Unit Roots Tests with Smooth Breaks: An Application to the Nelson-Plosser Data Set

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14. February 2008

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February 14, 2007

Abstract

This paper reconsiders the nature of the trends (i.e. deterministic or stochastic) in macroeconomic time series. For this purpose, the paper employs two new tests that display robustness to structural breaks of unknown forms, irrespective of the date and/or location of the breaks. These tests approximate structural changes as smooth processes via Flexible Fourier transforms. The tests deliver strong evidence in favor of a nonlinear deterministic trend for real GNP, real per capita GNP, employment, the unemployment rate, and stock prices. Further, the two tests confirm the existence of stochastic trends in nominal GNP, consumer prices, real wages, monetary aggregates, velocity, and bond yields. In general, it appears that real variables are stationary while nominal ones have a unit root.

Keywords: Unit Roots, Stationarity Tests, Structural Change
JEL-Classification: C50, E10

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*I am grateful to Walter Enders and Junsoo Lee for helpful comments and suggestions. Any remaining errors are my own.

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1 Introduction

One of the most studied and questioned topics in the applied unit root literature is whether macroeconomic time series, in particular those considered by Nelson and Plosser (1982), are random walks or stationary processes around a level or a trend. The issue of stochastic versus deterministic trends in time series models has important practical policy considerations.

The Nelson and Plosser (1982) article has generated a significant amount of literature as reflected by the numerous unit root and stationarity tests developed[1]. Until the empirical work of Nelson and Plosser (1982) (i.e., abbreviated as NP), the general view was that macroeconomic time series were stationary around either a deterministic trend or level (see Kydland and Prescott (1980); Blanchard (1981); Barro (1976)). However, using the unit root tests of Dickey and Fuller (1979) and Dickey and Fuller (1981) (abbreviated as DF and ADF), Nelson and Plosser (1982) find that with one exception (i.e., the unemployment rate) all historical time series have a unit root. This finding supports the real business cycle hypothesis and goes against the deterministic approach which separates business cycles from trend growth (see Rudebusch (1993)). The paper of Nelson and Plosser (1982) initiated a long debate, with subsequent research both partially confirming and challenging its findings. I attempt only a partial and brief review of this literature.

Phillips and Perron (1988) depart from the standard Dickey-Fuller assumptions (i.e. Dickey and Fuller (1979)) of independently and identically distributed (*i.i.d.*) errors and in a non-parametric fashion develop new unit root tests (abbreviated as PP tests) that are robust to heterogeneity and serial correlations in errors. However, both ADF and PP tests have the same limiting distribution and thus lead to the same qualitative results. Departing

[1] A test with the null of I(0) will be called a stationarity test, and as usual a test with the null of I(1) is called a unit root test

from the world of DF-type tests, Sargan and Bhargava (1983) and Bhargava (1986) suggest tests in the Durbin-Watson framework. Further, based on one of Bhargava's statistics, Schmidt and Phillips (1992) derive a Lagrange Multiplier test or score test (abbreviated as LM) which they argue is more powerful than the DF-type tests.

The stationarity test of Kwaitowski et al. (1992) (i.e. KPSS for short) applies the LM principle to a general error process similar to the one in the PP-type tests. Their model has the following representation:

$$y_t = \xi t + r_t + \varepsilon_t \tag{1}$$

where r_t is a random walk:

$$r_t = r_{t-1} + u_t \tag{2}$$

and u_t is *iid* (0,σ_u^2). The null hypothesis of stationarity is stated as $\sigma_u^2 = 0$. Kwaitowski et al. (1992) apply their test to NP data and find support for the trend stationarity hypothesis for six series (i.e., real per capita GNP, employment, the unemployment rate, the GNP deflator, wages, and the monetary aggregates). Using a similar set-up, Leybourne and McCabe (1994) modify the KPSS test to form a stationarity test in the DF-type framework. Leybourne and McCabe (1994) use the data set from Schwert (1987)[2] and in contrast to Kwaitowski et al. (1992), they find that monetary base, CPI, and wages clearly have a unit root. Kwaitowski et al. (1992) also compare their findings with the results from the ADF/PP unit root tests to perform a so-called confirmatory analysis. They conclude that the unemployment rate is stationary while CPI, real wages, velocity, and stock prices have

[2]The data in Schwert (1987) contains the following series: monetary base, bond yields, consumer prices, producer prices,wages, civilian population,labor force, employment, the unemployment rate, industrial production, price/earnings ratio, dividend yield for the S&P composite portfolio, volatility of stock returns, price deflator, GNP, and real GNP. These are either monthly or quarterly and start from 1947 to 1954 and end in 1985

a unit root. However, they find less evidence in favor of a unit root in real GNP, nominal GNP, and the interest rate. Finally, for real per capita GNP, industrial production, employment, GNP deflator, wages, and monetary aggregates they conclude the evidence is mixed. It is not possible to reject either the unit root or the trend stationarity hypothesis for these series.

Another line of research approaches the issue from a Bayesian perspective. In this regard, it appears that the results concerning the stationarity of NP data differ with the choice of the prior distribution. Following Sims (1988), DeJong and Whiteman (1991) use a flat prior and based on the estimated posterior probabilities of the dominant root they find evidence of a unit root only for bond yields, for velocity, and, marginally, for consumer prices. However, Phillips (1991) argues that these results are driven by the use of the flat prior, which he argues is not a satisfactory representation of "uninformativeness". He suggests an ignorance prior (i.e. Jeffrey's prior), that when applied to the NP data reverses the findings of DeJong and Whiteman (1991). Subsequent research has confirmed the importance of the choice for the form of the prior distribution (see DeJong (1992); Zivot and Phillips (1994)).

However, as Perron (1989) points out, all these tests can be misleading if one does not account for the possibility of structural breaks in the time trend or level. His seminal paper led to a new area of research which seeks to develop unit root tests that are robust to structural breaks and outliers in the data. Perron (1989) shows that if one is willing to assume an exogenous break[3] in 1929, then for eleven of the initial fourteen series of Nelson and Plosser (1982), the unit root hypothesis can be rejected at a high significance level. However, Christiano (1992) argues that when the break is allowed to be endogenous, then the evidence against unit roots diminishes significantly. Similarly, using a sequential

[3]He also considers an exogenous break for postwar quarterly GNP series in 1973 due to the first oil price shock

test with a single unknown break Zivot and Andrews (1992) fail to reject the null of unit root for four of the ten series which Perron (1989) finds to be I(0) (i.e., real per capita GNP, GNP deflator, money stock, and real wages). Also, for the original Nelson and Plosser (1982) data set, Stock (1994) finds more support for the I(1) than the I(0) hypothesis in a Bayesian framework with both linear detrending and piecewise linear detrending.

This discussion makes clear the importance of properly modeling structural breaks in testing for unit roots. This poses a serious problem for applied economists since the number, duration, and form of structural breaks may not be known. Moreover, detecting the number and locations of the break(s) may in turn cause an unknown pre-testing bias (Maddala and Kim (1998)). A complicating factor is that a break occurring in a given year need not manifest itself contemporaneously. Even major breaks, such as the Great Depression of 1929 and the oil price shocks of the 1970s, did not display their full impacts immediately. These arguments motivate the use of a recently developed set of unit root and stationarity tests that avoid this problem. Enders and Lee (2006) and Becker et al. (2006) (for short, EL and BEL respectively) develop tests which model any structural break of an unknown form as a smooth process via means of Flexible Fourier transforms (i.e., an expansion of a periodic function in terms of an infinite sum of sines and cosines). Several authors, including Gallant (1981), Becker et al. (2004), and Enders and Lee (2006), show that a Fourier approximation can often capture the behavior of an unknown function even if the function itself is not periodic. The authors argue that their testing framework requires only the specification of the proper frequency in the estimating equations. By reducing the number of estimated parameters, they ensure the tests have good size and power irrespective of the time or shape of the break. Moreover, they increase the tests' power by adopting the LM principle instead of the DF-type approach. EL and BEL will be discussed in more detail in the following section.

This paper reconsiders the nature of the trends in the NP time series using both the EL and BEL tests. They are applied both to the original and to an extended version of the NP data set. Thus, my analysis is of a "confirmatory" type. Several issues should be kept in mind. To begin with, a valid confirmatory analysis should not simultaneously reject or accept the respective nulls. Also, according to the Amano and van Norden (1992) Monte Carlo study, a joint-testing approach is most useful when both tests suggest that the data is stationary or has a unit root for small samples with a large truncation lag. According to Burke (1994), if the true model has a unit root, then the proportion of correct confirmations is high, whereas when the true model is stationary, the proportion is low. Finally, following Burke (1994), a confirmatory analysis favors the 10% significance level over the 5% significance level.

The results for the extended NP data set suggest a high confirmatory power. Thus, both the EL and the BEL tests confirm the stationarity hypothesis for real GNP, real per capita GNP, employment, the unemployment rate, and, surprisingly, stock prices. I find six more series to be I(1): nominal GNP, CPI, real wages, monetary aggregates, velocity, and bond yields. Uncertainty regarding the type of integration persists only for industrial production, GNP deflator, and wages. However, both the GNP deflator and wages seem closer to having a unit root than being stationary. The only ambiguous result concerns industrial production, where both the null of I(0) and I(1) are strongly rejected. For robustness checks, I apply the tests to the original NP data set as well. In this case, the findings suggest that only the unemployment rate and the monetary aggregates are stationary. Although the evidence is mixed, it suggests that real GNP, real per capita GNP, employment, and real wages are closer to having a deterministic trend. Excluding employment, the stationarity test suggests these series are I(0). Also, the unit root statistics for these series are close to being significant at the 10% level. Thus, this evidence favors the hypothesis of deterministic trends. For employment, the EL unit root test suggests the trend is deterministic, while the BEL stationarity test only marginally rejects this conclusion. The rest of the variables (i.e., nominal GNP, GNP deflator, CPI, wages, velocity, bond yields, and stock prices) are confirmed to have a unit root. The emerging picture is that real and workforce variables have deterministic trends while nominal variables have stochastic trends. Most likely, only real wages represent an exception to this rule. Compared to previous studies, there is a significant increase in confirmatory power.

The paper is structured as follows: section 2 discusses in more detail the performance of the Fourier series to approximate various types of structural breaks; section 3 introduces the EL and BEL tests; section 4 discusses the empirical results and section 5 concludes.

2 Nonlinear Trend Approximation with Fourier Transforms

As discussed, traditional unit root tests lose power if structural breaks are ignored in unit root testing. The general method to account for breaks is to approximate them using dummy variables. However, this approach has several undesirable consequences. First, one has to know the exact number and location of the breaks. These are not usually known and therefore need to be estimated. This in turn introduces an undesirable pre-selection bias (see Maddala and Kim (1998)). Second, current available tests account only for one to two breaks. Third, the use of dummies suggests sharp and sudden changes in the trend or level. However, for low frequency data it is more likely that structural changes take the form of large swings which cannot be captured well using only dummies. Therefore, a complicating factor is that a break occurring in a given year need not manifest itself contemporaneously. Even major breaks, such as the Great Depression of 1929 and the oil price shocks of the 1970s, did not display their full impacts immediately. Breaks should therefore be approximated as smooth and gradual processes (see Leybourne et al. (1998)

and Kapetianos et al. (2003)).

Both EL and BEL tests implement a variant of the Flexible Fourier transform (i.e., Gallant (1981)) to control for the unknown nature of the breaks. Fourier series are able to capture the essential characteristics of one or more structural breaks by using only a small number of low frequency components. This is true because a break tends to shift the spectral density function towards frequency zero. The ability of the Fourier series to capture nonlinear trends is illustrated below.

In a simple Dickey-Fuller setting, one can allow the intercept $\alpha(t)$ to be a deterministic function of time:

$$y_t = \alpha(t) + \beta y_{t-1} + \gamma t + \varepsilon_t \tag{3}$$

where the drift term is written as:

$$\alpha(t) = \alpha_0 + \sum_{k=1}^{n} \alpha_k \sin\left(2\Pi kt/T\right) + \sum_{k=1}^{n} \beta_k \cos\left(2\Pi kt/T\right); n \leq T/2 \tag{4}$$

In the above formulation, ε_t is a stationary disturbance term with variance σ_ε^2, n is the maximum number of frequencies, k is a particular frequency and T is the total number of observations. The drift term represents the Fourier approximation written as a deterministic function of the sine and cosine terms. Note that by imposing $\alpha_k = \beta_k = 0$ one gets the standard DF specification. In contrast to other possible series expansions (e.g., Taylor series), the Fourier expansion has the advantage of acting as a global approximation (see Gallant (1981)). This property is preserved even if one specifies a small number of frequencies. In fact, Enders and Lee (2006) argue that a large value of n in a regression framework uses a lot of the degrees of freedom and leads to an over-fitting problem.

To illustrate the approximation properties of a Fourier series, I consider first a single frequency in the Data Generating Process (DGP):

$$\alpha(t) = \alpha_0 + \alpha_k \sin(2\Pi kt/T) + \beta_k \cos(2\Pi kt/T) ; \tag{5}$$

where k is the single frequency selected in the approximation, and α_k and β_k represent the magnitudes of the sinusoidal terms.

I generate series with breaks similar to the ones employed in Becker et al. (2004) and Clements and Hendry (1999). Thus, for $T = 500$, I simulate one break, two breaks and trend breaks both in the middle and towards the extremes. Cases for temporary, permanent and reinforcing breaks are considered. The appendix displays the results in panels 1 through 9. As in Enders and Lee (2006), Panels 1 and 2 display approximations for breaks towards the end of a series. In panel 3, the series has a temporary, though long-lasting break. Panels 4 and 5 display permanent breaks in opposite directions, while in Panel 6 the breaks are in the same direction. Finally, Panels 7-9 depict breaks in the intercept and slope of a trending series. I estimate the coefficients of the sinusoidal terms by performing a simple regression of y_t on $\alpha(t)$ and a time trend.

Just from a simple inspection of the graphs, several assessments can be made. First, a single frequency $k = 1$ or two cumulative ones $n = 2$ can approximate a large variety of breaks. Second, the Fourier transform approximates accurately, even when the breaks are asymmetric (see Panels 1 and 2). Third, a Fourier series works best when the break is smooth over time, which means it may not be suited for abrupt and sharp breaks of short duration (see Panel 5). An additional frequency of $k = 2$ can improve the fit in these circumstances. Further discussion of the properties of the Fourier approximation is given by Enders and Lee (2006) and Becker et al. (2006). Next we review the Enders and Lee (2006) unit root test.

3 The Models

3.1 The Flexible Fourier Unit Root Test

Throughout the paper, I shall call the EL test the Flexible Fourier Unit Root test. As argued, in this framework one does not need to worry about the dates or number of breaks, but only about the proper specification of the best fitting frequency. Consequently, the smaller number of estimated parameters insures a more parsimonious representation.

Enders and Lee (2006) develop their unit root test using the LM principle. As mentioned, the LM has increased power over the DF approach. They assume that the data-generating process (DGP) has the following form:

$$y_t = \alpha_0 + \gamma t + \alpha_k \sin\left(2\Pi kt/T\right) + \beta_k \cos\left(2\Pi kt/T\right) + \epsilon_t; k \leq T/2 \tag{6}$$

$$\epsilon_t = \beta\epsilon_{t-1} + \varepsilon_t \tag{7}$$

Under the null of a unit root $\beta = 1$, while under the alternative hypothesis $\beta < 1$. EL employ the LM methodology of Schmidt and Phillips (1992) and Amsler and Lee (1995) by imposing the null restriction and estimating the following regression in first differences:

$$\triangle y_t = \delta_0 + \delta_1 \Delta \sin\left(2\Pi kt/T\right) + \delta_2 \Delta \sin\left(2\Pi kt/T\right) + \nu_t \tag{8}$$

The estimated coefficients $\widetilde{\delta}_0$, $\widetilde{\delta}_1$ and $\widetilde{\delta}_2$ are then used to construct the following detrended series:

$$\widetilde{S}_t = y_t - \widetilde{\varphi} - \widetilde{\delta}_0 t - \widetilde{\delta}_1 \sin\left(2\Pi kt/T\right) - \widetilde{\delta}_2 \cos\left(2\Pi kt/T\right), t = \overline{2, T} \tag{9}$$

where $\widetilde{\varphi} = y_1 - \widetilde{\delta_0} - \widetilde{\delta_1} \sin(2\Pi kt/T) - \widetilde{\delta_2} \cos(2\Pi kt/T)$, and y_1 is the first observation of y_t. The testing regression based on the detrended series has the following expression:

$$\Delta y_t = \theta \widetilde{S_{t-1}} + d_0 + d_1 \Delta \sin(2\Pi kt/T) + d_2 \Delta \cos(2\Pi kt/T) + \varepsilon_t \tag{10}$$

If y_t has a unit root then $\theta = 0$ and the LM test statistic (denoted τ_{LM}) is the t-test for the null hypothesis of $\theta = 0$. The innovation process ε_t is assumed to satisfy Phillips and Perron (1988)'s serial correlation and heterogeneity conditions. They augment equation (10) with lagged values of $\triangle\widetilde{S_{t-j}}$, $j = 1, \ldots p$ to get rid of the remaining serial correlation (see also Ng and Perron (1995)). Enders and Lee (2006) derive the properties of the asymptotic distribution of the τ_{LM} statistic and demonstrate that it depends only on the frequency k and is invariant to all other parameters in the DGP.

It has been shown that a single frequency can mimic a wide variety of breaks. However, when a researcher does not know the correct frequency to use, the solution is to employ cumulative frequencies to estimate the unknown functional form. Therefore, they provide critical values for up to five cumulative frequencies. Also, in applied work the sine and cosine terms should both be included; otherwise the test diverges under the null (see Enders and Lee (2006)).

The Monte Carlo simulations of Enders and Lee (2006) show that ignoring the possibility of nonlinear breaks in the linear LM test creates serious size distortions when $\beta = 1$, regardless of the magnitudes of α_k and β_k, sample size and the frequency in the DGP (see EL, Table 5). Additionally, when dummies similar to those of Perron (1997)'s endogenous-break tests are used, the fit is very poor if the DGP contains a nonlinear Fourier function. The outcome has poor size and power properties (Table 6 in EL). Moreover, an endogenous-break unit root test that is designed to approximate sudden sharp breaks does not perform substantially better than the Flexible Fourier unit root test in these cases (see Table 8 in

EL). Overall, Monte Carlo experiments indicate that the Enders and Lee (2006) unit root test is excellent for various types of breaks with good size and power in large samples. However, for small samples with values of n greater than two the power of the test is poor. As a rule, they indicate that for small samples one should use $k = 1$, while for larger samples one should use $n = 2$.

3.2 The Flexible Fourier Stationarity Test

To perform a so-called confirmatory analysis, I also use the stationarity test of Becker et al. (2006). For ease of reference I will call it the Flexible Fourier stationarity test. The test in Becker et al. (2006) uses a modified version of the KPSS framework to accommodate nonlinear breaks, under both the null and the alternative. The Fourier series properties regarding the specification, number, and shape of breaks outlined for the EL test are valid in this case as well. The BEL test works best in the presence of breaks that are gradual and has good power to detect u-shaped and smooth breaks.

The DGP considered has the following form:

$$y_t = \alpha_0 + \gamma t + \alpha_k \sin(2\Pi kt/T) + \beta_k \cos(2\Pi kt/T) + \eta_t + \xi_t \tag{11}$$

where the η_t process is described as:

$$\eta_t = \eta_{t-1} + \varepsilon_t \tag{12}$$

where ξ_t is assumed to be stationary. The error process ε_t is assumed to be *i.i.d.* and have a variance σ^2_η. Under the null of stationarity $\sigma^2_\eta = 0$ and the process described by equations (11) and (12) is stationary. Similar to the EL test, the asymptotic properties of the Flexible Fourier stationarity test are invariant to the values of γ, α_k and β_k. As the *DGP* in (11) nests the one used to generate the common *KPSS* test, the BEL test statistic needs only a slight modification of the KPSS statistic. First, one needs to obtain the residuals from the following equations:

$$y_t = \alpha_0 + \alpha_1 \sin(2\Pi kt/T) + \alpha_2 \cos(2\Pi kt/T) + \nu_t \tag{13}$$

and

$$y_t = \alpha_0 + \gamma t + \alpha_1 \sin(2\Pi kt/T) + \alpha_2 \cos(2\Pi kt/T) + \nu_t \tag{14}$$

Equation (13) tests the null of level stationarity while equation (14) tests the null of trend stationarity. The test statistic is given by:

$$\tau_\tau(k) = \frac{1}{T^2} \frac{\sum_{t=1}^{T} \widetilde{S}_t(k)^2}{\widetilde{\sigma}^2} \tag{15}$$

where $\widetilde{S}_t(k) = \sum_{j=1}^{t} \widetilde{\nu}_j$ and $\widetilde{\nu}_j$ are the OLS residuals from regressions (13) and (14), respectively.

As in the KPSS framework and following the PP-type approach, Becker et al. (2006) suggest that a nonparametric estimate of σ^2 be obtained by choosing a truncation lag parameter l and a set of weights ω_j, $j = \overline{1,l}$:

$$\sigma^2 = \widetilde{\alpha}_0 + 2 \sum_{j=1}^{l} \omega_j \widetilde{\alpha}_j \tag{16}$$

where $\widetilde{\alpha}_j$ is the jth sample autocovariance of the residuals $\widetilde{\nu}_t$ from equations (13) and (14), respectively. Becker et al. (2006) suggest that the frequencies in (13) and (14) should be obtained via the minimization of the sum of squared residuals. However, their Monte Carlo experiments suggest that no more than one or two cumulative frequencies should be used because of the loss of power associated with a larger number of frequencies.

Becker et al. (2006) checks the small sample performance of their test. They find that when the coefficients α_1 and α_2 and/or the sample size is small there is a mild size distortion causing the procedure to be too conservative. However, they show that for moderate values of T and/or α_1 and α_2 the size is relatively good. Becker et al. (2006) note that regarding the power properties of their test, one potential problem could be the inclusion of trigonometric functions which can hide the non stationarity of a time series. This results in a non-rejection of the null. Nevertheless, their Monte Carlo simulations suggest that this is a rather mild problem (see Table 3 in Becker et al. (2006) for details). Finally, for the case in which the correct frequency to incorporate in the model is not known, Becker et al. (2006) suggest the minimization of the residual sum of squares (SSR) approach. This method has clear advantages over using a fixed value or using two cumulative frequencies, and it applies to both EL and BEL tests. Therefore, the estimations of this paper use the data dependent frequency suggested by the SSR minimization.

I apply next the Enders and Lee (2006) and Becker et al. (2006) tests to both the original and to an extended version of the Nelson and Plosser (1982) data set.

4 Empirical Results

The original macroeconomic time series of Nelson and Plosser (1982) run from 1860 through 1970. The extended data set adds supplemental information through 1988. Given the longer time period available, one should emphasize the results for this data set. The additional use of the original NP data set is for robustness purposes. However, this section considers first the original and then the extended series.

4.1 Empirical Estimates for the Original NP Data Set

A grid-search is performed to find the best frequency, as there is no *a priori* knowledge concerning the shape of the breaks in the data. I estimate equations (13) and (14) for each integer $k = 1, ...5$ following the recommendations of Enders and Lee (2006) that a single frequency can capture a wide variety of breaks. Following the literature, equation (13) is employed for the unemployment rate and bond yields, while equation (14) is employed for the rest of the series, respectively. Table 1 displays the SSRs for each integer frequency in the interval $[1, 5]$. A single frequency works best for most series. For employment, GNP deflator, wages, and monetary aggregates, the SSR minimization criterion suggests a single frequency of two. Lastly, my findings suggest a single frequency of four for the unemployment rate.

Next, Table 2 employs the Flexible Fourier stationarity test for each series based on the estimated frequencies. I follow Burke (1994) and use a 10% significance level. Further, I choose a lag of eight for the truncation lag. Kwaitowski et al. (1992) choose eight lags for the original NP data. Also, Becker et al. (2006) choose the same number of truncation lags in their analysis of the Purchasing Power Parity hypothesis. They use quarterly data from 1973-2004, which amounts to 120 observations and is approximately the same size as the NP data. Therefore, a choice of eight for the truncation lag seems reasonable. The second column in Table 2 shows the critical values for each frequency at the 10% level. I find that the null of trend stationarity cannot be rejected for real GNP, real per capita GNP, industrial production, real wages, and monetary aggregates. Also, for the unemployment rate, the null of level stationarity cannot be rejected. Furthermore, I accept the alternative of a unit root for nominal GNP, GNP deflator, CPI, wages, velocity, bond yields, and stock prices.

Table 3 displays the results from the EL test. The second column shows the best frequency that results from the minimization of SSR. The third column shows the number of lags of ΔS_t needed to remove serial correlation in the residuals. In most cases one lag is sufficient. Fourth column displays the τ_{LM} statistic of the Flexible Fourier unit root test. It is significant for three series: employment, the unemployment rate, and monetary aggregates. Hence, I reject the null of a unit root for these series. For a single frequency, the critical value at the 10% level reported by Enders and Lee (2006) is -3.82. For five series (i.e., real GNP, real per capita GNP, industrial production, real wages, and stock prices), the test statistic is relatively close to that value. However, the τ_{LM} statistic is much smaller for the rest. In these cases, one cannot reject the null of a unit root. As Enders and Lee (2006) recommend, I also employ two cumulative frequencies to obtain a better fit. For most series, an additional lag of ΔS_t suffices to eliminate residual correlation. Surprisingly, for stock prices one can now reject the null of a unit root. This confirms that two cumulative frequencies can better approximate structural breaks. I find again the unemployment rate stationary around a level.

When results from the two tests are combined, one finds that the stationarity hypothesis (level and trend) is confirmed for the unemployment rate and the monetary aggregates, respectively. One confirms the presence of stochastic trends in six series: nominal GNP, GNP deflator, CPI, wages, velocity, and bond yields. Using just the best single frequency, stock prices appear to have a stochastic trend as well. However, as shown, the Flexible Fourier unit root test, contrary to the stationarity test, suggests the trend is deterministic for stock prices when one uses two cumulative frequencies. Nevertheless, for the extended sample both tests suggest that stock prices are trend stationary. For the rest (i.e., real GNP, real per capita GNP, industrial production, employment, and real wages), the results are not conclusive, although these series appear to be stationary rather than random walks. Confirmatory power appears much higher for the extended NP data set and given the longer time period available I emphasize this next set of results.

4.2 Empirical Estimates for the Extended NP Data Set

I adopt a similar approach for the extended data set. The second column of Table 4 displays the best fitting frequency which minimizes the sum of squared residuals in equations (13) and (14). One can note some changes compared to the original sample. A frequency of 1 gives the best fit for most series. The SSR criterion suggests a frequency different from 1 for industrial production (i.e. $\hat{k} = 2$), employment (i.e. $\hat{k} = 3$), unemployment (i.e. $\hat{k} = 5$), and monetary aggregates (i.e. $\hat{k} = 3$). The best fitting frequency increases by 1 for industrial production, employment, unemployment, and monetary aggregates. The best fitting frequency for GNP deflator and wages decreases by 1. This strengthens the possibility of structural breaks. Note that the sum of the squared residuals is roughly the same across the two samples for most series with the exception of bond yields. This is true for all k integer values in the interval [1,5]. One possible explanation could be the switch to the Friedman rule as the basis for monetary policy at the end of the 1970's and the beginning of the 1980's.

Table 5 displays the results of the Flexible Fourier stationarity test. As before, a common value of eight is used for the truncation lag. Real GNP, real per capita GNP, employment, and the unemployment rate are stationary at the 10% significance level. The test statistic for GNP deflator, wages, and stock prices is significant up to the second and third decimals respectively. However, one cannot reject the null of trend stationarity if one considers all four decimals provided by Becker et al. (2006). The final verdict is considered after we investigate the results from the EL test. The null of stationarity is strongly rejected (i.e. at least with 95% confidence) for remaining variables. Note that there are some changes from the results on the shorter sample. For instance, industrial production, real wages, and monetary aggregates now appear to have a unit root. On the other hand, employment and possibly the GNP deflator, wages, and stock prices appear stationary. For employment in particular, an increase in the best frequency accompanies this finding. As already noted, the best fitting frequency decreases both for wages and GNP deflator, a fact that is generally associated with structural breaks.

Finally, Table 6 shows the estimation results from the EL test. One lag at most is needed to obtain white noise residuals in the regression equation (10). Column four displays the derived τ_{LM} statistics for the extended sample. These values reject the null of a unit root for real GNP, real per capita GNP, industrial production, employment, the unemployment rate, and stock prices while for remaining variables the null cannot be rejected. Hence, the number of instances we reject the null doubles in contrast to the original sample. Therefore, the case for deterministic trends in macro series strengthens from the perspective of the Flexible Fourier unit root test. The number of stationary series reduces to three with two cumulative frequencies (i.e., industrial production, the unemployment rate, and stock prices).

Overall, when we combine the output in Tables 5 and 6, we are able to confirm the stationarity hypothesis for real GNP, real per capita GNP, employment, the unemployment rate, and stock prices. However, the stationarity test for stock prices validates this conclusion only when one accounts for the fourth decimal of the critical value. Nevertheless, the EL test with both one and two cumulative frequencies brings solid evidence in favor of stock prices stationarity. Six other series confirm a unit root: nominal GNP, CPI, real wages, monetary aggregates, velocity, and bond yields. This hypothesis probably holds true also for the GNP deflator and wages. In these cases, the τ_{LM} statistic of the EL test is much smaller than the corresponding critical value. Using the BEL test up to the second decimal, one rejects the null of trend stationarity. Industrial production is the only series with ambiguous results where both the null of I(0) and the null of I(1) are strongly rejected. Overall, the evidence suggests that real variables are either level or trend stationary, and nominal ones have a unit root. The only exception is real wages. Nevertheless, these results are more conclusive than previous findings in the literature (see Kwaitowski et al. (1992), Gil-Alana and Robinson (1997)). I confirm the nonstationarity of CPI, monetary aggregates, velocity, and bond yields and the stationarity of unemployment. Additionally, using the tests of Enders and Lee (2006) and Becker et al. (2006), we can determine the nature of nominal and real GNP, real per capita GNP, employment, and stock prices.

5 Conclusion

This paper employs two new sets of unit root and stationarity tests recently introduced in the literature by Enders and Lee (2006) and Becker et al. (2006). These tests have the ability to test for unit roots in the presence of various types of smooth structural breaks with an unknown form. The Flexible Fourier transform introduced by Gallant (1981) captures the unknown shape of the breaks. The Monte Carlo simulations of Enders and Lee (2006) and Becker et al. (2006) show that the tests do not suffer from low power and have good size properties. I apply the tests to both the original and to an extended version of the Nelson and Plosser (1982) data set. For the original data set, both unemployment and monetary aggregates appear stationary at the 10% level. Nominal GNP, GNP deflator, consumer prices, wages, velocity, and bond yields are confirmed to have a unit root; however, the results for real GNP, real per capita GNP, industrial production, employment, and real wages are less straightforward. Confirmatory power increases for the extended data set. Thus, real GNP, real per capita GNP, employment, the unemployment rate, and stock prices are confirmed to be stationary. I find evidence of stochastic trends in nominal GNP, consumer prices, real wages, monetary aggregates, velocity, and bond yields. Perhaps, the GNP deflator and nominal wages have a unit root as well. Industrial production is the only contradictory result. In general, real variables have deterministic trends while nominal ones have stochastic trends.

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6 Appendix

Results for the original NP data set

Table 1: Best frequency selected for each series in the original data set

Series	Best frequency	Deterministic term	SSR♣ at 1 freq.	SSR at 2 freq.	SSR at 3 freq.	SSR at 4 freq.	SSR at 5 freq.
Real GNP	1	Trend	**0.683**	0.906	0.893	0.898	1.028
Nominal GNP	1	Trend	**2.610**	2.830	3.619	4.202	4.132
Real per capita GNP	1	Trend	**0.724**	0.790	0.886	0.873	1.016
Industrial Production	1	Trend	**2.306**	2.867	3.218	3.430	3.432
Employment	2	Trend	0.408	**0.352**	0.532	0.427	0.538
Unemployment Rate	4	Drift	38.147	30.579	35.855	**28.092**	38.488
GNP Deflator	2	Trend	1.735	**0.875**	1.400	1.907	1.904
Consumer Prices	1	Trend	**2.944**	6.254	7.619	7.023	7.785
Wages	2	Trend	1.916	**1.522**	1.861	2.475	2.530
Real Wages	1	Trend	**0.196**	0.420	0.381	0.385	0.382
Monetary Aggregates	2	Trend	2.636	**1.890**	2.153	2.349	3.013
Velocity	1	Trend	**1.341**	2.995	3.711	3.710	3.597
Bond Yields	1	Drift	**43.934**	45.247	62.435	62.813	64.558
Stock Prices	1	Trend	**7.916**	11.435	14.343	14.954	14.932

♣ Sum of Squared Residuals

Table 2: Results using the Flexible Fourier Stationarity Test (the original data set)

		Lag truncation parameter								
Series	10% Critical Value	0	1	2	3	4	5	6	7	8
Real GNP	0.0471	0.132	0.073	0.054	0.046	0.043	0.042	0.042	0.043	0.045
Nominal GNP	0.0471	0.256	0.136	0.097	0.079	0.068	0.062	0.058	0.056	**0.054***
Real per capita GNP	0.0471	0.142	0.078	0.058	0.049	0.045	0.044	0.043	0.044	0.046
Industrial Production	0.0471	0.158	0.090	0.068	0.057	0.051	0.048	0.046	0.045	0.044
Employment	0.1034	0.407	0.222	0.163	0.136	0.121	0.113	0.108	0.106	**0.105***
Unemployment Rate	0.3476	0.231	0.137	0.108	0.094	0.085	0.080	0.076	0.072	0.070
GNP Deflator	0.1034	0.485	0.260	0.186	0.151	0.131	0.119	0.112	0.106	**0.103***
Consumer Prices	0.0471	0.353	0.184	0.129	0.102	0.086	0.076	0.068	0.063	**0.059***
Wages	0.1034	0.682	0.365	0.262	0.213	0.185	0.168	0.157	0.149	**0.144***
Real Wages	0.0471	0.101	0.057	0.043	0.038	0.035	0.035	0.036	0.037	0.040
Monetary Aggregates	0.1034	0.350	0.182	0.128	0.102	0.088	0.079	0.074	0.071	0.070
Velocity	0.0471	0.348	0.191	0.139	0.114	0.099	0.089	0.082	0.077	**0.072***
Bond Yields	0.1318	1.988	1.105	0.795	0.639	0.545	0.482	0.436	0.402	**0.374***
Stock Prices	0.0471	0.300	0.164	0.121	0.099	0.088	0.080	0.075	0.070	**0.067***

** Significance at the 10% level and so we can reject the null of stationarity

Table 3: Results using the Flexible Fourier Unit Root Test (the original data set)

Series	Single Freq.	Lags	τ_{LM}	Cumulative Freq.	Lags	τ_{LM}
Real GNP	1	1	-3.62	2	1	-3.86
Nominal GNP	1	1	-2.98	2	1	-4.23
Real per capita GNP	1	1	-3.49	2	1	-3.83
Industrial Production	1	0	-3.65	2	1	-4.51
Employment	2	1	**-3.42***	2	1	-4.60
Unemployment Rate	4	1	**-4.32****	2	1	**-4.63***
GNP Deflator	2	1	-3.06	2	1	-3.33
Consumer Prices	1	1	-3.08	2	1	-4.11
Wages	2	1	-3.16	2	1	-3.70
Real Wages	1	1	-3.70	2	1	-3.85
Monetary Aggregates	2	1	**-3.48***	2	1	-3.77
Velocity	1	0	-2.93	2	0	-4.37
Bond Yields	1	0	-2.69	2	0	-2.85
Stock Prices	1	1	-3.55	2	1	**-4.77***

[1]Critical values using the best single frequency for each series; [2]A single frequency of two is used to check whether results can be improved;
[3] Critical value using two cumulative frequencies; * Significance at the 10% level; *** Significance at the 1% level;

Results for the extended NP data set

Table 4: Best frequency selected for each series in the extended data set

Series	Best frequency	Deterministic term	SSR♣ at 1 freq.	SSR at 2 freq.	SSR at 3 freq.	SSR at 4 freq.	SSR at 5 freq.
Real GNP	1	Trend	**0.716**	0.935	0.941	1.020	0.949
Nominal GNP	1	Trend	**3.145**	6.003	5.889	6.620	7.036
Real per capita GNP	1	Trend	**0.793**	0.898	0.893	0.992	0.929
Industrial Production	2	Trend	3.017	**2.559**	3.665	3.537	3.678
Employment	3	Trend	0.422	0.505	**0.378**	0.553	0.460
Unemployment Rate	5	Drift	39.751	33.040	36.584	35.793	**32.262**
GNP Deflator	1	Trend	**2.166**	4.793	3.406	5.107	5.088
Consumer Prices	1	Trend	**5.854**	14.384	17.664	16.749	17.492
Wages	1	Trend	**2.098**	3.926	2.520	3.763	3.032
Real Wages	1	Trend	**0.275**	0.585	0.776	0.721	0.739
Monetary Aggregates	3	Trend	2.772	3.996	**2.186**	3.742	3.502
Velocity	1	Trend	**1.368**	6.652	7.086	7.237	7.238
Bond Yields	1	Drift	**302.251**	423.082	485.564	519.197	530.145
Stock Prices	1	Trend	**8.351**	19.861	20.895	20.191	21.679

♣ Sum of Squared Residuals

Table 5: Results using the Flexible Fourier Stationarity Test (the extended data set)

		Lag truncation parameter								
Series	10% Critical Value	0	1	2	3	4	5	6	7	8
Real GNP	0.0471	0.117	0.065	0.048	0.041	0.038	0.036	0.036	0.037	0.038
Nominal GNP	0.0471	0.252	0.132	0.093	0.075	0.064	0.057	0.053	0.050	**0.048****
Real per capita GNP	0.0471	0.153	0.083	0.062	0.052	0.047	0.045	0.044	0.043	0.044
Industrial Production	0.1034	0.577	0.327	0.244	0.204	0.182	0.169	0.161	0.155	**0.151****
Employment	0.1141	0.306	0.167	0.123	0.102	0.090	0.084	0.080	0.078	0.077
Unemployment Rate	0.3518	0.216	0.126	0.098	0.083	0.075	0.070	0.066	0.063	0.061
GNP Deflator	0.0471	0.244	0.127	0.089	0.070	0.059	0.052	0.048	0.045	0.043
Consumer Prices	0.0471	0.544	0.279	0.191	0.148	0.122	0.105	0.093	0.085	**0.079****
Wages	0.0471	0.220	0.115	0.080	0.064	0.054	0.048	0.045	0.042	0.040
Real Wages	0.0471	0.205	0.115	0.085	0.071	0.064	0.060	0.057	0.056	**0.056****
Monetary Aggregates	0.1141	0.646	0.336	0.235	0.186	0.158	0.141	0.129	0.122	**0.116****
Velocity	0.0471	0.364	0.200	0.145	0.118	0.103	0.092	0.084	0.078	**0.074****
Bond Yields	0.1318	2.658	1.387	0.961	0.746	0.619	0.537	0.479	0.437	**0.406*****
Stock Prices	0.0471	0.209	0.114	0.084	0.069	0.061	0.056	0.052	0.049	0.047

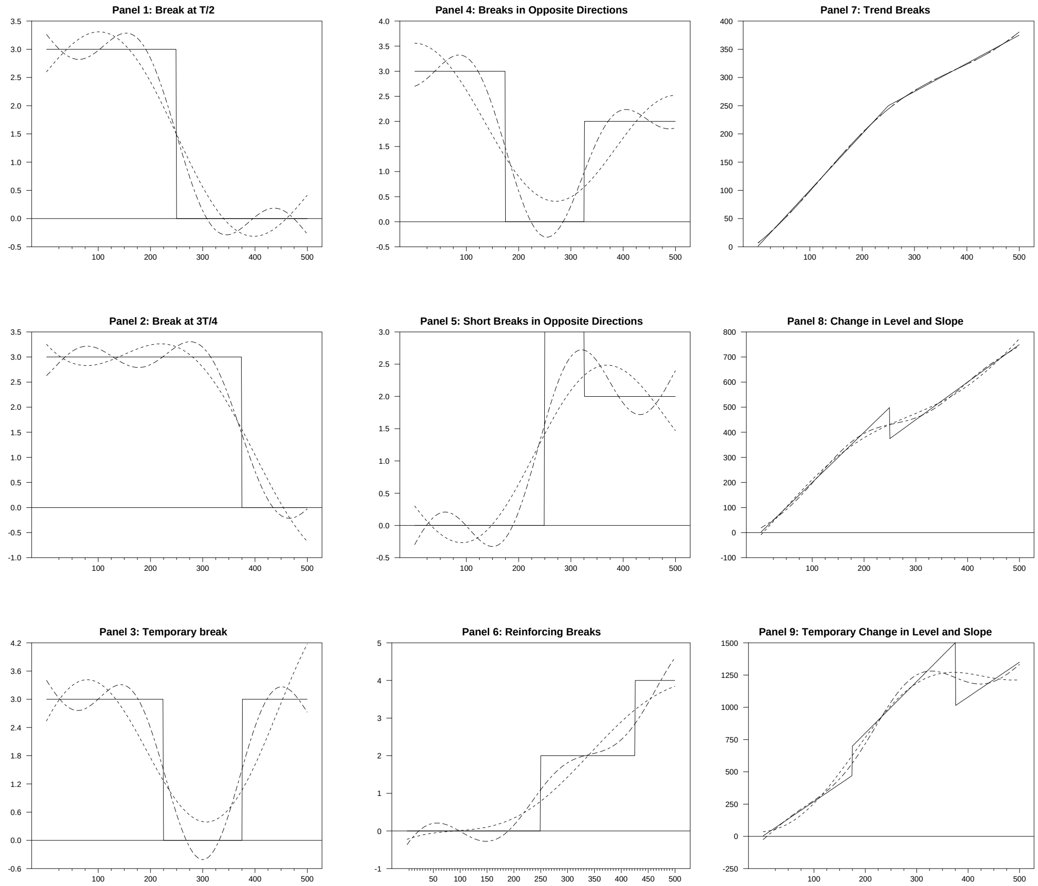
* Significance at the 10% ; ** Significance at the 5% level; *** Significance at the 1% level

Table 6: Results using the Flexible Fourier Unit Root Test (the extended data set)

Series	Single Freq.	Lags	τ_{LM}	Cumulative Freq.	Lags	τ_{LM}
Real GNP	1	1	**-4.09***	2	1	-4.23
Nominal GNP	1	1	-3.09	2	1	-3.88
Real per capita GNP	1	1	**-3.89***	2	1	-4.16
Industrial Production	2	0	**-3.83****	2	0	**-4.88***
Employment	3	1	**-3.23***	2	1	-4.31
Unemployment Rate	5	1	**-4.55****	2	1	**-4.94**
GNP Deflator	1	1	-2.44	2	1	-2.57
Consumer Prices	1	1	-2.82	2	0	-3.95
Wages	1	1	-3.15	2	1	-3.19
Real Wages	1	1	-3.70	2	1	-4.52
Monetary Aggregates	3	1	-2.75	2	1	-3.56
Velocity	1	0	-3.11	2	0	-4.39
Bond Yields	1	1	-3.60	2	1	-4.20
Stock Prices	1	1	**-4.17****	2	1	**-4.62***

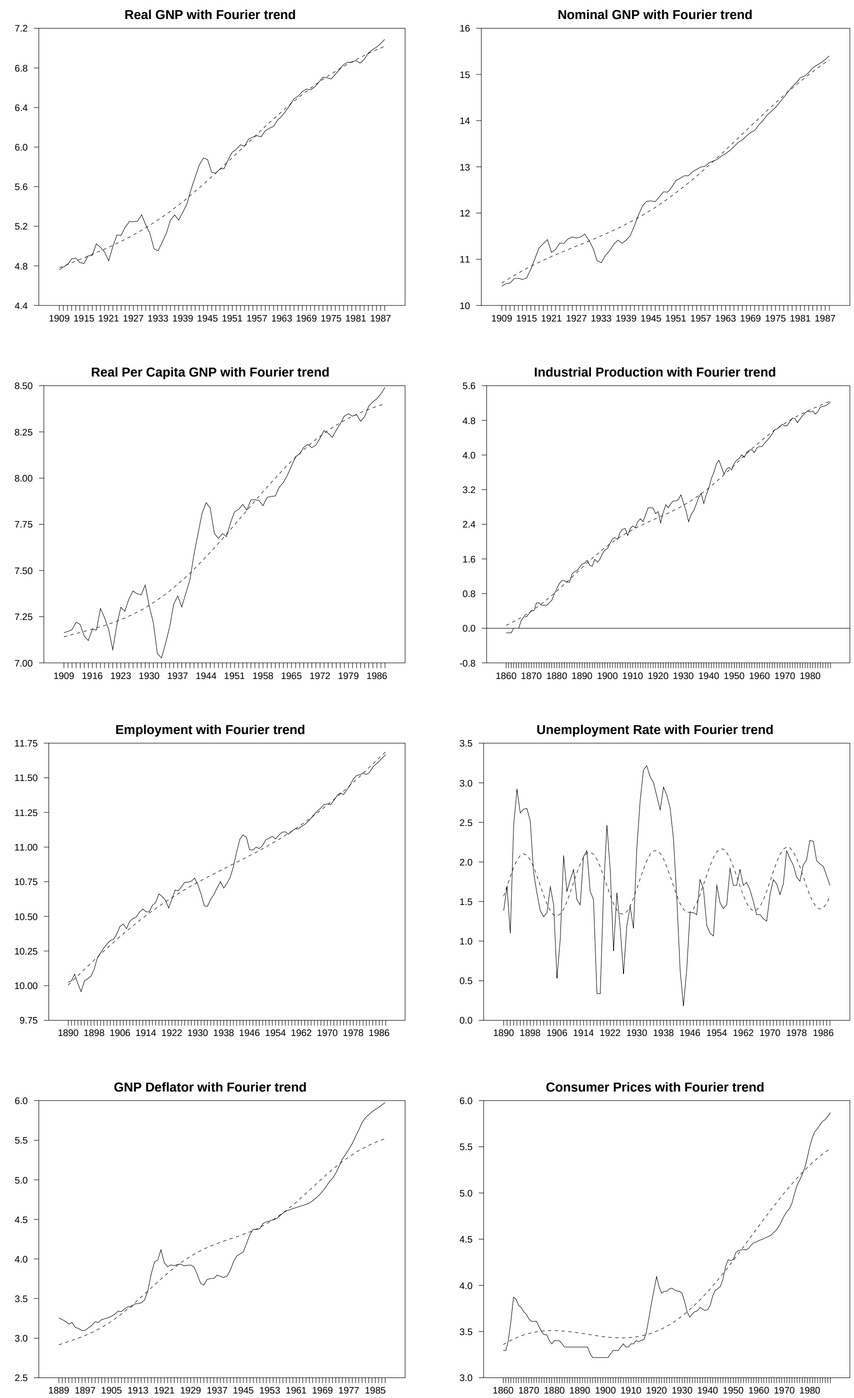
* Significance at the 10% level; ** Significance at the 5% level

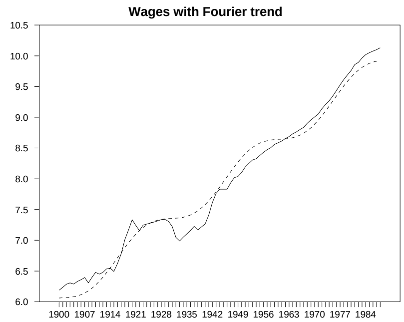
Approximation of Structural Breaks with Fourier Transforms



Series:__ ; One frequency: _ _ ; Two cumulative frequencies: __ _

Trend Approximation of NP Series

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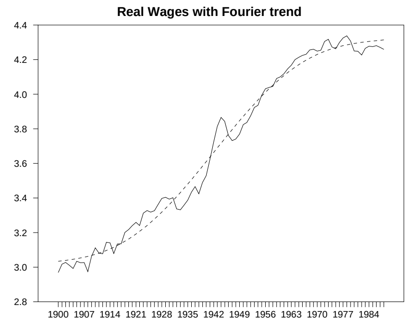
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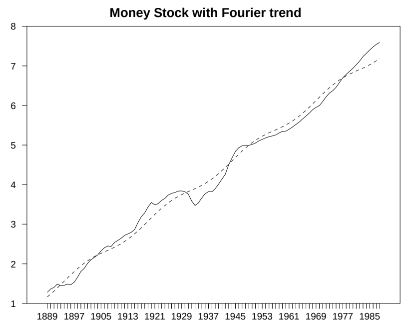
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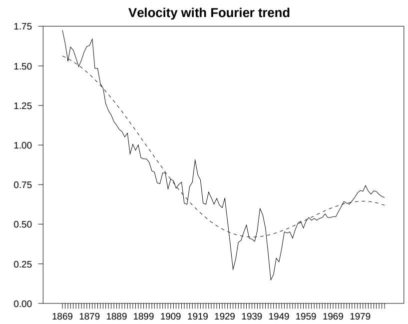
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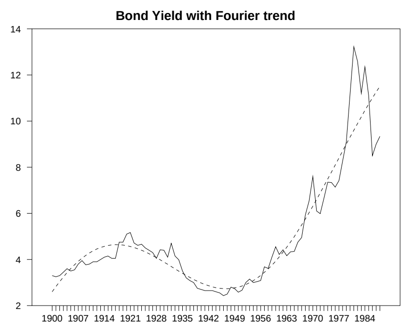
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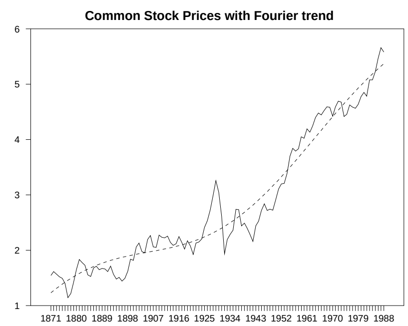
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