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Abstract

I develop a structural model of inflation by combining two different models of price setting behavior: the sticky price model of the New Keynesian literature and the sticky information model of Mankiw and Reis. In a framework similar to the Calvo model, I assume that there are two types of firms. One type of firm chooses its prices optimally through forward-looking behavior—as assumed in the sticky price model. It uses all available information when deciding on prices. The other type of firm sets its prices under the constraint that the information it uses is "sticky"—as assumed in the sticky information model. It collects and processes the information necessary to choose its optimal prices with a delay. This leads to the sticky price-sticky information (SP/SI) Phillips curve that nests the standard sticky price and sticky information models. Estimations of this structural model show that both sticky price and sticky information models are statistically and quantitatively important for price setting. However, the sticky price firms make up the majority of the firms in the economy. The resultant SP/SI Phillips curve models inflation better than either the sticky price or sticky information models. The results are robust to alternative sub-samples and estimation methods.

JEL classification: E10; E31; E37

Keywords: Inflation; Phillips Curve; Sticky prices; Sticky information;

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1. Introduction

Understanding short-run inflation dynamics is one of the main issues in macroeconomics. It is especially important for the conduct of monetary policy. In the literature, the current approach to theoretical modeling is to assume some kind of "sticky prices" (and/or wages) in the optimization problems of forward-looking individuals and firms.[1] Most of the models in this literature use the sticky price assumption in the framework of Calvo's (1983) model in which each firm adjusts its price with some probability in each period independent of waiting time. Those models have led to the derivation of the New Keynesian Phillips curve (NKPC). As an alternative to the NKPC, Mankiw and Reis (2002) proposed the "sticky information Phillips curve." The main premise of their model is that information about macroeconomic conditions spreads slowly throughout the population; although prices are set every period, information collecting and processing take time. In this model, a fraction of firms get complete information about the economy in each period randomly and independent of waiting time, and set their prices according to this new information, while the remaining firms set their prices according to old information. The dynamics of this model are similar to that of the backward-looking expectations model, and so it exhibits the inflation persistence observed in the data.[2]

The main question of this study is whether both price and information stickiness are necessary to model and explain inflation dynamics. Therefore, I develop a structural model of inflation by combining two different models of price setting behavior: the sticky price model of the New Keynesian literature and the sticky information model of Mankiw and Reis. In a framework similar to the Calvo model, I assume that there are two types of firms. One type of firm chooses its prices optimally through forward-looking behavior, as assumed in the sticky price model. It uses all available information when deciding on prices. The other type of firm sets its prices under the constraint that the information it uses is "sticky", as assumed in the sticky information model. It collects and processes the information necessary

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[1]Some early examples include Clarida, Gali and Gertler (1999), Rotemberg and Woodford (1997), McCallum and Nelson (1999).

[2]The dynamic implications of the sticky price and sticky information models are investigated and compared in Arslan (2005).

to choose its optimal prices with a delay. The resultant structural model of inflation is the Sticky Price-Sticky Information (SP/SI) Phillips curve that nests the standard sticky price and sticky information models as special cases. I estimate the SP/SI Phillips curve by a non-linear instrumental variables (GMM) method with the U.S. data, and using the output gap and unit labor cost as alternative measures of real marginal cost. Some robustness analysis is conducted to assess the reliability and validity of the results, including estimation by using different sub-samples, different numbers of lags for instrumental variables and different autocorrelation lags in calculating the weighting matrix in the GMM estimation, and also estimation with another method, Maximum likelihood estimation (MLE).

Although the NKPC model has been used as the "workhorse" in the literature and has an appealing theoretical structure, it has been criticized for producing implausible results regarding inflation dynamics. The standard NKPC model does not exhibit the inflation persistence and delayed and gradual effects of monetary shocks observed in the data. It is also unable to account for the correlation between inflation and the output gap. The NKPC model implies that inflation should lead the output gap over the cycle, although VAR studies have shown that the main effect of a shock on output precedes the effect on inflation.[3]

The empirical limitations and difficulties with the standard NKPC model have motivated researchers to extend the standard framework.[4] One such extension is allowing the delayed price changes, so policy shocks will not affect prices immediately and output will be affected earlier than inflation. Another extension is allowing for price changes between price revisions according to some mechanical rule. Some researchers, including Fuhrer and Moore (1995) and Gali and Gertler (1999), obtained a hybrid version of the NKPC and old Phillips curve by including a lagged inflation term into the NKPC.[5] However, these hybrid Phillips curves

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[3]See, among others, Ball (1994), Fuhrer and Moore (1995), Estrella and Fuhrer (1998), Lown and Rich (1997), Roberts (1998) and Gali and Gertler (1999) for empirical difficulties with the New Keynesian Phillips curve.

[4]For details of such extensions and their implications, see Woodford (2003).

[5]Fuhrer and Moore (1995) use the importance of relative real wages in Taylor's overlapping wage contracts model to obtain this hybrid specification. Roberts (1998) uses adaptive expectations for some fraction of price setters. Gali and Gertler (1999) derive a hybrid Phillips curve by allowing a subset of firms set their prices according to a backward-looking rule of thumb to obtain enough degree of persistence. A similar imposition of rule of thumb is also used by Steinsson (2003).

also have difficulties in characterizing inflation dynamics,[6] and they generally have no micro foundations; adding lagged inflation is just an ad-hoc approach in these models.

Past expectations of current economic conditions are important for the sticky information Phillips curve, while the current expectations of future economic conditions are important for the NKPC. Since both of those expectational approaches may be important and the existence of both sticky price and sticky information type firms in the economy is theoretically reasonable, a price setting model that contains all these aspects could potentially model inflation better and exhibit more realistic dynamics. The contribution of this paper is to develop such a model by combining the sticky price and sticky information models and to estimate it in order to see the relative importance (or unimportance) of these models.

There are two important structural parameters that I am interested in estimating. The first parameter of interest is the fraction of firms whose prices remain unchanged whether they are sticky price or sticky information type firms; it is interpreted as a measure of the degree of price stickiness in the economy. The other parameter of interest is the fraction of sticky price type firms; it is interpreted as a measure of the relative importance of the price setting models. In estimations, it is also important which variable to be used to measure real economic activity. Theoretical models of price setting in the New Keynesian literature imply real marginal cost to measure real economic activity. Most of the empirical literature has used the output gap as an appropriate proxy for real marginal cost. However, some recent literature has used the unit labor cost as another proxy for real marginal cost. Therefore, both measures of real marginal cost are used in the estimations.

The estimation results show that the structural parameters of the SP/SI Phillips curve are estimated much more reasonably when unit labor cost is used rather than the output gap. The price stickiness parameter is estimated to be between 0.76 and 0.85 by the GMM method (depending on the sample), which implies an average period for fixed prices between 4.2-6.5

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[6]There are difficulties especially in detecting a statistically significant effect of real activity on inflation with quarterly data. Also Fuhrer and Moore's (1995) model implies that expectations of future prices are empirically unimportant in explaining price and inflation behavior, so their model is consistent with the old Phillips curve (see Fuhrer, 1997). However, Gali and Gertler (1999) find the forward-looking model statistically and quantitatively very important in describing inflation; the backward-looking behavior, although statistically significant, has small quantitative importance.

quarters. These values are not far from generally accepted levels. The fraction of sticky price firms is estimated to be between 0.64 and 0.82 (again, depending on the sample). These results show that, although the sticky price firms form the majority of firms and so may have a more dominant role in price setting, the existence of sticky information firms is statistically and quantitatively important. Therefore, the sticky information model of price setting should be taken into account when inflation is modeled. Also, the SP/SI Phillips curve could perform and model inflation better than either the sticky price or the sticky information models; because, while the data cannot reject the SP/SI Phillips curve specification, it strongly rejects each of the sticky price and sticky information models within the SP/SI Phillips curve. These results are also robust to alternative sub-samples, different numbers of lags for instrumental variables, different numbers of autocorrelation lags in calculating the weighting matrix, and estimating by MLE.

The rest of the paper is organized as follows. In section 2, the price adjustment models are described and the SP/SI Phillips curve is derived. In section 3, the estimation methods are explained, and, in section 4, the estimation results are given. Some robustness analyses are performed in section 5, and section 6 concludes.

2.   Price Adjustment Models

2.1.   The Sticky Price and Sticky Information Models

Firms produce in a monopolistically competitive market. They are assumed to be identical except for the differentiated goods they produce, their price setting history and the constraints they face when setting their optimal prices. This difference in constraints divides the firms into two groups: sticky price and sticky information type firms. Firms behave optimally and try to maximize their profits by choosing prices. Each type of firm chooses the same price when it adjusts its prices under these assumptions.

Under the sticky price assumption, a fraction $1-\theta$ of firms adjust their prices in each period according to the expectations about future economic conditions, and the remaining fraction θ of firms keep their prices unchanged. The optimal price chosen by the representative firm that

has opportunity to change its price at t can be derived from the firm's profit maximization problem. This optimal price can be obtained as:[7]

$$p_t^{sp} = (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k \, E_t\{mc_{t+k}^n\}, \tag{1}$$

where mc_t^n is the log deviation of nominal marginal cost[8] from its steady state at t and β is a common discount factor. Thus, the sticky price firms set their optimal prices by taking the expected future path of nominal marginal costs into account.[9]

Under the sticky information assumption, when a firm changes its price at period t, the new price that applies beginning in period t is chosen on the basis of the last information it has at period $t - k$; that is, according to the state of the economy as of period $t - k$. Therefore, if a fraction $1 - \theta$ of firms get complete information about the economy and set their prices according to this new information and the remaining fraction θ of firms set their prices according to old information, the optimal price for a firm in period t can be obtained as:

$$p_t^{si} = (1 - \theta) \sum_{k=0}^{\infty} \theta^k \, E_{t-k}\{mc_t^n\}. \tag{2}$$

Therefore, the sticky information firms set their prices by taking into account all of the past expectations of current nominal marginal cost. Derivations of these optimal price equations are given in Appendix A.

2.2. Derivation of the SP/SI Phillips Curve

I assume that there two types of firms in the economy. They are identical except for the differentiated goods they produce, their price setting history and the constraints they face when setting their optimal prices. A fraction ω of firms are sticky price type and set their prices optimally through forward-looking behavior, as in the standard sticky price model.

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[7]Small letters for a variables denote the log deviation of the variables from their steady state value, such as $p_t = \log(P_t/P)$.

[8]It is the marginal cost of the representative firm in the economy.

[9]Firm's optimization problem results in optimal prices in terms of nominal marginal cost, mc_t^n, as given in Appendix A. However, these optimal prices lead to the SP/SI Phillips curve that is derived in terms of real marginal cost, mc_t, as explained in the next subsection.

However, the remaining fraction $1 - \omega$ of firms are sticky information type and set their prices as in the sticky information model.[10]

The price chosen by each firm, whether it is sticky price or sticky information firm, changes with some probability in each period. During each period, a randomly selected fraction $1 - \theta$ of firms are chosen. The sticky price firms among them choose their new optimal prices due to the opportunity they find to change their prices, while the sticky information ones obtain new information and use this when they set their optimal prices. The prices for the remaining fraction θ of firms do not change. The sticky price ones among these firms could not find the opportunity to adjust, while the sticky information type firms could not get the new information, and although they adjust every period, their prices do not change due to the old information they use.[11] The aggregate price index of the economy in such a framework can be written as follows :

$$P_t = [\theta P_{t-1}^{1-\varepsilon} + (1 - \theta) P_t^{*(1-\varepsilon)}]^{\frac{1}{1-\varepsilon}} , \tag{3}$$

where the price index P_t^* represents the optimal prices chosen by the adjusting sticky price firms and the newly-informed sticky information firms. The parameter ε is the elasticity of substitution among differentiated goods. This equation can be written in log-linear form as:

$$p_t = \theta p_{t-1} + (1 - \theta) p_t^* . \tag{4}$$

The optimal price index chosen in period t can be expressed as:

$$p_t^* = (1 - \omega) p_t^{si} + \omega p_t^{sp} , \tag{5}$$

where p_t^{si} and p_t^{sp} denote the optimal prices set by the sticky information and sticky price firms, respectively. Then, the aggregate price index can be written as:

$$p_t = \theta p_{t-1} + (1 - \theta)(1 - \omega) p_t^{si} + (1 - \theta) \omega p_t^{sp} . \tag{6}$$

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[10]It is possible that a firm can exhibit both sticky information and sticky price characteristics. But, in this study, it is assumed that a firm can only be one type to simplify the analysis.

[11]It is assumed that both types of firms have the same probability θ of keeping prices unchanged. However, different probabilities can be assumed for each type of firm, and this can be the subject of another study.

If the optimal prices p_t^{sp} and p_t^{si} given in equations (1) and (2) are substituted into (6), the aggregate price index can be rewritten as:

$$p_t - \theta p_{t-1} = (1-\theta)^2(1-\omega)\sum_{k=0}^{\infty}\theta^k \, E_{t-k}\{mc_t^n\} + (1-\theta)\omega(1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k \, E_t\{mc_{t+k}^n\}\,. \quad (7)$$

In this expression, the past expectations of current nominal marginal cost are all predetermined, so the expectational errors for nominal marginal cost are all observed. Therefore, one can derive the following expressions for the first summation term in equation (7):

$$\sum_{k=0}^{\infty}\theta^k \, E_{t-k}\{mc_t^n\} = \sum_{k=0}^{\infty}\theta^k(mc_t^n - e_{tk}) = \frac{mc_t^n}{1-\theta} - \sum_{k=1}^{\infty}\theta^k \, e_{tk} = \frac{mc_t^n}{1-\theta} - F_t\,, \quad (8)$$

where e_{tk} is the expectational error for nominal marginal cost between periods t and $t-k$, $e_{tk} = mc_t^n - E_{t-k}mc_t^n$, and $F_t = \sum_{k=1}^{\infty}\theta^k \, e_{tk}$. Then, the aggregate price index becomes:

$$p_t - \theta p_{t-1} = (1-\theta)(1-\omega)mc_t^n - (1-\theta)^2(1-\omega)F_t + (1-\theta)\omega(1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k \, E_t\{mc_{t+k}^n\}\,.$$

By using some algebraic manipulations and the identities $mc_t^n = mc_t + p_t$ and $\pi_t = p_t - p_{t-1}$, where mc_t and π_t are real marginal cost and inflation rate, respectively, the SP/SI Phillips curve can be obtained as:

$$\pi_t = C_t + \lambda_1 mc_t + \lambda_2 E_t \pi_{t+1} + \lambda_3 E_t mc_{t+1} + u_t\,, \quad (9)$$

where

$$\lambda_1 = \frac{1-\theta}{\theta}(1 - \omega\beta\theta)\,,$$

$$\lambda_2 = \beta[1 - (1-\theta)(1-\omega)]\,,$$

$$\lambda_3 = -\beta(1-\theta)(1-\omega)\,,$$

$$C_t = -(1-\theta)^2(1-\omega)F_t/\theta + \beta(1-\theta)^2(1-\omega)E_t F_{t+1}\,.$$

In this equation, u_t is the disturbance term in the model and can have different interpretations.[12] All coefficients depend on the structural parameters β, θ and ω of the model. This

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[12]In the empirical specification, this disturbance term may represent the errors in the forecast of expected variables under rational expectations, and errors due to the log-linearization or data measurement. This error term can also be introduced theoretically by using several approaches. Households can be assumed to be monopolistically competitive suppliers of their labor, so it can be interpreted to represent the bargaining power of households in the labor market as in Arslan (2007). It might also arise from the collusion among firms or from variable taxation as in Ball, Mankiw and Reis (2004).

is the SP/SI Phillips curve, which nests both the sticky price and sticky information Phillips curves. Therefore, when $\omega = 1$ (i.e. all firms are sticky price type and new price settings are done according to the sticky price model), the SP/SI Phillips curve reduces to the standard NKPC:

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{\theta} \, mc_t + u_t \,. \tag{10}$$

When $\omega = 0$ (i.e. all firms are sticky information type), the SP/SI Phillips curve becomes the pure sticky information Phillips curve:

$$\pi_t = C_t + \beta\theta \, E_t \pi_{t+1} + \frac{1-\theta}{\theta} \, mc_t - \beta(1-\theta) E_t mc_{t+1} + u_t \,. \tag{11}$$

The SP/SI Phillips curve is derived here in terms of real marginal cost, and the more familiar output gap does not appear in it.[13] As seen in the optimal price setting equations above, the theory implies that real economic activity should be measured by real marginal cost. However, most of the empirical literature has worked with the output gap as the measure of real marginal cost in price adjustment equations. This arises from the fact that, under certain conditions, real marginal cost and the output gap are proportional and can be written as $mc_t = \kappa \, x_t$, where x_t is the output gap and represents the log deviation of output from its potential level.[14]

3.   Estimation of the SP/SI Phillips Curve

The structural model of inflation is given in equation (9). Such models are estimated most frequently by instrumental variable techniques such as GMM in the literature. An alternative estimation approach would be full-information approach such as estimating by Maximum Likelihood Estimation (MLE) method. There is no way to know which approach is best, as emphasized in Cochrane (2001). Single equation methods may be sensitive to

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[13]The output gap is defined as real output relative to some measure of potential output, which is generally estimated as a linear or quadratic trend of output.

[14]Rotemberg and Woodford (1997) show that when capital is fixed, marginal cost and output are approximately proportional

the choice of instruments; however, maximum likelihood estimations are sensitive to imposed assumptions about the structure of the economy and the error term.[15] Therefore, both of these estimation methods have advantages and disadvantages when compared to each other.

In this study, the SP/SI Phillips curve in (9) can be put in the following form, and then is estimated by the GMM method. The robustness of the results is checked by re-estimating the model by MLE.

$$\pi_t - C_t = \lambda_1 mc_t + \lambda_2 E_t \pi_{t+1} + \lambda_3 E_t mc_{t+1} + u_t, \tag{12}$$

Since the main parameters of interest in this study are θ, which measures the degree of price stickiness, and ω, which is the fraction of sticky type firms, the other structural parameter β, which is discount factor, is calibrated to be 0.99 in the estimations.[16]

I use quarterly U.S. data over the period 1960:1 to 2004:3 to estimate the SP/SI Phillips curve. The data set includes the log of the non-farm business sector unit labor cost, s_t; the quadratically detrended log of the non-farm business sector output, x_t; change in the log of GDP deflator, π_t; the log of the commodity price inflation, $cpinf_t$; the log of wage inflation in the non-farm business sector, $winf_t$; and the federal fund rate, r_t.

The C_t term in the SP/SI Phillips curve is a function of the structural parameters β, θ and ω, and the sum of all past expectational errors for the current nominal marginal cost. This term can be thought as a time series variable. So, to be able to estimate the SP/SI Phillips curve, first F_t and F_{t+1} terms, which are the function of infinite past expectations, need to be calculated, and then the C_t series can be obtained by using these terms and the model's parameters. A VAR model is estimated to get the past expectational error terms e_{tk} and calculate the F_t and F_{t+1} terms for all possible k values. Since these terms include infinite sum of the past expectations but empirically only a finite number of them can be obtained, the C_t term calculated and used in the estimation would only be an approximation to the "true" C_t term. The details of this estimation and calculations are given in appendix B. However, even

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[15]The method used by Sbordone (2002) could also be used in the estimation of the structural model parameters. She constructs the theoretical path of prices for the structural equation of the model given some parameter values, and then chooses the values for those parameters that maximize the fit with the data as optimal values.

[16]The discount factor is generally estimated and assumed to be very close to 1 in the literature. So I calibrate it as 0.99.

after estimating the VAR and calculating the e_{tk} terms, the C_t term depends on the structural parameter θ, which needs to be estimated. The series C_t changes continuously depending on the value of θ in the estimations. Therefore, this dependence is explicitly modeled as part of the estimation procedure.[17]

3.1. Measurement of Real Marginal Cost

Theoretical models of price setting in the New Keynesian literature imply real marginal cost as the measure of real economic activity. Most of the empirical literature has used the output gap as the measure of real economic activity by assuming it to be an appropriate proxy for real marginal cost. However, it is difficult to measure the output gap because it depends on unobservable potential output. Thus the output gap used in empirical studies may not represent the "true" output gap. Also, estimates of the NKPC and the hybrid Phillips curves with the output gap have some empirical difficulties.

Therefore, the problems in measuring the output gap, the poor performance of the estimations with the output gap, and the direct implication of the theory have led some researchers to use another proxy for real marginal cost. Such as Gali and Gertler (1999), Gali, Gertler and López-Salido (2001), and Sbordone (2002) use unit labor cost to measure real marginal cost.[18] They conclude that the empirical difficulties of the NKPC model arise from using the output gap, which is not a good proxy for real marginal cost, so real activity should be measured by unit labor cost rather than by the output gap.

In this study, the SP/SI Phillips curve is estimated using both the output gap and unit labor cost as alternative measures of real marginal cost. So, real marginal cost becomes either $mc_t = s_t$ or $mc_t = x_t$ in estimations, where s_t is the real unit labor cost, x_t is the output gap, and both are expressed as log deviation from their steady states.

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[17]See Appendix B for details of C_t term calculation.

[18]One can easily obtain unit labor cost as a measure of real marginal cost by simply assuming a Cobb-Douglas technology. Firms' cost minimization problem yields that marginal cost equals the income share of labor (unit labor cost).

3.2. Estimation Method by GMM

The baseline estimation of the SP/SI Phillips curve is performed by the GMM method. Since under rational expectations, all information dated t and earlier is uncorrelated with the error in the forecast of π_{t+1} and mc_{t+1}, the orthogonality condition for the SP/SI Phillips curve can take the form of

$$E_t\left\{\left(\theta(\pi_t-C_t)-(1-\theta)(1-\omega\beta\theta)mc_t-\theta\beta(1-(1-\theta)(1-\omega))\pi_{t+1}+\theta\beta(1-\theta)(1-\omega)mc_{t+1}\right)\mathbf{Z_{t-1}}\right\}=0, \tag{13}$$

where $\mathbf{Z_{t-1}}$ is a vector of instrumental variables, which are dated $t-1$ and earlier, so they are supposed to be unrelated with the inflation surprise in period t. The orthogonality condition in equation (13) forms the basis for estimating the SP/SI Phillips curve by GMM. The instrument set includes six lags of inflation, the output gap, unit labor cost, commodity price inflation, wage inflation, and interest rate. The structural parameters are estimated by non-linear instrumental variables estimator. In the estimation, the weighting matrix is chosen according to Hansen (1982).[19]

3.3. Estimation Method by MLE

The SP/SI Phillips curve is also estimated by using the maximum likelihood estimation method, which is similar to the method used by Fuhrer and Moore (1995). For the purpose of this estimation, a VAR model of order four is estimated that comprises inflation, unit labor cost, output gap, commodity price inflation, wage inflation, and interest rate. The structural equation given by the SP/SI Phillips curve is combined with the estimated equations from the VAR model. Under the rational expectation, the structural equation of inflation and the

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[19]Hansen (1982) shows that when the weighting matrix W is chosen optimally, $u'\mathbf{Z}W\mathbf{Z}'u$ is distributed as χ^2. Hansen also shows that an efficient estimator can be obtained if the optimal weighting matrix is chosen as the inverse of

$$\sum_{k=-L}^{L}\sum_{t}\mathbf{Z'_t}u_t u_{t-k}\mathbf{Z_{t-k}}\,.$$

If there is serial correlation in the disturbances, L is non-zero, and this matrix may not be positive definite. Newey and West (1987) show that when the k terms in the above expression are multiplied by $1-\frac{|k|}{L+1}$, this guarantees a positive definite weighting matrix. This calculation gives the so-called L-lag Newey-West autocorrelation consistent estimator for the standard errors and the covariance matrix.

equations from the VAR model would be consistent. Then, the resultant dynamic system will be:

$$\pi_t = C_t + \lambda_1 mc_t + \lambda_2 E_t \pi_{t+1} + \lambda_3 E_t mc_{t+1} + \varepsilon_t$$

$$\mathbf{S_t} = \mathbf{c} + \sum_{k=1}^{p} \mathbf{\Phi_k S_{t-k}}, \tag{14}$$

where $\mathbf{S_t}$ is a vector of variables given above and excludes inflation, and $\mathbf{\Phi_k}$ is the coefficient matrices obtained from the VAR model. By taking the estimated coefficients in the VAR equations as given, only the structural parameters of the SP/SI Phillips curve need to be determined.

The system given in equation (14) is a forward-looking dynamic stochastic model and is solved by using the Anderson and Moore's (1985) AIM algorithm.[20] This algorithm results in a solution such that expectation of the future variables can be written in terms of the present and the past. After solving the system, the set of parameters, which produces the maximum value for the likelihood function, are obtained through a nonlinear optimization procedure. Appendix C gives details of the solution method and how the Anderson and Moore's algorithm is used.

4. Estimation Results

One parameter of interest, θ, is the fraction of firms whose prices remain unchanged each period. Therefore, this parameter measures the degree of price stickiness in the economy; such as, $\theta = 1$ represents the full stickiness, and $\theta = 0$ represents the full flexibility. The other parameter of interest ω, is defined as the fraction of the sticky price firms and measures the degree of the standard sticky pricing approach in price setting. If $\omega = 1$, all firms are forward-looking sticky price type and set their prices as assumed in the sticky price model. If $\omega = 0$, all firms are sticky information type and set their prices according to the information

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[20]This is a generalized saddlepoint analysis for perfect foresight models. The algorithm transforms the structural equations of the model into the state-space representation. Then, by using the stability and initial conditions, it excludes potential solutions which never converge to the steady state.

Table 1
Estimation of the SP/SI Phillips Curve by GMM

Real Marginal Cost	θ	ω	λ_1	λ_2	λ_3	J-Stat	R^2
ULC	0.832	0.637	0.096	0.930	-0.060	0.999	0.849
	(0.009)	(0.053)	(0.012)	0.010)	(0.002)		
YGAP	0.937	0.846	0.015	0.980	-0.010	0.999	0.797
	(0.018)	(0.128)	(0.010)	(0.009)	(0.001)		

Notes : θ : Measure of the price stickiness ($\theta = 0$: fully flexible price; $\theta = 1$: fixed prices)
ω : Fraction of sticky price firms
($\omega = 1$: all firms are sticky price type (NKPC); $\omega = 0$: all firms are sticky information type.)
λ_1 : Coefficient of real marginal cost in the SP/SI Phillips curve,
λ_2 : Coefficient of expected next period inflation in the SP/SI Phillips curve
λ_3 : Coefficient of expected next period real marginal cost in the SP/SI Phillips curve.
Standard errors are shown in parentheses, and they are calculated by using a 12-lag Newey-West estimate of the covariance matrix. The values in the J-Stat column represent the p-values of Hansen's J-test.

they have, as assumed in the sticky information model. These parameters are estimated alternatively using the output gap and unit labor cost as proxies for real marginal cost.

Estimations of the SP/SI Phillips curve by GMM, when real marginal cost is represented by either the output gap or unit labor cost, are given in Table 1. When unit labor cost is used as proxy for real marginal cost, the parameter θ is estimated to be 0.832, which implies six quarters for the average duration of fixed prices. This value is not far from generally accepted levels.[21] The parameter ω is estimated significantly to be 0.637 when unit labor cost is used. This implies a significant weight of 0.363 for the sticky information type firms. Therefore, roughly two-thirds of all the firms in the economy are forward-looking sticky price type, while the remaining one-third of the firms is sticky information type. This result shows that, although sticky price firms have a dominant role in price setting, sticky information firms are also important, so they should be taken into account in price setting models.

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[21] Gali and Gertler (1999) found this duration as five to six quarters, while Sbordone (2002) found as nine to 14 months, and survey evidence shows this duration is somewhat less than the above figures and around three to four quarters. For some survey evidence on the degree of the price stickiness, see Blinder et al. (1998), Bils and Klenow (2002).

Table 2
Test of Models within the SP/SI Phillips Curve

| Sticky Price Model | Sticky Information Model |
$\omega = 1$	$\omega = 0$
0.000	0.000

Notes : Tests are Wald tests, distributed as χ^2 with one degree of freedom for the indicated restrictions. The p-value for the null hypothesis is reported.

However, when the output gap is used as the measure of real marginal cost, ω is estimated significantly to be around 0.85, and θ is estimated to be around 0.94. This value of θ implies 16.7 quarters for the average duration of fixed prices. This value is unreasonable and much higher than generally accepted levels. The implied estimations of the reduced form coefficients of real marginal cost, λ_1 and λ_3, are, though they have the correct sign,[22] much smaller than both accepted levels and the ones estimated with unit labor cost. Therefore, these coefficients indicate only a very small effect of real activity on prices. Since estimations with the output gap do not yield sensible and plausible results,[23] only unit labor cost is used as proxy for real marginal cost in the remaining estimations of this study. Table 1 also reports p-values of the Hansen J-statistic for overidentifying restrictions. According to this J-test, null hypothesis of the overidentifying restrictions are satisfied cannot be rejected, so the instruments used in the estimations can be considered as valid.

Since the SP/SI Phillips curve nests both the sticky price and sticky information models of price adjustment, the validity of each model can be tested against the SP/SI Phillips curve. The results of such a test are given in Table 2 when unit labor cost is used as proxy for real marginal cost in GMM estimation. It shows that both the pure sticky price and the sticky information models are strongly rejected against the SP/SI Phillips curve. Therefore, both models are important for inflation dynamics, and the SP/SI Phillips curve could model

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[22]The coefficient of real marginal cost appears to be negative in the NKPC when the output gap is used, as opposed by the theory, as shown in Gali and Gerler (1999).

[23]This result is robust to different sub-samples and alternative estimation methods. In those cases, estimation with the output gap yields even worse results, such as in some cases, θ or ω is estimated greater than one or even as negative.

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Table 3
Estimation of the SP/SI Phillips Curve by GMM in Sub-Samples

Period	θ	ω	λ_1	λ_2	λ_3	J-Stat	R^2
1960:1-1980:4	0.765	0.650	0.156	0.908	-0.082	1.000	0.888
	(0.007)	(0.029)	(0.010)	0.007)	(0.002)		
1981:1-2004:3	0.847	0.823	0.056	0.963	-0.027	1.000	0.620
	(0.086)	(0.065)	(0.012)	0.010)	(0.001)		

Notes : See notes to Table 1.

inflation better than either of these models.

5. Robustness Analyses

To further assess the reliability of the previous results, some robustness exercises are conducted in the estimation of the SP/SI Phillips curve. They include exploring the validity of the results in sub-samples, estimating by using different numbers of lags for instrument variables and Newey-West estimates of covariance matrix in the GMM method, and also estimating by MLE.

5.1. Estimations with Different Sub-Samples

The whole sample for the period 1961:1-2004:3 is divided into two sub-samples, which are the periods 1960:1-1980:4 and 1981:1-2004:3. The first period is characterized by having high inflation and the second period by having low inflation. The estimation results by the GMM method for these sub-samples are given in Table 3.

The parameter ω is estimated significantly to be 0.650 for the first period and 0.823 for the second period. It shows that the fraction of sticky information firms (i.e. the weight of the sticky information modeling in price setting) has decreased during the last two decades when compared with the previous two decades. It might make sense because communication, obtaining and processing information have become easier due to improvement in information

technologies over the last two decades.[24] The parameter θ is estimated to be 0.765 for the first period, which implies that prices are fixed on average for 4.25 quarters; and it is estimated to be 0.847 for the second period that implies 6.5 quarters for fixed prices. Indeed these findings confirm inflation characteristics of the sub-samples. Since the first period is relatively inflationary compared to the second period, the parameter θ is expected to be lower (as well as the average duration for fixed prices) in the first period than in the second period.

Although the estimates of the structural parameter change with the samples, the main conclusion does not change. Therefore, the results for the sub-samples also confirm that there is a significant number of sticky information firms along with the sticky price firms in the economy; therefore, the sticky information model should be incorporated into price setting models. Also, the SP/SI Phillips curve that takes both models of price setting into account could perform better in modeling inflation than each separate model.

5.2. Robustness to Lag Numbers

In the GMM estimations, the instrument set includes six lags of the variables. It is generally accepted that, in choosing the lag number, at least one year or more is needed to capture the dynamics in the data. Therefore, in the baseline estimation, six lags of the variables are used as instruments. Also, when the parameters are estimated by GMM, 12-lag Newey-West estimate of the optimal weighting matrix is used. However, as shown by several studies, the results may depend on the chosen lag number for the Newey-West estimate of the variance-covariance matrix. Therefore, in this subsection, robustness of the results to different lag numbers is investigated. Table 4 shows the results of this analysis.

The first row of Table 4 shows the estimates of the parameters in the baseline estimation, which has six lags of instruments and 12 lag for the Newey-West estimate of the optimal weighting matrix. The second row gives the averages of the estimates of the parameters and their standard deviations when the GMM estimation is performed with the instrument set of four to 10 lags of variables. The last row also gives the averages and their standard deviations when the GMM estimation is performed with four to 16 lags for the Newey-West estimate

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[24]This process might also be affected by legislation and institutional improvements, which need to be investigated.

Table 4
Robustness of Estimates to Lag Numbers

	θ	ω	λ_1	λ_2	λ_3
Baseline Estimation	0.832	0.637	0.096	0.930	-0.060
Instrument Lags	0.831	0.678	0.090	0.936	-0.054
4 - 10	(0.008)	(0.032)	(0.007)	0.006)	(0.006)
Newey-West Lags	0.834	0.637	0.095	0.930	-0.060
4 - 16	(0.003)	(0.006)	(0.002)	0.001)	(0.001)

Notes : Baseline estimation is performed by using six lags of instruments and 12 lag for Newey-West estimates. The rows show the averages of the estimates for the corresponding lags. Standard deviations of these averages are given inside the parentheses.

of the covariance matrix. The averages of the estimates of the parameters show that the chosen lag numbers in the baseline estimation are not outside of the reasonable range and far from the optimal values, since the parameter values of the baseline estimation are very close to those averages. Also, the standard deviations of these average values are very small, so they imply that the parameters, including the ones in the baseline estimation, are estimated with very high precision. Therefore, these results show that the baseline estimation is very robust to the estimation with different numbers of lags for both the instrument set and the Newey-West estimate of the optimal weighting matrix.

5.3. Estimations by MLE

In this subsection, the SP/SI Phillips curve is estimated by using the MLE method described in section 3.3 above. Estimation results for the full sample and two sub-samples are given in Table 5.

The estimates of the structural parameters by MLE are higher than the corresponding estimates by GMM. Especially, the parameter θ and ω are estimated very high in the second sample and in the full sample, respectively. The parameter ω is estimated significantly to be

Table 5
Estimation of the SP/SI Phillips Curve by MLE

Period	θ	ω	λ_1	λ_2	λ_3
1960:1-2004:3	0.876	0.907	0.029	0.980	-0.010
	(0.036)	(0.051)	(0.014)	0.006)	(0.006)
1960:1-1980:4	0.857	0.740	0.062	0.953	-0.037
	(0.053)	(0.139)	(0.029)	(0.018)	(0.018)
1981:1-2004:3	0.918	0.833	0.0116	1.012	-0.009
	(0.093)	(0.118)	(0.028)	0.175)	(0.019)

0.907 in the full sample. This implies that the number of sticky price firms is much larger than the number of sticky information firms. Although the sticky information firms make up only a small fraction of all firms in the economy, their existence is still statistically significant. The estimates of the parameter ω are slightly larger than the corresponding GMM estimates in the two sub-samples. Also, similar to the results obtained using GMM, the number of sticky information firms decreases in the second period, and the parameter θ is estimated smaller in the first period, which is characterized by having a high level of inflation. The implied estimates of the reduced form coefficients of real marginal cost imply that real activity in the economy affects inflation much less in MLE than in GMM.

Although the structural parameters are estimated to be higher by the MLE method, and sticky price firms dominates sticky information firms, the results still show a significant number of the sticky information type firms in the economy, especially for the two sub-samples. Therefore, the main results of this study (i.e. the existence of both types of firms is statistically and qualitatively important, and the SP/SI Phillips curve may perform better than each model) are still valid and robust to the estimation by the MLE method.

6. Conclusions

This paper develops a structural model of inflation to investigate the relative importance (or unimportance) of sticky prices and sticky information approaches in price setting and

inflation modeling. This structural model is derived by combining two different models of price setting behavior: the sticky price model of the New Keynesian literature and the sticky information model of Mankiw and Reis. In a framework similar to the Calvo model, I assume that there are two types of firms: one type of firm sets its prices optimally through forward-looking behavior, and the other type of firm sets its prices under the constraint that the information it uses is sticky. I propose the "SP/SI Phillips curve" that nests the standard sticky price and sticky information models as particular cases. This structural model of inflation is estimated by nonlinear instrumental variables (GMM) for the period 1960-2004 using unit labor cost and the output gap as alternative proxies for real marginal cost.

The results show that unit labor cost is preferred over the output gap as a measure of real activity in the economy and a proxy for the real marginal cost when inflation is modeled and estimated by the SP/SI Phillips curve. The estimation results imply that the average duration of fixed prices is six quarters, which is in the accepted range for this duration. Also, the fraction of sticky information firms is estimated significantly to be about one-third of all firms. Therefore, the main result of this study is that, although the sticky price firms make up the majority, there is a significant amount of sticky information type firms in the economy. Both types of firms are statistically and quantitatively important, and so they both should be taken into account in inflation modeling. Also, in the structural model of inflation, the pure sticky price and sticky information models are both rejected by the data, while the SP/SI Phillips curve with a dominant role for the sticky price model cannot be rejected.

The results are robust to alternative sub-samples, different numbers of lags for instrumental variables, different numbers of autocorrelation lags used for calculating the optimal weighting matrix in the GMM estimation, and estimating by MLE.

The framework of this study might be extended by including the traditional Phillips curve, which requires a direct backward-looking term into the inflation equation. Also, the inflation dynamics of the proposed SP/SI Phillips curve could be investigated in future studies.

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Appendix

A. Derivation of the Optimal Price Setting Equations

A.1. The Sticky Price Model

In each period, a fraction $1 - \theta$ of firms adjust their prices during that period, while the remaining fraction θ of firms keep their prices unchanged. Firm j tries to maximize the current value of its profit by choosing its price P_{jt}^{sp}:

$$\max_{P_{jt}^{sp}} E_t \left(\theta^k \sum_{k=0}^{\infty} Q_{t,t+k} \, \Pi_{jt+k} \right) = \max_{P_{jt}^{sp}} \sum_{k=0}^{\infty} \theta^k \, E_t \left(Q_{t,t+k} \, \left[Y_{jt+k}(P_{jt+k}^{sp} - MC_{t+k}^n) \right] \right) , \quad \text{(A.1)}$$

subject to the demand curve

$$Y_{jt} = \left(\frac{P_{jt}^{sp}}{P_t} \right)^{-\varepsilon} Y_t . \quad \text{(A.2)}$$

In (A.1), $Q_{t,t+k}$ is the stochastic discount factor and MC_t^n is nominal marginal cost. If it is assumed that all firms in this group are the same except for differentiated goods they produce and their price setting history, then the firms will choose the same price when they have the chance to adjust their prices. So the firm's subscript j can be dropped from the above equations. Since P_{jt}^{sp} stays unchanged for k periods, the above maximization problem can be written as:

$$\max_{P_t^{sp}} \sum_{k=0}^{\infty} \theta^k \, E_t \left(Q_{t,t+k} \, \left[\left(\frac{P_t^{sp}}{P_{t+k}} \right)^{-\varepsilon} Y_{t+k} \, (P_t^{sp} - MC_{t+k}^n) \right] \right) . \quad \text{(A.3)}$$

The first order condition can be obtained as:

$$\sum_{k=0}^{\infty} \theta^k \, E_t \left(Q_{t,t+k} Y_{t+k} \, \left[P_t^{sp} - \frac{\varepsilon}{\varepsilon - 1} \, MC_{t+k}^n \right] \right) = 0 . \quad \text{(A.4)}$$

The stochastic discount factor $Q_{t,t+k}$ can be found as:

$$Q_{t,t+k} = \beta^k \left(\frac{C_{t+k}}{C_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+k}} \right) , \quad \text{(A.5)}$$

and if it is plugged into (A.4), the resultant expression becomes:

$$\sum_{k=0}^{\infty} (\beta\theta)^k \, E_t \left(P_{t+k}^{-1+\varepsilon} C_{t+k}^{-\sigma} Y_{t+k} \, \left[P_t^{sp} - \frac{\varepsilon}{\varepsilon - 1} \, MC_{t+k}^n \right] \right) = 0 . \quad \text{(A.6)}$$

Let $p_t = \log(P_t/P)$ and $mc_t^n = \log(MC_t^n/MC^n)$. If the above expression is log-linearized around the steady state values of P_t^{sp} and MC_{t+k}^n, and the identity of $MC^n \varepsilon/(\varepsilon - 1) = P$ is used, the equation for the optimal price p_t^{sp} can be obtained as[25]:

$$p_t^{sp} = (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k \, E_t\{mc_{t+k}^n\}. \tag{A.7}$$

A.2. The Sticky Information model

In each period, a fraction $1 - \theta$ of firms get complete information about the economy and set their prices according to this new information, while the remaining fraction θ of firms set their prices according to old information. When a firm j sets its price at period t, it will set it to its optimal expected price according to the last information it has at period $t - k$ as:

$$P_{jt}^k = E_{t-k} P_{jt}^{si}.$$

Similarly, when a firm j sets its price at period t, it will also set it according to the expected profit given the last information it has at period $t - k$. So, its maximization problem to find the optimal price P_{jt}^{si} may be written as:

$$\max_{P_{jt}^{si}} \sum_{k=0}^{\infty} \theta^k E_{t-k} \left(Y_{jt} \left(P_{jt}^{si} - MC_t^n \right) \right), \tag{A.8}$$

subject to the demand curve

$$Y_{jt} = \left(\frac{P_{jt}^{si}}{P_t} \right)^{-\varepsilon} Y_t. \tag{A.9}$$

The maximization problem becomes:

$$\max_{P_{jt}^{si}} \sum_{k=0}^{\infty} \theta^k E_{t-k} \left[Y_{jt} \frac{(P_{jt}^{si})^{1-\varepsilon}}{P_t^{-\varepsilon}} - \left(\frac{P_{jt}^{si}}{P_t} \right)^{-\varepsilon} MC_t^n Y_{jt} \right]. \tag{A.10}$$

The first order condition for this maximization can be found as:

$$\sum_{k=0}^{\infty} \theta^k E_{t-k} \left[Y_{jt} \left(P_{jt}^{si} - \frac{\varepsilon}{\varepsilon - 1} MC_t^n \right) \right] = 0. \tag{A.11}$$

---

[25]The log-linearized equations are derived under the assumption of zero-trend inflation to make calculations simpler.

Since this expression is valid for all firms, the j subscript can be dropped. If it is log-linearized around steady state values, and by using $p_t = \log(P_t/P)$ and $MC^n = P(\varepsilon - 1)/\varepsilon$, the optimal setting price for the sticky information model can be obtained as:

$$p_t^{si} = (1 - \theta) \sum_{k=0}^{\infty} \theta^k \, E_{t-k}\{mc_t^n\} \, . \tag{A.12}$$

B. Calculation of the Past Expectational Error Terms

The C_t term in the SP/SI Phillips curve is:

$$C_t = -(1 - \theta)^2 (1 - \omega) F_t / \theta + \beta (1 - \theta)^2 (1 - \omega) E_t F_{t+1} \, ,$$

where $F_t = \sum_{k=1}^{\infty} \theta^k \, e_{tk}$, and e_{tk} is the expectational error for nominal marginal cost between periods t and $t - k$, that is $e_{tk} = mc_t^n - E_{t-k} mc_t^n$. To be able to obtain the C_t series, first the error terms e_{tk} should be found for all k, then the F_t and $E_t F_{t+1}$ can be calculated depending on the parameter value of θ.

An unconditional VAR(p) model is estimated to get the expectational error terms e_{tk} for all possible k values. This VAR model includes nominal unit labor cost, nominal non-farm business sector output, GDP deflator, commodity prices, wages in non-farm business sector, and the federal fund rates. Nominal marginal cost is represented either by nominal unit labor cost or by the nominal output gap. The residuals of the VAR model are used to obtain the e_{tk} terms.[26]

A VAR(p) model can be written as a VAR(1) model as shown below:

$$X_t = A X_{t-1} + v_t \, , \qquad X_t \equiv \begin{bmatrix} x_t \\ x_{t-1} \\ \vdots \\ x_{t-p+1} \end{bmatrix} , \tag{B.1}$$

---

[26]The estimation results in this study are obtained when C_t is computed by a VAR(6) model according to AIC. But, the computation is also performed with the order of VAR between 4 and 12, and the results are very robust to this change in the order of the VAR model.

where x_t is the vector containing the variables in the VAR. The order of the variables in x_t changes depending on whether unit labor cost or the output gap is taken as the measure of nominal marginal cost. The variable that represents the nominal marginal cost is in the top row of the x_t vector. Therefore, the residual term v_t takes the form of

$$
v_t \equiv \begin{bmatrix} \varepsilon_t \\ 0 \\ \vdots \\ 0 \end{bmatrix},
$$

where ε_t is the residual vector from the estimation of the VAR(p) model, and its first row is the residual ε_t^{mc} for nominal marginal cost.

The F_t term can be expended as:

$$
F_t = \sum_{k=1}^{\infty} \theta^k \, e_{tk} = \theta e_{t1} + \theta^2 e_{t2} + \theta^3 e_{t3} + \cdots,
$$

and to be able to calculate it, the expectational error terms e_{tk} for all possible k's need to be calculated. By using (B.1), one can write

$$
X_t - E_{t-1} X_t = v_t.
$$

The first row of the left hand side on the above expression is $mc_t^n - E_{t-1} mc_t^n$, which is the expression for e_{t1}, and it is equal to the first row of the right hand side, which is ε_t^{mc}. So, $e_{t1} = \varepsilon_t^{mc}$ for $k=1$.

By a backward substitution, X_t can be written as

$$
X_t = A^2 X_{t-2} + A v_{t-1} + v_t,
$$

$$
X_t - E_{t-2} X_t = A v_{t-1} + v_t.
$$

The first row of the left hand side on the above expression is $mc_t^n - E_{t-2} mc_t^n$, which is the expression for e_{t2}, and it is equal to the first row of the right hand side, which is $a_1 v_{t-1} + \varepsilon_t^{mc}$, where a_1 is the first row of the A matrix. So, $e_{t2} = a_1 v_{t-1} + \varepsilon_t^{mc}$ for $k=2$.

If it is continued in this way, one can obtain the e_{tk} term for any k as:

$$
e_{tk} = a_1^{k-1} v_{k-1} + \cdots + a_1^2 v_{t-2} + a_1 v_{t-1} + \varepsilon_t^{mc}.
$$

Therefore, the F_t term can be written as:

$$F_t = \sum_{k=1}^{\infty} \theta^k \sum_{i=0}^{k-1} a_1^i \, v_{t-i}$$

where a_1^i is the first row of the $(A)^i$ matrix.

In the expression for the F_t term, k can go to infinity theoretically, and so the F_t term is an infinite summation of the past expectations. But this infinite values for k must be truncated at some point, and a finite value should be chosen for an empirical estimation. Therefore, the F_t term is approximated by a finite summation as:

$$F_t = \sum_{k=1}^{k^{max}} \theta^k \sum_{i=0}^{k-1} a_1^i \, v_{t-i}$$

where k^{max} is the truncation point. The approximation error in the calculation of the F_t term due to the truncation becomes smaller when k increases for a given θ. In the calculations k^{max} is chosen as 20, so the past expectational error terms e_{tk} are calculated by going 20 quarters back in time, similar to Khan and Zhu (2006).[27]

By following similar steps, one can find the $E_t F_{t+1}$ term as:

$$E_t F_{t+1} = \sum_{k=1}^{\infty} \theta^{k+1} \sum_{i=0}^{k-1} a_1^{i+1} v_{t-i},$$

where a_1^{i+1} is the first row of the $(A)^{i+1}$ matrix.

After having the F_t and $E_t F_{t+1}$ terms, the C_t series can easily be calculated with the given values of the structural parameters.

C. Details of the MLE Method

In solving the dynamic system given in equation (14) and calculating the maximum likelihood function, the method used by Fuhrer and Moore (1995) is closely followed here. Any forward-looking dynamic system with p lags and τ leads can be put into the following form

$$\sum_{k=-p}^{0} H_k X_{t+k} + \sum_{k=1}^{\tau} H_k E_t(X_{t+k}) = \epsilon_t \,, \tag{C.1}$$

---

[27]The truncation point k^{max} is chosen as 20 after the observation that the effect of higher k values on the past expectational error terms is very small and negligible.

where X_t is vector of variables, H_k are square coefficient matrices, and the disturbance term ϵ_t is i.i.d. with $N(0, \Omega)$. Constants in the system are represented by the help of the variable ONE_t, such that $ONE_t = ONE_{t-1}$. Constants are incorporated into the system by multiplying the coefficients with the variable ONE.

Since ϵ_t is white noise, we can write the above system in a deterministic form as:

$$\sum_{k=-p}^{\tau} H_k E_t(X_{t+i+k}) = 0, \qquad i > 0. \tag{C.2}$$

If this system has a unique solution, then the AIM algorithm can compute the vector autoregressive representation of the solution path as:

$$E_t(X_{t+i}) = \sum_{k=-p}^{-1} B_k E_t(X_{t+i+k}), \qquad i > 0. \tag{C.3}$$

If the roots of the above system, that is eigenvalues of the B matrix, are less than one, then the system does not diverge and has a stable solution, and if that solution is unique, then the algorithm produces a B matrix for the system. This equation can be used to derive the expectational terms in equation (C.1) in terms of the present and past realization of the data. If these derivations are substituted into (C.1), then we can obtain the *observable structure* of the model as:

$$\sum_{k=-p}^{0} S_k X_{t+k} = \epsilon_t, \tag{C.4}$$

where the coefficient matrix S_0 contains the contemporaneous relationships among the variables. We can also obtain the reduced form of the model from the observable structure as:

$$X_t = \sum_{k=-p}^{-1} B_k X_{t+k} + B_0 \epsilon_t, \tag{C.5}$$

where $B_0 = -S_0^{-1}$.

In this study, X_t and ϵ_t vectors take the following forms:

$$X_t = \begin{bmatrix} \pi_t \\ x_t \\ s_t \\ r_t \\ cinf_t \\ winf_t \\ one_t \end{bmatrix}, \qquad \epsilon_t = \begin{bmatrix} \varepsilon_t \\ \varepsilon_t^x \\ \varepsilon_t^s \\ \varepsilon_t^r \\ \varepsilon_t^{cinf} \\ \varepsilon_t^{winf} \\ 0 \end{bmatrix}. \qquad (C.6)$$

In order to use the MLE method, an unrestricted VAR of order four is estimated, and the equations for the output gap, unit labor cost, interest rate, commodity inflation and wage inflation are combined with the SP/SI Phillips curve, which results in a system given by equation (14). This system can be written easily in the form given by equation (C.1), and can be solved by the AIM algorithm as explained above. After solving the system, the parameters set, which produces the maximum value for the likelihood function are obtained through a nonlinear optimization procedure.