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Abstract

Linear demand formulations for price competition in horizontally differentiated products are sometimes used to compare situations where additional varieties become available, e.g. due to market entry of new firms. We derive a consistent demand system to analyze such situations.

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In industrial economics, often the following expression is used to model price competition in horizontally differentiated products: There are n product varieties with demand for variety $j = 1, .., n$, given by:

$$D_j(p) = \frac{1}{n}\left(1 - p_j - \gamma\left(p_j - \frac{\sum_{i=1}^{n} p_i}{n}\right)\right). \tag{1}$$

This formulation goes back to Shubik and Levitan (1971), is used in many models, and can be found in text books like e.g. Vives (2001), p. 163. The formulation is analytically tractable and has a very intuitive interpretation: Demand decreases directly in the own price but additionally if the own price increases above the price average, where the parameter γ describes how closely the different markets are

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linked. An important feature of this demand system is that it can be derived from a representative consumer with quasi-linear preferences that can be represented by the following utility function (where w denotes the initial wealth of the consumer):[1]

$$U=\sum_{j=1}^{n}q_j-\frac{1}{2}\left(\sum_{j=1}^{n}q_j\right)^2-\frac{n}{2\left(1+\gamma\right)}\left[\sum_{j=1}^{n}q_j^2-\frac{\left(\sum_{j=1}^{n}q_j\right)^2}{n}\right]+\left[w-\sum_{j=1}^{n}q_jp_j\right]. \quad (2)$$

It is often interesting to compare situations where different numbers of varieties are available. Consider, for instance, the decision problem of a consumer with preferences for different varieties of cereals who goes to the supermarket and realizes that the varieties $m+1,...,n$ are sold out. How much does she buy from the varieties that are available? Or consider a market where some varieties will be offered if and only if new firms enter the market. How do prices, quantities and welfare change if such market entry occurs? Suppose that $q_j=0$ for $j=m+1,...n$, i.e., some varieties are not available because they are sold out or entry did not occur. In this case, a consumer with preferences according to (2) maximizes:

$$U=\sum_{j=1}^{m}q_j-\frac{1}{2}\left(\sum_{j=1}^{m}q_j\right)^2-\frac{n}{2\left(1+\gamma\right)}\left[\sum_{j=1}^{m}q_j^2-\frac{\left(\sum_{j=1}^{m}q_j\right)^2}{n}\right]+\left[w-\sum_{j=1}^{m}q_jp_j\right]. \quad (3)$$

Note that (2) differs from the utility of a consumer who is not interested in goods $j=m+1,...n$. Such a consumer would also consume $q_j=0$ for $j=m+1,...n$, but in his utility function m instead on n appears in the product $\frac{n}{2(1+\gamma)}\sum_{j=1}^{m}q_j^2$. This subtle change affects the demand for all the other goods. A consumer with utility functions (2) who cares about goods $j=m+1,...,n$ that are not available and is constrained not to buy them has the following demand functions for the available varieties $j\leq m$:

$$D_j\left(p\right)=\frac{1+\gamma}{n}\left[1-p_j-\frac{\gamma}{n+m\gamma}\left(m-\sum_{j=1}^{m}p_j\right)\right] \text{ for } j\leq m\leq n. \quad (4)$$

[1]See Vives (2001), p. 163. Note that there is a typo, where for the last term in the utility function it reads $\sum_j q_j$, while correctly it should be $\left(\sum_j q_j\right)^2$, since only the latter results in the demand functions derived by Vives.

On the other hand, a consumer who does not care about goods $j = m+1, ..., n$ and would not buy them even if he could has different demand functions for varieties $j \leq m$:

$$D_j(p) = \frac{1}{m}\left(1 - p_j - \gamma\left(p_j - \frac{\sum_{i=1}^{m} p_j}{m}\right)\right) = \frac{1}{m}\left(1 - p_j - \gamma\left(p_j - \overline{p}\right)\right) \text{ for } j = 1, ..., m, \tag{5}$$

where $\overline{p}$ denotes the "average price". In some recent papers, e.g. Ordover and Shaffer (2007) or Bourreau, Hombert, Pouyet, and Schutz (2007), the latter demand system has been used to compare a situation in which some goods $j = m+1, ..., n$ are not available to one in which they are available and in which the consumer would buy them. This is not appropriate. It makes a difference for the demand system whether a consumer is constrained not to buy some goods or whether he voluntarily abstains from buying them. As an illustration assume $n = 3$, $m = 2$, $p_1 = p_2 = \frac{1}{2}$, and $\gamma = \frac{1}{2}$. Consider a situation where the price p_3 is so high that a consumer with utility function 3 demands $q_3\left(\frac{1}{2}, \frac{1}{2}, p_3\right) = 0$, which is the case for $p_3 \geq \frac{7}{8}$. The resulting other quantities are $q_1 = q_2 = \frac{3}{16}$. In contrast, demand system (5), which captures the situation where the consumer does not care about the third variety, yields as quantities for $p_1 = p_2 = \frac{1}{2}$: $\widehat{q}_1 = \widehat{q}_2 = \frac{1}{4}$. It is easy to check that for $p_1 = p_2 = \frac{1}{2}$ the formulation (4) yields quantities $q_1 = q_2 = \frac{3}{16}$.

Furthermore, a consistent welfare analysis[2] requires that the two demand functions in the two situations are derived from the same consumer, while the demand functions (4) and (5) are derived from different consumers. This is important, in particular when evaluating normatively the effects of market entry or entry deterrence.

References

Bourreau, M., J. Hombert, J. Pouyet, and N. Schutz (2007): "The Competitiveness of Integrated Goods Markets with Vertically Integrated Firms," CEPR Discussion Paper DP6224.

Ordover, J., and G. Shaffer (2007): "Wholesale Access in Multi-Firm Mar-

[2]The authors of the papers cited do not undertake a welfare analysis.

kets: When is It Profitable to Supply a Competitor?," *International Journal of Industrial Organization*, forthcoming.

SHUBIK, M., AND R. LEVITAN (1971): "Noncooperative Equilibria and Strategy Spaces in an Oligopolistic Market," in *Differential Games and Related Topics*, ed. by H. Kuhn, and G. Szego, pp. 429–447. North Holland Press, Amsterdam.

VIVES, X. (2001): *Oligoly Pricing: Old Ideas and New Tools.* MIT Press, Cambridge, MA, 2. edn.

Appendix

How to derive (4):

Assume, utility is given by (2), but impose that $q_j = 0$ for $j = m+1, ..., n$.Thus, the consumer maximizes:

$$U = \sum_{j=1}^{m} q_j - \frac{1}{2}\left(\sum_{j=1}^{m} q_j\right)^2 - \frac{n}{2(1+\gamma)}\left[\sum_{j=1}^{m} q_j^2 - \frac{\left(\sum_{j=1}^{m} q_j\right)^2}{n}\right] + \left[w - \sum_{j=1}^{m} q_j p_j\right].$$

The first order conditions for $j = 1, ..., m$ are:

$$\begin{aligned} p_j &= \frac{\partial U}{\partial q_j} \\ p_j &= 1 - \sum_{j=1}^{m} q_j - \frac{n}{2(1+\gamma)}\left[2q_j - \frac{2\left(\sum_{j=1}^{m} q_j\right)}{n}\right] \\ q_j &= \frac{1+\gamma}{n} - \frac{1+\gamma}{n} p_j + \frac{\gamma}{n}\underbrace{\sum_{j=1}^{m} q_j}_{=:X} \qquad (6) \end{aligned}$$

Summing over all m yields:

$$X = \frac{m(1+\gamma)}{n} - \frac{1+\gamma}{n}\sum_{j=1}^{m} p_j + \frac{m\gamma}{n} X \qquad (7)$$

$$X = \frac{m(1+\gamma)}{n-\gamma m} + \frac{1+\gamma}{n-\gamma m}\sum_{j=1}^{m} p_j. \qquad (8)$$

Plugging (8) into (6) then yields the result in the paper.