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Abstract

The results in this paper are relevant for the application of valuation studies in cost-benefit analysis in the presence of the willingness to pay - willingness to accept gap. We consider a consumer who makes choices based on choice preferences exhibiting reference-dependence and loss aversion. Choice preferences are related to underlying hedonic preferences through the marginal rates of substitution (MRS) at the reference. Our issue is the identification of hedonic preferences relevant to welfare economic analysis. We show that the hedonic MRS is identified from reference-dependent choices if loss aversion exhibits a certain symmetry. Moreover, we show that this symmetry is rational in the sense that it leads to maximal expected hedonic utility when choices are made under reference-dependent choice preferences.

Keywords: Behavioral public economics; valuation of non-market goods; prospect theory; loss aversion

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1 Introduction

A large literature has developed on the valuation of non-market goods in a variety of sectors, including health, transportation, environmental amenities, marketing, etc. A remarkably consistent finding is the large gap between measures of willingness to pay (WTP) and willingness to accept (WTA) in such valuation studies. Horowitz and McConnell (2003) review more than 200 valuation studies and find the mean of the ratio (WTA/WTP) to exceed 7. The size of this difference is hard to rationalize within a standard Hicksian framework and economists and psychologists have looked for alternative explanations. The theory of reference-dependent preferences (Tversky and Kahneman, 1991; Kahneman and Tversky, 1979) has been shown to provide an appealing and economically convincing explanation of the WTP/WTA gap. Their initial specification of reference-dependent preferences, often used in later work (Stott, 2006), is based on reference-dependent utility functions that are additively separable in value functions; the latter incorporate loss aversion. Reference-dependence implies that the slope of an indifference curve depends on the reference from which it is evaluated, and that kinks occur at the reference point. Loss aversion further implies that losses cause a greater loss of utility than same size gains. Within this framework, Bateman et al. (1997) theoretically show that WTA must necessarily exceed WTP, and they provide experimental evidence for the magnitude of the effect. Based on a large survey studying people's valuation of time, De Borger and Fosgerau (2008) confirm the power of reference-dependence in explaining differences in valuation measures.

As the original theory implies a drastic deviation from conventional consumer theory, Munro and Sugden (2003) have recently reformulated reference-dependent preferences in a way that is more closely related to standard theories of consumer behavior. They abandon the additivity assumption and allow for endogenous reference points. They consider a process of maximizing reference-dependent preferences in a series of trades, each time updating the reference, and they show that, under some mild conditions, the process converges to the set of 'reflexive' optima. A reflexively optimal bundle is defined for a given budget and prices such that, were the person endowed with this bundle, he would not want to consider any further trade.

Even if the various theories of reference-dependence have resolved the issue of large estimated differences between different valuation measures, they have raised another important question, namely how to obtain valuation measures that can be used in applied cost-benefit analysis? How to use the behavioral model of reference-dependence in a normative cost-benefit evaluation? In a more general setting, this relation between behavioral economic models and normative welfare economic models is a main focus of the recent literature on behavioral welfare economics (for a recent survey, see Bernheim and Rangel, 2007). Different views have been defended. Some authors argue (e.g., Gul and Pesendorfer, 2001, 2004) that, in case certain "anomalies" are observed, the best answer is to expand the preference domain to explain the observed behavior, and use the adapted behavioral model as the basis for a normative policy evaluation. Another school of thought suggests that, if choices cannot be explained by a set of coherent preferences or if people are observed to make systematic mistakes, it may be necessary to abandon the close relation between behavioral and normative economic models.[1]

In this paper, we study the question how to use the behavioral model of reference-dependence as the basis for obtaining valuation measures that are relevant for normative welfare economic evaluation such as cost-benefit analysis. It has been argued (e.g., Munro and Sugden, 2003; Köszegi and Rabin, 2006) that the reference may adapt quickly to new circumstances. In that case, it clearly seems inappropriate to use welfare measures for evaluation that have been directly obtained from reference-dependent preferences. We therefore take a different route and focus on recovering underlying hedonic reference-free preferences from estimates based on reference-dependent choice preference models, taking valuations pertaining to hedonic preferences as being those

[1] See Bernheim and Rangel (2007) for this discussion.

relevant for welfare economic evaluation. We follow Munro and Sugden (2003), Bernheim and Rangel (2007) and Köszegi and Rabin (2006) in explicitly linking choice preferences and reference-free hedonic preferences.[2] Moreover, we allow for endogenous reference points and adaptation of the reference in a series of trades. We do so, however, within the framework of the typical 'kinked at the reference'-value functions suggested by Tversky and Kahneman (1991). If choice preferences are reference-dependent in the sense just described, we then ask whether estimates of WTP and WTA provide information on the underlying reference-free hedonic trade-offs between goods. If choice preferences are reference-dependent, are estimates of people's WTP or WTA directly useful at all in a normative welfare economic setting such as cost-benefit analysis? This paper makes some progress in answering this question.

For the particular class of reference-dependent preferences considered, we obtain two important results. First, we show that imposing symmetric loss aversion (i.e., gains are underweighted by consumers by as much as losses are overweighted relative to the marginal hedonic utilities) allows us to recover the hedonic reference-free trade-off between goods from estimates of WTP and WTA. More specifically, we find that neither WTA nor WTP is an acceptable measure of the hedonic trade-off, but they do contain all necessary information to calculate the underlying hedonic valuation. This finding has important implications for cost-benefit analysis with reference-dependent preferences. Second, motivated by the importance of symmetric loss aversion for identification of the underlying reference-free trade-offs, we show that there are good economic reasons to justify imposing symmetry. We find that, within the Munro-Sugden (and others) philosophy of linking choice and hedonic preferences, a sufficient condition for the hedonic optimum to be a reflexive optimum is that the value functions exhibit symmetric loss aversion. This offers support for imposing symmetric loss aversion on Tversky-Kahnemann-type of value functions: if a series of reference-dependent

[2] This line of attack is also advocated by Beshears et al. (2007). See also Kahneman and Sugden (2005) on the distinction between hedonic and choice preferences.

choices are to converge to a point near the reference-free hedonic optimum, then the reference-free optimum must be a reflexive optimum, and symmetry is a sufficient condition for this to be the case. This result is especially relevant in view of recent experimental findings. Indeed, it has been argued (see, e.g., List, 2004; Plott and Zeiler, 2005) that the gap between WTP and WTA strongly declines when people gain more experience with the type of choices to be made (due to training, increasing familiarity with the choice environment, etc). This experimental evidence is consistent with our model only under symmetric loss aversion.

Finally, we show that symmetric loss aversion can be considered rational in a broader sense. We show that, if choice preferences are not symmetric, then a process of sequential choice utility maximization will lead to lower expected hedonic utility than would be the case under symmetry. Using a rationality argument like this to close a model is, of course, a fundamental strategy in economics. The novelty here is the setup where closure is obtained in a model that links hedonic and choice preferences.

The structure of the paper is the following. In Section 2 we present the model of reference-dependence that will be used, and we define the different valuation measures considered. In Section 3 we show that symmetric loss aversion allows identification of underlying hedonic trade-offs between goods on the basis of reference-dependent choice preferences, and the implications for cost benefit analysis are briefly discussed. Section 4 studies the rationality of symmetric loss aversion. A final section 5 concludes.

2 Reference-dependence and four valuation measures

We make minimal assumptions with respect to an individual's preferences. Consider a reference-dependent choice utility function defined over two goods; one of these may be money and the other may be a non-market good. Denote the reference-dependent choice utility function by $u(x|r)$, where x is the bundle under evaluation and r is the reference. Choice utility may have a kink at the reference. We therefore allow the left and right derivatives of $u(x|r)$ to be different, using the notation $u_i^+(x|r)$ for a partial derivative from the right and $u_i^-(x|r)$ for a partial derivative from the left. Note that loss aversion is just equivalent to $u_i^+ = u_i^+(r|r) < u_i^-(r|r) = u_i^-$. We also use the notation that $x^+ = x$ if $x > 0$ and 0 otherwise, while $x^- = -x$ if $x < 0$ and 0 otherwise. Consider then small changes x from the reference and differentiate $u(x + r|r)$ to obtain a linear approximation to utility at the reference

$$u(x + r|r) \simeq u(r|r) + u_1^+(r|r)x_1^+ - u_1^-(r|r)x_1^- + u_2^+(r|r)x_2^+ - u_2^-(r|r)x_2^-. \quad (1)$$

Now consider the four standard trade-offs used in economic valuation studies, given reference-dependence (Bateman et al., 1997). Define the willingness-to-pay (WTP) as how much the individual is willing to pay in terms of x_1 for a marginal increase in x_2, relative to the reference. Similarly, the willingness-to-accept (WTA) measures how much extra of x_1 would compensate for a marginal reduction (relative to the reference) in x_2. The equivalent gain (EG) measures indifference at the reference between receiving an increase in x_1 or in x_2. The equivalent loss (EL) is the corresponding measure for losses. Using (1), it is easy to show that the four valuation measures are given by:

$$WTP = \frac{u_2^+}{u_1^-} \quad , \quad WTA = \frac{u_2^-}{u_1^+}$$
$$EG = \frac{u_2^+}{u_1^+} \quad , \quad EL = \frac{u_2^-}{u_1^-}$$

In general, these will all be different. Observe that

$$WTP \cdot WTA = EG \cdot EL. \quad (2)$$

This equality follows just because reference-dependent utility is linear for small changes. If measurements of WTP, WTA, EG and EL are made in some experiment and subjects' choices are generated from maximisation of possibly reference-dependent utility, then

(2) will hold. This is true regardless of the properties of the utility function, as long as it has derivatives from the left and the right in each variable. What matters is that there is a function that generates choices. Empirical evidence in De Borger and Fosgerau (2008) shows that (2) holds with great precision in a large survey dataset from a discrete choice experiment designed to measure these four valuations simultaneously. This evidence then indicates that people in fact do make choices in a way that is consistent with them maximizing a function.[3]

Since loss aversion in this setting is equivalent to $u_i^+ < u_i^-$, it can easily be shown that loss aversion implies the following relationship between the four valuation measures: $WTP < EG, EL < WTA$ (Bateman et al., 1997; De Borger and Fosgerau, 2008).

We now assume the existence of an underlying reference-free hedonic utility function $u(r)$. It is not required that the individual is aware of his hedonic utility at all possible consumption bundles, it suffices that 'he has access' to his marginal utilities (marginal rates of substitution) at the reference when forming his reference-dependent preferences. To link reference-dependent choice preferences to the underlying reference-free hedonic utility function we specify marginal reference-dependent utilities, evaluated at the reference, as follows:

$$u_i^+(r|r) = u_i(r)\mu_i^+, u_i^-(r|r) = u_i(r)\mu_i^-.$$

Note that this is just a small elaboration of the specification of the linear reference-dependent utility in Tversky and Kahneman (1991). There is no loss of generality from (1). The formulation may also be derived from the definition of reference-dependent

[3]The survey comprised 2131 car drivers who were asked to choose between two alternative trips defined in terms of travel cost and time. Choice situations were designed relative to a recent trip subjects had made. Each subject made choices in 8 such situations: two WTP type choices, two WTA, two EG and two EL. The econometric model allowed for observed and unobserved heterogeneity and errors, and estimated first a parameter for the median in the sample of each of the four valuation measures. They were all very significant and very different. Imposing the restriction given by (2) cost about half a likelihood unit. Fosgerau et al. (2006) present similar evidence on datasets where subjects were recruited from other modes of transport.

utility in Köszegi and Rabin (2006), who are also concerned with the relation between choice and hedonic utility.

3 Symmetric loss aversion and the identification of reference-free trade-offs

To introduce the concept of symmetric loss aversion it will be instructive to consider a particular representation of choice preferences that makes asymmetry highly visible. Using the notation $S(x) = 1$ if $x > 0$ and $S(x) = -1$ otherwise, it is convenient to parameterize choice utility as

$$u(x + r|r) = u(r) + u_1(r)e^{-\eta_1 S(x_1)}x_1 + u_2(r)e^{\gamma-\eta_2 S(x_2)}x_2.$$

This formulation is equivalent to (1) since we are free to multiply utility by any positive number. It is easy to see that loss aversion is now equivalent to $\eta_i > 0$. The parameter γ generates asymmetry in reference-dependent utility; in the absence of loss aversion, γ measures the difference between the marginal rates of substitution of hedonic and choice preferences. Working out the four valuation measures for this parameterization, we find:

$$\begin{aligned} WTP &= \frac{u_2(r)}{u_1(r)}e^{\gamma-\eta_1-\eta_2} \quad , \quad WTA = \frac{u_2(r)}{u_1(r)}e^{\gamma+\eta_1+\eta_2} \\ EG &= \frac{u_2(r)}{u_1(r)}e^{\gamma+\eta_1-\eta_2} \quad , \quad EL = \frac{u_2(r)}{u_1(r)}e^{\gamma-\eta_1+\eta_2} \end{aligned}$$

Suppose we have estimates of the four valuation measures. Under what conditions do these estimates allow the identification of the trade-off implied by the underlying reference-free utility function? The answer immediately follows from the above expressions. Noting that $WTP \cdot WTA = EG \cdot EL = \left(\frac{u_2(r)}{u_1(r)}\right)^2 e^{2\gamma}$, we need to know γ in order to identify $\frac{u_2(r)}{u_1(r)}$ from observation of WTP and WTA (or equivalently from

EG and EL). Loss aversion is symmetric when $\gamma = 0$. Hence symmetry allows us to identify the marginal rate of substitution of the hedonic utility from estimates of the valuation measures.

What do we learn from this for applied cost-benefit analysis? We know that, if choice preferences are reference-dependent, a large difference may result between willingness to pay and willingness to accept (and a smaller difference between equivalent gain and loss). This in itself, however, does not provide any information on which value is most appropriate in applied cost-benefit analysis. But in this section, we showed that, if loss aversion is symmetric, the marginal rate of substitution implied by the underlying hedonic preferences can be recovered from the geometric mean of the estimates of WTA and WTP (or, alternatively, EL and EG). Indeed, the expressions just derived imply that, if $\gamma = 0$,

$$\frac{u_1(r)}{u_2(r)} = \sqrt{WTP \cdot WTA} = \sqrt{EL \cdot EG}.$$

4 Rationality arguments for symmetric loss aversion

In this section, we argue that there are good economic reasons for imposing symmetric loss aversion on Tversky-Kahneman type preferences. The first argument (section 4.1) builds upon Munro-Sugden and shows that, for arbitrary loss aversion, symmetry guarantees that the hedonic optimum is a reflexive optimum. The second argument (section 4.2) is a rationality argument under uncertainty. Specifically, we show that if loss aversion is asymmetric, then a sequence of reference-dependent choice utility maximizing choices will on average lead to lower hedonic utility than if loss aversion is symmetric.

4.1 Variable loss aversion, constant asymmetry

In this subsection, we build upon the interpretation of the relation between reference-dependent and reference-free utility suggested by Munro and Sugden (2003). They describe a trading process whereby individuals maximize reference-dependent preferences in a series of trades, each time updating the reference, until no further improvement can be found. They then define the set of endpoints of such a process as the set of reflexive optima.

Let us develop the implications of this idea for the interpretation of potentially asymmetric loss aversion. Consider reference-dependent choice utility maximization under a budget restriction reflecting given prices p and endowment Y. The set of reflexive optima then consists of all bundles r such that no small deviation x exists that maintains the budget ($px = 0$) and increases choice utility. The set of reflexive optima is defined by

$$X^* = \{r : u(x + r|r) \leq u(r|r) \forall x : px = 0\}.$$

Now reconsider the parametrization of reference-dependent preferences given above. Take an arbitrary reference point in the set of reflexive optima $r \in X^*$ and let x be a small budgetary neutral ($px = 0$) change from r to $r + x$. First looking at changes that marginally raise dimension 1 while reducing dimension 2 and then vice versa, it is easily shown, since $r \in X^*$ and therefore $u(x + r|r) \leq u(r|r)$, that

$$\ln \frac{p_1}{p_2} + \gamma - \eta_1 - \eta_2 \leq \ln \frac{u_1(r)}{u_2(r)} \leq \ln \frac{p_1}{p_2} + \gamma + \eta_1 + \eta_2. \tag{3}$$

It follows that the reference-free hedonic optimum belongs to the set of reflexive optima exactly if

$$-\eta_1 - \eta_2 \leq \gamma \leq \eta_1 + \eta_2, \tag{4}$$

since the reference-free optimum is characterized by $\frac{p_1}{p_2} = \frac{u_1}{u_2}$.

This has important implications. If the loss aversion parameters are small and asymmetry is substantial, then the set of reflexive optima excludes the hedonic optimum, and so the reference-dependent trading process will be sure *not* to converge to the hedonic optimum. Conversely, if there is a unique reflexive optimum (zero loss aversion), then symmetry implies that the trading process of sequentially updating the reference necessarily converges to the hedonic optimum. This finding has economic relevance, since it has been argued (List, 2004; Plott and Zeiler, 2005) that loss aversion may largely disappear as a result of training and experience. If the η_i are reduced by training and experience while γ is constant, we must have $\gamma = 0$ to guarantee the hedonic optimum to be a reflexive optimum. Starting from Tversky-Kahneman type reference-dependent preferences, we may therefore argue that rationality requires that loss aversion is symmetric.

4.2 Constant loss aversion, constant asymmetry

Despite its appeal, in one respect the above argument is not entirely convincing: there is no economic reason why asymmetry γ should be fixed as $\eta' s$ decline, e.g., due to experience. We therefore consider a broader and more general argument supporting symmetric loss aversion. We shall show that without symmetry, for fixed degrees of loss aversion, a process of reference-dependent utility maximization will on average lead to lower reference-free utility than if loss aversion is symmetric. In this sense, symmetric loss aversion is rational.

We take a perspective from behind a veil of ignorance: the optimal degree of asymmetry must be chosen not knowing the situations in which it is to be applied. We consider the expected hedonic utility given a budget y, random prices p, and a random hedonic utility function u. The degrees of loss aversion, η_1 and η_2, are constant. We consider then what choice of γ maximizes expected utility.

We assume (4) such that the hedonic optimum is always a reflexive optimum. The task is then to select γ to maximize

$$E(u(x)|px = y, x \in RO).$$

In other words, we select γ to maximize expected hedonic utility conditional on being in a reflexive optimum. We require no knowledge about the hedonic utility function, nor of income or of prices, except for some regularity conditions. We do however require that the distribution of preferences and prices, from our point of view behind the veil, is symmetric in dimensions. We will make this assumption more specific below. We may say that the assumption embodies a priori ignorance of the circumstances under which the individual is going to maximize his reference-dependent preferences. Since it is not known whether the γ is going to be applied on dimension 1 or dimension 2, we may assume that $\gamma = \gamma^+ \geq 0$ and $\gamma = -\gamma^+$ are equally likely.

Recall (3) which may be written as

$$\gamma - \eta_1 - \eta_2 \leq \ln \frac{u_1 p_2}{u_2 p_1} \leq \gamma + \eta_1 + \eta_2 \tag{5}$$

for any point in the set of reflexive optima. Let $\phi(t|px = y) = P(\ln \frac{u_1 p_2}{u_2 p_1} = t|px = y)$ be the density of $\ln \frac{u_1 p_2}{u_2 p_1}$ conditional on $px = y$. Then t is in the interval given by (5). Make the symmetry assumption that

$$\phi(t|px = y) = \phi(-t|px = y).$$

This assumption embodies ignorance by merely saying that reflexive optima on either side of the hedonic optimum are equally likely. We further assume that ϕ is unimodal, such that $t\phi'(t|px = y) < 0$.[4] Define for convenience

$$g(t) = E\left(u(x)|px = y, \ln \frac{u_1 p_2}{u_2 p_1} = t\right)$$

[4] Becker (1962) assumes just a uniform distribution in a similar situation.

and assume that $g(t)$ is symmetric. Again, this assumption merely states ignorance: the sign of t does not tell us anything about $u(x)$. Without loss of generality we also assume that $g(t) > 0$. Then, referring to (5) we may elaborate the expected utility as

$$\begin{aligned} E(u(x)|px = y, x \in RO) &= \frac{1}{2}\int_{\gamma^{+}-\eta_1-\eta_2}^{\gamma^{+}+\eta_1+\eta_2} g(t)\phi(t|px = y)dt \\ &+ \frac{1}{2}\int_{-\gamma^{+}-\eta_1-\eta_2}^{-\gamma^{+}+\eta_1+\eta_2} g(t)\phi(t|px = y)dt \\ &= \int_{\gamma^{+}-\eta_1-\eta_2}^{\gamma^{+}+\eta_1+\eta_2} g(t)\phi(t|px = y)dt. \end{aligned} \tag{6}$$

Assume finally that both goods are normal goods at all prices. This condition is equivalent to requiring that $\frac{u_j}{u_i}u_{ii} < u_{ij}$ for $i \neq j$. The condition is slightly stronger than strict convexity of the indifference curves, which (in two dimensions) is equivalent to assuming $\frac{u_2}{u_1}u_{11} - 2u_{12} + u_{22}\frac{u_1}{u_2} < 0$. From the assumptions made here, Appendix A then proves the following proposition.

Proposition 1 *With the assumptions above,* $0 = \operatorname{argmax}_{\gamma^{+}} E\left(u\left(x\right)|px = y, x \in RO\right)$.

This means that the expected hedonic utility from maximizing reference-dependent choice utility in a sequence of trades is maximized only if loss aversion is symmetric.

5 Conclusions

Starting from the observed large differences between different valuation measures (willingness to pay, willingness to accept, equivalent gain, and equivalent loss), we argue that it is not appropriate to use either of these measures in cost benefit analysis. This paper then makes two contributions. First, we show that if reference-dependent choice preferences stand in a particular relation to underlying reference-free hedonic preferences, then the assumption of symmetric loss aversion allows recovery of the hedonic marginal rate of substitutions between goods as the geometric average of WTA and

WTP (or equivalently of EL and EG). We suggest using this value in applied cost-benefit analysis. Second, as the previous argument requires symmetric loss aversion, we show that optimizing behavior under reference-dependent choice preferences will lead to maximal expected hedonic utility under symmetry. In this sense, symmetric loss aversion is rational.

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References

Bateman, I., Munro, A., Rhodes, B., Starmer, C. and Sugden, R. (1997) A Test of the Theory of Reference-Dependent Preferences *The Quarterly Journal of Economics* **112**(2), 479–505.

Becker, G. S. (1962) Irrational Behavior and Economic Theory *Journal of Political Economy* **70**(1), 1–13.

Bernheim, B. D. and Rangel, A. (2007) Behavioral Public Economics: Welfare and Policy Analysis with Nonstandard Decision-Makers *in* P. Diamond and H. Vartiainen (eds), *Behavioral Economics and Its Applications* Princeton University Press chapter 2, pp. 7–84.

Beshears, J., J.Choi, J., Laibson, D. and C.Madrian, B. (2007) How Are Preferences Revealed? *Working Paper* (http://www.economics.harvard.edu/faculty/laibson/files/How%20are%20preferences%20revealed%202007%2002%2019.doc).

De Borger, B. and Fosgerau, M. (2008) The trade-off between money and travel time: a test of the theory of reference-dependent preferences *Journal of Urban Economics* **Forthcoming**.

Fosgerau, M., Hjort, K. and Lyk-Jensen, S. V. (2006) *The Danish Value of Time Study - Results for Experiment 1* www.dtf.dk.

Gul, F. and Pesendorfer, W. (2001) Temptation and Self-Control *Econometrica* **69**(6), 1403–1435.

Gul, F. and Pesendorfer, W. (2004) Self-Control and the Theory of Consumption *Econometrica* **72**(1), 119–158.

Horowitz, J. K. and McConnell, K. E. (2003) Willingness to accept, willingness to pay and the income effect *Journal of Economic Behavior & Organization* **51**(4), 537–545.

Kahneman, D. and Sugden, R. (2005) Experienced Utility as a Standard of Policy Evaluation *Environmental and Resource Economics* **32**(1), 161–181.

Kahneman, D. and Tversky, A. (1979) Prospect Theory: An Analysis of Decision under Risk *Econometrica* **47**(2), 263–292.

Köszegi, B. and Rabin, M. (2006) A model of reference-dependent preferences *The Quarterly Journal of Economics* **121**(4), 1133–1166.

List, J. A. (2004) Neoclassical Theory Versus Prospect Theory: Evidence from the Marketplace *Econometrica* **72**(2), 615–625.

Munro, A. and Sugden, R. (2003) On the theory of reference-dependent preferences *Journal of Economic Behavior & Organization* **50**, 407–428.

Plott, C. R. and Zeiler, K. (2005) The Willingness to PayWillingness to Accept Gap, the "Endowment Effect," Subject Misconceptions, and Experimental Procedures for Eliciting Valuations *The American Economic Review* **95**(3), 530–545.

Stott, H. P. (2006) Cumulative prospect theory's functional menagerie *Journal of Risk and Uncertainty* **32**(2), 101–130.

Tversky, A. and Kahneman, D. (1991) Loss Aversion in Riskless Choice: A Reference-Dependent Model *The Quarterly Journal of Economics* **106**(4), 1039–1061.

A Proof of Proposition 1

This section is devoted to the proof of proposition 1. Differentiate (6) with respect to γ^+ to find the first order condition for a maximum.

$$\begin{aligned}\frac{dg(t)}{d\gamma^+} &= g(\gamma^+ + \eta_1 + \eta_2)\phi(\gamma^+ + \eta_1 + \eta_2|px = y) - g(\gamma^+ - \eta_1 - \eta_2)\phi(\gamma^+ - \eta_1 - \eta_2|px = y) \\ &= 0\end{aligned}$$

The first order condition is satisfied for $\gamma^+ = 0$. We note that $\gamma^+ - \eta_1 - \eta_2 < 0 < \gamma^+ + \eta_1 + \eta_2$ by assumption. Differentiation of the first order condition shows that second derivative of the expected utility $E(u(x)|px = y, x \in RO)$ is always negative, provided that $g'(t) < 0$ for $t > 0$. In this case, the expected utility has a unique maximum at $\gamma^+ = 0$. Our task is then reduced to showing that indeed $g'(t) < 0$ for $t > 0$.

The budget constraint, $px = y$, and the definition of $t = \ln \frac{u_1 p_2}{u_2 p_1}$ together identify a unique x as a function of y, p, t. Write just $x(t) = (x_1(t), x_2(t))$ since y, p are given under the expectation operator. We then consider $u(t) = u(x(t))$. Differentiating, we find

$$\begin{aligned}
u'(t) &= u_1 x_1' + u_2 x_2' \\
&= u_1 x_1' - \frac{p_1}{p_2} u_2 x_1' \\
&= \sqrt{u_1 u_2}\sqrt{\frac{p_1}{p_2}} e^{t/2} x_1' - \sqrt{u_1 u_2}\sqrt{\frac{p_1}{p_2}} e^{-t/2} x_1' \\
&= \left(e^{t/2} - e^{-t/2}\right)\sqrt{u_1 u_2}\sqrt{\frac{p_1}{p_2}} x_1',
\end{aligned}$$

where the second equality follows from the budget constraint and the third equality follows from the definition of t.

The problem is now to find x_1'. First differentiate the definition of t to find

$$u_{11} x_1' + u_{12} x_2' = \frac{p_1}{p_2}\left(u_{12} x_1' + u_{22} x_2'\right) e^t + \frac{p_1}{p_2} u_2 e^t.$$

We can then solve this equation for x_1'. Use first the budget constraint to find $x_2' = -\frac{p_1}{p_2} x_1'$, such that

$$\begin{aligned}
u_{11} x_1' - u_{12}\frac{p_1}{p_2} x_1' &= \frac{p_1}{p_2}\left(u_{12} x_1' - u_{22}\frac{p_1}{p_2} x_1'\right) e^t + \frac{p_1}{p_2} u_2 e^t \\
\frac{p_1}{p_2} u_2 e^t &= x_1'\left(u_{11} - u_{12}\frac{p_1}{p_2} - u_{12}\frac{p_1}{p_2} e^t + u_{22}\frac{p_1}{p_2}\frac{p_1}{p_2} e^t\right) \\
x_1' &= \frac{\frac{p_1}{p_2} u_2 e^t}{u_{11} - u_{12}\frac{p_1}{p_2} - u_{12}\frac{p_1}{p_2} e^t + u_{22}\frac{p_1}{p_2}\frac{p_1}{p_2} e^t} \\
x_1' &= \frac{u_2 e^t}{\frac{p_2}{p_1} u_{11} - u_{12} - u_{12} e^t + u_{22}\frac{p_1}{p_2} e^t}.
\end{aligned}$$

Inserting this into the expression for $u'(t)$ yields

$$\begin{aligned}
u'(t) &= \left(e^{t/2} - e^{-t/2}\right)\frac{u_1 u_2}{\frac{p_2}{p_1} u_{11} e^{-t/2} - u_{12} e^{-t/2} - u_{21} e^{t/2} + u_{22}\frac{p_1}{p_2} e^{t/2}} \\
&= \left(e^{t/2} - e^{-t/2}\right)\frac{u_1 u_2}{\frac{u_2}{u_1} u_{11} e^{t/2} - u_{12}\left(e^{-t/2} + e^{t/2}\right) + u_{22}\frac{u_1}{u_2} e^{-t/2}}
\end{aligned}$$

This expression is always negative by the assumption on the possible hedonic utility functions. Hence $g'(t) = Eu'(t) < 0$ as required. This concludes the proof of proposition 1.