we switch to lower frequencies of data sets, that is monthly or quarterly intervals, few analyses have been made so far. It is of common practice to assume that monthly and longer time span observations are independent over time. However, this issue is important since certain companies, insurance companies for instance, base their risk management system on a longer time span than one day or one week. The independence assumption can have a non-negligible impact on risk estimations. The lack of observations often leads to using resampling techniques as proposed in [28] but such methodologies are not consistent if any time-dependencies in data are highly present.

In this paper, we empirically test the presence of dependencies in monthly observations. The analysis is applied to major markets stock indices, bond indices, short-term interest rates and foreign exchange rates. We extend the analysis by adding an implied volatility index and two real estate indices. In parallel to the usual log-return exploration, we develop a more complex proxy for interest rates and foreign exchange rates. For long-term analysis there are other elements that enter into the characterization of the risk. To that purpose, we introduce the notion of *innovation*. We enhance dependence measures' accuracy by the use of the overlapping technique and use non-parametric correlation to get rid of underlying distributional assumptions. We want to emphasize the fact that our study is not model-centered. By applying standard tools to our data sets, we just want to detect dependencies: in the end, data decide by themselves. Main results of univariate and bivariate analyses can be summarized as follows:

univariate:

- The profile of dependencies is different between asset classes. Whereas autocorrelation is negligible for stock and bond indices, interest rates exhibit significant time-dependencies.
- GARCH effects are still present in monthly observations and ARMA-GARCH models give a better performance in terms of Akaike information criterion.

[1] In effect, if price changes exhibit a significant correlation, this correlation may be used to conceive a simple strategy with positive expected earnings. Such strategies will tend to reduce as the market reacts to new information.

bivariate:

- We observe significant dependencies within data classes, as expected.
- The one-month lead-lag analysis does not support any leader across data classes, apart US stock indices that lead the volatility index. This is also natural since market participants change their volatility expectation according to past movements.
- The tail dependencies analysis shows a strong presence of extreme co-movements between assets, especially for interest rates.

Univariate results warn us against the independence assumption. Even if autocorrelation is weak over 1-month time spans, GARCH effects are present and GARCH type models can be used to model innovations over time. For interest rates, both autocorrelation and volatility clustering are present. Therefore, the assumption of independence should not be applied like a simple rule of thumb. In addition to univariate results, bivariate findings strongly suggest incorporating tail dependencies measures in a monthly risk management framework.

The paper continues as follows: section 2 presents the data sets and introduces the notion of innovation. In section 3 we review the standard tools used in the empirical analysis to test dependencies. Section 4 presents the results for the time dependency analysis whereas section 5 gives the results for the bivariate analysis. Section 6 concludes.

2 Data

The low-frequency dependencies analysis is applied to a large class of securities for major markets. We test stock and bond indices as well as short-term interest rates and foreign exchange rates. The study is extended with an implied volatility index and two real estate indices. Data were downloaded from *www.finance.yahoo.com*, Bloomberg and Datastream. We present hereafter data sets in detail and introduce the notion of *innovation*.

2.1 Data description

2.1.1 Stock indices

We analyse equity markets using Morgan Stanley Capital International (MSCI) stock indices[2] for Switzerland, the United Kingdom, Japan and for the United States. In addition to these 'local' indices, we use the MSCI world index which measures the equity performance of 23 developed market countries[3]. The choice of MSCI indices has been made for two main reasons. First, these indices are computed in the same manner for each market with a rigid discipline in the index maintenance. Second, MSCI indices are free-float indices, i.e. the computation of the index takes into account the available shares on the market. This methodology is clearly useful for investors who seek a benchmark which accurately reflects the number of shares to invest in. Hence, we have an interesting reflection of institutional investors point of view. MSCI indices' data consist of end-of-the-month quotes and range from January 1970 to July 2003.

We complete the equity analysis by studying the well known S&P500. This index is one of the most widely used benchmark of U.S. equity performance. It consists of 500 stocks chosen for market size, liquidity and industry group representative sample of leading companies and leading industries of the U.S. market. It is a market-value weighted index where dividends are not reinvested in. S&P500 data have been available since 1950 and consist of end-of-the-day quotes.

2.1.2 Bond indices

To analyse medium- and long-term cash investors horizons, we focus on Government Bond indices. Our selection is made on Salomon Brother's Government bond indices for the same reasons that have led us to choose MSCI indices in the equity analysis: comparability across several markets, rigorous construction and maintenance[4]. Analyzed markets are Switzerland, the United Kingdom, Japan and the United States. We also analyse a global index, the Salomon Brother's Global Government bond index. Our data consist of end-of-the-month quotes. The period ranges from January 1985 to July 2003.

2.1.3 Interest rates

Analyses for interest rates (IR) are realized using Interbank 1- and 2-month spot rates. The currencies are the Swiss Franc (CHF), the British Pound (GBP), the Japanese Yen (JPY), the European ECU and the U.S. Dollar (USD). Short-term Interbank rates are chosen in order to take a short-term cash investor's point of view. Depending on

[2]Gross indices

[3]Countries in April 2002: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Hong Kong, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Singapore, Spain, Sweden, Switzerland, the United Kingdom and the United States.

[4]Indices do not change composition very often, and changes are made in order to be easily understood and highly predictable. In addition, since 1999, Salomon Brother's has introduced a new selection methodology whose aim is that indices remain investment-grade benchmarks. A sovereign issuer must have a minimum local (internal) debt rating of BBB-Baa3 from either Standard & Poor's Corporation or Moody's Investors Service. The Salomon Brothers indices measure the total rate-of-return performance for bond market with a remaining maturity of at least one year. The total return are market-capitalisation-weighted using the security's beginning-of-the period market value. Indices are reconstitued each month.

the underlying currency, we take middle, offered or bid rates in order to get the longest available history[5]. In GBP and ECU cases, 2-month spot rates are created by interpolation between the 1- and 3-month spot rates since the 2-month spot rate is not available for the whole history. ECU is chosen instead of Euro for convenience since Euro appeared on markets on the 31/12/1999. The observation window differs across currencies. Starting dates are 04/01/1977 for CHF, 03/11/82 for ECU, 02/01/1975 for GBP, 01/07/1986 for JPY and 02/01/1986 for the USD. All time series consist of end-of-the-day quotes and end on the 31/07/2003.

2.1.4 Foreign exchange rates

To analyse foreign exchange rates, we take the same currencies as for interest rates, all given against the USD. Hence, we have CHF/USD-, GPB/USD-, JPY/USD- and ECU/USD-FX spot rates. The observation window ranges from 02/01/1980 to 31/07/2003 and consists of end-of-the-day quotes.

2.1.5 Volatility index

One of the most popular measure of investors' expectations about future stock market volatility is the Chicago Board of Exchange volatility index (VIX). The VIX is computed on a minute-by-minute basis from the implied volatilities of eight near-the-money, nearby, and second nearby OEX options (S&P100 options) series. These implied volatilities are then weighted in such a manner that the VIX represents the implied volatility of a 30-calendar days (22-trading days) at-the-money OEX options[6]. Since the VIX index started on the 02/01/1986, we use the complete history until the 31/07/2003. Our data set consists of end-of-the-day quotes.

2.1.6 Real estate indices

For real estate's data analysis, we use EPRA/NAREIT real estate indices for Switzerland and Great Britain. These indices reflect the stock performance of companies engaged in specific aspects of Swiss and British real estate markets as perceived by institutional investors[7]. For both indices, the observation window ranges from 31/01/1990 to 30/06/2003 and data consist of end-of-month quotes.

[5] CHF: bid rate; ECU,GBP: middle rate; JPY, USD: offered rate

[6] We refer the reader to *http://faculty.fuqua.duke. edu/%7whaley/pubs/fear_trading.pdf* for more details on VIX's construction.

[7] The EPRA/NAREIT Real Estate Index is calculated using official closing share price from the home exchange of all the securities included in the index. The entire amount of issued share of a constituent company is included in the calculation of the company's market capitalisation, and adjusted by the free-float weighting of the company. The index construction's methodology ensures that the underlying constituents continue to meet the basic principles of the index, and that the index continues to reflect as closely as possible the value of the underlying share portfolio. The periodicity of rebalancing is quarterly. However, adjustment in the stocks and weightings in the index do not change the index value because of the divisor adjustment.

type	name	frequency	start	end
stock indices	MSCI	monthly	31/12/70	31/07/03
	S&P500	daily	03/01/50	31/07/03
bond indices	Salomon Brother's	monthly	31/01/85	30/06/03
interest rates	CHF	daily	04/01/77	31/07/03
	GBP	daily	02/01/75	31/07/03
	JPY	daily	01/07/86	31/07/03
	ECU	daily	03/11/82	31/07/03
	USD	daily	02/01/86	31/07/03
FX rates	CHF/-, GBP/-, JPY/-, ECU/-	daily	02/01/80	31/07/03
volatility index	VIX	daily	02/01/86	31/07/03
real estate indices	EPRA/NAREIT	monthly	31/01/90	30/06/03

Table 1: Data sets

We present in table 1 the data sets used in the empirical analysis. FX spot rates are given against the USD. The third column gives the frequency of observations. The fourth and last columns give dates of first and last observations. Data were downloaded from *www.finance.yahoo.com*, Bloomberg and Datastream.

2.2 Data cleaning

A good data quality is essential for a consistent empirical analysis: especially in small sample lengths, outliers can lead to significant bias in correlation estimations. Therefore, as a first step in our analysis, we check the quality of data and clean abnormal observations accordingly. We insist to the fact that outliers are abnormal observations which cannot be explained by market conditions. An automatic filter algorithm is applied to data sets but the decision to delete or to keep observations is taken on a case-by-case basis. This cleaning process has led to remove 0.2% of the data. We refer the reader to appendix A for more details on the filtering methodology as well as for filtering results. From cleaned data, we then construct innovations.

2.3 Innovations

In literature, most empirical analyses test log-returns dependencies. Whereas it is natural to look at log-returns for stock and bond indices, it seems that the complexity of interest rates and foreign exchange rates requires a different approach, especially for longer horizons than one day or one week. Therefore, we introduce the notion of *innovation* also referred to as *risk factor*. An innovation quantifies the deviation from the market expectation and hence characterizes the risk[8]. For IR and FX rates, this deviation can be measured by comparing forward and spot rates. We present hereafter the definition of the innovation for each class of securities tested in the empirical analysis.

2.3.1 Stock, bond and real estate indices

For stock, bond and real estate indices, innovation at time t is defined as

$$X_t := \ln(S_t) - \ln(S_{t-1}) \tag{1}$$

where S_t is the index level at time t. This definition approximates the relative change in the index (return). In effect, we have

$$\ln(S_t) - \ln(S_{t-1}) = \ln\left(\frac{S_t}{S_{t-1}}\right) = \ln\left(1 + \frac{S_t - S_{t-1}}{S_{t-1}}\right) \approx \frac{S_t - S_{t-1}}{S_{t-1}}$$

for a small relative change.

[8] 'Market expectation' has to be understood as a 'fair' value under the market conditions. This is directly related to the no arbitrage conditions.

2.3.2 Volatility

For the volatility, innovation is simply defined as the difference in volatility over a time interval, that is

$$X_t := S_t - S_{t-1}$$

where S_t is the level of the volatility (in percent) at time t.

2.3.3 Interest rates

For interest rates (IR), we use two kinds of innovations. The first one is defined the same way as for stock indices (def.1) applied to the one-month spot rates. This definition is the most used in empirical studies so far. However, as it was mentioned before, we need to go beyond this in order to capture 'deformation components' of the yield curve. To that aim, we develop a second definition which is suggested in [28]. It is based on spot rate and implied forward rate's comparison.

We start from the fact that the yield curve consists of annualized interest rates $r(\tau)$ as a function of maturity τ (expressed in years). Then, we transform rates in logarithmic form $R(\tau)$. Formally, the equivalent instantaneous interest rate $R(\tau)$ is given by

$$R(\tau) := \frac{1}{\tau} \ln\left(1 + r(\tau)\tau\right)$$

This has various advantages, mainly transforming the multiplicative compounding of interest rates to simple additive compounding. The forward interest rate from maturity τ_1 to τ_2 is given by $F(\tau_1, \tau_2)$. An implied forward interest rate can be constructed using basic interest compounding rules[9] as follows:

$$F(\tau_1, \tau_2) = \frac{\tau_2 R(\tau_2) - \tau_1 R(\tau_1)}{\tau_2 - \tau_1}$$

The next step consists of introducing the time dimension. We denote by $F_t(\tau_1, \tau_2)$ the value of the implied forward interest rate at time t for maturity τ_1 to τ_2. This rate is computed using $R_t(\tau_1)$ and $R_t(\tau_2)$, spot rates values at time t for maturities τ_1 and τ_2. To construct the innovation, we first notice that the price of a forward interest rate $F_t(\tau, \tau+\Delta\tau)$ reflects the current market consensus forecast of the spot rate $R_{t+\tau}(\Delta\tau)$. That is, at maturity $t+\tau$ of the forward contract, we can directly read the value $R_{t+\tau}(\Delta\tau)$ of the spot rate for the maturity $\Delta\tau$. Said differently, $F_t(\tau, \tau+\Delta\tau)$ is the market predictor for $R_{t+\tau}(\Delta\tau)$. Of course, this market consensus forecast is not static since as time moves, interest rates continuously improve and adapt market news, until maturity is reached. In a surprise-free market, we should observe that $F_t(\tau, \tau+\Delta\tau)$ equals $R_{t+\tau}(\Delta\tau)$ for all t. But the reality is a stream of unexpected news that lead to deviations in the forecast. This deviation is precisely our innovation. Hence, from the implied forward interest rate we define the innovation at time t by

$$X_t := R_t(\Delta\tau) - F_{t-\tau}(\tau, \tau+\Delta\tau)$$

From now on, we give the extension .new for these innovations. The usual method of taking log-returns of a one-month spot rate is given with the extension .old.

2.3.4 Foreign exchange rates

Foreign exchange (FX) rates can be treated in a simpler way than interest rates. The market's unbiased predictor of an FX spot rate at time t is the FX forward rate at time $t-\tau$ with maturity τ. An FX forward rate depends on

[9] See [19], pages 93-95.

the difference of interest rates of the two involved currency zones. This condition comes from the covered interest rate parity. The FX implied forward rate of a foreign currency $\alpha \in \{\mathrm{CHF}, \mathrm{GBP}, \mathrm{JPY}, \mathrm{ECU}\}$ against the USD, at time t and for a maturity τ is defined by

$$F_{\alpha/\mathrm{USD},t}(\tau) := S_{\alpha/\mathrm{USD},t}\left(\frac{1 + r_{\alpha,t}(\tau)\tau}{1 + r_{\mathrm{USD},t}(\tau)\tau}\right) \tag{2}$$

where $S_{\alpha/\mathrm{USD},t}$ is the FX spot rate at time t and $r_{\alpha,t}(\tau)$ is the spot rate at time t with maturity τ for the foreign currency α. From the relation (2), we define the innovation at time t by

$$X_t := S_{\alpha/\mathrm{USD},t} - F_{\alpha/\mathrm{USD},t-\tau}(\tau)$$

that is the deviation of the FX spot rate at t from its unbiased predictor, the FX implied forward rate with maturity τ evaluated at time $t-\tau$. These innovations are referred to as new innovations. As in the IR case, we analyse log-returns, here computed on FX spot rates. In that case, we give the extension .old to innovations time series.

2.4 Monthly innovations' construction

The lack of data in a low-frequency framework leads inevitably to a poor statistical accuracy. We can overcome this problem by taking a larger observation window. In this case however, the stationarity assumption can become questionable. Therefore, we require the use of overlapped observations in order to increase the sample size. From the original daily time series, we create five monthly time series. This methodology allows to decrease the error variance and leads to more accurate results in terms of estimation. However, since observations are not independent anymore we need to adjust test statistics.

2.4.1 An example

We start with S&P500 daily quotes ranging from Monday, March 3rd to Wednesday, April 30th, 2003. Our aim is to construct five monthly observations from this sample.

The first monthly observation is simply constructed by taking March and April's last valid quotes. In our example, this monthly observation is constructed using the quotes from Monday, March 31th and Wednesday, April 30th (MM31-AW30). Twenty-two valid days separate these two observations. Then, we create four additional monthly observations. We split both March and April into four intervals intervals and take the first valid days preceding splitting dates. In our example, we obtain the following results: for March, dates are Friday 7th, Thursday 13th, Wednesday 19th and Tuesday 25th. For April: Friday 4th, Friday 11th, Thursday 17th and Thursday 24th. From these four couples of days, we construct four additional monthly observations. We have MF7-AF4, MT13-AF11, MW19-AT19, MT25-AT24. Numbers of valid days within each monthly interval are 21, 22, 22 and 23 days.

2.4.2 Pros. and cons. of overlapping technique

Overlapping technique has both advantages and drawbacks. By increasing the number of observations it provides more accurate estimates. For instance, Müller [27] shows that, under Gaussian distribution assumption, overlapping technique reduces by one-third the stochastic error when computing the variance's estimation of log-returns. A similar result is obtained for the covariance between two time series. Such gains are certainly not negligible when dealing with small data sets. On the other hand, this method creates spurious autocorrelation effects. In a hypothetical framework where intervals are constant and observations are log-returns, these effects can be quantified precisely. The aggregation property of log-returns leads to the creation of moving-average (MA) effects.

In our particular case, both advantages and drawbacks are difficult to quantify since observation intervals have different lengths, Gaussian distribution is questionable and innovations do not necessarily aggregate. However, the gain lies between zero and one third, and is therefore not negligible. Since we cannot quantify precisely autocorrelation effects up to lag one, we ignore them in the empirical analysis. In addition, we adopt an *ad-hoc* procedure to adjust test statistics. In effect, statistics distributions are derived under the assumption that observations are independent. Since with the overlapping process we create dependent observations, we replace the number of independent observations by the *effective number* of observations, as suggested in [27]. This number is given by

$$n_{\text{eff}} := \frac{3mn}{2m^2+1}\left(1 - \frac{m(m^2-1)}{2(2m^2+1)n}\right) \tag{3}$$

where n is the total number of observations (with overlapping), and m is the number of overlap ($n \geq m$). In our case, $m = 5$ since we have five monthly observations constructed from a one-month daily sample.

We illustrate in appendix B that, on average, the number of significant lags given by the ad-hoc procedure is close to the number given by monthly non-overlapped observations, whereas using the whole number of observations leads to overestimating the number of significant results. Based on these results, we decide to focus on overlapped observations in order to get more accuracy and correct statistics by taking the number of effective observations instead of the total number of observations.

2.5 Innovations analysis

2.5.1 Descriptive analysis

In table 2 we present some descriptive statistics for innovations time series. First, we notice that the annualized mean is significantly positive at the 95% level for stock, bond and real estate indices whereas negative for IR new innovations. For IR.old and FX innovations, means are not significantly different from zero (beside IR.USD.old). In terms of Sharpe ratio[10], MSCI.UK outperforms stock indices. More surprisingly SLOMON.US overperforms both equity and bond indices with a Sharp ratio of 1.78. The skewness coefficient is significantly negative for stock indices. Skewness varies in sign and value across IR and FX rates. However, it is of the the same sign between new and old innovations (beside the IR.JPY). The kurtosis is significantly positive for stock indices and IR, indicating leptokurtic distributions. Distributions of new innovations for IR and FX are more centered and less heavy-tailed than for old innovations. Finally, we observe that VIX innovations' distribution is highly skewed[11].

In figure 1 we present quantile-quantile plots for SLOMON.US and IR.JPY.old innovations. On the left hand-side, points lying on the straight line indicate that SLOMON.US innovations' distribution is almost Gaussian. However, for IR.JPY.old we observe fat tails and peakdness in the empirical distribution: normal distribution is clearly rejected.

[10] The Sharpe ratio is defined as $\frac{\mu}{\sigma}$, i.e. the risk-free interest rate is set at 0 for simplicity.

[11] We can explain this fact by two phenomena. First, individuals overreact bad news which implies more frequent increases than decreases in volatility. Second, the volatility is bounded from below.

type	name	overlap	n	n_{ind}	$\hat{\mu}$	$\hat{\sigma}$	$\widehat{skew}$	$\widehat{kurt}$
stock indices	MSCI.CH	F	403	403	**0.079**	0.175	**-0.751**	**3.052**
	MSCI.US	F	403	403	**0.101**	0.158	**-0.561**	**2.349**
	MSCI.UK	F	403	403	**0.118**	0.206	**0.302**	**8.367**
	MSCI.JP	F	403	403	0.067	0.186	**-0.268**	**1.256**
	MSCI.GLOB	F	403	403	**0.089**	0.142	**-0.891**	**2.914**
	S&P500	T	3210	642	**0.077**	0.148	**-0.747**	**3.672**
bond indices	SLOMON.CH	F	221	221	**0.052**	0.034	-0.101	0.639
	SLOMON.US	F	221	221	**0.086**	0.048	0.006	0.008
	SLOMON.UK	F	221	221	**0.100**	0.063	-0.081	**1.473**
	SLOMON.JP	F	221	221	**0.055**	0.044	**-0.522**	**2.062**
	SLOMON.GLOB	F	221	221	**0.094**	0.070	0.171	-0.087
interest rates	IR.CHF.new	T	1590	318	**-0.020**	0.021	**-0.245**	**3.335**
	IR.USD.new	T	1110	222	**-0.013**	0.009	**-0.702**	**5.379**
	IR.GBP.new	T	1775	355	**-0.011**	0.021	**1.014**	**7.268**
	IR.JPY.new	T	1020	204	**-0.005**	0.006	**-0.570**	**2.888**
	IR.ECU.new	T	1240	248	**-0.013**	0.016	**-1.455**	**14.396**
	IR.CHF.old	T	1590	318	-0.073	1.137	**-0.434**	**12.702**
	IR.USD.old	T	1110	222	**-0.108**	0.218	**-0.802**	**4.705**
	IR.GBP.old	T	1775	355	-0.041	0.239	**0.986**	**6.456**
	IR.JPY.old	T	1020	204	-0.280	0.980	**0.941**	**25.797**
	IR.ECU.old	T	1240	248	-0.079	0.215	**-1.177**	**9.187**
FX rates	FX.CHFUSD.new	T	1050	210	-0.040	0.177	-0.204	0.099
	FX.GBPUSD.new	T	1050	210	-0.005	0.062	**0.657**	**1.823**
	FX.JPYUSD.new	T	1020	204	-2.308	14.513	**-0.470**	**1.066**
	FX.ECUUSD.new	T	1050	210	-0.013	0.095	-0.097	0.347
	FX.CHFUSD.old	T	1410	282	-0.007	0.125	**-0.259**	0.279
	FX.GBPUSD.old	T	1410	282	0.014	0.107	0.220	**2.176**
	FX.JPYUSD.old	T	1410	282	-0.030	0.119	**-0.604**	**1.058**
	FX.EURUSD.old	T	270	282	0.011	0.110	-0.098	-0.248
volatility	VIX	T	1050	210	0.163	25.889	**3.029**	**130.984**
real estate	REAL.ESTATE.CH	F	162	162	**0.077**	0.083	0.312	**1.583**
	REAL.ESTATE.UK	F	162	162	**0.062**	0.185	-0.325	-0.116

Table 2: Innovations data sets

We present in table 2 innovations' descriptive statistics. Column 3 indicates whether overlapping technique is used (T=true, F=false). Column 4 gives the total number of observations n. Column 5 gives the total of independent observations (end-of-the-month). Columns 6-9 give respectively the mean $\hat{\mu}$ (annualized), the standard deviation $\hat{\sigma}$ (annualized), the skewness $\widehat{skew}$ and the excess kurtosis $\widehat{kurt}$. Significant results at the 95% level are displayed for the mean, the skewness and the kurtosis in **bold face** format. To test the significance we compute the following statistics

$$t_{\hat{\mu}} := \frac{\hat{\mu}}{\hat{\sigma}/\sqrt{n_{\text{eff}}}} \qquad t_{\widehat{skew}} := \widehat{skew}\sqrt{\tfrac{6}{n_{\text{eff}}}} \qquad t_{\widehat{kurt}} := \widehat{kurt}\sqrt{\frac{24}{n_{\text{eff}}}}$$

We notice that we use the number of effective observations instead of the whole number of observations in order to avoid the overlapping bias. $t_{\hat{\mu}}$ follows a Student-t distribution with $n_{\text{eff}} - 1$ degrees-of-freedom, whereas $t_{\widehat{skew}}$ and $t_{\widehat{kurt}}$ follow a standard normal distribution. We conclude that the sample mean is significantly different from zero when $|t_{\hat{\mu}}|$ is higher than the upper 2.5%-quantile of the Student-t distribution with $n_{\text{eff}} - 1$ degrees-of-freedom.

Skewness is the asymmetric coefficient of the probability density function (PDF). The excess kurtosis is defined such that a positive value of $kurt$ indicates a 'fat tail', that is, a slow asymptotic decay of the PDF. We recall that skewness and excess kurtosis are 0 for a Gaussian distribution. Innovations' definitions are given in section 2.3. Observation windows vary across time series (see table 1).

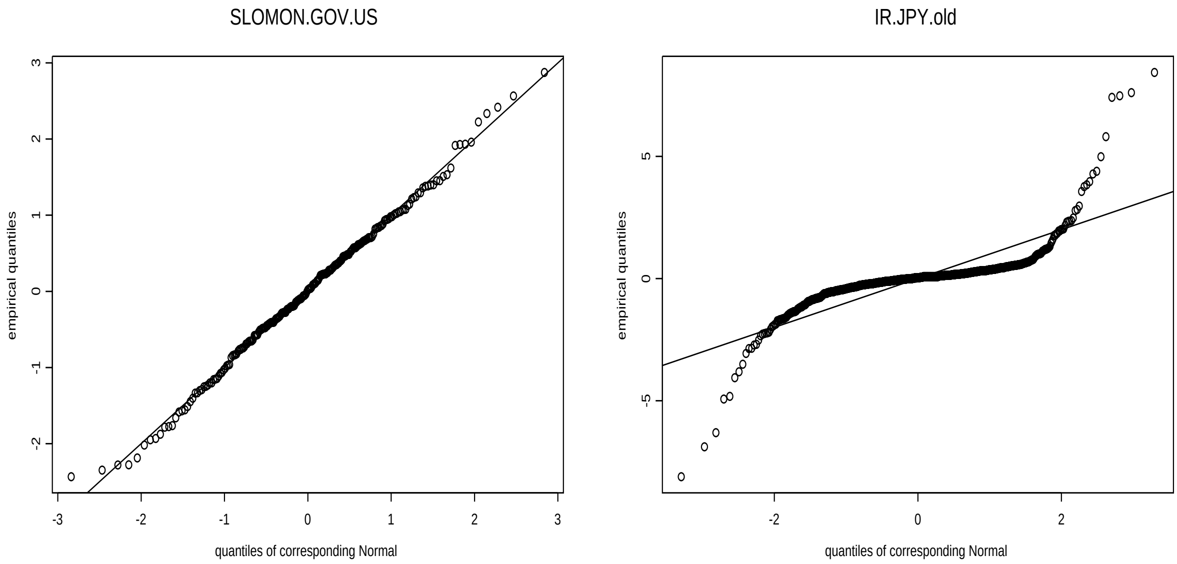


Figure 1: Quantile-quantile plots

In figure 1, we present normal quantile-quantile plots for SLOMON.GOV.US and IR.JPY.old innovations time series. The horizontal axis indicates quantiles of a normal distribution (with the same mean and standard deviation as the empirical distribution). The vertical axis indicates empirical quantiles.

2.5.2 Stationarity testing

We test the stationarity of innovations time series using Kwiatkowski, Phillips, Schmidt and Shin (KPSS) statistic[12]. On the whole sample, only two rejections of the $I(0)$ hypothesis at the 5% significance level have been detected: for IR.ECU.new (fig. 2) and IR.USD.old innovations time series. However, on the 32 innovations time series, 5% implies 1.6 theoretical rejections. Hence, we cannot conclude to a strong rejection of the stationary hypothesis for these time series. For more detailed explanations of the testing procedure we refer the reader to [35].

[12]Whereas the augmented Dickey-Fuller (ADF) and Phillips Perron (PP) unit root tests are for the null hypothesis that a time series is $I(1)$ (integrated of order 1), KPSS tests the null that the time series is $I(0)$. In general, the ADF and PP tests have very low power against $I(0)$ alternative that are close to being $I(1)$. That is, unit root tests cannot distinguish highly persistent stationary processes from nonstationary processes very well. The KPSS allows to overcome this problem.

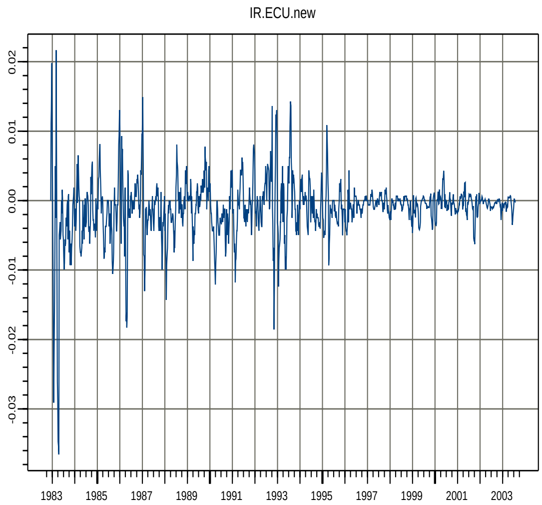


Figure 2: IR.ECU.new time series

In figure 2 we present IR.ECU.new innovation time series. This time series as well as IR.USD.old has been detected to be non-stationary by the KPSS statistic at the 5% level.

3 How to test dependencies ?

This section reviews some tools usually used to detect linear and non-linear dependencies between data. In addition to formal definitions we comment their advantages and drawbacks.

3.1 Pearson's correlation

3.1.1 Definition

The most widely used measure of association between variables is the *linear correlation* or *Pearson's correlation* coefficient. Given $\{\mathbf{X}_t, t \in \mathbb{Z}\} := \{X_{i,t}; t \in \mathbb{Z}; i = 1, ..., d\}$, a d-dimensional random process, the correlation at lag k between variable $X_{i,t}$ and $X_{j,t}$ is defined by

$$\rho_{i,j}(k) := \rho(X_{i,t}, X_{j,t-k}) := \frac{\langle X_{i,t}, X_{j,t-k}\rangle}{\sqrt{\langle X_{i,t}\rangle \langle X_{j,t-k}\rangle}} \tag{4}$$

where

$$\langle X_{i,t}, X_{j,t-k}\rangle := \mathbb{E}((X_{i,t} - \mathbb{E}(X_{i,t}))(X_{j,t-k} - \mathbb{E}(X_{j,t-k}))) \tag{5}$$

$$\langle X_{i,t}\rangle := \langle X_{i,t}, X_{i,t}\rangle \tag{6}$$

Relation (5) expresses the covariance between variables $X_{i,t}$ and $X_{j,t-k}$ whereas relation (6) expresses the variance of variable $X_{i,t}$. The function $\mathbb{E}$ is the expectation operator. When we assume stationarity of $\mathbf{X}_t$, the denominator does not depend on time anymore and variance of $X_{i,t}$ simplifies to $\langle X_i\rangle$. The value of ρ lies between -1 and 1, inclusive. It takes on a value of 1, termed "complete positive correlation" when the variable $X_{i,t}$ can be expressed as $\alpha + \beta X_{j,t}$ (almost surely) where $\beta = 1$. A value near zero indicates that variables are uncorrelated. The natural multivariate representation of the correlation coefficient is the correlation matrix. This matrix is denoted by $\Sigma(k)$ where the (i, j)-th component is defined by

$$[\Sigma(k)]_{i,j} := \rho_{i,j}(k) \quad 1 \leq i, j \leq d \tag{7}$$

This matrix is the *lead-lag correlation matrix* at lag k. It is neither symmetric nor positive semi-definite in general.

From definition (7), we can notice the two particular cases:

- $\Sigma(0)$ is the *contemporaneous cross-correlation* matrix between our set of random variables. This matrix is symmetric, positive semi-definite.
- $[\Sigma(k)]_{i,i}$ is the *autocorrelation* of random variable X_i at lag k.

To estimate this matrix empirically, we start from a d-dimensional sample with n observations denoted by $\mathbf{x} := \{x_{i,t}; t = 1, ...n; i = 1, ..., d\}$. The sample correlation estimate at lag k, between variables $X_{i,t}$ and $X_{j,t}$ is given by

$$\hat{\rho}_{i,j}(k) := \frac{\sum_{t=k+1}^{n} z_{i,t} z_{j,t-k}}{\sqrt{(\sum_{t=1}^{n} z_{i,t}^2)(\sum_{t=1}^{n} z_{j,t}^2)}} \tag{8}$$

with $z_{i,t} := (x_{i,t} - \overline{x}_i)$, where $\overline{x}_i := \frac{1}{n}\sum_{k=1}^{n} x_{i,k}$ denotes the sample mean. From these coefficients, we construct the *sample lead-lag correlation matrix* by

$$\left[\hat{\Sigma}(k)\right]_{i,j} := \hat{\rho}_{i,j}(k) \quad 1 \leq i, j \leq d \tag{9}$$

We notice that this definition implicitly assumes strict stationarity since it uses the whole information (n observations) to compute the variance estimates[13]. To test whether our sample estimate is not significantly different from zero we use the statistic defined by

$$\text{t} := \hat{\rho}_{i,j} \sqrt{\frac{n-2}{1-\hat{\rho}_{i,j}^2}} \tag{10}$$

where n stands for the number of independent observations[14]. Under the null hypothesis the statistic follows a Student's distribution with $n-2$ degrees-of-freedom.

3.1.2 Comments

Pearson's correlation coefficient has the advantage of being a real-number easy to compute and to interpret. However, it suffers from a large number of drawbacks. Firstly, it only detects linear dependencies in data; non-linear patterns, even simple ones, cannot be measured. Secondly, correlation is only defined when the variance is finite. Thirdly, Pearson's coefficient is not a distribution-free measure. It describes completely the dependence structure in a normal world. However, it is now well-known and empirically proved that the Gaussian framework does not describe reality, especially due to the presence of heavy-tails in empirical distributions. Finally, linear correlation is not invariant under non-linear strictly increasing transformations. For these reasons, we should be careful when drawing quantitative conclusions from this coefficient.

3.1.3 Transformations

From its definition, we clearly see that correlation is highly influenced by the variance. Hence, even few extreme observations can imply a high variance in the denominator, and therefore, can bias the correlation coefficient. Analyzing correlation through a range of power will attenuate this problem. Therefore, in the empirical analysis, the usual Pearson's correlation coefficient is calculated for observations x_t as well as for transformed observations $|x_t|^p$ where $p \in \{0.5, 1, 1.5, 2\}$. Low powers will attenuate the bias in estimation whereas high powers will shed light on correlation in tails.

3.2 Spearman's rank correlation

3.2.1 Definition

The incertainty in interpreting the significance of linear correlation leads us to the concept of Spearman's rank correlation. This correlation is a distribution-free analog of Pearson's correlation that measures the monotone association between variables. Given $\{\mathbf{X}_t, t \in \mathbb{Z}\}$, a d-dimensional random process, the rank correlation between variable $X_{i,t}$ and $X_{j,t}$ is given by

$$\varrho_{i,j}(k) := \rho(F_{X_i}(X_{i,t}), F_{X_j}(X_{j,t-k})) \tag{11}$$

where F_{X_i} denotes the distribution function of random variable X_i and ρ is given in (4). We estimate it empirically from the d-dimensional sample $\mathbf{x}$ by

$$\hat{\varrho}_{i,j}(k) := \hat{\rho}(\text{rank}(x_{i,t}), \text{rank}(x_{j,t-k})) \tag{12}$$

where rank($\bullet$) is the position in the empirical distribution and $\hat{\rho}$ the sample correlation coefficient defined in (8). Statistical testing for rank correlation is made in the same manner as in Pearson's case (10). The range of Spearman's correlation is from -1 to 1, inclusive.

[13]This methodology is defined by default in S-Plus. Sometimes however, the definition differs slightly in order to speed up numerical computations. This is the case for EViews software. These differences are negligible for large samples but can lead to significant differences in small samples. Since in our case, observations sets are quite small, the maximum of information is used in order to increase results accuracy.

[14]See [34] for further details.

3.2.2 Comments

The key concept of nonparametric correlation is to replace the value of each observation $x_{i,t}$ by the value of its rank in the sample, rank$(x_{i,t})$. Hence, the resulting list of numbers is drawn from a uniform distribution function, eliminating the problem of unknown underlying distribution. Therefore, nonparametric correlation is robust against underlying distribution. In addition, this measure is invariant under strictly monotonic transformation and is defined even when the variance is not finite. All these characteristics are desirable in analyzing dependencies in financial data.

3.3 Box-Pierce and Ljung-Box

The Box-Pierce (BP) [5] and Ljung-Box (LB) [23] tests are based on sample autocorrelation coefficients. However, instead of testing the randomness at each distinct lag, they test the 'overall' randomness based on a number of lags k. The Box-Pierce statistic is given by

$$BP(k) := n \sum_{j=1}^{k} \hat{\rho}(j)^2 \tag{13}$$

where n is the number of independent observations and $\hat{\rho}_x(j)$ is defined in (8)[15]. However, Ljung and Box show that substantially improved approximation can be made from the BP statistic, especially for small samples. They adapt the BP statistic by

$$LB(k) := n(n+2) \sum_{j=1}^{k} \frac{1}{n-j} \hat{\rho}(j)^2 \tag{14}$$

Asymptotically, both statistics are χ^2_k distributed. When the statistic is significant at lag k, this means that the statistic has detected presence of autocorrelation up to lag k. We can notice that LB gives more emphasis to later autocorrelation than BP does. Hence, this statistic will provide a more accurate estimation of autocorrelation if it occurs at a high lag.

3.4 Copulas

To move away from simple scalar measures of dependence as it is the case for Person's correlation and Spearman's rank correlation and capture the overall dependence structure contained within the joint distribution of innovations, we use the notion of Copulas.

Copulas: *A d-dimensional copula is a distribution function $C : [0,1]^d \mapsto [0,1]$ which satisfies*

1 *For all $u_i \in [0,1], C(1, ..., 1, u_i, 1, ..., 1) = u_i$*

2 *$C(u_1, ..., u_d)$ is increasing in each u_i*

Copulas capture the dependence structure of the multivariate distribution. This property comes from the following fundamental theorem:

[15] We can either use Pearson's or Spearman's correlation.

Sklar's theorem: *Given a joint distribution function H of a random vector $(X_1, ..., X_d)$ with continuous margins $F_1, ..., F_d$, then there exists a unique d-dimensional copula $C : [0,1]^d \mapsto [0,1]$ such that*

$$H(X_1, ..., X_d) = C(F_1(X_1), ..., F_d(X_d)) \tag{15}$$

Hence, from this theorem, we see that for continuous multivariate distribution functions, the univariate marginal distributions and the multivariate dependence structure can be separated, and the dependence structure can be represented by a copula. To spell how this unique copula is related to the joint distribution we use the following Corollary:

Corollary: *Let H be the joint distribution function of a random vector $(X_1, ..., X_d)$, with continuous margins $F_1, ..., F_d$ and copula C satisfying (15). Then, for any $(u_1, ..., u_d) \in [0,1]^d$, we have*

$$C(u_1, ..., u_d) = H(F_1^{-1}(u_1), ..., F_d^{-1}(u_d)) \tag{16}$$

where $F^{-1}(s) := \inf\{x \in \mathbb{R} : F(x) \geq s\}$ is the generalized inverse of F defined on $[0,1]$. Without the continuity assumption, this may not hold. Unlike the linear correlation that captures the full dependence structure in multivariate normal distributions, the copula summarizes this dependence structure for any joint distribution (with continuous marginals). In addition, if $(X_1, ..., X_d)$ has continuous margins, and $T_1, ..., T_d$ are strictly increasing transformations, then $(T_1(X_1), ..., T_d(X_d))$ has the same copula as $(X_1, ..., X_d)$ but not in general the same correlation matrix.

Many copula functions exist in literature, either belonging to the fundamental, the implicit parametric or explicit parametric family. We present hereafter the two copulas that are used in this study, namely the Gaussian and the Student-t copulas[16]. The fundamental difference between these two copulas lies in their respective tail dependence structure. The concept of tail dependence relates the amount of dependence in the upper-right-quadrant tail or lower-left-quadrant tail of a bivariate distribution. It turns out that tail dependence between two continuous random variables is a copula property. It can be shown that, in the Gaussian copula case, we have tail dependence (perfect tail dependence) only when the linear correlation is 1, otherwise, the tail dependence is zero. However, it is empirically observed that extreme co-movements happen even when the correlation is not perfect between securities.

It is therefore important to consider copulas that possess the property of non-trivial tail dependence. This is the case with the Student-t copula. In effect, for random variables which are linked via a Student-t copula, we can expect joint extreme movements to occur with non-negligible probability, even when the random variables exhibit small correlation. It is essentially the degree-of-freedom which controls the extent of tail dependence and tendency to exhibit extreme co-movements.

3.4.1 Gaussian copula

Let Φ denote the standard normal distribution function and let Φ_Σ^d denote the d-dimensional normal distribution function, with zero mean, unit variance for each margin and linear correlation matrix $\Sigma \in \mathbb{R}^{d \times d}$. That is, for $\mathbf{x} \in \mathbb{R}^d$

$$\Phi_\Sigma^d(\mathbf{x}) = \int_{-\infty}^{\mathbf{x}} \frac{1}{(2\pi)^{d/2}|\Sigma|^{0.5}} \exp(-0.5\mathbf{y}'\Sigma^{-1}\mathbf{y})d\mathbf{y}$$

where $|\Sigma|$ is the determinant of Σ. Then, for $\mathbf{u} := (u_1, ..., u_d) \in [0,1]^d$, we define the Gaussian copula as

$$C^G(\mathbf{u}; \Sigma) := \Phi_\Sigma^d(\Phi^{-1}(\mathbf{u}))$$

where $\Phi^{-1}(\mathbf{u}) := (\Phi^{-1}(u_1), ..., \Phi^{-1}(u_d))$.

[16] These copulas belong to the implicit parametric copulas family.

3.4.2 Student-t copula

Let t_ν denote the (standard) univariate Student-t distribution function with ν degrees-of-freedom, formally

$$t_\nu(x) := \int_{-\infty}^{x} \frac{\Gamma((\nu+1)/2)}{\Gamma(\nu/2)(\nu\pi)^{0.5}}(1+y^2/\nu)^{-(\nu+1)/2}dy$$

We denote the d-dimensional Student-t distribution, with ν degrees-of-freedom and shape parameter matrix $\Sigma \in \mathbb{R}^{d\times d}$ by

$$t^d_{\nu,\Sigma}(\mathbf{x}) := \int_{-\infty}^{\mathbf{x}} \frac{\Gamma((\nu+1)/2)}{\Gamma(\nu/2)(\nu\pi)^{0.5}|\Sigma|^{0.5}}(1+\mathbf{y}'\Sigma^{-1}\mathbf{y}/\nu)^{-(\nu+1)/2}d\mathbf{y}$$

where $\mathbf{x}\in\mathbb{R}^d$. Then, for $\mathbf{u}\in[0,1]^d$, the Student-t copula is defined by

$$C(\mathbf{u};\nu,\Sigma) := t^d_{\nu,\Sigma}(t_\nu^{-1}(\mathbf{u}))$$

where $t_\nu^{-1}(\mathbf{u}) := \left(t_\nu^{-1}(u_1),...,t_\nu(u_d)\right)$. If $\nu > 2$, the shape parameter matrix Σ can be interpreted as the linear correlation matrix. We recall that the multivariate t-distribution is a generalization of the multivariate normal in the sense that the normal distribution can be considered as a t-distribution with infinite degrees of freedom. Gaussian and Student-t-copulas are essentially indistinguishable for degrees-of-freedom that are greater than 10^4.

4 Time dependency analysis

We present in this section a time dependencies analysis of monthly innovations. The goal of our investigation is two fold. First of all, by using Pearson's and Spearman's rank correlation we aim to detect time dependencies in data sets. Secondly, if measures conclude to significant dependencies over time, we aim to determine which kind of model is able to better reproduce observed patterns.

4.1 Autocorrelation analysis

The univariate analysis consists mainly of computing autocorrelation for different lag horizons. An example of the sample autocorrelation function (SACF) is given for the S&P500 in figure 3. In addition to the computation of Pearson's autocorrelation for innovations themselves, we compute the autocorrelation for transformed innovations $|x_t|^p$, $p \in \{0.5, 1, 1.5, 2\}$ in order to diminish or reinforce extreme observations. Furthermore, Spearman's autocorrelation is computed to support or contradict Pearson's results. Since Spearman's autocorrelation is invariant under monotonic increasing transformation, we compute it for absolute values of innovations, i.e. p=1. Significant results are determined at the 99% level using (10).

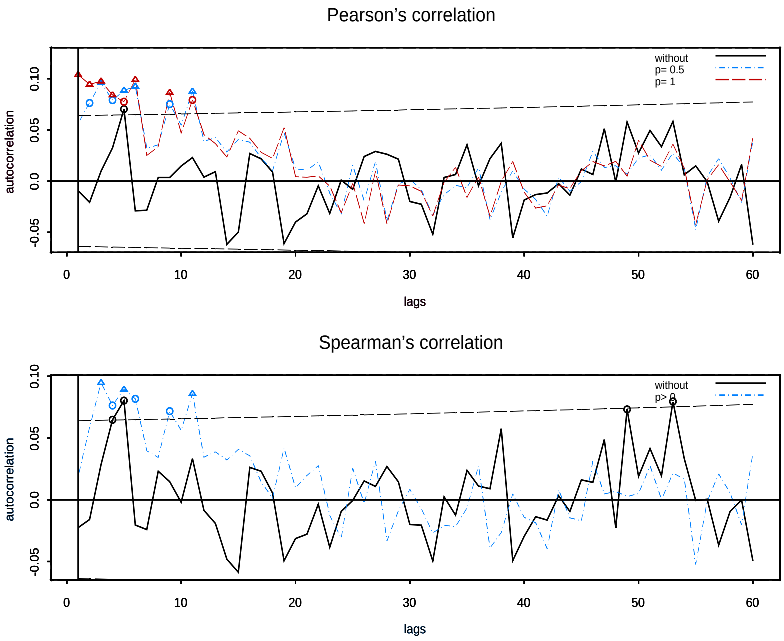


Figure 3: SACF – S&P500

In figure 3, we present the sample autocorrelation function (SACF) of the S&P500 over 60 months. Given observations $\{x_{i,t}; t = 1, \dots, n\}$ of the random process $\{X_{i,t}; t \in \mathbb{Z}\}$, the SACF function is the plot of $\{(k, \hat{\rho}_{i,i}(k)), k = 1, \dots, K\}$ where K is the maximum horizon analyzed. In the upper plot, Pearson's correlation is computed for innovations (full line) and transformed innovations $|x_{i,t}|^p$, for $p \in \{0.5, 1\}$ (dashed lines). Presence of autocorrelation in absolute values indicates volatility clustering effects. To avoid the problem of the normal distribution assumption, we compute on the lower plot, Spearman's rank autocorrelations (full line). Since this correlation is invariant under increasing transformations, we compute Spearman's rank correlation for absolute values of innovations (dashed line). Significant autocorrelations at the 95% level (99 % level) are indicated by a circle (triangle). The band about the zero autocorrelation line represent 99% significance of the hypothesis of independent Gaussian observations, that is $\pm 2.57/\sqrt{n_{\text{eff}}}$. They are not straight since, as the lag increases, the number of effective observations decreases.

We display in table 3 the overall lags which exhibit significant autocorrelation for each time series. At a first look, we can distinguish two groups of data.

For stock, bond and real estate indices and FX rates, neither systematic nor persistent autocorrelation patterns is observed. For autocorrelation of transformed innovations, Pearson's and Spearman's results give slightly more significant lags, indicating volatility clustering. We can notice the special case of SLOMON.JP innovations, where we observe a 3-month cycle in transformed innovations.

On the contrary, interest rates show a larger number of significant lags, for both original and transformed innovations. Sometimes, volatility clustering is present for up to 24 months! Differences emerge from new and old innovations. The use of new innovations decreases the autocorrelation but increases the volatility clustering effect. When analyzing autocorrelation relative to the powers, we observe that even for $p = 2$, numerous lags are significant and are supported by Spearman. This indicates that interest rates are more sensitive to clustering in tails than other assets. An example is given in figure 4 for the IR.CHF.new time series. The autocorrelation at lag 1 is computed for transformed innovations and $p \in]0, 8]$. In this case, even if the highest correlation is observed for powers which are close to one, autocorrelation decreases slowly and cancels only near $p = 7$.

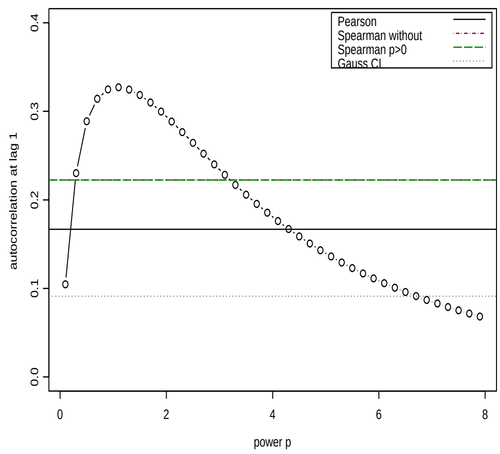


Figure 4: IR.CHF.new innovation

We present in figure 4 the autocorrelation at lag 1 of $|x_t|^p$ as a function of the power p for IR.CHF.new innovations. The dashed line is the upper limit of the 99% confidence band of the hypothesis of independent Gaussian observations.

Finally, we observe that the VIX index exhibits significant autocorrelation at lag 1, and autocorrelation of absolute values up to lag 4.

		Pearson's					Spearman's	
name	K	untransformed	$p=0.5$	$p=1$	$p=1.5$	$p=2$	untransformed	$p=1$
MSCI.CH	60	10,14	1,4	1,4,14	1,14	1,14		4
MSCI.US	60		3,10,28	3,28	3		44	3,10
MSCI.UK	60	23	2,4,5,30	1-5,10	1-3,5,10	1,2,10		19,30
MSCI.JP	60	27	2	2	2,37			2
MSCI.GLOB	60						44	49
S&P500	60		3,5-6,11	1-4,6,9	1-2	1		3,5,11
SLOMON.CH	60	1,17					1	8
SLOMON.US	60		1,13	13	3,13	3		13
SLOMON.UK	60		8	6	6	6		8
SLOMON.JP	60		1,3,6,9	3	3		1,58	1,3,6,9,11,19
SLOMON.GLOB	60	11	1	1	1		1,8,11	1,9,10
IR.CHF.new	24	1,3,8	1-8,10,12,14,16,18,22,24	1-13,16-18,22,24	1-13,16-18	1-13,17	1	1-8,10,12,14,16,18,20,22,24
IR.USD.new	24	13,18,20	1,7,14	1,3,7,10,14	1,3,10,14	1,10,14	1,13,20	1,7,14
IR.GBP.new	24	7,10	1-14,17,19,20,24	1,2,5-8,10,12,14,19,20,24	1,5-7,12	1,5,6	10,12,15	1-15,19,20,22-24
IR.JPYnew	24		1-3,5,6,14,23	1,2,5,12,14,24	12,24	12,24		1-6,8,13,14,17,23,24
IR.ECU.new	24	12	1,3-6,8,9,11,12,16,17,20-22	2-4,8,9,12	2	2		1,3-6,8,9,11,12,14-17,19-23
IR.CHF.old	24	5,12,14	1-20,23,24	1-20	1-18	1-12,14,15	3,6,12,15,18,24	1-20,23,24
IR.USD.old	24	3,6,7,9,12,24	1,2	1	1	1	3,6,9,12,13,24	1,2
IR.GBP.old	24	1,19	1-3,5-8,12,14,19,20	1,2,5-8,12-14,18-20	1,2,5-8,13,14,19	1,5-8,14,19	1	1-3,5,6
IR.JPY.old	24	1,3,12,24	1-5,7-9,11-16,23,24	1,3,4,7-9,11-16,24	1,7-9,12,15,16	1,8	3,6,9,12,15,21,23,24	1,3,4-8,12,13,18,23,24
IR.ECU.old	24	3	1	1	1	1	3	1,12
FX.JPYUSD.new	24	11		17	17	11	5,11	14
FX.GBPUSD.new	24		5,19	5,19	1,19	1,19		5,19
FX.CHFUSD.new	24				1,2	1,2		
FX.ECUUSD.new	24							
FX.CHFUSD.old	24							
FX.GBPUSD.old	24		3	3	1,3	1,3	21	10
FX.JPYUSD.old	24	11					8,11	
FX.ECUUSD.old	24							
VIX	24	1	1-3	1	1	1	1	1-4,23
REAL.ESTATE.CH	60	1	1,28	1,28	1	1	50,55	28,30,58
REAL.ESTATE.UK	60							

Table 3: Significant lags

We present in table 3 significant autocorrelation lags at the 99% level for innovations time series. Column 2 gives K, the maximum lag horizon analyzed. Columns 3-7 give results for Pearson's correlation (def.8) for the original innovations time series (untransformed) and for transformations ($p \in \{0.5, 1, 1.5, 2\}$). Last two columns give results for Spearman's rank correlation (def.12) applied to original innovations time series (untransformed) and for absolute values ($p=1$).

Now, we go further in the SACF analysis and present hereafter numerical values of Spearman's autocorrelation for S&P500, SLOMON.JP, and VIX innovations time series. Results in table 4 show that the S&P500 does not exhibit significant autocorrelation. In the case of SLOMON.JP, we observe significant autocorrelation at lag 1 and significant volatility clustering at lags $\{1, 3, 6, 9\}$ all positive. This indicates a 3 months-cycle pattern in volatility clustering. For the VIX index, the significant negative autocorrelation at lag 1 suggests a mean-reverting behaviour[17]. Level of autocorrelation for absolute value is almost constant up to lag 4 and indicates presence of clusters in volatility's changes.

	1	2	3	4	5	6	9	12	18	24
SP500	-0.02	-0.01	0.03	0.06	0.08	-0.02	0.01	0.00	0.00	0.00
	0.02	*0.06*	***0.09***	*0.07*	***0.09***	*0.08*	*0.03*	*0.03*	*0.00*	*0.00*
SLOMON.JP	**0.22**	0.13	-0.06	-0.09	-0.08	0.08	0.07	0.08	0.08	-0.07
	0.22	*0.16*	***0.35***	*0.09*	*0.16*	***0.22***	***0.21***	*0.11*	*0.10*	*0.06*
VIX	**-0.19**	-0.04	-0.08	-0.04	0.04	0.00	0.05	0.08	0.10	-0.07
	0.22	***0.15***	***0.16***	***0.16***	*0.03*	*0.11*	*0.12*	*0.06*	*-0.03*	*0.06*

Table 4: Autocorrelation estimates

We present in table 4 Spearman's autocorrelation (def.12) up to lag 24 for S&P500, SLOMON.JP and VIX innovations time series. Results for absolute values are given in *italic*. For both original and absolute innovations, significant autocorrelations at the 99% level are given in **bold face** format. 0.00 indicates that the value is lower than the second digit.

When analyzing interest rates in details, we observe from table 5 that autocorrelation is positive for significant lags. This is also the case for transformed innovation which argues for volatility clustering effects. All new innovations time series exhibit less autocorrelations than old definitions. In addition, if this autocorrelation is present in new innovations time series, it locates only at lag one. It is more difficult to give such a clear conclusion for volatility clustering since results differ across time series. However, when restricted to lags given in the table, IR.CHF, IR.USD and IR.JPY suggest that new innovations are less clustered that old one.

name		1	2	3	4	5	6	9	12	18	24
IR CHF	new	**0.22**	0.03	0.14	0.00	-0.02	-0.05	-0.06	-0.08	0.01	0.1
		0.22	***0.27***	***0.19***	***0.23***	*0.13*	***0.21***	*0.12*	***0.22***	***0.23***	***0.23***
	old	-0.06	-0.07	**0.18**	-0.04	-0.07	**0.12**	0.09	**0.21**	**0.12**	**0.22**
		0.34	***0.36***	***0.37***	***0.31***	***0.30***	***0.28***	***0.24***	***0.28***	***0.17***	***0.16***
IR USD	new	**0.18**	0.00	0.07	-0.03	-0.06	-0.02	0.03	0.00	0.13	0.12
		0.24	*0.12*	*0.14*	*0.09*	*0.08*	*0.09*	*0.04*	*0.03*	*-0.02*	*0.14*
	old	0.14	0.10	**0.26**	0.08	0.07	**0.27**	**0.18**	**0.21**	0.08	**0.20**
		0.37	***0.22***	*0.13*	*0.05*	*0.04*	*-0.01*	*0.05*	*0.12*	*0.02*	*0.13*
IR GBP	new	0.10	0.09	-0.05	0.01	0.09	0.10	0.00	**0.14**	0.04	0.00
		0.33	***0.23***	***0.19***	***0.21***	***0.27***	***0.19***	***0.15***	***0.22***	0.10	***0.18***
	old	**0.19**	0.11	0.04	0.00	0.05	0.03	0.02	0.02	0.02	0.04
		0.26	***0.13***	***0.15***	0.10	***0.15***	***0.15***	0.03	0.08	0.08	0.08
IR JPY	new	-0.05	0.01	0.00	-0.07	-0.06	0.07	0.00	0.11	0.07	0.10
		0.29	***0.34***	***0.22***	***0.20***	***0.27***	***0.30***	*0.15*	*0.10*	*0.14*	***0.19***
	old	-0.06	-0.02	**0.25**	-0.02	-0.15	**0.18**	**0.18**	**0.36**	0.10	**0.38**
		0.24	*0.13*	***0.20***	***0.17***	***0.16***	***0.19***	***0.20***	***0.34***	***0.15***	***0.27***
IR ECU	new	0.09	0.00	0.05	-0.08	-0.02	0.01	-0.04	0.08	-0.06	-0.01
		0.21	*0.10*	***0.30***	***0.26***	***0.22***	***0.19***	***0.25***	***0.21***	*0.12*	*0.12*
	old	0.08	0.06	**0.16**	0.12	0.04	0.06	0.00	0.10	-0.09	0.07
		0.17	*0.09*	*0.08*	*0.07*	*0.03*	*0.03*	*-0.01*	***0.14***	*0.03*	*-0.04*

Table 5: Autocorrelation estimates – interest rates

We present in table 5 Spearman's autocorrelation (def.12) up to lag 24 months for interest rates new and old innovations time series. Results for absolute values are given in *italic*. For both original and absolute innovations, significant autocorrelations at the 99% level are given in **bold face** format. 0.00 indicates that the value is lower than the second digit.

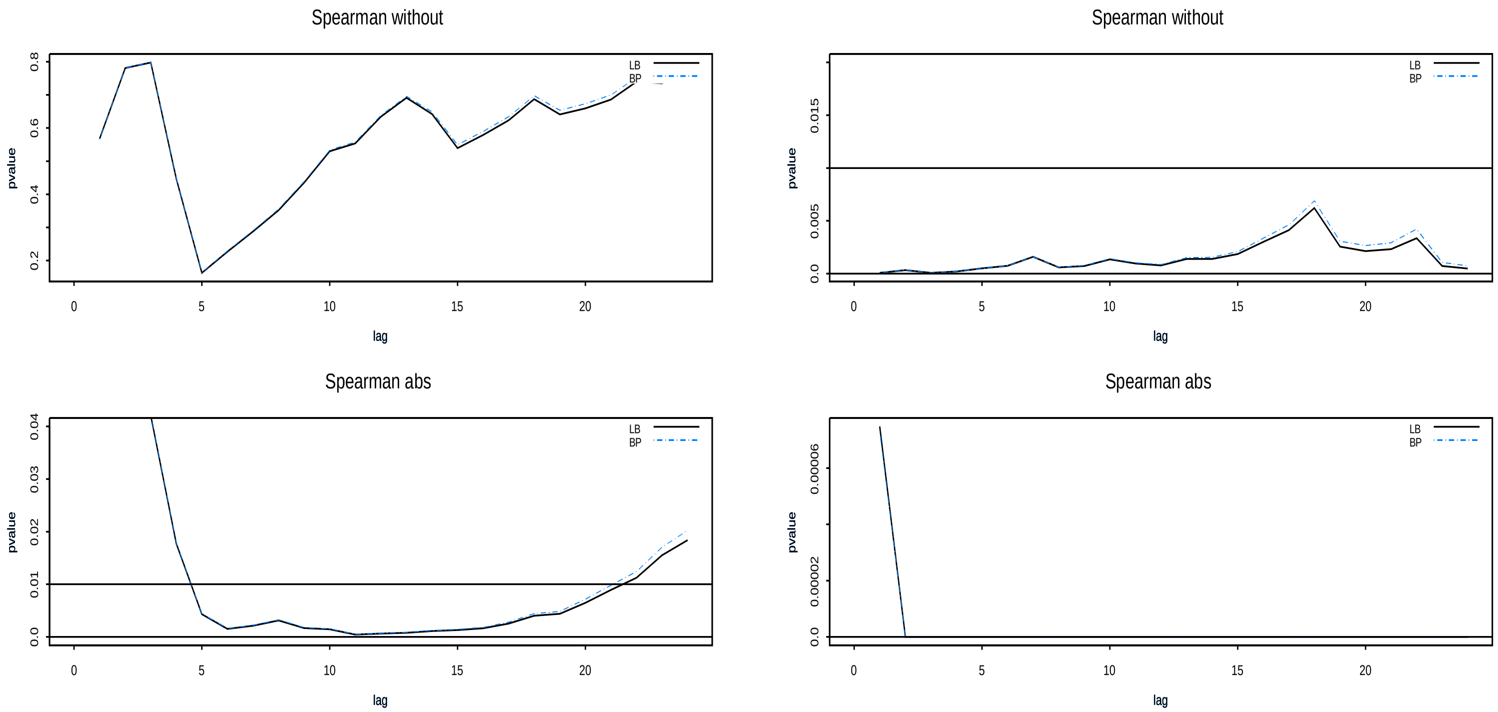
[17] also referred as to *overshooting*.

4.2 Box-Pierce and Ljung-Box

The SACF analysis can be extended by testing the overall presence of autocorrelation. To that purpose, we compute the Ljung-Box (LB) and Box-Pierce (BP) statistics in order to determine up to which lag autocorrelation is present.

Figure 5 plots LB and BP p-values for S&P500 (left) and IR.CHF.new (right) innovations time series. In the case of S&P500, Spearman's LB and BP do not exhibit significant autocorrelation in the original time series. On the other hand, LB and BP p-values lie below the 1% p-value for transformed innovations, indicating volatility clustering.

Results for volatility clustering is similar for IR.CHF.new innovations. In addition, p-value results for original innovations indicate that autocorrelation is also present. In this case, the time dependency is brought to light by both autocorrelation and volatility clustering. For other IR innovations time series, the same conclusion holds. This suggests the need for GARCH models to reproduce innovations' evolutions.



S&P500 IR.CHF.new

Figure 5: Box-Pierce and Ljung-Box p-values

We present in figure 5 p-values of LB (full line) and BP (dashed line) statistics computed with Spearman's rank correlation. Uppers graphics give results for original innovations whereas lower graphics give results for absolute values of innovations. The horizontal line above the horizontal axis represents the 1% p-value level. A LB or BP p-value lying below the 1% level at lag k implies the rejection of the hypothesis that there is no autocorrelation up to lag k. Formal definition of LB and BP are given in (def.14) and (def.13).

4.3 Time series models fitting

Univariate tools used previously suggest that time dependency is present, even in monthly time series. The autocorrelation for absolute values of innovations is stronger than for original time series themselves, suggesting volatility clustering patterns. In this section, we test the adequacy of selected time series models to fit non-overlapped monthly innovations. Models tested are of ARMA and GARCH types. The former models aim to reproduce autocorrelation whereas latter capture volatility clustering effects.

4.3.1 ARMA models

The covariance stationary[18] univariate process $\{X_t, t \in \mathbb{Z}\}$ is an ARMA(p,q) [19] model if, for all t, it satisfies

$$X_t - \sum_{i=1}^{p} \varphi_i X_{t-i} = \epsilon_t + \sum_{j=1}^{q} \theta_j \epsilon_{t-j} \tag{17}$$

where $\epsilon \sim WN(0, \sigma_\epsilon^2)$ is a white noise process[20] with mean zero and variance σ_ϵ^2. The condition $\sum_{i=1}^{p} \varphi_i < 1$ implies that the model is stationary.

Within this class of models, we test five particular cases: MA(1), MA(2), AR(1), AR(2) and ARMA(1,1)[21]. These models are fitted to non-overlapped monthly innovations by maximum likelihood using the function *arima.mle* in S-Plus[22]. To determine the best model for a particular data set we focus on the Akaike information criterion[23]. The smaller the AIC is, the better the model fits the data.

Coefficients estimates as well as AIC are given in table 6. We observe that, in general, models MA(1) and ARMA(1,1) give better AIC results. For stock indices, apart for in Switzerland, all models are of MA(1) or MA(2) types. This suggests that autocorrelation is only present up to lag 1 or 2 since MA(k) autocorrelations cut off at lag k. For the S&P 500, the MA(1) coefficient is not significantly different from zero at the 99% level. Here, in line with previous results, no autocorrelation patterns are observed.

For bond indices, the autocorrelation is captured by both AR and MA effects. In that class of security, the ARMA(1,1) model gives the better performance.

Finally, we can notice differences between FX old and new innovations. Whereas ARMA type gives better performance for new definitions, which suggests dependence over time, best models for old innovations are of MA(1) type. In addition, all estimates for this model are not significantly different from zero. Therefore we reject autocorrelation for old definition as our previous results have suggested.

[18] Covariance stationary stands for a process with constant mean and where covariance between variable only depends on the time interval's lenght.

[19] Autoregressive Moving-Average.

[20] A white noise process (WN) is a covariance stationary process where increments are uncorrelated.

[21] Particular cases of (17) are:

MA(1):	$p = 0, q = 1$
MA(2):	$p = 0, q = 2$
AR(1):	$p = 1, q = 0$
AR(2):	$p = 2, q = 0$
ARMA(1,1):	$p = 1, q = 1$

[22] The assumption of normality distribution for error terms ϵ in (17) is often violated in practice. By performing a maximum likelihood under the hypothesis that errors are normally distributed even if they are not, we estimate the parameters by quasi-maximum likelihood (QML).

[23] For a model M_j with k_j parameters $\theta_j := (\theta_{j,1}, ..., \theta_{j,k_j})$ and the likelihood $l_j(\theta_j; \mathbf{x})$, the Akaike information criterion is given by $AIC(M_j) := -2 \ln l_j(\hat{\theta}_j; \mathbf{x}) + 2k_j$

name	model	$\hat{\varphi}_1$	$\hat{\varphi}_2$	$\hat{\theta}_1$	$\hat{\theta}_2$	AIC
MSCI.CH	5	**-0.621**		**0.760**		-1260
MSCI.US	1			0.052		-1332
MSCI.UK	1			**0.131**	0.074	-1125
MSCI.JP	1			0.074		-1210
MSCI.GLOB	1			**0.125**		-1426
SP500	1			0.045		-2236
SLOMON.CH	5	**0.888**		**-0.674**		-1411
SLOMON.US	5	**0.997**		**-1.000**		-1247
SLOMON.UK	5	**0.997**		**-1.000**		-1131
SLOMON.JP	2			**0.210**	**-0.247**	-1296
SLOMON.GLOB	2			**0.234**	-0.151	-1085
IR.CHF.new	5	**0.829**		**-0.646**		-2323
IR.USD.new	1			**0.274**		-2003
IR.GBP.new	2			0.053	-0.108	-2605
IR.JPY.new	1			0.013		-2048
IR.ECU.new	4	0.154	**0.163**			-2030
IR.CHF.old	3	-0.082				181
IR.USD.old	5	**0.927**		**-0.827**		-594
IR.GBP.old	1			0.130		-908
IR.JPY.old	1			-0.175		44
IR.ECU.old	5	-0.666		0.723		-738
FX.CHFUSD.new	5	**-0.686**		**0.812**		-668
FX.GBPUSD.new	1			0.062		-1091
FX.JPYUSD.new	4	-0.004	0.077			1167
FX.ECUUSD.new	5	**-0.649**		**0.801**		-931
FX.CHFUSD.old	1			0.086		-1087
FX.GBPUSD.old	1			0.066		-1156
FX.JPYUSD.old	1			0.032		-1085
FX.ECUUSD.old	1			0.083		-1155
VIX	5	**0.608**		**-0.913**		1231
REAL.ESTATE.CH	2			0.284	-0.175	-749
REAL.ESTATE.UK	1			**0.219**		-495

Table 6: ARMA results

We present in table 6 the best ARMA model for each time series. Column 2 gives model's number: 1=MA(1), 2=MA(2), 3=AR(1), 4=AR(2), 5=ARMA(1,1). Columns 3-6 give parameters estimates. Parameters that are significantly different from zero at the 99% level are given in **bold face** format. Last column give the AIC information criterion value. Parameters are fitted by QML using *arima.mle* function in S-Plus.

name	model	$\hat{\mu}$	$\hat{\varphi}_1$	$\hat{\theta}_1$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\beta}_1$	AIC
MSCI.CH	10	0.010	**-0.661**	**0.768**	0.000	**0.084**		**0.876**	-1280
MSCI.US	10	**0.000**	**0.999**	**1.018**	0.000	**0.077**		**0.851**	-1357
MSCI.UK	9	**0.010**			0.000	**0.142**		**0.777**	-1181
MSCI.JP	9	0.007			0.000	**0.059**		**0.923**	-1229
MSCI.GLOB	9	**0.008**			0.000	**0.048**		**0.792**	-1432
SP500	9	**0.006**			0.000	**0.075**		**0.875**	-2269
SLOMON.CH	10	**0.002**	**0.557**	0.344	0.000	0.075		**0.908**	-1427
SLOMON.US	10	**0.008**	-0.158	-0.310	0.000	0.032		**0.931**	-1258
SLOMON.UK	9	**0.008**			0.000	**0.165**		**0.725**	-1151
SLOMON.JP	9	**0.004**			**0.000**	**0.384**		**0.590**	-1319
SLOMON.GLOB	10	0.005	0.261	0.090	0.000	0.020		**0.956**	-1099
IR.CHF.new	10	**-0.001**	0.205	-0.034	0.000	**0.126**		**0.856**	-2458
IR.USD.new	10	**-0.002**	**-0.595**	**-0.755**	0.000	**0.063**		**0.913**	-2062
IR.GBP.new	10	**0.000**	**0.984**	**1.018**	0.000	**0.101**		**0.886**	-2830
IR.JPY.new	9	**0.000**			0.000	**0.055**		**0.935**	-2137
IR.ECU.new	9	**-0.001**			0.000	**0.240**		**0.783**	-2155
IR.CHF.old	9	-0.011			**0.001**	**0.202**		**0.822**	-119
IR.USD.old	10	0.000	**0.938**	**0.844**	0.000	**0.088**		**0.865**	-601
IR.GBP.old	10	-0.001	**0.674**	0.557	0.000	**0.034**		**0.960**	-984
IR.JPY.old	10	-0.001	0.512	**0.716**	0.000	**0.312**		**0.784**	-134
IR.ECU.old	10	-0.001	**0.689**	0.527	0.001	**0.308**		**0.337**	-739
FX.CHFUSD.new	8				0.001	0.121		0.632	-654
FX.GBPUSD.new	8				0.000	**0.158**		**0.667**	-1101
FX.JPYUSD.new	7				**14.652**	0.010	0.206		1173
FX.ECUUSD.new	8				0.000	0.041		**0.925**	-918
FX.CHFUSD.old	6				**0.001**	0.136			-1087
FX.GBPUSD.old	8				0.000	**0.066**		**0.918**	-1169
FX.JPYUSD.old	6				**0.001**	0.024			-1083
FX.ECUUSD.old	8				0.000	0.019		**0.961**	-1152
VIX	7				**7.154**	**0.264**	**0.981**		1214
REAL.ESTATE.CH	9	**0.007**			**0.000**	**0.303**		**0.607**	-762
REAL.ESTATE.UK	10	0.003	0.571	0.413	0.000	0.046		**0.855**	-489

Table 7: ARMA-GARCH results

We present in table 7 the best ARMA-GARCH model for each time series. Column 2 gives the model's number: 6=ARCH(1), 7=ARCH(2), 8=GARCH(1,1), 9=C-GARCH(1,1), 10=ARMA(1,1)-GARCH(1,1). Columns 3-9 give parameters estimates. Parameter that are significantly different from zero at the 99% level are given in **bold face** format. Last column give the AIC information criterion value. Parameters are fitted by QML using *garch* function in S-Plus (FinMetrics library).

4.3.2 ARMA-GARCH models

The univariate process $\{X_t, t \in \mathbb{Z}\}$ is an ARMA(p_1, q_1)-GARCH(p_2, q_2)[24] model, if, for all t, it satisfies

$$\begin{aligned} X_t &= \mu_t + \epsilon_t \\ \mu_t &:= \mu + \sum_{i=1}^{p_1} \varphi_i (X_{t-i} - \mu) + \sum_{j=1}^{q_1} \theta_j \epsilon_{t-j} \\ \epsilon_t &:= \sigma_t Z_t \\ \sigma_t^2 &:= \alpha_0 + \sum_{i=1}^{p_2} \alpha_i (X_{t-i} - \mu_{t-i})^2 + \sum_{j=1}^{q_2} \beta_j \sigma_{t-j}^2 \end{aligned} \tag{18}$$

where $\alpha_0 > 0$, $\alpha_i \geq 0$ $(i = 1, .., p_2)$, $\beta_j \geq 0$ $(j = 1, ..., q_2)$ and $Z_t \sim SWN(0, 1)$ is a normalized strict white noise[25] independent of $\{X_s, s \leq t\}$. We notice that positive parameters ensure that the conditional variance σ_t^2 remains positive[26]. The process is covariance stationary when $\sum_{i=1}^{p_1} \varphi_i < 1$ and $\sum_{i=1}^{p_2} \alpha_i + \sum_{j=1}^{q_2} \beta_j < 1$. The unconditional variance of the process is given by $\alpha_0/(1 - \sum_{i=1}^{p_2} \alpha_i - \sum_{j=1}^{q_2} \beta_j)$.

Within this class of models, we test five particular cases: ARCH(1), ARCH(2), GARCH(1,1), C-GARCH(1,1) and ARMA(1,1)-GARCH(1,1)[27]. These models are fitted by QML to non-overlapped monthly observations using the function *garch* in S-Plus[28]. Since GARCH models can be treated as ARMA models for squared residuals, the traditional Akaike information criterion is used for selecting best models.

Coefficients estimates as well as AIC are given in table 7. For most time series, ARCH and GARCH coefficient are significantly different from zero, indicating volatility clustering patterns. In addition, ARMA coefficients are significant for some time series, in particular for interest rates. Here, both autoregressive and GARCH effects are captured by the ARMA-GARCH model.

Nevertheless, a more attentive look at coefficients estimates sheds light on an interesting phenomenon. In the case of VIX, IR.ECU.new, IR.CHF.old and IR.JP.old time series, the resulting unconditional variance is negative! This does not seem to be a small-sample effect since the other time series contain the same number of observations but estimated GARCH variances are well defined. For these four time series, the high kurtosis observed in table 2 is certainly at the origin of the misspecification in fitted GARCH models.

Comparison between ARMA and GARCH models is given in table 8. For each class of asset, we make the cumulative sum of AIC numbers[29]. Although the ARMA(1,1) model overperforms GARCH(1,1) for FX.new innovations, the ARMA-GARCH type gives a better performance in general.

Therefore, even over monthly intervals, autocorrelation and volatility clustering have to be taken into account. It is now clear that the independence's rule of thumb is too strong a hypothesis and can lead to underestimating the

[24] Autoregressive moving-average mean with Generalized Autoregressive Conditionally Heteroscedatic errors.

[25] A strict white noise is a process of independent, identically distributed, finite variance increments.

[26] It is a sufficient but not necessary condition.

[27] Particular cases of (18) are :

ARCH(1):	$p_2 = 1, p_1 = q_1 = q_2 = 0, \mu = 0$
ARCH(2):	$p_2 = 2, p_1 = q_1 = q_2 = 0, \mu = 0$
GARCH(1,1):	$p_2 = q_2 = 1, p_1 = q_1 = 0, \mu = 0$
C-GARCH(1,1):	$p_2 = q_2 = 1, p_1 = q_1 = 0$
ARMA(1,1)-GARCH(1,1):	$p_1 = q_1 = p_2 = q_2 = 1$

[28] FinMetrics library. We set the BHHH.control argument *positive* at *true* in order to constraint the coefficients in the variance equation of GARCH models to be positive during the optimization process.

[29] In addition to this methodology, we ranked each model and aggregated the rank within each security's classes. Conclusions are the same as with AIC numbers.

risks. In addition, we should be careful when applying GARCH models to time series with high kurtosis since the unconditional variance might not be defined.

	ARMA class		GARCH class	
	model	AIC	model	AIC
stock indices	MA(1)	-9401	**C-GARCH(1,1)**	**-9565**
bond indices	ARMA(1,1)	-6155	**ARMA(1,1)-GARCH(1,1)**	**-6249**
IR new	ARMA(1,1)	-10983	**ARMA(1,1)-GARCH(1,1)**	**-11633**
IR old	ARMA(1,1)	-2008	**ARMA(1,1)-GARCH(1,1)**	**-2574**
FX new	**ARMA(1,1)**	**-1517**	GARCH(1,1)	-1497
FX old	MA(1)	-4483	**ARCH(1)**	**-4487**
VIX	ARMA(1,1)	1231	**ARCH(2)**	**1214**
real estate	MA(2)	-1243	**C-GARCH(1,1)**	**-1249**

Table 8: Akaike information criterion comparison

We present in table 8 aggregated AIC numbers for ARMA and ARMA-GARCH best models within each security's class. Column 2 and 4 give the best model for both ARMA and GARCH types. Columns 3 and 5 give the total AIC number within each security's class. The best model between ARMA and ARMA-GARCH models is displayed in **bold face** format.

5 Bivariate analysis

The next step in our analysis is to determine cross-dependencies in monthly innovations. To this purpose, we compute the sample cross-correlation matrix defined in (def.9) to innovations time series[30]. We focus on Spearman's rank correlation to be robust against underlying distributions. The contemporaneous analysis is extended with a lead-lag analysis by the computation of the sample lead-lag correlation matrix. The goal here is to detect whether a given asset leads others over one month. Finally, we use Student-t copula to determine any presence of extreme co-movements between innovations.

5.1 Contemporaneous analysis

In table 9 we present the sample cross-correlation matrix for some stock, bond and interest rates innovations on US and UK markets. The volatility index VIX is also included in the basket. As we might have expected, we notice significant correlations between stock indices. Correlation between MSCI.US and SP500 is 0.999. Stock and bond indices are positively correlated, with a stronger dependence within the U.K. market[31]. Interest rates are negatively correlated with stock and bond indices. Finally, VIX index is negatively correlated with stock indices. This well-known empirical fact is referred to as the 'leverage effect'[32].

The cross-correlation between absolute values of innovations indicates whether assets' regimes are correlated, that is whether volatility clustering is observable across asset classes. Results suggest that it is the case within stock indices, within bond indices and within interest rates. In addition, we notice significant cross-correlation between stock and bond indices and between bond indices and interest rates. The same result holds between US stock indices and the VIX.

		stocks			bonds		IR	
		US	UK	SP500	US	UK	USD	GBP
volatility	VIX	**-0.48**	**-0.32**	**-0.49**	0.04	0.05	0.06	-0.03
		0.34	***0.17***	***0.31***	*0.01*	*-0.02*	*0.01*	*0.01*
stocks	MSCI.US	**1.00**	**0.56**	**1.00**	**0.19**	0.12	**-0.21**	-0.04
		1.00	***0.38***	***0.98***	***0.16***	*0.01*	*0.10*	*-0.09*
	MSCI.UK		**1.00**	**0.57**	0.13	**0.37**	***-0.17***	**-0.28**
			1.00	***0.40***	***0.16***	***0.26***	*0.03*	*0.06*
	SP500			**1.00**	**0.20**	0.12	**-0.19**	0.02
				1.00	***0.16***	*0.02*	*0.07*	*-0.02*
bonds	SLOMON.US				**1.00**	**0.44**	**-0.47**	0.06
					1.00	***0.24***	***0.22***	*0.06*
	SLOMON.UK					**1.00**	**-0.18**	**-0.35**
						1.00	*0.05*	***0.20***
IR	IR.USD						**1.00**	-0.01
							1.00	***0.23***

Table 9: Spearman's sample cross-correlation matrix

We present in table 9 Spearman's sample cross-correlation matrix (def.9) for stock, bond and interest rates (new) innovations on US and UK markets. The volatility index VIX is also included in the basket. Results displayed in **bold face** format indicate that they are significantly different from zero at the 95% level. To test it, we compute the t-stat given in (def.10) applied to the Spearman's sample cross-correlation $\hat{\varrho}(0)_{i,j}$. Cross-correlation estimates for absolute values of innovations are given in *italic* format. The construction of the matrix is made pairs-by-pairs in order to increase the number of observations and therefore may not be positive semi-definite.

[30] Since time series have different lengths and are sometimes enhanced by overlapped observations, matrix components are computed using different numbers of observations. This construction leads to more accurate estimates since we use the overall information at disposal. However, it does not lead necessarily to a positive semi-definite matrix. Numerical methods exist to turn the sample matrix into a correlation matrix which is positive semi-definite [22, 31]. This can be quite convenient in a factorial analysis for instance, where eigenvalues have to be positive.

[31] For other MSCI indices, only MSCI.CH presents significant positive correlation (0.19) with its bonds market index, SLOMON.GOV.CH.

[32] The first explanation to this empirical fact was given by Black [1] and Christie [8] in the sense that negative returns increase financial leverage which extend the risk of the company and therefore its volatility. Another possible explanation to the negative correlation is, that the fear induced by an increase of volatility produces a fall of demand and hence a price decrease.

We present in table 10 the cross-correlation estimates for interest rates. The upper-left quadrant gives cross-correlations between new innovations whereas the lower-right gives cross-correlations between old definitions. The upper-right quadrant gives cross-correlations between new and old innovations. From the latter, we see that the new and old innovations are around 0.7 correlated. Hence, new innovations behave differently than the simple log-return of one month spot rates; there are definitely not same things.

When comparing upper-left to lower-right quadrants, we notice that new innovations are less cross-correlated than old innovations. This suggests that the 1-2 months IR yield curve shifts are less correlated across markets than one-month spot rates themselves. On the other hand, the cross-correlation of absolute values is higher for new innovations. In that case, shifts magnitudes are more clustered across markets than one-month spot rates changes.

		new				old				
		USD	GBP	JPY	ECU	CHF	USD	GBP	JPY	ECU
new	CHF	**0.16**	**0.13**	0.06	**0.20**	**0.74**	**0.11**	0.00	0.02	0.09
		0.17	***0.25***	***0.22***	***0.22***	***0.53***	*0.06*	***0.14***	*-0.03*	***0.12***
	USD	**1.00**	-0.01	0.03	0.10	**0.16**	**0.66**	0.00	0.09	0.05
		1.00	***0.23***	***0.14***	***0.27***	*0.01*	***0.48***	***0.16***	*-0.10*	***0.15***
	GBP		**1.00**	0.09	**0.35**	***0.09***	0.00	**0.74**	0.10	**0.23**
			1.00	*0.14*	***0.26***	***0.16***	***0.16***	***0.61***	*-0.05*	***0.22***
	JPY			**1.00**	0.08	0.03	0.07	0.02	**0.51**	0.04
				1.00	***0.22***	*-0.03*	*0.07*	*0.06*	***0.23***	*0.09*
	ECU				**1.00**	**0.19**	0.10	**0.26**	0.04	**0.76**
					1.00	*-0.02*	***0.17***	***0.18***	***-0.16***	***0.56***
old	CHF					**1.00**	**0.35**	0.04	**0.20**	**0.27**
						1.00	*0.04*	***0.15***	***0.16***	*0.11*
	USD						**1.00**	0.12	**0.29**	**0.22**
							1.00	*0.17*	*0.02*	***0.21***
	GBP							**1.00**	**0.17**	**0.36**
								1.00	*0.00*	***0.19***
	JPY								**1.00**	**0.17**
									1.00	*0.05*

Table 10: Spearman's sample cross-correlation matrix – Interest rates

We present in table 10 Spearman's sample cross-correlation matrix (def.9) for interest rates new and old innovations. Results displayed in **bold face** format indicate that they are significantly different from zero at the 95% level. To test it, we compute the t-stat given in (def.10) applied to the Spearman's sample cross-correlation $\hat{\varrho}(0)_{i,j}$. Correlation estimates for absolute values of innovations are given in *italic* format. The construction of the matrix has been made pairs-by-pairs in order to increase the number of observations and therefore may not be positive semi-definite.

Finally, we apply the sample cross-correlation analysis to FX rates. On the upper-right quadrant of table 11, we notice a quasi perfect correlation between new and old innovations suggesting that, on the contrary of interest rates, new innovations for FX rates do not capture any additional risk component. Both original and absolute values of innovations exhibit significant positive cross-correlations. Currencies react together to USD changes. The highest correlation is observed between CHF and ECU currencies.

		new			old			
		GBP	JPY	ECU	CHF	GBP	JPY	ECU
new	CHF	**0.70**	**0.51**	**0.92**	**1.00**	**0.7**	**0.52**	**0.92**
		0.45	***0.24***	***0.75***	***0.99***	***0.44***	***0.25***	***0.75***
	GBP	**1.00**	**0.43**	**0.74**	**0.70**	**1.00**	**0.42**	**0.74**
		1.00	***0.21***	***0.46***	***0.43***	***1.00***	***0.20***	***0.48***
	JPY		**1.00**	**0.48**	**0.51**	**0.44**	**1.00**	**0.48**
			1.00	***0.19***	***0.24***	***0.22***	***0.99***	***0.20***
	ECU			**1.00**	**0.91**	**0.73**	**0.49**	**1.00**
				1.00	***0.72***	***0.46***	***0.21***	***0.99***
old	CHF				**1.00**	**0.70**	**0.57**	**0.91**
					1.00	***0.43***	***0.28***	***0.72***
	GBP					**1.00**	**0.47**	**0.75**
						1.00	***0.19***	***0.50***
	JPY						**1.00**	**0.54**
							1.00	***0.22***

Table 11: Spearman's cross-correlation matrix – FX rates

We present in table 11 Spearman's sample cross-correlation matrix (def.9) for FX rates old and new innovation definitions. Results displayed in **bold face** format indicate that they are significantly different from zero at the 95% level. To test it, we compute the t-stat given in (def.10) applied to the Spearman's sample cross-correlation $\hat{\varrho}(0)_{i,j}$. Correlation estimates for absolute values of innovations are given in *italic* format. The construction of the matrix has been made pairs-by-pairs in order to increase the number of observations and therefore may be not be positive semi-definite.

5.2 Lead-lag analysis

This section extends the contemporaneous analysis by the computation of the sample one-month lead-lag correlation matrix. The lagged correlation is a more powerful tool in investigating the relation between two time series. It reveals causal relation relations and information flow structures in the sense of Granger causality. If two time series were generated on the basis of a synchronous information flow, they would have a symmetric lagged correlation function; the symmetry would be violated only by insignificantly small, purely stochastic deviations. As soon as the deviations between $\rho_{i,j}(k)$ and $\rho_{j,i}(k)$ become significant, there is asymmetry in the information flow and a causal relation that requires an explanation.

In table 12 we present the results for the first basket of assets analyzed in section 5.1. Diagonal elements are auto-correlations at lag 1. Each off-diagonal element gives the correlation between the security at time $t-1$ (horizontal labels) and security at time t (vertical label). For instance, the 0.22 given in the first line, second column, indicates that MSCI.US innovations lead VIX innovations.

Most elements in the table are not different from zero at the 99% level. We observe however that the US stock market, expressed by both S&P500 and MSCI.US, leads the VIX index significantly and positively. In parallel with contemporaneous findings (leverage effect), this fact induces a mean reverting behaviour of the volatility over one month. This is in line with the overshooting effect detected in the univariate analysis.

More surprisingly, we observe that SLOMON.US leads the US stock market negatively. Since the contemporaneous correlation between SLOMON.US and US stock indices is positive, we obtain the same mean-reverting pattern for US stock markets. However, any economical interpretation is able to explain this finding in a satisfactory manner. This may be due to the few number of observations, here 210.

In spite of these two significant findings, general results speak against the presence of one-month cross dependencies. Whereas profits can be made from dependencies at a univariate level by ARMA or GARCH models, benefits of multivariate autoregressive models like VECC or BEKK types seem questionable for monthly observations.

			stocks			bonds		IR	
		VIX	US	UK	SP500	US	UK	USD	GBP
volatility	VIX	**-0.19**	**0.22**	0.10	**0.22**	0.08	0.09	0.01	-0.05
		0.22	*0.08*	*0.02*	*0.09*	*0.05*	*0.02*	*-0.04*	*0.02*
stock	MSCI.US	-0.05	0.00	0.05	0.00	0.05	-0.01	-0.05	0.06
		0.08	*0.01*	*0.08*	*0.01*	*-0.03*	*-0.07*	*0.03*	*-0.07*
	MSCI.UK	0.01	0.00	0.00	0.00	-0.07	0.04	0.00	0.05
		0.04	*-0.03*	*0.07*	*0.00*	*-0.06*	*0.00*	*0.04*	*0.00*
	SP500	-0.06	0.00	0.04	-0.02	0.04	-0.01	-0.05	0.06
		0.07	*0.00*	*0.09*	*0.01*	*-0.02*	*-0.07*	*0.03*	*-0.07*
bonds	SLOMON.US	0.00	**-0.14**	-0.10	**-0.14**	0.15	0.03	-0.09	0.06
		-0.10	*0.00*	*0.05*	*0.00*	*0.15*	*0.00*	*0.14*	*0.05*
	SLOMON.UK	0.08	-0.05	0.00	-0.05	*0.06*	0.11	-0.01	0.06
		-0.04	*-0.02*	*0.02*	*-0.01*	*-0.10*	*0.08*	*0.00*	*0.04*
IR	US	0.11	0.07	0.04	-0.06	-0.07	*0.05*	**0.18**	0.01
		-0.03	*0.08*	***0.23***	*0.08*	*0.09*	*0.00*	***0.22***	***0.21***
	GBP	-0.07	0.12	0.05	0.06	0.11	-0.01	0.06	0.10
		-0.08	*-0.01*	*0.06*	*0.00*	*-0.04*	*0.08*	***0.29***	***0.33***

Table 12: Spearman's lead-lag correlation matrix

We present in table 12 Spearman's sample one-month lead-lag-correlation (def.9) for stock, bond, IR (new) innovations for US and UK markets. The volatility index VIX is also included in the basket. Results displayed in **bold face** format indicate that they are significantly different from zero at the 99% level. To test it, we compute the t-stat given in (def.10) applied to the Spearman's sample one-month lead-lag-correlation $\hat{\varrho}(1)_{i,j}$. Correlation estimates for absolute values of innovations are given in *italic* format.

5.3 Tails dependencies

This section presents an analysis of tail dependencies based on copulas. We test the presence of extreme co-movements by fitting a Student-t copula on bivariate pseudo-samples and then test whether it better describes tail dependencies than a Gaussian copula. If we cannot reject the Gaussian copula hypothesis, we conclude to absence of tail dependencies. We extend the analysis by a tri-variate fitting to check whether co-movements are of the same magnitude across markets.

5.3.1 Methodology

Consider a d-multivariate random sample $\mathbf{X} := (X_1, ..., X_d)$, where random variables are assumed to be mutually independent and distributed according to a common distribution function H with continuous univariate margins $F_1, ..., F_d$. If the margins were known, then by Corollary 16, the copula C of H would be the distribution function of $(F_d(X_1), ..., F_d(X_d))$. To determine these univariate margins we use a parametric estimation. The ith marginal distribution F_i is estimated by

$$\hat{F}_i(s) := \frac{n}{n+1} \sum_{t=1}^{n} \mathbb{I}_{\{x_{i,t} \leq s\}} \tag{19}$$

where $x_{i,t}$ is the t-th observation of random process X_i and $\mathbb{I}$ denotes the indicator function[33]. From this definition, we construct the pseudo-observation $\hat{\mathbf{u}}_t$ at time t by

$$\hat{\mathbf{u}}_t := \left(\hat{F}_1(x_{1,t}), ..., \hat{F}_d(x_{d,t})\right) \tag{20}$$

[33] The ratio $n/(n+1)$ avoids "edge effects" that occur as some of the variables tend to one which may result in unboundedness of the log-likelihood function.

This approach has been suggested in particular by Bouyé et al. [4] in the context of calibrating copulas to observed financial data[34]. From the pseudo-sample $\hat{\mathbf{u}} := \{\hat{\mathbf{u}}_t; t = 1, .., n\}$, we can derive the pseudo log-likelihood by

$$l(\hat{\mathbf{u}}; \theta) := \sum_{t=1}^{n} \ln c_\theta(\hat{\mathbf{u}}_t) \tag{21}$$

where $c_\theta = \frac{\partial}{\partial \mathbf{u}} C_\theta$ is the density of the copula C.

In the case of a Student-t copula, the density parameter is $\theta := (\Sigma, \nu)$ as it is pointed in section 3.4.2. In order to estimate its components, we follow a two steps procedure. First we estimate the matrix Σ by the robust Kendall's τ estimator[35]. In effect, for elliptical multivariate distributions we have a one-to-one relation between Kendall's τ and usual correlation coefficient, $\rho_{i,j} = \sin(\frac{\pi}{2}\rho^\tau_{i,j})$. From correlation estimates, we construct the correlation matrix Σ. The estimated matrix may be not be positive semi-definite. However, they are simple methods to correct this [22, 31][36]. Then, we plug this matrix in the pseudo-ML function and maximize with respect to ν, formally

$$\hat{\nu} \in \arg\max \left\{ l(\hat{\mathbf{u}}; \hat{\Sigma}, \nu) : \nu \in [2, \infty[\right\} \tag{22}$$

The resulting parameter $\hat{\nu}$ is referred to as the pseudo-maximum log-likelihood estimator (p-MLE) . In order to test the null hypothesis that the Student-t copula degree-of-freedom is ν_0, we use the pseudo-likelihood ratio test statistic (p-LRT) given by

$$\Lambda(\hat{\nu}|\nu_0) := -2 \ln \frac{l(\hat{\mathbf{u}}; \hat{\Sigma}, \hat{\nu})}{l(\hat{\mathbf{u}}; \hat{\Sigma}, \nu_0)} \tag{23}$$

In our problem, the null hypothesis is that the Gaussian copula fits the pseudo-sample. Therefore, we fix $\nu_0 = 10^4$ since, at this level Gaussian and Student-t copulas are indistinguishable. Asymptotic derivations given in [26] suggest that $\Lambda \sim (1+\gamma)\chi^2_1$ as the sample length tends to infinity, where the constant γ depends on the null hypothesis. In their article, the authors use Monte-Carlo techniques to compute this constant. Their results belong to $]0, 0.1]$. However, since we deal with small data sets, we take a very conservative way and set $\gamma = 1$. Significant results are determined at the 99% level.

5.3.2 Results

In Figure 6 we show pseudo-samples for the basket of assets analyzed in section 5.1. We clearly notice presence of positive relation dependencies between stock indices, as well as for bond indices. On the other hand, VIX innovations are negatively correlated with stock indices. We zoom on the pseudo-sample scatter plot for MSCI.US and MSCI.UK in figure 7. We can observe non-negligible concentration of observations both in the upper-right and lower-left corners underlying extreme co-movements. The distribution of points between the two corners is almost symmetric: a bivariate Student-t copula seems appropriate to match these data points.

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[34] As it is pointed in [26], we notice that:

- When the sample size tends to infinity, then $\hat{F}_i$ converges to F_i uniformly on the real-line, almost surely (Glivenko-Cantelli lemma).
- Even if $\mathbf{X}$ is iid, $\hat{\mathbf{u}}$ would not be iid. This comes from the data dependence of the empirical marginal transformation. This problem is recurrent in any inference problem unless the margins are known a priori.

[35] Kendall's τ between variables X_i and X_j is given by $\rho^\tau_{i,j} := \mathbb{E}\left[\text{sign}((X_i - \tilde{X}_i)(X_j - \tilde{X}_j))\right]$ where $(\tilde{X}_i, \tilde{X}_j)$ is an independent vector of (X_i, X_j) with the same distribution. The coefficient is estimated by $\hat{\rho}^\tau_{i,j} := \left(\frac{2!(n-2)!}{n!}\right) \sum_{1 \le t \le s \le n} \text{sign}((x_{i,t} - x_{i,s})(x_{j,t} - x_{j,s}))$.

[36] We do not use such methods in this study since estimated correlation matrices are positive semi-definite.

In table 13 we give degree-of-freedom (DoF) estimates for respective pairs of data in the basket. Significant results indicate extreme co-movements within stock indices. In effect, estimated degrees of freedom are 5 between MSCI indices and 3 between S&P500 and MSCI.US. Between stock and bond indices, DoF are very low as well. However, the statistic gives significant results only in the $\gamma = 0$ case. We reject the presence of extreme co-movements between interest rates and bond indices.

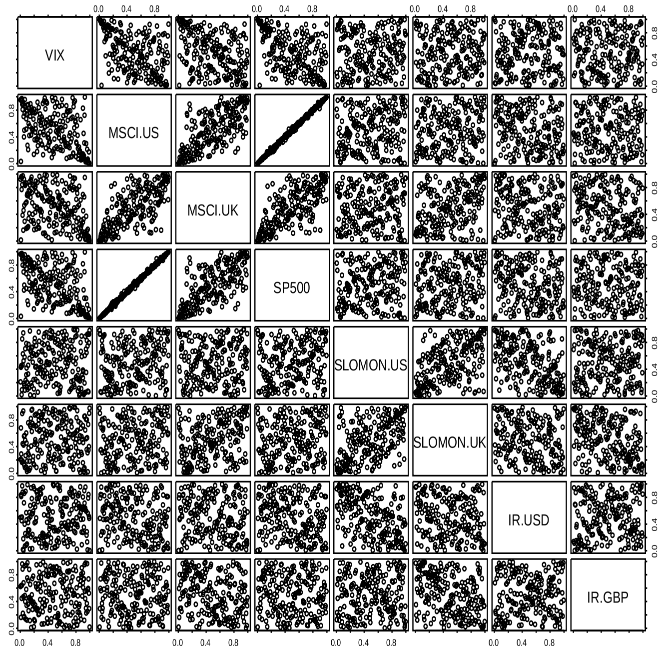

Figure 6: Pseudo-samples

	stocks			bonds		IR	
	US	UK	SP500	US	UK	USD	GBP
VIX	**2** [-0.51]	**3** [-0.39]	**2** [-0.52]	*5* [0.05]	*18* [0.06]	9 [0.06]	*14* [-0.02]
MSCI.US		5 [0.58]	**3** [0.99]	4 [0.20]	*8* [0.11]	*9* [-0.21]	*29* [-0.04]
MSCI.UK			5 [0.58]	*9* [0.09]	4 [0.35]	*7* [0.13]	*10* [-0.24]
SP500				4 [0.20]	*7* [0.12]	*8* [-0.21]	*32* [-0.04]
SLOMON.US					4 [0.46]	*24* [-0.48]	*6* [0.05]
SLOMON.UK						*6* [-0.20]	*14* [-0.36]
IR.USD.new							**3** [0.07]

Table 13: DoF of t-copulas

In figure 6, we present pseudo-samples for some innovations time series. We give in table 13 the estimated degree-of-freedom (DoF) of respective bivariate Student-t copulas. Estimations of DoF are computed using ML. Correlation estimates are given in squared brackets. The correlation matrix is computed using robust Kendall's τ. To determine whether pseudo-samples are better fitted by a Gaussian copula instead of a Student-t copula, we use the p-LRT given in (23). As an conservative way, we take $\gamma = 1$ and the level of significance is set to 99%. Results in **bold face** format give DoF which reject the Gaussian hypothesis. The normal format gives significant results for $\gamma = 0$. Results displayed in *italic* format do not reject the null hypothesis.

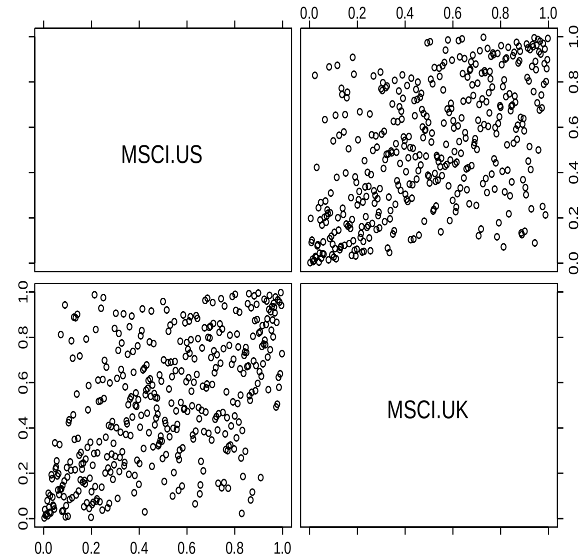

Figure 7: MSCI.US and MSCI UK

Special attention must be payed to VIX innovations. We present in figure 8 three bivariate pseudo-samples between VIX and S&P500 innovations and VIX's level. The upper-middle graphic clearly shows the negative and asymmetric relation between VIX and S&P500 innovations. VIX changes react more to S&P500 decreases than increases. The Clayton copula[37] has the property to reproduce such an asymmetric effect. When we turn to volatility levels instead of its changes, we observe a very interesting pattern. The U-shaped scatter plot (lower-middle) indicates that, under S&P500 positive and negative extreme market events, the level of volatility belongs to its highest values. This phenomenon can be explained by a high market excitation in both extremely positive or negative market conditions.

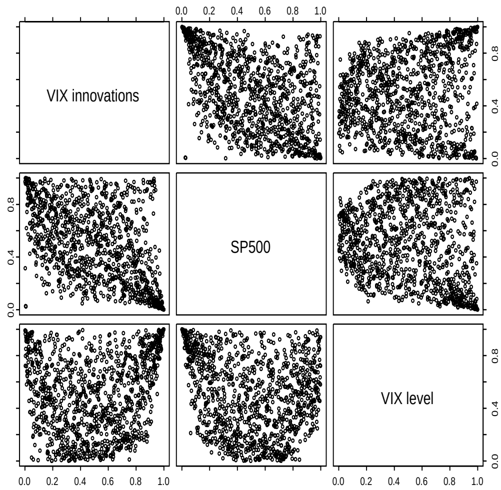

Figure 8: VIX pseudo-samples

We show in figures 9 and 10 the pseudo-samples for IR and FX rates innovations. In the tables below we give respective DoF of Student-t copula fittings.

Low degrees-of-freedom for all fittings indicate high tail dependencies between interest rates. In spite of the fact that cross-correlations between interest rates are lower than for FX rates, extreme co-movements are more likely to happen. This result clearly warns us on hidden risks that a simple correlation coefficient is not able to detect. For instance, the rank correlation between IR.USD and the IR.ECU is estimated to be 0.10 and the correlation derived from Kendall's τ is 0.11 (both not significant at the 95% level). At first glance, we might conclude that the time series are independent. However, the tail dependency analysis strongly rejects the Gaussian copula hypothesis. The estimated degree-of-freedom is 3, which clearly indicates extreme co-movements.

[37]Clayton copula is defined by $C_\theta^{CL}(\mathbf{u}) := \left(\sum_{i=1}^{d}(u_i^\theta - 1) + 1\right)^{-1/\theta}$, where $\theta \in]0, \infty[$ is the copula's parameter.

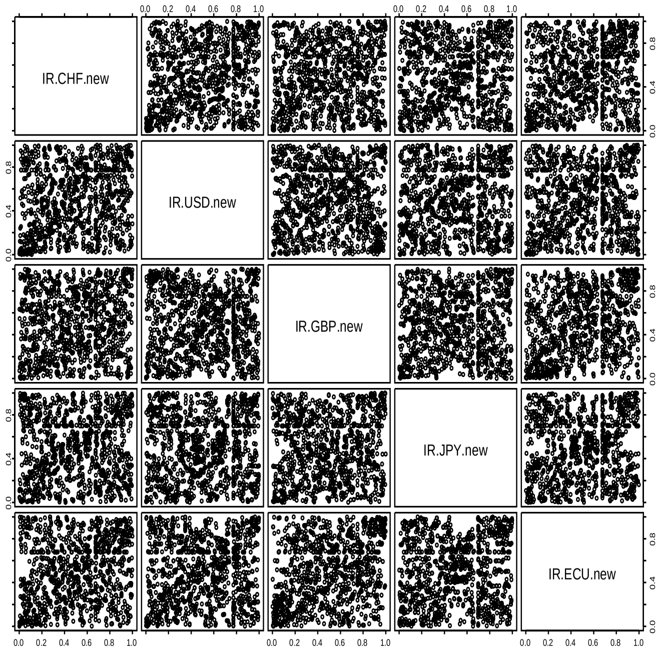

Figure 9: Pseudo-samples – Interest rates

	IR.USD	IR.GBP	IR.JPY	IR.ECU
IR.CHF	**4** [0.17]	**3** [0.14]	**4** [0.07]	**4** [0.22]
IR.USD		**4** [-0.01]	**7** [0.03]	**3** [0.11]
IR.GBP			**4** [0.10]	**5** [0.37]
IR.JPY				**6** [0.09]

Table 14: DoF – Interest rates

In figure 9, we present pseudo-samples for some IR innovations time series. We give in table 14 the estimated degree-of-freedom (DoF) of respective bivariate Student-t copulas. Estimations of DoF are computed using ML. Correlation estimates are given in squared brackets. The correlation matrix is computed using robust Kendall's τ. To determine whether pseudo-samples are better fitted by a Gaussian copula instead of a Student-t copula, we use the p-LRT given in (23). As an conservative way, we take $\gamma = 1$ and the level of significance is set to 99%. Results in **bold face** format give DoF which reject the Gaussian hypothesis. The normal format gives significant results for $\gamma = 0$. Results displayed in *italic* format do not reject the null hypothesis.

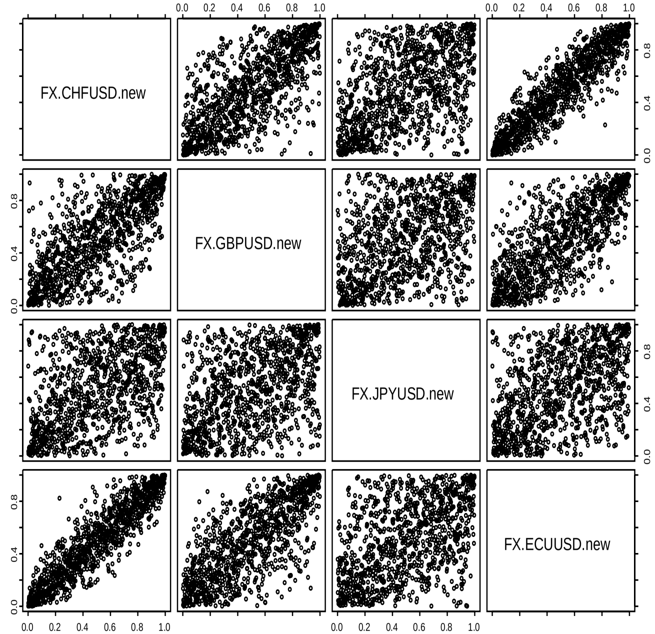

Figure 10: Pseudo-samples – FX rates

	GBP	JPY	ECU
CHF	**5** [0.73]	12 [0.54]	*31* [0.92]
GBP	-	**8** [0.46]	**8** [0.76]
JPY		-	12 [0.50]

Table 15: DoF – FX rates

In figure 10 we present pseudo-samples for some FX innovations time series. We give in table 15 the estimated degree-of-freedom (DoF) of respective bivariate Student-t copulas. Estimations of DoF are computed using ML. Correlation estimates are given in squared brackets. The correlation matrix is computed using robust Kendall's τ. To determine whether pseudo-samples are better fitted by a Gaussian copula instead of a Student-t copula, we use the p-LRT given in (23). As an conservative way, we take $\gamma = 1$ and the level of significance is set to 99%. Results in **bold face** format give DoF which reject the Gaussian hypothesis. The normal format gives significant results for $\gamma = 0$. Results displayed in *italic* format do not reject the null hypothesis.

5.4 Tri-variate extension

To finish the tail dependencies analysis, we test the stock, bond and interest rate basket for four major markets. Our aim is to determine whether monthly co-movements are of the same magnitude in Switzerland, the United Kingdom, Japan and the United States.

In the table 16 we give DoF estimates of tri-variate Student-t copula fittings. We notice that investors face different extreme co-movements depending on the underlying market. Whereas a Swiss investor cannot reject the 3-dimensional Gaussian copula for his portfolio, the Japanese investor is subject to a higher co-movements exposure.

	dim	# obs	$\hat{\nu}$
MSCI.US - SLOMON.US - IR.USD	3	209	8
MSCI.UK - SLOMON.UK - IR.GBP	3	220	*9*
MSCI.JP - SLOMON.JP - IR.JPY	3	144	**5**
MSCI.CH - SLOMON.CH - IR.CHF	3	221	*19*

Table 16: t-copula fitting – markets

In table 16 we present DoF estimates for MSCI-SLOMON and IR.new innovations for Switzerland, United Kingdom, Japan and United states. The number of observations is indicated in the second column. Estimations of DoF are computed using ML. To determine whether pseudo-samples are better fitted by a Gaussian copula instead of a Student-t copula, we use the p-LRT given in (23). As a conservative way, we take $\gamma = 1$ and the level of significance is set to 99%. Results in **bold face** format give DoF which reject the Gaussian hypothesis. The normal format gives significant results for $\gamma = 0$. Results displayed in *italic* format do not reject the null hypothesis.

6 Conclusion

So far, empirical studies of dependencies in financial data sets have focused on intradaily, daily or weekly horizons. Whereas dependence is clearly observable for these frequencies, illustrated for instance with the volatility clustering, it is of common practice to assume independence for monthly and longer time span observations. However, this issue is important since certain companies base their risk management system on longer intervals than one day or one week.

In this paper, we empirically test the presence of dependencies in monthly observations. The analysis is applied to major markets for stock indices, bond indices, short-term interest rates and foreign exchange rates. We use overlapping technique and non-parametric correlation estimation in order to increase both accuracy and consistency in this low-frequency framework. Copulas are used to test extreme co-movements between financial securities.

Our main findings both for the univariate time dependency and for the cross-dependency analysis can be summarized as follows:

univariate:

- The profile of dependencies is different between asset classes. Whereas autocorrelation is negligible for stock and bond indices, interest rates exhibit high and persistent time-dependencies.
- GARCH effects are still present in monthly observations and ARMA-GARCH models give the better performance in terms of Akaike information criterion.

bivariate:

- We observe significant dependencies within data classes, as expected.
- The one-month lead-lag analysis does not support any leader across data classes, apart US stock indices that lead the volatility index.
- The tail dependencies analysis shows a strong presence of extreme co-movements between assets.

Univariate results speak against the time-independence hypothesis and suggest the use of GARCH type models to fit monthly observations. The bivariate analysis clearly shows the need to go beyond the usual correlation coefficient. In effect, even if non-parametric rank correlation does not speak against independence, a further look at tails dependencies shows a systematic presence of extreme co-movements between assets.

To conclude we note that more sophisticated tools can capture non-linear patterns of data and test for independence. This is the case for instance with the correlation integral [25] or through non-parametric smoothing kernels [29]. The application of such tools to monthly financial observations may constitute a further field of research.

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A Filtering methodology

For a given daily time series $\{x_t, t=1, ..., n\}$, the filtering process is based on the first and second differences of x_t. For each observation, we test the following inequality

$$\frac{|x_{t+1} - 2x_t + x_{t-1}|}{|x_{t+1} - x_{t-1}| + adj} \leq tol$$

Coefficients adj and tol are defined specifically for each time series. In the case of CHF interest rates for instance, we take $adj = 0.005$ since we expect daily changes of a half percent magnitude. For stock, bond and real estate indices, the filter is applied to the log-return of the original series.

An observation is defined as an outlier when the above inequality is not satisfied. Each outlier is then carefully checked. When market conditions do not explain the abnormal observation, the outlier is deleted from its original sample. The filtering methodology has led to remove 55 outliers from the overall time series.

type	name	outliers	deleted
stock indices	S&P500	2	0
interest rates	CHF	13	13
	GBP	8	8
	JPY	1	1
	ECU	13	13
	USD	6	6
FX rates	GBP/USD	10	5
	JPY/USD	2	0
	ECU/USD	8	8
volatility	VIX	3	1

Table 17: Filtering results

B Ad-hoc procedure results

To illustrate the impact of the ad-hoc procedure on tests significance, we compute autocorrelation for a subset of innovations. We take independent observations (end-of-the-month quotes) as well as overlapped observations time series. For each of them, we compute the sample autocorrelation function (SACF) up to lags 24 of months (for the S&P500, we compute it for lags of up to 60 months). Pearson's autocorrelations are computed for innovations and transformed innovations, $p \in \{0.5, 1, 1.5, 2\}$, and Spearman's autocorrelations are computed for innovations and absolute values of innovations. For all these estimates, we count and aggregate the number of significant lags at the 99% level. To test the significance, we use the t-stat defined in (def.10) and compute it with n, n_{eff} and n_{indep}.

We show in table 18 that using the whole number of observations leads to overestimating the number of significant results whereas using the number of independent observations leads to underestimate it. On average, the numbers of significant lags given by the ad-hoc procedure are close to the numbers given by monthly non-overlapped observations. In addition, within the number of significant lags, the ad-hoc procedure most of the time matches lags given by the non-overlapped observations. Therefore, based on these results, we decide to focus on overlapped observations in order to get more accuracy and correct statistics by taking the number of effective observations instead of the total number of observations.

type	name	no overlap	n	n_{indep}	n_{eff}
volatility index	VIX	8	30 [8]	7 [4]	13 [7]
stock index	S&P500	5	69 [3]	3 [0]	16 [3]
interest rates	IR.USD.new	12	60 [8]	6 [4]	24 [10]
	IR.USD.old	17	56 [16]	16 [13]	19 [16]
FX rates	FX.JPYUSD.new	6	24 [4]	2 [2]	7 [5]
	FX.JPYUSD.old	3	23 [2]	1 [1]	3 [1]

Table 18: Statistical comparison

We present in table 18 results of the tests comparison. For each innovation time series given in the first column, we compute Pearson's and Spearman's autocorrelations up to lag 24 months (60 months for the S&P500). The correlation is calculated for innovations and transformed innovations. Then, significant lags, at the 99% level, are determined by (10) using n, the total number of observations, n_{eff} the number of effective observations and n_{indep} the number of independent observations in the test statistic Column 3 gives results for non-overlapped observations (end-of-the-month observations). Number in squared brackets indicate the number of lags which match the non-overlapped case.