# Evaluation of pairs trading strategy at the Brazilian financial market

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# Evaluation of Pairs Trading Strategy at the Brazilian Financial Market

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**Abstract:** Pairs trading is a popular trading strategy that tries to take advantage of market inefficiencies in order to obtain profit. The idea is simple: find two stocks that move together and take long/short positions when they diverge abnormally, hoping that the prices will converge in the future. From the academic point of view of weak market efficiency theory, pairs trading strategy shouldn't present positive performance since, according to it, the actual price of a stock reflects its past trading data, including historical prices. This leaves us with a question, does pairs trading strategy presents positive performance for the Brazilian market? The main objective of this research is to verify the performance and risk of pairs trading in the Brazilian financial market for different frequencies of the database, daily, weekly and monthly prices for the same time period. The main conclusion of this simulation is that pairs trading strategy was a profitable and market neutral strategy at the Brazilian Market. Such profitability was consistent over a region of the strategy's parameters. The best results were found for the highest frequency (daily), which is an intuitive result.

**Key Words**: pairs trading, quantitative strategy, asset allocation

## 1. Introduction

The market efficiency theory has been tested by different types of research. Such concept postulates, on its weak form, that the past trading information of a stock is reflected on its value, meaning that historical trading data has no potential for predicting future behavior of asset's prices. The main theorical consequence of this concept is that no logical rules of trading based on historical data should have a significant positive excessive return over some benchmark portfolio.

In opposition to the market efficiency theory, several papers have showed that past information is able, in some extent, to explain future stock market returns. Such predictability can appear in different ways, including time anomalies (day of the weak effect, French (1980)) and correlation between the asset's returns and others variables, Fama and French (1992). A substantial review on the market efficiency subject can be found at the papers of Fama (1991) and Dimson e Mussavian (1998).

A respectable amount of papers have tried to use quantitative tools in order to model the market and build trading rules. The basic idea of this type of research is to look for some kind of pattern in the historical stock price behavior and, using only historical information, take such pattern into account for the creation of long and short trading positions.

One of the most popular approaches to model the market and infer logical rules is technical analysis, Murphy (1999). Such technique is based on quantitative indicators[1] and also visual patterns[2] in order to identify entry and exit points on the short-term behavior of stock prices. The popularization of technical analysis lead to a number of tests that had as objective to verify if such tools were profitable or not. A good review of past works on that

[1] Such indicators includes moving averages, stochastics, and many others indicators. More details at Murphy (1999).

[2] Such patterns includes Head and Shoulders, Triple Top, etc. A complete description of such patterns can be found at Murphy (1999).

subject can be found at Park and Irwin (2004). Its worth to say that even thought the majority of papers have showed that technical analysis is profitable, several problems can be addressed with such studies, including data snooping problems, transaction costs and liquidity. All this incompleteness of the research still makes technical analysis a subject to be studied.

With the advent of computer power in the late 90's, more sophisticated mathematical methods could be employed in the case of trading rules. One example is the use of nearest neighbor algorithm in trading strategies, Fernandez-Rodrigues et al (2002), Fernandez-Rodrigues et al (1997), Fernandez-Rodrigues et al (2001) and Perlin (2006). The NN algorithm is a non parametric method of modelling time series that has an intuitive appealing based on chaos theory. The main conclusion drawn from the results presented on the predictability potential of this method is that it is able to predict correct market direction for most of the forecasted financial observations. But its important to say that the evidence wasn't strong in all studies.

For the case of trading strategies based on parametric models, there is the work of Efetkhari (1997) on stock market and Dueker et al (2006) at currency. Both papers based the forecasts on the regime switching model, where the results indicated that the method can predict the financial time series researched in each case. Others types of strategies using quantitative formulations includes timing the market with fundamentals or statistical models, Brooks at al (2005) and Anderson et al (2006), momentum strategies, Siganos et al (2006) and Balsara et al (2006). The results from these papers are also positive.

A popular strategy that has made its reputation in the early 80's is the so called pairs trading. Such methodology was designed by a team of scientists from different areas (mathematics, computer sciences, physics, etc), which were brought together by the Wall Street quant Nunzio Tartaglia. The main objective of such team was to use statistical methods to develop computer based trading platforms, where the human subjectivity had no influence whatsoever in the process of making the decision of buy or sell a particular stock. Such systems were quite successful for a period of time, but the performance wasn't consistent after a while and the team was dismantled after a couple periods of bad performance. More details about the origins of pairs trading can be found at Vidyamurthy (2004) and Gatev et al (1999).

Basically, the basic idea of pairs trading is to take advantage of market inefficiencies. The first step is to identify two stocks that move together and trade them every time the absolute distance between the price paths is above a particular threshold value. If the stocks, after the divergence, return to the historical behavior of symmetry, then is expected that the one with highest price is going to have a decrease in value and the one with the lowest price has an increase. All long and short positions are taken according with this logic. The specific details about choosing pairs and defining the threshold value in pairs trading are going to be given in the scope of the paper.

The main objective of this research is to investigate the profitability and risk of the pairs trading strategy for Brazilian stock market. This trading rule presented a positive performance in the studies of Gatev et al (1999) and Nath (2003), and this is one of the motivations of the study, along with the fact that this type of research hasn't, yet, been applied to the Brazilian market.  In order to reach such objective, is going to be used data from three different frequencies (daily, weekly and monthly), and all returns from the logical rules are going to be compared against a naïve strategy of buy&hold and also against a bootstrap method of random trading. The systematic risk and the filtered constant return (Jensen's Alpha) of such strategies are also part of the analysis.

The paper is organized as follows; the first part is related to the main guidelines of the methodology, including the way the pairs are going to be formed, the logical rules of trading and performance assessment. Second, the results are presented and discussed, followed by some with some concluding remarks.

## 2. Methodology

The methodology of this research is concerned with two points: 1) How to trigger a long/short position based on pairs trading strategy in each stock and 2) How to evaluate the performance of the trading signals. All of the calculations for this paper were conducted at Matlab[3]. The steps of the algorithm are covered next.

### 2.1 Pairs Selection

In the pairs formation phase, the basic idea is to bring all assets' prices to a particular unit and, after that, search two stocks that move together. Quantitatively speaking, this can be done in many different ways. The approach used on this paper is the minimum squared distance rule, meaning that, for each stock, will be searched a corresponding pair that offers the minimum squared distance between the normalized price series[4].

The reason for the unit transformation is straightforward. The use of original prices (without normalization) would be a problem for the case of minimum squared distance rule since two stocks can move together but still have a high squared distance between them. After the normalization, all stocks are brought to the same standard unit and this permits a quantitatively fair formation of pairs.

The transformation employed is the normalization of the price series based on its mean and standard deviation, Equation [1].

$$P_{it}^{*} = \frac{P_{it} - E(P_{it})}{\sigma_i} \qquad [1]$$

The value of $P_{it}^{*}$ is the normalized price of asset *i* at time *t*, $E(P_{it})$ is just the expectation of $P_{it}$, in this case the average, and $\sigma_i$ is the standard deviation of the respective stock price. Both indexes were calculated within a particular moving window of the time series. With

[3] The matlab functions used for this research, including pairs trading execution and the bootstrap method can be downloaded at: http://www.mathworks.com/matlabcentral/fileexchange/ , keywords: "pairs trading" and "monkey trading"

[4] The research was also executed using a maximum correlation criteria, but the results were quite similar.

the use of Equation [1], all prices are going to be transformed to the same normalized unit, which will permit the use of the minimum squared distance rule.

The next step is the chose, for each stock, a pair that has minimum squared distance between the normalized prices. This is a simple search on the database, using only past information up to time *t*. The normalized price for the pair of asset *i* is now addressed as $p_{it}^{*}$. After the pair of each stock is identified, the trading rule is going to create a trading signal every time that the absolute distance between $P_{it}^{*}$ and $p_{it}^{*}$ is higher than *d*. The value of *d* is arbitrary, and it represents the filter for the creation of a trading signal. It can't be very high, otherwise only a few trading signal are going to be created and it can't be to low or the rule is going to be too flexible and it will result in too many trades and, consequently, high value of transaction costs.

After a trading sign is created, the next step is to define the positions taken on the stocks. According to the pairs trading strategy, if the value of $P_{it}^{*}$ is higher (lower) than $p_{it}^{*}$ then a short (long) position is kept for asset *i* and a long (short) position is made for the pair of asset *i*. Such position is kept until the absolute difference between the normalized prices is lower than *d*[5].

The main logic behind the expected profits of pairs trading strategy is: if the correlated movement between the pairs is going to continue in the future then, when the distance between an asset and its pair is higher than a particular threshold value (*d*), there is a good possibility that such prices are going to converge in the future, and this can be explored for profit purposes. If the distance is positive, then the value of $P_{it}^{*}$, according to the logic expressed earlier, probably will reduce in the future (short position for asset *i*) and the value of $p_{it}^{*}$ is probably going to increase (long position for the pair of *i*). The same logic is true for the cases where the distance is negative. The situations where pairs trading fails to achieve profit are: a increase in the distance between $P_{it}^{*}$ and $p_{it}^{*}$, where the market goes the opposite way of the expectation and also a decrease (increase) on the price of the long (short) position.

As an example, Figure 1 shows the pairs trading strategy for weekly prices of asset TNLP4 and its pair, TNLP3.

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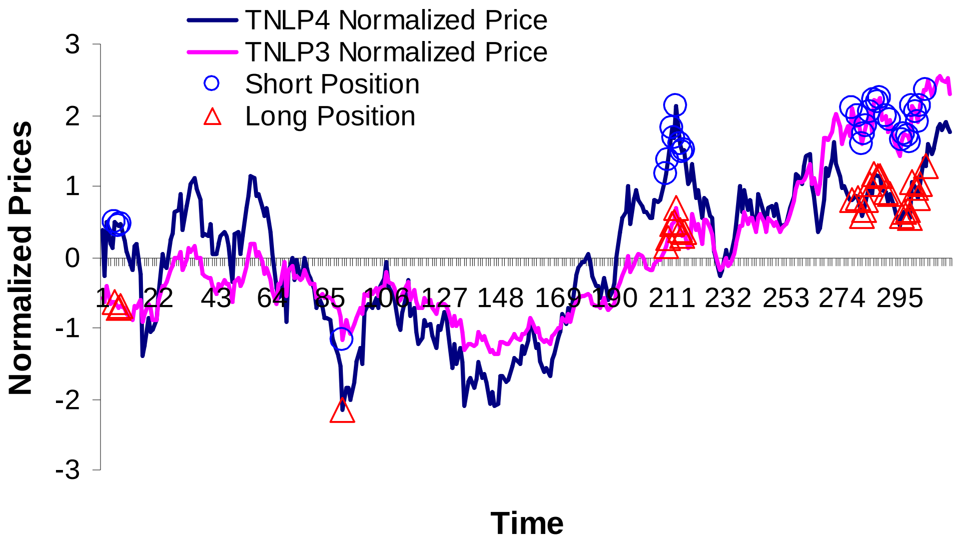
[5] This may sound counter intuitive, since, using continuous price behavior, if one buys when the distance is d and sells it when is the distance is again d, there is no profit. But remember that the prices were in discrete time, meaning that the buying price occurs when the distance is higher than d, therefore the expected profit is positive. For the case of pairs trading at approximate continuous time (eg. 5 min quotes), this can be easily adapted by setting a gap between the threshold for buying operation and for the sell operation

Figure 1 – Example of Pairs Trading with TNLP4 and TNLP3 with *d=1*




In Figure 1, TNLP3 is the found pair of TNLP4 based on the minimum squared distance criteria. It's possible to see that both normalized prices have a similar behavior. On the points that have a blue circle or red triangle the absolute difference in the normalized prices have crossed the value of *d*, meaning that a trade has taken place. The blue circles (red triangles) are the short (long) positions created. This happens every time the absolute distance is higher than 1 and the value of the analyzed asset is higher (lower) than it's pair. Every time the absolute difference uncrosses the value of *d*, the positions are closed. If the assets, after the opening of a position, move back to the historical relationship, then the one with the higher price should have a decrease in the prices and the one with the lower price should have an increase. Since a short position was made for the first asset and a long position for the second, then, if both prices behave historically, a profit will arise from this trading case, and that's the whole idea behind pairs trading, making profits out of market corrections.

### 2.2 Assessing Performance of the Strategy

One of the concerns of this study is to evaluate the performance of pairs trading strategy against a naïve approach. For that purpose, two methods are employed here. The first is the computation of the excessive return of the strategy over a properly weighted portfolio and the second is the use of bootstrap methods for evaluating the performance of the trading rule against the use of random pairs for each stock.

### 2.2.1 Calculation of Strategy's Returns

The calculation of the strategy's total return is going to be executed according to the next formula, Equation [2].

$$R_E = \sum_{t=1}^{T}\sum_{i=1}^{n} R_{it} I_{it}^{L\&S} W_{it} + \left(\sum_{t=1}^{T}\sum_{i=1}^{n} Tc_{it}\right)\left[\ln\left(\frac{1-C}{1+C}\right)\right] \qquad [2]$$

Where:

$R_{it}$ Real return of asset *i* on time *t*, calculated by $\ln\left(\frac{P_{it}}{P_{it-1}}\right)$;

$I_{it}^{L\&S}$ Dummy variable that takes value 1 if a long position is created for asset *i*, value -1 if a short position is created and 0 otherwise. When a long position is made at time *t*, this variable is going to be addressed as $I_{it}^{L}$ and as $I_{it}^{S}$ for short positions;

$W_{it}$ Weighting variable that controls for portfolio construction at time *t*. In this particular paper the simulated portfolio is *equal weight*, meaning that each trading position will have the same weight at time *t*, that is $W_{it} = \frac{1}{\sum_{i=1}^{n}\left|I_{it}^{L\&S}\right|}$. Naturally, the sum of $W_{it}$ for all assets is equal to 1 or zero (no trading position at time *t*);

$Tc_{it}$ Dummy variable that takes value 1 if a transaction is made for asset *i* on time *t* and zero otherwise[6]

$C$ Transaction cost per operation (in percentage);

$T$ Number of observations on the whole trading period;

For Equation [2], the basic idea is to calculate the returns from the strategy accounting for transaction costs. The first part of [2], $\sum_{t=1}^{T}\sum_{i=1}^{n} R_{it} I_{it}^{L\&S} W_{it}$, calculates the total raw return of the strategy. Every time a long and short position is created for asset *i*, the raw return of the

[6] It's important to distinguish the values of $I_{it}^{L\&S}$ (long and short positions) from $Tc_{it}$ (transaction dummy). The values of $Tc_{it}$ are derived from the vector $I_{it}^{L\&S}$, but they are not equal. For example, suppose a long position is created for asset *i* on time t-1 and also on time *t*, only. The vector of $I_{it}^{L}$ is going to have values of 1 to time *t-1* and *t*, but the vector of $Tc_{it}$ has only value 1 for time *t-1*, since for *t*, the asset was already in the portfolio, so there is no need to buy it again. The same is true for short positions.

simulated portfolio on time *t*, is $\sum_{i=1}^{n} R_{it} I_{it}^{L\&S} W_{it}$ , that is, the prospected returns multiplied by their corresponding weight in the portfolio. Since *t* goes from *1* to *T*, is necessary to sum such returns, which gives the final result for the first part of [1], $\sum_{t=1}^{T}\sum_{i=1}^{n} R_{it} I_{it}^{L\&S} W_{it}$ .

The second part of Equation [2] has the objective of accounting for transaction costs. As an example, suppose that the trading cost of buy and selling one stock is *C*, which is expressed as a percentage of the transaction price. If a stock is purchased at price $P_B$ and sold at price $P_S$ , then the real buy and sell prices, including transaction costs, are $P_B(1+C)$ and $P_S(1-C)$ . Taking the logarithm return of the operation results on the formula $R = \ln\left(\frac{P_S(1-C)}{P_B(1+C)}\right)$. Using logarithm properties, the previous equation becomes $R = \ln\left(\frac{P_S}{P_B}\right) + \ln\left(\frac{1-C}{1+C}\right)$. It's possible to see from this result that the return for this operation has two separate components, the logarithm return from difference between selling and buying price and also the term $\ln\left(\frac{1-C}{1+C}\right)$, which accounts for the transaction cost on the whole operation. This exemplified result basically states that the transaction cost for one operation (buy&sell) is $\ln\left(\frac{1-C}{1+C}\right)$.

Returning to the analysis of the second part of Equation [1], since $\ln\left(\frac{1-C}{1+C}\right)$ is the transaction cost of one operation, logically the term $\left(\sum_{t=1}^{T}\sum_{i=1}^{n} Tc_{it}\right)$ is just the number of operations made by the trading strategy. Is important to notes that, since $\frac{1-C}{1+C}$ is always less than one because C is always positive and higher than zero, then the value of $\ln\left(\frac{1-C}{1+C}\right)$ is always negative, meaning that the transaction costs are going to be subtracted from the strategy' returns, which is an intuitive result.

### 2.2.2 Evaluation of Strategy' Returns

In order to evaluate the performance of the strategy, is necessary to compare it to a naïve approach. If the strategy performs significantly better than an out-of-skill investor, then such trading rule has value. This is the main idea that will conduct both methods used in this research to evaluate the performance of pairs trading strategy for Brazilian financial market. The approaches described here are computation of excessive return over a naïve buy&hold rule and the more sophisticated bootstrap method of random trading.

#### 2.2.2.1 Computation of Excessive Return of a Naïve Portfolio

The calculation of excessive return is the simplest approach to evaluate a trading strategy. The idea is quite simple: verify how does the tested strategy exceeds a naïve trading rule in terms of profitability. In this case, the naïve rule is the buy&hold of a properly weighted portfolio for comparison with the long positions and a "sell&unhold" for the short positions.

The return of the naïve approach, over the whole number of assets, is based on the following formula, Equation [3].

$$R_{NE} = \sum_{i=1}^{n} P_i^L \sum_{t=1}^{T} R_{it} + \sum_{i=1}^{n} P_i^S \sum_{t=1}^{T} R_{it} + 2n \ln\left(\frac{1-C}{1+C}\right) \qquad [3]$$

For Equation [3], the value of $P_i^L$ and $P_i^S$ is just the proportion of days, related to the whole trading period, that the strategy created long and short positions for asset *i*. Formally, $P_i^L = \frac{\sum_{t=1}^{T} I_{it}^L}{T}$ and $P_i^S = \frac{\sum_{t=1}^{T} I_{it}^S}{T}$. Notes that, in the calculation of $P_i^S$, the sum of the short positions is always negative or equal to zero, since $I_{it}^S$ takes values -1 and 0, only.

Since pairs trading strategy uses two different types of positions in the stock market, long for the hope of a price increase and short for the hope of a price decrease, it's necessary to construct a naïve portfolios that also takes use of such positions. This is the function of the terms $\sum_{i=1}^{n} P_i^L \sum_{t=1}^{T} R_{it}$ and $\sum_{i=1}^{n} P_i^S \sum_{t=1}^{T} R_{it}$, where the first simulates a buy&hold (long positions) of a properly weighted portfolio and the second simulates a "sell&unhold" (short positions) scheme for another properly weighted portfolio. The weights in both terms are derived from the number of long and short positions taken on each asset, as was showed before. The higher the number of long and short signals a strategy makes for asset *i*, higher the weight that such stock will have on the simulated portfolio. It's clear to see from Equation [3] that, if $P_i^S = P_i^L$, which is a perfectly hedged position for asset *i* in the benchmark portfolio, the terms $\sum_{i=1}^{n} P_i^L \sum_{t=1}^{T} R_{it}$ $\sum_{i=1}^{n} P_i^S \sum_{t=1}^{T} R_{it}$ nulls each other and the contribution of accumulated return for this respective asset in the benchmark portfolio is just the transaction cost for setting up the portfolios.

It should be notes that the calculation of return at Equation [3] doesn't include $W_{it}$ variable as in Equation [2]. This happens because the refereed equation is calculating the sum of expected returns of a naïve long and short positions for all assets, and not the return of the simulated portfolio over time (Equation [2]).

As can be seen from Equation [3], one of the premises of the research is that the transaction cost per operation is the same for long and short positions. The last term of [3] is the transaction costs for opening positions (making the portfolio) and trade them at the end of the period. In this case, the number of trades required to form and close the two portfolios is 2*n*, where *n* is the number of researched assets.

The excessive return for the strategy is given by the difference between [2] and [3], which forms the final formula for computing excessive return, Equation [4].

$$R_E^* = \sum_{t=1}^{T}\sum_{i=1}^{n} R_{it} I_{it}^{L\&S} W_{it} - \sum_{i=1}^{n} P_i^L \sum_{t=1}^{T} R_{it} - \sum_{i=1}^{n} P_i^S \sum_{t=1}^{T} R_{it} + \left(\sum_{i=1}^{n}\sum_{t=1}^{T} Tc_{it} - 2n\right)\left[\ln\left(\frac{1-C}{1+C}\right)\right] \quad [4]$$

Analyzing Equation [4], the maximization of $R_E^*$, which is the objective of any trading strategy, is given by the maximization of $\sum_{t=1}^{T}\sum_{i=1}^{n} R_{it} I_{it}^{L\&S} W_{it}$, minimization of $\sum_{i=1}^{n} P_i^L \sum_{t=1}^{T} R_{it}$ and $\sum_{i=1}^{n} P_i^S \sum_{t=1}^{T} R_{it}$ and also minimization of $\left(\sum_{i=1}^{n}\sum_{t=1}^{T} Tc_{it} - 2n\right)$, since $\left[\ln\left(\frac{1-C}{1+C}\right)\right]$ is a constant. The conclusion about this analysis is intuitive because the strategy is only going to be successful if it efficiently creates long and short positions on the stocks, keeping the transaction costs and the benchmark returns at low values. Short story, make more money with less trades.

### 2.2.2.2 Bootstrap Method for Assessing Pairs Trading Performance

The bootstrap method represents a way to compare the trading signals of the strategy against pure chance. The basic idea is to simulate random entries in the market, save the values of a performance indicator for each simulation and count the percentage number of times that those random entries were worst than the performance obtained in the tested strategy. It should be notes that each trading strategy takes different number of long and short positions and for a different number of days. Such information is also taken in account at the random simulations. Before applying the algorithm, separately, for long and short position, it should be calculated the median number of days (*nDays_Long* and *nDays_Short*) that the strategy has been trading in the market and also the median number of assets (*nAssets_Long* and *nDays_Short*).

The steps are:

1. With the values of the *nDays* and *nAssets* for long and short, define *nDays* random entries in the market for *nAssets* number of assets. Again, making it clear, this procedure should be repeated for each type of trading position (long and short). The output from this step is a trading matrix which has, only, values 1 (long position), -1 (short position) or zero (no transaction).
2. Taking as input the trading matrix and the transaction costs, the portfolio is build with equal weights, resulting in a vector with the returns of the trading signals over time, $R_t = \sum_{i=1}^{n} R_{it} W_{it}^{RND}$, where $R_{it}$ is the return for asset *i at* time *t*, $W_{it}^{RND}$ is corresponding portfolio weight of asset *i* at time *t*, which is build with the random trading signals from last step. Such vector is then used for calculation of the performance indicators (eg. annualized raw return).
3. Repeat steps 1 and 2 *N* number of times, saving the performance indicator value for each simulation.

After a considerable number of simulations, for example *N=5000*, the result for the bootstrap method is going to be a distribution of performance indicators. The test here is to verify the percentage of cases that the tested strategy has beaten comparing with the use of random trading.

As an example, the next ilustraton is the histogram of the accumulated returns from the use of bootstrap algorithm[7] for the daily database with options: *N=5.000*, *nDays_Long=400*, *nDays_Long=250*, *nAssets_Long=5*, *nAssets_Short=3* and with zero transaction cost (*C=0*).

[7] The algorithm used is kindly called monkey trading and can be found at author's matlab's exchange site, together with the classical pairs trading algorithm.

Figure 2 – Histogram of the annualized raw returns from the Random Trading signals

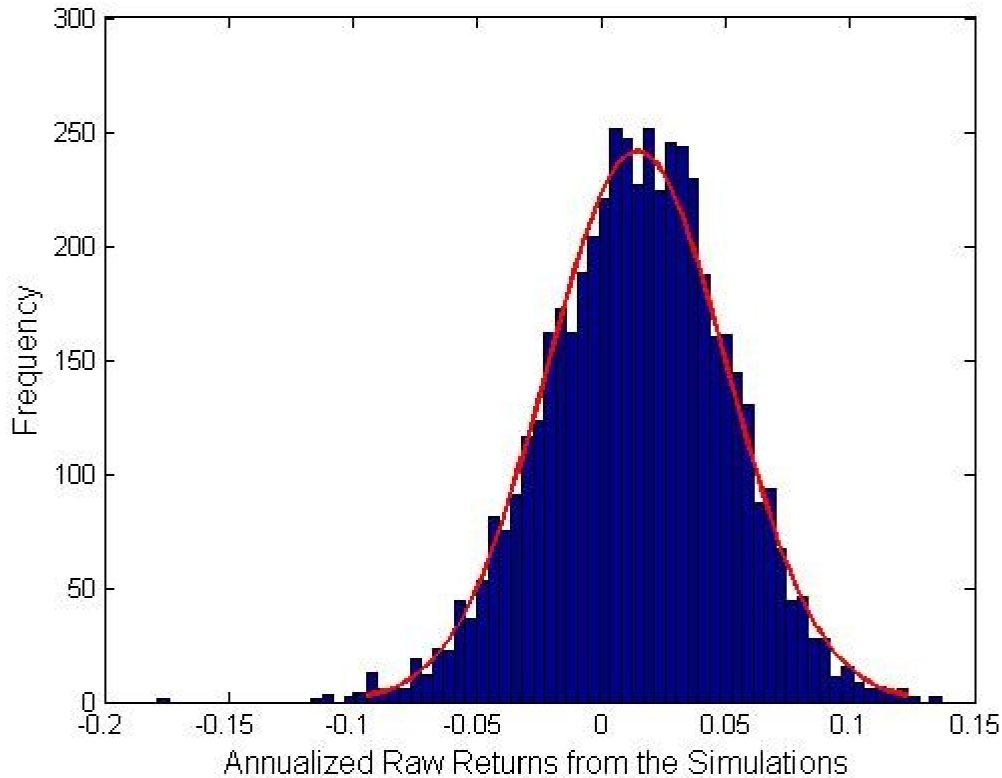



Figure 2 shows that, considering the options given to the algorithm, an out-of-skill investor would earn, in average, an annualized raw return of approximately 1.5%. The best case for the random trading signals is approximately 15% and the worst is -10%. One can also see that the distribution can be well pictured by a normal likelihood (the line).

The next step in using this bootstrap approach is to count the number of times that the performance indicator, in this case the annualized raw return, from the tested strategy is better[8] than the simulated performance indicators from the random trading signal and divide that by the number of simulations. The result is a percentage showing how many random signals the tested strategy has beaten. If such strategy has value, it would produce percentages close to 80%. If it is just a case of chance, it would give a percentage close to 50% and, if the strategy doesn't present any value, it would result in a percentage close to 20%, meaning that, in this case, it's possible to get higher returns by just using a random seed to select assets and days to trade. One way of analyzing the result of the bootstrap algorithm is that it compares the selections made by the trading strategy, that is, the days and assets to trade, against an expected value of the indicator for the same days and number of trades over the full researched data.

[8] Better could mean higher or lower, depending of which performance indicator is being calculated. For instance, a higher annualized return is better, while a lower annualized standard deviation is preferred

## 3. Database for the Research

The database for this research is based on the 100 most liquids stocks from the Brazilian financial market between the periods of 2000 and 2006.

The study is going to assess the performance and risk of pairs trading for different frequencies of the data: daily, weekly and monthly prices. Since there are some liquidity problems for a few cases, the database had to be reconfigured for each frequency. The rule here is to select the stocks that have at least 98% of valid closing prices. The resulting number of stocks after the application of the filter is presented next, Table 1.

Table 1 – Database According to Time Series Frequency

| Frequency | Number of Stocks Selected* | Number of Observations for each stock | Total Number of Observations |
|---|---|---|---|
| Daily | 57 | 1.491 | 84.987 |
| Weekly | 92 | 314 | 28.888 |
| Monthly | 100 | 73 | 7.300 |

* Out of 100 stocks

For Table 1, the major decrease in the researched database is for daily frequency, where only 57 stocks were selected after filtering for stocks with less than 98% of closing prices. For weekly and monthly prices, such filtering was not a problem, and most of the stocks from the original database were kept. For the cases of missing prices, they were just replaced by the past price, which sets the logarithm return in that date to zero.

Every test of trading strategy has two phases in the research data: the training period and the trading period. For this research, the training period is going to be a moving window composed of approximately 2 year of data for all frequencies[9]. For daily prices such moving windows has length 494, for weekly prices is 105 and for annual frequency is 24.

Another issue in executing the pairs trading strategy over data is that each stock may change its pairs along the time. In order to asses such possibility, the pair of each stock is updated at each month for all tested frequencies. For instance, for daily data, at each 25 observations the pairs are recalculated[10].

It's important to notes that the algorithm is not using future observation to build the trading rules. All aspects of the strategy are calculated using only past information, which is a necessary assumption for a realistic assessment of a quantitative trading strategy performance.

[9] One should be careful when choosing the training period of pairs trading. The first draft of this research was done using 1 year of trading data as a moving training window and found significant systematic risk for the tested strategy. When the moving training window was increased, such evidence vanished, as will be showed later in the paper.

[10] This is input *ut* at the matlab code, which changes the periodicity of recalculation of the pairs for each stock

## 4. Results

Next, Table 2, is presented the returns obtained from the pairs trading strategy at the different researched frequencies, with C=0.1%[11]

Table 2 – Evaluation of Pairs Trading Returns for Different Values of *d*

**Panel A - Pairs Trading for Daily Frequency**

| Value of d | Raw Return (with transaction costs) | | | Excessive Returns | | | % Number of Days in the Market | % of Beaten Rdn Port. |
|---|---|---|---|---|---|---|---|---|
| | Long Positions | Short Positions | Total | Long Positions | Short Positions | Total | | |
| 1.50 | 58.77% | -47.00% | -2.42% | -39.57% | 163.15% | 109.39% | 78.84% | 90.60% |
| 1.60 | 85.45% | -5.85% | 31.87% | 10.09% | 166.90% | 129.26% | 71.11% | 100.00% |
| 1.70 | 73.13% | 12.45% | 31.31% | 19.53% | 146.47% | 111.73% | 61.58% | 100.00% |
| 1.80 | 83.53% | 6.58% | 37.95% | 41.16% | 116.49% | 105.49% | 51.25% | 100.00% |
| 1.90 | 85.39% | -1.10% | 36.79% | 56.86% | 87.00% | 96.36% | 43.23% | 100.00% |
| 2.00 | 67.45% | -28.04% | 14.72% | 47.29% | 43.20% | 65.81% | 36.81% | 99.80% |
| 2.10 | 31.40% | -36.65% | -7.99% | 16.36% | 24.81% | 38.44% | 31.80% | 22.60% |
| 2.20 | -3.59% | -3.60% | -8.93% | -13.13% | 48.40% | 33.52% | 27.18% | 13.90% |
| 2.30 | -11.23% | -24.22% | -21.54% | -18.00% | 22.02% | 17.94% | 24.27% | 0.50% |
| 2.40 | 22.74% | -40.62% | -10.77% | 19.76% | -1.87% | 25.00% | 20.06% | 8.90% |
| 2.50 | 32.38% | -48.58% | -9.93% | 30.94% | -15.92% | 21.29% | 16.85% | 14.50% |
| 2.60 | -1.47% | -42.36% | -23.52% | 0.15% | -14.48% | 5.99% | 14.04% | 0.00% |
| 2.70 | 7.49% | -29.47% | -12.19% | 11.60% | -4.43% | 16.96% | 11.63% | 6.60% |
| 2.80 | 12.69% | -17.16% | -3.04% | 18.64% | 4.76% | 24.83% | 9.93% | 53.80% |
| 2.90 | 27.72% | -35.66% | -4.17% | 33.98% | -14.87% | 22.88% | 9.03% | 43.90% |
| 3.00 | 22.51% | -37.82% | -7.76% | 29.39% | -18.12% | 18.84% | 7.92% | 17.50% |

**Panel B - Pairs Trading for Weekly Frequency**

| Value of d | Raw Return (with transaction costs) | | | Excessive Returns | | | % Number of Weeks in the Market | % of Beaten Rdn Port. |
|---|---|---|---|---|---|---|---|---|
| | Long Positions | Short Positions | Total | Long Positions | Short Positions | Total | | |
| 1.50 | 70.13% | -177.22% | -49.40% | -79.06% | 124.85% | 103.47% | 89.95% | 0.00% |
| 1.60 | 27.29% | -150.74% | -59.44% | -87.88% | 97.74% | 73.87% | 84.21% | 0.00% |
| 1.70 | 7.24% | -168.54% | -78.46% | -77.84% | 33.54% | 38.54% | 80.38% | 0.00% |
| 1.80 | 30.31% | -139.23% | -52.61% | -39.56% | 28.55% | 45.31% | 73.68% | 0.00% |
| 1.90 | 37.10% | -169.91% | -61.47% | -13.37% | -30.78% | 27.18% | 68.42% | 0.00% |
| 2.00 | 20.71% | -140.47% | -54.18% | -15.39% | -22.56% | 27.64% | 61.72% | 0.00% |
| 2.10 | -15.11% | -135.60% | -69.55% | -42.09% | -36.74% | 2.33% | 54.55% | 0.00% |
| 2.20 | -41.07% | -115.35% | -72.40% | -60.10% | -31.07% | -7.15% | 45.45% | 0.00% |
| 2.30 | -31.58% | -85.29% | -53.51% | -44.46% | -14.22% | 4.68% | 40.19% | 0.00% |
| 2.40 | -17.29% | -67.88% | -37.46% | -22.26% | -9.60% | 15.84% | 33.01% | 0.00% |
| 2.50 | -27.26% | -77.75% | -47.38% | -28.99% | -26.12% | 2.52% | 28.23% | 0.00% |
| 2.60 | -20.91% | -54.69% | -32.68% | -19.66% | -10.65% | 12.60% | 24.88% | 0.10% |
| 2.70 | -41.68% | -63.69% | -47.47% | -37.93% | -24.33% | -4.35% | 22.01% | 0.00% |

[11] The trading cost of 0.1% per operation is a realistic value for the Brazilian market. It can be easily achieved with a relatively small amount of 15.000 R$, which, today (November 2006), is something close to 7.000 USD.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 2.80 | -0.12% | -41.00% | -20.13% | 5.27% | -8.87% | 17.39% | 16.27% | 0.10% |
| 2.90 | -2.55% | -43.39% | -22.54% | 4.08% | -12.40% | 15.07% | 15.31% | 0.10% |
| 3.00 | 7.18% | -9.18% | -0.47% | 15.23% | 19.79% | 36.54% | 13.40% | 56.20% |

**Panel C - Pairs Trading for Monthly Frequency**

| Value of d | Raw Return (with transaction costs) | | | Excessive Returns | | | % Number of Months in the Market | % of Beaten Rdn Port. |
|---|---|---|---|---|---|---|---|---|
| | Long Positions | Short Positions | Total | Long Positions | Short Positions | Total | | |
| 1.50 | 43.30% | -192.06% | -74.78% | 1.52% | -61.26% | 14.23% | 71.43% | 0.00% |
| 1.60 | 34.89% | -93.90% | -29.90% | 6.18% | 5.76% | 41.05% | 61.22% | 0.20% |
| 1.70 | 41.05% | -56.22% | -7.79% | 31.83% | 8.92% | 48.13% | 44.90% | 15.50% |
| 1.80 | 29.80% | -20.14% | 4.73% | 24.87% | 38.50% | 58.45% | 36.73% | 78.00% |
| 1.90 | 24.13% | 0.99% | 12.56% | 25.41% | 46.10% | 58.95% | 30.61% | 96.30% |
| 2.00 | 7.55% | -15.28% | -3.87% | 14.70% | 27.50% | 46.06% | 26.53% | 31.00% |
| 2.10 | 8.94% | -18.86% | -4.96% | 18.10% | 22.21% | 45.27% | 24.49% | 22.10% |
| 2.20 | 15.66% | -20.94% | -2.64% | 26.18% | 16.47% | 45.30% | 20.41% | 48.40% |
| 2.30 | 15.66% | -20.94% | -2.64% | 26.18% | 16.47% | 45.30% | 20.41% | 44.10% |
| 2.40 | -6.53% | 3.59% | -1.47% | 7.69% | 32.46% | 41.63% | 14.29% | 40.90% |
| 2.50 | -6.65% | 11.57% | 2.46% | 10.42% | 35.32% | 43.29% | 10.20% | 69.50% |
| 2.60 | 19.07% | 5.46% | 12.27% | 39.43% | 27.42% | 54.58% | 4.08% | 94.20% |
| 2.70 | 19.07% | 5.46% | 12.27% | 39.43% | 27.42% | 54.58% | 4.08% | 94.40% |
| 2.80 | 19.07% | 5.46% | 12.27% | 39.43% | 27.42% | 54.58% | 4.08% | 95.10% |
| 2.90 | 19.07% | 5.46% | 12.27% | 39.43% | 27.42% | 54.58% | 4.08% | 96.10% |
| 3.00 | 19.07% | 5.46% | 12.27% | 39.43% | 27.42% | 54.58% | 4.08% | 96.60% |

* The bootstrap method was based in 1.000 simulations for each value of *d*, always respecting the median number of days and assets that the strategy was in the market for each type of position.

Before the analysis of Table 2, an important observation is that the total return is not just the sum of the returns from the long and short positions. For the excessive and raw returns, if the column "Long Positions" was summed with the column "Short Positions", it will not equal the column "Total". This happens because one asset can be having a buy sign and also a sell sign for the same time *t*, since such stock can be the pair of other stock. If only short or long positions are analyzed, the respective trading positions are valid, but when analyzing the total return from both, a buy and sell sign, for the same asset at the same time, nulls each other. As can be seen, the difference is quite high, meaning that such event has happened very often.

For the case of raw return, Table 2, which is just the clean return of the strategy minus the transaction costs, it can be seen that the long positions were far more profitable than the short positions in all tested frequencies. This was expected, since the period of the study was clearly an upward trending market[12], meaning that a short position would not make much money, as can be seen in the raw returns for the short signals, at different frequencies.

Analyzing the excessive returns of Table 2, it's possible to see that the pairs trading strategy was able to beat a properly weighted naïve portfolio in most of the cases. Such

[12] From the period of 2001 to 2006, Ibovespa, which is the broad market index for Brazilian Market, grew from 17.672 to 38.382 points.

result is more consistent for the daily frequency in the interval of *d* between 1.5 and 2 and also for the monthly frequency in the whole tested interval of *d*.

Verifying the relationship between *d* and number of trades, it's very clear that they are negatively correlated, since in the execution of the trading rules high (low) values of *d* presented low (high) number of transactions. This can be easily explained by the fact that *d* is the threshold variable that controls when a price divergence is not considered normal. As *d* grows, less and less abnormal divergences are found, which consequently reduces the number of transactions made by the strategy.

The bootstrap method presented at Table 2 shows that pairs trading is superior to the use of random trading signals in just a few cases, more precisely when the database is daily with $1.5 \geq d \geq 2$, where the percentage ranges from 90% to 100%. There are also indications of positive performance of the bootstrap method over the monthly data with $2.6 \geq d \geq 3$. But, given that just a few trades were made for this particular interval (4.08% - 3 observations), the result can't be take seriously in the performance of the pairs trading strategy. A much clearer picture of positive performance was given at Panel A.

Another information given by Table 2 is that the bootstrap method is much more restrictive for positive performance than the benchmark portfolio approach. While the last presented positive excess return for almost all values over the different panels, the last only resulted in positive performance for a couple of cases. It could be argued that the benchmark method is a static way of assessing performance, and the bootstrap method is superior in the sense that the way to asses naïve performances is much more dynamic and, therefore, superior.

The best case of Table 2, when comparing returns and bootstrap methods, is for daily frequency, where the total raw returns presented high percentage of beaten random portfolios and also a positive and consistent excessive return at a particular domain of *d*. The performance of pairs trading for weekly prices were not very consistent for different values of *d* and, for monthly prices, positive values of excessive returns were found, but the simulation of random portfolios showed that most of the raw returns obtained at this frequency were just a case of chance, and not skill.

The result of the superiority of higher frequencies in the pairs trading framework is logically consistent since the objective of pairs trading is to take advantage of market corrections and such inefficiency would, in expectation, happen more at high frequencies. The next step in this type of research could be to study the performance of pairs trading at high frequency data (intraday quotes), and check if, again, the performance is higher at higher frequencies of the data.

The next analysis pursued in the paper is the evaluation of the risk in the tested strategies, Table 3.

Table 3 – Beta and Jensen's Alpha for Pairs Trading

| Value of d | Beta | | Jensen's Alpha | |
|---|---|---|---|---|
| **Panel A - Pairs Trading for Daily Frequency** | | | | |
| | Value | Prob | Value | Prob |
| 1.5 | 0.074 | 0.104 | 0.000 | 0.949 |
| 1.6 | 0.044 | 0.323 | 0.001 | 0.328 |
| 1.7 | 0.041 | 0.368 | 0.001 | 0.310 |
| 1.8 | 0.043 | 0.349 | 0.001 | 0.290 |
| 1.9 | 0.013 | 0.766 | 0.001 | 0.285 |
| 2.0 | 0.055 | 0.196 | 0.000 | 0.641 |
| 2.1 | 0.043 | 0.274 | 0.000 | 0.895 |
| 2.2 | 0.067 | 0.063* | 0.000 | 0.833 |
| 2.3 | 0.054 | 0.108 | 0.000 | 0.496 |
| 2.4 | 0.033 | 0.297 | 0.000 | 0.706 |
| 2.5 | 0.022 | 0.472 | 0.000 | 0.733 |
| 2.6 | 0.010 | 0.733 | 0.000 | 0.389 |
| 2.7 | 0.008 | 0.782 | 0.000 | 0.642 |
| 2.8 | -0.005 | 0.859 | 0.000 | 0.932 |
| 2.9 | -0.008 | 0.754 | 0.000 | 0.876 |
| 3.0 | -0.011 | 0.658 | 0.000 | 0.743 |
| **Panel B - Pairs Trading for Weekly Frequency** | | | | |
| Value of d | Beta | | Jensen's Alpha | |
| | Value | Prob | Value | Prob |
| 1.5 | 0.127 | 0.119 | -0.006 | 0.084* |
| 1.6 | 0.077 | 0.398 | -0.006 | 0.089* |
| 1.7 | 0.112 | 0.248 | -0.008 | 0.036* |
| 1.8 | 0.066 | 0.494 | -0.005 | 0.159 |
| 1.9 | 0.044 | 0.688 | -0.007 | 0.140 |
| 2.0 | 0.038 | 0.723 | -0.006 | 0.172 |
| 2.1 | 0.011 | 0.913 | -0.007 | 0.085* |
| 2.2 | 0.041 | 0.677 | -0.008 | 0.051* |
| 2.3 | 0.046 | 0.619 | -0.006 | 0.120 |
| 2.4 | -0.016 | 0.846 | -0.004 | 0.222 |
| 2.5 | -0.006 | 0.943 | -0.005 | 0.114 |
| 2.6 | 0.011 | 0.889 | -0.004 | 0.252 |
| 2.7 | 0.010 | 0.900 | -0.005 | 0.109 |
| 2.8 | 0.036 | 0.517 | -0.002 | 0.340 |
| 2.9 | 0.018 | 0.738 | -0.002 | 0.296 |
| 3.0 | 0.023 | 0.567 | 0.000 | 0.905 |
| **Panel C - Pairs Trading for Monthly Frequency** | | | | |
| Value of d | Beta | | Jensen's Alpha | |
| | Value | Prob | Value | Prob |
| 1.5 | -0.245 | 0.200 | -0.025 | 0.109 |
| 1.6 | -0.192 | 0.307 | -0.008 | 0.602 |
| 1.7 | -0.131 | 0.372 | 0.000 | 0.979 |
| 1.8 | 0.010 | 0.940 | 0.002 | 0.877 |
| 1.9 | -0.074 | 0.624 | 0.007 | 0.589 |
| 2.0 | 0.017 | 0.894 | -0.002 | 0.853 |
| 2.1 | 0.012 | 0.923 | -0.002 | 0.827 |
| 2.2 | -0.001 | 0.993 | -0.001 | 0.920 |

| | | | | |
|---|---|---|---|---|
| 2.3 | -0.001 | 0.993 | -0.001 | 0.920 |
| 2.4 | -0.004 | 0.976 | -0.001 | 0.960 |
| 2.5 | -0.007 | 0.957 | 0.001 | 0.912 |
| 2.6 | 0.019 | 0.706 | 0.005 | 0.271 |
| 2.7 | 0.019 | 0.706 | 0.005 | 0.271 |
| 2.8 | 0.019 | 0.706 | 0.005 | 0.271 |
| 2.9 | 0.019 | 0.706 | 0.005 | 0.271 |
| 3.0 | 0.019 | 0.706 | 0.005 | 0.271 |

* Significant at 10%

** The betas and alphas are obtained with a regression of the vector with the strategies returns over time against the returns from Ibovespa (Broad Brazilian Market Index).

Regarding the Jensen's Alphas in Table 3, which should be positive and statistically significant if the strategy has good performance independently of market conditions, it's possible to view that, for Panel A, most of them are positive but not statistically significant. This particular result shows that pairs trading strategy has a positive constant return after filtering for market conditions, but such coefficient is not statistically significant.

Another concern of this study is regarding the risk of pairs trading strategy. The values of systematic risk (beta) at Table 3 are very close to zero and just one of them is statistically significant at 10%. Such result corroborates with the fact that pairs trading is often called a market neutral rule, meaning that the returns from such strategy usually doesn't follow the market behavior. This is intuitive because, in the pairs trading framework, the number of long positions is equal to the number of short positions when there is no over lapping (short and long at the same time), which creates a natural hedge against the market movements.

After the analysis of the information given by Tables 2 and 3, is possible to say that, for the Brazilian financial market, the positions created by the pairs trading were a moderately profitable strategy in the past and, at the same time, neutral to the market systematic movements. The best results were found at the database with daily frequency. For this particular database, the excessive returns obtained were consistent over a particular region of *d* and the raw returns cannot be considered a simple case of chance. The conclusion about the profitability of pairs trading corroborates with the previous researches in the topic, Gatev et al (1999) and Nath (2003).

## Conclusions

The main objective of this research was to verify the performance (return) and also the risk of classical pairs trading at the Brazilian financial Market at different time frequencies (daily, weekly and monthly). Such analysis was also made considering different values for the threshold parameter *d*.

In order to achieve this objective, the returns from the strategies were compared against a properly weighted portfolio made with long and short positions at the beginning of the trading period and also against a variant of the bootstrap method for assessing performance. The risk of the trading signals was obtained with the analysis of the systematic risk (beta) of the strategies.

The main conclusion of this paper is that pairs trading had a good performance when applied to the Brazilian financial market, especially for the daily frequency. The tests performed showed that the market rules presented betas very close to zero and not statistically significant at 10%, which means that pairs trading may be called a market neutral rule. Regarding profitability, the best case was for daily frequency, where the interval of *d* between 1.5 and 2 presented consistent values of excessive return over a benchmark portfolio. The bootstrap approach also showed that the raw returns for this particular set of parameter were not given by chance, but by skill.

But, it's also important to address some weakness of the research. The framework used at the study didn't allowed for liquidity risk of the strategy, which may be a negative factor affecting the realizable (and not measurable) returns. Given that, the results of positive performance can only be assessed given the constraints of the research.

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