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TIME-TREND IN SPATIAL DEPENDENCE: SPECIFICATION STRATEGY IN THE FIRST-ORDER SPATIAL AUTOREGRESSIVE MODEL

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Abstract. The purpose of this article is to analyze if spatial dependence is a synchronic effect in the first-order spatial autoregressive model, SAR(1). Spatial dependence can be not only contemporary but also time-lagged in many socio-economic phenomena. In this paper, we use three Moran-based space-time autocorrelation statistics to evaluate the simultaneity of this spatial effect. A simulation study shed some light upon these issues, demonstrating the capacity of these tests to identify the structure (only instant, only time-lagged or both instant and time-lagged) of spatial dependence in most cases.

Key words: Space-time dependence, Spatial autoregressive models, Moran's ***I***.

Resumen. En este artículo, se analiza la instantaneidad de la dependencia especial en el modelo auto-regresivo espacial de primer orden o SAR(1). En muchos fenómenos socioeconómicos, el fenómeno de dependencia especial puede ser no sólo contemporáneo o instantáneo, sino también retardado en el tiempo. Con ayuda de tres estadísticos espacio-temporales basados en el test ***I*** de Moran, evaluaremos la instantaneidad de este efecto especial. Además, se demostrará la capacidad de dichos tests para identificar la estructura de dependencia espacial (sólo instantánea, sólo retardada en el tiempo y mixta) de la mayoría de las distribuciones, a través de un ejercicio de simulación de Monte-Carlo.

Key words: Dependencia espacio-temporal, Modelos espaciales auto-regresivos, ***I*** de Moran.

JEL classification: C15, C21, C51

1 INTRODUCTION

The purpose of this article is to analyze the time-trend of spatial dependence in the first-order spatial autoregressive model, SAR(1), making a differentiation between two types of spatial dependence: contemporary or instant and non-contemporary or time-lagged. The first type is the consequence of a very quick interaction of the process over the neighboring locations, while the second implies that a shock in a certain location needs some time to extend over its neighborhood. It is not easy to separate both types of spatial dependence but both are present very frequently and should be considered when specifying a spatial autoregressive model.

Spatial dependence has usually been defined as a spatial effect, which is related to the spatial interaction existing between geographic locations and *takes place in a particular moment of time*. In other words, spatial dependence is considered as the contemporary coincidence of value similarity with locational similarity. When spatial interaction, spatial spillovers or spatial hierarchies produce spatial dependence in the endogenous variable of a regression model, the spatial autoregressive model has been frequently mentioned as the solution in the literature (e.g. Florax *et al.* 2003). Analogous to the Box-Jenkings approach in the time-series analysis, spatial model specifications consider autoregressive processes. Particularly, in the first-order spatial autoregressive model, SAR(1), a variable is a function of its spatial lag (a weighted average of the value of this variable in the neighboring locations) for *a same moment of time*.

However, in most socio-economic phenomena, this coincidence in values-locations is not only an instant coincidence but also (or perhaps only) a final effect of some cause that happened in the past, one that has spread through geographic space during a certain period. In this sense, there are some authors that have considered this pure simultaneity of spatial dependence as problematic (Upton and Fingleton 1985, pp 369), suggesting the introduction of a time-lagged spatial dependence term. Moreover, Cressie (1993, pp 450) proposes a generalization of the STARIMA models presented in Martin and Oeppen (1975) and Pfeifer and Deutsch (1980), among others, such that they also include not only time-lagged but also “instantaneous spatial dependence”.

Recently, there are several contributions in this subject. For example, Elhorst (2001, 2003) presented several single equation models that include a wide range of substantive non-contemporary spatial dependence lags, not only in the endogenous but also in the exogenous variables. Anselin *et al.* (2005) present a brief taxonomy for panel data models with different kind of spatial dependence structure for the endogenous variable (space, time and space-time), referring to them as pure space-recursive, time-space recursive, time-space simultaneous and time-space dynamic models.

Space-time dependence has also been specified in spatial autoregressive models in either theoretical frameworks (Baltagi *et al.* 2003; Pace *et al.* 1998, 2000) or panel data applications (Case 1991; Yilmaz *et al.* 2002; Baltagi and Li 2003; Mobley 2003).

In this article, we analyze if spatial dependence is a synchronic effect in the SAR(1) model, allowing for not only horizontal (static) but also space-time interaction (dynamic). Similarly as in Pace *et al.* (1998, 2000), it is our aim to identify different components in the spatial lag term, splitting it into instant, space-time or both –instant and space-time- spatial dependence components. Therefore, we propose the identification and use –if necessary- of the space-time lagged endogenous variable in the SAR(1) model, since it reflects the effects due to spatial interaction as a spatial diffusion phenomena, which is not only "horizontal" – simultaneous – but also time-wise.

For this purpose, we present some space-time Moran-based statistics in order to identify the spatial dependence structure in the SAR(1) model. We illustrate the performance of these statistics with a simulation exercise.

The remainder of paper is organized as follows. In the next section, we derive three Moran-based autocorrelation statistics to evaluate the simultaneity of the SAR(1) model. In section 3, we present three different specifications of spatial dependence in this model, as well as the behavior of the identification statistics in each one. In section 4, we evaluate the power of the tests with a simulation analysis. Some summary conclusions and references complete the paper.

2 MORAN SPACE-TIME STATISTICS FOR THE EVALUATION OF SPATIAL DEPENDENCE IN THE FIRST-ORDER SPATIAL AUTOREGRESSIVE MODEL

The first-order spatial autoregressive model, SAR(1), or simultaneous model dates back to the work of Whittle (1954). In matrix notation, it takes the form:

$$\begin{aligned} z &= \rho W z + \varepsilon \\ \varepsilon &\sim N\left(0, \sigma^2 I_n\right) \end{aligned} \qquad (1)$$

where $z = [y - \overline{y}]/\sigma_y$ is a *n* by 1 vector of observations (variable vector *y* is expressed in deviations from the means form to eliminate the constant term in the model); *W* is the spatial weight matrix; ρ is the spatial autoregressive coefficient; and ε is a *n* by 1 vector of random error terms. *W* is the familiar spatial weight matrix that defines the neighborhood interactions existent in a spatial sample (Cliff and Ord 1981). In this context, the usual row-standardized form of the spatial weights matrix can be used, yielding an interpretation of the spatial lag (*Wz*) as an "average" of neighboring values.

The spatial term, *Wz*, is a way to assess the degree of spatial dependence of *z in a same moment of time* (from now on, it is denoted as Wz_t). Nevertheless, in most socio-economic phenomena, the relationship between z_t and Wz_t is not only synchronic but also –or perhaps only- a final effect of some cause that happened in the past (Wz_{t-k}; $k = 1, 2, ...$). Consequently, before estimating a SAR(1) model, we should identify correctly the form of the spatial effect, Wz_t, in this model.

In this section, we present some Moran-based statistics that are useful to detect the existence of time-lagged spatial dependence. First, we briefly present the space-time Moran's *I* statistic (STI), which evaluates spatial dependence in two instant of time. Secondly, we present two partial space-time Moran's functions: the partial time-lagged Moran's *I* (PLI) and the partial instant Moran's *I* (PII). Our goal is to contribute towards obtaining appropriate indicators to evaluate the temporal structure of spatial dependence in the SAR(1) model.

2.1 Space-time autocorrelation

When considering both space-time dimensions, some Moran-based statistics can be defined to analyze and visualize the space-time structure of a distribution (Anselin *et al*. 2002). This is the case of the **space-time Moran's *I* (STI)**. This instrument is similar to others already proposed in the literature (e.g. Cliff and Ord 1981, pp. 23).

The STI is an extension of Moran's *I*. It computes the relationship between the spatial lag, Wz_t, at time *t* and the original variable, *z*, at time $t-k$ (*k* is the order of the time lag). Therefore, this statistic quantifies the influence that a change in a spatial variable *z*, that operated in the past ($t-k$) in an individual location *i* (z_{t-k}) exerts over its neighborhood at present (Wz_t). Hence, it is possible to define it as follows:

$$I_{t-k,t} = \frac{z'_{t-k} W z_t}{z'_{t-k} z_{t-k}} \qquad (2)$$

where, the denominator can be substituted by n as this variable z is also standardized. The value adopted by this index, corresponds with the slope in the regression line of Wz_t on z_{t-k}. Note that for $k = 0$, this statistic coincides with the familiar univariate Moran's I that from now on, we denote as I_t.

The significance of this statistic can be assessed in the usual fashion by means of a randomization (or permutation) approach. In this case, the observed values for one of the variables are randomly reallocated to locations and the statistic is recomputed for each such random pattern.

A space-time Moran's I function could be considered. It is the result of plotting all the values of the STI statistic, adopted by a variable z in time t, for different time lags k. The first value corresponds to the contemporary case, $k = 0$, which is the univariate Moran's I (I_t), whereas the other ones are proper space-time Moran's I coefficients ($I_{t-k,t}$). This function is a particular case of the "full" space-time autocorrelation function (Pfeifer and Deutsch 1980, Bennett 1979), which is a 3-D plot that includes the correlation coefficients for all the space and time lags of a distribution.

2.2 Moran space-time partial autocorrelation statistics

There is no doubt that the spatial dependence measures that have been presented include different sources of dependence that are difficult to separate.

$$Cov(z_{it}, z_{js}) \neq 0 \qquad (3)$$

where sub-indexes i, j are different spatial locations and t, s are different instants of time.

Therefore, we consider the following types of dependencies:

(a) There is a dependence in expression (4) that is the result of time evolution:

$$Cov(z_{it}, z_{js}) \neq 0 \quad ; \quad \forall i = j \qquad (4)$$

This expression affirms that (for $s = t - k$) the value of the z variable in period t is more or less related to $t - k$. This assertion is more correct for lower values of k.

(b) There is a dependence in expression (4) that is the result of spatial interactions:

$$Cov(z_{it}, z_{js}) \neq 0 \qquad ; \qquad \forall t = s \tag{5}$$

This second type of dependence –spatial dependence- can be produced by two sources:

(b1) **Simultaneous or contemporary spatial dependence** constitutes the usual definition of spatial dependence in the literature and it is the consequence of an instant, very rapid, spatial diffusion of a phenomenon in geographic space. It can be connected to or the consequence of a lack of concordance between a spatial observation and the region in which the phenomenon is analyzed.

(b2) **Time-lagged or non-contemporary spatial dependence** is the result of a slower diffusion of a phenomenon towards the surrounding space. This kind of dependence is due to the usual interchange flows existing between neighboring areas, which requires of a certain time to be tested.

Although it is very difficult to divide spatial dependence into its two dimensions (instant and time-lagged), it is worth trying to compute them separately in order to correctly specify a spatial process that exhibits spatial dependence. One of the aims of this article is to show a new range of Moran-based statistics that allow justifying the inclusion of both kind of spatial lags, contemporary (Wz_t) and time-lagged (Wz_{t-k}) ones, to explain z_t in a spatial regression. Some coefficients can be defined to evaluate the inclusion of a space-time lag term in a spatial regression.

Since the space-time Moran's *I* (STI) –equation (2)- equals to the slope of the regression of Wz_{t-k} on z_t, it is possible to connect this statistic with the standard Pearson correlation coefficient between these two variables, as also derived by Lee (2001). So we can express the STI statistic as:

$$I_{t-k,t} = r_{z_{t-k}, Wz_t} \sqrt{Var(Wz_t)} \tag{6}$$

where r_{z_{t-k}, Wz_t} is the Pearson linear correlation coefficient between z_{t-k} and Wz_t.

The basic underlying idea consists of eliminating the influence of one of the dimensions in order to compute separately contemporary and non-contemporary spatial dependence. For this purpose, we substitute in (6) the space-time correlation coefficient by a partial correlation one. Two space-time partial autocorrelation statistics can be defined:

(a) Partial Time-Lagged Moran's *I* (PLI) computes the correlation between variable *z* in period *t-k* and its spatial lag *Wz* in period *t* removing the influence of *z* in *t*:

$$I_{t-k,t}^{P} = Corr(z_{t-k}, Wz_t \mid z_t)\sqrt{Var(Wz_t)} \quad ; \quad k = 1, 2, ..., t-1 \tag{7}$$

where $Corr(z_{t-k}, Wz_t \mid z_t) = \frac{r_{z_{t-k},Wz_t} - r_{z_{t-k},z_t} \cdot r_{Wz_t,z_t}}{\sqrt{1-r_{z_{t-k},z_t}^2} \cdot \sqrt{1-r_{Wz_t,z_t}^2}}$ is the partial correlation coefficient of variables z_{t-k} and Wz_t after eliminating the correlation from z_t. Therefore, it is possible to express this statistic as a function of both Moran's *I* and space-time Moran's *I*:

$$I_{t-k,t}^{P} = \frac{I_{t-k,t} - r_{z_{t-k},z_t} \cdot I_t}{\sqrt{1-r_{z_{t-k},z_t}^2} \cdot \sqrt{1-r_{z_t,Wz_t}^2}} \tag{8}$$

This indicator removes contemporary spatial dependence from the relationship between variables z_{t-k} and Wz_t. If the pattern of spatial dependence is one that can be totally captured by contemporary spatial dependence ($I_t \neq 0$), then the PLI will be close to zero. On the contrary, if the process is one that can be captured by non-contemporary spatial dependence, then $I_{t-k,t}^{P}$ will be significantly different from zero. Regarding to the sign, it is positive/negative depending on $I_{t-k,t}$ sign.

(b) Partial Instant Moran's *I* (PII) is the complementary expression that consists of computing contemporary –or instant- spatial dependence after removing time-lagged spatial dependence by means of an index:

$$I_t^{P_k} = Corr(z_t, Wz_t \mid z_{t-k})\sqrt{Var(Wz_t)} \quad ; \quad k = 1, 2, ..., t-1 \tag{9}$$

where $Corr(z_t, Wz_t \mid z_{t-k})$ is the partial correlation coefficient of variables z_t and Wz_t after eliminating the correlation from Wz_{t-k}. Therefore, it is possible to express this statistic as a function of both Moran's *I* (I_t) and space-time Moran's *I* ($I_{t-k,t}$).

$$I_t^{P_k} = \frac{I_t - r_{z_{t-k},z_t} \cdot I_{t-k,t}}{\sqrt{1-r^2_{z_{t-k},z_t}} \cdot \sqrt{1-r^2_{Wz_t,z_{t-k}}}} \qquad (10)$$

This indicator removes time-lagged spatial dependence from the contemporary spatial relationship between variables z_t and Wz_t. If the pattern of spatial dependence is one that can be totally captured by a time-lagged spatial autoregression, then the PPI will be close to zero. On the contrary, if the process is one that can be captured by contemporary spatial dependence, then PPI will be significantly different from zero.

From (8) and (10), it is easy to derive the following expression:

$$I_t = \frac{\sqrt{1-r^2_{Wz_t,z_{t-k}}}}{\sqrt{1-r^2_{z_{t-k},z_t}}} I_t^{P_k} + \frac{\sqrt{1-r^2_{Wz_t,z_t}} \cdot r_{z_{t-k},z_t}}{\sqrt{1-r^2_{z_{t-k},z_t}}} I^P_{t-k,t} \;\; ; \; k = 1, ..., t-1 \qquad (11)$$

In this expression, spatial dependence (measured with Moran's *I* statistic or I_t) is shown as the sum of two contributions: contemporary spatial dependence (PII or $I_t^{P_k}$) and non-contemporary spatial dependence (PLI or $I^P_{t-k,t}$), both weighted by a corresponding scalar.

In case of normal distribution, the inference of the common partial correlation coefficient can be applied to both Moran space-time partial autocorrelation statistics, as they are the result of multiplying the former by a constant. In case of non-normality, a permutation approach can be the solution to compute the moments.

3. IDENTIFICATION OF SPATIAL DEPENDENCE STRUCTURE IN THE FIRST-ORDER SPATIAL AUTOREGRESSIVE MODEL

The joint representation of the space-time Moran's *I* coefficient (STI) in combination with the Moran space-time partial autocorrelation statistics (PLI, PII) is a useful tool to identify the simultaneity of spatial dependence in the SAR(1) model. The identification of spatial dependence structure should be conducted in two steps as follows:

Step 1. The space-time Moran's *I* function (STI) indicates the existence (or non-existence) of a temporal trend in the spatial term of the SAR(1) model (*Wz*). If the STI values are significant (using the regular inference process), we can conclude that there is time-lagged spatial dependence in the corresponding distribution, and vice versa.

Step 2. Secondly, the Moran space-time partial autocorrelation statistics (PLI, PII) are the instrument to determine whether the existent spatial dependence contains an instant and/or time-lagged component.

Contemporary or instant spatial dependence is present in a variable if only the partial instant Moran's *I* (PII) has significant values. In this case, only the present values of the variable (z_t) can explain its present spatial lag (Wz_t). In a spatial regression, if an endogenous variable z_t exhibits significant STI and PII values, we could capture spatial autocorrelation by a contemporary spatial lag of z_t (Wz_t) as an explicative variable in the model.

$$z_t = \rho W z_t + \varepsilon_t \qquad (12)$$

where ρ is the spatial parameter to estimate and ε the error term. This is the SAR(1) model.

Non-contemporary or time-lagged spatial dependence is present in a variable if only the partial time-lagged Moran's *I* (PLI) has significant values. In fact, past values of variable *z* (z_{t-k}) completely explain its present spatial lag (Wz_t). In this case, we could capture spatial dependence in an endogenous variable z_t introduction a space-time lag of *z* (Wz_{t-k}) as an explicative, exogenous, variable in the model.

$$z_t = \rho W z_{t-k} + \varepsilon_t \qquad (13)$$

Mixed contemporary and non-contemporary spatial dependence is present in a variable if both partial functions have high significant values for the same periods. In this case, not only present but also past values of variable *z* can completely explain its present spatial lag. Therefore, we could capture spatial dependence in an endogenous variable z_t specifying both an instant and a time-lagged spatial lag of *z* (Wz_t, Wz_{t-k}) as explicative variables in the model.

$$z_t = \rho_1 W z_t + \rho_2 W z_{t-k} + \varepsilon_t \qquad (14)$$

where ρ_1, ρ_2 are spatial parameters to estimate. This model is the mixed regressive-spatial autoregressive model or space lag model, which includes as explicative not only the spatial-lagged endogenous variable (Wz_t), but also real exogenous variables (Wz_{t-k}).

4. MONTE CARLO SIMULATION STUDY

Next, we evaluate empirically the Moran space-time autocorrelation statistics (STI, PLI, PII) in a series of Monte Carlo simulation in order to obtain an initial assessment of their discriminating power of spatial dependence into instant and/or time-lagged.

5.1. Experimental design

We consider two different moments of time t, s, such that $s < t$. For time s, we generate an instant spatial dependence process (z_s) whereas for time t we set up three alternative processes (z_t): instant spatial dependence (*i*), time-lagged spatial dependence (*ii*) and both instant and time-lagged spatial dependence (*iii*). Formally:

i) For time t, instant spatial dependence only:

$$\begin{aligned} z_s &= \rho W z_s + \varepsilon_s \\ z_t &= \rho W z_t + \varepsilon_t \end{aligned} \tag{15}$$

ii) For time t, time-lagged spatial dependence only:

$$\begin{aligned} z_s &= \rho W z_s + \varepsilon_s \\ z_t &= \rho W z_s + \varepsilon_t \end{aligned} \tag{16}$$

iii) For time t, both instant and time-lagged spatial dependence:

$$\begin{aligned} z_s &= \rho W z_s + \varepsilon_s \\ z_t &= \rho_1 W z_t + \rho_2 W z_s + \varepsilon_t \end{aligned} \tag{17}$$

where ρ, ρ_1, ρ_2 are the spatial autoregressive coefficients and ε_s, ε_t are the error terms for time s, t, respectively. For the sake of simplicity, we specify the same spatial weight matrix, W, in all cases. It is a queen-case contiguity matrix, as it is defined in Anselin (1988, p. 18).

Clearly, we can obtain different degrees of instant and time-lagged spatial dependence by manipulating the values of the parameters and the characteristics of the error terms. In our experiments, we set the purely temporal dependence structure in variable *z* by means of the error terms (ε_s, ε_t), which are generated as a bivariate normal distribution with zero mean and $\Sigma = \left(\sigma_{ij}\right)$ covariance matrix, such that $\sigma_{ii} = 1$ and $\sigma_{ij} = r$. Temporal autocorrelation is defined by the *r* parameter considering three different cases: weak temporal autocorrelation ($r = 0.5$), moderate temporal autocorrelation ($r = 0.75$) and strong temporal autocorrelation ($r = 0.98$).

The spatial configuration we have consider is a regular lattice structure for a queen-type contiguity in square 10 by 10 (*N* = 100). The spatial weights matrix is used in row-standardized form.

In the case of expression (17) –both instant and time-lagged spatial dependence in time *t*, we also consider two different intensities in spatial dependence:

a) Stronger instant spatial dependence: $\rho_1 = 2\rho_2$ and $\rho_1 + \rho_2 = \rho$

b) Stronger time-lagged spatial dependence: $2\rho_1 = \rho_2$ and $\rho_1 + \rho_2 = \rho$

Therefore, we compute the space-time partial autocorrelation statistics in time *t*, with respect to time *s*, for the different values of parameters *r*, ρ. We have only considered positive spatial autocorrelation structures, which is the most common tendency found in empirical cases (Griffith and Arbia 2006). In the simulations, we let ρ vary from *0* to 1, with increments of 0.05, to assess the effect of spatial lag autocorrelation. We have generated 9,999 replications for each situation. We have evaluated the power of the test statistics (STI, PLI, PII) to discriminate between instant, time-lagged or both kind of spatial dependence structure in the SAR(1) model.

5.2. Power of the tests

Figures 1-4 present the simulation results. In the upper-left cell, we have plotted the power of the selection criterion as the number of times that a partial index is higher than the other one. The rest of the graphs show the average values of the STI, PLI and PII for the three types of temporal autocorrelation ($r = 0.5,\ 0.75,\ 0.98$). Each Figure specifies a different data generating process.

In Figure 1, which corresponds to model (15), the average values of PII are quite similar to STI ones, while PLI adopts almost zero values. This gap is higher for lower temporal autocorrelation ($r = 0.5,\ 0.75$), but in the case of $r = 0.98$, though PLI values are almost zero, PII coefficients move down away from STI. It is a natural effect, since such a strong temporal autocorrelation ($r = 0.98$) induces an even stronger correlation between z_t and z_s (0.998) what leads to a practical identification between both processes. Regarding the power of the partial tests (number of times $\mathrm{PII} > \mathrm{PLI}$) to select the correct model, it loose strength with higher temporal correlation (r), but it augments with stronger spatial autocorrelation (ρ).

When having the pure time-lagged spatial dependence situation shown in (16), both partial tests get positive average values (Figure 2). These figures decline with temporal autocorrelation so that in the extreme case of $r = 0.98$, both PII and PLI are nearly zero. In this situation, the number of times $\mathrm{PLI} > \mathrm{PII}$ is around 80% for moderate time-lagged spatial dependence ($\rho \geq 0.4$) and weak temporal correlation ($r = 0.5$). This percentage declines for higher levels of temporal correlation but increases with spatial autocorrelation.

The situation described in model (17) –mixed instant and time-lagged spatial dependence- is more difficult to detect because they are intermediate cases between models (15) and (16). As previously described, we have considered two states: stronger instant spatial dependence (Figure 3) and stronger time-lagged spatial dependence (Figure 4). In the first case, PII is always over PLI, though the gap is not as steeper as in the generating process shown in Figure 1. That is why the percentage of times that PII>PLI is smaller (about 70%). As in the other cases, higher temporal correlation produces an identification of both processes z_t and z_s, so that it is more difficult to differentiate between them.

5 CONCLUSIONS

The main aim of this paper is to analyze to what extent spatial dependence is an instantaneous effect in the SAR(1) model, making a differentiation between instant or contemporaneous and lagged or non-contemporaneous spatial dependence. The first is the consequence of a very quick diffusion of the process over the neighboring locations, while the second implies that a shock in a certain location needs of several periods to take place and be tested over its neighborhood. It is –without any doubt- a subject to consider when working with space-time distributions, though it is not easy to differentiate between both kinds of dependencies, especially when dealing with highly correlated time-series.

For the fulfillment of this aim, we propose the use of three Moran's ***I***-based statistics: space-time Moran's ***I***, partial instant Moran's ***I*** and partial time-lagged Moran's ***I***.

We illustrate the power of these tests for the identification of spatial dependence structure via a simulation exercise. Specifically, we build three data generating processes based on different spatial dependence structures: only instant, only time-lagged and both instant and time-lagged.

In the primary two cases, this experiment has clearly revealed the good performance of the Moran's tests, mainly in processes with strong spatial autocorrelation (instant and/or time-lagged) and weaker temporal correlation. Nevertheless, in the extreme case of (nearly) null spatial autocorrelation and (nearly) perfect temporal correlation, it is very difficult to split both kind of spatial dependence, since there is a practical identification between the process in both –past and present-periods.

Regarding the mixed spatial dependence, the space-time Moran's tests are less efficient in general. Only in the case of extreme spatial autocorrelation and weak temporal correlation, the performance of these statistics is acceptable in order to split spatial dependence into instant and time-lagged.

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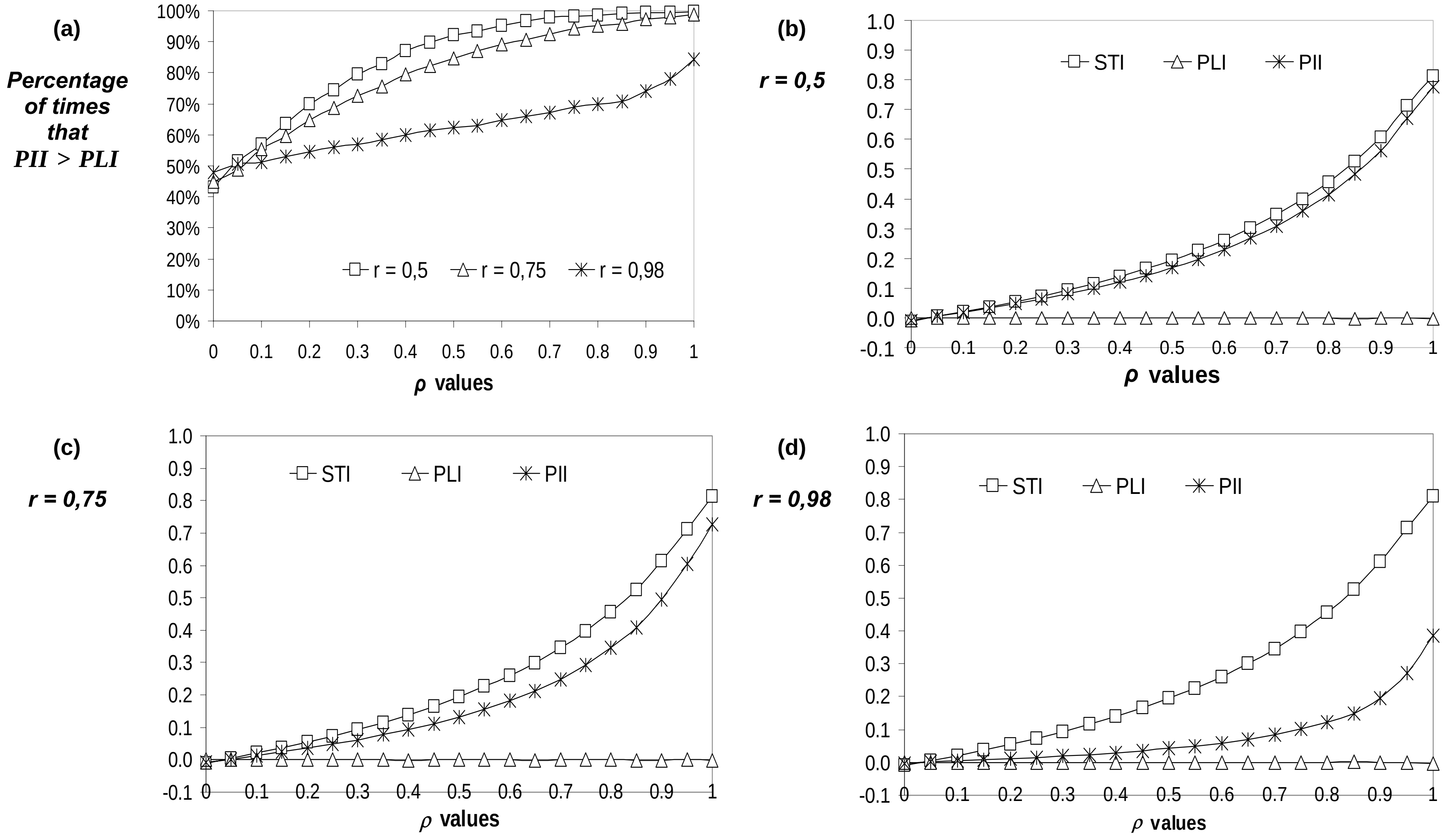

Fig. 1a-d. Power of space-time Moran's *I* coefficients against instant spatial dependence only (9,999 replications). STI is the space-time Moran's *I*, PII is the partial instant Moran's *I*, PLI is the partial time-lagged Moran's *I*. Case (**a**) shows the percentage of times that $PII > PLI$, case (**b**) computes the indices for weak temporal correlation ($r = 0.5$), case (**c**) computes them for moderate temporal correlation ($r = 0.75$) and case (**d**) corresponds to strong temporal autocorrelation ($r = 0.98$).

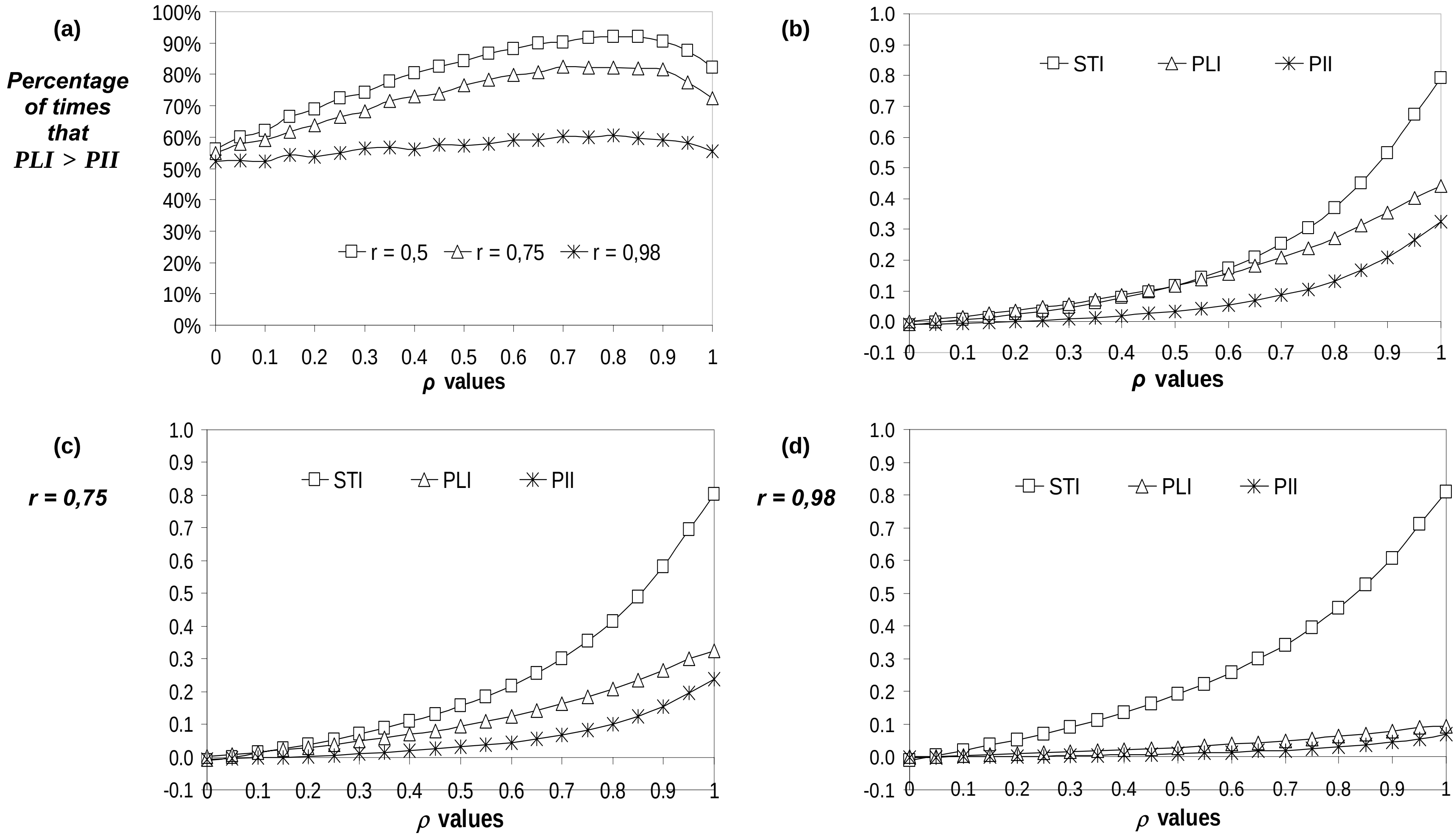

Fig. 2a-d. Power of space-time Moran's *I* coefficients against time-lagged spatial dependence only (9,999 replications). STI is the space-time Moran's *I*, PII is the partial instant Moran's *I*, PLI is the partial time-lagged Moran's *I*. Case (**a**) shows the percentage of times that $PLI > PII$, case (**b**) computes the indices for weak temporal correlation ($r = 0.5$), case (**c**) computes them for moderate temporal correlation ($r = 0.75$) and case (**d**) corresponds to strong temporal autocorrelation ($r = 0.98$).

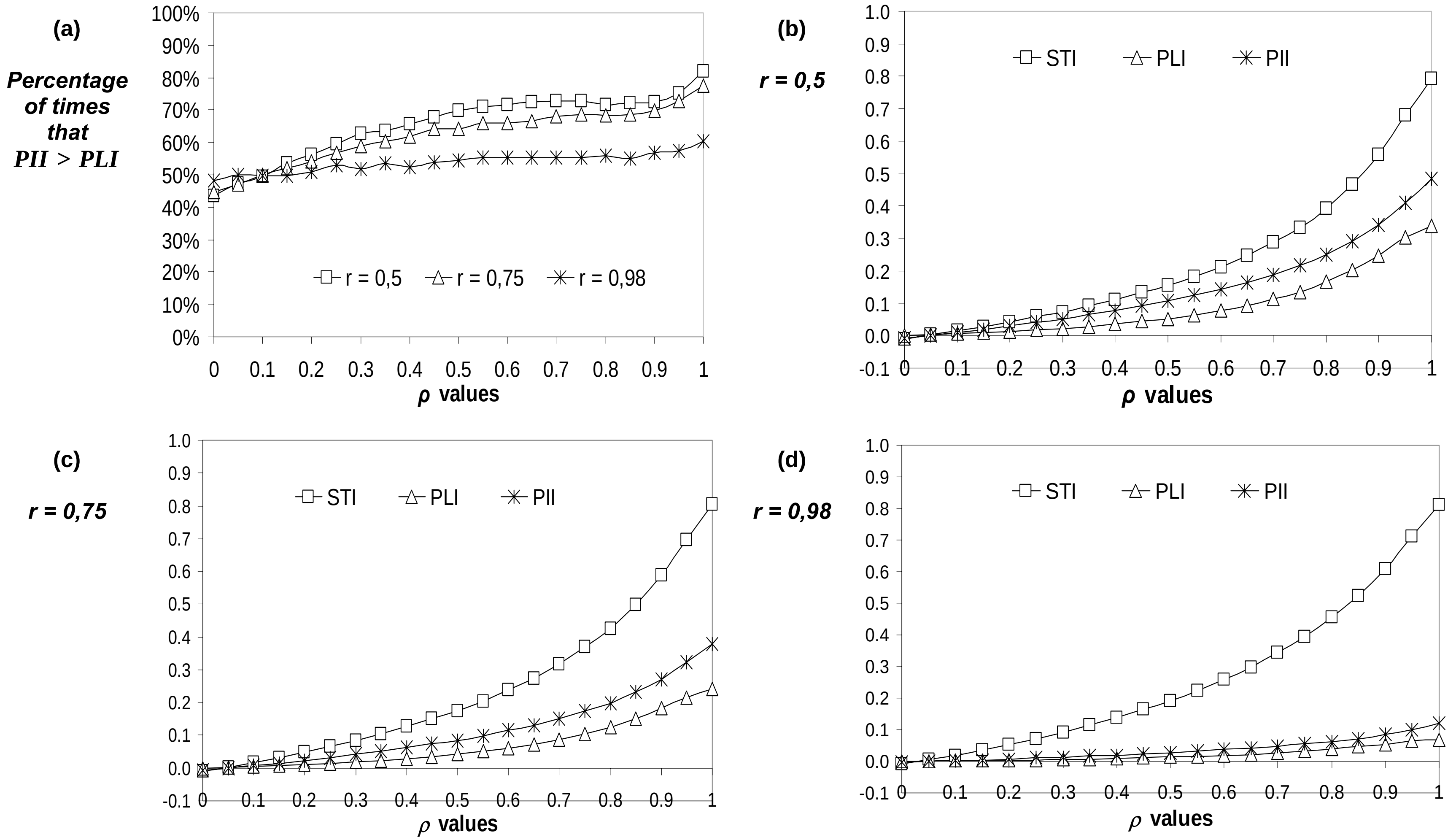

Fig. 3a-d. Power of space-time Moran's *I* coefficients against both instant and time-lagged spatial dependence, for stronger instant spatial dependence (9,999 replications). STI is the space-time Moran's *I*, PII is the partial instant Moran's *I*, PLI is the partial time-lagged Moran's *I*. Case (**a**) shows the percentage of times that $PII > PLI$, case (**b**) computes the indices for weak temporal correlation ($r = 0.5$), case (**c**) computes them for moderate temporal correlation ($r = 0.75$) and case (**d**) corresponds to strong temporal autocorrelation ($r = 0.98$).

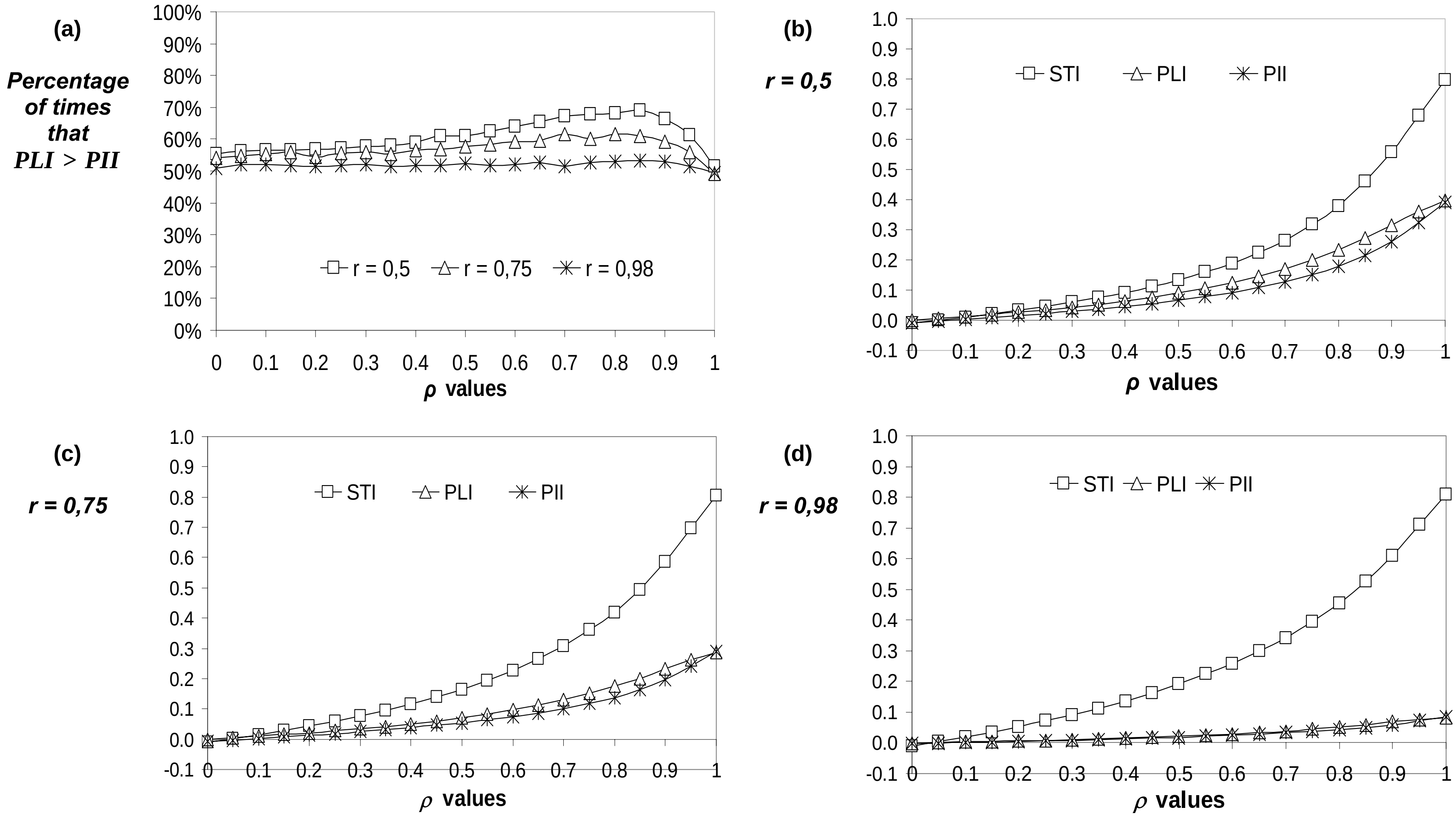

Fig. 4a-d. Power of space-time Moran's *I* coefficients against both instant and time-lagged spatial dependence, for stronger time-lagged spatial dependence (9,999 replications). STI is the space-time Moran's *I*, PII is the partial instant Moran's *I*, PLI is the partial time-lagged Moran's *I*. Case (**a**) shows the percentage of times that $PII > PLI$, case (**b**) computes the indices for weak temporal correlation ($r = 0.5$), case (**c**) computes them for moderate temporal correlation ($r = 0.75$) and case (**d**) corresponds to strong temporal autocorrelation ($r = 0.98$).