A dynamic model of renewable resource harvesting with Bertrand competition

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Abstract

In this paper a differential game model of renewable resource exploitation is considered in which firms compete in exploiting a common resource in a Bertrand price-setting game. The model characterizes a situation in which firms extract a common renewable resource which after harvesting may be considered a differentiated product. Firms then choose prices rather than harvest quantities. Quantities extracted are determined by consumer demand. Optimal price and harvest policies are determined in a linear state differential game for whichr open-loop and feedback strategies are known to be equuivalent. Furthermore, the case of search costs and capacity constraints is analysed and the role they play in determining the dynamics of the resource stock is considered. The results are compared to those of Cournot competition which has been analysed extensively in the literature. Previous studies of differential games applied to renewable resource harvesting have concentrated on quantity competition (see for example [12]) and the case of price competition has been largely ignored. the exceptions to this have been in the more empirical literature where evidence for price competition versus quantity competition for renewable resources such as fisheries is mounting [1]. Consequently the results presented here are not only new, but possibly of greater empirical relevance than existing results on quantity competition.

1 Introduction

Traditional microeconomic models of oligopolistic competition may be divided into two types: either firms compete by setting quantities that then impact on price via consumer demand, or firms compete by directly setting prices. The former is known as the Cournot oligopoly model, the latter as the Bertrand oligopoly model. Both models may be represented as strategic games in which the strategy space is either the space of possible firm outputs or the space of possible output prices respectively. In models of renewable resource exploitation firms are often also assumed to compete with each other for the exploitation of a common resource. This problem is frequently analyzed using differential game theory in which competing firms simultaneously choose the amount of the resource that they wish to harvest. Competition between firms is in terms of the quantities harvested. So called Cournot competition. The Cournot model makes sense if firms are competing with each other to produce a homogeneous good. The Cournot assumption may not always be valid in renewable resource industries. The catch of one fishermen often differs in quality to that of another fishermen in terms of both the quality and the size of the fish caught. Consequently, for a given tonnage of fish different fishing enterprises will often produce catches of varying quality. Fish may therefore be considered a heterogeneous good. Competition between firms producing heterogeneous goods is better understood in terms of price competition rather than quantity competition. This is because consumers will resist switching to a lower priced supplier if the lower priced supplier is also offering an inferior product, so that price competition becomes effective if the offered product is heterogeneous in nature.

In the model presented here a differentiated good duopoly in which two firms exploit a common renewable resource is studied. Four cases are distinguished corresponding to four different types of fisheries with different types of search costs. Firstly, a schooling fishery with no costs of harvesting is considered in which two firms exploit a common fishery. For reasons of analytical tractability it is assumed that the stock of the resource grows linearly. This corresponds to an assumption that the resource is being harvested at a rate that would maintain the stock far from it's natural equilibrium.

Renewable resource models employing differential game theory have concentrated on quantity competition between firms exploiting common renewable resources to the exclusion of price competition. Results on sustainable exploitation and extinction of species for exploitation of a common renewable resource are therefore for the most part based on the assumption that firms compete in terms of quantities. There has been little in the way of research analysing what impact price competition may have on the sustainability of resource stocks from either a theoretical or applied perspective. This is despite the fact that in static studies of particular fisheries there is some empirical evidence that competition between firms may be characterised by price competition. For example, Weninger [16] applies Bertrand pricing to vertically integrated fisheries in a theoretical model. Adelaja et al. [1] apply Bertrand pricing models to the study of the Atlantic clam quahog fisheries. The latter study concludes that the Bertrand model provides a good fit for the Atlantic surf clam fishery but not the ocean quahog fishery. In other work on the Marseilles fish market price dispersion between firms has been observed with prices also varying over time [8],[10]. It is possible that the model presented here could explain at least some of this behavior. So for example the schooling fishery is characterized by both price dispersion and prices varying over time at least along the adjustment path to steady-state.

Bertrand models have been employed elsewhere in the context of differential games to study price competition between firms engaged in research and development. So for example Cellini and Lambertini [3] study a differential game of R&D with both Cournot and Bertrand competition. In the field of advertising Cellini, Lambertini and Mantovani [4] apply Bertrand competition to a differential game of advertising. Elsehwere in resource economics Gaudet and Moreaux [7] have compared Bertrand and Cournot equilibria for differential games of non-renewable resource exploitation. However, to the best of my knowledge the Bertrand model has not been applied to differential game models of renewable resources such as fisheries.

In this paper I present a linear-state differential game model of renewable resource exploitation based on a fishery model. The case of oligopolistic price competition between two firms is analysed and solved for three different types of fisheries. The first case analyzes a two-player linear state differential game of Bertand competition in harvesting a renewable resource. The steady-state strategies are then analysed and the impact that increased competition between the firms has on the stock of the resource is studied. This case ignores harvesting costs so these are essentially symmetric. The next part of the anlysis considers the case of a schooling fishery without search costs in which harvesting costs differ between players. The results show that the steady-state price strategies will also differ under these circumstances. In steadys-state the results of the schooling fishery with and without search costs are indentical however the path to steady-state differs in these two cases. Finally, the case of a search fishery is considered. In this case tractability requires the introduction of a a dynamic equation for fleet cacpacity, essentially a capacity constraint. The results suggest that search fisheries tend to be characterized by overexpansion of the fleet compared to schooling fiheries. The reason for this that search costs are too low compared with harvest costs. Conseuquntly, schooling fisheries are likely to be more sustainable than search fisheries. this result differs considerably from received wisdom (cf. Neher [11]). The explanation lies in the need for search fisheries to be able to expand fleet capacity as stocks become small and therefore fish rare. If harvest costs are sufficiently low compared to search costs then this will slow the expansion of the fleet, otherwise the fleet will expand to maintain the search for fish. The result is due more to the search fishery nature of the problem than to Bertrand competition.

The paper is structured as follows firstly in section 2, a basic model is analysed with zero costs of harvesting, in section 3 a schooling fishery with and without search costs is considered and finally a search fishery is considered. In the case of the search fishery an analytical result is achieved by introducing a second state variable to capture capacity constraints in the industry. Finally the different types of fisheries are compared in section 4 in terms of the solution of the game and conclusions are drawn.

2 The Model

The basic model is that of a duopoly in which two firms exploit a common resource and sell the harvested product on an imperfectly competitive market. The particular good (for example fish) is considered to be of heterogeneous quality, so that quantity competition makes little sense. Instead firms compete by setting prices.

Consider for example a common fishery with two firms who face the following demand curves for their product:

$$q_i = a - bP_1 + cP_2 \tag{1}$$

the form of this demand function is standard and is based largely on that employed in the R&D literature where it is attributed to spence [13].

The dynamics of the resource is expreseed by the following differential equation:

$$\dot{x} = f(x) - q_1 - q_2 \tag{2}$$

Each firm's profit (ignoring fixed costs is given by:

$$\Pi_i = p_i q_i, i = 1, 2 \tag{3}$$

The firm's intertemporal profit maximization problem is therefore given by

$$\max_{P_i} J_i = \int_0^\infty \Pi_i e^{-rt} dt, i = 1, 2$$

subject to

$$\dot{x} = f(x) - q_1 - q_2 \tag{4}$$

Consider a renewable resource far from equilibrium (steady-state) so that the growth of the stock can be approximated by an exponential growth function (linear differential equation), consequently one may assume $f(x) = nx$.

3 The Basic Model

The current-value Hamiltonian is given by:

$$\tilde{H} = P_i q_i + \mu_i [nx - q_1 - q_2] \tag{5}$$

Let us consider a two player game (n=2). On substituting in the demand equations we get:

$$\tilde{H}_1 = P_1(a - bP_1 + cP_2) + \mu_1[f(x) - (a - bP_1 + cP_2) - (a - bP_2 + cP_1)] \tag{6}$$

and

$$\tilde{H}_2 = P_2(a - bP_2 + cP_1) + \mu_2[f(x) - (a - bP_1 + cP_2) - (a - bP_2 + cP_1)] \tag{7}$$

Note that the following conditions hold:

$$\frac{\partial^2 H_i}{\partial x \partial x} = 0 \tag{8}$$

and

$$\frac{\partial H^i}{\partial P_i} = 0 \Rightarrow \frac{\partial^2 H^i}{\partial P_i \partial x} = 0 \tag{9}$$

and therefore the game may be characterized as a linear-state differential game [5, p. 188]. A necessary condition for this to be case is linearity of the growth function for the resource.

Pontryagin's maximum principle results in:

$$\frac{\partial \tilde{H}_1}{\partial P_1} = (a - bP_1 + cP_2) - bP_1 + \mu_1(b - c) = 0 \tag{10}$$

$$\frac{\partial \tilde{H}_1}{\partial P_1} = (a - bP_2 + cP_1) - bP_2 + \mu_2(b - c) = 0 \tag{11}$$

with co-state equations:

$$\dot{\mu}_1 - r\mu_1 = -\mu_1 f^{'}(x) \tag{12}$$

$$\dot{\mu}_2 - r\mu_2 = -\mu_2 f^{'}(x) \tag{13}$$

Rearranging one obtains

$$P_1^* = \frac{a(c + 2b) + (b^2 - cb)2\mu_1(t) + (cb - c^2)\mu_2(t)}{4b^2 - c^2} \tag{14}$$

$$P_2^* = \frac{a(c + 2b) + (b^2 - cb)\mu_1(t) + (cb - c^2)2\mu_2(t)}{4b^2 - c^2} \tag{15}$$

The canonical system is obtained by substituting these into $\dot{x}, \dot{\mu}_1, \dot{\mu}_2$ to obtain:

$$\dot{x} = nx - (a - bP_1 + cP_2) - (a - bP_2 + cP_1) \tag{16}$$

$$\dot{\mu}_1 = \mu_1(r - n) \tag{17}$$

$$\dot{\mu}_2 = \mu_2(r - n) \tag{18}$$

Because the co-state equations are independent of the state variable this may be treated as a separate system and solved to yield:

$$\mu_1(t) = \mu_1(0)e^{(r-n)t} \tag{19}$$

and

$$\mu_2(t) = \mu_2(0)e^{(r-n)t} \tag{20}$$

Substituting into the state equation yields:

$$\dot{x} = nx - (a - bP_1^* + cP_2^*) - (a - bP_2^* + cP_1^*) \tag{21}$$

this is a linear non-homegeneous first order ordinary differential equation that may be solved, using an integrating factor. Denoting

$$g(t) = (a - bP_1^* + cP_2^*) + (a - bP_2^* + cP_1^*) \tag{22}$$

The state equation may be rewritten as:

$$\dot{x} - nx = g(t) \tag{23}$$

using the integrating factor e^{-nt} one obtains:

$$x(t) = \frac{-\frac{e^{-nt}}{n}g(t) + C}{e^{-nt}} \tag{24}$$

or

$$x(t) = Ce^{nt} - \frac{g(t)}{n} \tag{25}$$

using$x(0) = x_0$ yields the constant of integration $C = x_0 + \frac{g(0))}{n}$ which on substituting gives

$$x(t) = (x_0 + \frac{g(0)}{n})e^{nt} - \frac{g(t)}{n} \tag{26}$$

Let us now consider what happens in the steady-state. First we assume that $r < n$. This assumption is necessary for a steady-state to exist.

Proposition 1. *If $r < n$ then as $t \to \infty$*

$$P_1^\infty = \frac{a(c + 2b)}{4b^2 - c^2}$$

$$P_2^\infty * = \frac{a(c + 2b)}{4b^2 - c^2}$$

Proof. Evaluating $\lim_{t\to n} P_1^* = \frac{a(c+2b)}{4b^2-c^2}$ and $\lim_{t\to n} P_2^* = \frac{a(c+2b)}{4b^2-c^2}$ □

What is the steady-state stock of the resource? Solving for the steady-state value of x

$$x_{\infty} = \frac{2a + (c-b)P_1^{\infty} + (c-b)P_2^{\infty}}{n} \tag{27}$$

What impact does discounting the future have on prices? As $r \to n$ prices charged by both enterprises increase as long as $b > c$. If the cross-price effect on demand c is greater than the own-price effect on demand b then discounting will reduce prices. Consequently depending on which case one faces discounting could induce price wars between competitors. In considering the impact on the stock of the resource it is clear that regardless of which regime one is in the stock of the resource is reduced by discounting.

What impact does the extent of competition between firms have on the stock of the resource. In other words as c increases what happens to the steady-state stock of the resource. First consider what happens to the steady-state equilibrium prices when c increases.

This results in the following impact on prices:

$$\frac{\partial P^{\infty}}{\partial c} = \frac{a(4b^2 - c^2) - 2ca(c+2b)}{(4b^2 - c^2)^2} \tag{28}$$

Proposition 2. *As c increases x_{∞} also increases as long as $b > c$ and*

Proof. Differentiate and simplify to get

$$\frac{\partial x_{\infty}}{\partial c} = 2\frac{P^{\infty}}{n} + 2\frac{(c-b)}{n}\frac{\partial P^{\infty}}{\partial c}$$

this will be positive if $\frac{1}{P^{\infty}}\frac{\partial P^{\infty}}{\partial c} > \frac{1}{b-c}$. It is possible to demonstrate that $\frac{\partial P^{\infty}}{\partial c}$ is negative by a limit argument. Note that the condition $4b^2 > c^2$ must hold for positive retail prices. $4b^2 - c^2$ is positive implies $b > \frac{c}{2}$ Taking the limit of $\frac{\partial P^{\infty}}{\partial c}$ as b approaches $\frac{c}{2}$ shows that the derivative is negative. This is definitely true if $b > c$. □

It would seem therefore that price competition may be good for the stock of the resource if the own price effect on demand is large compared to the cross-price effect. If this is not the case price competition will drive up demand for the resource and consequently drive the stock of the resource down.

Fast growing species such as r-strategists (perhaps n-strategists here) tend to be characterised by lower steady-state stock than slow growing species (K-strategists). It is assumed here that harvest levels constrain the resource stock well away from the steady-state level that would be achieved by a natural population not subject to exploitation. Nevertheless the intuition appears clear.

4 Search costs and Capacity constraints

There is a considerable literature in resource economic on the role of search costs in renewable resource exploitation particularly with respect to fisheries. Most of this literature considers static models of fisheries and there has been little research analysing the impact of search costs on sustainability fo fisheries from a dynamic perspective. Part of the problem is tractability.

Three different kinds of fisheries can be identified with respect to search costs;

- schooling fishery without search costs,in this case costs are independent of stock size, e.g. $C(q,\theta) = c_q^i q_i + c_s^i, c_s > 0$
- a schooling fishery with search costs $C(q,x,\theta) = C(q,\theta) + C(x) = c_q q + c_s x, c_s < 0$
- a search fishery $C(q,x,\theta)$, such that $\frac{\partial^2 C}{\partial x \partial q} \neq 0$

In case three we specify the following cost function $C(q,x,\theta) = c_s \frac{q}{x}$. Again the linear-quadratic nature of the game is not affected by this specification however we are able to capture the impact of fishing effort on search costs and vice versa. Each of these will be compared.

4.1 Schooling fishery without search costs

In the first case we consider harvest costs and constant search costs. The previous Bertrand pricing model now needs to be modified to incorporate these costs. Instantaneous firm profit is now given by

$$\Pi(t) = p_i q_i - c_q q_i - c_s \tag{29}$$

the state equation remains

$$\dot{x} = nx - (a - bP_1 + cP_2) - (a - bP_2 + cP_1) \tag{30}$$

From this one obtains the Hamiltonian pair:

$$\tilde{H}_1 = p_1(a-bP_1+cP_2)-c_q^1(a-bP_1+cP_2)-c_s+\mu_1[nx-(a-bP_1+cP_2)-(a-bP_2+cP_1)] \tag{31}$$

and

$$\tilde{H}_2 = p_2(a-bP_2+cP_1)-c_q^2(a-bP_2+cP_1)-c_s+\mu_2[nx-(a-bP_1+cP_2)-(a-bP_2+cP_1)] \tag{32}$$

Pontryagin's maximum principle gives:

$$\frac{\partial \tilde{H}_1}{\partial P_1} = (a - bP_1 + cP_2) - bP_1 + c_q^1 b + \mu_1(b - c) = 0 \tag{33}$$

$$\frac{\partial \tilde{H}_1}{\partial P_1} = (a - bP_2 + cP_1) - bP_2 + c_q^2 b + \mu_2(b - c) = 0 \tag{34}$$

with co-state equations:

$$\dot{\mu}_1 - r\mu_1 = -\mu_1 n \tag{35}$$

$$\dot{\mu}_2 - r\mu_2 = -\mu_2 n \tag{36}$$

The canonical system is obtained by substituting these into $\dot{x}, \dot{\mu}_1, \dot{\mu}_2$ to obtain:

$$\dot{x} = nx - (a - bP_1 + cP_2) - (a - bP_2 + cP_1) \tag{37}$$

$$\dot{\mu}_1 = \mu_1(r - n) \tag{38}$$

$$\dot{\mu}_2 = \mu_2(r - n) \tag{39}$$

Because the co-state equations are independent of the state variable this may be treated as a separate system and solved to yield:

$$\mu_1(t) = \mu_1(0)e^{(r-n)t} \tag{40}$$

and

$$\mu_2(t) = \mu_2(0)e^{(r-n)t} \tag{41}$$

Solving the first-order conditions yields:

$$P_1^* = \frac{a(2b+c) + cc_q^2 b + \mu_2(bc - c^2) + 2c_q^1 b^2 + 2(b^2 - bc)\mu_1}{4b^2 - c^2} \tag{42}$$

$$P_2^* = \frac{a(2b+c) + cc_q^1 b + \mu_1(bc - c^2) + 2c_q^2 b^2 + 2(b^2 - bc)\mu_2}{4b^2 - c^2} \tag{43}$$

Substituting into the state equation yields:

$$\dot{x} = nx - (a - bP_1^* + cP_2^*) - (a - bP_2^* + cP_1^*) \tag{44}$$

This may be solved similarly to the previous sections to yield:

$$x(t) = (x_0 + \frac{g(0)}{n})e^{nt} - \frac{g(t)}{n} \tag{45}$$

Now let us examine the steady-state equilibria:

Proposition 3. *If $r < n$ then as $t \to \infty$*

$$P_1^\infty = \frac{a(2b+c) + cc_q^2 b + 2c_q^1 b^2}{4b^2 - c^2}$$

$$P_2^\infty * = \frac{a(2b+c) + cc_q^1 b + 2c_q^2 b^2}{4b^2 - c^2}$$

Proof. Evaluating $\lim_{t\to n} P_1^* = \frac{a(2b+c)+cc_q^2 b+2c_q^1 b^2}{4b^2-c^2}$ and $\lim_{t\to n} P_2^* = \frac{a(2b+c)+cc_q^1 b+2c_q^2 b^2}{4b^2-c^2}$ □

Now consider the impact of the strength of competition between firms on the steady-state stock of the resource. From the previous section one obtains the steady-state stock of the resource:

$$x_\infty = \frac{2a + (c-b)P_1^\infty + (c-b)P_2^\infty}{n} \tag{46}$$

This may be differentiated with respect to c to obtain the impact of competition on the stock of the resource.

Firatly, consider

This results in the following impact on prices:

$$\frac{\partial P_1^\infty}{\partial c} = \frac{(a + c_q^2 b)(4b^2 - c^2) - 2c(a(2b+c) + cc_q^2 b + 2c_q^1 b^2)}{(4b^2 - c^2)^2} \tag{47}$$

and

$$\frac{\partial P_1^\infty}{\partial c} = \frac{(a + c_q^1 b)(4b^2 - c^2) - 2c(a(2b + c) + cc_q^1 b + 2c_q^2 b^2)}{(4b^2 - c^2)^2} \tag{48}$$

Proposition 4. *As c increases x_∞ also increases as long as $b > c$*

Proof. Differentiate and simplify to get

$$\frac{\partial x_\infty}{\partial c} = \frac{P_1^\infty}{n} + \frac{c-b}{n}\frac{\partial P_1^\infty}{\partial c} + \frac{P_2^\infty}{n} + \frac{c-b}{n}\frac{\partial P_2^\infty}{\partial c}$$

The condition $4b^2 > c^2$ must hold for positive retail prices. The proof follows similarly to that employed in the previous section. Except that the asymmetry in costs requires taking the limit of each partial derivative. Taking the limit in the other direction demonstrates that as b becomes large the derivative approaches zero. □

In the next section we consider a schooling fishery with search costs.

4.2 Schooling fishery with search costs

Now consider a schooling fishery with search costs.

$$\Pi(t) = p_i q_i - c_q q_i - c_s x \tag{49}$$

$$\tilde{H}_1 = P_1(a - bP_1 + cP_2) - c_q^1(a - bP_1 + cP_2) - c_s x + \mu_1[nx - (a - bP_1 + cP_2) - (a - bP_2 + cP_1)] \tag{50}$$

and

$$\tilde{H}_2 = P_2(a - bP_2 + cP_1) - c_q^2(a - bP_2 + cP_1) - c_s x + \mu_2[nx - (a - bP_1 + cP_2) - (a - bP_2 + cP_1)] \tag{51}$$

Pontryagin's maximum principle gives:

$$\frac{\partial \tilde{H}_1}{\partial P_1} = (a - bP_1 + cP_2) - bP_1 + c_q^1 b + \mu_1(b - c) = 0 \tag{52}$$

$$\frac{\partial \tilde{H}_1}{\partial P_1} = (a - bP_2 + cP_1) - bP_2 + c_q^2 b + \mu_2(b - c) = 0 \tag{53}$$

with co-state equations:

$$\dot{\mu}_1 - r\mu_1 = c_s - \mu_1 n \tag{54}$$

$$\dot{\mu}_2 - r\mu_2 = c_s - \mu_2 n \tag{55}$$

The canonical system is obtained by substituting these into $\dot{x}, \dot{\mu}_1, \dot{\mu}_2$ to obtain:

$$\dot{x} = nx - (a - bP_1 + cP_2) - (a - bP_2 + cP_1) \tag{56}$$

$$\dot{\mu}_1 = c_s + \mu_1(r - n) \tag{57}$$

$$\dot{\mu}_2 = c_s + \mu_2(r - n) \tag{58}$$

Because the co-state equations are independent of the state variable this may be treated as a separate system and solved to yield:

$$\mu_1(t) = c_s t + \mu_1(0)e^{(r-n)t} \tag{59}$$

and

$$\mu_2(t) = c_s t + \mu_2(0)e^{(r-n)t} \tag{60}$$

Solving the first-order conditions yields:

$$P_1^* = \frac{a(2b + c) + cc_q^2 b + \mu_2(bc - c^2) + 2c_q^1 b^2 + 2(b^2 - bc)\mu_1}{4b^2 - c^2} \tag{61}$$

$$P_2^* = \frac{a(2b + c) + cc_q^1 b + \mu_1(bc - c^2) + 2c_q^2 b^2 + 2(b^2 - bc)\mu_2}{4b^2 - c^2} \tag{62}$$

Substituting into the state equation yields:

$$\dot{x} = nx - (a - bP_1^* + cP_2^*) - (a - bP_2^* + cP_1^*) \tag{63}$$

Solving as before one obtains:

$$x(t) = (x_0 + \frac{g(0)}{n})e^{nt} - \frac{g(t)}{n} \tag{64}$$

Proposition 5. *If $r < n$ then as $t \to \infty$*

$$P_1^\infty = \frac{a(2b + c) + cc_q^2 b + 2c_q^1 b^2}{4b^2 - c^2}$$

$$P_2^\infty * = \frac{a(2b + c) + cc_q^1 b + 2c_q^2 b^2}{4b^2 - c^2}$$

Proof. Evaluating $\lim_{t\to n} P_1^* = \frac{a(2b+c)+cc_q^2 b+2c_q^1 b^2}{4b^2-c^2}$ and $\lim_{t\to n} P_2^* = \frac{a(2b+c)+cc_q^1 b+2c_q^2 b^2}{4b^2-c^2}$ □

Now consider the impact of the strength of competition between firms on the steady-state stock of the resource. From the previous section one obtains the steady-state stock of the resource:

$$x_\infty = \frac{2a + (c-b)P_1^\infty + (c-b)P_2^\infty}{n} \tag{65}$$

This may be differentiated with respect to c to obtain the impact of competition on the stock of the resource. This results in the following impact on prices:

$$\frac{\partial P_1^\infty}{\partial c} = \frac{(a + c_q^2 b)(4b^2 - c^2) - 2c(a(2b+c) + cc_q^2 b + 2c_q^1 b^2)}{(4b^2 - c^2)^2} \tag{66}$$

and

$$\frac{\partial P_1^\infty}{\partial c} = \frac{(a + c_q^1 b)(4b^2 - c^2) - 2c(a(2b+c) + cc_q^1 b + 2c_q^2 b^2)}{(4b^2 - c^2)^2} \tag{67}$$

Proposition 6. *As c increases x_∞ also increases as long as $b > c$*

Proof. Differentiate and simplify to get

$$\frac{\partial x_\infty}{\partial c} = \frac{P_1^\infty}{n} + \frac{c-b}{n}\frac{\partial P_1^\infty}{\partial c} + \frac{P_2^\infty}{n} + \frac{c-b}{n}\frac{\partial P_2^\infty}{\partial c}$$

The condition $4b^2 > c^2$ must hold for positive retail prices. The proof follows similarly to that employed in the previous section. Taking the limit in the other direction demonstrates that as b becomes large the derivative approaches zero. □

4.3 Search Fishery

Now consider a search fishery. The profit function is given by:

$$\Pi(t) = p_i q_i - c_q \frac{q_i}{x} \tag{68}$$

The resultant game is no longer a linear state differential game. Consequently, the open loop and closed loop solutions may differ. As it stands this problem is not tractable. Instead of solving the problem in this form we introduce the idea of a catch per unit effort production function $q_i = kE_i x$ where k is a catchability coefficient and E_i is fishing effort of player i. The procedure we employ follows [15]. Recall the search costs are $c_q \frac{q}{x}$, it is easy to see that the search costs may now be rewritten as $c_q k E_i$. Now introduce an equation for fishing effort $\dot{E}_i = \nu_i$ where ν is a new control variable indicating the rate of entry or exit to the resource. this equation may be interpreted as a dynamic capacity constraint, which constrains the evolution of the effort measured for example by number of fishing boats. We now assume this results in quadratic adjustment costs $\frac{\beta}{2}\nu_i^2$.

The instantaneous profit function may now be rewritten:

$$\Pi(t) = p_i q_i - c_q k E_i - \frac{\beta}{2}\nu_i^2 \tag{69}$$

The resultant game is once again a linear state differential game. However it now posseses two state variables and two control variables:

$$\max_{P_i,\nu_i} J_i = \int_0^\infty \Pi_i e^{-rt} dt$$

subject to

$$\dot{x} = nx - q_1 - q_2 \tag{70}$$

$$\dot{E}_i = \nu_i - \delta E_i, i = 1, 2 \tag{71}$$

The corresponding Hamiltonians are:

$$\tilde{H}_1 = P_1(a - bP_1 + cP_2) - c_q k E_1 - \frac{\beta}{2}\nu_1^2 + \mu_1[nx - (a - bP_1 + cP_2) - (a - bP_2 + cP_1)] + \mu_3[\nu_1 - \delta E_1] \tag{72}$$

and

$$\tilde{H}_2 = P_2(a - bP_2 + cP_1) - c_q k E_2 - \frac{\beta}{2}\nu_2^2 + \mu_2[nx - (a - bP_1 + cP_2) - (a - bP_2 + cP_1)] + \mu_4[\nu_4 - \delta E_2] \tag{73}$$

Pontryagin's maximum principle gives the first-order consitions;

$$\frac{\partial \tilde{H}_1}{\partial P_1} = (a - bP_1 + cP_2) - bP_1 + \mu_1(b - c) = 0 \tag{74}$$

$$\frac{\partial \tilde{H}_1}{\partial P_1} = (a - bP_2 + cP_1) - bP_2 + \mu_2(b - c) = 0 \tag{75}$$

$$\frac{\partial \tilde{H}_1}{\partial \nu_1} = -\beta\nu_1 + \mu_3 = 0 \tag{76}$$

$$\frac{\partial \tilde{H}_1}{\partial \nu_2} = -\beta\nu_2 + \mu_4 = 0 \tag{77}$$

with co-state equations:

$$\dot{\mu}_1 - r\mu_1 = -\mu_1 n \tag{78}$$

$$\dot{\mu}_2 - r\mu_2 = -\mu_2 n \tag{79}$$

$$\dot{\mu}_3 - r\mu_3 = -\frac{\partial \tilde{H}}{\partial E_1} = c_q k + \mu_3\delta \tag{80}$$

$$\dot{\mu}_4 - r\mu_4 = -\frac{\partial \tilde{H}}{\partial E_2} = c_q k + \mu_4\delta \tag{81}$$

Solving the first-order conditions:

$$P_1^* = \frac{a(c + 2b) + (b^2 - cb)2\mu_1(t) + (cb - c^2)\mu_2(t)}{4b^2 - c^2} \tag{82}$$

$$P_2^* = \frac{a(c + 2b) + (b^2 - cb)\mu_1(t) + (cb - c^2)2\mu_2(t)}{4b^2 - c^2} \tag{83}$$

$$\nu_1^* = \frac{\mu_3}{\beta} \tag{84}$$

$$\nu_2^* = \frac{\mu_3}{\beta} \tag{85}$$

This results in the system of linear equations:

$$\dot{x} = nx - (a - bP_1^* + cP_2^*) - (a - bP_2^* + cP_1^*) \tag{86}$$

$$\dot{E}_1 = \nu_1^* - \delta E_1 \tag{87}$$

$$\dot{E}_2 = \nu_2^* - \delta E_2 \tag{88}$$

$$\dot{\mu}_1 = (r - n)\mu_1 \tag{89}$$

$$\dot{\mu}_2 = (r - n)\mu_2 \tag{90}$$

$$\dot{\mu}_3 = -\frac{\partial \tilde{H}}{\partial E_1} = c_q k + (r+\delta)\mu_3 \tag{91}$$

$$\dot{\mu}_4 = -\frac{\partial \tilde{H}}{\partial E_2} = c_q k + (r+\delta)\mu_4 \tag{92}$$

Once again this system may be solved recursively to find the solution:

$$\mu_1(t) = \mu_1(0)e^{(r-n)t} \tag{93}$$

$$\mu_2(t) = \mu_2(0)e^{(r-n)t} \tag{94}$$

$$\mu_3(t) = c_q kt + \mu_3(0)e^{(r+\delta)t} \tag{95}$$

$$\mu_4(t) = c_q kt + \mu_3(0)e^{(r+\delta)t} \tag{96}$$

$$x(t) = (x_0 + \frac{g(0)}{n})e^{nt} - \frac{g(t)}{n} \tag{97}$$

$$g(t) = (a - bP_1^* + cP_2^*) + (a - bP_2^* + cP_1^*) \tag{98}$$

and

$$E_1(t) = \frac{c_q kt^2}{2\beta} + \frac{\mu_3(0)e^{(r+\delta)t}}{(r+\delta)\beta} - E_1(0)e^{\delta t} \tag{99}$$

$$E_2(t) = \frac{c_q kt^2}{2\beta} + \frac{\mu_4(0)e^{(r+\delta)t}}{(r+\delta)\beta} - E_2(0)e^{\delta t} \tag{100}$$

Let us now consider what happens in the steady-state. First we assume that $r < n$. This assumption is necessary for a steady-state to exist.

Proposition 7. *If $r < n$ then as $t \to \infty$*

$$P_1^{\infty} = \frac{a(c+2b)}{4b^2 - c^2}$$

$$P_2^{\infty} * = \frac{a(c+2b)}{4b^2 - c^2}$$

Proof. Evaluating $\lim_{t \to n} P_1^* = \frac{a(c+2b)}{4b^2-c^2}$ and $\lim_{t \to n} P_2^* = \frac{a(c+2b)}{4b^2-c^2}$ □

The steady-state stock of the resource is once again given by:

$$x_\infty = \frac{2a + (c-b)P_1^\infty + (c-b)P_2^\infty}{n} \tag{101}$$

The impact of competition between firms on the stock of the resource is the same as that considered in section 1, in other words as c increases the steady-state stock of the resource also increases. First consider what happens to the steady-state equilibrium prices when c increases. This results in the following impact on prices:

$$\frac{\partial P^\infty}{\partial c} = \frac{a(4b^2 - c^2) - 2ca(c+2b)}{(4b^2 - c^2)^2} \tag{102}$$

Proposition 8. *As c increases x_∞ also increases as long as $b > c$ and*

Proof. Differentiate and simplify to get

$$\frac{\partial x_\infty}{\partial c} = 2\frac{P^\infty}{n} + 2\frac{(c-b)}{n}\frac{\partial P^\infty}{\partial c}$$

this will be positive if $\frac{1}{P^\infty}\frac{\partial P^\infty}{\partial c} > \frac{1}{b-c}$. It is possible to demonstrate that $\frac{\partial P^\infty}{\partial c}$ is negative by a limit argument. Note that the condition $4b^2 > c^2$ must hold for positive retail prices. $4b^2 - c^2$ is positive implies $b > \frac{c}{2}$ Taking the limit of $\frac{\partial P^\infty}{\partial c}$ as b approaches $\frac{c}{2}$ shows that the derivative is negative. This is definitely true if $b > c$. □

In the case of the search fishery we also need to consider the steady-state level of effort of each fishery:

$$E_1(t) = \frac{c_q kt + \mu_3(0)e^{(r+\delta)t}}{\delta\beta} \tag{103}$$

$$E_2(t) = \frac{c_q kt + \mu_3(0)e^{(r+\delta)t}}{\delta\beta} \tag{104}$$

As $t \to \infty$ these will approach infinity unless either the harvest costs are zero $c_s = 0$ and the search costs β very large. One possible conclusion therefore is that search fisheries tend to encourage an overexpansion of effort compared to school fisheries.

If however one confines oneself to steady-state analysis then because $q_i = kE_i x, i = 1, 2$ and $\dot{x} = nx - q_1 - q_2$ the stock dynamics may be rewritten as $\dot{x} = nx - kE_1 x - kE_2 x = nx - kx(E_1 + E_2)$ so that in steady state:

$$\frac{n}{k} = E_1(t) + E_2(t) \tag{105}$$

or

$$n = k(E_1(t) + E_2(t) \tag{106}$$

Consequently the accumulation of effort (capital) is bounded by the ratio of the growth rate of the stock to the catchability of fish. One interpretation of this is that this is a measure of escapement. Either way it placess an upper bound on fishing effort in steady-state.

Substituting this back into x_∞ enables the exptression of the steady-state stock in terms of both demand effort characteristics.

$$x_\infty = \frac{2a + (c-b)P_1^\infty + (c-b)P_2^\infty}{k(E_1(t) + E_2(t))} \tag{107}$$

Clearly as effort expands cetris-paribus the steady-state stock of fish declines. An example would be increased harvesting costs which would reduce the steady-state stock of fish, whereas an increase in search costs, e.g. fuel costs, would increase the steady-state stock of fish.

5 Conclusion

In this paper a model of exploitation of a common fishery has been presented in which two firms compete with each other via price competition rather than via quantity competition. this set-up is more suited to the analysis of natural resources that are characterised by product heterogeneity. The model was formulated as a linear-state differential game under a variety of different cost assumptions and closed form solutions for each of these scenarios were derived. Becaus of the linear-state nature of the problem closed loop and open loop strategies coincide in all cases. In all cases it can be seen that competitive pressures lead to prices being lowered and the steady-state stock of fish increasing. In the case of the search fishery there is a tendency for fishing effort to expand thereby driving the fishery to exctinction. For schooling fisheries with and without search costs the fishery remains sustainable as long as $b > c$, i.e. the own price effect on demand is greater than the cross-price effect. For the case where $\frac{c}{2} < b < c$ price competition could lead to extinction if the cross-price effect on demand c becomes sufficiently large.

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