

# MPRA

Munich Personal RePEc Archive

## Inattentive Consumers and Product Quality

Armstrong, Mark and Chen, Yongmin  
Department of Economics, University College London

September 2007

Online at <http://mpra.ub.uni-muenchen.de/4797/>  
MPRA Paper No. 4797, posted 07. November 2007 / 04:13

# Inattentive Consumers and Product Quality\*

Mark Armstrong<sup>†</sup>      Yongmin Chen<sup>‡</sup>

September 2007

## Abstract

This paper studies a model in which some consumers shop on the basis of price alone, without attention to potential differences in product quality. A firm may offer a low-quality product to exploit these inattentive consumers. In the unique symmetric equilibrium of the model, firms choose prices with mixed strategies, similarly to Varian (1980) in which some consumers purchase from a random seller without attention to market prices. In our model, though, firms also choose quality stochastically, and there is both price and quality dispersion. Two stylized policy interventions are considered: competition policy, which acts to increase the number of sellers, and market transparency reforms which act to increase the fraction of attentive consumers. With fewer inattentive consumers, firms are less likely to “cheat” (i.e., cut quality) which therefore improves welfare, but profit and consumer surplus can either increase or decrease. When there is a large number of sellers, approximately half the sellers cheat (regardless of the fraction of inattentive consumers), and introducing more sellers boosts consumers surplus and reduces profit, while the impact on welfare is ambiguous.

## 1 Introduction

Consumers may not possess all relevant information when purchasing a product from a particular seller. The missing information could be the price of the product sold by rival sellers, the quality of the product, or the availability of alternative sellers. Sometimes a consumer’s lack of information could be her rational choice in an environment of costly information gathering and processing. But at other times, a consumer’s inattention to information may be due to other factors. For instance, she may be unaware of such information, or she may mis-perceive the game being played by sellers.<sup>1</sup>

---

\*We are grateful to Benedikt Koehler and Priya Sinha for useful information. Armstrong is grateful to the Economic and Social Research Council (UK) for funding assistance.

<sup>†</sup>Department of Economics, University College London

<sup>‡</sup>Department of Economics, University of Colorado at Boulder

<sup>1</sup>See Stiglitz (1989) for a survey on imperfectly informed consumers.

Varian (1980) considers a market in which some consumers do not know market prices and simply purchase from a random seller, while other consumers know the prices of all sellers and buy at the lowest price.<sup>2</sup> A “rational consumer” interpretation of the inattentive consumers is that they incur extremely high search costs to sample a second seller. A “boundedly rational” interpretation might be that these consumers mis-perceive the market, and mistakenly believe it is perfectly competitive in the sense that all sellers offer the same price; in this situation there is no point in trying another seller. We describe Varian’s model in detail in the next section.

In this paper, we study a variant of Varian’s model in which all consumers know all prices, but only some consumers are attentive to a seller’s product quality. (By “quality” we include such financial items as “hidden charges”, price hikes after “introductory offers”, and so on.) In many markets the headline price is transparent and prominently displayed, while quality is more opaque. For instance, marketing material for a new credit card may emphasize a headline interest rate, but put relevant information about penalty charges, when interest rates are levied, and other details in the “small print” which some consumers may overlook. A mortgage seller may offer a generous deal for the first two years of the contract, but then move to a less-than-generous interest rate for the remainder of the term, which in some cases may force borrowers to default.<sup>3</sup> Also, a landscape contractor may quote a low price for a project, without highlighting the quality of material to be used; or a seller of insurance may advertise a headline premium, while details about excesses and exclusions are more hidden.<sup>4</sup>

Chapter 4 of Cruickshank (2000) describes a useful survey on consumer attentiveness in financial retail markets. For instance, Table 4.4 reports that 18% know the interest rate for unauthorized overdrafts on their own current account “exactly”, while 43% know this figure “not at all”. Table 4.5 reports that 29% of respondents know the notice period

---

<sup>2</sup>See Salop and Stiglitz (1977) and Rosenthal (1980) for related models.

<sup>3</sup>For instance, see the opening remarks made on March 22, 2007 by Senator Chris Dodd (Chairman of the Senate Committee on Banking, Housing and Urban Affairs) at a hearing on “Mortgage Market Turmoil: Causes and Consequences”. The statement referred to “Amy Dodd [...] who was promised a mortgage at \$927 per month and ended up with a mortgage costing her \$2,100.” See <http://dodd.senate.gov>.

<sup>4</sup>See Leland (1979) for a model where no consumers can observe quality, and Chan and Leland (1982) and Dranove and Satterthwaite (1992) for models where price and/or quality can be observed at a search cost. Gabaix and Laibson (2006) present a model of “add-ons”, where the price of a complementary item is “shrouded” when the principal product is purchased. A difference between their model and ours is that they assume consumers need not consume the add-on, or can find a close substitute for it. In our model, consumption is a one-off decision. Interestingly, Gabaix and Laibson find situations in which firms have no incentive to make the price of their add-on transparent.

on their savings account “exactly” while 23% know it “not at all”. (Similar figures apply to knowledge about charges for withdrawals.) That chapter also documents a wide degree of price dispersion in these markets. Other illustrative information is found in Financial Services Authority (2006, pages 99-100), which reports that when buying a financial product about half of survey respondents read the terms and conditions “in detail”, while about 10% did not read them at all. And in the context of choosing a broadband provider, a survey suggests that “most customers admit to being ‘dazzled’ by headline-grabbing ‘lures’ [...] rather than searching the small print”, and “over half of all broadband customers admit to not reading the contract before signing up”.<sup>5</sup>

Section 4.1 of Della Vigna (2007) suggests a theoretical framework in which the true total price of an item is  $V = v + i$ , where  $v$  is the “visible” price and  $i$  is a less visible component (such as “postage and packing” for online retailers or indirect taxes not factored into the displayed price). Due to limited attention to  $i$ , though, consumers behave as though the total price were instead  $\hat{V} = v + (1 - \theta)i$ , where  $\theta$  is an “inattention” parameter. Della Vigna then summarizes a series of empirical studies in terms of their implied estimates for  $\theta$ . These include the estimated inattention parameter being in the range  $0.18 \leq \theta \leq 0.45$  for sales of CDs on eBay, and in the range  $0.75 \leq \theta \leq 0.94$  (extremely high figures) for indirect US state taxes on beer and groceries.

Our model is a highly stylized representation of these kinds of markets. In particular, we suppose there are just two levels of quality which can be chosen by a seller, where the low-quality product is “useless” and would not be produced if there were no inattentive consumers in the market.<sup>6</sup> The reason a firm finds it profitable to offer a low-quality product is that some consumers, however small in number, would mistake it for the high-quality product. The model has a unique symmetric equilibrium, in which both prices and qualities are chosen stochastically by sellers. Consequently, there is both price and quality dispersion in our market. In contrast to Varian’s model, here the inattentive consumers are genuinely “ripped off” since they may receive a strictly negative payoff from participating in the market.

---

<sup>5</sup>See item “Small print catching out broadband customers”, posted 18 June 2007 at [www.FinanceNewsOnline.co.uk](http://www.FinanceNewsOnline.co.uk).

<sup>6</sup>That is, consumer utility derived from the low-quality product does not exceed its production cost. Thus, our model is not intended to be one of vertical production differentiation, where some well-informed consumers may choose to purchase the low-quality product which is offered with a lower price. We return to this point in Section 5, where we will give a “rational consumer” interpretation of our model in the context of vertical differentiation.

Different forms of public intervention could have different effects on market parameters: *competition policy*, through forbidding mergers and encouraging entry, may increase the number of rivals in the market, while *market transparency* reforms (e.g., improving information flows to consumers or regulating disclosure requirements) could increase the fraction of attentive consumers.<sup>7</sup> In Varian’s model, as we recapitulate in the next section, more market transparency is always good for both groups of consumers, but bad for profit, while competition policy has no impact on industry profit, helps the attentive consumers, but harms inattentive consumers.

In our model, market transparency reforms improve overall welfare (which is unaffected by policy in Varian’s model) and the surplus enjoyed by the attentive consumers. However, the impact on industry profits, on inattentive consumers and on aggregate consumer surplus is ambiguous. In situations where most consumers are attentive, however, the comparative statics are as in Varian’s model: profits fall and the inattentive consumers are better off when the fraction of attentive consumers is increased still further. Tighter competition policy, at least when there are a reasonably large number of sellers already, acts to reduce industry profits and improve aggregate consumer surplus. Total welfare can go up or down with more sellers. Attentive consumers generally are better off with more sellers, while inattentive consumers can be made worse off (as in Varian’s model). Strikingly, when there are many sellers, approximately half of them will cheat, regardless of other parameter values.

Section 2 below reviews Varian’s model. In Section 3 we formulate our model and characterize the market equilibrium, while the impact of policy interventions is described in Section 4. Section 5 discusses the results and concludes.

## 2 Varian’s Model: Inattention to Price

Before we present our model, recall Varian’s (1980) model of sales. There, a population of consumers each wish to buy a single unit of a homogeneous product. There are  $N$  identical risk-neutral firms in the market, each with marginal cost  $c$ . A fraction  $\lambda$  of consumers are “attentive”, in that they know all the prices in the market and buy from the lowest-price supplier. The remaining  $1 - \lambda$  consumers buy from a random supplier, so long as that supplier’s price is no greater than the reservation utility  $v > c$ . Each

---

<sup>7</sup>See DTI (2003) for various market transparency proposals in the consumer credit sector.

consumer has a unit demand, and the consumer population size is normalized to 1.

In this environment, it is clear there can be no pure strategy equilibrium. For instance, if a firm knows that each of its rivals is charging price  $p > c$  for sure, it can slightly undercut this price and so attract the entire pool of attentive consumers. Or if the other firms set a price equal to cost, a firm can make positive profit by exploiting its captive, inattentive consumers, who are  $(1 - \lambda)/N$  in number. Therefore, firms follow a mixed pricing strategy, and the reason for price dispersion in this model is to confuse rival firms, not consumers.

A symmetric mixed pricing strategy will involve each firm's price being taken from the cdf  $F(p)$ , where  $F$  satisfies

$$\left[ \lambda (1 - F(p))^{N-1} + \frac{1 - \lambda}{N} \right] (p - c) \equiv \frac{1 - \lambda}{N} (v - c). \quad (1)$$

The term in square brackets represents a firm's expected demand when it sets price  $p$ : it will attract the  $\lambda$  attentive consumers whenever it sets the lowest price, which occurs with probability  $(1 - F(p))^{N-1}$ ; and the firm will always attract its share of the inattentive consumers,  $(1 - \lambda)/N$ , as long as  $p \leq v$ . Therefore, the left-hand side is a firm's expected profit with the price  $p$ . The firm must be indifferent between all prices in the support of  $F$ . One can show that  $p = v$  is in the support, in which case a firm's profit is always equal to the right-hand side. Expression (1) can be rearranged to give the explicit solution

$$F(p) = 1 - \left( \frac{1 - \lambda}{N\lambda} \frac{v - p}{p - c} \right)^{\frac{1}{N-1}}.$$

The lower end of the support,  $p_0$ , is then the price which makes  $F(p) = 0$ .

One can calculate the expected prices paid by the attentive and the inattentive consumers, and see how these prices depend on the main parameters of interest,  $N$  and  $\lambda$ . As the number of suppliers becomes large, one can show that the attentive consumers pay an expected price close to cost, while the inattentive consumers pay an expected price close to the monopoly price  $v$ . This is depicted in the Figure 1 for a particular example. Competition policy—which acts to forbid mergers or easy the entry of new firms—will therefore help attentive consumers (as one would expect) but *harms* the inattentive consumers. Notice that industry profit (indicated by the dotted line on the figure) does not depend on the number of suppliers, since each firm's profit is equal to its share of the captive market. Industry profit is always  $(1 - \lambda)(v - c)$  in this market.

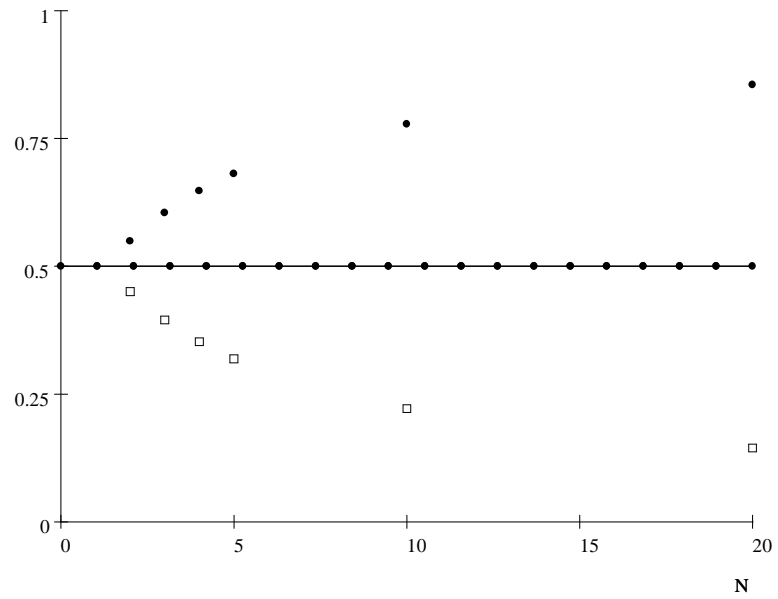


Figure 1: Expected prices paid by attentive and inattentive consumers for  $N = 2, 3, \dots$   
 $(c = 0, v = 1, \lambda = \frac{1}{2})$

Turning next to the impact of  $\lambda$  on market outcomes, consider Figure 2.

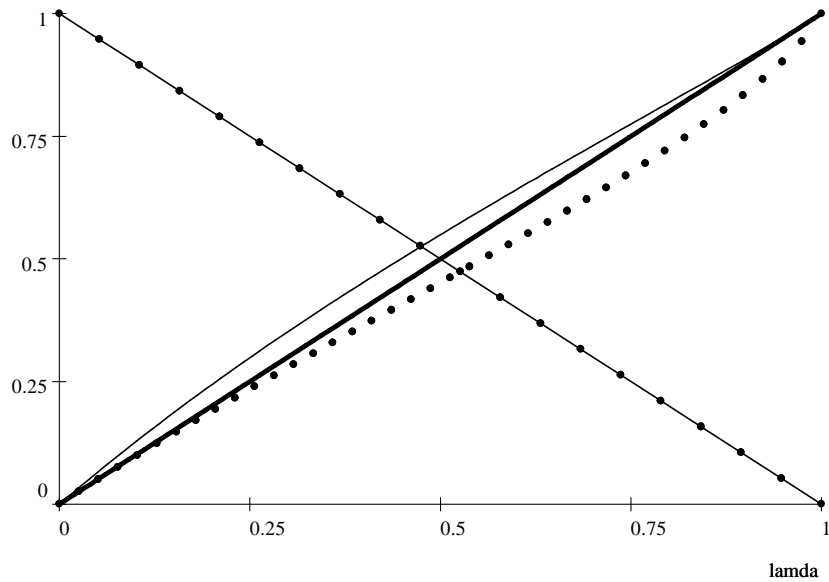


Figure 2: Consumer surplus and profit as a function of  $\lambda$  ( $N = 2, v = 1, c = 0$ )

Here, the thick line represents aggregate consumer surplus (which is linear in  $\lambda$ ), the thin line represents the surplus of attentive consumers, and the dotted line represents the

surplus of inattentive consumers. The downward-sloping line represents industry profit, which is linear in  $\lambda$ . Total welfare—the sum of aggregate consumer surplus and industry profit—is constant in Varian’s model. Obviously, attentive consumers obtain higher surplus than inattentive consumers who do not always pay the lowest available price. Here, increasing the fraction of attentive consumers benefits both groups of consumers, and reduces industry profit.

### 3 A Model with Inattention to Quality

Consider the following variant of Varian (1980) where, instead of being inattentive to price, some consumers do not pay attention to potential differences in the product’s quality. Suppose  $N \geq 2$  identical and risk-neutral firms supply a product which is homogeneous except for the fact that it may be offered with two quality levels, high ( $H$ ) or low ( $L$ ). For instance, in the context of insurance, “quality” may refer to a firm’s policy towards exclusions and excesses in the small print, which only the attentive consumers take into account. The consumer valuation of the low-quality product is assumed not to exceed its marginal cost; for convenience we set them equal and normalize them to zero. Thus, in the insurance context, this implies that a company offering a low quality service has small print which acts to avoid *any* payout in the event of a claim. The marginal cost of the high-quality product is  $c > 0$ , while each consumer’s valuation for the high-quality product is  $v > c$ . Each consumer has a unit demand, and the consumer population size is normalized to 1. (Market size plays no role in the determination of equilibrium prices derived below.) Each firm can choose just one quality level.

Suppose all consumers observe the product’s price (unlike the Varian model) but only a fraction  $\lambda$  also observe each firm’s quality. The remaining  $1 - \lambda$  do not observe any firm’s quality and we make the behavioral assumption that these inattentive consumers buy on the basis of price alone (as long as the price is no higher than  $v$ ). In effect, they (irrationally) hold the belief that unobserved quality is always  $H$ , and purchase from the seller with the lowest price.

When consumers are homogeneous, i.e.,  $\lambda = 0$  or  $\lambda = 1$ , the unique equilibrium involves marginal-cost pricing and zero profits. When  $\lambda = 1$ , in equilibrium all firms supply quality  $H$  and the price is  $c$ . When  $\lambda = 0$  all firms supply quality  $L$  and the price is zero.



From now on assume  $0 < \lambda < 1$ . Then one can show that with two or three firms in the market there is no pure strategy equilibrium (either symmetric or asymmetric). However, we are also interested in situations with more firms, in which case there is an asymmetric pure strategy equilibrium:

**Lemma 1** *If  $N \geq 4$ , the game has no pure strategy symmetric equilibrium, but it has a pure strategy asymmetric equilibrium of the following form: at least two firms offer  $\{p = c, \text{ quality} = H\}$  and at least two firms offer  $\{p = 0, \text{ quality} = L\}$ . Firms make zero profit at this asymmetric equilibrium and total welfare, which in this case is equal to aggregate consumer surplus, is*

$$W = \lambda(v - c).$$

**Proof.** At any potential pure strategy symmetric equilibrium, either all firms choose high quality, or all firms choose low quality. If all choose high quality, Bertrand competition implies that all firms charge  $p = c$ . But then a firm can benefit from deviating to low quality and a price slightly below  $c$ . If all choose low quality, Bertrand competition implies that all firms charge  $p = 0$ . But then a firm can benefit from deviating to high quality and  $p = v$ . Therefore the game has no pure strategy symmetric equilibrium.

On the other hand, if at least two firms offer  $\{p = c, \text{ quality} = H\}$  and at least two firms offer  $\{p = 0, \text{ quality} = L\}$ , then all firms earn zero profit and no firm has a profitable deviation. Thus the proposed set of strategies is indeed an equilibrium. The expression for total welfare follows, since only an attentive consumer will receive positive surplus, equal to  $v - c$ . ■

For any  $N \geq 2$ , the game has a symmetric mixed strategy equilibrium, which generates positive profits. (We will see that when the number of firms is large, this symmetric mixed strategy equilibrium converges to the asymmetric pure strategy outcome in Lemma 1.) One can interpret this equilibrium as firms attempting to confuse consumers in the small print. Here, each firm chooses its price  $p$  randomly on an interval  $[p_0, v]$ . There is a threshold price  $p_1 \in [p_0, v]$  such that a firm chooses low quality for sure when it chooses price  $p \in [p_0, p_1)$  and chooses high quality for sure when  $p \in (p_1, v]$ . (At the threshold  $p_1$  a firm is indifferent between the two quality levels.) Denote by  $P$  the probability that any given firm “cheats”, that is it sets  $p \in [p_0, p_1]$  and offers the low-quality product for

a positive price. We will show that this cheating probability is the unique solution in the interval  $[0, 1]$  of the equation<sup>8</sup>

$$\left(\frac{P}{1-P}\right)^{N-1} = \frac{1-\lambda}{\lambda} \frac{c}{v-c} \left[1 + \frac{1-\lambda}{\lambda} (1-P)^{N-1}\right]. \quad (2)$$

The boundary prices  $p_0$  and  $p_1$  are then given in terms of this  $P$  by

$$p_1 = c + \frac{\lambda P^{N-1}}{\lambda + (1-\lambda)(1-P)^{N-1}}(v-c) \leq v, \quad (3)$$

and

$$p_0 = p_1(1-P)^{N-1} \leq p_1. \quad (4)$$

The details of the equilibrium are described in our main result:

**Theorem 1** *Suppose there are  $N \geq 2$  suppliers. Let  $P$ ,  $p_1$  and  $p_0$  be defined in (2)–(4), and define  $F(p)$  to be the function such that when  $p_0 \leq p \leq p_1$*

$$F(p) = 1 - \left(\frac{p_0}{p}\right)^{\frac{1}{N-1}}, \quad (5)$$

and when  $p_1 \leq p \leq v$  it is given implicitly by

$$(1-\lambda)[1-F(p)]^{N-1} + \lambda[P+1-F(p)]^{N-1} = \lambda P^{N-1} \frac{v-c}{p-c}. \quad (6)$$

Then  $F(p)$  is a continuous cumulative distribution function with support  $[p_0, v]$ , where  $P = F(p_1)$ . The game has a unique symmetric mixed-strategy equilibrium where firms choose prices randomly with support  $[p_0, v]$  and cdf  $F(p)$ . A firm chooses quality  $H$  when  $p_1 < p \leq v$  and chooses quality  $L$  when  $p_0 \leq p < p_1$  (and is indifferent to quality when  $p = p_1$ ). The equilibrium expected profit of each firm is

$$\pi = \lambda(v-c)P^{N-1}, \quad (7)$$

equilibrium total welfare is

$$W = (v-c)[\lambda(1-P^N) + (1-\lambda)(1-P)^N], \quad (8)$$

and equilibrium aggregate consumer surplus is

$$S = (v-c)[\lambda(1-P^N) + (1-\lambda)(1-P)^N - \lambda NP^{N-1}]. \quad (9)$$

---

<sup>8</sup>The left-hand side of (2) is increasing in  $P$  and ranges from 0 to  $+\infty$  for  $P \in [0, 1]$ , while the right-hand side is positive and decreases in  $P$ . Therefore, there is a unique solution to (2) for  $P \in [0, 1]$ .

**Proof.** See Appendix A. ■

We shall focus on this symmetric equilibrium of the game for the rest of the paper. To illustrate the equilibrium strategy, consider the example with  $N = 2, \lambda = \frac{1}{2}, c = 1$  and  $v = 2$ . Then Theorem 1 implies that

$$p_1 = \sqrt{2}, P = p_0 = 2 - \sqrt{2}$$

and the cdf is

$$F(p) = \begin{cases} 1 - \frac{p_0}{p} & \text{if } p_0 \leq p \leq p_1 \\ 1 - \frac{1}{2} P \frac{2-p}{p-1} & \text{if } p_1 \leq p \leq 1 \end{cases}$$

which is depicted on this figure.

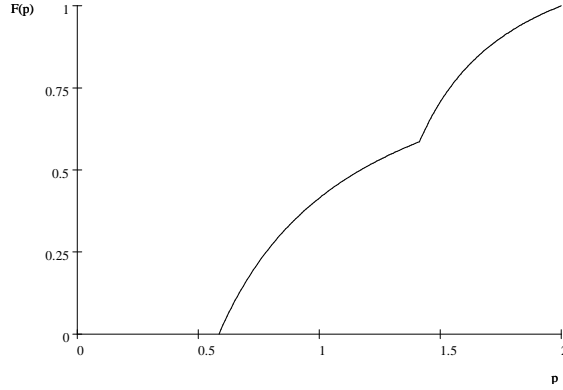


Figure 3: Cumulative distribution function for price  $p$  ( $v = 2, c = 1, \lambda = \frac{1}{2}, N = 2$ )

Finally, we observe that expression (2) can be used directly to obtain comparative statics results concerning the effect of  $\lambda$ , but it is less useful to generate corresponding results for changes in  $N$ . For that purpose, the analysis can sometimes be simplified by using the following approximation for the cheating probability, which is accurate when  $\lambda$  is close to 1 or  $N$  is large:

$$P \approx \frac{k^{\frac{1}{N-1}}}{k^{\frac{1}{N-1}} + 1}, \text{ where } k = \frac{1 - \lambda}{\lambda} \frac{c}{v - c}. \quad (10)$$

See Appendix B for further discussion of this approximation.

## 4 The Impact of Policy

In this section we investigate the impact on outcomes of changes in the market parameters  $\lambda$  and  $N$ . Changes in  $N$  may be caused by competition policy, which affects mergers and

the ease of entry. Changes in  $\lambda$  may be induced by policy towards market transparency, for instance. Recall that in Varian's model, increases in  $\lambda$  benefit all consumers, while increases in  $N$  benefit the attentive consumers but harm inattentive consumers.

#### 4.1 Market transparency policy

Consider first the impact of  $\lambda$  on outcomes. When almost all consumers are attentive, so that  $\lambda \approx 1$ , one can check from (2) (or (10)) that the high-quality product is offered almost all the time ( $P \approx 0$ ), and the equilibrium price is very likely to be close to cost  $c$ . The equilibrium profits are close to zero. When almost all consumers are inattentive ( $\lambda \approx 0$ ), then firms cheat almost all the time ( $P \approx 1$ ). Attentive consumers almost never buy, and firms compete to offer a low-quality product to only the inattentive consumers. The result is that the price is very likely to be close to zero. (In particular,  $p_0 \approx 0$ .)

The following result expands this discussion:

##### Proposition 1

- (i)  $P$  decreases with  $\lambda$ ,  $P = 1$  when  $\lambda = 0$  and  $P = 0$  when  $\lambda = 1$ ;
- (ii) welfare  $W$  increases with  $\lambda$ ,  $W = 0$  when  $\lambda = 0$  and  $W = v - c$  when  $\lambda = 1$ ;
- (iii) profit  $\pi$  increases with  $\lambda$  when  $\lambda$  is small, decreases with  $\lambda$  when  $\lambda$  is large, and  $\pi = 0$  if  $\lambda = 0$  or  $\lambda = 1$ ;
- (iv) aggregate consumer surplus  $S$  decreases with  $\lambda$  when  $\lambda$  is small, increases with  $\lambda$  when  $\lambda$  is large, and  $S = 0$  when  $\lambda = 0$  and  $S = v - c$  when  $\lambda = 1$ .

**Proof.** (i) Since the left-hand side of (2) is increasing in  $P$  and the right-hand side decreases with  $P$  and  $\lambda$ , it follows that  $P$  decreases with  $\lambda$ .

(ii) From (8), welfare is increasing in  $\lambda$  (for given  $P$ ) and decreasing in  $P$  (for given  $\lambda$ ). Hence

$$\frac{dW}{d\lambda} = \frac{\partial W}{\partial \lambda} + \frac{\partial W}{\partial P} \frac{dP}{d\lambda} > 0 .$$

(iii) For given  $\lambda$ , from (7) write  $\pi(\lambda) = \lambda(v - c)(P(\lambda))^{N-1}$  to be equilibrium profit for each firm, where  $P(\lambda)$  is the probability of cheating. Clearly,  $\pi(\lambda) \geq 0$  for all  $\lambda$ . Also, since  $P = 1$  when  $\lambda = 0$  and  $P = 0$  when  $\lambda = 1$ , we see that profit  $\pi = \lambda(v - c)P^{N-1}$  is zero when  $\lambda = 0$  or  $\lambda = 1$ . Therefore,  $\pi$  first increases in  $\lambda$  and eventually decreases in  $\lambda$  for  $\lambda \in [0, 1]$ .

(iv) From expression (9) we have  $S = 0$  when  $\lambda = 0$ , and  $S = v - c$  when  $\lambda = 1$ . Notice also that  $S \leq v - c$  for all  $0 \leq \lambda, P \leq 1$ . In addition,

$$\begin{aligned} \frac{1}{v-c} \frac{dS}{d\lambda} &= (1 - P^N) - (1 - P)^N - NP^{N-1} \\ &\quad - [\lambda NP^{N-1} + (1 - \lambda) N(1 - P)^{N-1} + \lambda N(N - 1) P^{N-2}] \frac{dP}{d\lambda}, \end{aligned}$$

which equals  $-N$  when  $\lambda = 0$ . Therefore,  $S$  slopes down for sufficiently small  $\lambda$ .

Similarly, when  $\lambda = 1$   $S$  reaches its maximum value  $v - c$ , and hence is locally increasing at  $\lambda = 1$ . ■

The behavior of welfare and profit as  $\lambda$  varies (parts (ii) and (iii) of the result) are illustrated in Figure 4:

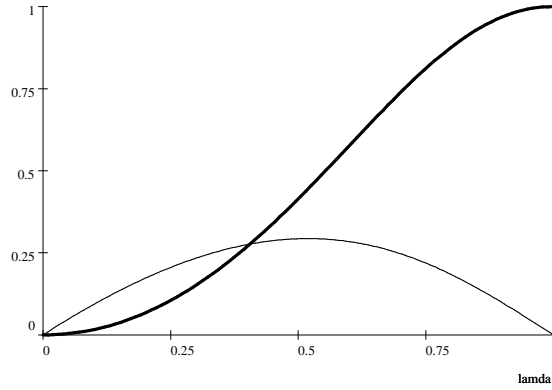


Figure 4: Welfare (thick line) and profit (thin line) ( $N = 2, c = 1, v = 2$ )

These comparative statics results have both intuitive and surprising aspects. The cheating probability  $P$  is decreasing in the proportion of attentive consumers, as seems intuitive. This also implies that the average product quality in the market is higher when more consumers are attentive to quality, which implies that welfare increases when there are greater numbers of attentive consumers.

It is more intriguing that profit is non-monotonic in the consumer attentiveness. The interactions between firms' quality choices and prices appear to be the key to understanding the non-monotonic relationship between profit and attentiveness. The presence of inattentive consumers creates product differentiation for otherwise homogeneous producers, even though firms have the same equilibrium price/quality strategy. There is more product differentiation when  $\lambda$  is at some intermediate level.

It is also surprising that aggregate consumer surplus is non-monotonic in the attentiveness of consumers in the market. (Recall that in the Varian model, all consumers were better off when there was a greater proportion of attentive consumers present in the market.) When  $\lambda$  is small, increasing the proportion of attentive consumers actually lowers aggregate consumer surplus. When  $\lambda$  is small, an increase in  $\lambda$  reduces cheating (increases quality) in the market, but it also increases prices, and the latter effect dominates when  $\lambda$  is small. Thus, when most consumers are inattentive, a market transparency policy which increases  $\lambda$  will harm the inattentive consumers (but always help the attentive consumers). When  $\lambda$  is large, further increases in  $\lambda$  not only increases quality in the market, but may also lower prices. This two effects are likely to work in the same direction for large  $\lambda$ , so that consumer surplus increases in  $\lambda$  when  $\lambda$  is large. In an example aggregate consumer surplus as a function of  $\lambda$  looks like the thick line on Figure 5:

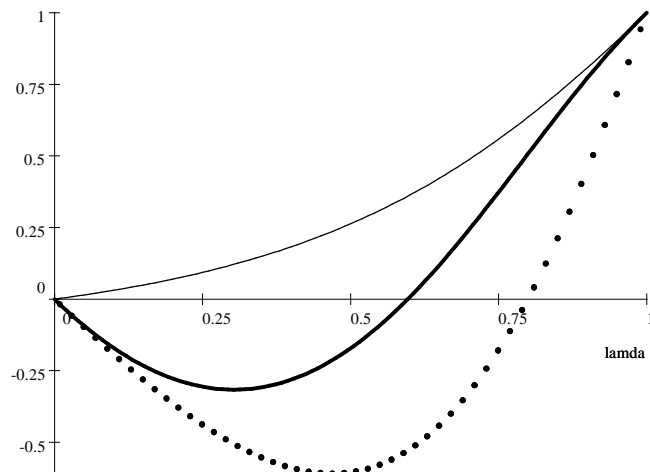


Figure 5: Consumer surplus as a function of  $\lambda$  ( $N = 2, v = 2, c = 1$ )

Of course, the impact differs on the two consumer groups. For instance, the surplus obtained by attentive consumers can never be negative. The thin solid line shows the consumer surplus (per consumer) enjoyed by the attentive consumers, while the dotted line shows the per-consumer surplus of the inattentive consumers. In particular, when there are relatively few inattentive consumers, those consumers obtain a positive surplus since firms so rarely cheat. Notice that attentive consumers are always harmed by the presence of inattentive consumers. (If there were no inattentive consumers then attentive consumers would obtain the maximum possible surplus,  $v - c$ .) This contrasts with the models of “add-on” pricing, where the sophisticated consumers can be cross-subsidised

by the high prices the naive consumers end up paying (see Ellison (2005) and Gabaix and Laibson (2006)).

## 4.2 Competition policy

Consider next the impact of  $N$  on market outcomes. This turns out to be less clear-cut than the impact of  $\lambda$ , and we do not try to describe the comparative statics in all situations, but rather focus on the cases where  $N$  and/or  $\lambda$  is reasonably large (so that we may use the approximate formula (10)).

First, it is straightforward to characterize the limit outcome for large  $N$ . From (2) we have

$$\frac{P}{1-P} = \left\{ k \left[ 1 + \frac{1-\lambda}{\lambda} (1-P)^{N-1} \right] \right\}^{\frac{1}{N-1}},$$

where recall  $k$  is given in (10). Since the term in brackets  $\{\cdot\}$  is positive and bounded above by  $k/\lambda$  and bounded below by  $k$ , the limit of the right-hand side of the above as  $N \rightarrow \infty$  always exists and is equal to 1. It follows that

$$P \rightarrow \frac{1}{2} \text{ as } N \rightarrow \infty .$$

(This can also be seen in the large- $N$  approximation given by expression (10).) Thus we see that, regardless of parameter values for  $v$ ,  $c$  and  $\lambda$ , in markets with many firms, each firm chooses to cheat approximately half the time!

Since  $P \approx \frac{1}{2}$  for large  $N$ , expression (7) implies that both an individual firm's profit and industry profit converge to zero as the number of firms becomes large. (This contrasts with the Varian model, where industry profit did not depend on the number of firms.) Similarly, (8) implies that welfare converges to  $\lambda(v-c)$  for large  $N$ . This in turn implies that aggregate consumer surplus also converges to  $\lambda(v-c)$ . The impact on the two types of consumer is a little more involved, but still clear-cut. From (3) we see that  $p_1 \rightarrow c$  and from (4) we see that  $p_0 \rightarrow 0$ , both as  $N \rightarrow \infty$ . Then (5) and (6) imply that the distribution for prices converges to a discrete two-point distribution, with probability half on  $p = 0$  and probability half on  $p = c$ . Therefore, the surplus of each of the attentive consumers converges to  $v - c$ , while the surplus of each of the inattentive consumers converges to zero. Notice that this limit outcome corresponds exactly to the asymmetric pure strategy equilibrium described in Lemma 1 above.

We next consider how  $N$  affects the cheating probability, industry profit, welfare, and aggregate consumer surplus:

**Cheating probability:** If  $N$  and/or  $\lambda$  is relatively large, we can use the formula (10) for the cheating probability  $P$ . Clearly,  $P$  in (10) increases (decreases) with  $N$  if  $k < 1$  (respectively,  $k > 1$ ). Note that  $k < 1$  is equivalent to the condition

$$\lambda > \frac{c}{v} .$$

In particular, if the fraction of attentive consumers is large enough, the cheating probability starts small and increases (with asymptotic limit  $\frac{1}{2}$ ) as the number of seller increases. If we interpret  $P$  as the degree of ethical behavior in the market, then increased competition in the sense of more firms in the market leads to less ethical behavior whenever there is a small fraction of inattentive consumers.<sup>9</sup>

**Industry Profits:** It turns out that industry profits can rise or fall with the addition of a further seller. For instance, for fixed  $\lambda \in (0, 1)$ , when  $N$  is large the approximation (10) is valid, and industry profit is proportional to

$$\frac{N}{(1 + k^{\frac{1}{N-1}})^{N-1}} \approx \frac{N}{2^{N-1}} .$$

This is decreasing in  $N$  for large  $N$ . By contrast, for fixed  $N$  (not necessarily large), when  $\lambda \approx 1$  the approximation (10) is valid, and the impact on industry profit of introducing one more seller is proportional to

$$\frac{N+1}{(1 + k^{\frac{1}{N}})^N} - \frac{N}{(1 + k^{\frac{1}{N-1}})^{N-1}} .$$

This term is positive when  $\lambda \approx 1$  (i.e., when  $k \approx 0$ ). Thus, in the “almost rational” case where only a small fraction of consumers are inattentive, the addition of more sellers boosts industry profit.

**Welfare:** In a similar way, it is possible to characterize the impact of an additional seller on welfare when  $N$  is large or  $\lambda$  is close to 1. Using the approximation (10), we have

$$\frac{W}{v-c} \approx \lambda + \frac{1 - \lambda - \lambda k^{\frac{N}{N-1}}}{\left(1 + k^{\frac{1}{N-1}}\right)^N} .$$

---

<sup>9</sup>See Shleifer (2004) for further discussion about the effect of competition on ethical behavior.



Therefore, the differential of  $W$  with respect to  $N$  has the same sign as

$$\frac{dW}{dN} \stackrel{\text{sign}}{=} (\ln k) \lambda \frac{k^{\frac{N}{N-1}}}{(N-1)^2} + \left(1 - \lambda - \lambda k^{\frac{N}{N-1}}\right) \left( N (\ln k) \frac{k^{\frac{1}{N-1}}}{(N-1)^2 (k^{\frac{1}{N-1}} + 1)} - \ln \left(k^{\frac{1}{N-1}} + 1\right) \right).$$

In particular, for given  $\lambda \in (0, 1)$ , as  $N$  becomes large the only term in the above which does not vanish is

$$- \left(1 - \lambda - \lambda k^{\frac{N}{N-1}}\right) \ln \left(k^{\frac{1}{N-1}} + 1\right) \approx (\lambda k - (1 - \lambda)) \ln 2.$$

Therefore, for large  $N$  welfare increases (decreases) with  $N$  when  $\lambda k > 1 - \lambda$  (respectively,  $\lambda k < 1 - \lambda$ ). Note that  $\lambda k > 1 - \lambda$  if and only if  $2c > v$  (which does not depend on  $\lambda$ ).

Similarly, for fixed  $N$ , as  $\lambda \rightarrow 1$  we can see that  $dW/dN$  is negative.<sup>10</sup> Thus, in markets where only a few consumers are inattentive, adding more competitors is bad for welfare. Intuitively, when  $\lambda$  is close to 1, all that matters for welfare in (8) is the payoff to the attentive consumers, which is proportional to  $1 - P^N$ . One can show that  $P^N$  is increasing in  $N$  whenever  $\lambda$  is sufficiently close to 1, and hence attentive consumers are harmed by the addition of a further seller.

**Consumer surplus:** Again, if either  $N$  is large or  $\lambda$  is close to 1, from (9) we have

$$\begin{aligned} \frac{S}{v-c} &\approx \lambda + \frac{1}{\left(1 + k^{\frac{1}{N-1}}\right)^N} \left[1 - \lambda - \lambda k^{\frac{N}{N-1}}\right] - \lambda \frac{Nk}{\left(1 + k^{\frac{1}{N-1}}\right)^{N-1}} \\ &= \lambda + \frac{1 - \lambda - \lambda k^{\frac{N}{N-1}} - \lambda Nk \left(1 + k^{\frac{1}{N-1}}\right)}{\left(1 + k^{\frac{1}{N-1}}\right)^N}. \end{aligned}$$

Therefore, the differential of  $S$  with respect to  $N$  has the sign of

$$\begin{aligned} \frac{dS}{dN} &\stackrel{\text{sign}}{=} \left( k\lambda \left(-k^{\frac{1}{N-1}} - 1\right) + (\ln k) \lambda \frac{k^{\frac{N}{N-1}}}{(N-1)^2} + N (\ln k) \frac{\lambda}{(N-1)^2} k^{\frac{N}{N-1}} \right) + \\ &\left( Nk\lambda \left(-k^{\frac{1}{N-1}} - 1\right) - \lambda k^{\frac{N}{N-1}} - \lambda + 1 \right) \left( N (\ln k) \frac{k^{\frac{1}{N-1}}}{(N-1)^2 (k^{\frac{1}{N-1}} + 1)} - \ln \left(k^{\frac{1}{N-1}} + 1\right) \right). \end{aligned}$$

---

<sup>10</sup>The easiest way to see this is to divide the expression for  $dW/dN$  by  $k^{\frac{N}{N-1}}$  and let  $k$  tend to zero.

For fixed  $\lambda \in (0, 1)$  one can show this expression is positive for sufficiently large  $N$ . (As  $N$  becomes large in the above, there is just one explosive term, which is positive, and this is  $-Nk\lambda \left(-k^{\frac{1}{N-1}} - 1\right) \ln \left(k^{\frac{1}{N-1}} + 1\right)$ .) In effect, welfare converges to its limit much faster than profit converges to zero, and consumers and firms end up playing a constant-sum game when  $N$  is reasonably large. Since firms' profits fall with  $N$ , it follows that consumers must gain. Finally, for fixed  $N$  (not necessarily large), consumers in aggregate are worse off with the introduction of an additional seller whenever  $\lambda$  is close enough to 1. Since we know that welfare decreases with  $N$  in this case, while industry profit rises, it necessarily follows that consumer surplus (equal to welfare minus profit) must fall.

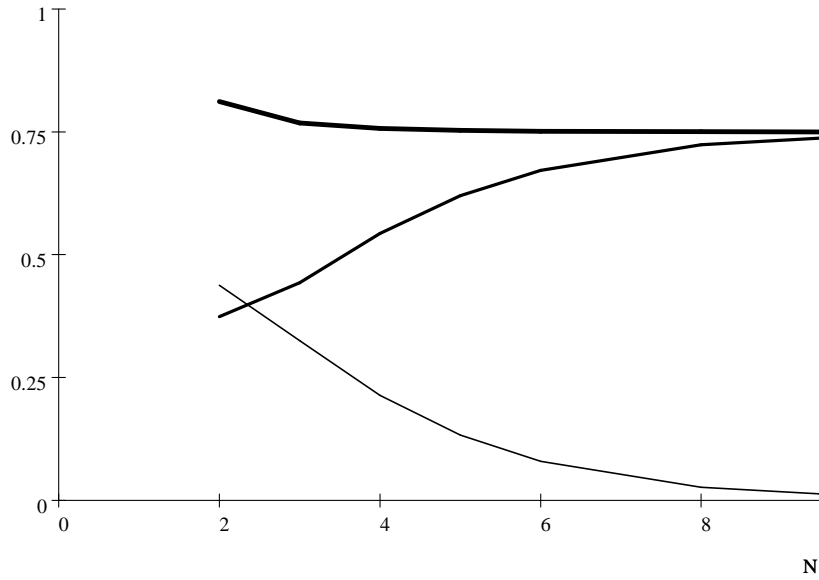


Figure 6: Welfare (thick line), industry profit (thin line) and aggregate consumer surplus (medium line) as the number of firms changes ( $\lambda = 0.75$ ,  $v = 2$ ,  $c = 1$ )

We summarize this discussion in the following two results:

**Proposition 2** *For fixed  $\lambda \in (0, 1)$  and  $N$  sufficiently large:*

- (i) *the outcome converges to the asymmetric pure strategy equilibrium described in Lemma 1, namely: industry profit is zero; welfare and aggregate consumer surplus is  $\lambda(v - c)$ , and inattentive consumers obtain zero surplus;*
- (ii) *each firm offers the low quality product with cheating probability approximately equal to  $\frac{1}{2}$ ;*
- (iii) *the cheating probability decreases with  $N$  if  $\lambda < c/v$  and increases with  $N$  if  $\lambda > c/v$ ;*

- (iv) industry profit falls with  $N$ ;
- (v) aggregate consumer surplus rises with  $N$ ; and
- (vi) welfare falls with  $N$  if  $v > 2c$  and rises with  $N$  if  $v < 2c$ .

These results are illustrated in Figure 6. Here, welfare decreases slightly with  $N$  but converges quickly to its limit, while profits fall and consumer surplus rises monotonically with  $N$ . Notice that  $N$  does not need to be especially “large” for the results reported in Proposition 2 to be observed.

Next, we summarize the results for  $\lambda \approx 1$ :

**Proposition 3** *Fix the initial number of firms  $N$ . If  $\lambda$  is sufficiently close to 1 then when an additional seller is introduced:*

- (i) the cheating probability rises;
- (ii) industry profit rises;
- (iii) welfare falls;
- (iv) aggregate consumer surplus falls.

To illustrate these large  $\lambda$  results, in this table we present the incremental effect on industry profit, welfare and consumer surplus of moving from two to three sellers. (These numbers use the exact formula (2) not the approximation (10).)

Table 1: The incremental effect of moving from two to three sellers ( $c = 1, v = 2$ )

	$\Delta(\text{profit})$	$\Delta(\text{welfare})$	$\Delta(\text{consumer surplus})$
$\lambda = 0.5$	-0.1707	0.668	0.2375
$\lambda = 0.75$	-0.1126	-0.0432	0.0694
$\lambda = 0.9$	-0.0195	-0.0426	-0.0231
$\lambda = 0.99$	0.005	-0.003	-0.008

We do not place too much weight on these “almost rational market” results in Proposition 3, where parts (ii) and (iv) are somewhat pathological. Numerical simulations suggest that the fraction of inattentive consumers needs to be *extremely* low in order for, say, profit to rise when there are more sellers. (In Table 1, which concerns duopoly, the fraction needs to be around 1%, but with more initial sellers the fraction needs to be much smaller still.) In any case, when  $\lambda \approx 1$ , profit, welfare and consumer surplus

are all essentially “flat” with respect to the number of sellers, and the impact (positive or negative) of changing the number of sellers is negligible.

We have not yet mentioned the impact of more competition on the two kinds of consumer. In general, this is a difficult task. However, it is fairly straightforward to compare the outcomes in the two extreme cases of  $N = 2$  and  $N = \infty$ . With duopoly, the outcomes for the two kinds of consumer are illustrated on Figure 5 above (in the particular case with  $v = 2$  and  $c = 1$ ). With many firms, the outcome for attentive consumers is that they each have surplus  $v - c$  while the outcome for inattentive consumers is that they have surplus of zero. Thus, comparing the two market structures shows that the attentive consumers are always better off in very competitive markets compared to duopoly (as in the Varian model). The impact on inattentive consumers depends on how many of them there are. When  $\lambda$  is relatively large, these consumers can “free ride” on the attentive consumers in the duopoly setting and obtain positive surplus, which is eroded when there are more suppliers. Thus, in this case increased competition harms the inattentive consumers (as in the Varian model). But if  $\lambda$  is relatively small, these consumers are strictly exploited with duopoly, whereas they are simply given nothing with many firms. More competition helps these consumers in this case (unlike the Varian model).

## 5 Discussion

We have studied a stylized model. A key departure of our model from the standard analysis is that not all consumers are fully rational when making purchases. We model this in a particularly simple form: some consumers are inattentive to details in a transaction and are not aware that the product could be of lower quality. We have found that the presence of such consumers in the market has a significant effect on the nature of market equilibrium and on the effects of public policy on consumer and social welfare.

Theorem 1 describes a mixed strategy equilibrium in which firms sometimes offer a high-quality product for a high price and sometimes offer a low-quality product for a low price. The outcome can really involve “bargains and rip-offs”, since the inattentive consumers often receive strictly negative utility. (The Varian model simply involved inattentive consumers paying up to their reservation price.)

It is important for the mixed strategy equilibrium to exist that a firm cannot price

discriminate between the two types of consumers, for instance, by offering a high-quality product aimed at the attentive consumers and a low-quality product aimed at the inattentive consumers. If firms could do this, they would compete separately in each market, and competition would force the high-quality product's price down to cost, and likewise for the low-quality product (just like the pure strategy equilibrium outlined in Lemma 1). Thus, the model applies to situations in which a potential consumer approaches a firm (e.g., by phone) and obtains a quote. If instead, a firm publishes its contracts (e.g., on the internet), inattentive consumers may be puzzled why some contracts have lower headline prices than others, and may enquire further why the contracts have different prices, and hence become more informed about quality. For this reason, a firm may prefer not to offer two contracts (so that inattentive consumers are not alerted).

The presence of inattentive consumers allows firms to earn positive profits, even though there is only one product which has positive social value. The fact that firms may cheat by cutting quality to exploit inattentive consumers allows a non-cheating firm to raise its price to attentive consumers. Expression (7) shows that in equilibrium a firm's profit is such that it extracts monopoly profit from each of the  $\lambda$  attentive consumers (equal to  $v - c$  per consumer) whenever all its rivals choose to cheat (which occurs with probability  $P^{N-1}$ ).

Price here is a perfect signal of quality. The product will be of high quality if and only if price is above  $p_1$ . If the product in question is purchased *frequently*, we expect that even "inattentive" consumers will learn not to buy when the price is low. Thus the model applies best to infrequently purchased (and infrequently experienced) items, such as life insurance and home improvement projects.

When there are at least four suppliers, there are multiple equilibria: the asymmetric pure strategy equilibrium in Lemma 1 and the symmetric mixed strategy equilibrium in Theorem 1. The former yields zero profit, whereas the latter (except in extreme cases) yields positive profit. Therefore, we might expect that firms find a way to coordinate on the more profitable equilibria. In addition, if there were some small cost of entering the market, then this would rule out the Bertrand-style equilibrium, and leave only the mixed strategy equilibrium.

While an increase in the proportion of attentive consumers reduces cheating and increases the expected product quality in the marketplace, as one's intuition would sug-

gest, it is surprising that consumer surplus is non-monotonic in this proportion: a greater number of attentive consumers sometimes reduces aggregate consumer surplus. Market structure has unusual effects on market performance. Perhaps most surprisingly, when nearly all consumers are fully rational, increased competition, in the sense having more suppliers, monotonically increases cheating in the market and monotonically reduces social welfare.

Like Varian's model, our model also has a "rational consumer" interpretation: all consumers have perfect information about prices and qualities in the market, but a fraction  $1 - \lambda$  of them do not suffer utility loss from consuming the low-quality product (their utility is not sensitive to quality changes). Then our main result, Theorem 1, would still hold, and the model would be one of vertical product differentiation with price and quality dispersion. However, this interpretation would change our consumer and social welfare analysis, and it would change the kinds of markets to which the model might apply.

In our model, the low-quality product has no social value. So it is perhaps best viewed as a model where low quality is supplied by "unethical" sellers, who profit by exploiting consumers' inattention to quality (or taking advantage of consumers' mistakes). An alternative situation would be that the low-quality product also generates consumer utility which is higher than its cost. Then, even if all consumers are attentive to quality, a firm may offer low quality to achieve vertical product differentiation in a market with heterogeneous consumer preferences for quality. Our analysis is complementary to this alternative approach. An ambitious aim would be to combine these two approaches, for instance by allowing some inattentive consumers in a setting of vertical differentiation. In this case there are two reasons why a low-quality product is chosen: a consumer does not pay attention to quality, or the consumer is careful but prefers this product due to a low premium for higher quality. In our simple framework, there is a clear role for policy: since the low-quality product has no real social value, welfare is surely improved if firms are legally forced to offer only the high-quality product (assuming this can be enforced). But in a richer framework where some attentive consumers actually prefer the low-quality product at its lower price, the role for policy is less clear: a ban on the low quality product will help the inattentive consumers but harm the attentive consumers

who only want the low quality item.<sup>11</sup> It would be desirable for future research to address such issues.

## References

- CHAN, Y.-S., AND H. LELAND (1982): “Prices and Qualities in Markets with Costly Information,” *Review of Economic Studies*, 49(4), 499–516.
- CRUICKSHANK, D. (2000): *Competition in UK Banking: A Report to the Chancellor of the Exchequer*. HM Treasury, London.
- DELLA VIGNA, S. (2007): “Psychology and Economics: Evidence from the Field,” mimeo.
- DRANOVE, D., AND M. SATTERTHWAITTE (1992): “Monopolistic Competition When Price and Quality are Imperfectly Observable,” *Rand Journal of Economics*, 23(4), 518–534.
- DTI (2003): *Fair, Clear and Competitive: The Consumer Credit Market in the 21st Century*. Department of Trade and Industry, London.
- ELLISON, G. (2005): “A Model of Add-on Pricing,” *Quarterly Journal of Economics*, 120(2), 585–637.
- FINANCIAL-SERVICES-AUTHORITY (2006): *Levels of Financial Capability in the UK: Results of a Baseline Survey*. Financial Services Authority, London.
- GABAIX, X., AND D. LAIBSON (2006): “Shrouded Attributes, Consumer Myopia, and Information Suppression in Competitive Markets,” *Quarterly Journal of Economics*, 121(2), 505–540.
- LELAND, H. (1979): “Quacks, Lemons and Licensing: A Theory of Minimum Quality Standards,” *Journal of Political Economy*, 87, 1328–1346.
- ROSENTHAL, R. (1980): “A Model in which an Increase in the Number of Sellers Leads to a Higher Price,” *Econometrica*, 48(6), 1575–1579.

---

<sup>11</sup>A similar trade-off is seen in Leland (1979).

- SALOP, S., AND J. STIGLITZ (1977): “Bargains and Ripoffs: A Model of Monopolistically Competitive Price Dispersion,” *Review of Economic Studies*, 44(3), 493–510.
- SHLEIFER, A. (2004): “Does Competition Destroy Ethical Behavior?,” *American Economic Review*, 94(2), 414–418.
- STIGLITZ, J. (1989): “Imperfect Information in the Product Market,” in *Handbook of Industrial Organization*, Vol. 1, ed. by R. Schmalensee, and R. Willig, pp. 771–848. North-Holland, Amsterdam.
- VARIAN, H. (1980): “A Model of Sales,” *American Economic Review*, 70(4), 651–659.

## TECHNICAL APPENDICES

### APPENDIX A: Proof of Theorem 1

First, we establish that  $F(\cdot)$  described in the statement of Theorem 1 is a continuous cdf with support  $[p_0, v]$ . Clearly,  $F$  in (5) is increasing in  $p$  and satisfies  $F(p_0) = 0$ . Similarly,  $F$  defined (implicitly) by (6) is an increasing function of  $p$ , where  $F(p)$  ranges from  $P$  to 1 as  $p$  ranges from  $p_1$  to  $v$ . (As  $F(p)$  on the left-hand side of (6) ranges from  $P$  to 1, the  $p$  which makes the right-hand side equal ranges from  $p_1$  (where this follows from the definition of  $p_1$  in (3)) to  $v$ .) Finally, we need to show that  $F(\cdot)$  is continuous at the point  $p_1$  which separates expressions (5) and (6). As we have said, if we set  $p = p_1$  in expression (6), the definition of  $p_1$  in (3) implies that  $F(p_1) = P$ . And if we set  $p = p_1$  into expression (5), from the definition of  $p_0$  in (4) we obtain that  $F(p_1) = P$ . Thus, the two parts of  $F(\cdot)$  join up appropriately, and  $F$  is indeed a cdf.

Suppose  $N - 1$  firms follow the strategy described. If the remaining firm chooses a price  $p$  with  $p_0 \leq p \leq p_1$  and quality  $L$ , its expected profit is

$$(1 - \lambda) p [1 - F(p)]^{N-1} .$$

(The firm will sell only to inattentive consumers, and then only if it sets the lowest price, which occurs with probability  $(1 - F(p))^{N-1}$ .) A firm is indifferent between any  $p \in [p_0, p_1]$  if

$$(1 - \lambda) p [1 - F(p)]^{N-1} = (1 - \lambda) p_0 = (1 - \lambda) p_1 (1 - P)^{N-1} ,$$

where if a firm sets  $p_0$ , it will surely sell to inattentive consumers, and if it sets  $p_1$  (and offers quality  $L$ ), it will sell to inattentive consumers whenever all other firms set a higher



price. The first equality above implies (5), while the second equality implies (4). Since  $0 \leq P \leq 1$  in (2), it follows that  $p_0 \leq p_1$  as required.

Alternatively, if a firm chooses a price  $p$  with  $p_1 \leq p \leq v$  and quality  $H$ , its expected profit is

$$(p - c) \left\{ (1 - \lambda) [1 - F(p)]^{N-1} + \lambda [P + 1 - F(p)]^{N-1} \right\}, \quad (11)$$

where if a firm charges  $p \geq p_1$  and sets quality  $H$ , it sells to inattentive consumers only if it has the lowest price, and it sells to the attentive consumers if no other firm sets a price between  $p_1$  and  $p$ . The probability that no other firm sets a price between  $p_1$  and  $p \geq p_1$  is

$$[F(p_1) + 1 - F(p)]^{N-1} = [P + 1 - F(p)]^{N-1}.$$

A firm is indifferent between any  $p \in [p_1, v]$  if expression (6) holds, where if a firm charges  $v$  it guarantees itself an expected profit  $\lambda(v - c)P^{N-1}$  (i.e., it sells to the attentive consumers at the monopoly price when the other firms have all cheated and chosen quality  $L$ ).

So far, all of this analysis is valid for arbitrary  $0 \leq P \leq 1$ . The equilibrium choice of  $P$  is determined by the requirement that a firm be indifferent between being a low-price/low-quality firm and being a high-price/high-quality firm. This is ensured if at the point  $p = p_1$  a firm is indifferent between offering a low or a high quality product. At this price, if a firm decides to offer a high-quality product its demand jumps (it then attracts all the attentive consumers), but its unit cost jumps too, from zero to  $c$ . For a firm to be indifferent, these two jumps must cancel out, so that

$$\underbrace{p_1(1 - \lambda)(1 - P)^{N-1}}_{\text{profit with low quality}} = \underbrace{(p_1 - c)(\lambda + (1 - \lambda)(1 - P)^{N-1})}_{\text{profit with high quality}}. \quad (12)$$

However, expression (12) when combined with (3) yields the formula (2) for  $P$ . In sum, a firm is indifferent between all strategies of the forms: (i) set price  $p_0 \leq p \leq p_1$  and offer low quality and (ii) set price  $p_1 \leq p \leq v$  and offer high quality. In all cases, the firm makes the profit obtained by choosing the highest price  $p = v$  and offering the high-quality product, which yields profit (7).

To complete our proof that the candidate equilibrium is indeed an equilibrium, we need to show that the remaining deviations are not profitable, and these deviations are (i) setting price  $p_0 \leq p < p_1$  and high quality and (ii) setting price  $p_1 < p \leq v$  and

low quality. (If these deviations are not profitable a firm also cannot benefit from a randomized deviation in quality choice.) If a firm chooses price  $p_0 \leq p < p_1$  and high quality, its profit is

$$(p - c) \left\{ (1 - \lambda) [1 - F(p)]^{N-1} + \lambda \right\} = (p - c) \left\{ (1 - \lambda) \frac{p_0}{p} + \lambda \right\},$$

which is increasing in  $p$  within this range. (The above equality follows from (5).) Therefore, the highest profit the firm can obtain using this form of strategy involves choosing  $p = p_1$ , in which case the firm obtains just the equilibrium profit  $\pi$  in (7). Therefore, this deviation cannot be profitable.

The argument for deviation (ii) is more complicated. If a firm sets price  $p_1 < p \leq v$  and low quality, its profit is

$$(1 - \lambda) p [1 - F(p)]^{N-1}.$$

From (6), if the cdf at the price  $p \geq p_1$  is  $F \geq P$ , the price in terms of  $F$  is

$$p = c + \frac{\lambda(v - c)P^{N-1}}{(1 - \lambda)(1 - F)^{N-1} + \lambda(P + 1 - F)^{N-1}} \geq p_1.$$

Therefore, the firm's profit in terms of  $F$  instead of  $p$  is

$$(1 - \lambda)(1 - F)^{N-1} \left[ c + \frac{\lambda(v - c)P^{N-1}}{(1 - \lambda)(1 - F)^{N-1} + \lambda(P + 1 - F)^{N-1}} \right]. \quad (13)$$

By construction this profit equals  $\pi$  in (7) when  $F = P$ . However, one can show that (13) is decreasing in  $F$ , and so the firm cannot strictly gain by such a deviation.

Welfare is zero whenever a low quality product (or no product) is supplied to a consumer; otherwise welfare is  $v - c$  for each consumer served. All  $\lambda$  attentive consumers are served with high quality when at least one firm does not cheat, which occurs with probability  $(1 - P^N)$ . The  $1 - \lambda$  inattentive consumers are served with high quality only when no firm cheats, which occurs with probability  $(1 - P)^N$ . This explains expression (8). Since aggregate consumer surplus is the difference between welfare and industry profit, (9) follows from (7) and (8).

Finally, we show that there can be no other symmetric equilibrium. From Lemma 1, there can be no pure strategy symmetric equilibrium, and it suffices to limit our attention to situations where  $\lambda \in (0, 1)$ . So suppose that there is another mixed-strategy symmetric equilibrium. We only need to consider strategies in which prices follow some

cdf  $G(p)$  and quality choices are some randomization between  $H$  and  $L$  for any  $p$ . Denote the lower and upper support of  $G$  by  $p_l$  and  $p_h$ , respectively. Then  $0 < p_l < p_h \leq v$ , and each firm's expected profit is positive. At such an equilibrium, it is not possible that each firm always chooses quality  $H$  or each firm always chooses quality  $L$ . Notice that at  $p_h$ , each firm must choose  $H$ , otherwise a firm's profit at  $p_h$  would be zero. Also, there cannot be a mass point at  $p_h$ . Thus there exists some  $p'' < p_h$  such that each firm chooses  $H$  for  $p \geq p''$ .

If for  $p \in [p_l, p'']$  each firm chooses  $L$ , then the equilibrium must be the same as the one we have already characterized. Otherwise, there exists some  $p' < p''$  such that each firm chooses  $L$  for  $p \in (p', p'')$  and chooses  $H$  at  $p' > c$ , where  $G(p)$  is continuous at  $p'$  and  $p''$ . Then, at prices  $p'$  and  $p''$ , the expected profit from choosing  $L$  must be the same, or

$$p'(1-\lambda)[1-G(p')]^{N-1} = p''(1-\lambda)[1-G(p'')]^{N-1}.$$

Similarly, at prices  $p'$  and  $p''$ , the expected profit from choosing  $H$  must also be the same, or

$$(p' - c) \left[ (1 - \lambda) [1 - G(p')]^{N-1} + \lambda\gamma \right] = (p'' - c) \left[ (1 - \lambda) [1 - G(p'')]^{N-1} + \lambda\gamma \right],$$

where  $\gamma$  is the probability that the firm will sell to attentive consumers at  $p''$ . Notice that since all firms choose  $L$  for  $p' < p < p''$ , the probability that the firm will sell to attentive consumers at  $p'$  is equal to that at  $p''$ . It follows that

$$\frac{p''}{p'} = \frac{[1 - G(p')]^{N-1}}{[1 - G(p'')]^{N-1}}$$

and

$$\frac{p'' - c}{p' - c} = \frac{(1 - \lambda) [1 - G(p')]^{N-1} + \lambda\gamma}{(1 - \lambda) [1 - G(p'')]^{N-1} + \lambda\gamma}.$$

Since  $[1 - G(p')] > [1 - G(p'')]$  and  $\lambda\gamma > 0$ , we have

$$\frac{(1 - \lambda) [1 - G(p')]^{N-1} + \lambda\gamma}{(1 - \lambda) [1 - G(p'')]^{N-1} + \lambda\gamma} < \frac{(1 - \lambda) [1 - G(p')]^{N-1}}{(1 - \lambda) [1 - G(p'')]^{N-1}} = \frac{[1 - G(p')]^{N-1}}{[1 - G(p'')]^{N-1}} = \frac{p''}{p'}.$$

On the other hand,

$$\frac{(1 - \lambda) [1 - G(p')]^{N-1} + \lambda\gamma}{(1 - \lambda) [1 - G(p'')]^{N-1} + \lambda\gamma} = \frac{p'' - c}{p' - c} > \frac{p''}{p'},$$

which is a contradiction. Therefore there can be no other symmetric equilibrium in mixed strategies.

## APPENDIX B: Justification for Expression (10)

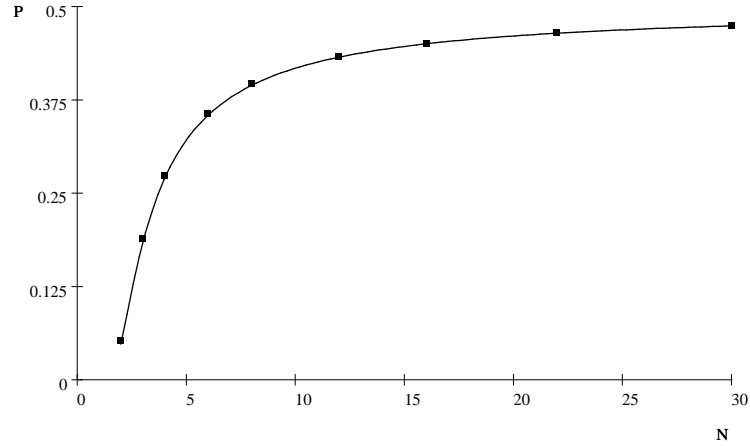


Figure 7A: Approximate  $P$  (continuous line) and the true values of  $P$  (dots) when  $\lambda = 0.95$ ,  $c = 1$ ,  $v = 2$

From (2), the approximation given in (10) is accurate whenever the term  $\frac{1-\lambda}{\lambda}(1-P)^{N-1}$  is negligible. This is true in two situations: (i) when  $\lambda \approx 1$  so that almost all consumers are attentive, or (ii) when  $N$  is large so that there are many suppliers. To see that (10) indeed gives a good approximation to the true value of  $P$  in (2) when  $1 - \lambda$  is small, see Figure 7A which depicts  $P$  as a function of  $N$ . To see the approximation is good when  $N$  is (relatively) large, see Figure 7B which depicts  $P$  as a function of  $\lambda$ .

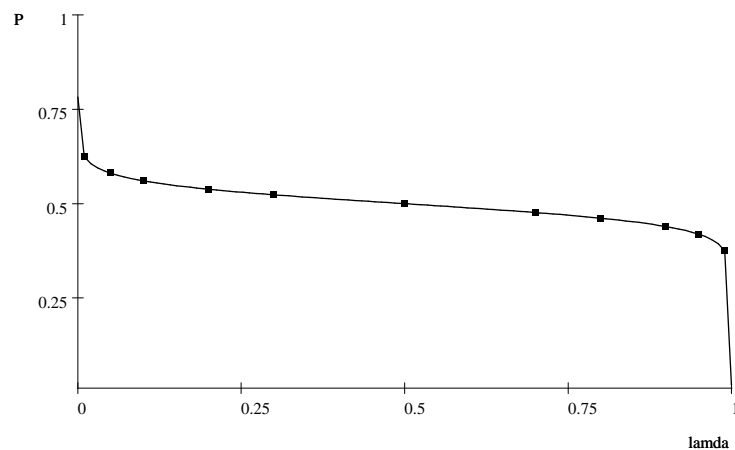


Figure 7B: Approximate  $P$  (continuous line) and the true values of  $P$  (dots) when  $N = 10$ ,  $v = 2$ ,  $c = 1$