Match Effects

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Abstract

We present an empirical model of earnings that controls for observable and unobservable characteristics of workers (person effects), unmeasured characteristics of their employers (firm effects), and unmeasured characteristics of worker-firm matches (match effects). We interpret these as the returns to general human capital, firm-specific human capital, and match-specific human capital, respectively. We stress the importance of match effects because the returns to match-specific human capital will be incorrectly attributed to general and/or firm-specific human capital when match effects are omitted, and because general and specific human capital have very different implications for the economic cost of job destruction. We find that slightly more than half of observed variation in log earnings is attributable to general human capital, 22 percent is attributable to firm-specific human capital, and 16 percent to match-specific human capital. Specifications that omit match effects over-estimate the returns to experience by as much as 50 percent, over-estimate the returns to a college education by as much as 8 percent, attribute too much variation to person effects, and too little to firm effects. Our results suggest that considerable earnings variation previously attributed to general human capital – both observed and unobserved – is in fact attributable to workers sorting into higher-paying firms and better worker-firm matches.

JEL Classification: C23, J21

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1 Introduction

It is well known that observable characteristics of workers and firms explain little of the observed variation in employment earnings. One possible explanation is that unexplained wage dispersion reflects unmeasured productivity differences across workers, firms, and worker-firm matches. Theory suggests several possibilities for the source of these productivity differences. Human capital theory (Becker, 1964) suggests both general human capital (which is transferable between employers) and specific human capital (which is not transferable) are potentially important. Matching models further emphasize the importance of match-specific human capital in determining wages and employment mobility. In reality, general human capital, firm-specific human capital, and match-specific human capital are all potentially important determinants of labor earnings. The distinction between general and specific human capital is important because they have different implications for the cost of employment re-allocation over the business cycle. To date, however, empirical attempts to distinguish between them have been hampered by data limitations. It is only with the recent advent of longitudinal linked data on employers and employees that we can credibly hope to identify the separate contribution of general, firm-specific, and match-specific human capital to earnings variation.

We present an empirical model of earnings that controls for observable and unobservable characteristics of workers (person effects), unmeasured characteristics of their employers (firm effects), and unmeasured characteristics of worker-firm matches (match effects). We call this the match effects model. It allows us, for the first time, to measure the relative importance of general human capital, firm-specific human capital, and match-specific human capital in labor earnings. Using the US Census Bureau's Longitudinal Employer-Household Dynamics database, we find that over half of observed variation in log earnings is attributable to general human capital, 22 percent is attributable to firm-specific human capital, and 16 percent to match-specific human capital.

Most recent empirical work using linked employer-employee data has focused on an empirical model of earnings that controls for person and firm effects (Abowd et. al., 1999, AKM hereafter). The match effects model generalizes the person and firm effects model by introducing an interaction between worker and firm, which we call the match effect. The match effect has a straightforward interpretation. It measures persistent within-match differences in log earnings between two workers who possess the same level of general human capital (i.e., share the same measured and unmeasured characteristics) and the same level of firm-specific human capital (i.e., are employed in otherwise identical firms). We interpret this as the value of match-specific human capital, both productive (e.g., match-specific skills, or "match quality") and unproductive (e.g., accumulated knowledge about match quality as in Jovanovic (1979) and other learning models).

Although the match effects model seems a straightforward generalization of the person and firm effects model, it makes two important contributions. The first contribution is to decompose earnings variation into person, firm, and match-specific components. This decomposition is of substantive economic interest because general, firm-specific, and match-specific human capital have different implications for the cost of employment re-allocation over the business cycle. Consider the termination of an employment relationship. Match-specific human capital accumulated over the course of the employment relationship is permanently destroyed when the worker and firm separate. Its value is lost – both to match participants and to society as a whole. Firm-specific human capital is also destroyed. However, it is replaceable (at some cost) because the firm can hire and train a new worker to have the same skills. In contrast, general human capital is fully transferable. It is returned to service when the worker finds new employment, so that the income it generates is only lost for the period of unemployment. Because of these differences, knowing the relative importance of general and specific human capital can usefully inform labor market policy. For instance, it can illuminate the relative value of subsidizing general training versus on-the-job training in specific skills or subsidizing job search.

The second contribution of the match effects model is to correct potential biases in the person and firm effects model. The person and firm effects model has proved very useful for measuring the returns to general versus firm-specific human capital (see, e.g., Abowd et al. (2003) for an application). However, its usefulness is limited if it yields biased estimates of quantities of interest. We find evidence that it does.

There are two related ways to conceptualize this bias. The first is a standard omitted variable bias. Omitted match effects will bias the estimated coefficients of observable characteristics that are correlated with the match effect. This will manifest itself, for example, if workers with some characteristics are more successful at finding good worker-firm matches than others. We find considerable evidence of this bias for standard measures of general human capital, e.g., labor market experience and education. We find that the person and firm effects model over-estimates the returns to 25 years of experience by as much as 50 percent for men and 37 percent for women. Likewise, it over-estimates the returns to a college degree by about 6 percent for men and 8 percent for women. Our results suggest that much of the returns traditionally associated with the accumulation of general human capital are in fact attributable to the accumulation of match-specific human capital. A potential explanation is that workers sort into increasingly good matches over the course of their career, and that more educated workers sort into better employment matches than less educated workers. We find corroborating evidence in the determinants of earnings growth when individuals change employers.

We obtain an even stronger result for the omitted variable bias in estimated person and firm effects. In general, the estimated person and firm effects are unbiased only if the excluded match effects are all zero. We find substantial evidence to the contrary. In fact, we easily reject the hypothesis of no match effects. As a consequence, the person and firm effects model substantially overestimates the proportion of variation attributable to person effects, and underestimates the proportion of attributable to firm effects.

There is a second way to conceptualize bias in the person and firm effects model. An identifying assumption of this model is that employment mobility is conditionally exogenous given observable characteristics and the person and firm effects. All parameter estimates are potentially biased if the exogenous mobility assumption is violated. This would be the case, for instance, if a worker and firm separate due to a "bad" match. Introducing the match effect adds an additional dimension on which to condition the exogenous mobility assumption: unmeasured characteristics of worker-firm matches, including match quality. We examine the sources of earnings growth when individuals change employers, and find evidence that the person and firm effects model violates the exogenous mobility assumption but that the match effects model does not.

Although the match effects model is conceptually straightforward, estimating it is not. In choosing an estimator, we have two objectives: correcting the bias due to omitted match effects, and obtaining a meaningful decomposition of wage variation into person, firm, and match-specific components. We consider both fixed and mixed effect estimators of the match effects model. These provide similar results for the bias correction, but quite different results for the variance decomposition.

The fixed effect estimator provides an easily computed bias correction based on ordinary least squares. However, separately identifying the person, firm, and match effects using this estimator requires ancillary identifying assumptions. Intuitively, these are required to distinguish "good" workers and firms (i.e., those with large person/firm effects) from "lucky" ones (i.e., those with large match effects). As a consequence, interpreting fixed effect estimates of the variance decomposition – and even the person, firm, and match effects themselves – is open to the choice of ancillary assumptions. We consider several possibilities, none of which is wholly satisfactory.

Because of these problems of identification and interpretation, we prefer a mixed effect estimator that treats the person, firm, and match effects as random. This approach yields a straightforward bias correction based on generalized least squares, and identification does not require ancillary assumptions. Instead, it relies on conditional moment restrictions on the random effects. In addition to a traditional mixed effect estimator, we also present a novel "hybrid" estimator based on a combination of fixed effect and mixed effect identifying assumptions. The hybrid estimator allows arbitrary correlation between time-varying observable characteristics and the random effects. It is thus in the spirit of the Hausman and Taylor (1981) correlated random effects estimator.

The remainder of the paper is organized as follows. To provide some context, we briefly review the person and firm effects model and formalize the exogenous mobility assumption in Section 2. In Section 3, we derive the bias due to omitted match effects, and develop our estimators of the match effects model. Section 4 describes the data used in the empirical application, and Section 5 presents the estimation results. We conclude with some brief remarks in Section 6.

2 The Person and Firm Effects Model

To make ideas concrete, it is helpful to review the person and firm effects model. The basic specification is

$$y_{ijt} = \mu + x'_{ijt}\beta + \theta_i + \psi_j + \varepsilon_{ijt} \tag{1}$$

where y_{ijt} is a measure of log compensation for worker i at firm j in period t; μ is the grand mean; x_{ijt} is a vector of time-varying observable characteristics that earn returns β; θ_i is the person effect; ψ_j is the firm effect; and ε_{ijt} is stochastic error. The portable component of compensation, i.e., the returns to individual characteristics plus the person effect, is usually interpreted as measuring the value of general human capital. The firm effect is usually interpreted as a measure of firm-specific human capital.[1]

Several estimators have been proposed for the person and firm effects model. AKM develop approximate solutions for least squares (fixed effect) estimates. Abowd et al. (2002, ACK hereafter) present exact least squares solutions, estimated via a conjugate gradient algorithm. Woodcock (2005a) presents a mixed effect estimator, of which the least squares estimator is a special case.

[1] See Woodcock (2005a) for an equilibrium matching model that yields this interpretation of the person and firm effects. In general, the person effect will measure persistent differences in compensation between individuals, conditional on observable characteristics and firm effects. Likewise, the firm effect will measure persistent differences in compensation across firms, conditional on measured and unmeasured characteristics of workers. These persistent differences in compensation could arise for reasons other than productivity differences. For instance, the person effect could reflect the workers' skill in negotiating compensation. Likewise, compensation may vary across firms because of product market conditions, monopsony power, compensating differentials for non-pecuniary aspects of the job, or firm-specific compensation policies. With this caveat in mind, and without any way to identify these alternate interpretations from the data, we interpret person and firm effects as general and firm-specific human capital, respectively.

Each of these estimators depends critically on employment mobility to identify the various effects. The person effect measures the component of earnings that is common to all of an individual's employment spells (i.e., portable), and that is not due to observable characteristics. Identifying this effect therefore requires repeated observations on the individual at different employers. Likewise, the firm effect measures the component of earnings that is common to all employees of the firm, and that is distinct from variation due to x_{ijt} and the person effects. Thus identifying the firm effect requires observations on multiple employees of the firm. Identifying both effects requires mobility of workers between firms.

To obtain *unbiased* estimates of the various effects, however, requires more. Specifically, the identity of the firm j at which worker i is employed in period t, which we represent by the function $j = \mathcal{J}(i,t)$, must be unrelated to omitted determinants of earnings, i.e., unrelated to statistical error ε_{ijt}. Consequently, when workers change employers, so that $\mathcal{J}(i,t) \neq \mathcal{J}(i,t+1)$, this must also be unrelated to $\varepsilon_{i\mathcal{J}(i,t)t}$ and $\varepsilon_{i\mathcal{J}(i,t+1)t+1}$. These requirements are neatly summarized by the standard assumption that errors have zero conditional mean:

$$E\left[\varepsilon_{ijt} | i, j, t, x_{ijt}\right] = 0 \tag{2}$$

and note that we condition on $j = \mathcal{J}(i,t)$. In the context of the person and firm effects model, assumption (2) has become known as the exogenous mobility assumption. It requires that employment mobility depend only on observable characteristics, the person effect, and the firm effect, and precludes mobility determined by omitted factors (ε_{ijt}).

There are a number of empirically relevant situations where actual employment mobility may violate the exogenous mobility assumption.[2] The match effects model addresses one such situation: where employment mobility depends on unobserved match-specific components of wages. In fact, we argue that exogenous mobility will be violated if productivity depends in any meaningful way on match-specific human capital. The argument is simple. Suppose productivity depends on match-specific human capital. If workers capture any of its returns, then earnings also depend on match-specific human capital. If match-specific human capital is not directly observable, its influence on labor earnings will be absorbed into the error term. When employment mobility depends on wages,[3] it consequently depends on unobserved match-specific human capital. This violates the exogenous mobility assumption.

For those readers who are not convinced by a verbal argument, we present a formal

[2] Gruetter and Lalive (2004) estimate the person and firm effects model on a sample of job-to-job employment transitions, where mobility is arguably endogenous, and a sample of job-unemployment-job transitions, where mobility is arguably exogenous, and find substantial differences.

[3] Empirical evidence on the relationship between wages and mobility dates to the advent on longitudinal data (if not earlier), e.g., Mincer and Jovanovic (1981) and Bartel and Borjas (1981). Dostie (2005) provides more recent evidence.

one in Appendix A. There, we develop a simple two-period model of wage bargaining with on-the-job search. Under rudimentary assumptions, wages and mobility jointly depend on productivity. The implication is that when productivity depends on match-specific human capital, so too do wages and mobility.

3 The Match Effects Model

We consider the empirical specification

$$y_{ijt} = \mu + x'_{ijt}\beta + \theta_i + \psi_j + \phi_{ij} + \varepsilon_{ijt} \tag{3}$$

where ϕ_{ij} is a match effect and all other terms are as defined in Section 2. From a statistical perspective, the match effect has a simple interpretation: it is the interaction effect between worker and firm. Its economic interpretation is also straightforward. The match effect measures the returns to unobserved time-invariant characteristics of worker-firm matches, which we interpret as the return to match-specific human capital.[4] Note these returns are distinct from returns to unmeasured individual and firm characteristics.

We further decompose the person effect θ_i into components observed and unobserved by the econometrician:

$$\theta_i = \alpha_i + u'_i\eta \tag{4}$$

where u_i is a vector of time-invariant observable individual characteristics, η measures returns to those characteristics, and α_i is the unobservable component. The firm and match effects can be similarly decomposed, but we do not consider that case here.

As in the case of the person and firm effects model, identification requires assumptions about the error distribution. We continue to assume that errors have zero conditional mean:

$$E\left[\varepsilon_{ijt}|i, j, t, x_{ijt}\right] = 0 \tag{5}$$

which simply restates the exogenous mobility assumption. Note, however, that introducing the match effect in (3) fundamentally changes the interpretation of this assumption. Specifically, mobility based on unobserved characteristics of worker-firm matches no longer violates exogenous mobility. That is, (5) now requires that employment mobility depend only on observable characteristics, the person effect, the firm effect, and the match effect.[5]

[4] Another possible interpretation is that the match effect measures the value of production complementarities between the worker and firm. This has essentially the same implications for its predicted impact on wages and mobility.

[5] Of course, introducing the match effect only makes the exogenous mobility assumption robust to mobility

In addition to the zero conditional mean assumption, we assume errors are spherical:

$$E\left[\varepsilon_{ijt}\varepsilon_{mns}|i,j,t,m,n,s,x_{ijt},x_{mns}\right] = \begin{cases} \sigma_\varepsilon^2 \text{ for } i=m,\ j=n,\ t=s \\ 0 \text{ otherwise.} \end{cases} \quad (6)$$

These assumptions can be relaxed, but doing so complicates estimation. See Woodcock (2005a) for an application of the person and firm effects model with non-spherical errors.

Let N^* denote the total number of observations; N is the number of individuals; J is the number of firms; $M \leq NJ$ is the number of worker-firm employment matches; k is the number of time-varying covariates; and q is the number of time-invariant observable individual characteristics. We can rewrite the match effects model in matrix notation:

$$y = \mu + X\beta + D\theta + F\psi + G\phi + \varepsilon \quad (7)$$

$$\theta = \alpha + U\eta \quad (8)$$

where y is the $N^* \times 1$ vector of log compensation; μ is now the $N^* \times 1$ mean vector; X is the $N^* \times k$ matrix of time-varying covariates; β is a $k \times 1$ parameter vector; D is the $N^* \times N$ design matrix of the person effects; θ is the $N \times 1$ vector of person effects; F is the $N^* \times J$ design matrix of the firm effects; ψ is the $J \times 1$ vector of firm effects; G is the $N^* \times M$ design matrix of the match effects; ϕ is the $M \times 1$ vector of match effects; α is the $N \times 1$ vector of unobserved components of the person effect; U is the $N \times q$ matrix of time-invariant individual characteristics; η is a $q \times 1$ parameter vector; and ε is the $N^* \times 1$ error vector. There is a simple relationship between D, F, and G. Specifically, the column of G corresponding to the match between worker i and firm j, which we call column ij, is the elementwise product of the i^{th} column of D and the j^{th} column of F.

Identification and estimation of the various effects is nontrivial. Before turning to these matters, however, we first derive the bias that arises from omitting match effects. In doing so, we focus on the fixed effect estimator of the person and firm effects model because virtually all prior research is based on this specification.

3.1 Omitted Match Effects

When the data generating process is given by equation (3) but the estimated equation excludes the match effect ϕ_{ij}, the estimated parameters β^*, θ_i^*, and ψ_j^* are biased. Specifically,

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decisions based on *time-invariant* unobserved characteristics of worker-firm matches. It is plausible that time-varying characteristics (e.g., the accumulation of match-specific human capital) also matter. In principle, this could be accomodated via a tenure effect. However, to enable a straightforward comparison between the match effects model and the person and firm effects model, we defer such considerations for future research.

least squares estimates of the mis-specified model satisfy

$$\begin{aligned} E\left[\beta^*\right] &= \beta + \left(X'M_{[D\ F]}X\right)^{-1} X'M_{[D\ F]}G\phi \\ E\left[\theta^*\right] &= \theta + \left(D'M_{[X\ F]}D\right)^{-} D'M_{[X\ F]}G\phi \\ E\left[\psi^*\right] &= \psi + \left(F'M_{[X\ D]}F\right)^{-} F'M_{[X\ D]}G\phi \end{aligned} \tag{9}$$

where A^- denotes a generalized inverse of A,[6] and $M_A \equiv I - A\left(A'A\right)^{-} A'$ projects onto the column null space of A.

In expectation, the estimated returns to observable characteristics, β^*, equal the true vector of returns plus an employment-duration weighted average of the match effects in the individual's employment history, conditional on the design of the person and firm effects. The sign and magnitude of the bias depends on the conditional covariance between X and G, given D and F.

The expected value of the estimated person effects in the mis-specified model, θ^*, equal the true vector of person effects plus the employment-duration weighted average of match effects in the worker's employment history, conditional on observable time-varying characteristics and firm effects. Because of the simple relationship between D, F, and G noted above, in general $D'M_{[X\ F]}G\phi = 0$ only if $\phi = 0$. In fact, in the simplest case where X and F are orthogonal to D and G, so that $D'M_{[X\ F]}D = D'D$ and $D'M_{[X\ F]}G = D'G$, the omitted variable bias is a vector of employment duration-weighted average match effects,[7] so that

$$E\left[\theta_i^*\right] - \theta_i = \frac{1}{T_i} \sum_{t=t_i^1}^{T_i} \phi_{i\mathcal{J}(i,t)} \tag{10}$$

where we denote the periods that person i appears in the sample by $t_i^1, t_i^2, ..., T_i$.[8]

In similar fashion, the omitted variable bias in ψ^* is zero only when $F'M_{[X\ D]}G\phi = 0$, which again requires $\phi = 0$ in general. If X and D are orthogonal to F and G, so that $F'M_{[X\ D]}F = F'F$ and $F'M_{[X\ D]}G = F'G$, the omitted variable bias in ψ^* is a vector of

[6] For simplicity, we assume X has full column rank k. However D, F, and G do not, in general, have full column rank without additional identifying restrictions, e.g., exclusion of one column per connected group of workers and firms. See Searle (1987, Ch. 5) for a general statistical discussion of connected data, or ACK for a discussion in the context of linked employer-employee data.

[7] $D'G$ is an $N \times M$ matrix whose entry in row i and column ij is the duration of the match between worker i and firm j.

[8] We implicitly assume each worker has only one employer per period. The extension to multiple employers per period is straightforward, but complicates notation.

employment duration-weighted average match effects,[9] so that

$$E\left[\psi_j^*\right] - \psi_j = \frac{1}{N_j} \sum_{i \in \mathcal{I}_j} \sum_{t=t_i^1}^{T_i} \phi_{i\mathcal{J}(i,t)} \tag{11}$$

where we use $\mathcal{I}_j = \{i : \mathcal{J}(i,t) = j \text{ for some } t\}$ to denote the set of all employees of firm j, $N_j = \sum_{i=1}^{N} \sum_{t=t_i^1}^{T_i} \mathbf{1}(\mathcal{J}(i,t) = j)$ is the total number of observations on firm j, and $\mathbf{1}(A)$ is the indicator function that takes value one when A is true and zero otherwise.

The preceding illustrates that if match effects are nonzero, the person and firm effects model will attribute variation to person and firm effects that is actually due to omitted match effects. The returns to observable characteristics will also be biased if workers with certain characteristics (e.g., more education or experience) sort into better employment matches than others.

3.2 Fixed Effect Estimators

Economists often prefer fixed effect estimators to mixed (random) effect estimators because they are perceived to embody fewer assumptions about the relationship between observables and unobservables. Indeed, almost all estimates of the person and firm effects model are based on the fixed effect estimator, so we begin here.

Estimating β is straightforward and requires no further assumptions. Applying standard results for partitioned regression, the least squares estimator of β is:

$$\hat{\beta} = \left(X' M_{[D\ F\ G]} X\right)^{-1} X' M_{[D\ F\ G]} y. \tag{12}$$

Some algebra verifies that $M_{[D\ F\ G]}$ takes deviations from match-specific means.[10] So we can recover $\hat{\beta}$ from the regression of y_{ijt} on x_{ijt}, both in deviations from match-specific means:

$$y_{ijt} - \bar{y}_{ij\cdot} = \left(x_{ijt} - \bar{x}_{ij\cdot}\right)' \beta + \nu_{ijt} \tag{13}$$

where $\bar{y}_{ij\cdot} = \frac{1}{T_{ij}} \sum_{t=t_i^1}^{T_i} \mathbf{1}(\mathcal{J}(i,t) = j)\, y_{i\mathcal{J}(i,t)t}$, $\bar{x}_{ij\cdot} = \frac{1}{T_{ij}} \sum_{t=t_i^1}^{T_i} \mathbf{1}(\mathcal{J}(i,t) = j)\, x_{i\mathcal{J}(i,t)t}$, and ν_{ijt}

[9] $F'G$ is a $J \times M$ matrix whose entry in row j and columnn ij is the duration of the match between worker i and firm j.

[10] $M_{[D\ F\ G]}$ projects onto the column null space of $[D\ F\ G]$. It is a block diagonal matrix with N^* rows and columns, where the M diagonal blocks correspond to each of the M worker-firm matches. The ij^{th} diagonal block is zero if worker i never works at firm j. Otherwise, it is the $T_{ij} \times T_{ij}$ submatrix $M_{[D\ F\ G]}^{ij} = I_{T_{ij}} - \frac{1}{T_{ij}} \iota_{T_{ij}} \iota_{T_{ij}}'$ where $T_{ij} = \sum_{t=t_i^1}^{T_i} \mathbf{1}(\mathcal{J}(i,t) = j)$ is the duration of the match between worker i and firm j; I_A is the identity matrix of order A; and ι_A is an $A \times 1$ vector of ones. Each $M_{[D\ F\ G]}^{ij}$ takes deviations from means in the match between worker i and firm j.

is statistical error. Note this simple method to recover the least squares estimate of β is only valid when the model includes match effects.[11]

3.2.1 Identifying the Person, Firm, and Match Effects

Separately identifying the person, firm, and match effects is less trivial than estimating β. At its core, the identification problem is to distinguish "good" workers and firms (i.e., those with larger person/firm effects) from "lucky" ones (i.e., those with large match effects). In the case of the fixed effect estimator, this is complicated by the sheer number of parameters to estimate (k elements of β, N person effects, J firm effects, M match effects, and the intercept).[12] Beyond this, however, there is a fundamental identification problem: the fixed effect formulation of the match effects model is over-parameterized. There are $N+J+M+1$ person effects, firm effects, match effects, and a constant term to estimate, but only M worker-firm matches ("cell means") from which to estimate them.[13] Alternately put, the only estimable functions of $\theta_i, \psi_j, \phi_{ij}$ and μ in equation (3) are the M population cell means $\mu_{ij\cdot} = \mu + \theta_i + \psi_j + \phi_{ij}$ (Searle, 1987 p. 331).[14]

To see the identification problem, note that with $\hat{\beta}$ in hand, the least squares estimator of $\theta_i, \psi_j, \phi_{ij}$ and μ solves the remaining normal equations from the partitioned regression. This is equivalent to regressing $y - X\hat{\beta}$ on D, F, G, and an intercept. Predicted values from this regression are the $N^* \times 1$ vector $\left(I - M_{[D\ F\ G]}\right)\left(y - X\hat{\beta}\right) = \hat{\mu} + D\hat{\theta} + F\hat{\psi} + G\hat{\phi}$. There are only M distinct elements in the vector of predicted values, the sample cell means

$$\bar{\mu}_{ij\cdot} = \frac{1}{T_{ij}} \sum_{t=t_i^1}^{T_i} \left(y_{ijt} - x'_{ijt}\hat{\beta}\right) = \hat{\mu} + \hat{\theta}_i + \hat{\psi}_j + \hat{\phi}_{ij}. \tag{14}$$

And yet we are tasked with decomposing the M sample cell means into $N + J + M + 1$ parameters. This requires ancillary assumptions.

One solution is to impose linear restrictions on the estimated coefficients. A candidate

[11] That is, whereas $M_{[D\ F\ G]}$ takes deviations from match match-specific means, $M_{[D\ F]}$ does not.

[12] The typical application involves millions of workers and matches, and hundreds of thousands of firms.

[13] The term "cell mean" is adopted from the statisical literature on estimation of the two-way crossed classification with interaction, of which the match effects model is an example. It arises from representing the data as a table with rows defined by the levels of i (workers), and columns defined by the levels of j (firms). The entry in row i and column j is the mean earnings of worker i at firm j.

[14] In practice, there are only M estimable functions of the person, firm, and match effects, the overall constant, and a set of group means. The group means are defined for connected groups of observations in the sample. When the sample consists of $\mathcal{G}$ connected groups of observations, the number of estimable functions of the other effects is reduced by a corresponding amount. For clarity of exposition, I asbtract from these considerations in the main text, and presume the sample consists of a single connected group. See ACK for further discussion of connectedness, including a graph-theoretic algorithm for determining connected groups of observations and identification conditions in the person and firm effect model.

collection of restrictions is

$$\sum_{i=1}^{N} \hat{\theta}_i = 0, \quad \sum_{j=1}^{J} \hat{\psi}_j = 0, \quad \sum_{i=1}^{N} \hat{\phi}_{ij} = 0 \; \forall j, \text{ and } \sum_{j=1}^{J} \hat{\phi}_{ij} = 0 \; \forall i. \tag{15}$$

These simply normalize the person and firm effects to have zero mean, and the match effects to have zero mean for each person and firm.[15] We use an algorithm suggested by Searle (1987, pp. 328-332) to solve (14) and (15). We do not, however, report the results because the estimates are difficult to interpret.[16] Notably, we would like to be able to compare match effects across workers or firms. However the restrictions (15) preclude any such comparison because match effects are measured relative to person and firm-specific means. This interpretability problem is not due to the linear restrictions (15) per se. Any other other collection of linear restrictions will rule out some types of meaningful comparisons.

More importantly, however, least squares estimates of the match effects model rule out interpersonal comparisons of person effects and interfirm comparisons of firm effects. This is because only the cell means are estimable, and hence the only estimable linear contrasts are those involving the cell means. For example, in the case of two employees i and m of firm j, the linear contrast

$$\mu_{ij} - \mu_{mj} = \left(\mu + \theta_i + \psi_j + \phi_{ij}\right) - \left(\mu + \theta_m + \psi_j + \phi_{mj}\right) = \left(\theta_i - \theta_m\right) + \left(\phi_{ij} - \phi_{mj}\right) \tag{16}$$

is estimable. However, linear constrasts like $\theta_i - \theta_m$ and $\psi_j - \psi_n$ (for $i \neq m$ and $j \neq n$) are not estimable in this framework because there is no way to eliminate match effects from (16). Of course these contrasts *are* estimable in the person and firm effects model. Consequently, terms such as "high wage workers" and "high wage firms" are meaningful in the person and firm effects model, but meaningless in least squares estimates of the match effects model.

Because of these interpretability problems, we take a different approach. We define the match effects to be orthogonal to person and firm effects. This permits meaningful comparison of the person, firm, and match effects across workers and firms. In fact, the match effect is identified whenever the corresponding person and firm effects are identified in the model without match effects. We can therefore base identification on conditions developed by ACK for the person and firm effect model. We can also use the ACK conjugate gradient algorithm to decompose the cell means into least squares estimates of the intercept, the person effect, the firm effect, and an orthogonal match effect, as follows.

[15] These restrictions require slight modification when the data consist of $\mathcal{G}$ connected groups of workers and firms. In particular, we need to normalize the person and firm effects to have zero mean in each group, as well as zero overall mean.

[16] These results are available on request.

Let $\bar{\mu}$ denote the $N^* \times 1$ vector of cell means (14). The orthogonal match effect estimator is defined by the least squares regression of $\bar{\mu}$ on an intercept, D, and F. The implied estimate of the intercept, $\hat{\mu}$, is the sample mean of the cell means: $\hat{\mu} = \frac{1}{N^*} \sum \bar{\mu}_{ij\cdot}$, and the estimated person and firm effects solve

$$\begin{bmatrix} D'D & D'F \\ F'D & F'F \end{bmatrix}^{-} \begin{bmatrix} \hat{\theta} \\ \hat{\psi} \end{bmatrix} = \begin{bmatrix} D' \\ F' \end{bmatrix} (\bar{\mu} - \hat{\mu}) \tag{17}$$

subject to the grouping conditions of ACK.[17] The least squares estimator of the orthogonal match effect is $\hat{\phi} = M_{[D\ F]} (\bar{\mu} - \hat{\mu}) = \bar{\mu} - \hat{\mu} - D\hat{\theta} - F\hat{\psi}$, which is just the residual in the regression of $\bar{\mu}$ on D, F, and an intercept.

Given the estimated effects, there remains to decompose the person effect into its observable and unobservable components as in (4). This is straightforward. We estimate the least squares regression of $\hat{\theta}_i$ on observable characteristics u_i. Residuals from this regression define an estimator of the unobserved component α_i that is orthogonal to u_i.

3.3 Mixed Effect Estimators

An alternative identification strategy is to assume the person, firm, and match effects are random. In this case, identification relies on restrictions on the conditional moments of the random effects. These are like Bayes prior information on the distribution of the random effects (see Searle et al. (1992) for a Bayesian interpretation of the mixed effect estimator).[18]

We consider two mixed (random) effect estimators. Both treat β and η as fixed, and α, ψ, and ϕ as random. To facilitate comparison with prior research, we estimate mixed models with and without match effects.

[17] ACK derive necessary and sufficient conditions to identify $\hat{\theta}$ and $\hat{\psi}$ in the person and firm effects model. They are only identified up to a group mean in each group of connected workers and firms. Hence a sufficient condition for identification of $\hat{\theta}$ and $\hat{\psi}$ is $\sum_{i \in g} \hat{\theta}_i = 0$ and $\sum_{j \in g} \hat{\psi}_j = 0$ in each group g.

[18] There is another difference between fixed and mixed effect identification when the data consist of $\mathcal{G} > 1$ connected groups of observations. The mixed effects estimator spreads identification across all groups. Fixed effect estimates of the person, firm, and match effects are only identified within a connected group, i.e., they are measured relative to $\mathcal{G}$ group means and an overall mean. This implies that at most $M - \mathcal{G} - 1$ fixed effect estimates of person, firm, and match effects are identified. In contrast, all $N + J + M$ random person, firm, and match effects are identified, though each effect is normalized to have zero conditional mean.

The first estimator is a traditional mixed model based on the moment conditions

$$E\left[\alpha_i|x_{ijt},u_i\right] = E\left[\psi_j|x_{ijt},u_i\right] = E\left[\phi_{ij}|x_{ijt},u_i\right] = 0 \tag{18}$$

$$Cov\left[\begin{array}{c|c} \alpha_i & \\ \psi_j & x_{ijt},u_i \\ \phi_{ij} & \end{array}\right] = \begin{bmatrix} \sigma^2_\alpha & 0 & 0 \\ 0 & \sigma^2_\psi & 0 \\ 0 & 0 & \sigma^2_\phi \end{bmatrix}. \tag{19}$$

Estimation follows a Feasible GLS strategy. We first estimate the variance components $\left(\sigma^2_\alpha, \sigma^2_\psi, \sigma^2_\phi\right)$ and the error variance σ^2_ε by Restricted Maximum Likelihood (REML).[19] REML is often described as maximizing the part of the likelihood that is invariant to the values of the fixed effects and is akin to partitioned regression.[20] The REML estimator has many attractive properties: estimates are invariant to the value of β and η, consistent, asymptotically normal, and asymptotically efficient in the Cramer-Rao sense.

We estimate β, η, and the realized random effects in a second stage. Given the moment conditions (18) and (19), the Best Linear Unbiased Estimator (BLUE) of the fixed effect and Best Linear Unbiased Predictor (BLUP) of the random effects solve the Henderson et al. (1959) mixed model equations.[21] In the match effects model, these are

$$\begin{bmatrix} X'X & X'U & X'D & X'F & X'G \\ U'X & U'U & U'D & U'F & U'G \\ D'X & D'U & D'D+\left(\tilde\sigma^2_\varepsilon/\tilde\sigma^2_\alpha\right)I_N & D'F & D'G \\ F'X & F'U & F'D & F'F+\left(\tilde\sigma^2_\varepsilon/\tilde\sigma^2_\psi\right)I_J & F'G \\ G'X & G'U & G'D & G'F & G'G+\left(\tilde\sigma^2_\varepsilon/\tilde\sigma^2_\phi\right)I_M \end{bmatrix}\begin{bmatrix} \tilde\beta \\ \tilde\eta \\ \tilde\alpha \\ \tilde\psi \\ \tilde\phi \end{bmatrix} = \begin{bmatrix} X'y \\ U'y \\ D'y \\ F'y \\ G'y \end{bmatrix} \tag{20}$$

where $\left(\tilde\sigma^2_\varepsilon, \tilde\sigma^2_\alpha, \tilde\sigma^2_\psi, \tilde\sigma^2_\phi\right)$ are REML estimates and $\left(\tilde\beta, \tilde\eta, \tilde\alpha, \tilde\psi, \tilde\phi\right)$ denote solutions for the various effects. As $\left(\tilde\sigma^2_\alpha, \tilde\sigma^2_\psi, \tilde\sigma^2_\phi\right) \to \infty$, the mixed model equations converge to the least squares normal equations solved by the fixed effect estimator. In this sense, the least squares estimator is a special case of the mixed effect estimator.

We also estimate a novel "hybrid" mixed effect estimator that combines identification conditions of the traditional mixed and fixed effect estimators. The main advantage of this approach is that it relaxes the zero-conditional-mean assumption (18). It is in the spirit of

[19] We compute REML estimates using the Average Information (AI) algorithm of Gilmour et al. (1995).

[20] Formally, REML is maximum likelihood on linear combinations of y under the assumption of normally distributed errors. The linear combinations $K'y$ are chosen so that $K'\left(X\beta + U\eta\right) = 0$ for all values of β and η, which implies $K'\left[X \;\; U\right] = 0$. Thus K' projects onto the column null space of $\left[X \;\; U\right]$ and is of the form $K' = C'M_{[X \;\; U]}$ for arbitrary C'.

[21] The BLUPs $\tilde\alpha, \tilde\psi$, and $\tilde\phi$ are *best* in the sense of minimizing the mean square error of prediction among linear unbiased estimators, and *unbiased* in the sense $E\left[\tilde\alpha\right] = E\left[\alpha\right]$, $E\left[\tilde\psi\right] = E\left[\psi\right]$, and $E\left[\tilde\phi\right] = E\left[\phi\right]$. See Robinson (1991) for details.

the Hausman and Taylor (1981) correlated random effects estimator. Estimation proceeds in three stages. In the first stage, we estimate β under the identifying assumptions of the fixed effect model, so that $\hat{\beta}$ is given by the "within" estimator (12). In the second stage, we estimate the variance components and error variance via REML on the "gross residuals" $y_{ijt} - x'_{ijt}\hat{\beta}$. The implied conditional moment restrictions are now:

$$E\left[\alpha_i|u_i\right] = E\left[\psi_j|u_i\right] = E\left[\phi_{ij}|u_i\right] = 0 \tag{21}$$

$$Cov\left[\begin{array}{c|c} \alpha_i & \\ \psi_j & , u_i \\ \phi_{ij} & \end{array}\right] = \begin{bmatrix} \sigma^2_\alpha & 0 & 0 \\ 0 & \sigma^2_\psi & 0 \\ 0 & 0 & \sigma^2_\phi \end{bmatrix}. \tag{22}$$

Note that unlike (18) and (19), the moment restrictions (21) and (22) no longer condition on x_{ijt}. This allows correlation between x_{ijt} and the person, firm, and match effects. In the third stage we solve the mixed model equations:

$$\begin{bmatrix} U'U & U'D & U'F & U'G \\ D'U & D'D + \left(\tilde{\sigma}^2_\varepsilon/\tilde{\sigma}^2_\alpha\right) I_N & D'F & D'G \\ F'U & F'D & F'F + \left(\tilde{\sigma}^2_\varepsilon/\tilde{\sigma}^2_\psi\right) I_J & F'G \\ G'U & G'D & G'F & G'G + \left(\tilde{\sigma}^2_\varepsilon/\tilde{\sigma}^2_\phi\right) I_M \end{bmatrix} \begin{bmatrix} \check{\eta} \\ \check{\alpha} \\ \check{\psi} \\ \check{\phi} \end{bmatrix} = \begin{bmatrix} U' \\ D' \\ F' \\ G' \end{bmatrix} \left(y - X\hat{\beta}\right). \tag{23}$$

for $\check{\eta}, \check{\alpha}, \check{\psi}$, and $\check{\phi}$.

The hybrid mixed effect estimator has the following properties. $\hat{\beta}$ is the BLUE of β given the minimal assumptions (5) and (6) on ε. Given the additional stochastic assumptions (21) and (22), $\check{\eta}$ is the BLUE of η and $\left(\check{\alpha}, \check{\psi}, \check{\phi}\right)$ are BLUPs of the random effects.

4 Data

Identifying the person, firm, and match effects requires longitudinal data on employers and employees. We use data from the US Census Bureau's Longitudinal Employer-Household Dynamics (LEHD) database. These data span thirty-two states that represent the majority of American employment. We use data from two participating states, whose identity is confidential.

The LEHD data are administrative, constructed from Unemployment Insurance (UI) system employment reports. These are collected by each state's Employment Security agency to manage the unemployment compensation program. Employers are required to report total payments to all employees on a quarterly basis. These payments (earnings) include gross wages and salary, bonuses, stock options, tips and gratuities, and the value of meals and lodging when these are supplied (Bureau of Labor Statistics (1997, p. 44)).

The coverage of UI data varies slightly from state to state, though the Bureau of Labor Statistics (1997, p. 42) claims that UI coverage is "broad and basically comparable from state to state" and that "over 96 percent of total wage and salary civilian jobs" were covered in 1994. See Stevens (2002) and Abowd et al. (2006) for further details. With the UI employment records as its frame, the LEHD data comprise the universe of employment at firms required to file UI reports — that is, all employment potentially covered by the UI system in participating states.

Individuals are uniquely identified in the data by a Protected Identity Key (PIK). Employers are identified by an unemployment insurance account number (SEIN). The UI employment records contain only limited information: PIK, SEIN, and earnings. The LEHD database integrates these with internal Census Bureau data to obtain additional demographic and firm characteristics, including sex, race, date of birth, industry, and geography.

Though the underlying data are quarterly, they are aggregated to the annual level for estimation. The full sample consists of over 49 million annualized employment records on full-time workers between 25 and 65 years of age who were employed at private-sector non-agricultural firms between 1990 and 1999.

Missing values are imputed from the posterior predictive distribution of a parametric missing data model. Specifics on the imputation models, and further details on sample construction and variable creation, are given in the Data Appendix to Woodcock (2005a).

Because the fixed effect estimators described in Section 3.2 do not solve the least squares normal equations directly, it is possible to estimate the fixed effect specifications on very large samples.[22] Unfortunately, there currently exists no similar computational alternative to solving the mixed model equations. We must therefore estimate the mixed effect specifications on a subsample of observations. Sampling from linked employer-employee data is nontrivial because the sample must be sufficiently connected to precisely estimate the person, firm, and match effects. We therefore draw a ten percent subsample of individuals employed in 1997 using the dense sampling algorithm of Woodcock (2005b). This algorithm ensures that each worker is connected to at least five others by a common employer, but is otherwise representative of the population of individuals employed in 1997. That is, all individuals employed in 1997 have an equal probability of being sampled.[23] The dense subsample consists

[22] The cross-products matrix in the least squares normal equations has $N + J + M + k + 1$ rows and columns. Solving the normal equations requires inverting this matrix. This is infeasible for samples of the size considered here. Our fixed effect estimates are based on the ACK conjugate gradient algorithm, which does not invert this matrix.

[23] The dense sampling algorithm ensures that individuals are connected to a specified minimum number of other workers by a common employer. This is achieved by first sampling firms with probabilities proportional to employment in a reference period, and then sampling workers within firms with probabilities inversely

of the full work history of each sampled individual. To enable direct comparison of results between the fixed and mixed effect specifications, we estimate the fixed effect specifications on the full work histories of all individuals employed in 1997.

Table 1 presents characteristics of the samples. The sample of individuals employed in 1997 is largely representative of the full sample of observations. Some slight differences indicate that individuals employed in 1997 have a slightly stronger labor force attachment than the sample of all individuals employed between 1990 and 1999: males are slightly over-represented, as are individuals with higher educational attainment and individuals who work four full quarters in an average calendar year. The ten percent dense subsample has characteristics virtually identical to the sample of all individuals employed in 1997.

5 Estimation Results

In discussing the empirical estimates, we focus on two comparisons. Because most prior empirical work is based on the fixed effect specification of the person and firm effects model, we take this as our baseline specification. We compare the baseline specification to mixed effect estimates of the person and firm effects model. This comparison highlights the difference between fixed and mixed effect estimation methods. Our second comparison is between mixed effect estimates of the person and firm effects model and mixed effect estimates of the match effects model. This comparison highlights the importance of match effects.

Table 2 presents estimated coefficients (β, η) for fixed and mixed effect specifications of the person and firm effects model. The fixed effect estimates are consistent with earlier work. The fixed effect estimator produces somewhat steeper experience and education profiles than the mixed effect estimator does. We discuss this further below. The other estimated coefficients are very similar across specifications, with the exception of coefficients on several missing data indicators.

Table 3 presents estimated coefficients for fixed and mixed effect specifications of the match effects model. The estimated coefficients are broadly similar across specifications of the match effects model, and broadly similar to the person and firm effects model. There are some notable exceptions, however. As illustrated in Figures 1 and 2, the person and firm effects model consistently over-estimates the returns to experience. For instance, fixed effect estimates of the person and firm effects model imply that a male worker with 25 years of labor market experience earns 0.78 log points (118 percent) more than a labor market entrant, all

proportional to firm employment. A minimum of 5 employees are sampled from each firm. All workers employed in the reference period have an equal probability of being sampled, but the algorithm guarantees that each worker is connected to at least 5 others by a common employer.

else equal. The mixed effect estimator of the person and firm effects model reduces this estimate to 0.70 log points (101 percent), and introducing the match effect reduces it further to 0.65 log points (92 percent). For women, the earnings differential accruing to 25 years experience is 0.59 log points (80 percent) in the fixed effect specification of the person and firm effects model, 0.43 log points in the comparable mixed effect specification, and 0.39 log points (48 percent) in the mixed model with match effects. The "within" estimator (12) on which the orthogonal match effects and hybrid mixed effect estimators are based yields an even flatter experience profile. Here, the 25 year earnings gap is 0.52 log points (68 percent) for men and 0.36 log points (43 percent) for women.[24] Hence the baseline specification over-estimates the returns to 25 years of experience by as much as 0.26 log points (50 percent) for men, and 0.23 log points (37 percent) for women.

Because introducing the match effect flattens the experience profile so markedly, it seems that a considerable fraction of the returns traditionally attributed to labor market experience (i.e., the accumulation of general human capital) actually reflects the acquisition of match-specific human capital. A possible explanation is that individuals sort into better matches over the course of their career. When match effects are omitted, the higher earnings associated with sorting are attributed to labor market experience. We return to this idea below, when we investigate the sources of earnings growth when individuals change employer.

To a lesser degree, the baseline model also over-estimates the returns to education. It estimates that men with a college degree earn 0.25 log points (29 percent) more than male high-school graduates, all else equal, compared to 0.21 log points (23 percent) in the mixed model with match effects. The comparable estimates are 0.29 log points (33 percent) and 0.23 log points (25 percent), respectively, for women. Here too, it seems that some of the returns traditionally associated with general human capital (education) actually reflect match-specific human capital. A possible explanation is that more educated workers sort into better matches than less educated workers. When match effects are omitted, the returns to sorting into good matches are incorrectly attributed to education.

Table 4 presents the estimated variance of log earnings components. In all specifications, person effects exhibit the greatest dispersion and the returns to time-varying characteristics exhibit the least. This is consistent with earlier estimates of the person and firm effects model, e.g., AKM, ACK, and Woodcock (2005a). The traditional mixed model and hybrid mixed model give very similar results, so we focus on results for the traditional mixed model.

The fixed effect estimator of the person and firm effects model exhibits the greatest dispersion in person effects (0.274 squared log points) and time-varying covariates (0.031),

[24] Note this estimate is based entirely on within-job variation in earnings, i.e., it ignores earnings growth that occurs when individuals change employer.

and the least dispersion in firm effects (0.065). In contrast, the mixed effect estimator of the person and firm effects model exhibits slightly less dispersion in person effects (0.258) and time-varying covariates (0.029), and considerably more dispersion in firm effects (0.158). Introducing the match effect reduces variation in the person effect further (to 0.189), and reduces dispersion in the firm effect to 0.104. These imply a one standard deviation increase in the value of the person effect increases log earnings by 0.435, and a one standard deviation increase in the value of the firm effect increases log earnings by 0.322. The variance of the match effect itself is 0.079, so that a one standard deviation increase in the value of the match effect increases earnings by 0.28 log points. This is also very substantial and nearly as large as the firm effect.

These results imply that some of the variation attributed to the match effect is incorrectly attributed to person and firm effects in prior work. This not surprising, given the expression we derived for the bias due to omitted match effects. However, some of the variation attributed to the match effect was formerly unexplained, as we see from the reduced error variance (from 0.052 to 0.036) when the match effect is introduced.

The orthogonal match effect estimator produces quite different results. The estimates are very similar to the person and firm effects model, with only trivial variation in the match effect (0.022 squared log points). This is not surprising given the orthogonality assumption.

Before proceeding further, we formally test for the presence of match effects. The test is straightforward and the results are in Table 4. In the fixed model, the null hypothesis is $H_0 : \phi_{ij} = 0$ for each i, j pair in the data, i.e., that all match effects are zero. This is a test of $M - N - J = 4,176,870$ linear restrictions.[25] We test this hypothesis with a conventional Wald test. Given the number of restrictions, it is no surprise that we easily reject the null of no match effects at conventional significance levels.[26]

In the mixed model specifications, the null of no match effects is $H_0 : \sigma^2_\phi = 0$. We test this hypothesis with a likelihood ratio test based on the REML log-likelihoods of specifications with and without match effects. Because the null hypothesis places σ^2_ϕ on the boundary of the parameter space, the test statistic has a non-standard asymptotic distribution. Stram and Lee (1994) show its asymptotic distribution is a 50:50 mixture of a χ^2_0 and a χ^2_1. Once again, we easily reject the null of no match effects at conventional significance levels.[27]

[25] When the data consist of $\mathcal{G}$ connected groups of observations, there are $N^* + G - N - J - k - 1$ degrees of freedom in the model without match effects, and $N^* + G - M - k - 1$ degrees of freedom in the model with match effects. The model without match effects therefore imposes

$$(N^* + G - N - J - k - 1) - (N^* + G - M - k - 1) = M - N - J$$

linearly independent restictions on the estimated effects.

[26] The value of the Wald statistic is around 18 million.

[27] The value of the likelihood ratio statistic is over 35 thousand for both mixed models.

Table 5 presents sample correlations between the estimated effects in the person and firm effects model. Table 6 presents the same information for the match effects model. In each case, the person effect is most strongly correlated with log earnings (between 0.79 and 0.89). The firm effect is also strongly correlated with log earnings: between 0.41 and 0.57, depending on specification. The returns to time-varying covariates are less strongly correlated with log earnings (between 0.25 and 0.30). There is considerable variation in the estimated correlation between match effects and log earnings across specifications. The correlation is 0.23 in the orthogonal match effect specification, which is comparable to the correlation between observable characteristics and log earnings. In contrast, the correlation between the match effect and log earnings is around 0.60 in both mixed effect specifications, which is second only to the correlation between the person effect and log earnings.

There are several other items of note in Tables 5 and 6. One is that introducing the match effect strengthens the correlation between the person effect and log earnings in all specifications. It also strengthens the correlation between the firm effect and log earnings in both mixed effect specifications. The match effect evidently helps disentangle person- and firm-specific components of log earnings. Furthermore, notice that the estimated match effect is positively correlated with person and firm effects in both mixed effect specifications. The correlation with the person effect, in particular, is sufficiently strong (0.49) that the orthogonal match effect specification seems dubious.[28]

The correlation between person and firm effects is approximately zero in the fixed effect specification of the person and firm effects model and the orthogonal match effects model. This is consistent with earlier estimates of the person and firm effects model based on US data (e.g., ACK and Woodcock (2005a)). Recently, Andrews et al. (2004) have argued that when the true correlation between person and firm effects is positive, the estimated correlation based on least squares estimates of the person and firm effects model is biased downward. In light of this, it is interesting to note that the correlation is noticeably larger in both mixed effect specifications of the person and firm effects model (0.09). Introducing the match effect further increases the correlation between person and firm effects by a factor of two. This suggests the bias noted by Andrews et al. (2004) is partly a characteristic of the fixed effect estimator, and partly due to the omission of match effects.

[28]The substantial correlation between person and match effects might seem to contradict the assumed conditional covariance of the random effects in (19). The Bayesian interpretation of the mixed effect model is helpful in understanding this result. In the Bayesian formulation, the conditional moment restrictions (18) and (19) are the mean and variance of informative priors on the distribution of the random effects. An estimated non-zero correlation between random effects is evidence that the priors are swamped by data.

5.1 Decomposing the Variance of log Earnings

The match effects model defines a formal decomposition of the variance of log earnings into components attributable to time-varying observables, person effects, firm effects, match effects, and a residual component. Specifically,

$$\begin{aligned} Var\left(y_{ijt}\right) &= Cov\left(y_{ijt}, y_{ijt}\right) = Cov\left(y_{ijt}, \hat{\mu} + x'_{ijt}\hat{\beta} + \hat{\theta}_i + \hat{\psi}_j + \hat{\phi}_{ij} + e_{ijt}\right) \\ &= Cov\left(y_{ijt}, x'_{ijt}\hat{\beta}\right) + Cov\left(y_{ijt}, \hat{\theta}_i\right) + Cov\left(y_{ijt}, \hat{\psi}_j\right) + Cov\left(y_{ijt}, \hat{\phi}_{ij}\right) + Cov\left(y_{ijt}, e_{ijt}\right) \end{aligned} \tag{24}$$

where $\hat{\beta}, \hat{\theta}_i, \hat{\psi}_j, \hat{\phi}_{ij}$ are sample estimates defined by any of the fixed or mixed effect estimators, and e_{ijt} is the corresponding residual. Gruetter and Lalive (2004) present a similar decomposition for the person and firm effects model. We have the proportional decomposition:

$$\frac{Cov\left(y_{ijt}, x'_{ijt}\hat{\beta}\right)}{Var\left(y_{ijt}\right)} + \frac{Cov\left(y_{ijt}, \hat{\theta}_i\right)}{Var\left(y_{ijt}\right)} + \frac{Cov\left(y_{ijt}, \hat{\psi}_j\right)}{Var\left(y_{ijt}\right)} + \frac{Cov\left(y_{ijt}, \hat{\phi}_{ij}\right)}{Var\left(y_{ijt}\right)} + \frac{Cov\left(y_{ijt}, e_{ijt}\right)}{Var\left(y_{ijt}\right)} = 1. \tag{25}$$

Of course we can further decompose $Cov\left(y_{ijt}, \hat{\theta}_i\right) = Cov\left(y_{ijt}, \hat{\alpha}_i\right) + Cov\left(y_{ijt}, u'_i\hat{\eta}\right)$.

We present the proportional decomposition (25) in Table 7. In our baseline specification, nearly 64 percent of the variance of log earnings is attributed to person effects. Firm effects contribute the next largest component, about 16 percent. Conditional on person and firm effects, time-varying covariates explain only 6.7 percent of the variance of log earnings, leaving more than 13 percent unexplained. Results for mixed models with person and firm effects are very similar, though both mixed effect estimators attribute slightly more variation to firm effects and slightly less to person effects.

Introducing the match effect dramatically reduces the proportion of the variance of log earnings attributed to person effects: it is below 46 percent in the traditional mixed model and about 48 percent in the hybrid mixed model. Introducing the match effect also increases the proportion attributed to firm effects to 22 percent – now roughly half of the proportion attributed to person effects. Once again, it seems that introducing match effects helps disentangle person- and firm-specific components of earnings. The match effect explains about 16 percent of the variance of log earnings. This is substantial and more than twice the variation explained by time-varying observables. Finally, the unexplained component falls below 9 percent of the variance of log earnings. Thus about a quarter of the variance explained by match effects was unexplained by the baseline specification. The remaining three quarters were incorrectly attributed to other components of earnings.

5.2 Earnings Growth and Job Mobility

The match effects model also provides a formal decomposition of the sources of earnings growth when individuals change employers. For an individual i who changes employers (from employer j to employer n in periods t and s, respectively), the gross change in earnings is

$$\begin{aligned}\Delta y &= y_{ins} - y_{ijt} \\ &= \left(x'_{ins} - x'_{ijt}\right)\hat{\beta} + \left(\hat{\psi}_n - \hat{\psi}_j\right) + \left(\hat{\phi}_{in} - \hat{\phi}_{ij}\right) + \left(e_{ins} - e_{ijt}\right) \\ &\equiv \Delta x\hat{\beta} + \Delta\psi + \Delta\phi + \Delta e.\end{aligned}$$

This defines a simple decomposition of earnings changes into components attributable to the change in time-varying observables, firm effects, match effects, and a residual component. Again, we define a proportional decomposition

$$\frac{\Delta x\hat{\beta}}{\Delta y} + \frac{\Delta\psi}{\Delta y} + \frac{\Delta\phi}{\Delta y} + \frac{\Delta e}{\Delta y} = 1 \tag{26}$$

that aggregates linearly over job transitions. We use (26) to decompose the mean change in log earnings when individuals change employers into its respective components.

To decompose wage changes via (26), we focus on job-to-job transitions for two reasons. First, non-employment in the LEHD data is only identified by the absence of a UI record. Periods of non-employment may therefore reflect unemployment, withdrawal from the labor force, employment not covered by the UI reporting system, or employment in a state other than the two in our sample. These may confound our ability to identify a genuine transition from one employer to another. Job-to-job transitions, which we define as employment spells that overlap by at least one quarter, are less subject to these confounding influences. Second, job-to-job transitions are arguably more likely to violate the exogenous mobility assumption than those with an intervening period of unemployment because they are more likely to be driven by "good" and "bad" matches. Gruetter and Lalive (2004) argue this point at length. By focusing on job-to-job transitions, we can look for evidence of exogenous mobility in specifications with and without match effects. If the proportion of earnings growth attributed to the residual component is statistically significant, we take this as evidence that the exogenous mobility assumption is violated. We formalize this with the null hypothesis $H_0 : \frac{1}{M^*}\sum\left(e_{ins} - e_{ijt}\right) = 0$ where the summation is over all job-to-job employment transitions, and M^* is the number of transitions.

Table 8 presents the results of the decomposition (26). The mean annual change in real log earnings is 0.03 log points. Individuals that change jobs, in contrast, experience an average increase in log earnings of 0.045 log points (0.049 in the dense subsample). Average log earnings growth in the subset of job-to-job transitions is even larger: about 0.08 log points. Of this, our baseline specification attributes the largest component (about 40 percent) to time-varying covariates $X\hat{\beta}$. Firm effects also contribute significantly (31.6 percent). This suggests that workers ascend a "firm ladder" when they change jobs by moving into employment at higher paying firms. But notice that a large component of log earnings growth remains unexplained in the person and firm effects model: nearly 29 percent of log earnings growth is due to the residual component. Consequently, we easily reject the null that the residual component is zero at conventional significance levels. We take this as evidence that the person and firm effects model violates the exogenous mobility assumption.

The decomposition is similar for the mixed effect specification of the person and firm effects model. This specification attributes a larger proportion of log earnings growth to firm effects (nearly 40 percent) and a smaller proportion to the residual component (21.5 percent). Nevertheless, we still easily reject the null that the residual component is zero.

Introducing the match effect overturns this result. Time-varying covariates and firm effects still contribute about equally to log earnings growth when individuals change jobs – about 40 percent each in both mixed effect specifications. Match effects explain nearly all of the remainder: about 18 percent in the mixed effect specifications and 30 percent in the orthogonal match effects specification. Hence individuals sort into better matches when changing jobs, as well as into higher paying firms. This corroborates our earlier claim that individuals sort into better matches over the course of a career, which we posited as an explanation for the biased experience effect in the person and firm effects model. The substantial earnings growth accounted for by match effects leaves little variation unexplained: in both mixed effect models the residual component explains only about 1 percent of average growth in log earnings, and about 4 percent in the orthogonal match effects specification. Indeed, in both mixed effect specifications we fail to reject the null hypothesis that the residual component is zero at the five percent level. We take this as evidence that the mixed effect estimator of the match effects model satisfies the exogenous mobility assumption.

6 Conclusion

The match effects model illuminates the relative importance of worker-specific, firm-specific, and match-specific components of labor earnings. We interpret these as the returns to general, firm-specific, and match-specific human capital, respectively. The model attributes just over half of observed variation in log earnings to the combined effect of time-varying and time-invariant components of general human capital. Firm-specific human capital contributes an additional 22 percent, and match-specific human capital another 16 percent.

Introducing the match effect helps to disentangle person and firm-specific components of earnings, and corrects several biases in the person and firm effects. The person and firm effects model overestimates the returns to labor market experience and education, attributes too much variation to person effects and too little to firm effects, and underestimates the correlation between person and firm effects. Taken together, these suggest that some earnings variation previously attributed to general human capital is in fact attributable to workers sorting into higher-paying firms and better worker-firm matches. Consequently, the person and firm effects model underestimates the implied cost of employment re-allocation over the business cycle because it understates the importance of specific human capital and overstates the importance of general human capital.

These biases arise because employment mobility is not exogenous conditional on observable characteristics, person effects, and firm effects. We find evidence, however, that exogeneity holds when we control for match effects. Indeed, match effects explain a substantial portion of the change in log earnings when individuals change employers.

Our specification treats firm and match effects as time-invariant. In reality, firm-specific human capital and match-specific human capital probably accumulate over the course of an employment relationship. A fruitful avenue for future research is to examine the evolution firm and match effects over time.

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A Appendix: A Model of Human Capital, Wages, and Mobility

This appendix develops a simple two-period model of wage bargaining with on-the-job search. The model illustrates several key points argued in the text regarding the relationship between match-specific human capital, wages, and mobility. Specifically, it demonstrates that when productivity depends on match-specific human capital, so do mobility and wages. This violates the exogenous mobility assumption of the person and firm effects model.

Workers live for two periods. In each period, they are endowed with a single indivisible unit of labor that they supply to production at home or at a firm. Home production generates income h. Workers maximize the expected present value of income.

In each period, the worker meets a firm in a matching market. In the first period she meets "Firm 1," and in the second period she meets "Firm 2." Firms produce a homogeneous good with price normalized to one. They only produce output when matched with a worker. The worker produces output q at Firm 1 q' at Firm 2. Both q and q' are random variables distributed according to F on support $\left[\underline{q}, \overline{q}\right]$. Productivity is unknown until the worker and firm meet, but is observable thereafter.

Firms maximize the net revenues from a match. For Firm 1, this is $q - w_t$ where w_t denotes the period t wage payment to the worker. For Firm 2, net revenues are $q' - w'_t$. Wages are determined by a Nash bargain between worker and firm. The worker's share of the surplus is $\gamma \in (0,1)$. Workers and firms discount future income at the common rate ρ.

The worker's value of being employed in period t is J_t. The firm's value of employing the worker is Π_t. The firm's outside option is to forego production, whose value is normalized to zero. The value of the worker's best alternative to employment at the firm is U_t. The worker and firm mutually agree to engage in production if the joint surplus is non-negative, i.e., if $J_t + \Pi_t \geq U_t$.In this case, the wage payment w_t (or w'_t) solves

$$\max_{w_t} \quad (J_t - U_t)^{\gamma} \, \Pi_t^{1-\gamma}. \tag{27}$$

The model's solution consists of the worker's optimal mobility strategy and a schedule of wage offers. These are summarized in the following proposition. The proof is in Appendix B.

Proposition 1 *In the first period, the worker's optimal strategy is to accept employment at Firm 1 if $q \geq h$ and remain unemployed otherwise. If she accepts employment, she is paid*

$$w_1 = \gamma q + (1-\gamma)\, h + \rho \frac{(1-\gamma)^2}{2-\gamma} \int_h^{\overline{q}} \left[(2-\gamma)\,(h - q') + (q' - q)\right] dF. \tag{28}$$

If the worker begins the second period unemployed, she optimally accepts employment at Firm 2 if $q' \geq h$ *(Case 0), and remains unemployed otherwise. If she accepts, she is paid* $w'_{2,C0} = \gamma q' + (1-\gamma) h$.

If the worker begins the second period employed at Firm 1, her optimal strategy is as follows. If $q' \leq h \leq q$ *(Case 1) she remains employed at Firm 1 and is paid* $w_{2,C1} = \gamma q + (1-\gamma) h$. *If* $h \leq q' \leq q$ *(Case 2), she remains employed at Firm 1 and is paid* $w_{2,C2} = (2-\gamma)^{-1} [q + (1-\gamma) q']$. *Finally, if* $h \leq q < q'$ *(Case 3), she quits employment at Firm 1 and accepts employment at Firm 2. In this case, the wage is* $w'_{2,C3} = (2-\gamma)^{-1} [q' + (1-\gamma) q]$.

The first-period wage (28) is the sum of three components: [1] the worker's share γ of output, [2] compensation for foregoing the income generated by home production, and [3] the option value of employment (the expectation term). In the proof, we show that the option value is non-positive. The intuition is simple: the worker's second-period bargaining position is weakly improved if she is already employed at Firm 1, and she is consequently willing to accept a reduced first-period wage. To see this, note that the second-period wage is the bargaining-strength weighted average of match productivity and the worker's outside option. If she begins the second period unemployed (Case 0), or if she begins the second period employed but is less productive at Firm 2 than in home production (Case 1), then her outside option is h. When she is more productive at Firm 2 than in home production (Cases 2 and 3), her outside option is employment at the other firm, at a wage greater than h. Hence employment weakly improves her second period bargaining position.

Notice the worker changes employers if $q' > q$. Hence wages and mobility both depend on productivity. If productivity depends on match-specific human capital, then so do wages and mobility. This violates the exogenous mobility assumption underlying the person and firm effect specification (1) if the econometrician cannot observe match-specific human capital directly.

There are two limiting cases of this simple model that give rise to the match effects model. First, normalize h to zero and let worker i's productivity at firm j in period t be given by

$$q_{ijt} = e^{m + x'_{ijt}\beta + \theta_i + \psi_j + \phi_{ij}} \tag{29}$$

where m is the mean of log-productivity (common to all matches) and other terms are as defined in the main text.

The first case arises when the worker captures the entire match surplus, so that she is paid the value of her marginal product. That is, as $\gamma \to 1$ the period t wage at firm j is $w_{ijt} \to q_{ijt}$. Taking logarithms gives the match effects model.

The second case is more subtle. As the difference between the worker's productivity at Firm 1 and Firm 2 vanishes (i.e., as $q' - q \to 0$) the second-period wage is $w_{ij2} \to \gamma q_{ij2}$ in Cases 0 and 1, and $w_{ij2} \to q_{ij2}$ in Cases 2 and 3. Again, taking logarithms gives the match effects model.[29] Furthermore, as the expected difference between her productivity at Firm 1 and Firm 2 vanishes, i.e., as

$$\rho \frac{(1-\gamma)^3}{2-\gamma} \int_0^{\bar{q}} (q' - q)\, dF \to 0,$$

the first period wage is $w_{ij1} \to q_{ij1} \left[\gamma - \rho (1-\gamma)^2 \int_0^{\bar{q}} dF \right]$. This case arises, for example, as the worker's productivity at Firm 1 approaches the conditional mean of productivity given x_{ijt} and θ_i. Once again, taking logarithms gives the match effects model. The option value term, $q_{ij1}\rho (1-\gamma)^2 \int_0^{\bar{q}} dF$, will be reflected in the estimated returns to experience.

B Appendix: Proofs

Proof of Proposition 1. We solve the model backward. The worker begins the second period in one of two states. She is either unemployed or employed at Firm 1. If she is unemployed and meets Firm 2 in the matching market (Case 0), she must decide whether to remain unemployed or accept employment. Therefore $U_2 = h$, $J_2 = w_2'$, and $\Pi_2' = q' - w_2'$. The wage payment that solves the Nash bargain is $w_2' = \gamma q' + (1-\gamma) h$. The worker accepts employment at Firm 2 if $J_2 + \Pi_2' \geq U_2$, which implies $q' \geq h$.

If the worker begins the second period employed at Firm 1, she must choose between unemployment, employment at Firm 1, and employment at Firm 2. The optimal action depends on q, q', and h. There are three relevant subcases where $q \geq h$. We establish below that subcases where $q < h$ are irrelevant because the worker will not accept employment in the first period under these conditions.

The first subcase (Case 1) is $q' \leq h \leq q$. The maximum wage that Firm 2 can offer is $w_2' = q' \leq h$. The worker prefers unemployment to employment at Firm 2, so she chooses between unemployment and employment at Firm 1. Therefore $U_2 = h$, $J_2 = w_2$, and $\Pi_2 = q - w_2$. The wage payment that solves (27) is $w_2 = \gamma q + (1-\gamma) h$. Since $J_2 + \Pi_2 = q \geq h = U_2$ the worker accepts the offer and remains employed at Firm 1.

[29] The intercept differs in these two cases: it is $\ln \gamma + m$ in Cases 0 and 1, and m in Cases 2 and 3. Case 1 is likely to be empirically indistinguishable from Case 2, since in either case the worker remains at Firm 1 (her wage only changes because of the change in her outside option.) This will bias the estimated intercept, but this is rarely a concern. However, because the value of the intercept depends on her employment history (whether or not she was employed in period 1, i.e., Case 0 vs. Cases 2 and 3), the difference in intercepts may be partly reflected in the estimated returns to experience.

The second possibility is $h < q' \leq q$ (Case 2). In this case, both firms can offer wages greater than h. The worker prefers employment at either firm to unemployment, and she must choose whether to remain employed at Firm 1 or move to Firm 2. When bargaining with Firm 1, $U_2 = w_2'$, $J_2 = w_2$, and $\Pi_2 = q - w_2$; and when bargaining with Firm 2, $U_2 = w_2$, $J_2 = w_2'$, and $\Pi_2' = q' - w_2'$. The firms' wage offers solve the system of equations:

$$\begin{aligned} w_2 &= \gamma q + (1-\gamma) w_2' \\ w_2' &= \gamma q' + (1-\gamma) w_2. \end{aligned} \tag{30}$$

The unique solution is:

$$w_2 = (2-\gamma)^{-1} [q + (1-\gamma) q'] \tag{31}$$

$$w_2' = (2-\gamma)^{-1} [q' + (1-\gamma) q] . \tag{32}$$

Because $q \geq q'$ and $\gamma \in (0,1)$ it follows immediately that $w_2 \geq w_2'$. Thus the worker optimally chooses to remain at Firm 1.

The final possibility is $h \leq q < q'$ (Case 3). Again, both firms can offer wages greater than h, so the worker chooses between continued employment at Firm 1 and moving to Firm 2. As in the previous case, the firms' wage offers solve (30), resulting in the wage offers (31) and (32). Now $q' > q$, which implies $w_2' > w_2$, so the worker moves to Firm 2.

In the first period, the worker chooses between unemployment and employment at Firm 1. The value of being unemployed in period 1 and behaving optimally thereafter is

$$U_1 = h + \rho E [\max \{J_2, U_2\}; A = 0] = h + \rho E [\max \{w_2', h\}; A = 0] \tag{33}$$

where the expectation is taken over q', and where $A = 1$ if the worker accepts employment in the first period and zero otherwise. The worker and firm value employment in period 1 as follows

$$J_1 = w_1 + \rho E [\max \{J_2, U_2\}; A = 1] = w_1 + \rho E [\max \{w_2, w_2', h\}; A = 1] \tag{34}$$

$$\Pi_1 = q - w_1 + \rho E [\max \{\Pi_2, 0\}; A = 1] = q - w_1 + \rho E [\max \{q - w_2, 0\}; A = 1] . \tag{35}$$

The worker and Firm 1 engage in production if $J_t + \Pi_t \geq U_t$, which implies

$$q \geq h - \rho \left(\begin{array}{c} E [\max \{w_2, w_2', h\}; A = 1] + E [\max \{q - w_2, 0\}; A = 1] \\ -E [\max \{w_2', h\}; A = 0] \end{array} \right) \equiv q_1^*. \tag{36}$$

Here, q_1^* is the reservation productivity above which the worker and firm mutually agree that employment is beneficial, and below which they prefer to separate. Lemma 2 establishes $q_1^* = h$. Hence the worker accepts employment in the first period if $q \geq h$. The wage payment that solves (27) is

$$\begin{aligned} w_1 &= \gamma \left(q + \rho E \left[\max\left\{q - w_2, 0\right\}; A = 1\right]\right) \\ &\quad + (1-\gamma)\left(h + \rho E\left[\max\left\{w_2', h\right\}; A = 0\right] - \rho E\left[\max\left\{w_2, w_2', h\right\}; A = 1\right]\right). \quad (37) \end{aligned}$$

The various expectations are

$$\begin{aligned} E\left[\max\left\{q - w_2, 0\right\}; A = 1\right] &= \int_{\underline{q}}^{h} (1-\gamma)(q-h)\, dF + \int_{h}^{q} \frac{1-\gamma}{2-\gamma}(q - q')\, dF \\ E\left[\max\left\{w_2', h\right\}; A = 0\right] &= \int_{\underline{q}}^{h} h dF + \int_{h}^{\overline{q}} \left[\gamma q' + (1-\gamma) h\right] dF \\ E\left[\max\left\{w_2, w_2', h\right\}; A = 1\right] &= \int_{\underline{q}}^{h} \left[\gamma q + (1-\gamma) h\right] dF + \int_{h}^{q} \frac{1}{2-\gamma}\left[q + (1-\gamma) q'\right] dF \\ &\quad + \int_{q}^{\overline{q}} \frac{1}{2-\gamma}\left[q' + (1-\gamma) q\right] dF. \end{aligned}$$

Collecting terms gives

$$\begin{aligned} w_1 &= \gamma q + (1-\gamma) h + \rho \frac{(1-\gamma)^2}{2-\gamma} \int_{h}^{q} \left[(2-\gamma)(h - q') + (q' - q)\right] dF \\ &\quad + \rho \frac{(1-\gamma)^2}{2-\gamma} \int_{q}^{\overline{q}} \left[(1-\gamma)(h - q') + (h - q)\right] dF \quad (38) \\ &= \gamma q + (1-\gamma) h + \rho \frac{(1-\gamma)^2}{2-\gamma} \int_{h}^{\overline{q}} \left[(2-\gamma)(h - q') + (q' - q)\right] dF. \end{aligned}$$

The integral terms reflect the option value of employment. In (38), it is easy to see that the option value is non-positive: in the first integral, $h \leq q' \leq q$; and in the second integral, $h \leq q < q'$. ■

Lemma 2 $q_1^* = h$.

Proof of Lemma 2. Let

$$Z = E\left[\max\left\{w_2', h\right\}; A = 0\right] - E\left[\max\left\{w_2, w_2', h\right\}; A = 1\right] - E\left[\max\left\{q - w_2, 0\right\}; A = 1\right]$$

and suppose to the contrary that $q_1^* > h$. This implies $Z > 0$. Let $h < q_1^* \leq q$ so the worker accepts employment in the first period. Because $q > h$ we know Firm 1 will offer a second period wage $w_2 \geq h$, and hence $\max\{w_2, w_2', h\} = \max\{w_2, w_2'\}$. Therefore

$$E[\max\{w_2', h\}; A = 0] = \int_{\underline{q}}^{h} h dF + \int_{h}^{\overline{q}} [(1-\gamma) h + \gamma q'] dF \quad (39)$$

$$\begin{aligned} E[\max\{w_2, w_2', h\}; A = 1] &= \int_{\underline{q}}^{h} [(1-\gamma) h + \gamma q] dF + \int_{h}^{q} \frac{1}{2-\gamma} [q + (1-\gamma) q'] dF \\ &\quad + \int_{q}^{\overline{q}} \frac{1}{2-\gamma} [q' + (1-\gamma) q] dF \end{aligned} \quad (40)$$

$$\begin{aligned} E[\max\{q - w_2, 0\}; A = 1] &= \int_{\underline{q}}^{h} (1-\gamma)(q-h) dF \\ &\quad + \int_{h}^{q} \left[q - \frac{1}{2-\gamma} [q + (1-\gamma) q'] \right] dF. \end{aligned} \quad (41)$$

Rearranging gives:

$$\begin{aligned} Z &= \int_{\underline{q}}^{h} (h-q) dF + \int_{h}^{q} [(1-\gamma)(h-q) + \gamma (q'-q)] dF \\ &\quad + \int_{q}^{\overline{q}} \left[(1-\gamma)^2 (h-q') + (1-\gamma)(h-q)\right] dF \\ &< 0, \end{aligned} \quad (42)$$

a contradiction.

Suppose instead that $q_1^* < h$. This implies $Z < 0$. Let $q_1^* \leq q < h$ so the worker accepts employment in the first period. Because $q < h$, the largest second period wage that Firm 1 can offer is $w_2 < h$. Hence $\max\{w_2, w_2', h\} = \max\{w_2', h\}$ and $\max\{q - w_2, 0\} = 0$. Thus $Z = 0$, another contradiction. ■

TABLE 1
SUMMARY STATISTICS
(Sample Proportions Unless Otherwise Stated)

	FULL SAMPLE		ALL INDIVIDUALS EMPLOYED IN 1997		TEN PERCENT DENSE SUBSAMPLE	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
Demographic Characteristics						
Male	0.56	0.50	0.58	0.49	0.57	0.50
Age (Years)	40.6	10.2	40.3	9.6	40.3	9.6
Men						
Nonwhite	0.21	0.57	0.20	0.55	0.20	0.56
Race Missing	0.04	0.25	0.03	0.24	0.03	0.24
Less Than High School	0.12	0.45	0.11	0.43	0.11	0.43
High School	0.30	0.67	0.30	0.65	0.29	0.66
Some College	0.23	0.60	0.23	0.59	0.23	0.59
Associate or Bachelor's Degree	0.25	0.62	0.25	0.61	0.25	0.62
Graduate or Professional Degree	0.10	0.42	0.11	0.42	0.11	0.42
Women						
Nonwhite	0.24	0.69	0.24	0.71	0.25	0.72
Race Missing	0.02	0.22	0.02	0.22	0.02	0.22
Less Than High School	0.09	0.45	0.09	0.45	0.09	0.44
High School	0.31	0.78	0.30	0.79	0.30	0.78
Some College	0.25	0.71	0.25	0.73	0.25	0.72
Associate or Bachelor's Degree	0.26	0.72	0.27	0.75	0.27	0.75
Graduate or Professional Degree	0.08	0.42	0.09	0.44	0.09	0.44
Work History Characteristics						
Real Annualized Earnings (1990 Dollars)	41,107	38,849	43,183	39,324	43,528	38,782
Men						
Labor Market Experience (Years)	11.8	13.1	11.9	12.7	11.8	12.7
Worked 0 Full Quarters in Calendar Year	0.08	0.36	0.06	0.32	0.06	0.32
Worked 1 Full Quarter in Calendar Year	0.15	0.49	0.12	0.44	0.12	0.44
Worked 2 Full Quarters in Calendar Year	0.13	0.47	0.12	0.44	0.12	0.44
Worked 3 Full Quarters in Calendar Year	0.14	0.48	0.13	0.46	0.14	0.47
Worked 4 Full Quarters in Calendar Year	0.50	0.80	0.56	0.81	0.57	0.00
Women						
Labor Market Experience (Years)	9.5	13.0	9.0	12.5	9.2	12.6
Worked 0 Full Quarters in Calendar Year	0.07	0.39	0.06	0.36	0.05	0.35
Worked 1 Full Quarter in Calendar Year	0.14	0.54	0.11	0.50	0.11	0.50
Worked 2 Full Quarters in Calendar Year	0.13	0.53	0.12	0.51	0.11	0.50
Worked 3 Full Quarters in Calendar Year	0.14	0.55	0.13	0.54	0.13	0.54
Worked 4 Full Quarters in Calendar Year	0.52	0.96	0.58	1.02	0.59	1.01
Year						
1990	0.09	0.29	0.07	0.26	0.07	0.26
1991	0.09	0.29	0.08	0.27	0.08	0.27
1992	0.09	0.29	0.08	0.27	0.08	0.28
1993	0.10	0.29	0.09	0.28	0.09	0.28
1994	0.10	0.30	0.10	0.29	0.10	0.29
1995	0.10	0.30	0.10	0.31	0.10	0.31
1996	0.10	0.31	0.11	0.32	0.11	0.32
1997	0.11	0.31	0.14	0.35	0.14	0.34
1998	0.11	0.31	0.12	0.32	0.12	0.32
1999	0.11	0.31	0.11	0.31	0.11	0.31
Number of Observations	49,291,205		37,688,492		3,652,544	
Number of Workers	9,272,529		5,235,887		503,179	
Number of Firms	573,307		476,745		121,227	
Number of Worker-Firm Matches	15,309,134		9,889,502		947,883	
Number of Connected Groups	84,748		46,829		1,460	

TABLE 2
ESTIMATED COEFFICIENTS: PERSON AND FIRM EFFECTS MODEL

	FIXED MODEL		MIXED MODEL		HYBRID MIXED MODEL	
	Estimate	SE	Estimate	SE	Estimate	SE
Time-Varying Characteristics (β)						
Male x Experience	0.074	0.000	0.066	0.001		
Male x Experience^2 / 100	-0.243	0.001	-0.213	0.005		
Male x Experience^3 / 1000	0.036	0.000	0.029	0.001		
Male x Experience^4 / 10000	-0.002	0.000	-0.002	0.000		
Male x Worked 0 Full Quarters	0.035	0.000	0.030	0.001		
Male x Worked 1 Full Quarters	-0.004	0.000	-0.007	0.001		
Male x Worked 2 Full Quarters	-0.013	0.000	-0.014	0.001		
Male x Worked 3 Full Quarters	-0.015	0.000	-0.014	0.001		
Female x Experience	0.031	0.000	0.029	0.001		
Female x Experience^2 / 100	-0.020	0.001	-0.052	0.005		
Female x Experience^3 / 1000	-0.006	0.000	0.002	0.001		
Female x Experience^4 / 10000	0.001	0.000	0.000	0.000		
Female x Worked 0 Full Quarters	0.010	0.000	-0.001	0.001		
Female x Worked 1 Full Quarters	-0.006	0.000	-0.012	0.001		
Female x Worked 2 Full Quarters	-0.014	0.000	-0.019	0.001		
Female x Worked 3 Full Quarters	-0.020	0.000	-0.029	0.001		
Time-Invariant Characteristics‡ (η)						
Male x High School	0.075	0.000	0.079	0.003	0.056	0.003
Male x Some College	0.168	0.000	0.169	0.003	0.143	0.003
Male x Associate or Bachelor's Degree	0.329	0.000	0.316	0.003	0.282	0.003
Male x Graduate or Professional Degree	0.526	0.000	0.493	0.004	0.458	0.004
Male x Nonwhite	-0.326	0.000	-0.351	0.002	-0.369	0.002
Male x Race Missing	0.006	0.001	-0.061	0.005	-0.067	0.005
Male x First Period Potential Experience <0	-0.074	0.001	-0.080	0.005	-0.199	0.005
Female x Less Than High School	-0.254	0.001	-0.166	0.007	-0.227	0.004
Female x High School	-0.073	0.000	-0.048	0.006	-0.152	0.003
Female x Some College	0.033	0.000	0.042	0.006	-0.064	0.003
Female x Bachelor or Associate's Degree	0.212	0.000	0.200	0.006	0.089	0.003
Female x Graduate or Professional Degree	0.396	0.001	0.374	0.007	0.261	0.005
Female x Nonwhite	-0.121	0.000	-0.127	0.002	-0.136	0.002
Female x Race Missing	-0.004	0.001	-0.041	0.007	-0.047	0.007
Female x First Period Potential Experience <0	0.092	0.001	0.032	0.006	-0.032	0.006
Intercept	9.84	0.001	9.69	0.005	9.87	0.003
Year Effects	YES		YES		YES	

Notes: Fixed model is estimated on the sample of all individuals employed in 1997. Both mixed model specifications are estimated on the ten percent dense subsample. Time-varying coefficients in the hybrid mixed model are the same as fixed effects estimates of the match effects model (Table 3).

TABLE 3
ESTIMATED COEFFICIENTS: MATCH EFFECTS MODEL

	FIXED MODEL		MIXED MODEL		HYBRID MIXED MODEL	
	Estimate	SE	Estimate	SE	Estimate	SE
Time-Varying Characteristics (β)						
Male x Experience	0.058	0.000	0.061	0.001		
Male x Experience^2 / 100	-0.215	0.001	-0.200	0.005		
Male x Experience^3 / 1000	0.033	0.000	0.028	0.001		
Male x Experience^4 / 10000	-0.002	0.000	-0.002	0.000		
Male x Worked 0 Full Quarters	0.053	0.000	0.042	0.001		
Male x Worked 1 Full Quarters	0.007	0.000	0.001	0.001		
Male x Worked 2 Full Quarters	-0.007	0.000	-0.010	0.001		
Male x Worked 3 Full Quarters	-0.013	0.000	-0.012	0.000		
Female x Experience	0.018	0.000	0.024	0.001		
Female x Experience^2 / 100	0.004	0.001	-0.033	0.005		
Female x Experience^3 / 1000	-0.010	0.000	-0.001	0.001		
Female x Experience^4 / 10000	0.001	0.000	0.000	0.000		
Female x Worked 0 Full Quarters	0.029	0.000	0.012	0.001		
Female x Worked 1 Full Quarters	0.008	0.000	-0.003	0.001		
Female x Worked 2 Full Quarters	-0.009	0.000	-0.016	0.001		
Female x Worked 3 Full Quarters	-0.017	0.000	-0.027	0.001		
Time-Invariant Characteristics(η)						
Male x High School	0.054	0.000	0.074	0.003	0.051	0.003
Male x Some College	0.143	0.000	0.158	0.003	0.131	0.003
Male x Associate or Bachelor's Degree	0.294	0.000	0.300	0.003	0.265	0.003
Male x Graduate or Professional Degree	0.491	0.637	0.473	0.004	0.437	0.004
Male x Nonwhite	-0.339	0.467	-0.343	0.002	-0.360	0.002
Male x Race Missing	0.003	0.000	-0.056	0.005	-0.062	0.005
Male x First Period Potential Experience <0	-0.196	0.348	-0.089	0.005	-0.186	0.005
Female x Less Than High School	-0.222	0.000	-0.188	0.007	-0.236	0.004
Female x High School	-0.120	0.000	-0.072	0.006	-0.163	0.003
Female x Some College	-0.020	0.000	0.013	0.006	-0.081	0.003
Female x Bachelor or Associate's Degree	0.146	0.000	0.162	0.006	0.063	0.003
Female x Graduate or Professional Degree	0.323	0.000	0.328	0.007	0.227	0.005
Female x Nonwhite	-0.131	0.000	-0.122	0.002	-0.130	0.002
Female x Race Missing	-0.009	0.000	-0.033	0.007	-0.038	0.007
Female x First Period Potential Experience <0	-0.021	0.000	0.027	0.006	-0.025	0.006
Intercept	10.00	0.000	9.71	0.004	9.86	0.003
Year Effects	YES		YES		YES	

Notes: Fixed model estimates are based on the orthogonal match effects specification and estimated on the sample of all individuals employed in 1997. Both mixed model specifications are estimated on the ten percent dense subsample. Time-varying coefficients in the hybrid mixed model are the same as the fixed model.

FIGURE 1

Estimated Returns to Experience: Men

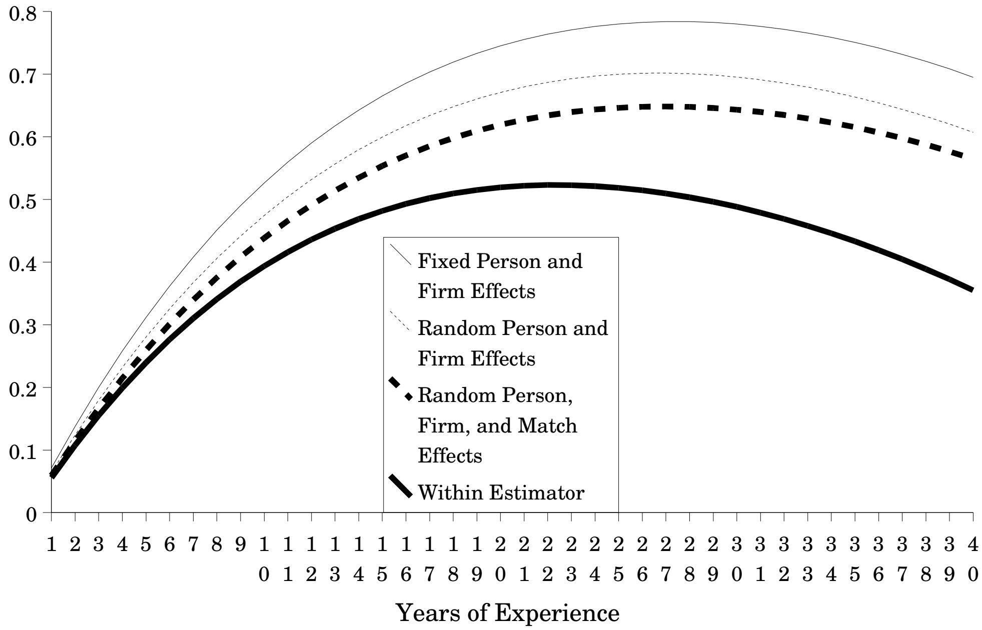

FIGURE 2

Estimated Returns to Experience: Women

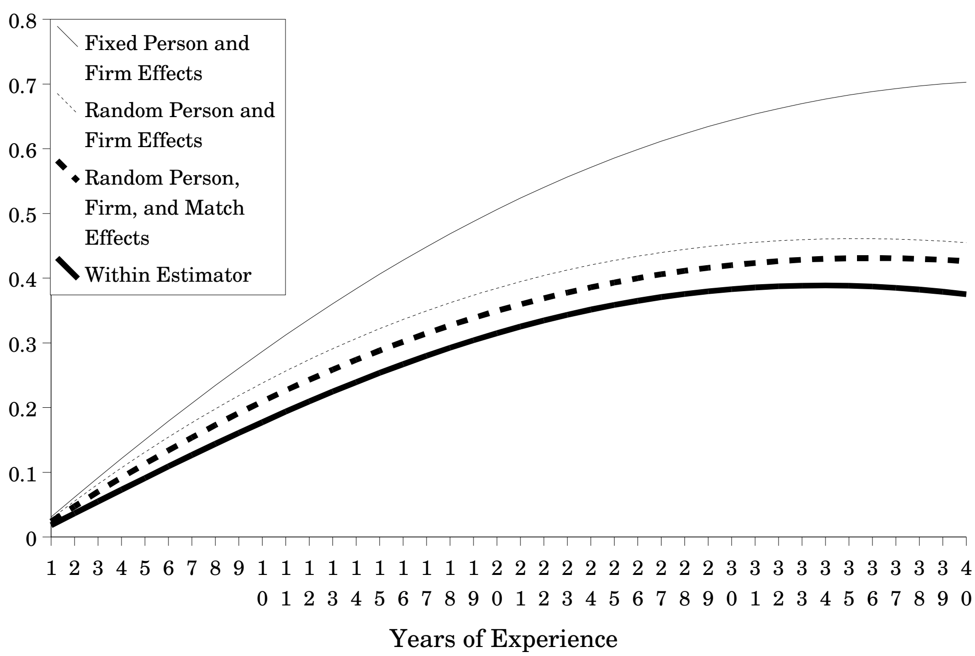

TABLE 4
VARIANCE OF ESTIMATED COMPONENTS OF LOG EARNINGS

	FIXED MODEL*		MIXED MODEL†		HYBRID MIXED MODEL†	
	Person and Firm Effects	Orthogonal Match Effects	Person and Firm Effects	Match Effects	Person and Firm Effects	Match Effects
Variance of Log Real Annualized Earnings (y)	0.422		0.410		0.410	
Variance of Time-Varying Covariates (Xβ)	0.031	0.017	0.029	0.027	0.017	0.017
Variance of Pure Person Effect (θ)	0.274	0.273	0.258	0.189	0.269	0.198
Time-Invariant Covariates (Uη)	0.043	0.041	0.038	0.036	0.041	0.039
Unobserved Heterogeneity (α)	0.232	0.233	0.220	0.153	0.228	0.159
Variance of Firm Effect (ψ)	0.065	0.066	0.158	0.104	0.158	0.102
Variance of Match Effect (ϕ)		0.022		0.079		0.079
Error Variance (ε)	0.066	0.040	0.052	0.036	0.052	0.036
H_0: No Match Effects (p-value)		<0.00001		<0.00001		<0.00001
R^2	0.867	0.919	0.894	0.933	0.894	0.933
Model Degrees of Freedom	32,022,609	27,798,909	3,652,501	3,652,500	3,652,501	3,652,500

* Estimates are based on full sample of individuals employed in 1997. Values in the table are sample variances of the estimated effects. The estimated error variance is corrected for degrees of freedom.

† Estimates are based on ten percent dense subsample of individuals employed in 1997. For the rows labeled y, Xβ, Uη, values in the table are sample variances. For the rows labeled θ, ψ, ϕ, ε, values in the table are REML estimates of variance components.

TABLE 5
SAMPLE CORRELATIONS AMONG ESTIMATED EFFECTS
Person and Firm Effects Model

Fixed Model[*]

	y	Xβ	θ	Uη	α	ψ
Log Real Annualized Earnings (y)	1					
Time-Varying Covariates (Xβ)	0.25	1				
Pure Person Effect (θ)	0.79	-0.06	1			
Time-Invariant Covariates (Uη)	0.36	0.02	0.39	1		
Unobserved Heterogeneity (α)	0.71	-0.08	0.92	0.00	1	
Firm Effect (ψ)	0.41	0.07	0.00	0.08	-0.03	1

Mixed Model[†]

	y	Xβ	θ	Uη	α	ψ
Log Real Annualized Earnings (y)	1					
Time-Varying Covariates (Xβ)	0.30	1				
Pure Person Effect (θ)	0.80	0.01	1			
Time-Invariant Covariates (Uη)	0.36	0.09	0.40	1		
Unobserved Heterogeneity (α)	0.71	-0.03	0.91	-0.01	1	
Firm Effect (ψ)	0.50	0.07	0.09	0.10	0.06	1

Hybrid Mixed Model[†]

	y	Xβ	θ	Uη	α	ψ
Log Real Annualized Earnings (y)	1					
Time-Varying Covariates (Xβ)	0.25	1				
Pure Person Effect (θ)	0.83	0.04	1			
Time-Invariant Covariates (Uη)	0.40	0.22	0.41	1		
Unobserved Heterogeneity (α)	0.72	-0.05	0.91	-0.01	1	
Firm Effect (ψ)	0.49	0.05	0.09	0.11	0.05	1

[*] Estimates are based on full sample of individuals employed in 1997.

[†] Estimates are based on ten percent dense subsample of individuals employed in 1997.

TABLE 6
SAMPLE CORRELATIONS AMONG ESTIMATED EFFECTS
Match Effects Model

*Orthogonal Match Effects Model**

	y	Xβ	θ	Uη	α	ψ	φ
Log Real Annualized Earnings (y)	1						
Time-Varying Covariates (Xβ)	0.25	1					
Pure Person Effect (θ)	0.81	0.02	1				
Time-Invariant Covariates (Uη)	0.38	0.14	0.39	1			
Unobserved Heterogeneity (α)	0.72	-0.04	0.92	0.00	1		
Firm Effect (ψ)	0.41	0.08	0.01	0.09	-0.03	1	
Match Effect (φ)	0.23	0.00	0.00	0.00	0.00	0.00	1

Mixed Model†

	y	Xβ	θ	Uη	α	ψ	φ
Log Real Annualized Earnings (y)	1						
Time-Varying Covariates (Xβ)	0.29	1					
Pure Person Effect (θ)	0.80	0.04	1				
Time-Invariant Covariates (Uη)	0.37	0.13	0.51	1			
Unobserved Heterogeneity (α)	0.71	-0.03	0.85	-0.01	1		
Firm Effect (ψ)	0.54	0.05	0.18	0.13	0.14	1	
Match Effect (φ)	0.59	-0.01	0.49	-0.01	0.57	0.04	1

Hybrid Mixed Model†

	y	Xβ	θ	Uη	α	ψ	φ
Log Real Annualized Earnings (y)	1						
Time-Varying Covariates (Xβ)	0.25	1					
Pure Person Effect (θ)	0.82	0.08	1				
Time-Invariant Covariates (Uη)	0.40	0.24	0.51	1			
Unobserved Heterogeneity (α)	0.71	-0.05	0.85	-0.01	1		
Firm Effect (ψ)	0.54	0.04	0.19	0.13	0.14	1	
Match Effect (φ)	0.60	-0.04	0.49	-0.01	0.57	0.04	1

* Estimates are based on full sample of individuals employed in 1997.

† Estimates are based on ten percent dense subsample of individuals employed in 1997.

TABLE 7
DECOMPOSITION OF THE VARIANCE OF LOG EARNINGS

	FIXED MODEL*		MIXED MODEL†		HYBRID MIXED MODEL†	
	Person and Firm Effects	Orthogonal Match Effects	Person and Firm Effects	Match Effects	Person and Firm Effects	Match Effects
Proportion of Variance of Log Earnings:						
Time-Varying Covariates (Xβ)	0.067	0.050	0.080	0.075	0.051	0.051
Pure Person Effect (θ)	0.637	0.653	0.595	0.457	0.625	0.482
Time-Invariant Covariates (Uη)	0.115	0.117	0.110	0.111	0.126	0.124
Unobserved Heterogeneity (α)	0.523	0.536	0.485	0.346	0.499	0.358
Firm Effect (ψ)	0.163	0.164	0.198	0.223	0.198	0.222
Match Effect (ϕ)		0.052		0.157		0.157
Residual (e)	0.133	0.081	0.126	0.088	0.126	0.087
TOTAL	1.000	1.000	1.000	1.000	1.000	1.000

* Estimates are based on full sample of individuals employed in 1997.

† Estimates are based on ten percent dense subsample of individuals employed in 1997.

TABLE 8
DECOMPOSITION OF CHANGES IN LOG EARNINGS DUE TO JOB MOBILITY

	FIXED MODEL‡		MIXED MODEL†		HYBRID MIXED MODEL†	
	Person and Firm Effects	Orthogonal Match Effects	Person and Firm Effects	Match Effects	Person and Firm Effects	Match Effects
Mean Annual Change in Real Log Earnings	0.030		0.032		0.032	
Mean Change in Log Earnings, All Job Transitions	0.045		0.049		0.049	
Mean Change in Log Earnings, Job-to-Job Transitions	0.080		0.082		0.082	
Proportion Attributed to (Job-to-Job Transitions):						
Time-Varying Covariates ($X\beta$)	0.395	0.410	0.391	0.401	0.415	0.415
Firm Effect (ψ)	0.316	0.305	0.395	0.405	0.373	0.390
Match Effect (ϕ)		0.329		0.183		0.185
Residual (e)	0.289	-0.044	0.215	0.011*	0.212	0.010*
TOTAL	1.000	1.000	1.000	1.000	1.000	1.000
Total Number of Job Transitions	4,822,691		461,397		461,397	
Number of Job-to-Job Transitions	2,233,456		213,763		213,763	

Asterisk (*) indicates estimate is **not** statisitically significant at the 5% level.

‡ Estimates are based on full sample of individuals employed in 1997.

† Estimates are based on ten percent dense subsample of individuals employed in 1997.