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The Effects of Income Inequality on Trade Patterns

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Abstract

This paper builds a two-country-two-sector trade model with a monopolistically competitive sector and non-homothetic preferences. It assumes the existence of two types of goods: necessities (which are homogeneous) and luxuries (which are differentiated) and heterogeneous labor. The implications of income inequality on trade patterns are examined. It also considers the effects of redistributive policies on the production structure and welfare of countries and concludes that: First, in autarky, the more unequal country produces a larger number of varieties; Second, the opening to trade will unambiguously increase the number of varieties consumed by any country, and hence, welfare; Third, the more equal country benefits more from trade liberalization. Fourth, a redistributive policy may harm some consumers not only by diminishing disposable income, but also by diminishing the number of varieties produced.

Keywords: Income inequality, monopolistic competition, non-homothetic preferences, labor heterogeneity

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1 Introduction

The topic of income inequality and trade has received great attention in recent years as levels of income inequality in many countries have been increasing. Rich countries, particularly, have been concerned that international trade may be the driving force behind increasing concentration of income. The argument is based on the fact that trade liberalization allows cheap imports to substitute domestic products and, as a result, domestic unskilled labor lose jobs or become underemployed, which widens the gap between skilled and unskilled labor, or, between the rich and the poor. Income inequality, however, may be a result of different factors such as skill inequality, labor standards, market imperfections, social values, etc. If, at one hand, international trade may affect domestic income inequality, on the other hand, inherent inequality may affect trade patterns.[1] This paper is particularly interested in the latter case.

According to the Linder (1961) hypothesis, people with similar income levels would consume similar goods. Linder argued that income levels were decisive in determining consumption patterns. The intra-industry trade literature, however, have paid little attention to this fact either due to modeling difficulties or to the wide use of the utility homotheticity assumption. In the aggregation of demand, a homothetic utility function nulls the effects arising from heterogeneity of income because every consumer spends the same fraction of income in a certain good. In that case, there is no difference between countries of equal income per capita, even if they have completely different inequality levels.

Nonhomotheticity is a necessary feature in order to have income inequality to play a role in aggregate demand. Incorporating monopolistic competition, as in Markusen (1986), it is possible to analyze intra-industry related trade patterns. Markusen (1986) incorporates a Dixit-Stiglitz-Krugman type industry in a Stone-Geary utility function to analyze the volume of trade. Despite obtaining a quasi-homothetic demand with varying expenditure shares, his focus was on intra-industry trade between countries rather than labor heterogeneity.[2] Labor heterogeneity is thought to be a primary factor

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[1]High levels of inequality can create sufficient demand for luxury or sophisticated goods such that these industries may develop in poor unequal countries rather than in more equal rich societies. For instance, Brazil exports airplanes to Japan.

[2]Dinopolous, Fujiwara and Shimomura (2007) use a quasi-linear utility function to analyze the effects

determining income inequality and can be modeled by assuming differences in levels of human capital as in Bougheas and Riezman (2007),[3] differences in labor productivity as in Yeaple (2005), differences in kinds of labor (skilled and unskilled) that enter separately in the production function, or differences in physical capital as in Mitra and Trindade (2005).[4] Although the existing literature has focused on varying fraction of income, it is the usual assumption that even the poorest consumer purchases a certain amount of all goods. A few exceptions include Matsuyama (2000) and Foellmi, Hepenstrick and Zweimuller (2007) that include non-homothetic utility function with 0/1 preferences. In their model, consumers choose the number of varieties instead of quantity, as opposed to the standard variety model but heterogeneity in labor is not considered.

In order to analyze the effects of income inequality on trade patterns, we consider a tractable two-country-two-sector trade model with quasi-linear preferences and labor heterogeneity.[5] One sector is monopolistically competitive in the Dixit-Stiglitz fashion with differentiated products, and the other is a competitive sector with homogeneous goods. Setting apart from the existing literature, it is assumed that the homogeneous good is a necessity good and suffers no income effects, while the differentiated product is considered a luxury good, that is, only consumers with sufficiently high income will consume them.[6] Consumers are explicitly distributed over a range of skills (or productivity) so that more efficient workers have higher incomes. The analysis compares the production and consumption of varieties before and after trade and considers welfare and redistributive issues on an individual basis.[7]

The following results are derived. When two countries are identical except for their level of income inequality, the more unequal country has a more disperse population, thus the demand for variety goods is higher than the more equal country. Consequently, in autarky, the more unequal country produces a larger number of varieties. Trade

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of income distribution and trade patterns on a Hecksher-Ohlin model.

[3] In a two-country-two-sector model with equal human capital endowment but different distributions. They show that the human capital abundant country exports the human capital good.

[4] They incorporate general non-homothetic preferences in a Hecksher-Ohlin model and analyze the volume of trade and the shock transmission to other countries. They also consider a model of monopolistic competition to focus in the volume of inter and intraindustry trade.

[5] We adopt the Pareto distribution as it is used to construct standard measures of inequality such as the GINI index.

[6] See Francois and Kaplan (1996) for empirical evidence.

[7] Trade liberalization and redistributive policies affect consumers with different skills and income in different ways.

liberalization increases the number of varieties produced and consumed by any country due to the increased number of traded varieties that lower the price index, which in turn creates more demand by a "poor-inclusive" effect. As the aggregate price of differentiated goods decreases, "poor" consumers (that do not consume differentiated products) start to consume. Therefore, as the total number of varieties under free trade increases, both countries gain from trade liberalization. The more equal country benefits more due to a relatively higher increase in the number of varieties consumed. A redistributive policy that reallocates income from "rich" consumers to "poor" consumers may decrease individual welfare not only by diminishing disposable income, but also by diminishing the number of varieties available.

In order to see the direct effects of inequality on trade, this paper provides a very simple framework that includes heterogeneous labor combined with non-homothetic preferences, which, surprisingly, is not present in the existing literature. This paper also gives a new focus on the "poor" consumers as an important factor that magnifies the benefits of trade liberalization. The simplicity of the model facilitates extensions that could include trade barriers, many sectors, differences in technology among countries or more general distribution functions. Also, political issues could be included to endogenize income distribution.

The next section introduces the model, Section 3 analyzes the case of free trade and section 4 concludes this work.

2   The Model

In this section a tractable model with non-homothetic preferences and income inequality is built. There are two countries, Home and Foreign, and two sectors: a competitive homogeneous good sector and a monopolistically competitive sector with differentiated goods.

Home and Foreign have labor mass L and L^*, respectively.[8] Each individual has efficiency level $s \in [s_0, \infty]$ with a nonnegative support s_0 and probability given by a

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[8]Foreign variables and parameters will be denoted by the superscript (*).

density function $l(s)$ such that:

$$\int_{s_0}^{\infty} l(s)ds = 1 \quad \text{and} \quad \int_{s_0}^{\infty} sl(s)ds = \bar{s}. \tag{1}$$

Here, $l(s)$ determines the fraction of population with productivity level s, and $\bar{s}$ is the average productivity of labor.[9] Thus the total amount of effective labor is given by $L\bar{s}$.

2.1 Consumers

Consumers with skill level s derive utility from both homogeneous and differentiated goods in the following manner:

$$U^s = \ln Y^s + D^s, \tag{2}$$

with Y as the amount consumed of the homogeneous (or necessity) good and D the quantity index of differentiated (or luxury) goods. The quasi-linear function guarantees that the homogeneous good Y is not subject to income effects, reflecting the necessity feature. The usual assumption of differentiated goods not subject to income effect is inverted since it is somehow more intuitive to think of varieties as luxury goods since product differentiation is more likely to occur in products of lower need.[10]

The subutility derived form luxury goods takes the following Dixit-Stliglitz utility form:

$$D^s = \left(\sum_{i=1}^{n} (d_i^s)^\theta + \sum_{i^*=1}^{n^*} (d_{i^*}^s)^\theta \right)^{\frac{1}{\theta}}, \quad 0 < \theta < 1, \tag{3}$$

where i (i^*) denotes a differentiated good produced at Home (Foreign), n (n^*) is the total number of Home (Foreign) differentiated goods, d_i denotes Home consumption of Home variety i and d_{i*} denotes Home consumption of Foreign variety i^*. Here, $1/1-\theta > 1$ is the elasticity of substitution between every pair of differentiated goods. The price index

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[9]In this model, labor heterogeneity is depicted as differences in labor efficiency, that is, the quality of labor is homogeneous but each consumer is able to produce different amounts of output.

[10]Take the example of food and electronics, for instance. Food is clearly a necessity compared to electronics as a matter of demand and electronics are relatively more susceptible to product differentiation than food is. As a result it is common to observe the fraction of income spent in food declining and that of electronics growing as income increases.

P_D takes the form

$$P_D = \left(\sum_{i=1}^{n} (p_i)^{\frac{\theta}{\theta-1}} + \sum_{i^*=1}^{n^*} (p_{i^*})^{\frac{\theta}{\theta-1}} \right)^{\frac{\theta-1}{\theta}}, \tag{4}$$

where p_i and p_{i^*} denote the prices paid by Home consumers for Home variety i and Foreign variety i^* respectively.

The utility maximization problem can be solved in two steps. First, by solving the subutility maximization problem we obtain the Home demand for Home variety i and Foreign variety i^*, respectively:

$$d_i^s = \left(\frac{p_i}{P_D} \right)^{\frac{1}{\theta-1}} D^s \tag{5}$$

$$d_{i^*}^s = \left(\frac{p_{i*}}{P_D} \right)^{\frac{1}{\theta-1}} D^s. \tag{6}$$

In the second step consumers maximize utility over the homogenous good and the basket of varieties. The following demands are obtained:

$$Y^s = \frac{P_D}{P_Y} \tag{7}$$

$$D^s = \frac{I(s)}{P_D} - 1. \tag{8}$$

Note, however, that each consumer with labor efficiency s derives income $I(s)$ from labor, such that different efficiency levels imply different income levels and, consequently, different demand levels. Particularly, we obtain from (7) the following demand function of consumer s for the necessity good:

$$Y^s = \begin{cases} \frac{P_D}{P_Y} & \text{if } I(s) \geq P_D \\ \frac{I(s)}{P_Y} & \text{if } I(s) < P_D. \end{cases} \tag{9}$$

It is clear that the consumer with a sufficiently high income will not consume more than $\frac{P_D}{P_Y}$ of the necessity good. Now we turn to the demand for luxury goods. Note from (8) that only consumers with a sufficiently high skill level will demand differentiated products. Rearranging (5), (6), and (8) we derive the demand functions for Home

variety i and Foreign variety i^* of a consumer with skill level s:

$$d_i^s = \begin{cases} \left(\frac{p_i}{P_D}\right)^{\frac{1}{\theta-1}}\left[\frac{I(s)}{P_D} - 1\right] & \text{if } I(s) \geq P_D \\ 0 & \text{if } I(s) < P_D, \end{cases} \tag{10}$$

$$d_{i^*}^s = \begin{cases} \left(\frac{p_i^*}{P_D}\right)^{\frac{1}{\theta-1}}\left[\frac{I(s)}{P_D} - 1\right] & \text{if } I(s) \geq P_D \\ 0 & \text{if } I(s) < P_D. \end{cases} \tag{11}$$

The above demand functions give rise to the necessity and luxury feature we need to analyze income distribution issues. Consumers with low-income levels will spend all their incomes in the necessity good but, as income increases and reaches a certain level, they will stop spending more on the necessity good to spend all the remaining income on differentiated products.

Note that there is a consumer that will have income level exactly equal to the expenditure of the fixed amount of necessity. This is the *marginal consumer* with skill level s_m and it is appropriate to give a definition:

$$s_m \equiv \left\{ s \in [s_0, \infty] \mid I(s) = P_D \right\}. \tag{12}$$

Thus, consumers with income levels below s_m consume only homogeneous products and consumers with higher income levels consume a fixed amount of the homogeneous good and spend the remaining income on differentiated goods. The marginal consumer is completely defined by the price index P_D and the wage rate.

Figure 1 denotes a typical income expansion path under given prices P_Y and P_D. We obtain an inverted L-shaped income-expansion path that differs considerably from paths derived from homothetic utility functions. Note that the fraction of income spent on each type of good changes according to the income level.[11]

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[11]Markusen (1986) also suggests a quasi-homothetic utility function that gives rise to varying fraction of income spent on different goods.
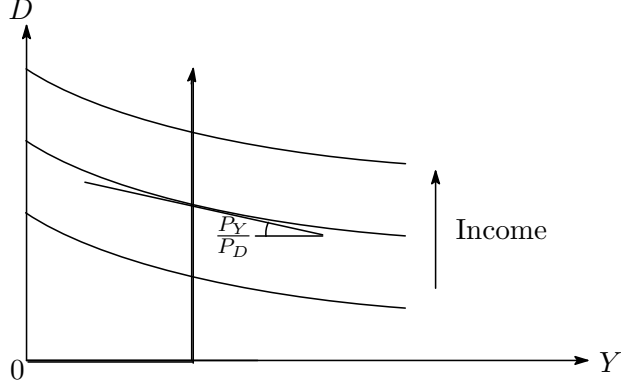
Figure 1: Income Expansion Path

We derive now the aggregate demand for the homogeneous good and varieties. But first, let us derive the following relations for the "rich" consumers, that is, consumers demanding differentiated products (in contrast with "poor" consumers demanding only homogeneous products). Defining $\bar{s}_R$ as the average efficiency level of rich consumers and δ_R as the fraction of rich consumers in the population, we have $\delta_R \times \bar{s}_R \equiv \int_{s_m}^{\infty} s l(s) ds$ and $\delta_R \equiv \int_{s_m}^{\infty} l(s) ds$.

Given that w is the wage paid for one unit of effective labor, then $I(s) = ws$ is the income of consumer with efficiency level s. Thus, using the above definitions and equations (9) to (11), the Home aggregate demand for homogeneous good, Y, aggregate demand for Home variety i, d_i, and aggregate demand for Foreign variety i^*, d_{i*}, are derived:

$$Y = L\frac{w}{P_Y}(\bar{s} - \delta_R \times \bar{s}_R) + L\left(\frac{P_D}{P_Y}\right)\delta_R \tag{13}$$

$$d_i = L\left(\frac{p_i}{P_D}\right)^{\frac{1}{\theta-1}}\left[\frac{w}{P_D}\delta_R \times \bar{s}_R - \delta_R\right] \tag{14}$$

$$d_{i*} = L\left(\frac{p_{i*}}{P_D}\right)^{\frac{1}{\theta-1}}\left[\frac{w}{P_D}\delta_R \times \bar{s}_R - \delta_R\right]. \tag{15}$$

Similarly, we have the Foreign aggregate demands Y^*, d_i^*, and d_{i*}^*:

$$Y^* = L^*\frac{w^*}{P_Y^*}(\bar{s^*} - \delta_R^* \times \bar{s}_R^*) + L^*\left(\frac{P_D^*}{P_Y^*}\right)\delta_R^* \tag{16}$$

$$d_i^* = L^*\left(\frac{p_i^*}{P_D^*}\right)^{\frac{1}{\theta-1}}\left[\frac{w^*}{P_D^*}\delta_R^* \times \bar{s}_R^* - \delta_R^*\right] \tag{17}$$

$$d_{i*}^* = L^*\left(\frac{p_{i*}^*}{P_D^*}\right)^{\frac{1}{\theta-1}}\left[\frac{w^*}{P_D^*}\delta_R^* \times \bar{s}_R^* - \delta_R^*\right], \tag{18}$$

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where p_i^* and p_{i*}^* denote de price paid by Foreign consumers for Home variety i and Foreign variety i^*, respectively. Under a given skill distribution curve, the aggregate demand, the average skill as well the fraction of rich consumers will be entirely defined by wage and price levels.

2.2 Production

Now let us turn to the supply side. As we have assumed, Y is a homogeneous good produced under constant returns to scale in a competitive sector. One unity of Y is produced with one unity of labor. Homogeneous goods are priced the average cost, that is, $P_Y = w$.

Differentiated goods are produced in a monopolistically competitive sector with production requiring a fixed amount μ and a variable amount β of labor. Countries share the same production technology. Then, the pricing rule becomes:[12]

$$p_i = p_i^* = \frac{\beta w}{\theta}, \quad \text{and} \quad p_{i*} = p_{i*}^* = \frac{\beta w^*}{\theta}.$$

Given the above pricing rule and assuming free entry and exit of firms in the long-run, we obtain the following zero-profit conditions:

$$(1 - \theta)\frac{\beta w}{\theta}(d_i + d_i^*) - w\mu = 0, \tag{19}$$

$$(1 - \theta)\frac{\beta w^*}{\theta}(d_{i*} + d_{i*}^*) - w^*\mu = 0. \tag{20}$$

Now we are ready to discuss the effects of skill inequalities. In the next section we analyze the autarky case.

3 The Autarky Economy

As a benchmark, let us consider the case in which there is no trade between Home and Foreign, and both homogeneous and differentiated products are produced in each country. We take the homogeneous good as numeraire, thus $P_Y = w = 11$.[13] Given the

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[12]Although we only consider the free trade case, an extension of the model with trade barriers is possible.

[13]We only analyze the case of Home, but the Foreign case is analogous.

symmetry of firms, we derive the price index and the marginal consumer $P_D = s_m = n^{\frac{\theta-1}{\theta}}\beta/\theta$.

Note that, if the number of firms in equilibrium is sufficiently high, the marginal consumer may be smaller than the support of the distribution function, that is, $s_m < s_0$, implying that all consumers are rich. First, however, we analyze the case in which there are poor and rich people in equilibrium, that is, $s_m > s_0$, then we continue with the rich-only case.

3.1   The Poor-and-Rich Case

Using the price index expression and the marginal consumer definition we obtain from (14) and (19) the following equilibrium condition:

$$(1-\theta)LP_D^{\frac{1}{1-\theta}}\left(\frac{\beta}{\theta}\right)^{\frac{\theta}{\theta-1}}\left[\frac{\delta_R \times \bar{s}_R}{P_D} - \delta_R\right] = \mu \tag{21}$$

The magnitude of $\bar{s}_R$ and δ_R are determined by the skill level distribution and prices. For our analysis we use a standard Pareto distribution function:

$$l(s; s_0, k) = \frac{ks_0^k}{s^{k+1}}, \quad \text{with} \quad s_0 > 0, \quad k > 1$$

where s_0 is the support of the function (minimum skill) and k is a parameter denoting skill inequality.[14] The number of firms in the poor-and-rich equilibrium is obtained by the following three expressions:

$$n = \frac{Ls_0^k(1-\theta)}{(k-1)\mu s_m^{k-1}} \tag{22}$$

$$s_m = n^{\frac{\theta-1}{\theta}}\left(\frac{\beta}{\theta}\right) \tag{23}$$

$$s_m > s_0. \tag{24}$$

Equations (22) and (23) constitute loci of which intersection uniquely determines the

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[14]If $k = 1$ there is complete inequality and if $k = \infty$ there is perfect equality. Note that when there is no redistributive policy, skill inequality and income inequality are equivalent. The average skill and the fraction of rich consumers is given by $\delta_R\bar{s}_R = \frac{k}{k-1}\frac{s_0^k}{s_m^{k-1}}$, $\delta_R = \frac{s_0^k}{s_m^k}$. They are functions of the marginal consumer and the level of inequality k only. It is important to note that, if s_0 is kept constant, one cannot change the level of inequality and keep the same average productivity in a Pareto distribution.

equilibrium number of firms. However, the intersection point is eligible as an equilibrium point only if it is stable and satisfies condition (24). Figure 2 denotes a case in which the intersection point e is eligible as an equilibrium point since $s_m^e > s_0$ and the point is stable (curve s_m cuts curve n from below).
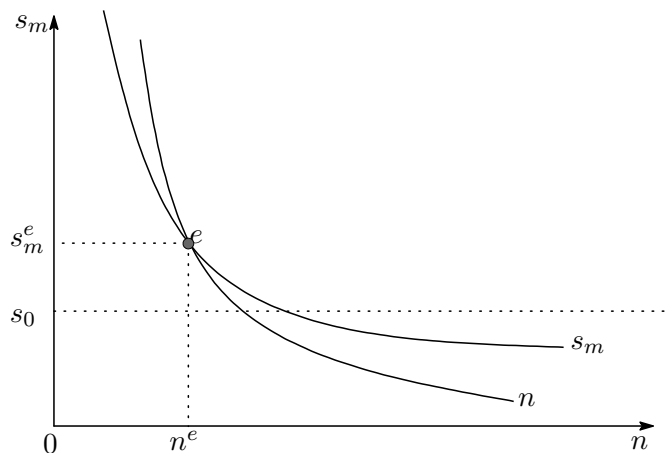

Figure 2: Poor-and-Rich Autarky Equilibrium

With lower levels of inequality, particularly if $k > 1/(1 - \theta)$, curve s_m will cut curve n from above making the intersection point unstable.

3.2 The Rich-Only Case

If the number of varieties is sufficiently high, all consumers will be able to purchase variety goods. In such case the demand for homogeneous products is equal to all consumers and s_m does not stand for the marginal consumer anymore, that is, even the consumer with the lowest skill level s_0 consumes a positive amount of differentiated products. The equilibrium conditions now change to:

$$n = \frac{(1 - \theta)L}{\mu} \left[\frac{ks_0}{k - 1} - s_m \right] \tag{25}$$

$$s_m = n^{\frac{\theta - 1}{\theta}} \left(\frac{\beta}{\theta} \right) \tag{26}$$

$$s_m \leq s_0. \tag{27}$$

The right and left-hand side of (25) can be depicted separately so that the intersections of the two loci determine the equilibrium points. Analogous to the poor-and-rich

case, however, s_0 is the maximum value the marginal consumer can assume, if she existed, so that the intersection points may be eligible as rich-only equilibrium points. Two cases are illustrated in Figure 3 and 4.
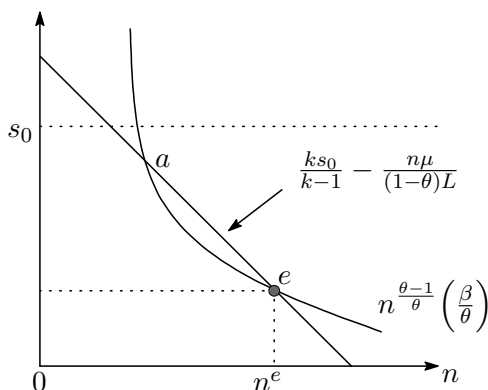

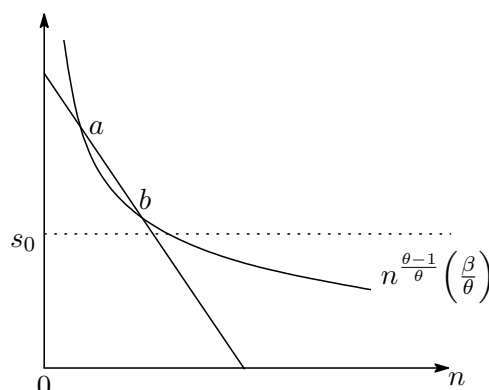Figure 3 - Unstable and Stable Equilibria   Figure 4 - No Equilibrium

In Figure 3, point a, is not eligible as an equilibrium point since it is not stable, leaving e as the only equilibrium point. In Figure 4 neither point a or b satisfies condition (27). Note that for sufficiently low levels of s_o or large fixed cost μ there may be no intersection points between the curve and the descending line.

3.3   Autarky Equilibrium

So far we have seen the poor-and-rich and the rich-only cases separately. However, in order to make a full analysis of the equilibrium it is necessary to consider both cases at once. From conditions (22) and (25) we note that curve n will invariably tangent the descending line when $s_m = s_0$.[15] It is possible to have several different equilibrium patterns as depicted in Figures 5 and 6.
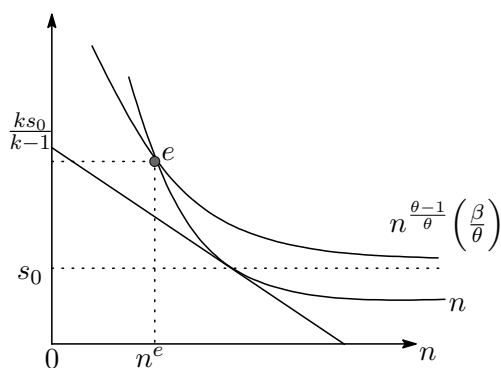

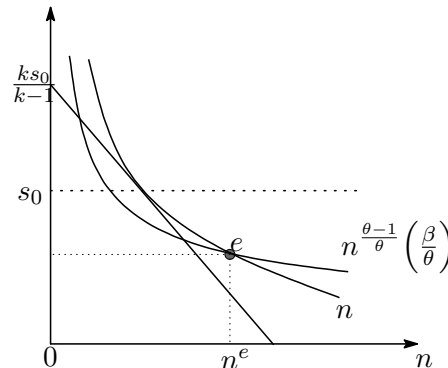Figure 5 - A Poor-and-Rich Equilibrium   Figure 6 - A Rich-Only Equilibrium

[15]See Appendix.

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When $k < 1/(1-\theta)$ an equilibrium can be either poor-and-rich or rich-only, always exists and is unique. When $k > 1/(1-\theta)$, a poor-and-rich equilibrium is never stable, thus the economy diverges to a state of either no production of varieties or rich-only equilibrium.[16] Under certain conditions, however, curve s_m may not intersect the descending line, and no equilibrium with varieties exists. The following condition guarantees that the marginal consumer curve intersects the descending line when $k > 1/(1-\theta)$:[17]

Lemma 1. *If $\frac{ks_0}{(k-1)} \geq \frac{\beta^\theta \mu^{1-\theta}}{\theta^{2\theta}(1-\theta)^{2(1-\theta)}L^{1-\theta}}$ then a stable equilibrium with production of varieties always exists.*

It is more likely to have equilibrium with production of varieties in economies with high average skill levels (or higher inequality levels) or sufficiently small marginal or fixed costs (more efficient technology).

3.4   Differences in Inequality Levels

Now we focus on the differences in inequality level k. Our primary focus is on trade patterns that may be observed between two countries that are identical except for the level of inequality. However, a simple change in the level of inequality with other variables kept constant alters the total amount of labor or the average skill, hindering any effective comparison. Therefore, in order to derive consistent implications, we need to keep either the range of skills or the average skill constant.

First, we consider a mean-preserving difference in inequality levels. We assume two countries in autarky that have the same labor masses $(L = L^*)$ and average skills $(\bar{s} = \bar{s}^*)$ but different skill distributions $(k \neq k^*)$. In order to keep the same average skill we need to adjust the support of the function s_0 so that the following equation holds:

$$\frac{ks_0}{k-1} = \frac{k^* s_0^*}{k^* - 1}.$$

The above relation implies that if $k^* > k$ then $s_0^* > s_0$. Taking k as the reference level of inequality, from (22), (23) and (25) we know that with a mean-preserving higher k^*, the

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[16]This stems from the fact that more equal economies have the population concentrated in a range of skill levels close to s_0, thus its likely to have either large demand or no demand for varieties depending on the proximity of the marginal consumer to s_0. Increased variety number will generate more demand, which, in turn, will generate more varieties and so on. The negative chain effect also happens.

[17]See Appendix.

descending line and the marginal consumer curve do not change and the n curve shifts up tangent to the descending line in the point $s_m^* = s_0^*$. Figure 7 depicts the case where k and k^* are smaller than $\frac{1}{1-\theta}$. In the poor-and-rich case (intersections with the upper s_m curve), a lower inequality level induces a larger number of firms ($n^e > n^{e*}$). In the rich-only case (intersection with the upper s_m curve), the number of firms is the same in both economies ($n^e = n^{e*}$).
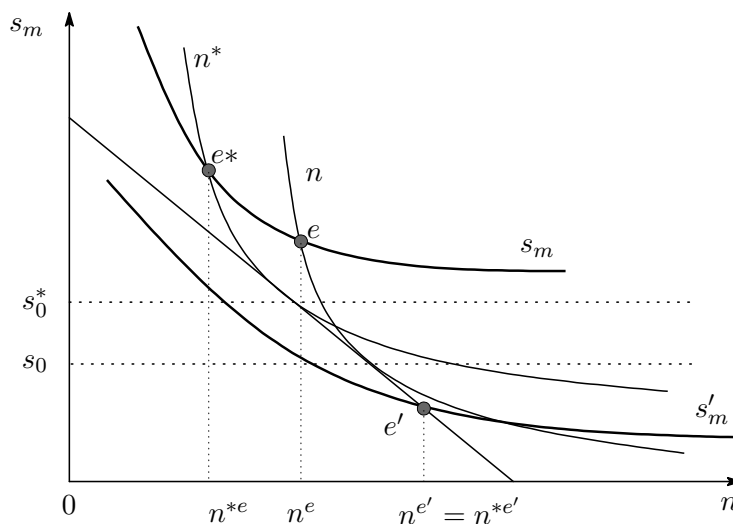

Figure 7 - Mean-Preserving Differences in Inequality

When $k > \frac{1}{1-\theta}$ the equilibrium number of firms is determined by the intersection of the s_m curve and the descending line. Thus, the number of firms does not change with a higher level of k (when there is an equilibrium).

From the above analysis, it is clear that the number of firms in the more unequal society is larger in the poor-and-rich equilibrium and does not change in the rich-only case. We conclude that, although there might be more consumers with low skill levels, more unequal economies are likely to have a larger number of firms than a more equal economy.[18] In order to keep the same average skill, the more equal country needs a higher support s^0 and concentrates the population on lower levels of skill. This diminishes the demand for differentiated products and the number of varieties.

Now let us examine the case of equal range of skills and different inequality levels. To maintain the same amount of labor supply with the same lowest skill level, the following

[18]The position of the marginal consumer curve determines the type of equilibrium and the number of firms. There might be changes in the type of equilibrium if the marginal consumer curve passes between the two curves. In that case the more equal society might have only rich consumers while the more unequal both poor and rich.

condition is necessary:

$$\frac{Lk}{k-1} = \frac{L^*k^*}{k^*-1}.$$

This implies that if $k^* > k$, then $L^* > L$, that is, the more equal economy have larger population. Again, from the equilibrium conditions (22), (23) and (25) it is clear that a larger k^* and a larger labor mass L^* will cause a parallel downward shift of the descending line and an upward shift of the n curve, which tangents the descending line when $s_m = s_0$ as depicted in Figure 8. There is no change in the marginal consumer curve.
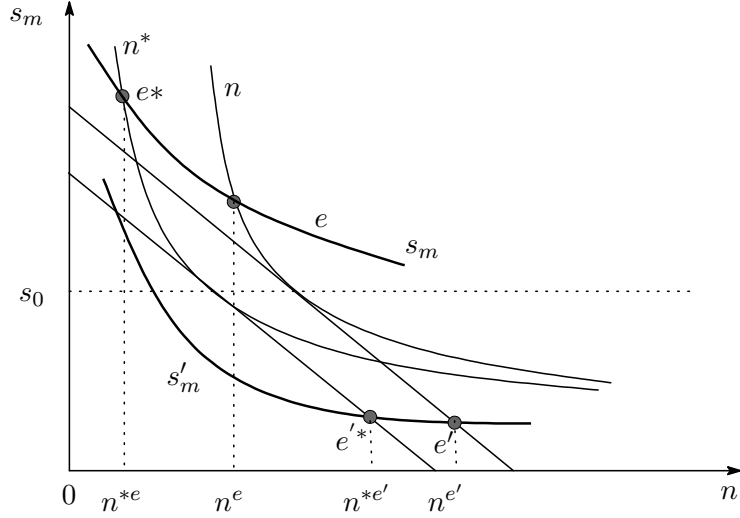

Figure 8: Differences in Inequality with Equal Skill Range

When $k < \frac{1}{1-\theta}$, the more equal economy have a poor-and-rich equilibrium with a smaller number of firms than the more unequal country. In the rich-only equilibrium, the more equal economy will have the same number of firms as the more unequal economy. This also applies to the rich-only case with $k > \frac{1}{1-\theta}$. These results are summarized as follows:

Proposition 1. *Suppose two countries have equal production technologies, quasilinear preferences, and amount of effective labor but differ in the levels of inequality. Then, in autarky, the more unequal country will produce the same or a larger number of varieties compared to the more equal country.*

As the more unequal country has consumers less concentrated in the skill space, demand for variety goods is higher when compared to the more equal country.

An interesting result arises from the case of different inequality levels with equal skill range. As depicted in Figure 9, there is the possibility that there are poor and rich

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consumers in the more equal country, while, in the more unequal country, all consumers
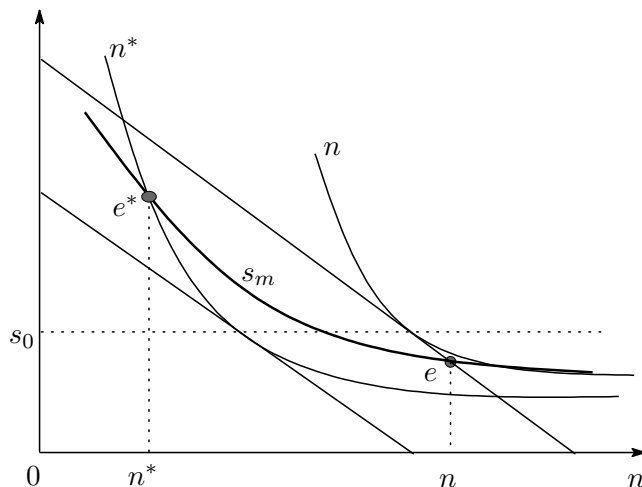are rich.

Figure 9: Poor consumers in the more equal economy, rich consumers in the more
unequal economy.

As the more equal country produces fewer varieties, the price index is higher and some
consumers do not consume varieties.

4   The Free Trading Economy

In this section the effects of trade liberalization and income inequality are examined. We
assume that Home and Foreign have the same production technologies and trade freely.
Also, we assume that both countries produce both differentiated and homogeneous goods
so that wages are equalized. Taking wage as the numeraire, the price of the homogeneous
good is one. From the symmetry of firms, the price index becomes $P_D = N^{T\frac{(\theta-1)}{\theta}}\frac{\beta w}{\theta}$,
where N^T is the total number of varieties and is consisted of the sum of n^T Home
varieties and n^{*T} Foreign varieties under free trade ($N^T = n^T + n^{*T}$).

   With the opening to trade, firms in any country face exactly the same demands
(although aggregate demand is higher in country with higher inequality level). Moreover,
since countries are identical in terms of total labor endowment, the number of varieties
produced in each country is equalized. Now that varieties from the other country are
available there is an overall increase in the number of varietes consumed. Consider both
mean-preserving and equal efficiency range cases with Home more unequal than Foreign,

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it is possible to identify three resulting cases after trade liberalization: poor-and-rich Home and Foreign, poor-and-rich Home and rich-only-Foreign and rich-only Home and Foreign.[19] Then the total number of varieties under free trade can be expressed as:[20]

$$N^T = \frac{L s_0^k (1 - \theta)}{(k-1)\mu s_m^{k-1}} \times A, \quad A > 1. \tag{28}$$

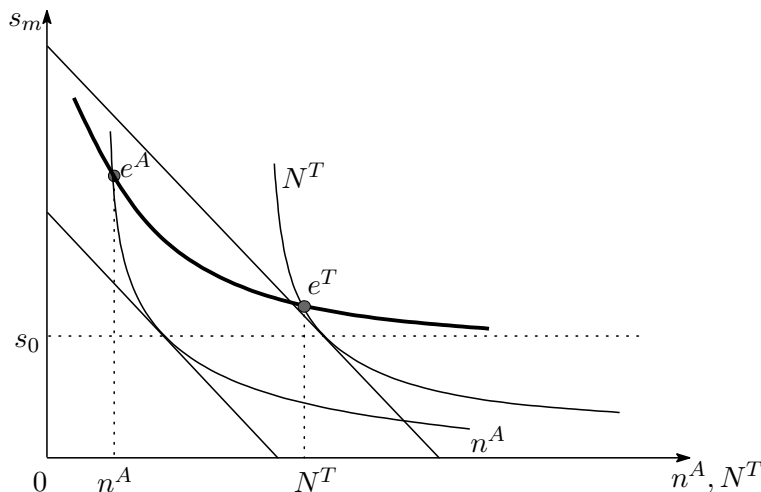Since $A > 1$, the curves will shift outward as depicted in Figure 10.

Figure 10: Trade Liberalization and the Number of Varieties

Figure 10 depicts the case of poor-and-rich Home and Foreign. There is an outward shift in both curves, resulting in an increase in the total number of varieties consumed under free trade, N^T. We summarize the above result in the folowing proposition.

Proposition 2. *If two countries have equal production technologies, quasilinear preferences, and amount of effective labor but differ in the levels of inequality, then, under free trade, the number of varieties produced by each country is equal and the total number of varieties consumed by each country increases.*

Notice that an increase in the number of varieties decreases the price index, which, in turn, widens the range of consumers that can afford variety goods. As a result, the demand for varieties also increases. This "poor-inclusive" effect caused by the non-homothetic nature of the preferences we assumed magnifies the effect of trade liberalization. Another important result is derived.

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[19]Remember that, prior to trade liberalization, Home can be either rich-only or rich-and-poor.

[20]See Appendix for a proof.

Proposition 3. *If Foreign has poor and rich consumers under free trade, then the number of total varieties increases with the level of inequality of Foreign. If Foreign has only rich consumers under free trade, then the number of varieties is not affected by the level of inequality of Foreign.*

5 Trade Liberalization and Welfare

In this section we consider changes in welfare on the opening to trade. We have seen that trade liberalization unambiguously increases the number of varieties produced and consumed by each country. Due to problems inherent in discussing aggregate welfare in the presence of heterogeneous labor, the analysis is conducted on an individual basis, that is, the welfare of individuals with different consumption patterns are studied separately.

First, we analyze the case of poor consumers. Consumers that were poor in autarky may stay poor or "become" rich after trade liberalization. The indirect utility of consumers that are poor in autarky is calculated from (2) and (9):

$$v^A = \ln \frac{ws}{P_Y} = \ln s. \tag{29}$$

Thus the indirect utility is a function of the skill level (which does not change). If the poor consumer stays poor, there is no change in utility. If the poor consumer becomes rich after trade liberalization, that is, $s > s_m^T$, the indirect utility can be calculated from (7) and (8):

$$v^T = \ln \frac{P_D}{P_Y} + \frac{ws}{P_D} = \ln s_m^T + \frac{s}{s_m^T} - 1. \tag{30}$$

The utility now is a function of the individual's skill level and the marginal consumer's skill level. Subtracting (29) from (28) we obtain:

$$v^T - v^A = \ln \frac{s_m^T}{s} + \frac{s}{s_m^T} - 1 > 0. \tag{31}$$

The above equation implies that poor consumers that become rich after trade liberalization are necessarily better off.

Given that the price index is lower in free trade $(s_m^T < s_m^A)$ we can derive the utility

change of consumers that were rich in autarky and remained rich after trade.

$$v^T - v^A = \ln \frac{s_m^T}{s_m^A} + \frac{s}{s_m^T} - \frac{s}{s_m^A} > 0. \tag{32}$$

The same applies for the other country and it does not matter if she produces or not any varieties. Thus we summarize these results in the following proposition:

Proposition 4. *Trade liberalization improves welfare of both countries by increasing the total number of varieties consumed.*

It is also possible to conclude that the more equal country has relatively larger gains from trade since the increase in the number of varieties consumed is relatively larger.

Proposition 5. *The more equal country is likely to obtain larger gains from trade liberalization since the increase in the number of varieties consumed is higher when compared to the more unequal country.*

6   Redistributive Policy

In this section we analyze the effects of redistributive policies on the economy. Initially we assume a government that undertakes a rather extreme form of income redistribution policy: it implements a perfect redistribution schedule in which every individual end up sharing exactly the same income without changing labor supply.[21]

Under this setting, each individual will have income equivalent to total population average skill multiplied by wage, that is, $I = \bar{s}w$. Therefore, all consumers are either poor or rich. If the average skill is sufficiently high to be greater than that of the marginal consumer - if she existed - then everybody is rich. Otherwise everybody is poor.

When all consumers are poor, there will be no demand for Home or Foreign varieties and welfare is derived solely from the consumption of homogeneous goods. For consumers that were poor before redistribution, the change in welfare is equivalent to $[\ln \bar{s} - \ln s]$. Thus if s is smaller than the average then welfare improves and if s is larger than the average skill there is welfare loss. On the other hand, for consumers that were rich before

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[21]Here, we ignore the incentive aspects of labor supply assuming that individuals have equal incentive to work or have only one unit of indivisible labor.

the redistributive policy the welfare change is equivalent to $[\bar{s} - \ln s_m - \frac{s}{s_m} + 1]$. Thus if the skill level of the marginal consumer was sufficiently smaller than the average skill then there is the possibility of welfare gains.[22]

If the average skill is higher than the skill level of the marginal consumer then all consumers are rich. If there were poor consumers before redistribution then the number of firms in equilibrium increases. If all consumers were already rich before redistribution then the equilibrium number of firms is the same as before. On the individual basis, consumers with high skill levels will always lose.

Now we turn to the case of a less extreme redistributive policy. Suppose the government redistributes income so as to increase k (decrease inequality) while keeping the average skill $\bar{s}$ unchanged. Since labor mass L is constant, the support of the function s_0 has to increase. That is exactly the case we have seen in Figure 7. Thus we can summarize this result as follows:

Proposition 6. *A redistributive policy decreases the number of firms in equilibrium.*

Given the above result, it is possible to identify the following types of consumers in terms of welfare change: poor that remained poor that gain from redistribution, rich that became poor and suffer welfare loss, rich that remained rich and gain from income redistribution but lose from the decrease in varieties, and rich that lose from both income redistribution and decrease in varieties.

Note that a redistributive policy diminishes the number of varieties produced and increases the price index. This also affects the consumption of differentiated products in the foreign country by diminishing demand and increasing the number of poor consumers.

7   Concluding Remarks

This paper built a simple trade model with quasi-linear preferences and monopolistic competition. There are two sectors: one competitive sector with homogeneous goods and one monopolistically competitive sector with differentiated products. The homogeneous good is considered a necessity and the differentiated product a luxury good. Consumers

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[22]In this case, there exists the possibility that Home produces varieties although it does not consume them. There will be specialization in consumption.

derive income solely from labor, which skill level is distributed unevenly among the population.

We derived the following results. In autarky, he more unequal country produces a larger number of varieties due to a larger demand. From the same logic, there is the possibility that a less unequal country may have poor consumers while the more unequal country only rich consumers. The opening to trade equalizes the number of varieties produced by any country and increases the number of varieties consumed. Trade has "poor-inclusive" effects that arises from the quasi-linear assumption. In terms of welfare, both countries gain with trade liberalization but the more equal country benefits more due to a higher increase in the number of varieties consumed. A redistributive policy may harm some consumers not only by diminishing disposable income, but also by diminishing the number of varieties produced.

This paper provided through a very simple framework a systematic way to deal with inequality and trade. We introduced income inequality in a more explicit way and focused on labor heterogeneity that was not a major issue in the existing literature. Further research could include trade barriers or to generalize the model to feature many sectors, differences in technology among countries or more general distribution functions. Also, political issues could be included to endogenize income distribution.

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Appendix

A1

We show that the curve n tangents the descending line. Rearranging conditions (23) and (26) we have:

$$n_{P+R} = \frac{Ls_0^k(1-\theta)}{(k-1)\mu s_m^{k-1}} \tag{33}$$

$$n_R = \frac{ks_0}{k-1}\frac{(1-\theta)L}{\mu} - \frac{s_m(1-\theta)L}{\mu}, \tag{34}$$

with (38) as the poor-and-rich equilibrium condition and (39) as the rich-only equilibrium. Subtracting (39) from (38) we obtain:

$$n_{P+R} - n_R = \frac{L(1-\theta)}{\mu(k-1)s_m^{k-1}}[(s_0^k - s_m^k) - k(s_0 s_m^{k-1} - s_m^k)]. \tag{35}$$

The above expression will be equal to zero only when $s_0 = s_m$, for any value of k, and will be positive over all other values of s_m. Then we conclude that the curve n will tangent the descending line.

A2

We need to guarantee that the s_m curve intersects the descending line at least once. Rearranging condition (26) we obtain:

$$n^{\frac{\theta-1}{\theta}}\left(\frac{\beta}{\theta}\right) + \frac{n\mu}{(1-\theta)L} = \frac{ks_0}{k-1}. \tag{36}$$

Note that the left-hand side of equation (41) has a U-shaped form, thus it has a minimum value. We need this minimum value to be smaller than or equal the right-hand side of the equation. Derivating the LHS by n we obtain the number of firms $\bar{n}$ that gives the minimum value:

$$\bar{n} = \left[\frac{(1-\theta)^2 L\beta}{\theta^2\mu}\right]^\theta. \tag{37}$$

Substituting this result into equation (41) and rearranging, we obtain the condition:

$$s_0 \geq \frac{(k-1)\beta^\theta \mu^{1-\theta}}{k\theta^{2\theta}(1-\theta)^{2(1-\theta)}L^{1-\theta}}. \tag{38}$$

Thus if the following condition holds the s_m curve intersects the descending line at least once and an stable equilibrium exists.