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Bayesian Model Averaging and Identification of Structural Breaks in Time Series.

Abstract.

Bayesian model averaging is used for testing for multiple break points in univariate series using conjugate normal-gamma priors. This approach can test for the number of structural breaks and produce posterior probabilities for a break at each point in time. Results are averaged over specifications including: stationary; stationary around trend; and, unit root models, each containing different types and numbers of breaks and different lag lengths. The procedures are used to test for structural breaks on 14 annual macroeconomic series and 11 natural resource price series. The results indicate that there are structural breaks in all of the natural resource series and most of the macroeconomic series. Many of the series had multiple breaks. Our findings regarding the existence of unit roots, having allowed for structural breaks in the data, are largely consistent with previous work.

Keywords: Bayesian Model Averaging, Structural Breaks, Unit Root, Macroeconomic Data, Natural Resource data.

1. INTRODUCTION

Marriott and Newbold (2000) was one of the first papers in the econometrics literature to apply the concept of Bayesian model averaging (BMA) to the problem of detecting structural breaks. Key to this approach is the idea that results from a number of models can be aggregated provided the marginal likelihood of each model can be calculated. In effect, Marriott and Newbold (2000) estimate a large number of models treating the break points as discrete parameters to be estimated. Models with and without breaks are compared on the basis of posterior odds, and if models containing breaks have higher odds, then it is concluded that the series contain breaks. The BMA approach avoids the need to pretest for unit roots or number of lags since the results of models with and without unit roots, or different numbers of lags, can be aggregated.

This paper builds on the Bayesian approach of Marriott and Newbold (2000) but constructs the marginal likelihoods using normal-gamma priors. The numerical simplifications that this approach affords allows us to deal easily with 3 breaks which would be impractical using the Marriott and Newbold (2000) approach since they require the numerical calculation of integrals. Furthermore, in addition to testing for structural breaks we also construct break probabilities for every point in the sample.

Although the central aim of this paper is to examine structural breaks, as with other papers in this area, we also address the issue of whether the data contain unit roots. This is imperative since the behaviour of stationary series with breaks can sometimes mimic the behaviour of unit root series and vice versa. Even when using BMA, priors may give different weights to models with

and without unit roots, and may play a significant role in testing for structural breaks.

The existing Bayesian literature on unit roots, and/or structural break testing has not adopted a conjugate normal gamma prior approach. The debate has mainly pivoted around the issue of what form a 'non-informative' or 'objective' prior should take in the time series setting. Applying the linear normal-gamma conjugate approach in a time series setting is recognised to have drawbacks. If the central aim is to discriminate between unit root processes and non-unit root processes, adopting normal-gamma priors will not, in some senses, be optimal since normal-gamma priors will not always lead to a good discrimination between unit root processes and highly autocorrelated, stationary (or trend stationary) processes relative to an analysis using certain other priors. Moreover, Bayes Ratios (BRs) and posterior odds are known to be highly sensitive to prior choice. In this paper we do not attempt to be 'objective' in the sense of adopting priors that have no effect on subsequent findings. Our main aim is to elucidate and use a Bayesian approach to structural break identification using BMA. However, in doing so we also aim to demonstrate that, providing the models and the priors are constructed carefully, the normal-gamma conjugate approach has the ability to discriminate between alternative data generating processes, including those with and without unit roots.

The strength of the approach developed and adopted here is that it enables a larger number of different specifications to be estimated (compared to Classical specifications), all of which will play some role in identifying structural breaks. Therefore, we are not forced to rely on a fragile pretesting procedure. Our point here is not to dismiss 'objective' or 'robust' Bayesian analysis. Instead it is to recognise that these approaches come with greater computational cost, thus limiting the number of models and breaks that can averaged.

The paper will proceeds as follows. The main theoretical and philosophical strands in the unit root and structural break literature will be reviewed in Section 2. Section 3 will discuss model construction and estimation along with the presentation of Monte Carlo evidence to illustrate the performance of our approach. Next, in Section 4 we will analyse macroeconomic and natural resource data using the theory developed in Section 3. We apply our procedures to two well known data sets. The first is the extended Nelson and Plosser (1982) data set (extended by Schotman and van Dijk, 1991). The second data set is the natural resource prices in Ahrens and Sharma (1997) and Lee et al. (2006). In Section 5 will conclude.

2. BACKGROUND

Since the work of Chow (1960), economists have frequently tested for structural change in regressions, requiring the separation of the sample into two or more parts with equality tests of the coefficients across equations, or the removal of dummy variables in models. In the case of time series regressions this type of test requires that the point where a structural break may have occurred is pre-specified. Unfortunately, the location of the structural break can be pivotal in determining the outcome of tests, and often the potential break point is

unknown.

The literature on 'structural breaks' has been intrinsically entwined with the literature on whether unit roots exist in the lag polynomials that characterise the autoregressive behaviour of series. Nelson and Plosser (1982) highlighted the fact that many macroeconomic series may have unit roots. However, a popular alternative to the unit root hypothesis is that the variables are stationary around a broken trend (or broken trends). This implies that there may be structural changes in the equations that are used to characterise the univariate behaviour of the data, and these structural changes may bias tests for unit roots, if they are not accounted for. Perron (1989) and Rapporport and Reichlan (1989) presented evidence that once structural breaks were dealt with, the majority of the Nelson and Plosser series set rejected unit roots.

The consequences of structural shifts on unit root tests have been extensively studied by, among others, Perron (1989, 1994), Perron and Vogelsang (1992), Banerjee *et al.* (1992), Lee (1996), and Zivot and Andrews (1992). Approaches that assume that the data are stationary with potential breaks are pursued in Bai and Perron (1998) and Bai (1999). A recent review of Classical approaches is given in Perron (2005).

In spite of the impressive array of techniques, problems with Classical approaches remain. Many Classical practitioners remain content to adopt model selection procedures that allow their prior beliefs to influence their results, seeing no contradiction in maintaining that informative priors should play no role in econometric analysis. However, most applied investigators also know that they are often faced with a choice between competing hypotheses that cannot be rejected with confidence, each potentially leading to different subsequent conclusions. BMA frees the investigator from some of the dilemmas of model selection. Using BMA the investigator does not need to make an explicit decision about unit roots or the type of break or the number of lags in a model. Moreover, a Bayesian approach can deliver a *probability* that a break has occurred in a given period. A Classical approach cannot construct such a measure.

Bayesian approaches to unit root testing have been investigated since the early 1990s and two comprehensive reviews are given in Chapter 8 of Maddala and Kim (1998) and Chapter 6 of Bauwens et al. (1999). However, relative to the large Classical literature on unit root vs structural break determination, the Bayesian econometrics literature is relatively small. Chib (1998) develops a general approach to the estimation of multiple change points that requires a Monte Carlo Markov Chain (MCMC) algorithms. Marriott and Newbold (2000) and Kim et al. (2003) use approaches that require either numerical integration or MCMC. Koop and Potter (2004) explore model selection and BMA in dynamic models using normal-gamma priors (with G-Priors), but do not explicitly address structural breaks or unit roots.

As with Koop and Potter (2004) and Fernandez et al. (2001) (who do not deal with structural breaks) our approach relies on the use of normal gamma conjugate priors. It is the use of these priors that allows the closed form expression for the marginal likelihood to be derived and avoids the need for Gibbs sampling or numerical integration since the posterior is also of a known form.

This enables us to allow for up to 3 breaks for series of around 100 observations, or 2 breaks if the series are much longer (e.g. 1000), without the need to resort to MCMC. This is as many breaks as current Classical approaches practically permit and more than those in current Bayesian approaches[1]. The number of breaks must be limited if a brute force approach is adopted because a large number of models require estimation if one allows breaks to be at any point in the series. The number of potential models increases rapidly with the number of breaks and approaches that require numerical integration (e.g. Marriott and Newbold, 2000) cannot practically deal with BMA using 3 breaks, so our approach is advantageous in this respect. Similar to Chib (1998) the marginal likelihoods are used here to construct the posterior probabilities of a break at each point. This approach is able to identify breaks even when the number of breaks is larger than 3 or 4 because many points in the series may be identified as having high break probabilities, even if the models explicitly allow for fewer breaks.

Readers familiar with the area unit root testing literature will know that the paper by Sims and Uhlig (1991) started a sometimes heated debate around the issue, by claiming that 'unit root discontinuity' present in classical theory was not present in Bayesian theory. Moreover, they showed that the adoption of a uniform prior delivered results that were unfavorable to the existence of unit roots in macroeconomic series. Responses by Phillips (1991), Schotman and van Dijk (1991), Berger and Yang (1994) and Lubrano (1995), and more recently Marriott and Newbold (2000) and Marriott et al. (2003) recognised that a uniform prior was informative in this context, and could be biased towards the rejection of unit roots[2]. Phillips (1995) also offers criteria that are essentially based on limiting Bayes Factors, that also offers insights into the impact of non-stationarity on selection procedures. Through this debate it was recognised that the Bayesian approach did not perhaps offer the universal approach that was seen as a major advantage by Sims and Uhlig (1991). The importance of this issue cannot be ignored when testing for structural breaks, because the decision about whether the data contains unit roots has implications for break testing. Nevertheless, as in Marriott and Newbold (2003) and Marriott et al. (2003) we argue that employing informative priors is justifiable. Conjugate priors, in particular, open up avenues that could not be explored without having closed form solutions for the marginal likelihood.

In view of the fact that prior specification can influence the results we experiment with a number of priors in order to assess their impact. We also examine the support for the unit root hypothesis using these priors. Evidence is presented that a normal-gamma approach does not lead to drastic reductions in

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[1] Chib's (1998) method allows for many breaks but he illustrates this method for Binary and Poisson data only.

[2] Readers should not jump to the conclusion that relatively flat priors such as a normal with a high variance always favour a stationary model. This depends on how a test is constructed. Using the marginal likelihood, a diffuse priors will tend to favour the model with the least parameters. Therefore, if the unit root model has the fewest parameters a diffuse prior will favour rather than discriminate against a unit root model.

the ability of the procedures to discriminate between unit root and stationary processes relative to priors that are 'less informative'. Our Monte Carlo evidence also shows that, given our priors, break models are seldom selected if the underlying data generating process has no breaks. On the other hand, when we apply our methods to real data, strong support for structural breaks are commonly found.

3. MODEL CONSTRUCTION AND ESTIMATION
3.1. Model Specification

For a variable of interest (y_t) the autoregressive model used herein is of the form:

$$\theta\left(L\right)y_t = \mu + \alpha t + B_t + e_t \tag{1}$$

for $t = 1, ...T$, where $\theta\left(L\right)$ is a polynomial lag, e_t is assumed to be normally distributed iid random variable. B_t is a 'break function', the construction of which is discussed below. [3]

3.2. Break functions

Define the following functions:

$$\text{Spike:} \qquad \delta_{j,t} = 1 \text{ if } t = j \text{ and 0 otherwise} \tag{2}$$
$$\text{Intercept shift:} \qquad \varepsilon_{j,t} = 1 \text{ if } t \geq j \text{ and 0 otherwise}$$
$$\text{Trend Shift} \quad : \quad \lambda_{j,t} = \frac{t}{T} \text{ if } t \geq j \text{ and 0 otherwise}$$

The general forms of 'breaks' considered in this paper are linear combinations of the components in [2]. Allowing for several types of break at a particular point, define:

$$b_{j,t} = \varphi_{1,j}\delta_{j,t} + \varphi_{2,j}\varepsilon_{j,t} + \varphi_{3,j}\lambda_{j,t} \tag{3}$$

A break function need not contain all three components, since $\varphi_{p,j}$ may be zero for $p = 1, 2, or\ 3$. The break function B_t in [1] is then defined as

$$B_t = \sum_{j \in \tau} b_{j,t} \tag{4}$$

where τ is a vector of integers (strictly increasing from 1 to T) identifying the break points. For example, if $\tau = \{3, 8\}$ the model would have break points at periods $t = 3$ and $t = 8$.

3.3. Model Features

Models can usefully be differentiated according to:

- The type of breaks included in the model (choice of $\varphi_{p,j}$);

- The number of breaks (choice of n);

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[3] Other exogenous variables could be included in the model, but we ignore them for simplicity of exposition.

- The timing of the breaks (choice of τ given n); and,

- Other attributes of the model (By 'other attributes', we mean restrictions on the coefficients within the polynomial lag and restrictions on the values of μ and/or α.)

3.4. Narrowing the Candidate Models

The model space will be reduced by considering subclasses that are popular in the literature. With regard to the 'other attributes' we will not consider restrictions on μ. That is, all models will contain an intercept. The number of lags in the autoregressive equation will be 3, 2, or 1. In common with much of the literature, model [1] is reparameterised as:

$$\Delta y_t = \mu + \rho y_{t-1} + \alpha \frac{t}{T} + B_t + \sum_{i=1}^{l} \theta_i^* \Delta y_{t-i} + e_t. \tag{5}$$

Readers should note that the restriction $\rho = 0$ has important implications for the long run impact of B_t (along with the other deterministic components) on y_t. For example, a 'spike' $\delta_{j,t}$ will have a transitory impact if $-2 < \rho < 0$, but will result in a permanent shift in y_t should the series contain a unit root $\rho = 0$. Likewise an intercept shift $\varepsilon_{j,t}$ will result in a mean shift in the former case, but a change in the trend of the series in the later.

Three basic submodels are considered: the Unit Root (UR); Stationary (ST); and, Stationary Around Trend (STAT), as follows:

$$\text{1) UR:} \qquad \Delta y_t = \mu + B_t + \sum_{i=1}^{l} \theta_i^* \Delta y_{t-i} + e_t \tag{6}$$

$$\text{2) ST:} \qquad \Delta y_t = \mu + \rho y_{t-1} + B_t + \sum_{i=1}^{l} \theta_i^* \Delta y_{t-i} + e_t \tag{7}$$

$$\text{3) STAT:} \qquad \Delta y_t = \mu + \rho y_{t-1} + \alpha \frac{t}{T} + B_t + \sum_{i=1}^{l} \theta_i^* \Delta y_{t-i} + e_t \tag{8}$$

Strictly speaking, for models 2) and 3) to be stationary and stationary around trend respectively, ρ is required to obey the condition that $-1 < \rho + 1 < 1$. Otherwise, the series may contain unit roots or be "explosive". We do not employ inequality restrictions that exclude the possibility of unitary or explosive roots in models 2) and 3). Therefore, the labels ST and STAT for models 2) and 3) are not strictly correct. However, we continue to refer to these models as stationary and trend stationary because we believe that there is little probability that the series display explosive behaviour. Accordingly, any empirical support for models 2) and 3) over 1) is indicative of stationary or trend stationary behaviour. Also, when adopting priors for ρ in models 2) and 3) the competing ST and STAT models will be estimated informatively with little prior mass outside the stationary region. Nevertheless, formally models 2) and 3) permit the

possibility of explosive behaviour and unit roots. The UR and ST models are nested within the STAT model, so these models are distinguished according to their restrictions. Within each possible model we allow for differences in the number of lags, and the number of breaks, but restrict the break types. This leads a further differentiation of the models. In order to be more explicit, define a vector

$$(u, k, v_1, v_2, v_3, n, l) \tag{9}$$

where: $u = 1$ if there is a unit root restriction and 0 otherwise; $k = 1$ if there is a trend in the model and zero otherwise; $v_p = 1$ if $\varphi_{p,j}$ is **not** restricted to zero and 0 otherwise (for all j); and, n is the number of breaks. Each model is defined by this vector and denoted $M(u, k, v_1, v_2, v_3, n, l)$. If $n = 0$ then the values of v_i become redundant (since they define non-existent breaks). Our candidate models are:

$$UR : M(1, 0, 1, v_2, 0, n, l) \text{ where } v_2 = 0 \text{ or } 1 \tag{10}$$

$$ST : M(0, 0, 0, 1, 0, n, l) \tag{11}$$

$$STAT : M(0, 1, 0, 1, v_3, n, l) \text{ where } v_3 = 0 \text{ or } 1 \tag{12}$$

Trend shifts in the UR models are not considered because we discount the possibility that the series in question have trends in their first differences. The reason why there are no spike shifts in the ST or STAT models is that spikes in these models could not characterised as breaks, because they would only have a transitory impact on y_t. For the breaks and the number of lags in each of the models we consider, n is restricted to the values 0,1,2 or 3 and l is restricted to the values 0,1,2 (1,2,3 lags in the autoregressive form [1]) For T=100 there are around 2.5 million sub models, all of which require estimation. Evidently, it becomes problematic to extend this to 4 breaks since the number of models increases to around 50 million, hence, the fact that we limit the number of breaks to 3.

3.5. The Marginal Likelihood
The model [5] can be more succinctly expressed as:

$$\Delta y_t = x'_{t,m} \beta_m + z'_t \gamma + e_t \tag{13}$$

with the elements of z'_t being included in all models. The elements of $x'_{t,m}$ vary over models. Herein, only the intercept is treated as common to all models (*i.e.* $z'_t = 1$). Therefore:

$$x'_{t,m} = \left(y_{t-1}, t/T, \{\delta_{j,t}\}_j, \{\varepsilon_{j,t}\}_j, \{\lambda_{j,t}\}_j, \Delta y_{t-1}, \ldots, \Delta y_{t-k} \right) \tag{14}$$

The m denotes a particular model associated with a choice of regressors. The model [13] can be expressed in linear regression form:

$$\begin{aligned} \Delta y_t &= w'_{t,m} \psi_m + e_t \\ w'_{t,m} &= \left(x'_{t,m}, z'_t \right) \end{aligned}$$

and in matrix form as:

$$\Delta Y = W_m \psi_m + e \tag{15}$$
$$W_m = (X_m : Z)$$
$$\psi'_m = (\beta'_m, \gamma')$$

The log likelihood function (treating the first $l+1$ lags of the dependent variable as being fixed points of initialisation) is:

$$\ln L\left(\psi_m, \sigma^2\right) = -\frac{T}{2}\ln\left(2\pi\sigma^2\right) - \frac{\left(\Delta Y - W_m\psi_m\right)'\left(\Delta Y - W_m\psi_m\right)}{2\sigma^2} \tag{16}$$

Specifying conjugate normal-gamma priors:

$$f\left(\psi_m | \sigma^2\right) = f_N\left(\psi_m | \bar{\psi}_m, \sigma^2 V_m\right)$$
$$f\left(\sigma^{-2}\right) \sim f_G\left(\sigma^{-2} | c_0, d_0\right)$$
$$V_m = \begin{pmatrix} Q_m & 0 \\ 0 & V_\gamma \end{pmatrix} \tag{17}$$
$$\left(\bar{\psi}_m\right)' = \left(\bar{\beta}'_m, \bar{\gamma}'\right)$$

(where $f_G\left(\sigma^2 | c_0, d_0\right)$ is the gamma distribution with mean $\frac{c_0}{d_0}$ and $f_N\left(x | \mu, V_0\right)$ specifies a normal distribution for x with mean μ and variance V_0). The hyper parameters for the priors are $\bar{\psi}_m, V_m, c_0$, and d_0.

Comparing models with common $\{z_t\}$, with the same priors on γ, the marginal likelihood of the model is:

$$l_m \propto \left(\frac{|Q_m|}{\left|\left(Q_m^{-1} + X'_m M_z X_m\right)^{-1}\right|}\right)^{-\frac{1}{2}} \tag{18}$$
$$\times \left(2d_0 + \Delta Y'_m \left(M_z - M_z X_m \left(Q_m^{-1} + X'_m M_z X_m\right)^{-1} X'_m M_z\right) \Delta Y_m\right)^{-c_0 - \frac{T}{2}}$$

where $\Delta Y_m = \Delta Y - W_m \bar{\psi}_m$ and $M_z = I - Z\left(V_\gamma^{-1} + Z'Z\right)^{-1}Z'$. Given equal prior odds on the two models, the Bayes Ratio

$$B_{m,j} = \frac{l_m}{l_j} \tag{19}$$

represents the posterior odds in favour of model m over model j. Improper priors can be placed on the coefficients of the variables that are included in all models since V_γ^{-1} can be set to zero and the marginal likelihood above can still be computed. However, Q_m^{-1} needs to be set to some positive definite matrix in order for the marginal likelihood to be valid. Priors are discussed in more depth in Section 3.8.

3.6. Identifying Break Points

Define the quantity l_m which is proportional to the marginal likelihood of the mth model, defined by a set of regressors X_m:

$$l_m = l(X_m) \tag{20}$$

For the models $m \in \mathcal{M}$ ($\mathcal{M}$ being all the models that are assigned a positive probability), the posterior probability of a model is:

$$p(m) = \frac{\pi(m) l_m}{\sum_{s \in \mathcal{M}} \pi(s) l_s} \tag{21}$$

where $\pi(s)$ is the prior probability attached to the model s and $\sum_{s \in \mathcal{M}} \pi(s) = 1$. Herein we assign equal prior weight to all models. Therefore, $\pi(s) = \pi(j)$ for all s and j so that $\pi(m)$ plays no role in the calculation of [21].

The posterior probability of a break at time t is the sum of the probabilities over all models that contain a break at that point. Defining $\mathbb{M}(t)$ as being the set of all models within $\mathcal{M}$ that contain a break at t, the posterior probability of a break at t is:

$$\Pi(t) = \sum_{m \in \mathbb{M}(t)} p(m). \tag{22}$$

This approach is somewhat different to Classical methods since it does not attempt to estimate a unique point for the structural break. Even with only one permissible break, posterior probabilities are constructed for a break at each point.

3.7. Testing for Breaks and Other Model Attributes

Higher posterior probabilities of a break appearing at one or more break points would support the contention that a series contains at least one structural break. However this does not constitute a test for a break (or breaks). A Bayesian test for breaks can be constructed by treating the break points as discrete parameters. The marginal likelihood of models with a set number of breaks can then be obtained by integrating out the discrete parameters (the break points). Accordingly, assuming that all models have the same prior probability, the marginal likelihood of the 'n break model' can be obtained using:

$$l(n) \propto \frac{\sum_{m \in \text{models with } n \text{ breaks}} l_m}{\text{number of models with } n \text{ breaks}} \tag{23}$$

Support for a given number of breaks can then be compared by examining the Bayes Ratios (BRs) using $l(0), \ldots . l(n)$. The same approach can be used to assess the support for other attributes of the model. For example, the marginal likelihood for the 'unit root model' is obtained using:

$$l(UR) \propto \frac{\sum_{m \in \text{models with Unit Root}} l_m}{\text{number of models with a Unit Root}} \tag{24}$$

Therefore, the BRs for UR vs ST, ST vs STAT, UR vs STAT can be constructed. Together this provides a rich set of information about the behaviour of the data.

3.8. Priors

The impact of V_γ on the marginal likelihood becomes negligible as its limits go to infinity (and $\bar{\gamma}'$ becomes irrelevant also). Likewise, d_0 and c_0 have a negligible impact on the marginal likelihood as their values tend to zero. Therefore, V_γ^{-1}, c_0, d_0 are set to zero. However, Q_m^{-1} cannot be set to zero. In one sense large variance values for the diagonal values for Q_m is 'non-informative'. However, it is not 'non-informative' if non-informative were to mean that our priors played little or no role in model selection. Large values for Q_m effectively translate into large penalties for additional parameters in the model. In finite samples the values of Q_m can always be set sufficiently large so that the most parsimonious model will be selected. The reasons for this type of outcome, that are not specific to time series, are discussed in Poirier, 1999, Chapter 9. (Using [5] this would be a unit root model with no structural breaks and no lags).

One possible way to set the priors is to use the 'G-Prior' approach (the approach used by Fernandez et al. 2001, and in a time series setting by Koop and Potter, 2004). However, there are a number of objections that can be raised about this approach. The use of G-Priors requires the moment matrix to form the variance matrix of the normal prior for the coefficients in the model. Using lags of the dependent variable to formulate the prior is problematic from a purist Bayesian perspective. This objection aside, the rationale for the G-Prior (apart from the further simplifications of the marginal likelihood that it affords) is usually premised on the assumption of stationary regressors However, in a time series setting, the regressors may contain trends. Moreover, where the data contains deterministic 'spikes' it is not invalid to make the argument that the prior is somehow proportional to 'one observation' of the data. Therefore, the G-Prior approach can lead to setting the prior precision of some parameters close to zero and others to infinity. In the absence of a clear understanding of what G-Priors represent in the the current setting this approach is unattractive.

An alternative approach to prior specification is to use 'training data' (e.g. Conigliani and Spezzaferri, 2007). This method allows investigators to be non-informative in the sense that the hyper parameters are based on a subset of the data. Training priors can give more robust results with regard to unit roots. However, training priors are problematic in the case where there are structural changes. For example, if a break point is at the end of the sample, then one cannot use an early part of the data set to construct a prior for the coefficient on this break variable. For this reason we do not use training priors.

The first step we take in our approach is to adopt the reparameterisation of our model. Assume that the original series is y_t^*, and that the data is transformed as:

$$y_t = \frac{y_t^* - \min y_t^*}{\max y_t^* - \min y_t^*} \tag{25}$$

The model is, as before,

$$\Delta y_t = \mu + \rho y_{t-1} + \alpha \frac{t}{T} + B_t + \sum_{i=1}^{l} \theta_i^* \Delta y_{t-i} + e_t. \tag{26}$$

Whereas (using y_t^*) before we had no clear basis to formulate a prior view about the parameters α and those that determine B_t, such a basis now exists since y_t is now bounded between 0 and 1. Accordingly, the value of α is likely to lie within the interval (1,-1), as are the parameters $\{\varphi_i\}$, ρ and θ_i^*. However, this still leaves room for variation in priors. Following earlier work (Marriott and Newbold, 1988, 2000) we explored the performance of priors using a Monte Carlo study. Each of the marginal priors were of the form [27], with the joint priors being the product of the marginals:

$$
\begin{aligned}
\alpha &\sim N(0, v_\alpha) \\
\rho &\sim N(\rho_0, v_\rho) \\
\theta_i^* &\sim N\left(0, \frac{1}{(i+1)^2}\right) \\
\varphi_i &\sim N(0, v_\varphi).
\end{aligned}
\tag{27}
$$

The variances for the lags are recommended by Doan et al. (1984). For ρ_0 we experimented with values equal to $\frac{1}{2}, -\frac{1}{4}, -\frac{1}{2}$ and $-\frac{3}{4}$. For the prior variances $\{v\}$ we experimented with values of $\frac{1}{10}, \frac{1}{5}, \frac{1}{2}$ and 1. After extensive experimentation we adopted the values of $\rho_0 = -0.5$, $v_\rho = \frac{1}{2}$, $v_\alpha = \frac{1}{2}$, and $v_\varphi = \frac{1}{10}$. Our selection criteria were the degree to which BRs could discriminate between unit root, stationary and stationary around trend series (with varying degrees of serial correlation). We also required that the priors did not spuriously detect breaks, but were able to detect moderately large breaks. The priors in [27] cannot be directly used as priors within the normal-gamma framework because these require specification of $v = \sigma^2 h$. Therefore, having specified v, $h = \sigma^{-2}v$ needs to be chosen. Therefore, we use a plug in method $h = \hat{\sigma}^{-2}v$ where $\hat{\sigma}$ is the standard deviation of the differenced series (the estimate of $\hat{\sigma}$ assuming the series is a random walk). Using this plug-in means that our method would be characterised as an 'Empirical Bayes' procedure rather than purely Bayesian. Alternative approaches to obtain $\hat{\sigma}$ could be to use estimates of the errors from an autoregression, or from an autoregression with trend. However, in our Monte-Carlo studies this had a relatively small effect on the results, even when the generating process was no longer a random walk. A subset of the Monte-Carlo results are discussed in Section 3.9.

3.9. Monte Carlo Experiments

Readers are reminded that our procedures do not require us to make a definitive decision about whether there is a unit root in the data because we are taking a BMA approach. Nevertheless, models that have high BRs will receive a greater weight when averaging. For this reason it is important to set hyper parameters for the priors, in order to have a good ability to discriminate between different data generating processes. In order to ensure this, and to guide us in our selection hyper parameters ρ_0, v_ρ, v_α, v_φ , we conducted a wide range of Monte Carlo experiments.

In Table 1 we present the results for a subset of the Monte-Carlo trials using

priors that we use in the empirical section ($\rho_0 = -0.5$, $v_\rho = \frac{1}{2}$, $v_\alpha = \frac{1}{2}$, and $v_\varphi = \frac{1}{10}$). We only present these results due to the need for brevity.

{Approximate Position of Table 1}

The BRs were derived using the model averaging procedures discussed in the preceding sections. The data generating process is as specified in [1] with no breaks and one lag (and AR(1) with and without a trend) with 100 observations (the approximate length of our data sets in the empirical section). In Monte Carlo trials (available on request) we also established that using the priors in [27] demonstrated a good ability to correctly predict lag lengths. The results in Table 1 relate only to the ability of our procedures to discriminate between unit root, stationary and stationary around trend processes. The data generating process is described in the first three columns. *Rho* refers to the value of the autoregressive coefficient in the model. Therefore, if *Rho* equals to one, this represents a unit root process. For values of *Rho* less than one, the process is stationary if *Alpha* is 0 and trend stationary if *Alpha* is 1. The value of the variance of the error for all trials was set equal to one. The results were approximately invariant for larger and smaller values of *Alpha* with the exception that as the ratio of *Alpha* to the variance of the error became small they tended to be more similar to the stationary (rather than trend stationary) results. Different values of *Mu* delivered similar results to those below for the stationary models (*Mu* is the intercept). For the unit root model, as *Mu* tended to zero the results tended to those where *Mu=0.*

The numbers in the remaining columns give the proportion of the time that the BR favoured one model over another at a given level (2, $\frac{1}{2}$ and 1). The column labelled UR/ST compares the unit root (UR) model relative to the stationary (ST) model. The column labelled UR/STAT compares the UR model relative to the STAT Model etc. The inequalities columns labelled > 2, $> \frac{1}{2}$ and > 1 indicate whether the BR was larger than 2, 0.5 or 1.respectively. For example, the value in the fourth column associated with the third line (*Theta*=1, *Mu*=0 and *Alpha*=0) indicates that in 68% of trials the BR for UR model relative to the ST model was more than 2 (2,500 trials were conducted to produce each cell in Table 1).

The salient points from Table 1 are:

- Breaks are spuriously detected (in the sense that they have higher BRs) in a relatively small proportion of the trials. For example, using a BR of one or more to indicate preference for a given model, models with breaks are incorrectly preferred less than 14% of the time (the figures in the last column).

- The procedures have a relatively good ability to discriminate between unit root processes and stationary processes if the generating process has a unit root. A unit root is correctly preferred (using BR>1) in around 80% of trials when the series has no drift and around 96% of trials if the series contains drift as well as a unit root.

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- Stationary, highly serially correlated series with an autoregressive coefficient of 0.9 or above are classified relatively frequently as unit root processes. This occurs particularly if there is a deterministic trend in the series also. In order to be able to recognise a stationary process relative to a unit root process, the autoregressive coefficient has to be less than 0.8, and if there is a trend, less than 0.7.

Alternative values for the prior hyper parameters can alter the results in Table 1. However, priors that uniformly improved the performance could not be found. Priors that lead to an improvement in the ability to correctly prefer a stationary model or trend stationary model lead to a decline in the ability to recognise a unit root process and vice versa The performance of the approach above is comparable to existing results reported in the literature. For example, the Monte Carlo work in Marriott and Newbold (1998) using priors that, in theory, should be better at discriminating between unit root and stationary processes seem to perform only slightly better (see also Table 1 of Gonigliani and Spezzaferri, 2007). Therefore, we proceed using these priors in the empirical section with one proviso: stationary series or trend stationary series that are highly serially correlated will tend to receive a high weight from a unit root model using the BMA procedure.

4. EMPIRICAL SECTION
4.1 Data.
The data is in two parts: The macroeconomic series (the extended Nelson and Plosser data) and the natural resource series series. Both these sets of data have already been described and extensively analysed in the economics literature. For a description of the macroeconomic data, readers are referred to Nelson and Plosser, (1982) Schotman and van Dijk (1991), and for the natural resource data, Arhens and Sharma (1997) For brevity we do not repeat these descriptions herein. All series are annual and are logged prior to estimation.

4.2. Results.
The results of our empirical study are contained in Table 2 and Figures 1 and 2. Table 2 contains the BRs that indicate the relative support for the models outlined in Section 3. Readers are reminded that each of these BRs result from integrating across all possible models and do not indicate that one particular model has been selected. A high BR for a particular model or model attribute indicates only that models with these features were given a high weight in the averaging procedure. For example, the BR of 9.81 for a model with 3 lags (in the autoregressive representation) for consumer prices in the second row of Table 2, indicates that at least some models with 3 lags were highly supported and therefore received greater weight. Figures 1 and 2 are plots of the data and the posterior probabilities for a structural break at each point in time (calculated as outlined in Section 3.6). Figure 1 contains plots for the macroeconomic data and Figure 2 contains plots for the natural resource data. For these figures, the axis values on the left gives the probability of a break, associated with the bars

within the plot. The axis values on the right give the logged value of the series, which is also included in the plot.

4.2.1 Macro Data
These results are presented in rows 2 to 15 of Table 2 and Figure 1.

Approximate Position of Table 2, Figures 1 and 2 (Note - some of the data in Table 2 need to be corrected - real wage/gas preferred models)

With regard to lag length, 6 of the 14 series support 2 lags, 5 support 3 lags and 3 support 1 lag. In previous studies the macro data have been for the most part been characterised by AR(2) or A(3) processes, depending on the lag selection procedures that have been adopted. Therefore, these results are in accordance with previous work in this respect. Readers are reminded that the Doan et al. (1984) priors that we adopt give declining prior variances to the parameters on the lags (where they have a mean of zero), which also tend to decrease the penalty for an additional lag. Without these declining variances, the BRs are likely to support fewer rather than more lags.

Moving to the summary of the BRs (the last two columns), we find that 8 of the series are characterised as having unit roots, with the other 6 being stationary around trend. Only one of the series (Unemployment) supports stationarity (ST) over a unit root (UR) (0.01 in column 7), but the stationary model (ST) is dominated by the stationary around trend model (STAT) (6.31 in column 5). Although the procedures herein allow for structural breaks, these results are in striking conformity with what would be found using a standard Augmented Dickey Fuller (ADF) test. For the ADF results presented for these series in Bauwens et al. (1999) the characterisation for all variables (vis-a-vis trends) is the same except for Unemployment. The ADF characterises Unemployment as ST (here STAT is preferred). In view of the fact that the Monte Carlo results suggest our priors tend to favour unit root processes, this result is not likely to be due to bias imparted from our priors. Our results regarding unit roots also largely concur with Zivot and Andrews (1992) who conclude that 8 out of the 14 series contain unit roots. The difference being that our procedures support a unit root for Nominal GNP, and reject it for the Employment series.

With regard to breaks, 8 of the macro series support the existence of breaks, 4 of which support 3 breaks, 3 of which support 2 breaks and 1 supports only one break. The posterior break probabilities in Figure 1 indicate the points at which there are high probabilities for such breaks. Figure 1 demonstrates that, once again, the findings with regard to structural breaks are similar to previous work. Zivot and Andrews (1992) find potential structural breaks at 1929 for 8 of the 14 series. Our analysis identifies evidence of potential breaks at, or close to 1930[4] in around 10 of the 14 series (the beginning of the Great Depression,

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[4]In our approach a break in 1930 will be indicated if there is a large shift between the 1929 and 1930 period (not 1930-31). Thus, a break indicated in 1930 could equally well be labelled as a 1929 break. Therefore, our results are consistent with Zivot and Andrews.

after the Crash of October 1929). However, we also find that there is evidence of a break at 1921 for a number of the series (Consumer prices, Money Stock, Nominal GNP, Nominal Wages). The remaining breaks vary, but with war years figuring commonly (1917, 1939, 1941 and 1946). Therefore, even when allowing for multiple structural breaks, the evidence suggests that, for the US at least, large structural breaks are a pre-Second World War phenomena.

4.2.2 Natural Resource Series

There is evidence to indicate that natural resource prices are stationary (around broken means) in 4 out of the 11 series. These results fall in between the empirical findings of Agbeyegbe (1993), Berck and Roberts (1996) who find predominantly unit root behaviour and Lee et al. (2006) who allow for two structural breaks and quadratic trends, and Ahrens and Sharma (1997), who found that 6 out of 11 natural resource price series reject unit roots having allowed for a structural break. Notably, we find that 7 out of the 11 series contain at least 3 breaks. Given that previous work has allowed for at most two breaks, this is a significant finding.

The behaviour of natural resource prices is different to the macroeconomic series. This is clear from visual plots, but also from the modelling results. The macroeconomic series tend to be characterised by smoother evolution, either through deterministic or stochastic trends, whereas the natural resource prices are more erratic and punctuated by very large 'episodic' shifts. The variation in the series, whether it be through noise or through 'structural shifts', makes any judgement about the direction of trends hazardous, as will be any forecasting procedure that is based on the historical behaviour of the data. The increasing demand for natural resources will inevitably be a force to drive natural resource prices upwards in the long run. However, the supply of natural resources, though destined for ultimate decline, is punctuated by discoveries of new stocks, innovation and productivity improvements in extraction methods and by the strategies of participants with market power.

The importance of market power is illustrated by the timing of the breaks. The posterior break probabilities are presented in Figure 2. These do not display the consistent pattern of the macroeconomic series, except that Coal, Gas and Petroleum (and Silver) all indicate potential breaks in 1974. This coincides with the first 'oil shock' associated with the OPEC oil embargo of October 1973[5]. We do not find great similarities to the work of Lee et al. (2006). However, we contend that an inspection of the plots in Figure 2 supports the breaks identified herein, rather than that of Lee et al. For example, in the case of coal, Lee et al. consistently choose 1972 as the break point. along with 1902, 1915, 1949 depending on the methods that they use. However, examining the middle pane on the top row of plots in Figure 2, the big shift (for coal) clearly seems to occur between the 1973-74 period, as suggested by our results. Likewise, the very big changes in Gas and Petroleum prices occur between 1973-74, which is again identified using the procedures herein.

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[5] As with the macro series we remind readers that a break in 74 is indicative of a shift between 73 and 74 (not 74 and 75).

5. CONCLUSIONS

This paper introduces a BMA approach to testing for multiple break points in univariate series using conjugate normal-gamma priors. This approach is able to test for a number of structural breaks, averaging over a number of specifications including stationary, stationary around trend and unit root models, each containing different types of breaks and different lag lengths. Unlike most previous papers in the literature, we also estimate posterior break probabilities at every point in the sample.

The normal-gamma approach, though not optimal in some respects, is shown to be highly advantageous since it can deal with up to 3 structural breaks, through the marginal likelihoods analytically expressed, thus removing the need for MCMC procedures or numerical integration. Monte Carlo work demonstrated that the normal-gamma approach has reasonable power to discriminate between unit root and stationary processes relative to other methods.

The procedures are used to test for structural breaks on 14 annual macroeconomic series and 11 natural resource price series over the last century and to construct posterior break probabilities. The results indicate that there are structural breaks in all of the natural resource series and most of the macroeconomic series. Most of the series with structural breaks have multiple breaks. Our findings regarding unit roots, having allowed for structural breaks in the data, are largely consistent with previous work. For example, with regard to the macroeconomic series, our conclusions about unit roots vs (trend) stationarity are the same as a standard Augmented Dickey Fuller test in 12 of the 14 series. Consistent with previous work, we find that a number of the natural resource series are consistent to being stationary around broken means or trends.

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Table 1: Monte Carlo Results														
Data Generating			Proportion BR greater than 0.5, 1 and 2												
Process															
Rho	Drift	Trend	UR/ST			UR/STAT			STAT/ST			Break/No Break		
			2	0.5	1	2	0.5	1	2	0.5	1	2	0.5	1
1	No	.	0.7	0.9	0.8	0.7	0.9	0.8	0.2	0.6	0.4	0.0	0.03	0.02
1	Yes	.	1.0	1.0	1.0	0.9	1.0	1.0	0.6	0.9	0.8	0.0	0.02	0.02
0.9	No	No	0.2	0.5	0.3	0.3	0.6	0.5	0.08	0.3	0.2	0.08	0.3	0.1
0.9	No	Yes	1.0	1.0	1.0	0.8	0.9	0.9	0.9	1.0	1.0	0.0	0.01	0.0
0.8	No	No	0.0	0.1	0.1	0.04	0.2	0.1	0.04	0.2	0.1	0.07	0.3	0.1
0.8	No	Yes	1.0	1.0	1.0	0.5	0.7	0.6	1.0	1.0	1.0	0.0	0.02	0.01
0.7	No	No	0.0	0.0	0.0	0.0	0.02	0.0	0.03	0.2	0.1	0.06	0.3	0.1
0.7	No	Yes	1.0	1.0	1.0	0.08	0.3	0.2	1.0	1.0	1.0	0.0	0.01	0.01
0.6	No	No	0.0	0.0	0.0	0.0	0.0	0.0	0.02	0.2	0.1	0.05	0.3	0.1
0.6	No	Yes	1.0	1.0	1.0	0.0	0.04	0.02	1.0	1.0	1.0	0.0	0.02	0.0
0.5	No	No	0.0	0.0	0.0	0.0	0.0	0.0	0.02	0.2	0.05	0.05	0.3	0.01
0.5	No	Yes	1.0	1.0	1.0	0.0	0.0	0.0	1.0	1.0	1.0	0.0	0.02	0.0
0.0	No	No	0.0	0.0	0.0	0.0	0.0	0.0	0.01	0.2	0.04	0.02	0.4	0.1
0.0	No	Yes	1.0	1.0	1.0	0.0	0.0	0.0	1.0	1.0	1.0	0.0	0.01	0.0
-0.5	No	No	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.3	0.03	0.02	0.5	0.2
-0.5	No	Yes	1.0	1.0	1.0	0.0	0.0	0.0	1.0	1.0	1.0	0.01	0.04	0.01
UR,ST and STAT refer to models as they are described in the text														
All figures are the proportion of trials that one model is preferred over another using the Bayes Ratio														
>2 indicates that the numerator model had Bayes Ratio twice that of the denominated model														

	Table 2: Structural Break Results - Macro and Natural Resources									
	No of Lags*			$\frac{STAT}{ST}$	$\frac{UR}{STAT}$	$\frac{UR}{ST}$	$\frac{1\ BRK}{0\ BRK}$	$\frac{2\ BRK}{0\ BRK}$	$\frac{3\ BRK}{0\ BRK}$	Best Model (Breaks)
	1	2	3							
Macro Series										
Cons Price	0.0	0.4	9.8	$2.0e^{+5}$	214.9	$4.31e^{+7}$	4575.2	$4.32e^{+7}$	$1.92e^{+09}$	UR (3)
Velocity	3.9	0.7	0.2	1.15	1.36	1.57	0.2	0.08	0.04	UR (0)
Unemploy	0.0	3.6	1.1	6.31	0	0.01	1.3	1.52	69.3	STAT(3)
Real Wage	0.9	1.9	0.6	4	1.8	7.2	0.11	0.03	0.01	UR (0)
GNP PC	0.0	1.4	2.8	4111	0.01	23.61	0.78	8.9	4.92	STAT (2)
Stock Pri	1.7	0.3	1.5	1.6	79.1	126.37	17.02	14.42	7.2	UR (1)
Real GNP	0.0	1.9	2.0	8451	0.01	113.04	0.24	2.72	1.13	STAT (2)
Money	0.0	3.4	1.2	8.83	1266	$1.1e^{+04}$	0.14	0.06	0.02	UR (0)
GNP DEF	0.0	5.2	0.8	9.22	2127	$1.9e^{+04}$	$1.8e^{+04}$	$2.8e^{+04}$	$1.99e^{+04}$	UR (2)
Wages	0.1	1.9	1.9	5.63	17.1	96.5	0.32	0.15	0.12	UR (0)
Interest	1.3	0.6	1.1	0.37	621.5	228.4	22.8	451.6	$2.37e^{+04}$	UR (3)
Employ	0.1	3.4	1.0	219.5	0.17	37.61	0.3	0.06	0.04	STAT (0)
Nom GNP	0.0	1.6	2.4	117.1	976	$1.1e^{+05}$	6.06	35.85	44.55	UR (0)
Ind Outp	0.8	2.0	0.5	1439	0.04	58.82	0.08	0.18	0.11	STAT (0)
Natural Res Prices										
Aluminium	0.0	0.3	12.6	1.2	150.9	180.9	193.9	2309	$8.7e^{+04}$	UR (3)
Coal	1.1	1.1	0.8	0.72	$3.9e^{+04}$	27701	3698	$4.1e^{+04}$	$2.2e^{+05}$	UR (3)
Copper	0.7	1.7	0.8	0.39	0.04	0.02	1.16	1.51	1.79	ST (3)
Iron	1.1	0.5	1.6	0.59	0.7	0.42	1.96	1.65	1.34	ST (1)
Lead	4.9	0.5	0.2	0.39	0.o	0.0	0.77	1.39	1.33	ST (2)
Gas	0.0	1.0	4.0	3.28	0.07	0.22	5.23	18.15	153.8	STAT(3)
Nickel	0.8	0.3	2.9	0.49	2260	1098	7803	$4.2e^{+05}$	$4.2e^{+05}$	UR (2)
Petroleum	1.0	2.0	0.4	0.92	0.01	0.01	5.73	3.89	4.5	ST (1)
Silver	0.0	0.0	1434	0.6	7556	4511	$2.3e^{+05}$	$1.5e^{+11}$	$3.7e^{+11}$	UR (3)
Tin	3.1	0.7	0.4	3.55	6.76	23.98	1.96	4.84	9.26	UR (3)
Zinc	0.0	0.1	117	558.9	$1.4e^{+04}$	$7.8e^{+06}$	372	$1.8e^{+11}$	$1.6e^{+13}$	UR (3)

* $1 = \frac{1\ Lag}{2\ or\ 3\ Lags}$ $2 = \frac{2\ lags}{1\ or\ 3\ lags}$ $3 = \frac{3\ lags}{1\ or\ 2\ lags}$

Consumer

Break Probabilities — Logged Value

1865 1921

1863 1869 1875 1881 1887 1893 1899 1905 1911 1917 1923 1929 1935 1941 1947 1953 1959 1965 1971 1977 1983 #N/A

Velocity

Break Probabilities — Logged Value

1881 1930 1945

1872 1878 1884 1890 1896 1902 1908 1914 1920 1926 1932 1938 1944 1950 1956 1962 1968 1974 1980 1986 #N/A

Unemployment

Break Probabilities — Logged Value

1930 1899 1941

1893 1899 1905 1911 1917 1923 1929 1935 1941 1947 1953 1959 1965 1971 1977 1983 #N/A #N/A #N/A #N/A

Real Wage

Break Probabilities — Logged Value

1941 1946

1903 1909 1915 1921 1927 1933 1939 1945 1951 1957 1963 1969 1975 1981 1987 #N/A #N/A #N/A #N/A #N/A

GNP Per Captia

Break Probabilities — Logged Value

1930 1939

1912 1918 1924 1930 1936 1942 1948 1954 1960 1966 1972 1978 1984 #N/A #N/A #N/A #N/A #N/A #N/A

Stock Prices

Break Probabilities — Logged Value

1931-32

1874 1880 1886 1892 1898 1904 1910 1916 1922 1928 1934 1940 1946 1952 1958 1964 1970 1976 1982 1988 #N/A

Real GNP

Break Probabilities — Logged Value

1930 1939

1912 1918 1924 1930 1936 1942 1948 1954 1960 1966 1972 1978 1984 #N/A #N/A #N/A #N/A #N/A #N/A

Money Stock

Break Probabilities — Logged Value

192 1932 1943

1892 1900 1908 1916 1924 1932 1940 1948 1956 1964 1972 1980 1988 #NA #NA

GNP Deflator

Break Probabilities — Logged Value

1921 1917

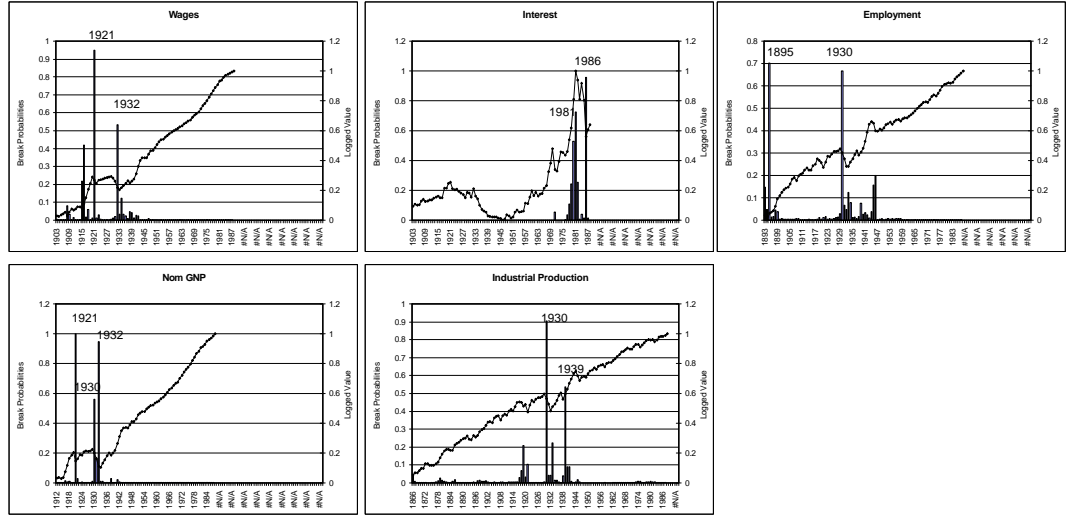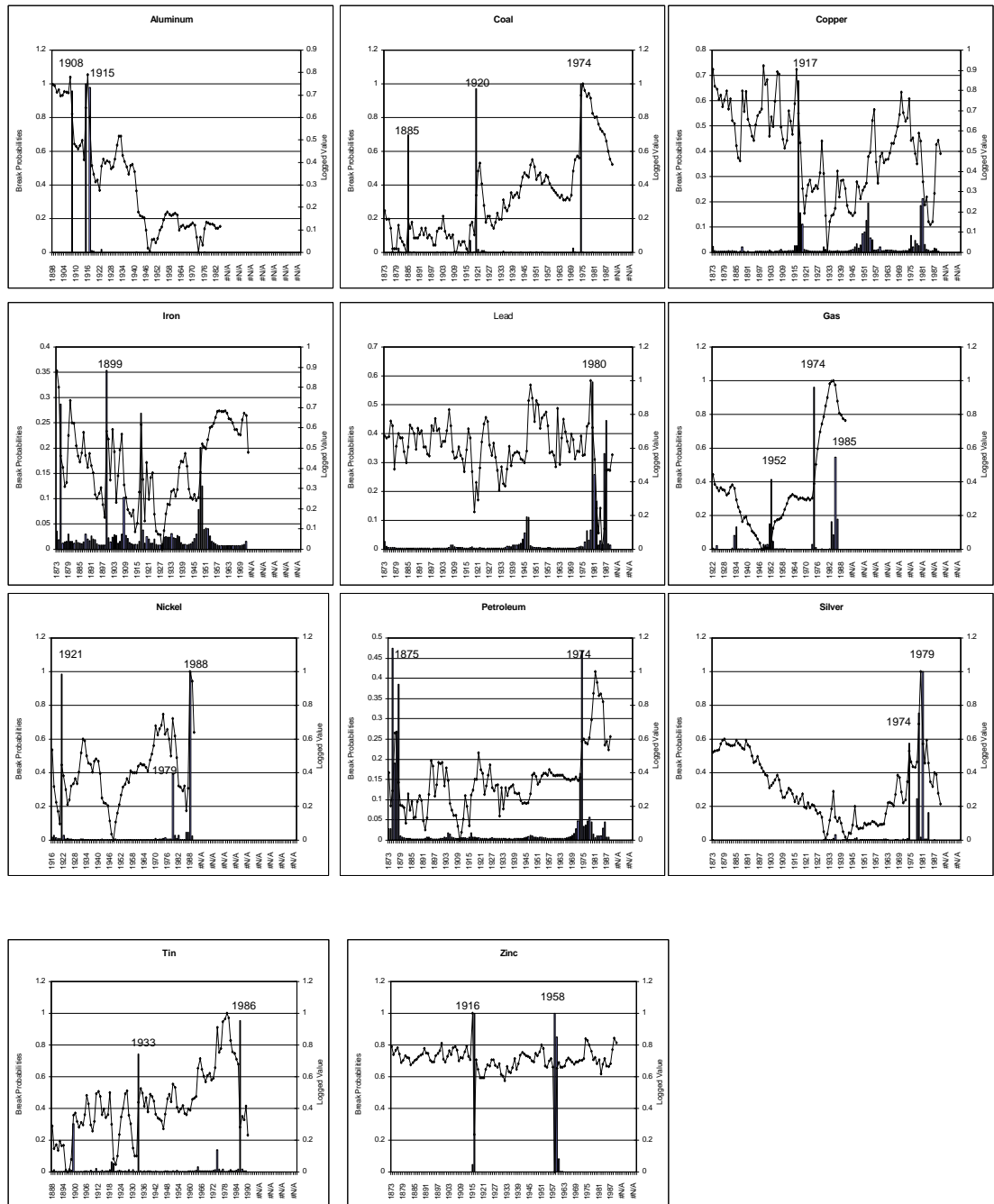1892 1898 1904 1910 1916 1922 1928 1934 1940 1946 1952 1958 1964 1970 1976 1982 1988 #N/A #N/A #N/A

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Figure 1.

Figure 2

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