Filtered Extreme Value Theory for Value-At-Risk Estimation

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Abstract

Extreme returns in stock returns need to be captured for a successful risk management function to estimate unexpected loss in portfolio. Traditional value-at-risk models based on parametric models are not able to capture the extremes in emerging markets where high volatility and nonlinear behaviors in returns are observed. The Extreme Value Theory (EVT) with conditional quantile proposed by McNeil and Frey (2000) is based on the central limit theorem applied to the extremes rater than mean of the return distribution. It limits the distribution of extreme returns always has the same form without relying on the distribution of the parent variable. This paper uses 8 filtered EVT models created with conditional quantile to estimate value-at-risk for the Istanbul Stock Exchange (ISE). The performances of the filtered expected shortfall models are compared to those of GARCH, GARCH with student-t distribution, GARCH with skewed student-t distribution and FIGARCH by using alternative back-testing algorithms, namely, Kupiec test (1995), Christoffersen test (1998), Lopez test (1999), RMSE (70 days) h-step ahead forecasting RMSE (70 days), number of exception and h-step ahead number of exception. The test results show that the filtered expected shortfall has better performance on capturing fat-tails in the stock returns than parametric value-at-risk models do. Besides increase in conditional quantile decreases h-step ahead number of exceptions and this shows that filtered expected shortfall with higher conditional quantile such as 40 days should be used for forward looking forecasting.

JEL classification: G0, C52, C32, C22

Keywords: Value at-Risk, Filtered Expected shortfall, Extreme value theory, emerging markets

1. Introduction

Estimating loss of financial investments has become the crucial task in the market risk management in the current global economy. The importance of that task is more critical in the emerging financial markets where fluctuations in the volume of hot money from international portfolio investments and hedge funds, unstable regulatory and political environment, and lack of informational efficiency create high volatility and extremes in the returns.

Complex and volatile market conditions in the emerging markets require dynamic and flexible econometric models being able to capture the extremes in the changes in the financial variables. In this research paper, we use filtered (conditional quantile) expected shortfall as filtered extreme value approach for value-at-risk estimation to capture the extremes in the returns. Extreme value theory (EVT) follows the central limit theorem in mathematics arguing that if the sum of the variables has a finite variance, then it follows Gaussian distribution. The EVT focuses on the extremes rater than mean. The distribution of extreme returns is limited into having the same form without relying on the distribution of the parent variable.

There are some important reasons to choose EVT against the parametric volatility models. Firstly, the return of distributions is heavy-tailed and asymmetric in most of the financial time series. EVT which approximates the tail areas asymptotically might be more powerful than imposing an explicit functional form. What is more, extremes in the returns might be caused by mechanisms that are structurally different from the usual dynamics of financial markets. For example, extremes might be the result of a major default or a speculative bubble. In those extreme conditions, the distributional characteristics of the financial time series might shift and requires separating tail estimation from estimation of the rest of the distribution (Neftci, 2000).

EVT has been used in financial risk estimation in recent years. The originality of our paper is that we use conditional quantile expected shortfall with different lags and compare them to find the optimal model capturing the extremes. As another original work, we also apply h-step-ahead root mean square error and number of exceptions to measure performance of the filtered expected shortfall. The performance of the model is also empirically compared with those of the parametric models with Kupiec(1995), Lopez(1999) and Christoffersen (1998) back-test algorithm.

We use the time series of daily returns of the Istanbul Stock Index-100 (ISE-100) from 02.01.2002 to 18.04.2007 for our empirical research. As an emerging market with dramatic macroeconomic and regulatory changes in recent years, the ISE-100 gives us an opportunity to work with a high volatile and heavy tailed data set. The back-test results show that the filtered expected shortfall has superior performance in estimating the extremes and presents a new, dynamic and flexible perspective in value-at-risk estimation.

2. Literature Review

Estimation value-at-risk has become crucial task of risk management functions of the banks and financial institutions since the Basle Committee stated that banks should be able to cover losses on their trading portfolios over a ten-day horizon, 99 percent of the time. However, classical value-at-risk models focus on the whole empirical distribution of the returns rather than that of extreme returns. On the other hand, managing extreme risk requires estimation of quantiles that usually are not directly captured from the time series data.

The distinguishing characteristic of EVT is to quantify the stochastic behavior of a process at unusually levels. Especially in bear markets, fat-tails are usually observed. Poon *et al.* (2004) show that extreme value dependence is usually stronger in bear markets (left tails) than in bull markets (right tails). Longin (2000), McNeil and Frey (2000), and Bali (2003) empirically show that the traditional parametric value-at-risk models with normal density fail to estimate loss during financial crises.

In general, EVT has been seen as an alternative to GARCH models. EVT with conditional quantile is constructed by McNeil and Frey (2000) under the assumption that the tail of the conditional distribution of the GARCH is approximated by a heavy-tailed distribution. They underline the conditional quantile problem and apply EVT to the conditional return distribution by using a two-stage method, which combines GARCH model with EVT in applying the residuals from the GARCH process.

In the literature, EVT is compared to GARCH based parametric value-at-risk estimation models. Yamai and Yossiba (2005) find out the empirical fact that value-at-risk models do not give the proper risk estimation in volatile market conditions while the EVT has more successful prediction performance. Kuester *et al*. (2005). Acerbi (2002), Inui and Kijima (2005) and Martins and Yao (2006) also empirically show that EVT has superior in risk estimation with financial time series. By using more than 30 years of the daily return data on the NASDAQ Composite Index, Kuester *et al*. (2005) compare the out-of-sample performance of value-at-risk models and extreme value theory. They state that a hybrid method, combining a heavy-tailed GARCH filter with an extreme value theory-based approach, performs best overall.

Extremes in returns are observed in time series data from hedge funds and emerging markets where high volatility and unstable money flows occurs. In the literature, we point out that the empirical evidence on the EVT is generally based on the data from hedge funds and emerging financial markets. Amin and Kat (2003) empirically show that while hedge funds combine well with stocks and bonds in the mean-variance framework, this is no longer the case when skewness is considered. By using hedge funds data, Liang and Park (2007) empirically show that EVT is able to foresight the fat-tails in returns especially in high volatility in negative direction. Blum *et al*. 2003), Lhabitant (2003) and Gupta and Liang (2005) also proof that the EVT works with hedge fund indices.

Empirical evidence from the emerging markets is also in favor of the EVT. Kalyvas *et al*. (2007) present evidence from three former emerging and currently transition countries along with two EU member countries of South and Eastern Europe using historical simulation, conditional historical simulation, EVT, and Conditional EVT. They show that Hungary exhibits higher risk under extreme conditions indicating that its market is much more vulnerable than all other markets under study.

Assaf (2006) use the EVT to examine four emerging financial markets belonging to the MENA region, namely Egypt, Jordan, Morocco and Turkey. He focuses on the tails of the unconditional distribution of returns in each market and provides estimates of their tail index behavior. The empirical evidence shows that the returns have significantly fatter tails than the normal distribution and therefore introduce the extreme value theory. Tolikas and Brown (2006) use EVT to examine the asymptotic distribution of the lower tail for daily returns in the Athens Stock Exchange over the period 1986 to 2001. They show that the parameters of this distribution appear to vary with a tendency to become less fat tailed over time. A more comprehensive literature review on the EVT with methodological concerns can be followed in the works of Embrechts *et al.* (1997), Focardi and Fabozzi (2003, 2004).

With Turkish data, Cifter *et al.* (2007) use as conditional quantile expected shortfall and generalized pareto distribution for interest rates and they find that conditional extreme value theory (EVT) improves forecasting. Gencay *et al.* (2003), Gencay and Selcuk (2004), Altay and Kucukozmen (2006) and Eksi *et al*. (2006) use EVT with unconditional quantile EVT to estimate fat-tails in the stock returns in Turkey and they find that EVT performs better than the classical value-at-risk models.

3. Methodology

3.a. Filtered Extreme Value Theory

Value-at-risk reflects the change in a portfolio with a confidence level on a time period. In this description, $\Delta P_{\Delta t}$, masures changes in the market value of P portfolio on Δt time period with π probability (Dowd, 2004):

$$P\,[\,\Delta P_{\Delta t} \leq RMD\,] = \pi \qquad (1)$$

In the equation, since F (ΔP_{Δ}) is a distribution fuction of changes in the portfolio value, it is possible to create an equation like (RMD) = F^{-1} (π). Obviously, F^{-1} is the reverse of the distribution function. From that perspective, estimated value-at-risk will depend on the distribution of F function. Risk for one day should be equal to RMD ((1- π)%). When we include the time as a variable in the equation, for T time period, risk is equal to TD- RMD ((1- π)%).

Semi-parametric models, like EVT, aim at estimating the returns that being not within the confidence level (π) but extremes and fat-tails.

EVT employes the generalized pareto distribution with tresholds. In the perspective of generalized pareto distribution, for pre-defined ξ and β, the following equation holds.

$$F_u = P[X - u \leq y \backslash X > u] = GPD_{\xi,\beta}(y) \quad (2)$$

For negative returns, under the assumptions that x = u+y, the tail estimation can be received with the Equation (3).

$$F(x) = 1 - (N^u/n)((1+\xi(x-u)/\beta)^{\xi-1} \quad (3)$$

In the equation, n is the total data set, N^u is the violations (extremes) above u. For pre-defined q> F(u) distribution, value-at-risk for one day, RMD (q%), is calculated with the Equation (4).

$$RMD = u + (\beta/\xi)\,[((n/N^u)(1-q))^{\xi} - 1] \quad (4)$$

In the Equation (4), by defining u with either constant or conditional quantile, GPD with constant or conditional quantile is obtained. From similar perspective, Artzner *et al.* (1999) evaluate expected shortfall as an alternative for value-at-risk. In expected shortfall, the expected value of the portfolio return is taken into consideration if there is violiation. Expected shortfall can be constructed with the following equation (Gilli ve Kellezi, 2000).

$$ES_p = E[X \backslash X > RMD_p] \quad (5)$$

The second nomination on the right side of the equation is the mean of the violation distribution of $F_{RMDp}(y)$ on the RMD_p treshold. For GPD, we can express the mean violation function with $\xi < 1$ parameters as in the Equation (6).

$$e(u) = E(X-u \backslash X > u) = (\sigma+\xi u)/(1-\xi) \quad (6)$$

$$\sigma+\xi u > 0$$

From that point of view, the expected shortfall is

$$ES_p = RMD_p + ((\sigma+\xi)(RMD_p - u))/(1-\xi) \quad (7)$$

$$ES_p = RMD_p(1-\xi) + ((\sigma-\xi u)/1-\xi) \quad (8)$$

If X is GPD; then for all r < 1/ξ integer, (r), the first moment of each r exists.

In this research paper, 8 different filtered expected shortfall estimation with 2, 3, 4, 5, 10, 15, 20 and 40 days rolling quantile are estimated. The rolling quantile days are randomly selected and maximum rolling is estimated as 40 days since conditional EVT approximate to unconditional EVT more than 40 days rolling.

We use parametric models like Garch, Garch-t, Garch-Skewed Student-t and Figarch for performance comparison of the filtered expected shortfall. The methodologies for GARCH models are not examined here, but detailed examinations can be found in Chung (1999), Baillie *et al.*(1996), Davidson (2002) and Laurent and Peters (2002).

3.b. Methodologies Of Alternative Back-Tests

Alternative back-testing algorithms are employed to compare the performance of the models. Kupiec test (1995), Christoffersen test (1998), Lopez test (1999), RMSE (70 days) and h-step ahead forecasting RMSE (70 days), number of exceptions and h-step ahead number of exceptions are used as the back-tests.

3.b.1. Kupiec Test

Kupiec test (1995) defines the failure ratio (f) as the excess values from VaR (*x*) to the total observations (*T*). When we nominate the pre-defined VaR with α, likelihood ratio statistics with Chi-square distribution for the Kupiec test can be given in Equation (9) (Kupiec, 1995).

$$LR = 2\left\{\log\left|f^x(1-f)^{T-x}\right|\right\} - \log\left|\alpha^x(1-\alpha)^{T-x}\right| \quad (9)$$

3.b.2. Christoffersen Test

According to Christoffersen test (Christoffersen, 1998), the probability of failure rate in the value-at-risk estimation is the important point for back-testing. To conduct the test, one should firstly define $p^{\alpha} = \Pr(y_t < VaR_t(\alpha))$ and test $H_0 : p^{\alpha} = \alpha$ against $H_1 : p^{\alpha} \neq \alpha$.

The condition of $\{1(y_t < VaR(\alpha)\}$ has a binomial likelihood and can be given in Equation (10).

$$L(p^{\alpha}) = (1-p^{\alpha})^{n_0}(p^{\alpha})^{n_1} \quad (10)$$

where $n_0 = \sum_{t=R}^{T} 1(y_t > VaR_t(\alpha))$ and $n_1 = \sum_{t=R}^{T} 1(y_t < VaR_t(\alpha))$ (Saltoglu, 2003).

Under the null hypothesis, it becomes $L(\alpha) = (1-\alpha)^{n_0}\alpha^{n_1}$. The likelihood ratio test statistics can be given in Equation (11).

$$LR = -2In(L(\alpha))/L(\hat{p}) \xrightarrow{d} \chi(1) \quad (11)$$

3.b.3. Lopez Test

Lopez (1999) performs the back-test in three steps. In the first step, the proper distribution of returns and statistics model is chosen. Secondly, from the model created in the first step, by using historical losses/gains, VaR and VaR(α) are constructed and L_i, a mean for losses/gains is obtained. In the last step, the process described above is repeated in many times, for example, 10.000 times to reach an estimated value for a mean of loss distribution, $L_{i=1}^{i=10,000}$.

Lopez (1999) defines the violation function, *L* (*VaRt*(α), *xt;t*+1) that can be given in Equation (12).

for (1 + (*xt;t*+1 − *V aRt*(α)) (12)
if *xt;t*+1< −*V aRt* (α) 1
if *xt;t*+1 > −*V aRt* (α) 0

By using that methodology, back-test is conducted with Equation (13)

$$L = 1/T \sum_{t=1}^{T} L\,(Var\,(\alpha),\ xt;t{+}1) \quad (13)$$

3.c. H-Step Ahead RMSE and Number of Exceptions

The root mean squared error (RMSE) is a scaled dependent comparison algorithm for forecasts. The smaller its values, the more accurate are the forecasts. The test value is calculated as the deviation of the h-step ahead forecasts of a variable, $E(y_{t+h})$, from its observed time path, y_{t+h}. The RMSE of $E(y_{t+h})$ equals to the square root of the Equation (14).

$$1/(T_2-T_1-h+1) \sum_{t=T1+h}^{T1} (E(y_{t+h}) - y_{t+h})^2 \quad (14)$$

In the function, T_1+h is the beginning of the testing sample, while T_2 is the end of the testing sample.

H-step ahead number of exceptions is calculated with the same methodology but where RMSE is replaced with number of exceptions. H-step ahead number of exceptions is more sensitive measure of forecasting rather than h-step ahead RMSE as considers tail loss directly.

3.d. Diebold And Marino Test Of Forecast Accuracy

Diebold and Marino (1995) developed the forecast comparison between a benchmark and selected models based on forecast errors. The main advantage of this statistic is that there is not any assumption on the distribution of forecast errors.

Define $\hat{u}_{1,t+1}^2$ and $\hat{u}_{2,t+1}^2$ as two forecast errors and estimate $d_{t+1} = \hat{u}_{1,t+1}^2 - \hat{u}_{2,t+1}^2$ and $\bar{d} = P^{-1}\sum_t d_{t+1} = MSE_1 - MSE_2$ where MSE represents mean squared errors of forecasting models.

Diebold-Mariano test for equal MSE is defined as in Equation 15.

$$DM = \frac{\bar{d}}{\sqrt{P^{-2}\sum_t (d_{t+1} - \bar{d})^2}} \quad (14)$$

4. Data And Empirical Results

4.a. Data

Istanbul Stock Exchange Rate (ISE-100 Index) is received from Bloomberg. Our dataset covers 1325 daily observations where 610 observations includes negative returns from 02.01.2002 to 18.04.2007. We constituted the series in log-differenced level. Figure 1 shows ISE Index in log-differenced series where Figure 2 shows negative and positive returns separately. By performing Augmented Dickey–Fuller (Dickey and Fuller, 1981) and Phillips-Peron test (Phillips and Peron, 1988) we found that ISE Index is stationary at log differenced level as shown in Table 1.

Main Statistical Properties of the log-differenced index is shown in Table 1. Although χ^2 normality test and Jargue-besa stat indicates that index is normally distributed kurtosis and skewness values shows that distribution is skewed and heavy-tailed. As a result filtered extreme value theory like filtered expected shortfall may capture tail loss better compare to Garch and alternative Garch models.

Table 1
Unit Root Test and Main Statistical Properties

	ISE
Unit Root tests	
ADF Test	-36.5121*
P-P Test	-36.5267*
Main Stats.	
Asymptotic test: (χ^2)	876.03 [0.0000]**
Normality test: (χ^2)	399.71 [0.0000]**
Mean (μ)	0.00091553C
Std.Devn. (σ)	0.0212336
Skewness (S)	-0.0707095
Kurtosis (K)	6.98092
Minimum	-0.133408 at obs. 289
Maximum	0.11794 at obs. 215
Jarque-Bera statistic	876.031

* Denotes statistical significance at the %5 level (at least). [] denotes t-value

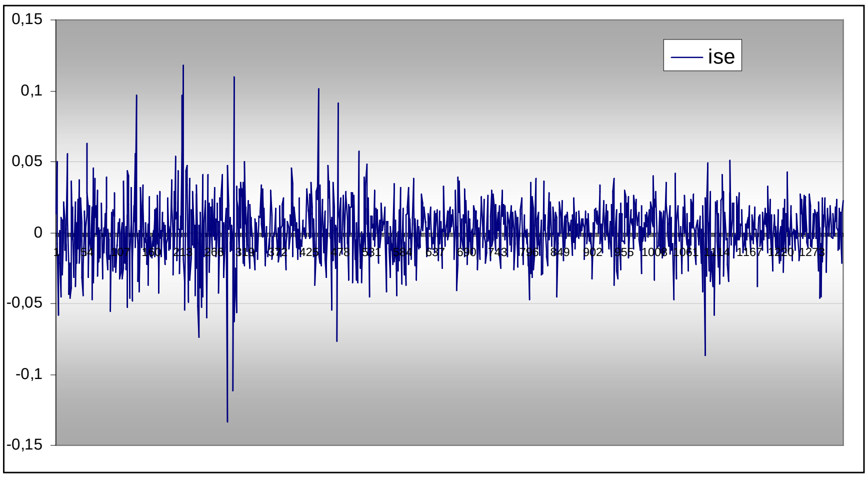

Figure 1. ISE Log-differenced series

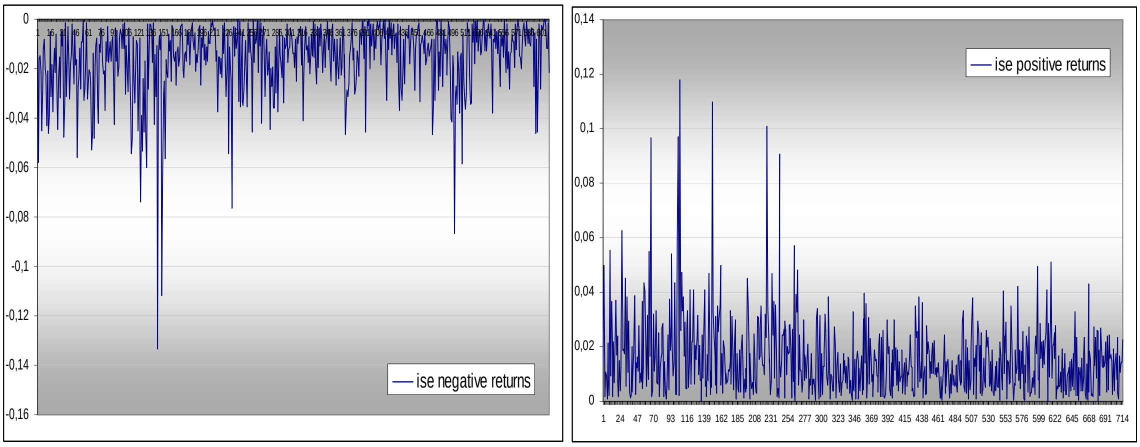

Figure 2. ISE negative and positive returns

4.b. Empirical Results

Filtered expected shortfall with conditional quantile of 2,3,4,5,10,15,20 and 40 days rolling and volatility models as Garch(1,1), Garch(1,1)-student t, Garch(1,1)-skewed student t and Figarch(1,1) are estimated. Kupiec(1995), Lopez(1999) and Christoffersen(1998) backtesting procedures and h-step ahead forecasting of RMSE and number of exceptions are applied to compare predictive performance of the models. Back testing is done with 95% and 99 confidence interval and Basel requires using 99% confidence interval.

Table 2 reports Garch(1,1), Garch(1,1)-student t, Garch(1,1)-skewed student t and Figarch(1,1) estimates for ISE index. All the parameters of the Garch models are statistically significant and according to log-likelihood stat Garch(1,1) fits better than the other Garch models. Table 3 reports filtered expected shortfall parameters as shape (ξ) and scale (β) parameters of lower and upper tail. In this paper we only estimate lower tail of value-at-risk estimation based on Garch and filtered expected shortfall models so only lower tail parameters are used to estimate filtered expected shortfall models[1].

[1] Value-at-risk results of upper tail estimation can be obtained from the authors.

Figure 3 and Figure 4 shows filtered expected shortfall and Garch models graphs. Aso reported in Table 4 Diebold-Mario test (Diebold and Mariano, 1995) shows that Garch-student t and Garch-skewed student-t are not statistically different from Garch with gaussian distribution where Figarch, filtered expected shortfall with 2, 3, 4 and 5 days conditional quantile are statistically different at 5% confidence interval and filtered expected shortfall with 10, 15, 20 and 40 days conditional quantile are statistically different from Garch model at 1% confidence interval.

The predictive performance of filtered expected shortfall and Garch models are reported in Table 5 and Table 6 with 95% and 99% confidence interval. According to 95% confidence interval, filtered expected shortfall with 2 days conditional quantile is the best based on Lopez test, filtered expected shortfall with 15 and 40 days conditional quantile performs best based on Christoffersen and Kupiec tests. According to 99% confidence interval, filtered expected shortfall with 10 and 20 days conditional quantile performs best based on Lopez test, filtered expected shortfall with 2 days conditional quantile performs best one based on Christoffersen test, filtered expected shortfall with 15 and 40 days conditional quantile are the best ones based on Kupiec tests. Since 99% confidence interval is more significant and

Basel requires using 99% confidence interval we also consider 99% confidence interval for back testing results. According to all back testing procedures filtered expected shortfall models predictive performance is better than Garch models. There is not one filtered expected shortfall model that beats other ones based on Lopez, Christoffersen and Kupiec tests therefore we applied h-step ahead forecasting of RMSE and number of exceptions. Based on h-step ahead forecasting of RMSE Garch(1,1) is the best one[2]. Table 7 shows that based on h-step ahead forecasting of number of exceptions up to 70 days filtered expected shortfall with 40 days conditional quantile is the best one. We observed that increase in conditional quantile decreases h-step ahead forecasting of number of exceptions and this shows that filtered expected shortfall with 40 days conditional quantile should be used for forward looking forecasting such as more than one month forecasting. Christoffersen ve Diebold(2000) shows that volatility models such as Garch and other Garch models can be used for forecasting up to 15-20 days ahead for USA financial instruments and Çifter(2004) shows that volatility models can be used for forecasting up to 10-14 days ahead for Turkish interest rates.

Figure 5 shows h-step ahead forecasting of number of exceptions and Figure 6 and Figure 7 shows h-step ahead forecasting of RMSE up to 70 steps. H-step ahead forecasting of number of exceptions shows that filtered expected shortfall from 15 days to 40 days conditional quantile beats all Garch and filtered expected shortfall less than 15 days conditional quantile. Diebold-Marino test of equal forecast accuracy(Diebold and Marino, 1995) is also applied to reveal statistical difference between filtered expected shortfall models and found that filtered expected shortfall models with less than 10 days conditional quantile is not statistically different than with 2 days conditional quantile estimation (Table 8). Thus indicate that filtering less than 10 days conditional quantile may imitate Garch models.

Table 2
Estimation Results from Volatility Models

	Garch	Garch-t	Garch-Skew	Figarch
ω	0.001**	0.001**	0.0014*	0.001**
	(2.738)	(3.395)	(2.912)	(2.973)
α	0.089**	0.084**	0.085**	0.204*
	(5.325)	(4.500)	(4.490)	(2.853)
β_1	0.889**	0.886**	0.884**	0.600**
	(44.21)	(37.29)	(36.15)	(6.051)
v -St. t	-	8.125**	-	-
		(5.314)		
ξ -Ske.	-	-	-0.0606	-
			(1.505)	
v -Skew	-	-	8.215**	-
			(5.266)	
d	-	-	-	0.5043
				(6.022)
$\alpha + \beta_1$	0.97840	0.97146	0.96958	0.80434
Loglike	3321.90	3349.99	3351.13	3325.44

[2] Based on standard RMSE(t+1) Garch(1,1) also the best one.

Table 3
Extreme Value Parameters

Models	Lower Tail Shape (ξ)	Lower Tail Scale (β)	Upper Tail Shape (ξ)	Upper Tail Scale (β)
Filtered ES - 2 Days	0,064	0,011	0,072	0,011
Filtered ES - 3 Days	0,081	0,011	0,115	0,010
Filtered ES - 4 Days	0,114	0,009	0,134	0,010
Filtered ES - 5 Days	0,034	0,011	0,187	0,008
Filtered ES - 10 Days	0,170	0,010	0,300	0,007
Filtered ES - 15 Days	0,131	0,011	0,238	0,009
Filtered ES - 20 Days	0,139	0,010	0,226	0,009
Filtered ES - 40 Days	0,184	0,009	0,348	0,008

Table 4
Comparing predictive accuracy with the Diebold–Mariano statistic[§]

Models	Ratio DM
Garch	-
Garch-t	0.5092
Garch-Skew	0.5194
Figarch	-0.09754*
Filtered ES - 2 Days	0.094686*
Filtered ES - 3 Days	0.148612*
Filtered ES - 4 Days	0.191513*
Filtered ES - 5 Days	0.185461*
Filtered ES - 10 Days	0.32498*
Filtered ES - 15 Days	0.047025**
Filtered ES - 20 Days	0.077021**
Filtered ES - 40 Days	0.085852**

[§] Benchmark model is Garch(1,1) with gaussian distribution.
Notes: * indicate significance at the 5% confidence level and ** stands for significance at the 1% level.

Table 5
Back testing (%95 confidence interval)

Models	Lopez	Christoffersen	Kupiec
Garch	0.68934	0.00315	0.00249
Garch-t	1.00409	0.00025	0.00023
Garch-Skew	0.78774	0.00146	0.00120
Figarch	0.89262	0.00063	0.00054
Filtered ES - 2 Days	0,15879	0,17591	0,11457
Filtered ES - 3 Days	0,52972	0,00958	0,00766
Filtered ES - 4 Days	0,85641	0,00065	0,00061
Filtered ES - 5 Days	0,98265	0,00021	0,00022
Filtered ES - 10 Days	1,41345	0,00000	0,00000
Filtered ES - 15 Days	1,74404	0,00000	0,00000
Filtered ES - 20 Days	1,41345	0,00000	0,00000
Filtered ES - 40 Days	1,74404	0,00000	0,00000

Table 6
Back testing (%99 confidence interval)

Models	Lopez	Christoffersen	Kupiec
Garch	0.32134	0.00080	0.99980
Garch-t	0.15609	0.01469	0.99601
Garch-Skew	0.25970	0.00226	0.99943
Figarch	0.20462	0.00595	0.99846
Filtered ES - 2 Days	0,66599	0,00000	0,99999
Filtered ES - 3 Days	0,23692	0,00396	0,99907
Filtered ES - 4 Days	0,08361	0,06909	0,98078
Filtered ES - 5 Days	0,04985	0,15074	0,95528
Filtered ES - 10 Days	0,00065	0,85707	0,68437
Filtered ES - 15 Days	0,01124	0,42083	0,32276
Filtered ES - 20 Days	0,00065	0,85707	0,68437
Filtered ES - 40 Days	0,01124	0,42083	0,32276

Table 7
H-Step Ahead Forecasting*

	No.of Exceptions		RMSE	
Models	t+1	Avrg.	t+1	Avrg.
Garch	15	15.136	0.0373	0.0402
Garch-t	13	11.454	0.0405	0.0433
Garch-Skew	14	13.500	0.0383	0.0411
Figarch	14	13.257	0.0389	0.0419
Filtered ES - 2 Days	22	25.196	0.0449	0.0479
Filtered ES - 3 Days	16	17.575	0.0481	0.0515
Filtered ES - 4 Days	13	14.515	0.0501	0.0537
Filtered ES - 5 Days	12	13.712	0.0498	0.0533
Filtered ES - 10 Days	8	7.696	0.0586	0.0617
Filtered ES - 15 Days	5	5.7272	0.0602	0.0630
Filtered ES - 20 Days	8	6.500	0.06051	0.0633
Filtered ES - 40 Days	4	3.0151	0.0614	0.0640

*Average no. of exceptions and RMSE is estimated with 70 days step-ahead forecasting.

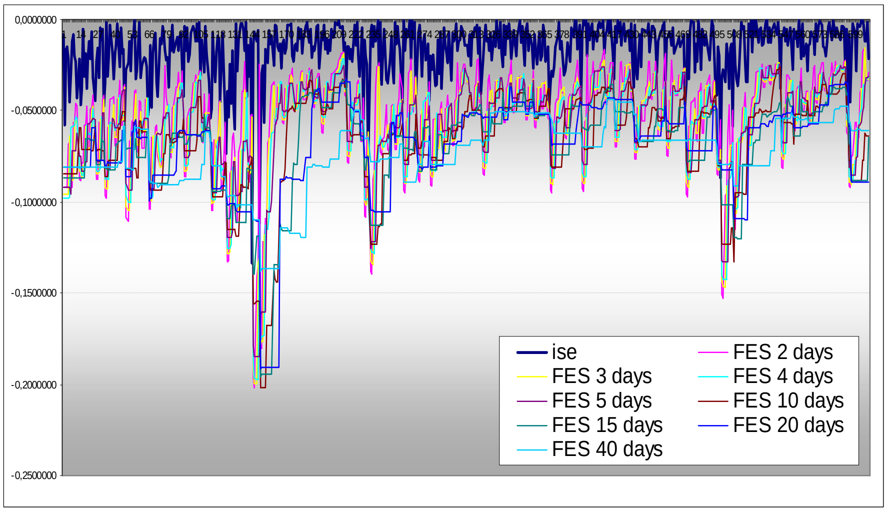

Figure 3. Filtered Expected Shortfall Models

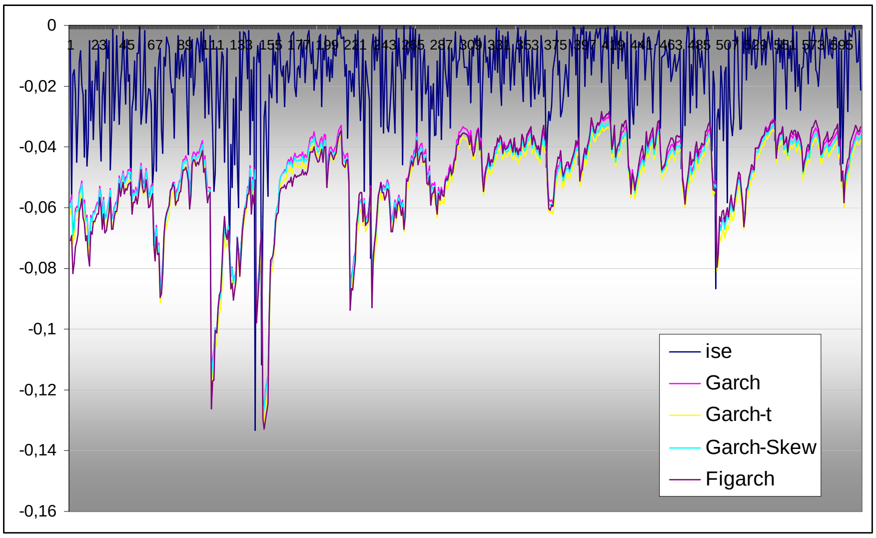

Figure 4. Garch Models

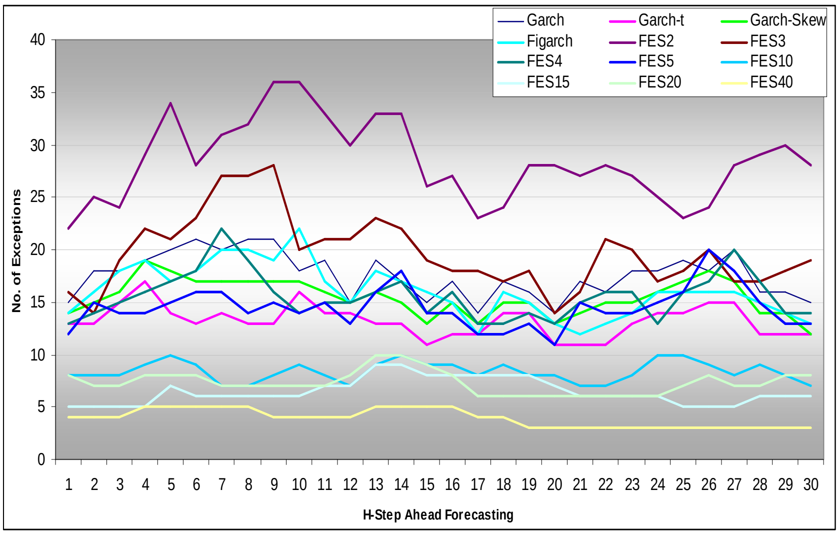

Figure 5. H-Step Ahead Forecasting (No. of Exceptions)

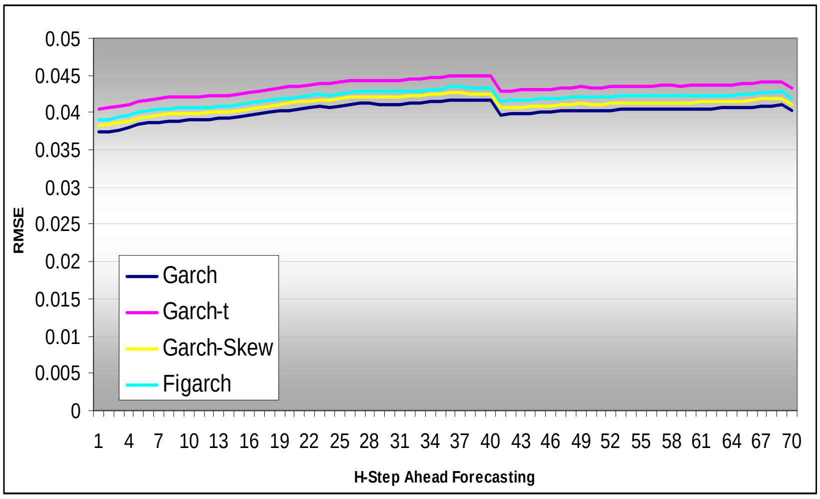

Figure 6. H-Step Ahead Forecasting (RMSE, Garch Models)

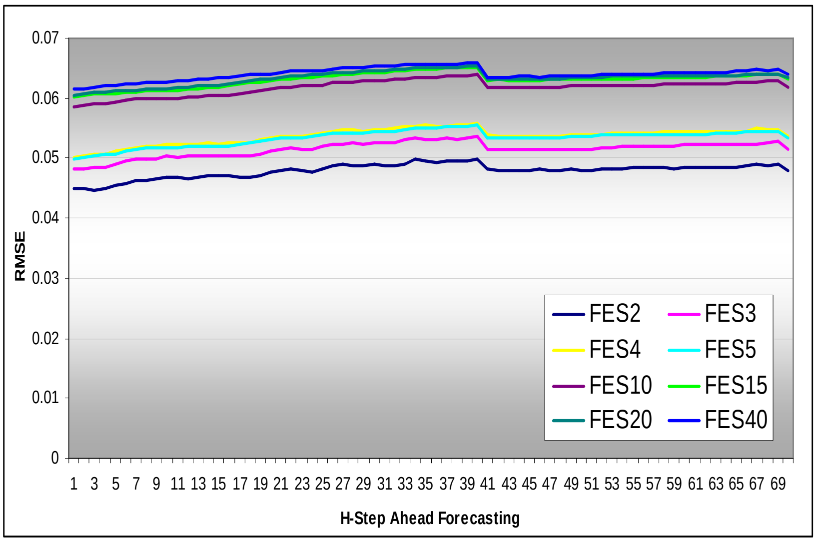

Figure 7. H-Step Ahead Forecasting (RMSE, Filtered Expected Shortfall Models)

Table 8
Comparing predictive accuracy with the Diebold–Mariano statistic[§]

Models	Ratio DM
Filtered ES - 2 Days Rolling	-
Filtered ES - 3 Days Rolling	1.2342
Filtered ES - 4 Days Rolling	0.612275
Filtered ES - 5 Days Rolling	0.627426
Filtered ES - 10 Days Rolling	0.323022*
Filtered ES - 15 Days Rolling	0.264849*
Filtered ES - 20 Days Rolling	0.309795*
Filtered ES - 40 Days Rolling	0.297323*

[§] Benchmark model is Filtered Expected shortfall with 2 days conditional quantile.
Notes: * indicate significance at the 5% confidence level.

5. Conclusion

The dynamic and chaotic features of financial markets in emerging economies make successful financial forecasting almost impossible with parametric models. Observed extremes and fat-tails in returns need to be estimated with relatively more flexible models. Parametric models have certain strict assumptions on the distribution function of the returns. Those restrictions, either normality or asymmetric distributional ones, are not able to make statistically significant estimations.

Extreme Value Theory, on the other hand, employs the central limit theorem for risk estimation. According to the theorem, if the sum of the variables has a finite variance, then it follows Gaussian distribution. The distribution of extremes in returns is limited into having the same form without relying on the distribution of the parent variable.

In this research paper, we write an algorithm with Matlab to conduct filtered EVT with different rolling quantile to estimate value-at-risk. By using daily returns of the Istanbul Stock Exchange National 100 Index, we estimate risk with filtered EVT. For comparison of the model performance, we also estimate value-at-risk with parametric models, namely, GARCH, GARCH with student-t distribution, GARCH with skewed student-t distribution and FIGARCH. The success of the estimation of the models are compared by using Kupiec test (1995), Christoffersen test (1998), Lopez test (1999), RMSE (70 days) and h-step ahead forecasting RMSE (70 days). The results of the back-tests show that the filtered EVT has better risk forecasting performance than parametric value-at-risk models.

We think that financial forecasting especially in dynamic markets needs flexible models. From that perspective, new semi-parametric models should be conducted in the future researches without ignoring the econometric methodological concerns.

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