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Abstract

This work presents a north-south endogenous-growth model that reproduces some recent EU stylized facts: convergence between countries, divergence within the same countries, more spatial concentration of economic activity and higher growth rates.

We claim that the ongoing technological reduction of transaction costs can conceivably spur those phenomena, specially if a regional productive duality within the less-developed countries were reinforced by a biased incidence of that fall in transaction costs.

A key element is Grossman and Helpman's complementarity between innovation and imitation. The channels that allow for higher growth-rates are migrations and scale-effects in the industrialized regions of the poorest countries.

1 Introduction.

Can we expect European countries and regions to converge in the long run? Is there any role for regional policies to reshape the relative position of local steady states?

Boldrin and Canova (2001)'s recent empirical research suggests a negative answer to both questions. They found no significant correlation between labor-productivity growth and the structural-fund contribution to capital endowments in recipient regions. Though cautiously, they used these

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results as a key to select between competing economic theories concerning EU stylized facts. Which were these alternative theories?

First, neoclassical (exogenous-growth) models imply that the local steady-state relative positions depend on microeconomic features, unaffected by current levels of cumulative factors. Therefore, policies aimed at an even distribution of cumulative stocks are irrelevant in the long run.[1] On the other hand, divergence-models[2] allow for agglomeration externalities, which make initial cumulative stocks really crucial with respect to the relative position of steady states. As a consequence, policy is extremely relevant, and in the dynamic (endogenous-growth) models a trade-off arises between regional cohesion and global growth rates. Consequently, Boldrin and Canova's findings seemed to support neoclassical models, typically associated with less activist regional policies, at the expense of the case for divergence models.

In our work we try to emphasize two ideas related to that controversy: first, available empirical evidence does not exclude any theoretical paradigm; second, and more importantly, maybe the whole dilemma between both paradigms is not the right discussion.

Why not? As J. S. Pischke noted in a comment to Boldrin and Canova (2001), *decreasing − return structures and local agglomeration nodes may be alternating, not only in time for a given place, but also in space for a given period of time*. In that case, it may be interesting to consider some duality in the regional production functions within countries, unlike the usual practice followed in both neoclassical and divergence models.

Certainly, both paradigms exhibit symmetry in local production functions but for the initial stocks of capital and public knowledge. However, recent growth-accounting exercises[3] detect fast $\beta-$convergence rates towards rather diverse steady states, emphasizing sectoral specialization and TFP differences as crucial elements that keep local steady-states apart. We have exploited that image of dually-structured countries to construct a model that respects Boldrin and Canova's findings. Moreover, our model reproduces some recent EU stylized facts: international convergence trends accompanied by regional divergence within the same countries.[4]

Our modelling tool is a north-south framework with an exogenous fragmentation within the southern (and poorest) country. This one consists of an industrialized core, which could potentially

[1] See, for example, Sala i Martin (96).

[2] See (for example) Krugman (91), Krugman and Venables (95), Krugman and Livas Elizondo (96), Puga (99), Martin and Ottaviano (99) or Baldwin and Forslid (2000).

[3] See Islam (95), Canova and Marcet (95), De la Fuente and Domenech (2001) or De la Fuente (2002b).

[4] Martin (98, 99) and Esteban (94) have documented these phenomena for the EU as a whole: in most member countries the distance between better-off and worse-off regions grows larger and, at the same time, both groups of affluent and lagged regions become more and more internally homogeneous.

host local agglomeration economies, and a periphery doomed to host just primary sectors under perfect competition. We assume that international-trade barriers for our homogeneous good do not decay at the same pace of those for manufactures, as if biased technological changes affected differently the transaction costs of both sectors[5]. Under some conditions, since the northern aggregate income is larger, a marginal increase in trade openness for manufactures induces a net inflow of demand for southern manufactures: this raises the relative wage of the core with respect to the north. Simultaneously, the relative wage of the core with respect to the periphery also increases, since primary goods remain barely as attractive to foreign consumers as before. Then, these widening income differentials within the south give rise to migrations, which also enhances peripheral wages and favors north-south convergence (reproducing our stylized facts).

Concerning a growth evaluation, such an agglomeration of labor force in the core turns out to be beneficial for the growth rate of the global economy. In our framework (based in Grossman and Helpman (91)) imitation and innovation are complementary activities. Therefore, given that some of the new immigrants in the core will undertake research activities, they will enlarge the southern catch-up potential and the growth rate of the global economy. The last effect holds because a stronger imitation will reduce northern wages and increase the value of a patent, raising the natural incentives to innovate. Taking all this into account, any policy measure that restricts periphery-core migrations proves to be harmful in terms of steady-state growth. But not necessarily in terms of regional cohesion, since a higher catch-up potential may boost core-periphery divergence patterns.

Our theoretical results conform with Boldrin and Canova's empirics, but they are at odds with neoclassical views about the irrelevance of regional policies in the long run. On the other hand, our dual structure within the south differentiates our conclusions from those of many divergence models: first, there is a trade-off between long-run growth and *core-periphery* (instead of north-south) cohesion; and secondly, unlike Puga (99) or Krugman and Livas Elizondo (96), regional inequalities do not fade away as trade openness becomes almost perfect. This final difference arises because trade-openness shocks are sectorally biased, acting as centripetal forces that drive economic activity towards the core.

In this model there is an interaction between an R&D sector, where patents are either copied or conceived, and a manufacturing sector whose varieties compete horizontally under monopolistic competition. The expected stream of profits for those manufacturing varieties is equal in equilibrium to the corresponding value of the patent. Moreover, our locations -north, core and periphery- are institutionally distinct. This distinction justifies, by assumption, two noticeable facts: firms can not move from north to core (and viceversa); and patents can not be traded from north to core or

[5]For an empirical study that confirms this tendency, linked to the recent breakthrough of telecommunications, see Rauch (99).

viceversa either, because there are specific features in every location that can not be successfully replicated abroad.

We do next a comparison between 2 different steady states, each of which is characterized by a different level of trade openness for manufactures. In section 2 we derive the properties of a generic steady-state. Section 3 contains the comparative-statics exercise that reproduces our stylized facts. Section 4 concludes.

2 The model without migration

As in Grossman and Helpman (91), we consider 2 countries - north and south - and the competitive interaction of firms from both of them. One important novelty is the existence of a periphery within the south. Researchers from the core can only replicate northern patents to sell the corresponding products at lower cost, whereas the institutional atmosphere in the periphery impedes the production of manufactures. The competitive environment for industrial varieties shows horizontal differentiation, monopolistic competition and no temporal obsolescence.

In the global economy there is a continuum of industrial varieties with measure n, and $n = n_c + n_n$ (the addition of the measures from the north and the core). This degree of product variety expands over time due to innovation. Moreover, an increase in the local measure of manufactures enlarges the stock of public knowledge and reduces future R&D-costs. Grossman and Helpman's local stocks of knowledge are equal to n in the north - since all patents were originally made up there - and to n_c in the core.

Then, by free entry in the innovative activity, the value of a patent is at most equal to the labor-cost of its imitation (in the core) or of its creation (in the north). Given the linear specification of the externality, that value decreases with the local stock of public knowledge in this way:

$$v_c \leq \frac{a_m w_c}{n_c}, \text{ with equality when } \dot{n}_c > 0 \tag{1}$$

$$v_n \leq \frac{a w_n}{n}, \text{ with equality when } \dot{n} > 0 \tag{2}$$

where $\frac{a_m}{n_c}$ and $\frac{a}{n}$ stand for the number of researchers needed to imitate a northern patent in the core and to create a new variety in the north, respectively. Our variables w_a, w_c and w_n denote the nominal wage in the periphery, the core and the north, respectively. Later we will establish some necessary and sufficient parameter restrictions so that imitation and innovation coexist, which implies that

$$w_n = \frac{nv_n}{a}; w_c = \frac{n_c v_c}{a_m} \tag{3}$$

Aggregate populations in the north and the south (L_n and L_s, respectively) are fixed by assumption, but there can be migrations within the same country. That means, in short, that Southerners can move from periphery to core (and viceversa) in response to economic-opportunity variables; i.e. $L_s = L_a + L_c$, where L$_a$ (the peripheral population) is an endogenous variable.

Any representative household (or individual) k, living in that location k, maximizes (in every period t) an intertemporal utility function W_t^k such as

$$W_t^k = \int_t^\infty e^{-\rho(s-t)} \log \left[U_s \left(X_s^i, A_s \right) \right] \, ds \tag{4}$$

W$_t^k$ reflects the discounted utility flow that household k expects to obtain from period t onwards by acquiring manufactures (grouped into the composite X) and the homogeneous agricultural good (A). On the other hand, the particular form of U_s reveals the relative weight assigned to food and manufactures in the following way:

$$U_s = X_s^\mu A_s^{1-\mu}, \text{ where } 0 < \mu < 1 \tag{5}$$

The composite of manufactures X_s is a Dixit and Stiglitz subutility function over the aggregate measure of varieties invented up to period s,

$$X_s = \left[\int_0^{n(s)} x_j \left(s \right)^\alpha dj \right]^{\frac{1}{\alpha}} \tag{6}$$

where 0< α < 1 is a positive measure of the substitutability between manufactures and $x_j\left(s\right)$ quantifies the household demand for variety j at time s, $\forall s \geq t$.

The production function for every particular manufacture and the primary good is identical and very simple: 1 unit of labor generates 1 unit of final output. Prior to the production of any manufacture it is necessary to incur a fixed cost (to buy, invent or imitate the corresponding patent), which is financed by means of gross savings. On the contrary, labor is the only factor in the production of food.

The function W$_s^k$ is intertemporally maximized with respect to its ultimate arguments ($x_j(s), \forall j$, $\forall s \geq t$; $A(s)$ $\forall s \geq t$) at every period t, taking as given the expected temporal paths $v_n\left(s\right)$, $v_c\left(s\right)$, $n(s)$, $p_j\left(s\right)$ $\forall j$ and $p_a\left(s\right)$, $\forall s \geq t$. As Grossman and Helpman do, this problem can be decomposed into 2 parts:

- The static allocation of a given per-household expenditure E$_s^k$ among the primary good and all kind of manufactures, which gives rise to demand function for each of these commodities.

- The choice of an optimal path for E_s^k, given the possibility of saving and investing in equity of southern and northern firms.

2.1 Static optimization.

Let us denote by E the aggregate world expenditure and by γ the proportion of E spent by Northerners, which is an endogenous variable. The parameter $\tau \geq 1$ introduces the classical iceberg-notion of international trade costs: it is necessary to buy τ units of that good abroad to consume 1 unit at home. Considering that demand for any variety comes from both northern and southern consumers who face different c.i.f. prices, we can derive the aggregate demand for any northern (x_n) and southern manufacture (x_c), taking into account (5), (6) and our previous definition of γ as follows:

$$x_n = \mu p_n^{1-\in} \left[\frac{\gamma E}{n_n p_n^{1-\in} + \delta\ n_c\ p_c^{1-\in}} + \frac{(1-\gamma)\ \delta\ E}{\delta\ n_n p_n^{1-\in} +\ n_c\ p_c^{1-\in}} \right] \tag{7}$$

$$x_c = \mu p_c^{1-\in} \left[\frac{\gamma\ \delta\ E}{n_n p_n^{1-\in} + \delta\ n_c\ p_c^{1-\in}} + \frac{(1-\gamma)\ E}{\delta\ n_n p_n^{1-\in} +\ n_c\ p_c^{1-\in}} \right] \tag{8}$$

where $\in = \frac{1}{1-\alpha}$. In expressions (7) and (8), as in Martin and Ottaviano (99), $\delta = \tau^{1-\in}$ $(0 \leq \delta \leq 1)$ is a measure of trade openness in the global economy with respect to manufactures.

Concerning firms, they maximize profits at any period s taking into account a demand of the type (7) or (8) and the simple production function described above. As a result, both utility and profit maximization from expressions (6), (7) and (8) result in a common optimal price for all industrial firms in location k, which is a constant mark-up over marginal costs:

$$p_k = \frac{w_k}{\alpha}, \text{ for } k = \text{ north, core.} \tag{9}$$

Then, from (9), per-period operating profits for any manufacturing firm in location k are

$$\pi_k = \left(\frac{1-\alpha}{\alpha} \right) w_k x_k \text{ for k=north, core} \tag{10}$$

On the other hand, we assume that the wage differential between north and core is high enough for southern imitators to quote the unconstrained optimal mark-up. Therefore, this wide-gap assumption will only be satisfied if the original manufacturer can not undercut the southern firm without incurring losses, i.e. iff

$$\frac{w_c}{\alpha} \tau \leq w_n \tag{11}$$

Given that the primary sector is characterized by perfect competition and free entry, the agricultural price is equal to the peripheral wage and per-firm operating profits are zero. We assume that international transaction costs for primary products remain unaltered. So, without loss of generality, we state that these costs are just nil. Taking all this into account,

$$p_a = w_a = \frac{(1-\mu)\,E}{L_a} \tag{12}$$

2.2 Dynamic optimization: system of differential equations.

Now we have to face the intertemporal allocation of expenditure and savings, not only to distribute consumption across the time horizon, but also to finance new start-ups in the north and the core. In the appendix we solve the general continuous-time optimization program for a representative household living in location k (k= north, core, periphery).

Moreover, we must follow the evolution of the aggregate measure of manufactures in the core and the global economy $(\frac{\dot{n}_c}{n_c}, \frac{\dot{n}}{n})$ by looking at the labor-market-clearing conditions. These equilibrium conditions in the core and the north can be specified considering the available production function and the technology in the imitation and innovation processes:

$$L_c = a_m \frac{\dot{n}_c}{n_c} + n_c x_c \tag{13}$$

$$L_n = a \frac{\dot{n}}{n} + n_n x_n \tag{14}$$

An important point in Grossman and Helpman (91) is the choice of a numeraire to evaluate wages and prices at any moment in time. We follow their normalization and take current aggregate expenditure as the numeraire:

$$E(t) = 1 \ \forall t \tag{15}$$

This implies that all wages and prices are always measured in units of current aggregate expenditure.

Our definition of steady state is made explicit in three differential equations (see the appendix). If we combine these 3 differential equations with expressions (3) and (15), we can redefine our steady state as a situation in which the values of w_n, w_c and c=$\frac{n}{n_c}$ remain stable, i.e. our system of differential equations becomes

$$\frac{\dot{w}_n}{w_n} = \frac{\dot{n}}{n} + \frac{\dot{v}_n}{v_n} - \frac{\mathring{\mathrm{E}}}{E} = \frac{L_n}{a} - \frac{(n-n_c)\,x_n}{a}\left[1 + \frac{(1-\alpha)}{\alpha}\frac{c}{(c-1)}\right] + \frac{1}{(c-1)}\left[\frac{L_c}{a_m} - \frac{n_c x_c}{a_m}\right] + \rho \tag{16}$$

$$\frac{\dot{w}_c}{w_c} = \frac{\dot{n}_c}{n_c} + \frac{v_c}{v_c} - \frac{\mathring{\mathrm{E}}}{E} = \frac{L_c}{a_m} - \frac{n_c x_c}{a_m} + \rho \tag{17}$$

$$\frac{\mathring{\mathrm{c}}}{c} = \frac{\dot{n}}{n} - \frac{\dot{n}_c}{n_c} = \frac{L_n}{a} - \frac{n_n x_n}{a} - \frac{L_c}{a_m} + \frac{n_c x_c}{a_m} \tag{18}$$

2.3 Steady state without migration.

If we could prove that there are some values c^{*}, $w_n{}^{*}$ and $w_c{}^{*}$ for which $\mathring{c} = \mathring{w}_c = \mathring{w}_n = 0$, this would imply that there exists a steady state for our system of differential equations established in (16), (17) and (18). From (17) , in our candidate to steady state

$$\frac{n_c x_c}{a_m} = \alpha\left(\frac{L_c}{a_m} + \rho\right) \tag{19}$$

and from (13), (18) and (19)

$$g = \frac{\mathring{n}}{n} = \frac{\mathring{n}_c}{n_c} = (1-\alpha)\frac{L_c}{a_m} - \alpha\rho > 0 \tag{20}$$

We can observe that our innovation growth rate is exclusively determined by the monopoly power, the discount rate and the imitation capacity of the core.

Therefore, from (13) and (20),

$$\frac{\mathring{v}_c}{v_c} = \frac{\mathring{v}_n}{v} = -g \tag{21}$$

Now, from equations (15), (21) and also the arbitrage condition (50) in the appendix, we are ready to obtain reduced-form equations for the profits of any northern and southern industrial firm:

$$\pi_n = (\rho + m + g)\, v_n; \;\; \pi_c = (\rho + g)\, v_c \tag{22}$$

Let us denote by ξ_c the steady-state proportion of southern industrial firms. It is useful, as Grossman and Helpman do, to express ξ_c as a function of m and g, where $m = \frac{\dot{n}_c}{n_n}$ is our imitation rate. Since $m = g\frac{\xi_c}{(1-\xi_c)}$, we can solve now for ξ_c :

$$\xi_c = \frac{m}{m+g} \tag{23}$$

As a consequence, from (3), (10), (14), (22) and (23), we can restate the arbitrage condition corresponding to northern manufactures as follows:

$$\frac{\pi_n}{v_n} = \left(\frac{1-\alpha}{\alpha}\right)\left[\frac{L_n}{a} - g\right]\left(\frac{m+g}{g}\right) = \rho + m + g \tag{24}$$

By combining (20) and (24), we can already derive a formal expression for the steady-state imitation rate m:

$$m = \left\{ \begin{array}{ll} 0, & \text{if } \frac{L_n}{a} \geq \frac{L_c}{a_m} \\ \frac{(1-\alpha)\left[\frac{L_c}{a_m} - \frac{L_n}{a}\right]\left((1-\alpha)\frac{L_c}{a_m} - \alpha\rho\right)}{\alpha\rho - (1-\alpha)\left[\frac{L_c}{a_m} - \frac{L_n}{a}\right]}, & \text{if } \frac{L_c}{a_m} \geq \frac{L_n}{a} \geq \frac{L_c}{a_m} - \frac{\alpha\rho}{1-\alpha} \\ \infty, & \text{if } \frac{L_n}{a} \leq \frac{L_c}{a_m} - \frac{\alpha\rho}{1-\alpha} \end{array} \right\} \tag{25}$$

As could be expected, m rises with the imitation potential of the core relative to the northern innovation capacity: $\left(\frac{L_c}{a_m} - \frac{L_n}{a}\right)$. We can already establish a first set of parameter restrictions so that the global economy exhibits a positive innovation rate and a positive measure of manufactures operate in both countries. That is, we want that $0< \xi_c < 1$, which requires $0< m < \infty$ and $0< g < \infty$. As we prove in the appendix, this initial condition can be simply summarized as follows:

$$0 < \left[\frac{L_c}{a_m} - \frac{L_n}{a}\right] < \frac{\alpha\rho}{1-\alpha} \tag{26}$$

2.4 Steady-state absolute and relative wages.

There are still several endogenous variables to be determined that are crucial for our comparative-statics exercise. Two of them are the relative wage of the core with respect to the north ($\omega = \frac{w_c}{w_n}$) and γ. From equations (7), (8), (9), (13), (14) and (15), we can get an idea of the determinants of ω as follows:

$$\frac{x_n}{x_c} = \frac{L_n - ag}{L_c - a_m g}\frac{m}{g} = \omega^{\in} C(\delta, L_c, \omega) \tag{27}$$

$$\text{where } C(\delta, L_c, \omega) = \left[\frac{\frac{\gamma}{(g/m)\omega^{\in-1}+\delta} + \frac{(1-\gamma)\delta}{\delta(g/m)\omega^{\in-1}+1}}{\frac{\gamma\delta}{(g/m)\omega^{\in-1}+\delta} + \frac{(1-\gamma)}{\delta(g/m)\omega^{\in-1}+1}}\right] \tag{28}$$

We can see from the left-hand side of (27) that only the supply-side fundamentals - i.e. industrial workforces in both countries and innovation and imitation long-term capacities - can modify $\frac{x_n}{x_c}$. That means that any variation in international trade openness (δ) will be exactly offset in the long run by a countervailing adjustment of ω.

Our term $C(\delta, L_c, \omega)$ is a direct measure of the home-market advantage of one of the countries to offer higher wages for similar supply-side fundamentals. The country with a higher demand capacity (i.e. the north if $\gamma > 1/2$) will be able to reward better the labor force, since less demand will be wasted paying transaction costs there. Before we explore the relative-wage consequences of a rise in δ, we need to express γ in terms of the parameters for a steady-state situation. Next lemma will be of considerable help.

2.4.1 Lemma 1 :

In any steady state without net migratory flows, any household's expenditure is identical to that household's income period by period. Therefore, the steady-state aggregate northern and southern incomes are equal to γ and 1-γ, respectively, and there are no net savings.

Proof. See appendix. ■

Subsequently, let us derive some formal expressions of northern and southern aggregate income. From (12), (15), (72) and our definition of γ it is possible to come out with a neat expression of this variable as a fraction between zero and one:

$$\gamma = \frac{1}{1 + \frac{\left(\frac{1-\mu}{w_n}\right) + \omega L_c + \rho\left[(1-\theta_{nn}L_n)a\left(\frac{g}{m+g}\right) + (1-\theta_{cn}L_n)a_m\omega\right]}{L_n\left[1+\rho\left(\theta_{nn}a\left(\frac{g}{m+g}\right)+\theta_{cn}a_m\omega\right)\right]}} \tag{29}$$

In the denominator of (29), w_n is an endogenous variable that has not been fully specified yet in terms of the parameters. So, we need to obtain an expression for local absolute wages as well. Let's define first

$$Q = \frac{m}{g}\omega^{1-\in} \tag{30}$$

Now, if we plug (7) into (14), divide numerator and denominator of the latter expression by $\left(\frac{w_n}{\alpha}\right)^{1-\in}$ and rearrange, eventually we find that

$$w_n = \frac{\alpha\mu}{(L_n - ag)}\left[\frac{\gamma}{1+\delta Q} + \frac{(1-\gamma)\,\delta}{\delta + Q}\right] \tag{31}$$

Proceeding in a similar way, we can solve for w_c from (13) as follows:

$$w_c = \frac{\mu}{(L_c + a_m\rho)}\left[\frac{\gamma\delta}{1+\delta Q} + \frac{(1-\gamma)}{\delta + Q}\right]Q \tag{32}$$

Now we can really derive a necessary and sufficient condition for an increase in ω in response to a marginal rise in trade openness (δ). In order to provide a benchmark that discloses the main determinants of convergence, we start adopting an extreme assumption: the imitation capacity of the core and its share in the aggregate measure of manufactures is infinitesimal. That means that $\mathrm{L}_c \to a_m \frac{L_n}{a}^+$.

We also adopt the following simplifying assumptions concerning the distribution of financial wealth:

$$\theta_{nn}L_n \to 1^-;\ \theta_{cn}L_n \to 0^+; \theta_{cc} = \theta_{ca} = 1/L_s \tag{33}$$

where θ_{kl} is equal to the proportion of aggregate wealth from location k owned by any household living in location l. That is, although people own some shares of foreign equity, the aggregate magnitude of those shares is negligible. We can argue that very small international capital movements suffice to preserve the arbitrage condition (22).

2.4.2 Proposition 1:

$\lim_{L_c \to a_m \frac{L_n}{a}^+}\left(\frac{d\omega}{d\delta}\right) > 0$ iff $\delta^2 > \frac{1-\mu}{\mu}$

Proof. Let's rewrite the second part of expression (27) as follows:

$$C\left(\delta, L_c, \omega\right) = \frac{\gamma\left(m\omega^{1-\in} + \delta g\right) + (1-\gamma)\,\delta\left(\delta m\omega^{1-\in} + g\right)}{\gamma\delta\left(m\omega^{1-\in} + \delta g\right) + (1-\gamma)\left(\delta m\omega^{1-\in} + g\right)} \tag{34}$$

After a marginal increase in δ, the right-hand side of (27) has to remain constant, because nothing is altered in the left-hand side of the equality. Therefore,

$$\lim_{L_c \to a_m \frac{L_n}{a}^+} \frac{(dC/d\delta)}{C} = - \lim_{L_c \to a_m \frac{L_n}{a}^+} \frac{\in \frac{d\omega}{d\delta}}{\omega} \tag{35}$$

Then, if we take logs of (34) and compute the total derivative, we can get that

$$\frac{(dC/d\delta)}{C} = \frac{\left(\gamma\delta^2 - (1-\gamma)\right)}{\delta\left[1 - \gamma\left(1-\delta^2\right)\right]} -$$

$$-\frac{\frac{d\omega}{d\delta}\left[Q\left(\in -1\right)\left(\delta^2 - \left(\gamma + (1-\gamma)\,\delta^2\right)\left(1-\gamma\left(1-\delta^2\right)\right)\right) + \frac{d\gamma}{d\omega}\delta\left(1-\delta^2\right)\right]}{\delta\left[1-\gamma\left(1-\delta^2\right)\right]} \tag{36}$$

From (35) and (36),

$$\frac{d\omega}{d\delta} = \frac{\omega\left(\gamma\delta^2 - (1-\gamma)\right)}{\in \delta\left(1-\gamma\left(1-\delta^2\right)\right) + (\in -1)\,Q\left[\delta^2 - \left(\gamma + (1-\gamma)\,\delta^2\right)\left(1-\gamma\left(1-\delta^2\right)\right)\right] + \frac{d\gamma}{d\omega}\delta\omega\left(1-\delta^2\right)} \tag{37}$$

In order to determine the sign of $\lim_{L_c \to a_m \frac{L_n}{a}^+} \frac{\frac{d\omega}{d\delta}}{\omega}$, it is useful to know the limit-value of ω when $\mathrm{L}_c \to a_m \frac{L_n}{a}^+$. From (27),

$$\lim_{L_c \to a_m \frac{L_n}{a}^+} \frac{L_n - ag}{L_c - a_m g}\frac{m}{g} = \left[\lim_{L_c \to a_m \frac{L_n}{a}^+} \omega^{\in}\right]\left[\lim_{L_c \to a_m \frac{L_n}{a}^+} C(\delta, L_c, \omega)\right] \tag{38}$$

Our parameter restriction (26) guarantees that g > 0 and then, from (25), (34) and (38), $0 = \left[\lim_{L_c \to a_m \frac{L_n}{a}^+} \omega^{\in}\right]\left[\frac{\delta}{1-\gamma(1-\delta^2)}\right]$. As we can infer from (29), $0 < \frac{\delta}{1-\gamma(1-\delta^2)} < \infty$ provided that $\delta > 0$. Then, as a consequence,

$$\lim_{L_c \to a_m \frac{L_n}{a}^+} \omega = 0^+ \tag{39}$$

Moreover, since we can easily check that $\lim_{L_c \to a_m \frac{L_n}{a}^+}\left(\frac{d\gamma}{d\omega}\right)$ is finite, from $(27)\,, (30)$ and (39) it is possible to conclude that $\lim_{L_c \to a_m \frac{L_n}{a}^+}\left(\frac{d\gamma}{d\omega}\right)\omega = \lim_{L_c \to a_m \frac{L_n}{a}^+}(Q) = 0$, and therefore, by (37),

$$\lim_{L_c \to a_m \frac{L_n}{a}^+} \frac{\frac{d\omega}{d\delta}}{\omega} = \frac{\gamma\delta^2 - (1-\gamma)}{\in \delta\left(1-\gamma\left(1-\delta^2\right)\right)} \tag{40}$$

Since the denominator of (40) is positive,

$$lim_{L_c \to a_m \frac{L_n}{a}^+}\left(\frac{d\omega}{d\delta}\right) > 0 \text{ iff } \gamma\delta^2 > (1-\gamma) \tag{41}$$

Next, from (29) and (31) we can obtain that

$$\lim_{L_c \to a_m \frac{L_n}{a}^+} \frac{(1-\mu)}{w_n} = \frac{(1-\mu)}{\mu}\left(L_n + a\rho\right) \tag{42}$$

Now, if we plug (42) into (29), we can restate condition (41) only in terms of the parameters:

$$lim_{L_c \rightarrow a_m \frac{L_n}{a}^+}\left(\frac{d\omega}{d\delta}\right) > 0 \; iff \; \theta_{nn} > \frac{1}{\mu\left(1+\delta^2\right)}\left[\frac{\left(1-\mu\left(1+\delta^2\right)\right)}{a\rho}+\frac{1}{L_n}\right] \tag{43}$$

Finally, taking into account our assumptions in (33),

$$lim_{L_c \rightarrow a_m \frac{L_n}{a}^+}\left(\frac{d\omega}{d\delta}\right) > 0 \quad iff \quad \delta^2 > \frac{1-\mu}{\mu}$$

■

There are 2 opposite effects of a reduction of international transaction costs on the relative wage ω. The first one has to do with the difference in aggregate income between north and south: a wealthier north will be likely to raise its demand for every southern manufacture beyond the increase in aggregate southern demand for any northern good. This would result in a rise of ω if there were no other active forces. Let's call this the relative-size effect.

But there is still another effect. Since most differentiated products are initially produced in the north, the southern price-index will decrease sharply with a rise in δ, whereas the northern one will remain almost unaltered. This phenomenon tends to reduce ω in steady state to keep $\frac{x_n}{x_c}$ according to the supply-side fundamentals.

The strength of this price-index effect decreases with the initial degree of trade openness (δ), since higher values of δ imply more symmetry in the relative impact of new trade liberalizations on the local price indices. Therefore, for $\frac{d\omega}{d\delta}$ to be positive we do not only need a large differential in the size of both countries, but a high enough initial value of δ. Under the assumptions of Proposition 1, a very high relative-size effect has been guaranteed, which makes trade openness the only determinant of the evolution of relative wages.

But we would like to know what happens to relative incomes out of this extreme situation, i.e. for any initial distribution of Southern population between core and periphery. Our next objective will be obtaining the function $\omega_c = \frac{w_c}{w_a} = f\left(L_a, \delta\right)$ that determines the labor-market-clearing relative wage in the south as a function of L_a and δ. The intersection of this curve with an exogenous migration function $\omega_c = h(L_a)$, which yields the amount of people willing to live in the periphery as a function of the relative wage, will offer the final-steady-state values ($L^*_a(\delta)$,$\omega_c{}^*(\delta)$).

3 Steady state with migration

3.1 The role played by migratory movements.

Since we want to reproduce some stylized facts, it is convenient for us to rule out any price-index effect threatening to abort north-south convergence. Then, the relative-size effect will remain as the single driving force. Therefore, $\gamma > 1/2$ appears as a natural fact for a north-south structure that (together with $\delta \longrightarrow 1^-$) could be enough to achieve international convergence in per-capita income. But let's provide first a sufficient condition for $\gamma > 1/2$ in terms of the parameters.

Lemma 2 :

Given our assumptions in (33), $\lim_{\delta \longrightarrow 1^-} (\gamma) > 1/2$ if $\mathrm{L}_n > \hat{L}_n (L_c)$, where $\hat{L}_n (L_c)$ is a monotone increasing function.

Proof. See appendix. ■

The assumption made explicit in (11) involves that $\lim_{\delta \longrightarrow 1^-} (\omega) < \alpha$, from which we can also derive the following lemma.

Lemma 3:

There exists a unique upper-bound $\mathrm{L}_c^* \geq L_c$ such that the wide-gap assumption holds together with the coexistence of a positive measure of northern and southern manufactures; i.e. $\exists$ a unique L_c^* such that (11) and (26) are simultaneously satisfied iff

$$a_m \frac{L_n}{a} < L_c \leq L_c^* < a_m \left[\frac{L_n}{a} + \frac{\alpha\rho}{1-\alpha} \right] \; \forall a, a_m$$

Proof. See appendix. ■

Our notion of steady state is partially characterized by the following equality:

$$\omega_c = f(L_a, \delta) = h(L_a) \tag{44}$$

where $\omega_c = h(L_a)$ is our migration function, for which we adopt a convectional convex, downward-sloping shape (as in Faini (96)).

In this model we just take as given the main features of the migration function, but we will endogenously determine the curve $\omega_c = f(L_a, \delta)$. From (12), (20) and (32) we can obtain that

$$\lim_{\delta \to 1^-} \omega_c = \lim_{\delta \to 1^-} f(L_a, \delta) = \lim_{\delta \to 1^-} \left[\frac{L_a}{(1-\mu)} \frac{\mu \left(\frac{Q(L_a,\delta)}{1+Q(L_a,\delta)} \right)}{(L_s - L_a + a_m \rho)} \right] \tag{45}$$

where

$$\lim_{\delta \to 1^-} \mathrm{Q}(\mathrm{L}_a, \delta) = \left[\frac{(1-\alpha)\left[\frac{L_s - L_a}{a_m} - \frac{L_n}{a}\right]}{\alpha\rho - (1-\alpha)\left[\frac{L_s - L_a}{a_m} - \frac{L_n}{a}\right]} \right]^{1-\alpha} \left[\frac{\alpha\left(L_s - L_a + a_m\rho\right)}{L_n - \frac{a(1-\alpha)}{a_m}\left(L_s - L_a\right) + a\alpha\rho} \right]^{\alpha} \tag{46}$$

Here we can appreciate the two basic effects of a declining peripheral labor force ($\downarrow L_a$) on ω_c :

- First, the numerator and denominator of (45) directly capture the straightforward *labor-supply effect*: if new immigrants come from periphery to core, ω_c will tend to decrease for a given value of Q.

- Secondly, the quotient $\frac{Q(L_a,\delta)}{1+Q(L_a,\delta)}$ is decreasing in L_a because it reflects the gain in *imitation potential* of the core after an inflow of former peripheral workers. This force tends to increase the fraction of the total measure of manufactures produced in the core, which channels world demand to this location and can potentially raise w_c.

The relative strength of these two effects varies along the relevant range of values of L_a : $\left[L_s - a_m \frac{L_n}{a},\ \mathrm{L}_s - L_c^*\right]$. In fact, $\mathrm{Q}(\mathrm{L}_a, \delta)$ acts as a positive measure of the imitation potential in the core. Moreover, additional migration reinforces much more that potential the lower $\mathrm{Q}(\mathrm{L}_a, \delta)$ is. In other words, once you have copied a high proportion of northern varieties, it is harder for you to raise your local wage by further imitating: you have to compete - every time more toughly - with more and more producers in your own location.

In fact, since by (45) $f(0,\delta) = f\left(L_s - a_m \frac{L_n}{a}, \delta\right) = 0 \ \ \forall\delta$ and our function f is continuous in L_a,we know for sure that $f(L_a, \delta)$ shows an inverted-U shape $\forall\delta$. That is, we can observe both an upward-sloping part of the curve - where the labor-supply effect is stronger - and a downward-sloping one, with a dominant imitation-potential effect[6] (see figure 1).

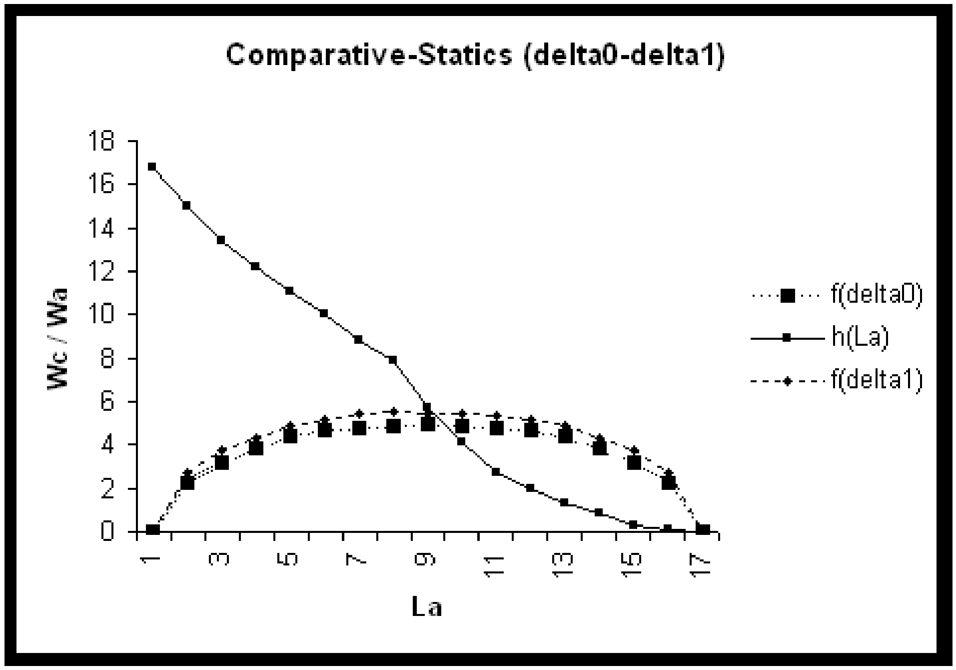
3.2 Main results.

Now we will obtain a sufficient condition for the ratio $\mathrm{R}_{ca} = \frac{\text{per-capita income in the core}}{\text{per-capita income in periphery}}$ to increase in response to a marginal rise in δ.

Proposition 2:

Let $\mathrm{R}_{ca} = \frac{Y_c/(L_s - L_a)}{Y_a/L_a}$ be the core-periphery relative per-capita income. If in the original steady state the following conditions are satisfied: a) $\mathrm{L}_n > \hat{L}_n(L_s)$; b) $\frac{a_m L_n}{a} < L_c < L_c^*$; c) $\delta \to 1^-$; d)

[6] Provided that the whole range of values of La satisfies the wide-gap assumption, i.e. if $\mathrm{L}_s - a_m \frac{L_n}{a} < L_c^*$.



$\frac{dh}{dL_a} < \lim_{\delta \to 1^-} \left(\frac{\partial f}{\partial L_a} \right)$, then :

$$\frac{dR_{ca}}{d\delta} > 0; \ \frac{d\omega_c}{d\delta} > 0; \frac{dg}{d\delta} > 0; \frac{dL_a}{d\delta} < 0$$

Proof. See appendix. ■

With a sudden rise in δ, the dominance of the relative-size effect - when we are close to full openness - weakens the home-market advantage of the north. The subsequent rise in ω_c attracts a net migratory flow from periphery to core and increases our southern imitation potential. Hence, the increase in ξ_c caused by migrations channels more world demand towards southern manufactures and exerts an upward pressure on the labor costs in the core. This force countervails the labor-supply effect, which usually happens when industrial competition within the core is soft enough and southern labor force is sufficiently sticky.

Given the significant agglomeration effects on labor productivity detected in the EU by Ciccone (99), accepting that $\frac{\partial f}{\partial L_a} < 0$ does nor seem counterfactual. Neither does the extreme stickiness of labor in many European countries (see Bentolila (99)). In that case, restraining migrations would be likely to mitigate core-periphery divergence, though at the expense of foregone growth-effects in the global economy.

Let's try to face now the north-south convergence issue in a similar fashion.

Proposition 3:

If in our initial steady state $a_m \frac{L_n}{a} \leq L_c \leq L_c^*$, $L_n > \hat{L}_n(L_s)$, $\delta \to 1^-$ and $\frac{dh}{dL_a} < \frac{\partial f}{\partial L_a}$, then necessarily $\frac{dR_{ns}}{d\delta} < 0$, where R_{ns} is the relative per-capita income of the north with respect to the south.

Proof. See appendix. ■

There are three forces involved in the comparative-statics evolution of relative north-south per-capita income, two of which exactly offset each other. These 2 opposite forces, whose joint effect is nil, can be described as follows:

- First, the net inflow of workers to the core enhances the innovation rate and, consequently, also the demand for labor in the north, which tends to raise w_n.

- At the same time, although the global economy innovates faster, a higher imitation potential raises the proportion of southern manufactures. Hence, a lower proportion of total financial wealth owned by the Northerners exactly makes up for the higher demand for researchers in that country. Therefore, the only effect capable of modifying γ comes from the aggregate demand for manufactures produced in the north. This aggregate demand goes down in terms of our numeraire, since the northern home-market advantage weakens.

Corollary:

If in our initial steady state $a_m \frac{L_n}{a} \leq L_c \leq L_c^* < a_m \left[\frac{L_n}{a} + \frac{\alpha\rho}{1-\alpha}\right]$, $L_n > \hat{L}_n(L_s)$, $\delta \to 1^-$ and $\frac{dh}{dL_a} < \lim_{\delta \to 1^-} \left(\frac{\partial f}{\partial L_a}\right)$, then, in our comparative-statics exercise

$$\frac{dR_{ca}}{d\delta} > 0; \frac{dg}{d\delta} > 0; \frac{dL_a}{d\delta} < 0; \frac{d\gamma}{d\delta} < 0 \text{ and } \frac{d\omega_c}{d\delta} > 0.$$

Proof. Straightforward from the last 2 propositions. ■

4 Conclusions.

We have studied a north-south endogenous growth model where exogenous institutional features play a major role: they determine the relative incidence of a biased shock in trade openness on 2 distinct southern regions. Within our southern country, we have considered a perfectly-competitive market structure for the periphery together with some sources of agglomeration economies in the core. As a result, we have reproduced our stylized facts, i.e. the coexistence of per-capita income convergence between countries and divergence within the same countries.

The existence of scale effects generates a trade-off between core-periphery convergence and global steady-state growth. But not necessarily a trade-off between long-run growth rates and north-south convergence.

We conclude that, no matter how generous interregional transfers are, if they do not help transform peripheral productive structures they can not prevent an asymmetric exposure to trade shocks. If transfers also restrained migratory flows, they could reduce the core-periphery gap, though only by lowering all Southerners' labor income.

On the other hand, if transfers helped to industrialize the periphery the scale-effects would be larger. This looks an argument to advocate structural changes in the periphery as opposed to direct transfers to household consumption. But, in order to elaborate on this, we need to do some welfare analysis requiring transitional dynamics and an explicit formulation of both migratory and structural-change costs.

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6 Appendix.

6.1 Household's Intertemporal Optimization.

In order to allocate expenditure and savings over time, any household k must choose in every period s a variation in its portfolio composition, buying or selling equity from northern and southern firms. During that process we have to keep in mind that, in every period s, a fraction $m = \frac{\dot{n}_c}{n_n}$ of the northern measure of varieties is copied by southern imitators.

Let π_n and π_c denote the current operating profits of any northern and southern industrial firm, respectively. At every period t, household k owns a measure $\beta_{nk}(s)$ of northern firms and $\beta_{ck}(s)$ of southern firms. Moreover, f_{nk} stands for the proportion of gross-savings devoted to buying northern equity. We will explore the properties of an interior equilibrium in which new start-ups from both countries are financed (i.e. $0< f_{nk} < 1$).

Our control variables are E_k (household's expenditure) and $f_{nk}(s)$, whereas the state variables are $\beta_{nk}(s)$ and $\beta_{ck}(s)$. Then, the present-value Hamiltonian faced by any household in location k at time t for the period s is the following:

$$\begin{aligned} H_k(s) \quad &= \quad e^{-\rho(s-t)} \log E_k(s) + \Phi_{nk}(s) \left[\frac{(w_k + \beta_{nk}\pi_n + \beta_{ck}\pi_c - E_k) f_{nk}(s)}{v_n} - m\beta_{nk} \right] + \\ &+\Phi_{ck}(s) \left[\frac{(w_k + \beta_{nk}\pi_n + \beta_{ck}\pi_c - E_k)(1 - f_{nk}(s))}{v_c} \right] \end{aligned} \tag{47}$$

The first-order condition for an interior solution for $f_{nk}(s)$ is the following:

$$\frac{\Phi_{nk}(s)}{v_n(s)} = \frac{\Phi_{ck}(s)}{v_c(s)}, \ \forall s \tag{48}$$

The first-order condition with respect to $E_k(s)$ yields, due to equation (48), that

$$e^{-\rho(s-t)} \frac{1}{E_k(s)} = \frac{\Phi_{nk}(s)}{v_n(s)} = \frac{\Phi_{ck}(s)}{v_c(s)}, \ \forall s \tag{49}$$

And therefore, by differentiating and using the first-order conditions with respect to the state variables,

$$\frac{\mathring{E}}{E} = \frac{\mathring{E}_c}{E_c} = \frac{\mathring{E}_n}{E_n} = \frac{\mathring{E}_a}{E_a} = \frac{\pi_n}{v_n} - m - \rho + \frac{\dot{v}_n}{v_n} = \frac{\pi_c}{v_c} - \rho + \frac{\dot{v}_c}{v_c} \tag{50}$$

Now, by grouping terms, we can define $A=\frac{E}{nv_n}$ and $B=\frac{E}{n_c v_c}$. From (13), (14) and (50), it is possible to obtain a system of 3 differential equations in A, B and $c=\frac{n}{n_c}$.Our steady state without migrations will be defined by equating these differential equations to zero.

On the other hand, the system describes the dynamics of A, B and c, but the separate evolutions of E, v_c and v_n can not be disentangled. As a consequence, Grossman and Helpman have one degree of freedom to normalize E=1, which implies (by equation (3)) that

$$A = \frac{1}{aw_n};\ B = \frac{1}{a_m w_c} \tag{51}$$

From the last expression and the definitions of A, B and c above, we could specify before the wage dynamics in (16) and (17). Consequently, by (16), (20), (51) and our definition of steady state,

$$\left[1+\frac{1-\alpha}{\alpha}\frac{c}{(c-1)}\right]\left[\frac{L_n}{a}-\left((1-\alpha)\frac{L_c}{a_m}-\alpha\rho\right)\right]-\left[(1-\alpha)\frac{L_c}{a_m}-\alpha\rho\right]\frac{1}{(c-1)}=\rho+\frac{L_n}{a} \tag{52}$$

Finally, solving for c in (52) we can get that

$$c^* = \frac{\alpha\rho}{(1-\alpha)\left[\frac{L_c}{a_m}-\frac{L_n}{a}\right]} \tag{53}$$

The trivial fact that n$\geq$n$_c$, i.e. c$\geq$1, imposes our restriction (26) on the value of the parameters.

6.2 Proof of Proposition 2.

Proof. From our definition of R$_{ca}$, assumption (33) and lemma 1 we can derive that in any steady state

$$R_{ca} = \frac{\omega_c\left[(L_s-L_a)+\rho a_m(1-\theta_{ca}L_a)\right]}{(1+\rho\theta_{ca}a_m\omega_c)(L_s-L_a)} \tag{54}$$

From (44), any marginal variation in δ must yield the following migratory reaction between steady states:

$$\lim_{\delta\to1^-}\frac{dL_a}{d\delta} = \lim_{\delta\to1^-}\left[\frac{\frac{\partial f}{\partial\delta}}{\left(\frac{\partial h}{\partial L_a}-\frac{\partial f}{\partial L_a}\right)}\right] \tag{55}$$

The assumptions of the proposition guarantee that the denominator in (55) is negative. As to the numerator, from (30) and (46) we can obtain that

$$\lim_{\delta\to1^-}\frac{\partial f}{\partial\delta} = \frac{L_a}{(1-\mu)}\frac{\mu}{(L_s-L_a)}\left[\frac{(2\gamma-1)Q+\frac{\partial Q}{\partial\delta}}{(1+Q)^2}\right] \tag{56}$$

and

$$\lim_{\delta\to1^-}\frac{\partial Q}{\partial\delta} = -\left(\frac{m}{g}\right)(\in -1)\left[\lim_{\delta\to1^-}\omega^{-\in}\right]\left[\lim_{\delta\to1^-}\frac{\partial\omega}{\partial\delta}\right] \tag{57}$$

Now, from (27) and (34) we can conclude that

$$\omega = C^{-\frac{1}{\in}}(\delta, L_a, \omega)\,.\left[\lim_{\delta\to1^-}\omega\right]\ \forall\delta,\text{ since }\lim_{\delta\to1^-}C(\delta, L_a, \omega)=1 \tag{58}$$

After some computations, we can additionally get from Lemma 3 and (34) that

$$\lim_{\delta\to1^-}\frac{\partial C}{\partial\delta} = 1-2\gamma<0 \tag{59}$$

Finally, expressions (58) and (59) imply that

$$\lim_{\delta\to1^-}\frac{\partial\omega}{\partial\delta}=\left[\lim_{\delta\to1^-}\omega^{-\in}\right].\frac{(2\gamma-1)}{\in}>0 \tag{60}$$

If we now go backwards, plugging (60) into (57) and then (57) into (56), our final result after rearranging is that $\lim_{\delta\to1^-}\frac{\partial f}{\partial\delta}=\frac{L_a}{(1-\mu)}\frac{\mu}{(L_s-L_a)}.\lim_{\delta\to1^-}\left[\frac{\left(\frac{m}{g}\right)(2\gamma-1)\omega^{1-\in}}{(1+Q)^2\in}\right]>0$. This positive sign means, by (55), that $\frac{dL_a}{d\delta}<0$. And hence, from (20), $\frac{dg}{d\delta}>0$. Since

$$\frac{dR_{ca}}{d\delta}=\frac{\partial R_{ca}}{\partial\omega_c}\frac{\partial\omega_c}{\partial\delta}+\left[\frac{\partial R_{ca}}{\partial\omega_c}\frac{\partial\omega_c}{\partial L_a}+\frac{\partial R_{ca}}{\partial L_a}\right].\frac{dL_a}{d\delta} \tag{61}$$

we must obtain now the expressions for $\frac{\partial R_{ca}}{\partial\omega_c}$ and $\frac{\partial R_{ca}}{\partial L_a}$ to clarify unambiguously which is the sign of (61). Then, from (33) and (54),

$$\frac{\partial R_{ca}}{\partial\omega_c}=\left[1+\frac{\rho a_m\left(1-\theta_{ca}L_a\right)}{(L_s-L_a)}\right].\frac{1}{\left(1+\theta_{ca}a_m\rho\omega_c\right)^2} \tag{62}$$

$$\frac{\partial R_{ca}}{\partial L_a}=\frac{\rho a_m\left(1-\theta_{ca}L_a\right)}{\left(L_s-L_a\right)^2}.\frac{\omega_c}{\left(1+\theta_{ca}a_m\rho\omega_c\right)} \tag{63}$$

If we consider simultaneously (61) and (62), we can easily observe that

$$\lim_{\delta\to1^-}\frac{dR_{ca}}{d\delta}>\lim_{\delta\to1^-}\frac{dL_a}{d\delta}\left[\frac{\partial R_{ca}}{\partial\omega_c}\frac{\partial\omega_c}{\partial L_a}+\frac{\partial R_{ca}}{\partial L_a}\right]$$

which means that $\lim_{\delta\to1^-}\frac{dR_{ca}}{d\delta}>0$ if $\frac{dh}{dL_a}<\lim_{\delta\to1^-}\left(\frac{\partial f}{\partial L_a}\right)$. Finally, if we focus on the evolution of ω_c, its total derivative can be proved to be positive provided that $\gamma>1/2$ and $\frac{dh}{dL_a}<\left(\frac{\partial f}{\partial L_a}\right)$, since

$$\frac{d\omega_c}{d\delta}=\frac{\partial f}{\partial\delta}.\left[\frac{\frac{dh}{\partial L_a}}{\left(\frac{\partial h}{\partial L_a}-\frac{\partial f}{\partial L_a}\right)}\right] \tag{64}$$

■

6.3 Proof of Proposition 3.

Proof. Since L_n and L_s are invariant in our model, from Lemma 1 we can infer that $\frac{dR_{ns}}{d\delta}<0$ iff $\frac{d\gamma}{d\delta}<0$.

The easiest way to compute $\frac{d\gamma}{d\delta}$ is by considering expressions (31) and (33). Let

$$D(L_c,L_n)=\left[L_n+\rho a\left(\frac{g}{m+g}\right)\right] \tag{65}$$

From (20), (25), (33) and (72), $\gamma=w_n\left[L_n+a\left[\rho-\frac{1}{(\in-1)}\left(\frac{L_c}{a_m}-\frac{L_n}{a}\right)\right]\right]$, and by taking logs and differentiating

$$\lim_{\delta\to1^-}\frac{d\gamma}{d\delta}\frac{1}{\gamma}=\frac{-a_m\left(\frac{dL_a}{d\delta}\right)}{a_m\in(L_n-ag)}-\lim_{\delta\to1^-}\frac{\left[\frac{dQ}{d\delta}+(2\gamma-1)\,Q\right]}{(1+Q)}+\frac{dD}{d\delta}\frac{1}{D} \tag{66}$$

It is easy to show that, precisely,

$$\frac{dD}{d\delta}\frac{1}{D} = \frac{a_m\left(\frac{dL_a}{d\delta}\right)}{a_m \in (L_n - ag)} \tag{67}$$

and therefore, by (57), (60), (66) and (67),

$$\lim_{\delta\to 1^-}\frac{d\gamma}{d\delta}\frac{1}{\gamma} = -\lim_{\delta\to 1^-}\frac{\left[\frac{\partial Q}{\partial L_a}\frac{dL_a}{d\delta} + \frac{(2\gamma-1)Q}{\in}\right]}{(1+Q)} < 0 \tag{68}$$

Apart from the assumptions of this proposition, expressions (46) and (68) ensure that $\lim_{\delta\to 1^-}\frac{d\gamma}{d\delta} < 0$. ∎

6.4 Proof of Lemma 1.

Proof. Let $\theta_{nk} = \frac{\beta_{nk}}{n_n}$ and $\theta_{ck} = \frac{\beta_{ck}}{n_c}$ be the proportion of northern and southern equity, respectively, owned by a representative household living in location k, where β_{nk} and β_{ck} are the absolute measures of northern and southern firms owned by that household. Then, the amount of gross savings for any household living in k can be expressed as follows:

$$(Gross\ \text{Savings})_k = GS_k = w_k + \theta_{ck}n_c\pi_c + \theta_{nk}n_n\pi_n - E_k \tag{69}$$

We know that in our steady state $\frac{\dot\theta_{jk}}{\theta_{jk}} = \frac{\dot\beta_{jk}}{\beta_{jk}} - g = 0$, i.e. $\frac{\dot\beta_{jk}}{\beta_{jk}} = g$ $\forall j$ = north, core; ∀k= north, core, periphery. Therefore,

$$\frac{\dot\beta_{nk}}{\beta_{nk}} = \frac{GS_k f_{nk}}{v_n\beta_{nk}} - m = \frac{\dot\beta_{ck}}{\beta_{ck}} = \frac{GS_k\,(1-f_{nk})}{v_c\beta_{ck}} = g \tag{70}$$

where f_{nk} is the proportion of total gross savings devoted to the purchase of northern equity. Then, from (70),(3) and (24), we can easily solve for GS_k:

$$GS_k = (m+g)\theta_{nk}a\left(\frac{g}{m+g}\right)w_n + g\theta_{ck}a_m w_c \tag{71}$$

On the other hand, it is easy to see from (3) and (21) that the instantaneous variation in the value of previously-owned assets, considering also the effect of imitation, is the following:

$$\frac{\partial V_k}{\partial t} = -\left(m+g\right)\theta_{nk}a\left(\frac{g}{m+g}\right)w_n - g\theta_{ck}a_m w_c$$

where V_k is the value of previously-owned assets by a household in location k. Since, by (71) and the last equation, $(\text{Net Savings})_k = GS_k + \frac{\partial V_k}{\partial t} = 0$ ∀t in any steady state, any household's wealth is kept constant along the balanced growth path, i.e.

$$y_k = E_k = w_k + \rho\left[\theta_{ck}a_m w_c + \theta_{nk}a\left(\frac{g}{m+g}\right)w_n\right] \tag{72}$$

where y_k is household k's income, ∀k= north, core, periphery in steady state. ∎

6.5 Proof of Lemma 2.

Proof. From (29) we can check that

$$\lim_{\delta\to 1^-} \gamma > 1/2 \text{ iff } (1-\mu) < \lim_{\delta\to 1^-} \left[w_n \left(L_n + a\rho \left(\frac{g}{m+g} \right) \right) - w_c \left(L_c + a_m \rho \right) \right] \tag{73}$$

As we can conclude after inspecting expressions (20), (25), (31) and (32), condition $\lim_{\delta\to 1^-} \gamma > 1/2$ can only be satisfied iff (73) holds. Now we just have to look for a sufficient condition that guarantees (73). From our definition of Q in expression (30), condition (73) can be restated as follows:

$$\frac{(1-\alpha)\left[\frac{L_c}{a_m} - \frac{L_n}{a}\right]}{\alpha\rho - (1-\alpha)\left[\frac{L_c}{a_m} - \frac{L_n}{a}\right]} \cdot \left[\frac{\alpha\left(L_c + a_m\rho\right)}{L_n - ag}\right]^{\in -1} < P^{\in} \tag{74}$$

By the assumptions established in this lemma, necessarily $\mathrm{P}^{\in} > 0$. Let's now define the function

$$H(L_c, L_n) = \frac{(1-\alpha)\left[\frac{L_c}{a_m} - \frac{L_n}{a}\right]}{\alpha\rho - (1-\alpha)\left[\frac{L_c}{a_m} - \frac{L_n}{a}\right]} \cdot \left[\frac{\alpha\left(L_c + a_m\rho\right)}{L_n - ag}\right]^{\in -1} - P^{\in} \tag{75}$$

It is easy to see that $\frac{\partial H}{\partial L_c} \geq 0$ and $\frac{\partial H}{\partial L_n} \leq 0 \; \forall L_c, L_n$. Therefore, a sufficient condition for (73) follows from any situation in which $\mathrm{H}(\mathrm{L}_c, L_n) < 0$. We want to search for a relation between the initial values of L_c and L_n that ensures that $\mathrm{H}(\mathrm{L}_s, L_n) < 0$ and hence that $\lim_{\delta\to 1^-} \gamma > 1/2$. For any initial value of L_c that satisfies (20) and (26), we can determine that, from (75),

$$H(L_c, \frac{aL_c}{a_m}) = -P^{\in} < 0 \text{ and } \mathrm{H}(\mathrm{L}_c, a\left[\frac{L_c}{a_m} - \frac{\alpha\rho}{1-\alpha}\right]) > 0 \tag{76}$$

Since the equality $\mathrm{H}(\mathrm{L}_c, L_n) = 0$ contains an implicit function $\hat{\mathrm{L}}_n(L_c)$ for which $\frac{\partial L_n}{\partial L_s} = -\frac{\partial Q/\partial L_s}{\partial Q/\partial L_n} > 0$ $\forall L_c, L_n$, then $\hat{L}_n\left(L_c\right)$ is an increasing function in L_c. Since $\mathrm{H}(\mathrm{L}_s, L_n)$ is a monotone and continuous function in L_n, from (76) we can apply Bolzano's theorem to state that

$$\exists \text{ a unique function } \hat{\mathrm{L}}_n(L_c) \text{ such that } \mathrm{H}(\mathrm{L}_c, \hat{L}_n(L_c)) = 0 \; \forall L_c \tag{77}$$

Finally, from the sign of the partial derivatives above, we can say with certainty that $\forall L_c$, if $\mathrm{L}_n > \hat{L}_n(L_c)$ then $\mathrm{H}(\mathrm{L}_c, L_n) < 0$, which means that Q < P and hence that $\lim_{\delta\to 1^-} \gamma > 1/2$. ■

6.6 Proof of Lemma 3.

Proof. From (11) we can express the wide-gap assumption when $\delta \to 1^-$ as

$$\frac{\left(L_n - \frac{(1-\alpha)L_c}{a_m} + \alpha\rho\right)}{\alpha\left(L_c + a_m\rho\right)} \cdot \left[\frac{(1-\alpha)\left[\frac{L_c}{a_m} - \frac{L_n}{a}\right]}{\alpha\rho - (1-\alpha)\left[\frac{L_c}{a_m} - \frac{L_n}{a}\right]}\right] \leq \alpha^{\in} \tag{78}$$

Rearranging and rewriting (78) with an equality, we get the following quadratic equation in L_c :

$$\left[\frac{L_c}{a_m} - \frac{L_n}{a}\right] = \frac{\frac{\alpha\rho}{(1-\alpha)}}{\left(1 + \frac{\left(L_n - a\frac{(1-\alpha)}{a_m} + a\alpha\rho\right)}{\alpha^{\frac{2-\alpha}{1-\alpha}}(L_c + a_m\rho)}\right)} \tag{79}$$

Since, from condition (26), $L_c > 0$ and $L_n > \frac{a}{a_m}L_c - \frac{a\alpha\rho}{(1-\alpha)}$, we can conclude that the denominator of the right-hand side of (79) is bigger than 1. This means that at least one root $L^*_{c1}$of (79) satisfies for sure the inequality $a_m\frac{L_n}{a} < L^*_{c1} < a_m\left[\frac{L_n}{a} + \frac{\alpha\rho}{1-\alpha}\right]$, because the right-hand side is positive and smaller than $\frac{\alpha\rho}{1-\alpha}$. Now we have to make sure that $L^*_{c1}$is a unique root within the interval $(a_m\frac{L_n}{a}, a_m\left[\frac{L_n}{a} + \frac{\alpha\rho}{1-\alpha}\right])$.

If we formally restate (78) we can obtain the following inequality:

$$Z(L_c) = EL_c^2 + FL_c + G \leq 0 \tag{80}$$

where

$$E = \frac{a_m\alpha^{\frac{2-\alpha}{1-\alpha}} - a(1-\alpha)}{a_m^2} \tag{81}$$

$$F = \frac{L_n\left(\frac{2-\alpha}{1-\alpha} - \alpha^{\frac{2-\alpha}{1-\alpha}}\frac{a_m}{a}\right) + \alpha\rho\left[a + \alpha^{\frac{1}{1-\alpha}}a_m\left(2 - \frac{1}{1-\alpha}\right)\right]}{a_m}$$

$$G = -\left[\frac{L_n}{a}\left[\alpha\rho\left(a + \alpha^{\frac{1}{1-\alpha}}a_m\right) + L_n\right] + \alpha^{\frac{3-2\alpha}{1-\alpha}}.\frac{1}{1-\alpha}a_m\rho^2\right]$$

We can see that, in principle, the signs of E and F are undetermined but that of G is clearly negative, which implies that $Z(0) < 0$. Let's explore now the implications of the 2 possibilities concerning the sign of A:

-If $E > 0$ then, since $Z(0) < 0$, $Z(L_c)$ is necessarily a quadratic function with one positive and one negative root. Therefore, we know for sure that there is a unique L^*_{c1} such that $Z(L^*_{c1}) = 0$ and $a_m\frac{L_n}{a} < L^*_{c1} < a_m\left[\frac{L_n}{a} + \frac{\alpha\rho}{1-\alpha}\right]$. Given that this curve cuts the horizontal axis from below, conditions (80) and (26) will be satisfied.

-If $E < 0$, $Z(L_c)$ will be now a concave function with at least one positive root L^*_{c1}, but in principle it could have another one within our particular interval $\left[a_m\frac{L_n}{a}, a_m\left[\frac{L_n}{a} + \frac{\alpha\rho}{1-\alpha}\right]\right]$. In order to reject this latter possibility, it will be enough to show that $Z(a_m\frac{L_n}{a}) < 0$ and $Z(a_m\left[\frac{L_n}{a} + \frac{\alpha\rho}{1-\alpha}\right]) > 0$,which would imply that the other root is out of our interval.

It is possible to check that

$$Z\left(a_m\frac{L_n}{a}\right) = -\frac{(\in -1)^{\in+2}}{\in^{\in+1}}.\rho a_m.\left(\frac{L_n}{a} + \rho\right) < 0$$

$$Z(a_m\left[\frac{L_n}{a} + \frac{\alpha\rho}{1-\alpha}\right]) = \left(\frac{\in -1}{\in}\right)\rho L_n\left(\in^2 + \in -2\right) > 0$$

Again, since this curve intersects the horizontal axis from below, if E $<$ 0 the wide-gap case is compatible with positive measures of manufactures in both countries iff $a_m \frac{L_n}{a} < L_c \leq L_c^* < a_m \left[\frac{L_n}{a} + \frac{\alpha\rho}{1-\alpha}\right]$.

To summarize, if $\delta \to 1^-$, $\forall a_m$ and a, $\forall \alpha \in (0,1)$, $\exists$ a unique L_c^* such that both (11) and (26) hold iff $a_m \frac{L_n}{a} < L_c \leq L_c^* < a_m \left[\frac{L_n}{a} + \frac{\alpha\rho}{1-\alpha}\right]$, where L_c^* is the smallest positive root of equation (79). ■

7 Legend

7.1 Endogenous variables.

A_s^k : Amount of the homogeneous (primary) good consumed by a reprsentative individual from location k at time s (k =north, core, periphery).

$\beta_{nk}(s)$: Measure of northern firms owned by a representative household from location k at time s (k =north, core, periphery).

$\beta_{ck}(s)$: Measure of southern firms owned by a representative household from location k at time s (k =north, core, periphery).

$c = \frac{n}{n_c}$: Inverse of the proportion of the aggregate measure of manufactures produced in the core.

$C(\delta, L_c, \omega)$: Measure of the home-market advantage of the north to offer higher aggregate demand to firms located there.

$D(L_c, L_n) = L_n + \rho a \left(\frac{g}{m+g}\right)$

E_s^k : Total expenditure allocated to consumption by a representative individual from location k at time s (k =north, core, periphery).

ξ_c : Steady-state proportion of southern manufactures ($\xi_c = \frac{1}{c} = \frac{n_c}{n}$ in steady state).

f_{nk} : Fraction of savings devoted to buying new norther firms (start-ups) by a household from location k.

$g = \frac{\dot{n}}{n} = \frac{\dot{n}_c}{n_c}$: Steady-state innovation growth-rate.

GS_k : Amount of gross savings of any household living in location k (k =north, core, periphery).

L_c : Aggregate measure of population currently living in the core.

L_a : Aggregate measure of population currently living in the periphery.

$m = \frac{\dot{n}_c}{n_n}$: Rate of imitation of northern varieties by southern researchers.

n_k : Total measure of manufactures currently produced in location k (k =north, core, periphery).

n : Total aggregate measure of manufactures existing currently in the global economy.

$p_j\,(s)$: Market price of variety j at time s.

$p_a\,(s)$: Market price of the homogeneous (primary) good at time s.

$p_k(s)$: Market price of any variety produced in location k at time s (k =north, core).

π_k : Current operating profits for any manufacturing firm in location k (k =north, core).

$Q = \frac{m}{g}\omega^{1-\varepsilon}$: Measure of the imitation potential of the core to copy northern patents (attracting world demand to southern varieties).

R_{ca} : Relative per-capita income in the core with respect to the periphery.

R_{ns} : Relative per-capita income in the north with respect to the south.

$U_s(X_s^k, A_s^k)$: Value of the 'felicity function' at time s for a representative consumer from location k.

V_k : Aggregate value of the assets owned by a representative household from location k.

v_k : Value of the patent of any variety produced in location k.

w_c : Nominal wage of any worker living in the core, employed either in manufacturing or research.

w_n : Nominal wage of any worker living in the north, employed either in manufacturing or research.

w_a : Nominal wage of any worker living in the periphery, employed in primary production.

W_t^k : Discounted flow of lifetime utility obtained from period t onwards by a representative household/individual in location k (k =north, core, periphery).

$\omega = \frac{w_c}{w_n}$: Relative wage of the core with respect to the north.

$\omega_c = \frac{w_c}{w_a}$: Relative wage of the core with respect to the periphery.

X_s^k : Subutility function (aggregator) derived from the consumption of manufactures at period s by a reprsentative individual from location k (k =north, core, periphery).

$x_j(s)$: Individual demand for variety j at time s.

x_n : Aggregate demand for any northern manufacture (variety).

x_c : Aggregate demand for any variety produced in the core.

y_k : Income of any household from location k (k =north, core, periphery).

γ : Proportion of aggregate consumption spent by Northerners.

7.2 Parameters and exogenous variables.

a_m : Indicator of research costs of imitation in the core.

a : Indicator of research costs of innovation in the north.

α : Positive measure of substitutability among varieties of manufactures.

$\delta = \tau^{1-\varepsilon}$: Measure of international trade-openness.

E : Aggregate expenditure in the global economy (taken as numeraire).

$\epsilon = \frac{1}{1-\alpha}$: Elasticity of substitution among varieties of manufactures.

L_s : Aggregate measure of population living in the south.

L_n : Aggregate measure of population living in the north.

τ : Positive measure of international-trade costs (classical iceberg-notion).

μ : Relative weight assigned to manufactures in the felicity function.