Poor's behaviour and inequality traps: the role of human capital

Lombardo, Vincenzo

November 2008

Poor's behaviour and inequality traps: the role of human capital*

Vincenzo Lombardo†

November, 2008

Abstract

We evaluate whether and how the persistence of inequality and the presence of inequality traps carry over the persistence of poverty and possible aggregate economic inefficiencies. We propose a microeconomic formalization of one possible definition of poverty and of the behaviour of poor and rich agents. Poverty is defined as lack of or low societal participation, which is source of both a direct private benefit and an indirect gain. Analysing poverty under the individual rational choice, we are able to endogeneize the threshold amount of the participation good needed to join the additional indirect benefit as well as the threshold level of income - the poverty line - needed to buy that amount; further we find the conditions for their *existence* which in turn determine also their *level.* In an overlapping generation structure we show that in economies starting largely poor two equilibria exist; one low locally stable equilibrium to which low and middle-income class converge and one upper locally unstable over the which richer classes of income enter an explosive path, with unbounded growth rates of personal income. In the rich regime the whole population enter the explosive path, with unbounded growth rates. We are able to enrich the dynamics *a là* Galor and Zeira (1993) in an economic environment with perfect capital markets, instead of the assumption of markets imperfections, by introducing a methodological innovation which connects the presence of the externality in the human capital accumulation of the children directly to the preference of parents. By further restricting the functional form of the initial income distribution to be lognormal we find that the mean income and the variance (i.e. inequality) of the richer part of the distribution are always higher than the ones of the lower part; moreover, while within the poor class inequality tends to zero in the very long-run, within the richer class inequality is increasing at an increasing rate. Finally, a negative relation between initial inequality and economic growth is observed. The inequality traps which cause the poor regime to emerge are also sources of aggregate economic inefficiencies, which can be eliminated by reducing income disparities accordingly.

Keywords: Poverty, economic growth, inequality traps, human capital, endogenous preference.

JEL: D31, D63, I32, J24, J62, O41

*I am extremely indebted to Maria Rosaria Carillo and Giuseppe Freni for their continuous support and precious insights. I would thank for useful comments Massimo Giannini, Oded Galor, Matthias Doepke and participants at the Jerusalem Summer School on Economic Growth, Jerusalem, 2008, Conference on Income Distribution and the Family, Kiel, 2008, ESRC Conference on Development Economics, Brighton, 2008, and SUN Workshop, Capua, 2008. All remaining errors are mine.

†E-mail: `vincenzo.lombardo@uniparthenope.it`. Department of Economics, University "Parthenope", Naples, Italy.

"...any city, however small, is in fact divided into two, one the city of the poor, the other of the rich; these are at war with one another; and in either there are many smaller divisions, and you would be altogether beside the mark if you treated them all as a single State. But if you deal with them as many, and give the wealth or power or persons of the one to the others, you will always have a great many friends and not many enemies. And your State, while the wise order which has now been prescribed continues to prevail in her, will be the greatest of States, I do not mean to say in reputation or appearance, but in deed and truth, though she number not more than a thousand defenders." [Plato, The Republic, Book IV, 422e-423a]

1 Introduction

The first sentence of the above Plato's quote often circulates in the development literature (Benabou, 1993). In this sentence Socrates does emphasize two elements. High inequality and polarization does split most of the contemporary places, either cities or States, in essentially two or more ones and this division may not fuel the well-being of the whole society. While much of the economic literature is still debating on the validity of this claim, we would address the following part of the quote, where Socrates explicitly describes the two, or more, parts of the society as inherently different. The poor have their own intrinsic characteristics and behavioral traits which differentiate them from the rich[1] such that the former behave and choose differently from the latter and both accordingly to their own status. In this paper we take up this issue by considering the effects for positive economics of assuming that agents do behave accordingly to their specific income status and how this particular behaviour does influence the more general issue of the relation between economic growth and income distribution, and how this latter interplay may fuel poverty patterns. We carry out this task by formalizing on the microeconomic ground the diverse behaviour of the agents under different income condition and the consequences of this behaviour. At this regard, this paper tries to connect two very wide literatures; the one focused on the elaboration of the ideas of poverty and inequality and the other focused on economic growth and development economics. On the former side – the conceptualization of poverty and inequality ideas – no large consensus has yet been reached on a large number of issues. Do we have to treat poverty as an absolute or relative concept? What do these two approaches imply and how they may be formalized? Is poverty different from inequality and if so, under which logic? On the other side, it does appear that the notion of poverty traps is not fully able to capture the relevant role of the relative position within a society and how this may influence and determine both individual macroeconomic dynamics. Since Banerjee and Duflo (2007) have shown that even the extremely poor have a lot of choices, we introduce poverty ideas in the individual rationality of the agents. This

[1]This position is not recognized in the whole literature. There are authors who maintains, otherwise, that the poor have no intrinsic features and no particular behavioural trait, but they fail more often than the rich simply since they are more vulnerable as their budget constraints are much more and much more often stringent (Bertrand, Mullainathan and Shafir, 2004).

is crucial not only for the endogeneization of the threshold amount of the participation good needed to join an additional indirect benefit as well as the threshold level of income - the poverty line - needed to buy that amount, but also for the endogeneization of the conditions which guarantee their *existence* and which fix their *level*.

We then employ this microeconomic structure in an overlapping generation model *a là* Galor and Zeira (1993). Differently from the latter, we are able to couple and enrich their dynamics in an economic environment with perfect capital markets, without needing the assumption of market imperfections, by introducing a methodological novelty which connects the presence of the externality in the production function of the human capital of the children to the preference of the parents.

The paper follows in the second section by introducing the literature related to these issues. In section 3 the basic structure of the model, with the definitions of the population structure, poverty, human capital accumulation and production is explored, so that in section 4 the statically individual equilibrium is computed. In section 5 the individual dynamics as well as the macroeconomic equilibrium are analysed in the light of the notion of inequality traps. The last section concludes.

2 Related Literature

2.1 Poverty and inequality: background ideas

Since the seventies a great bulk of literature (Atkinson, 1970; Foster, 1998; Foster et al., 1984; Sen, 1976, 1997) has made great advances on the issues related to the identification of the poor and the aggregation of poverty and inequality measures in tractable and reasonable functional forms[2]. These concepts have produced intense debates among advocates of an absolute view of poverty on a sideand sustainers of a relative position on the other one, strictly concerned with the suitability, goodness and validity of either one or other approach. Already Adam Smith dealt with the choice of which approach might be the more appropriate, expressing a preference for a relative view of poverty, claiming that not only the physical goods, necessary for the support of the life, but also the commodities that become necessary due to the societal influence might be taken into account when fixing the reference standard. He maintains that a relative position within a society does matter as long as societal customs force agents in consuming goods otherwise not necessary for the basic surviving.

In his defense of the absolutist approach, Sen (1983; 1985) does maintain that the two positions may be conciliated, since a relative position is necessary to achieve an absolute benefit. The idea is that holding a relatively advantageous position is a source not only of a direct benefit, in that it allows consuming a specific bundle of goods, but also a source of a more important gain; namely, a relatively advantageous position in the society allows to have information that other individuals, the ones falling behind, do not have. He or she does not, or at least not exclusively, want to be relatively better than others per se. What

[2]For a survey and more rigorous treatment of both definitional and measurement issues of poverty and inequality see Ch 3 and Ch 6 in Atkinson and Bourguignon (2000).

he or she actually desires is to be absolutely well; a relative advantage is then functional to the achievement of this absolute gain. Townsend (1985) sustains that a relative view of poverty must be the main guide for the assessment of welfare judgments, especially in rich areas. This position tends to criticize the absolutist approach, viewing in this latter a constant inclination in confusing and associating relative poverty to inequality. Although poverty and inequality may be closely linked (Foster, 1998), especially when a relative approach is adopted, they still deserve different treatment for their own intrinsic features. "[T]he fact that some people have a lower standard of living than others is certainly proof of inequality, but by itself it cannot be a proof of poverty unless we know something more about the standard of living that these people do in fact enjoy" (Sen, 1983).

Another source of confusion is likely due to the notion of relative deprivation (Runciman, 1966). Relative deprivation or deprivation with regard to some specific reference group does not necessarily entail equalization of poverty and inequality. Once the appropriate reference space is taken into account, it might be argued that absolute deprivation over some space (i.e. information, opportunity, and so forth) may be source of a relative deprivation over other spaces (i.e. income, consumption, and so forth), and both being still very different from inequality, as - for instance - "a sharp fall in general prosperity causing widespread starvation and hardship must be seen by any acceptable criterion of poverty as an intensification of poverty. But the stated view of poverty as an issue of "inequality" can easily miss this if the relative distribution is unchanged and there is no change in the differences between the bottom 20 or 10 per cent and the rest of the society" (Sen, 1983).

These extensive issues have been fitted in literatures which attempt to track the effects of poverty or unequal positions on positive economics, through microeconomic studies of the intrinsic characteristics of the poor. A large bulk of literature has included the ideas of minimum consumption requirements in growth models to assess the role of income distribution on economic growth and vice versa (Chatterjee, 1994; Chatterjee and Ravikumar, 1997). More recently, an effort is being accomplished in the microfoundation of poor's behaviour. Banerjee and Newman (1994) focus on the idea of "closeness to the lower bounds" to show how this affect behaviour, choices and incentives of the agents. Banerjee (2000) stresses the consequences for positive and normative economics of two notions of poverty: namely, poverty as "desperation" and poverty as "vulnerability"[3]. In the former case, the threat of punishment does not work, or at least it works less well, since the poor behaves as they had nothing to lose. On the other side, vulnerable agents are afraid of any losses since any potential failing causes them too much pain. This difference, it is shown, to decisively affect not only their own behaviour and choices, but also the aggregate dynamics. Banerjee and Mullainathan (2007) have presented new ideas to formalize, on a microeconomic ground, the behaviour of the poor. In an economy with "temptation goods", the authors supply a model in which the degree of temptedness is endogenous and may vary between rich and poor. This setting should allow explain several behavioural puzzles empirically found in the choices of the poor.

We proceed on this branch of research by relating our work closely to the one by Lewis

[3]On these issues see also Banerjee, Benabou and Mookherjee (2006) and Morduch (1994).

and Ulph (1988) who offer a microeconomic model of poverty, suggesting microeconomic principles for building utility functions, which can take into account the above discussion on the meaning of poverty as relative or absolute concepts. Feature of the poor is the lack of, or low, societal participation, which is source of two benefits. A direct benefit deriving from its consumption and, more importantly, an indirect advantage stemming from the fact that its consumption is source of relevant information acquisition as well. In order to buy this good a minimum level of income – the poverty line – is needed. The shortage is, in turn, due to an absolute lack of resources (i.e. income), which negatively affects their opportunities and their absolute welfare. This feature tends to capture a particular aspects of being poor; namely, "...being poor is discretely different from being non-poor, and that this is associated with discrete changes in consumer behaviour and, possibly, utility" (Lewis and Ulph, 1988). We capture this feature linking the discrete change in the income level with a discrete jump in the indirect utility function. The analysis on the extent of poverty is hence based upon the individual rational choice. If individuals are extremely poor as they are unable to consume some specific goods (i.e. the participation good), this aspect should not be exogenously described, but directly derived from the optimizing, rational behaviour of the agents. Was this good so important, why agents should rationally decide of not consuming it? Hence, the discrete jump in the indirect utility function is formalized by allowing the agents to make a choice between two kind of goods. A divisible normal good whose consumption ensures only direct benefit and another kind of good – the participation good, which may be either divisible or not and whose consumption increases utility in two ways; directly, as its consumption yields immediate private benefits and indirectly as its consumption over a threshold (i.e. the poverty line) adds up an additional gain. Participating in social activities does, in fact, entail a cost; only who is able to afford this cost may exploit both the advantages this commodity furnishes. This participation has two effects on utility. A direct one, since everyone, to say, might be happier in going out with friends, and an indirect one, since this participation is source of another and greater benefit; namely, it is source of relevant information acquisition. Although a minimum amount of participation activities may entails low costs, the consumption of such a low level does not allow agents in exploiting the further more important benefit of information acquisition. Further, despite the fact that the feature of this kind of good may be quite common (i.e. club's membership, and so forth) – namely goods which may be consumed only over a minimum income level, the specific characteristic of this participation good is that its consumption over a threshold (the poverty line) entails an advantage so high that consumers are willing to sacrifice all the consumption of the discretionary good to enjoy the benefits of the former. As soon as income level reaches the threshold, agent is well willing to renounce to the discretionary good to exploit the benefits deriving from the consumption of the participation good. It is this characteristic that makes this participation good a "poverty good", in that not owning it makes agents so discontented that they never renounce to it, whenever they can. We are hence referring not to extreme poverty situations, in which agents does not have the basic needs for surviving, but a broader aspect of poverty definition.

2.2 Human Capital and "Inequality Traps"

Based upon this microeconomic framework we construct a dynamic model in the attempts to analyse whether and the extent to which the persistence of inequality may augment the persistence of poverty, even in growing economies. We build an overlapping generation model in which the dynamics of income distribution of the generations are mapped one-to-one to those of human capital accumulation, as the production function, on the firm side, depends only on this last factor. Individuals are supposed initially heterogeneous in human capital. Given that there exists a one-to-one mapping from human capital to income dynamics, in each period this is equivalent to assume that individuals differs only for different income – or human capital – endowments[4]. In each period each parent has to choose how to split his budget into the consumption of the discretionary good, the participation good and investment for education of their children, which is supposed to occur in the standard form of "joy of giving". Each parent cares of the future well-being of the children, which depends on the amount of human capital these latter may accumulate in the future period. Human capital production of the children is function of parental expenditures in education and of parental human capital background. This latter depends in turn upon the amount of participation good consumed by the parents. When parents are able to afford to consume the participation good they will do as soon as they have not to renounce to the investment in educational expenditures, as physical investments in education and parental background are complementary in the production of children human capital, while they may decide to completely renounce in consuming the discretionary good. As result, human capital production presents a strong non-convex technology in that the consumption of the participation good, over the poverty line, does increase its final production, at an increasing rate. This setting does entail to assume that the poor enjoy relatively lower net marginal returns from education than the rich do. This assumption, while enough common in great part of the literature (Benabou, 1996; Durlauf, 1996; Galor and Tsiddon, 1997a; Galor and Zeira, 1993) is criticized as it is sustained that empirically it is less often observed this kind of non-convexities in the technology of the production of human capital (Ceroni, 2001). Nevertheless, some caveats are worthwhile to be explored. Human capital marginal returns are commonly assumed to be decreasing in educational investments, as by its nature human capital is embodied in human being, and its "physical" accumulation is obviously bounded. However, this observation cannot imply the renounce to the assumption of global increasing returns from human capital accumulation, whenever this production increases, at increasing rates, due to a complementary factor, which in our case depends on initial income distribution of the parents. Summing up, human capital production function shows decreasing marginal returns when parents are not able to spread its production with the accumulation of the participation good – the case of the poor dynasties. It presents, otherwise, an increasing returns technology in the case of rich dynasties, which are able to speed up its production by allowing their children to better exploit the "physical" investment in education as a result of the higher information obtained from the consumption of the participation

[4]In the rest of the paper, we will then use interchangeably the two words.

good. This jump in the curvature of the production function of human capital derives from the discrete jumps in the indirect utility function of the parents, and it introduces a methodological innovation which connects the externality in the human capital production function of the children with the preference of the agents. Given that also the relative position in the distribution does matter, the externality stems not only from the absolute economic possibilities of the agents, but also from the knowledge these ones have on its effect and from the value they attach to this externality with respect to the other parts of the distribution. We are hence able to couple and enrich the dynamics a là Galor and Zeira (1993) in an economic environment characterised by perfect capital markets, instead of needing to assume market imperfections.

In each period individual income distributions are linked to the previous ones through the intergenerational transmission of both educational expenditures and participation commodities. Two cases can be distinguished. When poverty is strikingly high two equilibria do exist; one locally stable and the other unstable. Generations that start with an income below the poverty line are condemned in the low equilibrium, while those that start rich enough to afford both educational investment and participation good deserve a further analysis. Richer dynasties present, in fact, a dual dynamics. Who is sufficient rich to consume an enough high amount of both educational investments and participation goods will end up behind the locally unstable steady state on an explosive path, on which their wealth will grow unbounded. On the other side, middle income dynasties, which are closer to the poverty line, will not be able to grow along the explosive path and will be condemned to end up in the lower steady state, even if they achieve in buying both the goods. In this case poverty reduction is positively correlated with reduction in inequality so that the persistence in inequality carries over the persistence in poverty as well, even in growing economies (Durlauf, 1996). In the other case, when poverty is quite low, there does exist no steady state as the whole population ends up on the explosive path, with unbounded growth rates, without any segmentation within the society.

In the former case, the economy is trapped in "inequality traps" (Bourguignon, Ferreira and Walton, 2007) rather than in poverty traps. The notion of inequality traps is still crude as this is a very new branch of research, opened up to answer some questions mainly relating to the John Roemer's idea of equality of opportunity (Roemer, 1998). This very infant literature has not yet precisely clarified the notion of inequality trap as well as its difference from poverty traps. For the first time, we propose to integrate these notions in an economic growth and development model, which takes contemporaneously into account the group-specific behaviour, in order to start filling an apparent weakness of the literature on poverty traps. We refer to the idea of considering poor simply the ones who are below some equilibrium thresholds, without taking into account that the poor have their own particular behaviour as well as that the condition of being poor might derive and perpetuate from a condition of relatively disadvantage with respect to other parts of the distribution. "...Inequality traps...describe situations where the entire distribution is stable because the various dimension of inequality (in wealth, power, and social status) interact to protect the rich from downward mobility and to prevent the poor from being upward mobile" (Rao, as quoted in Bourguignon et al., 2007). What distinguishes the inequality traps from the more classic poverty traps is that the latter refer to poverty without looking at the other part of the distribution (i.e. top and middle income), but only looking at the poor, individually and without any reference to group dynamics.

> "In a poverty trap, the incomes of the poor do not grow beyond some fixed threshold: the poor remain forever poor...An inequality trap, on the other hand, does allow for the advantages of the poor to grow over time, as long as patterns of unequal relative advantage persist in the long run. The dynamics of such persistent differences in opportunities are affected by the entire distribution of advantage, reflecting (economic, political and socio-cultural) interactions across groups" (Bourguignon et al., 2007).

Further insights are then derived looking at the intra-group dynamics through the restriction of the functional form of the initial distribution of human capital. If initial human capital is assumed to be lognormally distributed (Glomm and Ravikumar, 1992; Gradstein and Justman, 1997) it is possible to show that the mean income as well as the variance (i.e. inequality) of the richer dynasties grows faster than that of the poorer ones. These dynamics may furnish positive support to the current debate on the demise of the middle class and the constantly polarization of the societies. At aggregate level, finally, the inverse relationship between initial income distribution and aggregate economic growth is observed, as lower initial inequality fosters higher growth rates.

3 Model

3.1 Population

A continuum of individuals, indexed by i, is modelled in an overlapping generation economy, in which each of them lives for two periods, dying at the end of the second one. In each period t, each family is composed by a parent and a child and at the end of the second period each individual gives birth to another such that total population is constant over time. In each period the economy is then inhabited by heterogeneous families differentiated by the initial income of the parents, which are distributed according to an initial probability distribution $g_0()$. In the first period of their life, children obtain education, financed out by their parents. Parents have to choose how to split their budget among three goods. A discretionary good c, which gives only a direct private benefit, a participation good z, which furnishes a direct benefit by increasing utility of the parents as well as an indirect benefit, whenever consumed over a threshold $\underline{z} \geq 0$, by augmenting both the parental utility, linearly, and the human capital accumulation function of the children, exponentially, through a stock parameter θ, and finally educational expenditures e. In the second period of their life, depending on the amount of educational investment e and the amount of the participation good consumed by their parents, children accumulate human capital which is one-to-one mapped to income.

3.2 Preference

Individuals have identical preferences, which are defined by

$$U_t^i(c, z, e; \theta) = \alpha \log\left(\hat{c}_t^i\right) + (1-\alpha)\log\left(z_t^i + \theta\left(z_t^i\right)\right) + \gamma \log\left(e_t^i\right) \tag{1}$$

with

$$\theta\left(z_t^i\right) = \begin{cases} 0 & \text{if} \quad z_t^i < \underline{z} \\ 1 & \text{if} \quad z_t^i \geq \underline{z} \end{cases} \tag{2}$$

$$\hat{c}_t^i = \max\left[1, c_t^i\right] \tag{3}$$

with $0 < \alpha < 1$ and $\gamma > 0$ is the degree of altruism of the parents. Condition (2) describes the feature of the participation good z; its consumption over the threshold implies an additional benefit which makes the utility non-homothetic. Condition (3) is a simplifying normalization, needed to make utility function defined when the parents find optimum to bring consumption of the discretionary good, c, at zero.

In the utility in (1) it is explicit that the non-homotheticity of the function is not global but it appears only for a range of high incomes. Given that this non-homotheticity is due to the characteristics of the participation good, it is clearer the idea that closer the consumers are to the income poverty line, which will be determined as function of the threshold $\underline{z}$ of the participation good z, higher are their incentives in changing behaviour towards the purchase of that commodity, as they correctly know the additional benefits it furnishes.

The structure in (1) implies that $\forall c \geq 0, z \geq 0, e \geq 0$, $U(c, z, e; 0)$ and $U(c, z, e; \theta)$ are defined and twice continuously differentiable, strictly increasing and concave in c, z and e. Further, $\forall c \geq 0, z \geq 0, e \geq 0$, $U(c, z, e; 1) > U(c, z, e; 0)$ implies that participation is good. Finally, we make the following key assumption on the behaviour of the agents.

Assumption 1 *For each individual i and each period t,*

$$U(0, \underline{z}, e^*; \theta) > U(c, z, e^*; 0) \tag{4}$$

with

$$e^* = \underset{c\geq 0, z\geq 0, e\geq 0}{\arg\max} \left\{U(c, z, e; \theta) = \alpha \log(\hat{c}) + (1-\alpha)\log(z+\theta) + \gamma \log(e)\right\}$$

subject to

$$c + z + e \leq y$$

$$\theta = 0 \Rightarrow z < \underline{z}$$

and y is the consumer disposable income, over the minimum amount needed for the basic needs.

This is the main assumption needed for describing consumer behaviour and formalizing the definition of poverty we have sketched in the above sections, following which the gain from participation are so large that consumer is perfectly disposed to renounce to consume all the discretionary good c in order to purchase the threshold amount, $\underline{z}$, of the participation good z.

The role of the investments in education, e, must be clarified. In assumption 1 we assume that while for each agent it is perfectly rational to sacrifice the consumption of all the discretionary good, it is not rational and then she will not bring at zero educational investments neither she will decide to decrease its consumption at a level lower than the optimum amount chosen just before reaching the poverty line. This hypothesis on the behaviour of the agent is due to a couple of reasons. Firstly, had we assumed that rational agent might decide to reduce a bit the consumption in educational investment to be able to purchase the participation good, we would very likely lose the discrete jump, which characterizes our definition of poor. More precisely, the poor is different from the rich in that reaching the poverty line means that she becomes able in buying relevant goods *in excess* to the ones she was purchasing just before reaching that poverty line; in this sense, the jump tries to capture the figure by which the agent, once reached the poverty, becomes able to buy goods that before she cannot. The other rationale for this choice is that, as we will show soon, there does exist a complementarity between educational investments and societal participation in the human capital accumulation, such that it is neither worthwhile to bring educational expenditures at zero neither reduce them at a level lower that its optimal amount, only for purchasing an additional commodity, even though this furnishes a great benefit. Indeed, if it would be the case that, when reached the poverty line, the agent find optimum to reduce educational investments, it would imply that the choice made by the agent just before reaching that poverty line was not an optimum. As it will be shown in the next section, this setting produces a discrete jump in the choice of the goods, expressed by a sharp discontinuity in the indirect utility function of the agents. Finally, it results now clearer why we need to impose condition (3) above to make the utility function defined for consumption of the discretionary good c at zero.

3.3 Human Capital Accumulation

Human capital, and hence income, of the children is accumulated in each period and for each family through the following technology

$$h_{t+1}^{i} = \left(e_t^i\right)^{\beta} \left(h_t^i\right)^{\theta\left(z_t^i\right)} \tag{5}$$

where $0 < \beta < 1$.

In each period, parents have a double role in the production of the human capital of their children; through a directly "physical" investment in education and by passing on to the children their human capital background. This latter is augmented, at an increasing rate, by the opportunity they have to participate in social activities at a level higher than the threshold, such to be able to exploit the additional benefit given by the parameter θ. So, while human capital accumulation shows decreasing marginal returns from "physical" education, it has globally increasing marginal returns for high income levels, due to the presence of the complementary factor, h^{θ}. This hypothesis means that, overall, the poor show lower net marginal returns from human capital accumulation than the rich, as they are "excluded" from that kind of social activities, which spread up that accumulation by furnishing relevant information. This information would, for instance, refer to the opportunities opened by studying or yet to the way through which improving the outcomes of educational effort. So a poor family which is only able to afford educational expenditures, but not social activities above the threshold level, will have a decreasing returns to scale technology for production of human capital (i.e. $\theta = 0$), while richer families (i.e. $\theta = 1$) will have an increasing returns to scale technology.

3.4 Production

The aggregate production is very simple. The unique factor of production is human capital, so that

$$Y_t = H_t = \int_I h_t^i g_t\left(h_t^i\right) dh_t^i \tag{6}$$

where H_t is the aggregate stock of human capital at time t, expressed in efficiency units of labour, and $g_t\left(h_t^i\right)$ is the density function describing the distribution of the parent's human capital at time t, with $\int_I g_t\left(h_t^i\right) dh_t^i = 1$. In each period, parents supply inelastically their efficiency units of labour receiving a wage, such that their income is $y_t^i = h_t^i$.

4 Individual behaviour

In this section we compute the individual equilibrium based upon the characteristics of the utility function and the assumptions we made in the previous section.

In each period t, each agent i will face on with the following programme

$$\max U_t^i\left(c_t^i, z_t^i, e_t^i; \theta\right) = \alpha \log\left(\hat{c}_t^i\right) + (1-\alpha)\log\left(z_t^i + \theta\left(z_t^i\right)\right) + \gamma \log\left(e_t^i\right) \tag{7}$$

subject to

$$c_t^i + z_t^i + e_t^i \leq y_t^i \tag{8}$$

$$\theta\left(z_t^i\right) = \begin{cases} 0 & \text{if} \quad z_t^i < \underline{z} \\ 1 & \text{if} \quad z_t^i \geq \underline{z} \end{cases} \tag{9}$$

$$\hat{c}_t^i = \max\left[1, c_t^i\right] \tag{10}$$

$$c_t^i \geq 0, z_t^i \geq 0, e_t^i \geq 0 \tag{11}$$

Parents care of their children in the form of "joy of giving". They do not internalize the overall welfare of their children (Becker, 1974, 1979), in which case they would have maximized over their whole future human capital, h_{t+1}. They, instead, choose how much of their budget in (8) to spend on investment for education of their children. We use this latter approach, since in the former one we would have implicitly assumed parents had a perfectly direct control also on the stock parameter θ, while they do not, as they can only control it indirectly through their disposable income.

4.1 Conditional indirect utility functions

In order to find the consumer optimum choices we need to determine the income threshold, the poverty line, at which the agent does consider rationally optimal to change behaviour by purchasing the amount while keeping on consuming his optimal choice of educational investment. At this end we proceed in two steps, by computing the indirect utility functions *conditioned* on the specific income status, evaluating the optimum for each of them, and finally the general optimal solution.

4.1.1 Poor dynasties

Poor generations starting with a level of income, y_t^i, which does not allow to buy the minimum level of the participation good, $\underline{z}$, to accede to the additional benefits will choose c, z and e such to solve the following programme

$$\max U_t^i\left(c_t^i, z_t^i, e_t^i; \theta = 0\right) = \alpha \log\left(\hat{c}_t^i\right) + (1-\alpha)\log\left(z_t^i\right) + \gamma \log\left(e_t^i\right) \tag{12}$$

subject to

$$c_t^i + z_t^i + e_t^i \leq y_t^i \tag{13}$$

$$\hat{c}_t^i = \max\left[1, c_t^i\right] \tag{14}$$

$$c_t^i \geq 0, z_t^i \geq 0, e_t^i \geq 0 \tag{15}$$

The first order conditions (FOC) are

$$\left(c_t^i\right)_{NP}^* = \frac{\alpha}{1+\gamma} y_t^i, \quad \left(z_t^i\right)_{NP}^* = \frac{1-\alpha}{1+\gamma} y_t^i, \quad \left(e_t^i\right)_{NP}^* = \frac{\gamma}{1+\gamma} y_t^i \tag{16}$$

Consumers do split proportionally their budget across the three goods – i.e. the function is homothetic for this range of income. To this conditions is it associated the following indirect utility function for the poor, that do not participate ("NP"), $V^{NP}(y;\theta=0)$

$$V^{NP}(y;\theta=0) = \alpha \log\left(\frac{\alpha}{1+\gamma} y\right) + (1-\alpha)\left(\frac{1-\alpha}{1+\gamma} y\right) + \gamma \log\left(\frac{\gamma}{1+\gamma} y\right) \tag{17}$$

where "x_{NP}" are the optimum solutions in the poor regime, where the individuals are unable to fully participate in social activities and do not accede to the additional benefit

θ. Marginal utility of each good is infinite at zero and increasing in income; it is further verified that $c_{NP}^{'}(y) > 0, z_{NP}^{'}(y) > 0, e_{NP}^{'}(y) > 0$ and $c_{NP}(y=0) = 0, z_{NP}(y=0) = 0, e_{NP}(y=0) = 0$.

4.1.2 Rich Dynasties

Correspondingly, rich dynasties will solve a programme similar to the one in (12)-(15), but being able to exploit the additional benefit from the participation good; namely,

$$\max U_t^i \left(c_t^i, z_t^i, e_t^i; \theta = 1\right) = \alpha \log\left(\hat{c}_t^i\right) + (1-\alpha)\log\left(z_t^i + 1\right) + \gamma \log\left(e_t^i\right) \tag{18}$$

subject to

$$c_t^i + z_t^i + e_t^i \leq y_t^i \tag{19}$$

$$\hat{c}_t^i = \max\left[1, c_t^i\right] \tag{20}$$

$$c_t^i \geq 0, z_t^i \geq 0, e_t^i \geq 0 \tag{21}$$

where in this case income y is high enough to let the consumer buying each of the three good over the consumption of the amount $\underline{z}$.

The FOC are

$$\left(c_t^i\right)_P^* = \frac{\alpha}{1+\gamma}y_t^i + \frac{\alpha}{1+\gamma}, \quad \left(z_t^i\right)_P^* = \frac{1-\alpha}{1+\gamma}y_t^i - \frac{\alpha+\gamma}{1+\gamma}, \quad \left(e_t^i\right)_P^* = \frac{\gamma}{1+\gamma}y_t^i + \frac{\gamma}{1+\gamma} \tag{22}$$

Obviously, all the three goods are consumed in larger amounts; even the good z, which does appear having a corner solution. Indeed, as these solutions correspond to very rich agents which are consuming over the threshold amount $\underline{z}$, the corner solution implied in (22) does not means that the actual consumption of the good z is at zero, as it is at the threshold level $\underline{z}$. As before, the optimal choices are increasing in income and to them it is associate a correspondingly indirect utility functions for the rich, who are able to fully participate in social activities and then can exploit the additional benefit θ, $V^P(y; \theta = 1)$.

4.1.3 Static equilibrium

Looking at the marginal consumer, we derive the conditions under which a threshold amount of the good z does exist and hence the poverty line - the threshold level of income needed to buy the threshold amount of the participation good - and we prove that assumption 1 actually characterizes the behaviour of the agents.

Proposition 1 (Existence and level of the threshold) *Whenever either the degree of altruism (γ) is not extremely low or the consumption share on the good z is not extremely low or both, a threshold amount of the participation good $z \equiv \underline{z} \in [z^{\circ}, z^{\circ\circ}]$ such that assumption 1 holds and actually describes the behaviour of the agents does exist and it is bounded in the interval $[z^{\circ}, z^{\circ\circ}]$, with $z^{\circ\circ} > z^{\circ}$.*

Proof. See appendix.

Proposition 1 not only does ensure that a threshold which shapes the behaviour of the agents such as initially described in assumption 1 does in fact exist, but it also binds the *level* of the threshold in a closed interval such that $\underline{z} \in [z^{\circ}, z^{\circ\circ}]$. The width of this interval depends in turn on the degree of altruism of the parents (γ) and on the consumption shares on the goods z $(1 - \alpha)$ and c (α); indeed, such a threshold does exist whenever either the degree of altruism (γ) is not extremely low or the consumption share on the good z $(1 - \alpha)$ (c (α)) is not extremely low (high) or both. In the appendix (Figure A.2) we provide simulations from which it results that for a wide and very reasonable range of the parameters γ and α proposition 1 results verified. In a recent study on the customs of the poor in 13 countries Banerjee and Duflo (2007) report that unexpectedly the poor do not spend their whole income in food, while a large share of it is indeed spent on non-food consumption. In particular, they document that participation goods account for a large share of the income of the extremely poor; in most of the surveyed countries spendings on festivals - for instance - account for a share of the income of the extremely poor households higher than the 40%, reaching in many cases percentages ranging between the 70% and the 99% of the total income. Contrary to the conventional wisdom which claims that the poor do not have choices as they are forced to spend all their income in basic goods, Banerjee and Duflo (2007) show that even the extremely poor have a lot of choices which they decide to exercise not in the direction of more basic goods. This conclusion fits perfectly our theoretical strategy of analysing poverty under the individual rational choice of the agents, allowing these to rationally choose whether and how much of the discretionary good either to consume or to renounce. Coupling the individual rationality with the fact that the linkage between the parents utility and the children accumulation of human capital is driven also by the initially relative position of the household in the distribution of income we can explain why even the *existence* and the *level* of the threshold are endogenously determined. A threshold amount of the poverty good (i.e. the participation good z) does indeed not always exist; its existence will depend on the particular relation between the degree of altruism of the parents and the shares of income which they decide to spend on the discretionary and the participation goods; once a threshold exists, its level will be hence bounded in a closed interval since a considerably low threshold would not relevantly influence agents behaviour, while a considerably high threshold would reveal that agents do not forsee the possibility of reaching that threshold and hence they do not take actually into account the existence of such a threshold. Higher the degree of altruism and lower (higher) the share of income spent on the discretionary good (the participation good), larger the likelihood that a threshold does actually exist. Lower is the share of income spent on the discretionary good, more difficult it results for the agents to renounce to such a good, and higher is the degree of altruism, more important it is for the parents the accumulation of the children human capital.

Whenever the agents see the consumption of the participation good as entailing a threshold effect, we can endogenously determine also the threshold level of income - the poverty line - needed to buy this minimum amount of the good z, as

Proposition 2 (Poverty line and optimal choices) $\forall z \equiv \underline{z} \in [z^{\circ}, z^{\circ\circ}]$*, the threshold level of income – the poverty line in terms of the amount* $\underline{z}$ *of the good z, at which she find rationally optimum to sacrifice all the consumption of the discretionary good c to buy the minimum amount of good z,* $\underline{z}$*, while keeping on investing in education at its optimal level* e^*_{NP}*, is* $y_t \equiv \underline{y} = \underline{z}\,(1+\gamma)$*. For level of income lower than* $\underline{y}$ *the optimal decision is given by (16), while for level of income equal to or higher than* $\underline{y}$ *the optimal choice is given by (22).*

Proof. See appendix.

What characterizes the poor is the discrete jump they do at the poverty line. The general optimal solution to the problem (7)-(11) can be formally expressed, for each individuals i and period t, from the triple

$$(c^*, z^*, e^*) = \begin{cases} (c^*_{NP}, z^*_{NP}, e^*_{NP}) & \text{if} \quad y < \underline{y} \\ (c^*_{P}, z^*_{P}, e^*_{P}) & \text{if} \quad y \geq \underline{y} \end{cases} \tag{23}$$

where $\underline{y} \equiv \underline{z}\,(1+\gamma)$.

The condition $y \geq \underline{y}$ might imply that for values of y close to the poverty line, $\underline{y}$, the consumer would restrict the consumption of z at $\underline{z}$. However, even this possibility is excluded when noticing that:

Lemma 1 $\forall y \geq \underline{y}$*,* c_P *and* e_P *are strictly increasing and* z_P *non-decreasing in* y*. Moreover for values of* $z_P > z$*,* z_P *is strictly increasing in* y *as well.*

5 Dynamics and "Inequality Traps"

In each period the links within generations are described by the evolution of the human capital accumulation and hence income distribution dynamics through the intergenerational transmission of educational expenditures and parental human capital background. At any point in time, the current distribution of income (i.e. human capital) shapes the distribution tomorrow. We explore how the presence of "inequality traps" carries over the persistence of poverty by firstly evaluating the dynamics governing individual accumulation of human capital and hence personal income, then analysing their evolution within groups, by restricting the functional form of the income distribution and finally by commenting on the macroeconomic equilibrium. Two main cases will be throughout distinguished, depending on whether the initial level of poverty is high or low.

5.1 Individual Dynamics

From the static equilibrium above, we know that in each period t parents pass on their children the following optimal educational investments

$$\left(e_t^i\right)^* \equiv \begin{cases} \left(e_t^i\right)^*_{NP} = \dfrac{\gamma}{1+\gamma} h_t^i & \text{if} \quad h_t^i < \underline{h} \\ \\ \left(e_t^i\right)^*_{P} = \dfrac{\gamma}{1+\gamma} h_t^i + \dfrac{\gamma}{1+\gamma} & \text{if} \quad h_t^i \geq \underline{h} \end{cases} \tag{24}$$

with $\underline{h} \equiv \underline{y} = \underline{z}\,(1+\gamma)$ the poverty line expressed in terms of the good z.

The transition equation of the personal human capital accumulation of their children is determined by (5) as

$$\left(h_{t+1}^i\right) = \phi\left(h_t^i\right) \equiv \begin{cases} \phi_{NP}\left(h_t^i\right) = \left(\dfrac{\gamma}{1+\gamma}\right)^{\beta} \left(h_t^i\right)^{\beta} & \text{if} \quad h_t^i < \underline{h} \\ \\ \phi_{P}\left(h_t^i\right) = \left(\dfrac{\gamma}{1+\gamma}\right)^{\beta} \left(h_t^i+1\right)^{\beta} h_t^i & \text{if} \quad h_t^i \geq \underline{h} \end{cases} \tag{25}$$

The transition equation presents a jump in its curvature corresponding to the discrete jump in the indirect utility function of the individuals we took as our definition of poverty. In particular, while in the first part of its support it shows decreasing returns to scale (i.e. concave), in the range of higher incomes it is convex with increasing returns to human capital accumulation. This feature captures the idea that rich families are in better position – have better opportunities – to better exploit the gains from education as they are able to have additional benefits in terms of relevant information acquisition which in turn do speed up the overall human capital accumulation.

In order to analyse these dynamics let's notice that for each individual i, the followings are verified

Lemma 2 *$h_{t+1} = \phi_{NP}\left(h_t\right) = \left(\frac{\gamma}{1+\gamma}\right)^{\beta} \left(h_t\right)^{\beta}$ is defined only for $h_t \in [0, \underline{h})$. Over this support it is strictly increasing and strictly concave; moreover, $\phi_{NP}(0) = 0$ and $\lim_{h_t \to 0} \phi'_{NP}\left(h_t\right) = \infty$ imply that that whenever it intersects the 45-degree line it will do it from above. This implies that whenever a steady state does exists it is locally stable, since $\phi'_{NP}\left(h_L^*\right) < 1$, where this low-steady state h_L^* is defined as $h_L^* = \left(\frac{\gamma}{1+\gamma}\right)^{(\beta/1-\beta)}$.*

Lemma 3 *$h_{t+1} = \phi_P\left(h_t\right) = \left(\frac{\gamma}{1+\gamma}\right)^{\beta} \left(h_t+1\right)^{\beta} h_t$ is defined over the support for $h_t \in [\underline{h}, \infty)$ and is strictly increasing and strictly convex. The high steady state, whenever it exists, is then locally unstable, since $\phi'_P\left(h_H^*\right) > 1$, with h_H^* defined by $h_H^* = \frac{1}{\gamma}$.*

These two lemma allow to completely define the individual dynamics, distinguishing two key cases, depending on whether the initial level of poverty is either low (poverty regime) or high (rich regime).

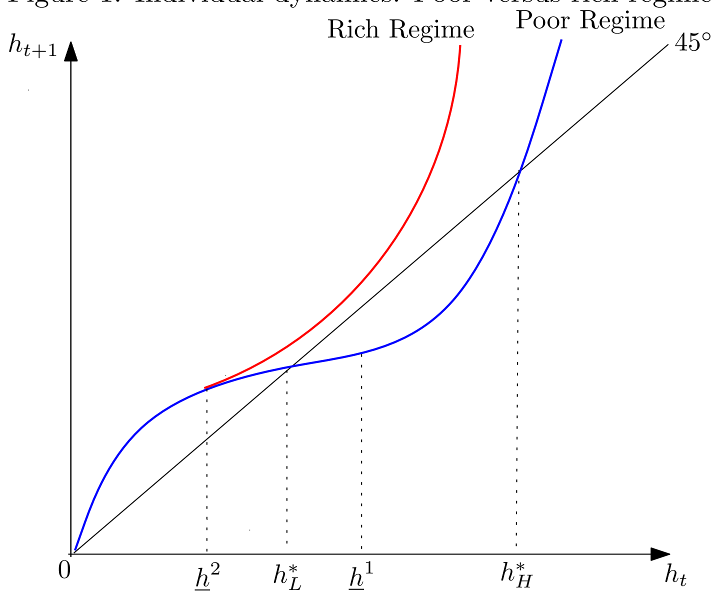
Proposition 3 (Poverty Regime) *If $\underline{h} > h_L^*$, that is if the level of poverty is initially high, two equilibria exist; a low locally stable $\left(h_L^*\right)$ and an upper locally unstable $\left(h_H^*\right)$. Poor* *and middle-income dynasties are condemned to end up in the lower stable equilibrium – h_L^*, while the richer ones, with initial income equal to or above h_H^*, will enter an explosive path, with unbounded income growth.*

Proof. If $\underline{h} > h_L^*$, the transition equation $\phi_{NP}(h_t)$ intersects the 45-degree line at h_L^* from above, implying stability of equilibrium. Correspondingly, under assumption 2, the transition equation $\phi_P(h_t)$ it will strictly increase starting from $\underline{z}$, intersecting the 45-degree line from below at h_H^*, implying both unstable equilibrium – h_H^* – and explosive path above this equilibrium.■

Proposition 4 (Rich Regime) *If $\underline{h} < h_L^*$ that is if the level of poverty is initially low, no equilibria exist. The whole population will enter the explosive path, with unbounded growth.*

In this case, indeed, even the poorer part of the distribution is able to buy the minimum amount of the participation good such that their children may accumulate human capital following $h_{t+1} = \phi_P(h_t)$ in (25) (figure 1).

Figure 1: Individual dynamics. Poor versus rich regime



If the level of societal poverty is quite high, $\underline{h} \equiv \underline{h}^1 > h_L^*$, two classes of equilibrium are possible. Dynasties starting with an income h lower than the low-equilibrium converge to it as well as middle-income dynasties that start with a level of income h higher than h_L^* but not high enough to sustain a positive consumption of both the educational investments and participation goods. Indeed, while very rich families, with income $h > h_H^*$, will grow unboundedly over an explosive path, middle-income or vulnerable dynasties, $h \in (\underline{h}^1, h_H^*)$, that are above the poverty line, but very close to it, are condemned to end up on the lower steady state. On the other side, in an economy starting very rich, $\underline{h} \equiv \underline{h}^2 < h_L^*$, dynasties evolve along the explosive path, with "globally" unbounded growth rates.

These results belong to the tradition of the Stiglitz's (1969) ones, following which economies starting very poor are characterised by multiple equilibria, with the lower part of the distribution showing a continuous worsening with respect to the upper part of the distribution. This consideration brings about another direct one. It is often assumed that reduction in inequality may be a powerful tool for reducing poverty especially in low poverty environments. Otherwise, we have shown that it is not the case, as the higher the initial level of poverty the higher the power of egalitarian policies in reducing poverty, improving aggregate economic efficiency and hence increasing aggregate output. In turn these results do appear to confirm the claim and the needs of a recent part of the literature focusing on pro-poor growth policies.

5.2 Inequality Traps and Intra-group dynamics

The low-equilibrium arising in the poor regime is closer to the notion of "inequality traps" than to the more classic poverty traps. As explained in the introduction, with the term inequality trap (Bourguignon, Ferreira and Walton, 2007) the literature is attempting to pointing out the importance of looking at the whole distribution rather than only at specific kind of individuals – poor versus rich. A specific characteristic of the inequality traps is the permanent non-convergence in the opportunities opened to some social groups, verified also, but not only, by the existence of multiple limiting distributions. Although this new branch of the literature has not fully explored the notion of inequality trap, a feature which differentiates it from the poverty traps is that "there must be persistence in relative positions in a distribution across time periods, and that this be (partly) a product of features of the overall distribution – or of relations between groups" (Bourguignon, Ferreira and Walton, 2007). A consequence is that this persistence often does lead to efficiency losses, "resulting in an economic equilibrium that is inferior to some feasible alternatives" (Bourguignon, Ferreira and Walton, 2007), as it does happen in our case, where the poor regime is inferior to the other equilibrium – the rich regime, which would be feasible by reducing inequality and allowing the poorer part of the distribution in participating in social activities in the relevant range.

In order to preliminarily catch this point, let's consider what would happen by increasing the level of income of the poor and middle classes or, equivalently, by reducing the poverty line, which would correspond to reduce the price at which the participation good might be bought. The economy might enter the rich regime, under which every individual enter the explosive path with unbounded growth rates. Graphically (figure 1), let's suppose that at time $t = 0$ the economy is trapped in a poor regime, with a level of poverty determined by the poverty line $\underline{h}^1 \equiv \underline{z}\,(1+\gamma)$. Either redistributing income from the upper classes to lower and middle ones, which would imply increasing the individual level of income of the latter classes and then a shifting up of the lower equilibrium, h_L^*, or by reducing the price, $\underline{z}$, at which it is possible to buy the societal participation or both, the economy would end up in the rich regime, with no poverty and higher aggregate output.

In a poor and polarized society, redistribution of income from the upper to the lower-middle classes as well as making the access to the societal participation less stringent would allow the entire economy to both grow richer and to substantially reduce individual poverty. That is, we would have got another feasible equilibrium which is superior to the former one. At this regard and as shown below at aggregate level, the efficiency losses in the poor regime stem from the fact that: 1) the aggregate output is lower under poor regime than under the rich one; 2) this is in turn due to the fact that in the poor regime not only the poor class is unable to accumulate the same amount of human capital accumulated by the richer families, but also the middle-income it is not, even though it has the same access to the participation good. Indeed, the vulnerable class ends up in the lower equilibrium even if it is able to spend how the richer class for educational investments. At aggregate level, this wastage of economic resources produces the above economic inefficiency. Reducing inequality, in this case, would generate both the reduction of poverty and the macroeconomic efficiency.

In order to deepen the analysis of the group-specific income dynamics, we proceed by restricting the functional form of the initial density function to be lognormal (Glomm and Ravikumar, 1992; Gradstein and Justman, 1997). We can show the following

Proposition 5 (Intra-group dynamics) *If the initial distribution of human capital (i.e. income) is lognormally distributed, the mean and the variance of the richer part of the distribution are always higher than that of the poorer one. While inequality of the poor group is strictly decreasing over time, tending to zero in the very long-run (i.e. infinity), it is strictly increasing over time, at a rate greater than 1, among the richer part of distribution.*

Proof.

Let's suppose that at time $t = 0$, human capital (i.e. income) is lognormally distributed with mean μ_0 and variance σ_0^2. Namely, $h_0^i \sim LN\left(\mu_0, \sigma_0^2\right)$ which also implies that the logarithm of the human capital is normally distributed with same mean and same variance, $\log\left(h_0^i\right) \sim N\left(\mu_0, \sigma_0^2\right)$. It is hence possible to compute the dynamics of the mean and the variance *within* the groups, i.e. poor and rich. From (25) let's loglinearize the transition equations for the two groups, as follow

$$\log\left(h_{NP,t+1}^i\right) = \beta \log\left(\frac{\gamma}{1+\gamma}\right) + \beta \log\left(h_{NP,t}^i\right) \tag{26}$$

and

$$\log\left(h_{P,t+1}^i\right) = \beta \log\left(\frac{\gamma}{1+\gamma}\right) + \beta \log\left(h_{P,t}^i + 1\right) + \log\left(h_{P,t}^i\right) \tag{27}$$

In this latter case (27) it would be not possible to compute mean and variance of $\log\left(h_{P,t+1}^i\right)$ due to the presence of the term $\log\left(h_{P,t}^i + 1\right)$, since the expected value of this latter does not coincide with the expected value of $\log\left(h_{P,t}^i\right)$, and we would be neither sure that there is a specific distribution describing it. Nonetheless, we can approximate, by first-order Taylor expansion, the term to get a tractable functional form. In particular, approximating (see appendix)

$$\log\left(h^i_{P,t}+1\right) \approx \log\left(h^i_{P,t}\right)$$

we can rewrite (27) as follow

$$\log\left(h^i_{P,t+1}\right) = \beta \log\left(\frac{\gamma}{1+\gamma}\right) + (1+\beta)\log\left(h^i_{P,t}\right) \tag{28}$$

Hence, mean and variance of the $\log\left(h^i_{X,t+1}\right)$, with $X = (NP, P)$, are given for the poor by

$$\mu^{NP}_{t+1} = E\left(\log\left(h^i_{NP,t+1}\right)\right) = \beta \log\left(\frac{\gamma}{1+\gamma}\right) + \beta\mu^{NP}_t \tag{29}$$

$$\sigma^2_{NP,t+1} = Var\left(\log\left(h^i_{NP,t+1}\right)\right) = \beta^2\sigma^2_{NP,t} \tag{30}$$

and for the rich by

$$\mu^{P}_{t+1} = E\left(\log\left(h^i_{P,t+1}\right)\right) = \beta \log\left(\frac{\gamma}{1+\gamma}\right) + (1+\beta)\,\mu^{P}_t \tag{31}$$

$$\sigma^2_{P,t+1} = Var\left(\log\left(h^i_{P,t+1}\right)\right) = \left(1+\beta^2\right)\sigma^2_{P,t} \tag{32}$$

■

Mean and variance for the variable h would be easily computed from (29)-(32), without changing the qualitative analysis (see appendix). Mean income as well as inequality of the poor group evolves being always lower than those of the richer part of the distribution. Moreover, while *within* the former group inequality is decreasing over time tending at zero in the very long-run, *within* the richer part of the distribution inequality is strictly increasing over time, at a rate $(1+\beta)^2$ greater than 1. These results couple the ones obtained in the more general setting above, showing a channel through which polarization and marginalization are actually possibilities in very poor societies. This effect might in turn explain both the continuous marginalization observed in less developed as well as in developed countries and the demise of the middle class observed especially in the latter case. This is mainly due to the fact that polarization and marginalization do fuel the inequality traps, excluding a part of the population from fully exploiting the available opportunities.

5.3 Macroeconomic equilibrium and aggregate economic inefficiencies

Finally, at aggregate level, we can see that the above results are consistent with the macroeconomic equilibrium, which is characterised on a side by the negative correlation between inequality and economic growth and on the other by aggregate economic inefficiency, in the poor regime, due to the wastage of economic resources. This latter economic inefficiency might be removed by reducing inequality, which would produce both the poverty reduction and the increase in the overall output. In each period, indeed, the current income distribution, today, is determined by the previous one, yesterday.

In the case of an initially poor economy, $\underline{h} \equiv \underline{h}^1 > h_L^*$, the evolution of aggregate educational investments and incomes is given by

$$E_t^{poor} = \frac{\gamma}{1+\gamma} \int_0^\infty h_t^i g\left(h_t^i\right) dh_t^i + \frac{\gamma}{1+\gamma}\left[1 - G_t\left(\underline{h}^1\right)\right] \tag{33}$$

where $G_t\left(\underline{h}^1\right) = \int_0^{\underline{h}} g_t\left(h_t^i\right) dh_t^i$ is the cumulative distribution function at $\underline{z}$.

$$Y_t^{poor} = \left(\frac{\gamma}{1+\gamma}\right)^\beta \int_0^{h_H^*} \left(h_{t-1}^i\right)^\beta g_{t-1}\left(h_{t-1}^i\right) dh_{t-1}^i + \\ + \left(\frac{\gamma}{1+\gamma}\right)^\beta \int_{h_H^*}^\infty \left(h_{t-1}^i + 1\right)^\beta \left(h_{t-1}^i\right) g_{t-1}\left(h_{t-1}^i\right) dh_{t-1}^i \tag{34}$$

Firstly, the negative relation between inequality and economic growth stems from considering that in (34) the final aggregate output would be higher by allowing more people to reach the level of steady state h_H^*. Starting from this point, indeed, the economy grows at a higher rate than below it.

In order to evaluate more rigorously the loss in economic efficiency due to the inequality traps, let's consider what follows:

1. Poor class: the proportion of people owning an income $h_t \in \left[0, \underline{h}^1\right]$ makes educational investments following $e_{NP}(.)$ in (24) and ends up in the low equilibrium in h_L^*, defined from $\phi_{NP}(.)$ in (25);

2. Middle-"Vulnerable" class: the proportion of people owning initially an income $h_t \in \left[\underline{h}^1, h_H^*\right]$ can spend and indeed they spend in educational investments the higher amount $e_P(.)$, but they are still condemned in the low-equilibrium h_L^*;

3. Rich class: the proportion of people owning, finally, an initially income $h_t \in [h_H^*, \infty)$ even spending for educational investment in the same proportion as the middle class, end up on the explosive path, growing without bounds.

The wastage of economic resources for the economy as a whole is quantifiable from (33) and (34) in the amount

$$\frac{\gamma}{1+\gamma} G_t^M\left(h_t^M\right)$$

with $G_t^M\left(h_t^M\right) = \int_{\underline{h}^1}^{h_H^*} g_t\left(h_t^i\right) dh_t^i$.

This wastage, due to the inequality traps, is persistent over time and is the source of the economic inefficiency, which might be removed by reducing initial inequality. A reduction in inequality would, indeed, allow an higher proportion of the population to be able not only to spend an higher amount for educational expenditure, but also to access the societal participation at the relevant level, such to produce contemporaneously a reduction of poverty and an increase in the aggregate output.

6 Conclusion

In this paper we evaluate how the persistence of inequality and the presence of inequality traps carry over the persistence of poverty and possible aggregate economic inefficiencies. We propose a microeconomic formalization of one possible definition of poverty and of the behaviour of poor and rich agents. Poverty, net of minimum basic needs, is defined as lack of or low societal participation, which is source of both a direct private benefit which augments the utility of the parents linearly and an indirect gain, taking the form of a factor increasing the human capital accumulation of the children, exponentially. This additional benefit is so large that agents are well willing in renouncing to the consumption of a discretionary good in order to purchase the participation good. In order to achieve this additional gain, a specific minimum amount of the participation good must be purchased. We hence introduce poverty ideas in the individual rationality of the agents, defining the movements within and from poverty status as a discrete jump in the indirect utility function to which corresponds a jump in the curvature of the human capital production function. This is crucial not only for the endogeneization of the threshold amount of the participation good needed to join an additional indirect benefit as well as the threshold level of income - the poverty line - needed to buy that amount, but also for the endogeneization of the conditions which guarantee their *existence* and which fix their *level*. We assemble an overlapping generation structure, in which in each period parent choose among three goods; a discretionary one, the participation good, and educational expenditures of their children. Human capital is the only factor of production which is then mapped one-to-one to individual income. We are able to couple and enrich the dynamics *a là* Galor and Zeira (1993) in an economic environment with perfect capital markets, without needing the assumption of market imperfections, by introducing a methodological novelty which connects the presence of the externality in the production function of the human capital of the children to the preference of the parents.

We show that two regimes may be distinguished; a poor and a rich regime. In economies starting largely poor, the poor regime, two equilibria exists; one low locally stable equilibrium and one upper locally unstable. Individuals whose income is under the poverty line converge to the low equilibrium as well as the vulnerable individuals of the middle class, defined by the proportion of individuals owning an income higher than the poverty line, but lower than the upper unstable equilibrium. Richer individuals, whose initial income is equal to or higher than the upper equilibrium end up on an explosive path, with unbounded growth rates of personal income. In the rich regime, instead, no steady state equilibrium does exist, as the whole population enter the explosive path, with unbounded growth rates. We further evaluate the intra-group dynamics, by restricting the functional form of the initial income distribution to be lognormal. It is shown that the mean income and the variance (i.e. inequality) of the richer part of the distribution are always higher than the ones of the lower part; moreover, while inequality in the poor class tend to zero in the very long-run (i.e. infinity), in the richer class of income inequality is increasing at an increasing rate. These dynamics do appear to firstly suggest a positive support for the current debate on the demise of the middle class and the continuous polarization of the modern society. Further, the low equilibrium of the poor regime is closer to the notion of inequality traps rather than poverty traps, in that the persistence of poverty is caused by the continuously relative disadvantageous positions of the lower classes of income in exploiting the benefits of societal participation. We find theoretical support for sustaining that policies aimed in reducing inequality are more powerful tools for poverty reduction when initially economies are very poor, contrary to an established claim, following which reduction in inequality induce higher decreasing in poverty in initially richer economies. On this ground, this paper tries to start the microeconomic and macroeconomic formalization of the so called pro-poor growth theory, firstly developed in policy institutes. At macroeconomic level, two conclusions may be furnished. Firstly, a negative relation between initial inequality and economic growth is observed, since by reducing inequality in poor regime economies, the whole population, or higher proportions of it, is able to enter the explosive path, producing a greater aggregate output as well. Finally, the inequality traps which cause the poor regime to emerge are also sources of aggregate economic inefficiencies, which can be eliminated by reducing income disparities accordingly. Indeed, economies trapped in the poor regime show that: 1) poor families, unable to purchase the minimum amount of participation good, end up in the low equilibrium; 2) middle-income families converge to the same equilibrium, even though they are able to purchase that good, but they have not enough initial income to reach the upper equilibrium; 3) richer families, able to purchase the "right" amount of all the bundle of goods grow richer. At aggregate level, this implies that there is a wastage of economic resources, as although the middle income class spends for accumulating human capital quite a lot, the final aggregate output does not increase accordingly, as people in this class are still marginalised with respect to an upper elite.

Appendix

Proof of Proposition 1 and Proposition 2

The proofs proceed in three steps and they apply for each individuals i and each period t.

Step 1

Let's consider the marginal consumer which owns the exact amount of income, in terms of the amount $\underline{z}$ of good z, needed to buy both $\underline{z}$ and e^*_{NP}. The indirect utility function V^P is defined only when the minimum level $\underline{z}$ of good z is purchased; namely, $\forall z \geq \underline{z}$. Given that $e^*_{NP} = \frac{\gamma}{1+\gamma}y$, this level y^1 is $y \equiv y^1 = \underline{z} + \frac{\gamma}{1+\gamma}y \Rightarrow y^1 = \underline{z}(1+\gamma)$. In order to be optimum for her to buy this bundle of goods instead of splitting the same amount of income over the three goods, it must be verified that

$$V^P\left(y^1; \theta \mid \theta = 1\right) > V^{NP}\left(y^1; \theta \mid \theta = 0\right) \tag{A.1}$$

That is, it must be verified that for the same level of income y^1 the indirect utility, V^P, obtained by consuming the minimum amount to accede to the extra-benefit from participation $\underline{z}$ must be greater than the one, V^{NP}, obtained by splitting it across the three goods, not purchasing that threshold amount and then not obtaining those extra-benefits. That is

$$V^P\left(y^1; 1\right) \equiv (1-\alpha)\log\left(\underline{z}+1\right) + \gamma\log\left(\frac{\gamma}{1+\gamma}y^1\right) > \alpha\log\left(\frac{\alpha}{1+\gamma}y^1\right) + \\ + (1-\alpha)\log\left(\frac{1-\alpha}{1+\gamma}y^1\right) + \gamma\log\left(\frac{\gamma}{1+\gamma}y^1\right) \equiv V^{NP}\left(y^1; 0\right) \tag{A.2}$$

Hence,

$$(1-\alpha)\log\left(\underline{z}+1\right) > \alpha\log\left(\alpha\underline{z}\right) + (1-\alpha)\log\left((1-\alpha)\underline{z}\right) \Rightarrow$$

by simplifying and taking the inverse function (i.e. exponential)

$$\Rightarrow \underline{z}+1 > \left(\alpha\underline{z}\right)^{(\alpha/1-\alpha)}(1-\alpha)\underline{z} \Rightarrow \underline{z}+1 > B\underline{z}^{(\alpha/1-\alpha)}\underline{z}$$

with $B \equiv \alpha^{(\alpha/1-\alpha)}(1-\alpha) = \left[\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}\right]^{1/1-\alpha}$; and $0 < B < 1$.

$$\Rightarrow \underline{z}+1 > B\underline{z}^{(1/1-\alpha)} \tag{A.3}$$

Step 2

It must be proved furthermore that for levels of income lower than y^1 that allow to buy the threshold amount $\underline{z}$ of good z, the consumer does not find optimum purchasing that minimum amount by not only sacrificing the entire consumption of the discretionary good c, but also by reducing the consumption of educational expenditures, e.

Let's suppose to assign to the consumer a level of income y^2 lower than y^1.
Given that $\gamma > 0$ only, let's define this level of income $y^2 = \underline{z}$. It must be the case that

$$V^P\left(y^2;\theta \mid \theta = 1\right) < V^{NP}\left(y^2;\theta \mid \theta = 0\right) \tag{A.4}$$

This would prove that the indirect utility obtained by buying only the amount $\underline{z}$ of good z and obtaining the additional benefit is lower than the indirect utility obtained by spending the same budget over the three goods and not achieving the benefits from participation.

$$\begin{aligned} V^P\left(y^2;\theta \mid \theta = 1\right) \equiv (1-\alpha)\log\left(\underline{z}+1\right) < \alpha \log\left(\frac{\alpha}{1+\gamma}y^2\right) + \\ + (1-\alpha)\log\left(\frac{1-\alpha}{1+\gamma}y^2\right) + \gamma\log\left(\frac{\gamma}{1+\gamma}y^2\right) \equiv V^{NP}\left(y^2;\theta \mid \theta = 0\right) \end{aligned} \tag{A.5}$$

Hence,

$$\log\left(\underline{z}+1\right) < \frac{\alpha}{1-\alpha}\log\left(\frac{\alpha}{1+\gamma}\underline{z}\right) + \log\left(\frac{1-\alpha}{1+\gamma}\underline{z}\right) + \frac{\gamma}{1-\alpha}\log\left(\frac{\gamma}{1+\gamma}\underline{z}\right) \Rightarrow$$

$$\Rightarrow \underline{z}+1 < \left(\frac{\alpha}{1+\gamma}\underline{z}\right)^{(\alpha/1-\alpha)}\left(\frac{1-\alpha}{1+\gamma}\underline{z}\right)\left(\frac{\gamma}{1+\gamma}\underline{z}\right)^{(\gamma/1-\alpha)} \Rightarrow$$

$$\Rightarrow \underline{z}+1 < \left(\frac{\alpha}{1+\gamma}\right)^{(\alpha/1-\alpha)}\left(\frac{1-\alpha}{1+\gamma}\right)\left(\frac{\gamma}{1+\gamma}\right)^{(\gamma/1-\alpha)} \cdot \underline{z}^{(\alpha/1-\alpha)} \cdot \underline{z} \cdot \underline{z}^{(\gamma/1-\alpha)} \Rightarrow$$

$$\underline{z}+1 < C \cdot \underline{z}^{((1+\gamma)/(1-\alpha))} \tag{A.6}$$

with $C \equiv \dfrac{\alpha^{(\alpha/1-\alpha)} \cdot (1-\alpha) \cdot \gamma^{(\gamma/1-\alpha)}}{(1+\gamma)^{((1+\gamma)/(1-\alpha))}} = \left[\dfrac{\alpha^\alpha (1-\alpha)^{(1-\alpha)}\gamma^\gamma}{(1+\gamma)^{1+\gamma}}\right]^{1/1-\alpha} > 0$ and $C < B$.

Step 3. Existence and level of the threshold

In order to completely prove the proposition 1, it must showed that there exists a value $\underline{z}$ such that (A.1) and (A.4) are *contemporaneously* verified. This amounts to prove that inequalities in (A.3) and in (A.6) are *contemporaneously* verified as well. Let's define three functions as:

$$f_1\left(\underline{z}\right) = \underline{z}+1$$

$$f_2\left(\underline{z}\right) = B\underline{z}^{(1/1-\alpha)}$$

$$f_3(\underline{z}) = C \cdot \underline{z}^{((1+\gamma)/(1-\alpha))}$$

It must be the case that

$$f_2(\underline{z}) < f_1(\underline{z}) < f_3(\underline{z}) \tag{A.7}$$

Let's consider the following properties of the functions $f_2(.)$ and $f_3(.)$:

- $f_2(0) = f_3(0) = 0$;
- The two functions are strictly increasing in z and strictly convex, namely: $\forall z > 0, f_2^{'}(.) > 0, f_2^{''}(.) > 0, f_3^{'}(.) > 0, f_3^{''}(.) > 0$;
- The slope of the function $f_2(.)$ is greater than the slope of the function $f_3(.)$, namely: $B > C$;
- The curvature of the function $f_3(.)$ is greater than the curvature of the function $f_2(.)$, namely: $\dfrac{1+\gamma}{1-\alpha} > \dfrac{1}{1-\alpha}$.

From these properties it results that $\exists z \equiv z^{\dagger} : f_2(z^{\dagger}) = f_3(z^{\dagger})$; moreover $\forall z < z^{\dagger}, f_2(z) > f_3(z)$ and $\forall z > z^{\dagger}, f_2(z) < f_3(z)$.

In particular, whenever either the degree of altruism (γ) is not extremely low or the consumption share on the good z is not extremely low or both, the point $z \equiv z^{\dagger}$ is such that $f_2(z^{\dagger}) = f_3(z^{\dagger}) < f_1(z^{\dagger})$. It follows that a threshold $z \equiv \underline{z} \in [z^{\circ}, z^{\circ\circ}]$ does exist, where z° is such that $f_3(z^{\circ}) = f_1(z^{\circ})$ and $z^{\circ\circ} > z^{\circ}$ is such that $f_2(z^{\circ\circ}) = f_1(z^{\circ\circ})$.

Let's consider the point $z \equiv z^{\dagger}$ at which the functions $f_2(.)$ and $f_3(.)$ equal each other, namely:

$$f_2\left(z^{\dagger}\right) \equiv B z^{\dagger^{(1/1-\alpha)}} = C z^{\dagger^{(1+\gamma/1-\alpha)}} \equiv f_3\left(z^{\dagger}\right)$$

after simple arithmetics such a point is defined as

$$z^{\dagger} = \frac{(1+\gamma)^{(1+\gamma/\gamma)}}{\gamma}$$

The existence of the threshold is guaranteed whenever the value of the functions $f_2(z^{\dagger})$ and $f_3(z^{\dagger})$ is lower than the value of the function $f_1(z^{\dagger})$ at the point $z^{\dagger}$ where the functions $f_2(z^{\dagger})$ and $f_3(z^{\dagger})$ do intersect each other - i.e. at the point where the functions $f_2(z^{\dagger})$ and $f_3(z^{\dagger})$ are equal. For this condition to be verified, it must hence be the case that

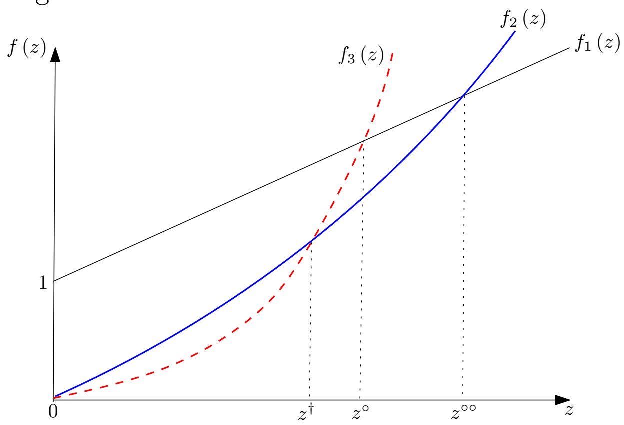
$$f_2\left(z^{\dagger}\right) = f_3\left(z^{\dagger}\right) < f_1\left(z^{\dagger}\right) \tag{A.8}$$

By substituting back the value of the point $z^{\dagger}$ in either $f_2(z)$ or $f_3(z)$ and in $f_1(z)$, it follows that inequality in (A.8) is verified whenever

$$B\left[\frac{(1+\gamma)^{(1+\gamma/\gamma)}}{\gamma}\right]^{1/1-\alpha} < \frac{(1+\gamma)^{(1+\gamma/\gamma)}}{\gamma}+1 \quad \text{(A.9)}$$

which is verified whenever either the degree of altruism (γ) is not extremely low or the consumption share on the good z is not extremely low or both. Whenever condition (A.9) is verified, it does exist a value $z \equiv \underline{z} \in [z^{\circ}, z^{\circ\circ}]$ such that inequality (A.7) and hence proposition 1 are both verified. This can be shown graphically as follow

Figure A.1: Existence and level of the threshold



Finally, it suffices to note that from the properties of these indirect utility functions (see also the text):

Given that for $y \geq \underline{y}$ $V^{P}(y;1) > V^{NP}(y;0)$ it implies that for all levels of income $y^{'} > y$ it will be also the case that $V^{P}\left(y^{'};1\right) > V^{NP}\left(y^{'};0\right)$. This property implies that whenever she can she always decide to buy the participation good. ■

Proofs for Lemma 2

The properties of the function $h_{t+1} = \phi_{NP}(h_t) = \left(\frac{\gamma}{1+\gamma}\right)^{\beta} h_t^{\beta}$ are easily derived from its first and second derivates.

$$\phi_{NP}^{'}(h_t) = \beta\left(\frac{\gamma}{1+\gamma}\right)^{\beta} h_{t-1}^{\beta} > 0$$

$$\phi_{NP}^{''}(h_t) = \beta(\beta-1)\left(\frac{\gamma}{1+\gamma}\right)^{\beta} h_t^{\beta-2} < 0$$

The steady state h_L^{*} is derived from

$$h_L^{*} = \phi_{NP}(h_L^{*}) = \left(\frac{\gamma}{1+\gamma}\right)^{\beta}(h_L^{*})^{\beta} \Rightarrow$$

$$h_L^* = \left(\frac{\gamma}{1+\gamma}\right)^{\beta/1-\beta}$$

and the stability conditions from

$$\phi_{NP}'(h_L^*) = \beta\left(\frac{\gamma}{1+\gamma}\right)^{\beta}(h_L^*)^{\beta-1} =$$
$$= \beta\left(\frac{\gamma}{1+\gamma}\right)^{\beta}\left[\left(\frac{\gamma}{1+\gamma}\right)^{\beta/1-\beta}\right]^{\beta-1} =$$
$$= \beta\left(\frac{\gamma}{1+\gamma}\right)^{\beta}\left(\frac{\gamma}{1+\gamma}\right)^{(\beta(\beta-1)/1-\beta)} = \beta < 1$$

Proofs for Lemma 3

The properties of the transition function $h_{t+1} = \phi_P(h_t) = \left(\frac{\gamma}{1+\gamma}\right)^{\beta}(h_t+1)^{\beta} h_t$ are derived from its first and second derivatives.

$$\phi_P'(h_t) = \left(\frac{\gamma}{1+\gamma}\right)\left[(h_t+1)^{\beta} + \beta h_t (h_t+1)^{\beta-1}\right] > 0$$

$$\phi_P''(h_t) = \left(\frac{\gamma}{1+\gamma}\right)\left[\beta(h_t+1)^{\beta-1} + \beta(h_t+1)^{\beta-1} + \beta h_t(\beta-1)(h_t+1)^{\beta-2}\right] =$$
$$= \left(\frac{\gamma}{1+\gamma}\right)^{\beta}\beta\left[2(h_t+1)^{\beta-1} - (1-\beta)h_t(h_t+1)^{\beta-2}\right] > 0$$

since $2(h_t+1)^{\beta-1} > (1-\beta)h_t(h_t+1)^{\beta-2} \Rightarrow 2(h_t+1) > (1-\beta)h_t \Rightarrow h_t(1+\beta)+2 > 0$, which is always verified.

The steady state h_H^* is derived from

$$h_H^* = \phi_P(h_H^*) = \left(\frac{\gamma}{1+\gamma}\right)^{\beta}(h_H^*+1)^{\beta} h_H^* \Rightarrow$$
$$\Rightarrow 1 = \left(\frac{\gamma}{1+\gamma}\right)^{\beta}(h_H^*+1)^{\beta} \Rightarrow h_H^*+1 = \left(\frac{1+\gamma}{\gamma}\right) \Rightarrow$$
$$\Rightarrow h_H^* = \frac{1}{\gamma}$$

and the stability condition from

$$\phi_P^{'}\left(h_H^*\right)=\left(\frac{\gamma}{1+\gamma}\right)^{\beta}\times$$
$$\times\left[\left(\frac{1+\gamma}{\gamma}\right)^{\beta}+\beta\left(\frac{1+\gamma}{\gamma}\right)^{\beta-1}\cdot\left(\frac{1+\gamma}{\gamma}-1\right)\right]=$$
$$=\left(\frac{\gamma}{1+\gamma}\right)^{\beta}\left[\left(\frac{1+\gamma}{\gamma}\right)^{\beta}+\beta\left(\frac{1+\gamma}{\gamma}\right)^{\beta}-\beta\left(\frac{1+\gamma}{\gamma}\right)^{\beta-1}\right]=$$
$$=1+\beta-\beta\left(\frac{1+\gamma}{\gamma}\right)^{-1}\Rightarrow$$

$$\Rightarrow\phi_P^{'}\left(h_H^*\right)=1+\beta\left[1-\left(\frac{1+\gamma}{\gamma}\right)^{-1}\right]>1$$

Lognormal distribution properties

It is shown (Aitchison and Brown, 1957; De La Croix and Michel, 2002; Glomm and Ravikumar, 1992; Gradstein and Justman, 1997) that if a variable h is lognormally distributed with mean μ_0 and variance σ_0^2, namely if

$$h^i\sim LN\left(\mu,\sigma^2\right)$$

which also implies that

$$\log\left(h^i\right)\sim N\left(\mu,\sigma^2\right)$$

then, the mean and the variance of the variable h are given by

$$E\left(h^i\right)=\exp\left(\mu+\frac{\sigma^2}{2}\right)$$

$$Var\left(h^i\right)=\left[\exp\left(\sigma^2\right)-1\right]\exp\left(2\mu+\sigma^2\right)$$

Lognormal approximation

$$\log\left(h_{P,t+1}^i\right)\approx\log\left(h_{P,t}^i\right)\tag{A.10}$$

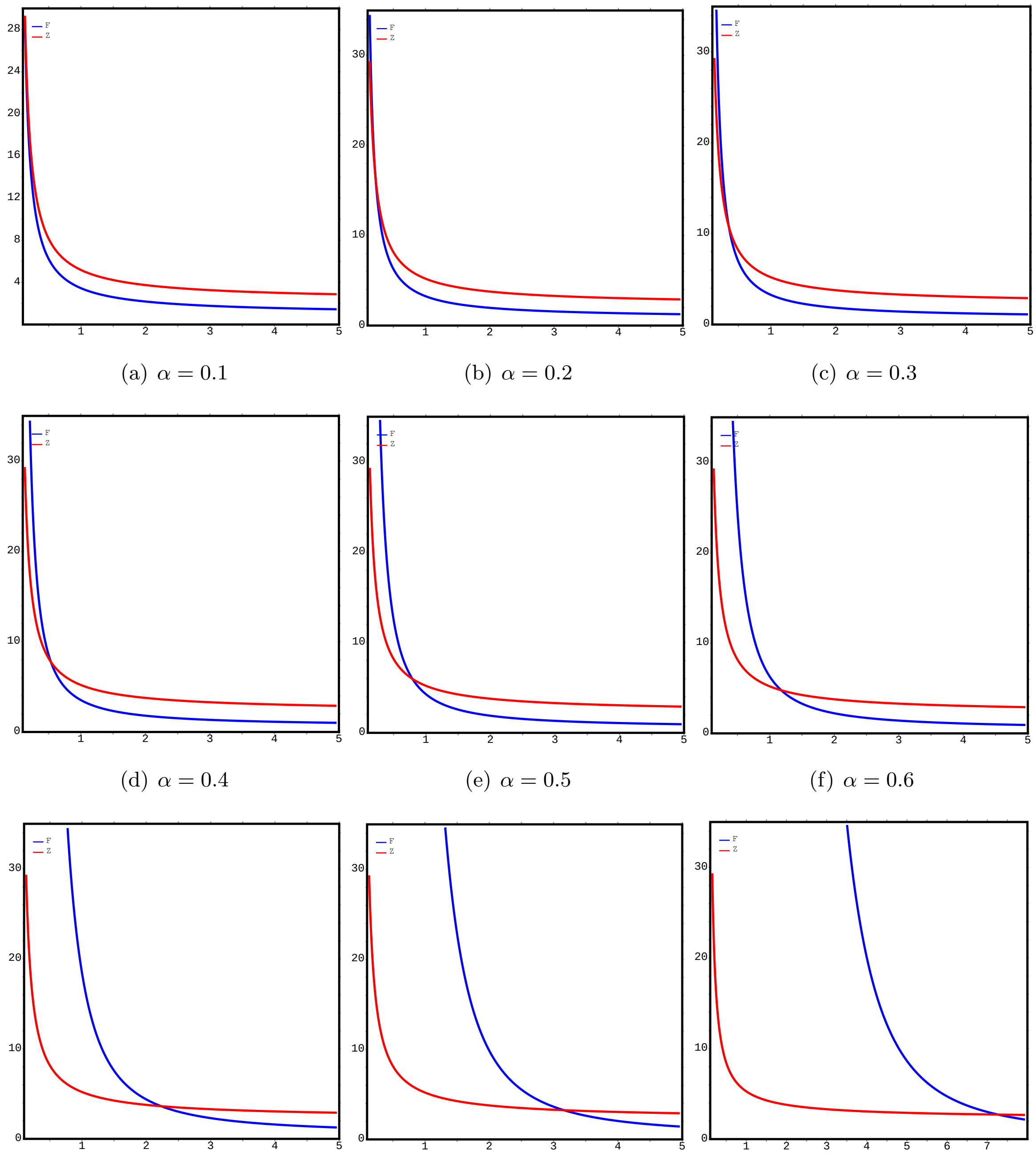
Taking the first-order Taylor approximation, it results that

$$\log\left(h_{P,t+1}^i\right)=\log\left(h_{P,t}^i\right)+\frac{1}{h_{P,t}^i}\tag{A.11}$$

For very large values of $h_{P,t}^i$ the approximation (A.10) is consistent when considering for instance that $\lim_{h\to\infty}\log\left(h_{P,t}^i\right)+\frac{1}{h_{P,t}^i}=\log\left(h_{P,t}^i\right)$

Simulations

Figure A.2: Simulation of condition in (A.9)



(a) $\alpha = 0.1$ (b) $\alpha = 0.2$ (c) $\alpha = 0.3$

(d) $\alpha = 0.4$ (e) $\alpha = 0.5$ (f) $\alpha = 0.6$

(g) $\alpha = 0.7$ (h) $\alpha = 0.8$ (i) $\alpha = 0.9$

Note: Simulations of condition in (A.9) with γ on the x-axis and the functions of γ for each given level of α on the y-axis. The red line and the blue line represent respectively the right hand side and the left hand side of the condition in equation (A.9).

References

Aitchison, J. and J. A. C. Brown (1957), *The Lognormal Distribution*, Cambridge, Cambridge University Press.

Atkinson, A. B. (1970), "On the measurement of inequality", *Journal of economic theory*, No. 2, pp. 244-263.

Atkinson, A. B. (1987), "On the measurement of poverty", *Econometrica*, 55(4), pp.749-764.

Atkinson, A. B. and F. Bourguignon (eds., 2000), *Handbook of Income Distribution*, Vol. 1, Elsevier Science Ltd., North-Holland

Azariadis, C. (1993), *Intertemporal Macroeconomics*, Oxford: Blackwell Publishers.

Azariadis, C. (1996), "The economics of poverty traps part one: Complete markets", *Journal of Economic Growth*, 1(4), pp. 449–486.

Azariadis, C. and A. Drazen (1990), "Threshold externalities in economic development", *Quarterly Journal of Economics*, 105(2), pp. 501-526.

Banerjee, A. V. (2000), "The Two Poverties", *Nordic Journal of Political Economy*, Vol. 26, pp. 132-140.

Banerjee, A. V., Benabou, R. and D. Mookherjee (2006), *Understanding Poverty*, Oxford University Press.

Banerjee, A. V. and E. Duflo (2007), "The Economic Lives of the Poor", *Journal of Economic Perspectives*, 21 (1), pp. 141-167.

Banerjee, A. V. and S. Mullainathan (2007), "Climbing Out of Poverty: Long Term Decisions Under Income Stress", mimeo, BREAD conference at CEPR (London) on Oct. 5th, 2007.

Banerjee, A. V. and A. F. Newman (1993), "Occupational choice and the process of development", *The Journal of Political Economy*, Vol. 101, No. 2, pp. 274-298.

Banerjee, A. V. and A. F. Newman (1994), "Poverty, Incentives, and Development", *The American Economic Review*, Vol. 84, No. 2, Papers and Proceedings, pp. 211-215

Becker, G. S. (1974), "A Theory of Social Interactions", *The Journal of Political Economy*, Vol. 82, No. 6, pp. 1063-1093.

Becker, G. S. and N. Tomes (1979), "An Equilibrium Theory of the Distribution of Income and Intergenerational Mobility", *The Journal of Political Economy*, Vol. 87, No. 6, pp. 1153-1189.

Benabou, R. (1993), "Working of a city: Location, Education and Production", *The Quarterly Journal of Economics*, 108 (3), pp. 619-652.

Benabou, R. (1996), "Equity and Efficiency in Human Capital Investment: The Local Connection", *Review of Economic Studies*, 63, 237–264.

Bertrand, M., Mullainathan, S. and E. Shafir (2004), "A behavioural-economics view of poverty", *The American Economic Review*, Vol. 94, No. 2, Papers and Proceedings, pp. 419-423.

Bourguignon, F., Ferreira F. and M. Walton (2007), "Equity, Efficiency and inequality traps: A research agenda", *Journal of Economic Inequality*, 5, pp. 235-256.

Ceroni, C. B. (2001), "Poverty traps and Human Capital Accumulation", *Economica*, Vol 68, N°270, pp. 203-219.

Chatterjee, S. (1994), "Transitional Dynamics and the Distribution of Wealth in a Neoclassical Growth Model", *Journal of Public Economics*, 54, pp. 97-119.

Chatterjee, S. and B. Ravikumar (1997), "Minimum Consumption Requirements: Theoretical and Quantitative Implications for Growth and Distribution", Working paper no. 97-15, Federal Reserve Bank of Philadelphia.

Cohen, G. A. (1989), "On the currency of egalitarian justice", *Ethics*, 99, pp. 906-944..

De La Croix, D. and P. Michel (2002), *A Theory of Economic Growth. Dynamics and Policy in Overlapping Generations*, Cambridge University Press.

Durlauf, S., (1996), "Theory of persistent income inequality", *Journal of Economic Growth* 1 (1), 75–94.

Dworkin R. (1981a), "What is equality? Part 1: Equality of welfare", *Philosophy and Public Affairs*, 10, pp. 185-246.

Dworkin R. (1981b), "What is equality? Part 2: Equality of resources", *Philosophy and Public Affairs*, 10, pp. 283-345.

Foster, J. (1998), "Absolute versus Relative Poverty", *The American Economic Review Papers and Proceedings*, Vol. 88, No. 2, pp. 335-341.

Foster, J., Greer J. and E. Thorbecke (1984), "A class of decomposable poverty measures", *Econometrica*, 52, pp. 761– 765.

Galor, O. and D. Tsiddon (1997), "The Distribution of Human Capital and Economic Growth", *Journal of Economic Growth*, 2, pp. 93-124.

Galor, O. and D. Tsiddon (1997a), "Technological Progress, Mobility, and Economic Growth", *American Economic Review*, 87, 363-382.

Galor, O. and J. Zeira (1993), "Income Distribution and Macroeconomics", *Review of Economic Studies*, 60, 35-52.

Glomm, G. and B. Ravikumar (1992), "Public Versus Private Investment In Human Capital: Endogenous Growth and Income Inequality", *The Journal of Political Economy*, Vol. 100, No. 4. (August), pp. 818-834.

Gradstein, M. and M. Justman (1997), "Democratic Choice of an Education System: Implications for Growth and Income Distribution", *Journal of Economic Growth*, 2, pp. 169-183.

Lewis, G. W. and D. T. Ulph (1988), "Poverty, Inequality and Welfare", *The Economic Journal*, Vol. 98, No. 390, pp. 117-131.

Morduch, J. (1994), "Poverty and Vulnerability", *The American Economic Review*, Vol. 84, No. 2, Papers and Proceedings, pp. 221-225.

Roemer, J. E. (1998), *Equality of Opportunity*, Cambridge: MA, Harvard University Press.

Runciman, W. G. (1966), *Relative deprivation and social justice*, London: Routledge & Kegan Paul

Sen, A. K. (1976), "Poverty: An Ordinal Approach to Measurement", *Econometrica*, Vol. 44, No. 2, pp. 219-231.

Sen, A. K. (1983), "Poor relatively speaking", *Oxford Economic Papers*, Vol. 35(2), pp. 153-169.

Sen, A. K. (1985), "A Sociological Approach to the Measurement of Poverty: A Reply to Professor Peter Townsend", *Oxford Economic Papers*, New Series, Vol. 37, No. 4, pp. 669-676.

Sen, A. K. (1997), *On economic inequality*, expanded ed. with J. E. Foster, Oxford University Press.

Stiglitz, J. E. (1969), "Distribution of income and wealth among individuals", *Econometrica*, Vol. 37, pp. 382-97

Townsend, P. (1985), "A Sociological Approach to the Measurement of Poverty-A Rejoinder to Professor Amartya Sen", *Oxford Economic Papers*, New Series, Vol. 37, No. 4, pp. 659-668.

Van de Gaer, D., Schokkaert, E. and M. Martinez (2001), "Three meanings of intergenerational mobility", *Economica*, 68, pp. 519-537.