

# MPRA

Munich Personal RePEc Archive

## **An Empirical Model of Factor Adjustment Dynamics**

Contreras, Juan  
Congressional Budget Office

November 2006

Online at <http://mpra.ub.uni-muenchen.de/9797/>  
MPRA Paper No. 9797, posted 01. August 2008 / 16:24

**Working Paper Series  
Congressional Budget Office  
Washington, D.C.**

**AN EMPIRICAL MODEL OF FACTOR ADJUSTMENT DYNAMICS**

Juan M. Contreras  
Macroeconomic Analysis Division  
Congressional Budget Office  
Washington, D.C.  
E-mail: [juan.contreras@cbo.gov](mailto:juan.contreras@cbo.gov)

November 2006  
2006-13

**Working papers in this series are preliminary and are circulated to stimulate discussion and critical comment. These papers are not subject to CBO's formal review and editing processes. The analysis and conclusions expressed in them are those of the authors and should not be interpreted as those of the Congressional Budget Office. References in publications should be cleared with the authors. Papers in this series can be obtained at <http://www.cbo.gov/publications/>.**

# An Empirical Model of Factor Adjustment Dynamics

Juan M. Contreras<sup>1</sup>  
Congressional Budget Office  
November 10, 2006

## Abstract

This paper investigates how firms dynamically adjust their use of capital, labor, energy, and materials when there are both smooth and lumpy adjustment possibilities and interrelation among adjustments. The Colombian Annual Census of Manufacturing provides evidence of these kinds of adjustment. The innovation of this paper lies in three areas: in considering the joint adjustment and interrelation of labor and capital at the establishment level; in describing the dynamic adjustment of all the production factors; and in a rich description of adjustment costs, which includes disruption of the production process and reallocation of internal resources, and fixed costs of installing capital and creating or discontinuing a job vacancy. The model also includes both a convex cost component, aimed at capturing smooth adjustments, and congestion effects, which means that it is more costly for firms to adjust capital and labor at the same time than it is to adjust them separately. Using a simulated method of moments, the study finds empirical support for the existence of disruption costs for capital and labor, the existence of convex costs for capital but not for labor, and the existence of congestion effects. An important implication of the model is that, in response to shocks, firms decide to adjust either capital or labor alone or both, depending on the initial capital to labor ratio and the magnitude of the shocks.

JEL CODES: E22, E24, J23, D24, L60

Key Words: Factor adjustment, capital and employment adjustment, simulated method of moments, capital and employment interaction adjustment costs.

---

<sup>1</sup>This paper is based on the first chapter of my Ph.D. thesis at the University of Maryland. I want to thank John Rust and John Haltiwanger for their help and support and John Shea for detailed comments on previous versions. Discussions with Iñez Tristao, Alex Whalley, Mike Pries, and Bob Dennis and comments from participants at several seminars helped me to improve this paper. I also want to thank Marcela Eslava for helping provide access to the data, and the Central Bank of Colombia for financial support. The ideas expressed in this paper are those of the author and should not be interpreted as those of the Congressional Budget Office.

Author e-mail: [juan.contreras@cbo.gov](mailto:juan.contreras@cbo.gov).

# 1 Introduction

It was not until a decade ago, with the availability of new firm-level data sets, that lumpiness and infrequent adjustment in capital and labor<sup>2</sup> could be observed in firms' behavior; this was in stark contrast to the smooth adjustment shown in aggregate data for those factors.<sup>3</sup> At the same time, models attempting to explain the aggregate behavior of these variables had to be revised to account for the microeconomic facts if models with micro foundations were to be useful in terms of policy and predictions at the firm and aggregate levels. However, previous studies of adjustment have tended to analyze one factor at a time, which has made it difficult to understand the joint adjustment of capital and labor at the micro level and its macroeconomic implications.

Looking for a better understanding of firms' factor adjustment behavior, this paper analyzes to what extent it is important to consider joint capital and labor decisions at the firm level from an empirical and theoretical point of view. Specifically, it asks whether firms adjust labor independently of capital, if there exist interactions in this adjustment, and what the nature of such interrelations is. Even if these questions are not new, there is no satisfactory explanation of the interrelation between capital and labor adjustments at the firm level and none of the previous models have been able to explain the factor adjustment dynamic patterns, in part because the existence of this type of empirical evidence is very recent and not complete for all the production factors, and in part because of the difficulty in the analysis and estimation of such a model (analytically or numerically). In this paper, although capital and labor movements are the main focus, I also incorporate materials and energy adjustments in the analysis.

The implicit view in the early investment<sup>4</sup> and labor demand literature was that profit-maximizing firms would adjust factor demand constantly in response to shocks in demand and productivity. It was natural to think that way since the aggregate

---

<sup>2</sup>The term labor is used in this paper to indicate the number of workers and is synonymous with employment.

<sup>3</sup>In order to clarify terms, lumpiness refers to coexistence of inaction and large adjustments with little in between; this is the opposite of a smooth adjustment that occurs when the adjustment is done in a continuous way. The main distinctive feature between them is the existence of long inaction periods and large adjustments in the case of lumpy adjustments, and the nonexistence of inaction periods in the case of smooth adjustments; note, however, that one might observe large and smooth adjustments at the same time.

<sup>4</sup>See Hall and Jorgenson (1967), Tobin (1969), and Abel (1979), among others.

series were smooth, and the literature instead focused on issues such as the role of the cost of capital, the serial correlation in investment, and the aggregate dynamics of labor demand. Recent firm-level empirical studies have shown, however, that firms do not adjust as often as they should under convex costs, and when they do adjust, this response comes often simultaneously from adjustment in several margins. Moreover, new and emerging evidence, such as that presented in this paper, shows that capital and labor adjustment distributions have fat tails, a mix of small and large adjustments, and a mass point around the inaction region, all of which has been taken as evidence of lumpy adjustment.<sup>5</sup> The recent evidence also shows that these adjustments are interrelated.<sup>6</sup>

Subsequent models of investment and labor demand made efforts to incorporate this emerging empirical evidence and especially the lumpy and infrequent adjustment.<sup>7</sup> An important point they neglect, however, is that firms adjust not just along one but along several margins, particularly for capital and labor. The few studies that consider capital and labor together either do so at an aggregate level, which does not exploit the rich and heterogeneous adjustment observed at the micro level<sup>8</sup>, or does not account for the correlation among adjustments.<sup>9</sup> As a consequence, estimated parameters governing the adjustment of capital and labor in response to shocks may be biased<sup>10</sup>.

---

<sup>5</sup>Davis and Haltiwanger (1992), Davis et al. (1996), Caballero et al. (1995, 1997), and Cooper and Haltiwanger (2005) have shown the mentioned patterns in the separate analysis of labor and capital adjustment distributions using micro-level data. Rust (1987) shows the lumpy nature of the adjustment for a single agent in the case of machine replacement.

<sup>6</sup>Nadiri and Rosen (1969), using aggregate data, and Sakellaris (2004), Eslava et al. (2004) and Narazani (2004), using micro data, find that the capital and labor adjustments are interrelated.

<sup>7</sup>These facts were incorporated in the early theoretical literature as models of infrequent adjustment, while later models also attempted to reproduce the lumpiness in such adjustments. See for example Dixit and Pindyck (1994) and Abel and Eberly (1994, 1998) in the case of investment, and Hamermesh (1989) and Hamermesh and Pfann (1996) in the case of labor. More recent work includes Cooper and Haltiwanger (2005) and Cooper et al. (2004), who consider also the small and smooth adjustments in conjunction with episodes of large and infrequent adjustments.

<sup>8</sup>See Shapiro (1986) and Hall (2004).

<sup>9</sup>Caballero et al. (1995, 1997), Cooper and Haltiwanger (2005), and Cooper et al. (2004) analyze capital and labor adjustment in a separate way. Rust (1987) analyzes investment for a single firm. Abel and Eberly (1998) model capital and labor adjustment but do not take into account the interactions between them. Rendon (2005) features a model with a simpler adjustment cost structure, to determine whether liquidity constraints restrict job creation. Bloom (2006) analyze capital and labor decisions but neither incorporates the interaction among adjustments nor the disruption in productivity, both crucial features in explaining the input factor movements as proposed in this paper

<sup>10</sup>The biases arise because the models analyze the response of either capital or labor to shocks, incorporating all the other factor decisions into the shocks. For example, movements in one factor such as capital would affect the shock in a labor adjustment model, and the real response of labor to shocks would be overstated.

The lack of understanding of factor adjustment at the firm level has important implications. For example, higher firing costs may affect capital formation. Or a policy aiming to increase investment may not be as effective as expected if hiring/firing costs are unaffected or if the elasticity of factor use with respect to its relative price is not properly estimated. Nonconvexities arising from adjustment costs, understood as infrequent and lumpy movements resulting from discrete payments or reductions in productivity during adjustment, may lead to very different industry responses to public policies aimed at job creation, destruction, or investment. It is necessary to estimate those responses in a consistent and realistic way, not only for the sake of predicting the effects of public policies but also because the behavior of aggregate investment and job creation and destruction is directly affected by the microeconomic response of firms to shocks in demand and technology.

In exploring these issues, this paper uses firm-level data from the Colombian Annual Manufacturing Census, covering the period 1982 to 1998. This is a unique data set because it contains firm-level data on the value of production, energy and materials, prices for each product and each material used, the number of workers and payroll, and book values of equipment and structures. The existence of firm-level prices opens a wide range of empirical possibilities;<sup>11</sup> in this paper they are useful because they allow the precise identification of technology and demand shocks and of input factor elasticities and considerably reduce the measurement error in factor demand due to confusion between prices and quantities.<sup>12</sup>

In line with recent studies, the empirical analysis reveals the mix of small and large adjustments in the capital and labor adjustment distributions (which also present fat tails and large inaction periods) and their interrelated nature. Going beyond these studies, the analysis reveals the adjustment patterns in energy and materials and their relation to capital and labor adjustments. The picture that emerges is that, in response to shocks, firms adjust capital and labor in a nontrivial, interrelated way. Firms also adjust energy and materials when they are hit by demand and technology shocks. This is not surprising if we think that firms face a profit maximization problem over all inputs. What is

---

<sup>11</sup>Like the analysis of price stickiness or the effects of technology shocks in the business cycle.

<sup>12</sup>Note that while other studies, dealing with different issues, have used a similar Colombian data set, all but Eslava et al. (2004, 2005) use a shorter period (up to 1991) and do not have price information.

interesting is the observed dynamics of the adjustment.

The data analysis undergirds the main contribution of this paper, which is to propose, analyze, and estimate a theoretical model of firm behavior that combines a labor decision problem with a machine replacement problem along with a rich specification of adjustment costs. The proposed adjustment cost structure captures key features of the data such as reductions in output and reallocation of resources during adjustment (disruption costs), the cost of installing capital and creating or destroying a job vacancy (fixed costs), and a convex cost component introduced to capture the observed mix of smooth and lumpy adjustment<sup>13</sup>.

Another key feature in the model is the presence of interaction effects in the adjustment of capital and labor. Interaction effects are precisely defined as the extra cost or benefit<sup>14</sup> of jointly adjusting capital and labor. If the interaction effect is a cost I call it a congestion effect, and if it is a benefit I call it a complementarity effect. For example, the interaction effect may be present as congestion if firms have to train new workers to operate new machines, thus incurring in an extra cost. Another example of interaction effects would be if disruption in the production process occurs while incorporating new workers and machines simultaneously, incurring production losses that may be greater (congestion) or less (complementarities) than if hiring new workers or making new investments independently. Take the case of a company expanding an existing plant. The company has the option of building the additional space, investing in new machines and hiring and training all the needed workers at the same time; or it has the option of buying the machines, expand the shift of some existing workers to operate the new machines and hiring later more workers as needed. If the first option is more expensive than the second one (due for example, to higher drops in productivity) there is a congestion effect; otherwise there is a complementarity effect.

The dynamic nature of the optimization problem helps explaining why is that, even if

---

<sup>13</sup>An important point not addressed here relates to the current debate about the production factors' response to technology shocks. Besides the serious identification issues faced in the literature that tries to estimate technology shocks (see Alexopoulos (2004) and her references), previous work has not considered the possibility that, while adjusting, firms decrease production, suggesting the presence of an adjustment cost in productivity shock estimates that may result in misleading conclusions about the effects of pure technology shocks on factor adjustment.

<sup>14</sup>The model proposed in this paper does not restrict this effect to be a cost or a benefit. It is the empirical analysis that determines it as a cost.

there is a congestion effect in the joint adjustment, firms are willing to pay this cost: if firms optimally adjust both factors at the same time, they can earn more profits today and in the future. Firms decide what to adjust comparing the foregone profit due to the sub-optimal level of capital or labor with the cost of the joint adjustment.

After observing in the data the adjustment patterns for the input factors and setting up a model with the features mentioned above, the paper goes on exploring if the decision rules generated by this model are able to produce similar firm behavior. In this initial stage, the adjustment cost parameters are arbitrarily chosen with ex-ante plausible values. This part of the paper is the core analytical contribution because it gives an idea of how labor and capital adjustments interact (or not) at the firm level, depending on the capital labor ratio a particular firm has.

In a second stage, these adjustment cost parameters are estimated with a minimum distance algorithm in order to match key moments that comprehensively describe firms' adjustment patterns in capital and employment. The model that best fits the data includes the interaction term in the adjustment costs for capital and labor, present as a congestion effect, which suggests that investment influences hiring decisions in an important way (and vice versa) because of the extra cost that this joint decision implies. The congestion effects are key, especially to match the contemporaneous correlation between capital and labor adjustment. The structural methodology allows me to reject statistically the existence of fixed costs and to accept the existence of disruption costs for capital and labor, the existence of convex costs for capital but not for labor and the existence of congestion effects. Finally, the benchmark model of convex costs, widely used in the macroeconomic literature, is not able to explain by itself the type of behavior observed in firms's investment and employment decisions.

The paper proceeds as follows. Section 2 presents the empirical evidence, which is descriptive and with minimal structure imposed on the data. Section 3 introduces the model used to explain the empirical patterns. Section 4 presents the solution method and analyzes the decision rules emerging from an illustrative parametrization of the model. Section 5 estimates the parameters governing the joint adjustment of capital and labor and describes tests of the goodness of fit in the estimated parameters. Section 6 concludes.



## 2 Factor Adjustment: Facts from the Microdata

Because the variables of interest are adjustments, I look at the gross investment rate and at the growth in demand for labor, energy, and materials. I start by showing the distribution of factor adjustments, then present basic correlations among them and analyze the conditional probability of inaction or adjustment<sup>15</sup> during large adjustment and inaction episodes.<sup>16</sup> Finally, I run a VAR(1) to get a sense of the magnitude of the dynamic interactions between factor adjustments at the firm level for the whole range of inputs. These empirical exercises are built around the issue of interest, which is the interrelation between capital and employment adjustment.<sup>17</sup>

The distributions show a mix of small and large adjustments of capital and labor with large inaction periods at the firm level; this has been interpreted in the literature as lumpy adjustment, but it misses the point that infrequent adjustment periods can be followed by smooth and small adjustment periods. For materials and energy the adjustment is more continuous, but they show also a mix of small and large adjustments; in this sense, their adjustment is not lumpy as that term is used to describe capital and labor adjustment. This micro evidence is in contrast to the smooth aggregate series that have been extensively analyzed in the literature. The contemporaneous correlations, and the analysis of the episodes of large adjustments and inaction, show that these adjustments indeed have a statistically significant degree of interrelation. The VAR(1) aims to show that the dynamic interrelation between factor adjustments holds during

---

<sup>15</sup>I also carried out additional analyses of labor and capital dynamics during periods of large adjustments. In particular, I used simple correlations among the incidence of the episodes and a variation of the methodology of Sakellaris (2004) and Letterie et al. (2004) to analyze the interrelated adjustment between capital, labor, energy, and materials. These analyses confirmed the results in this section, namely the interrelated adjustment, the mix of small and large adjustments, and a mix of smooth and lumpy factor adjustments. The results also show evidence of adjustment costs based on the observation of the decrease in productivity and output after periods of adjustment, especially in the case of capital, suggesting a cost in terms of forgone profits.

<sup>16</sup>The criteria used to define inaction and large episodes are discussed later in the paper.

<sup>17</sup>This section is in the same spirit as other recent work showing evidence on the interrelation between capital and labor adjustment using micro data. Narazani (2004) focuses on a small subset of large Italian firms to study capital and labor adjustment. Sakellaris (2004) shows evidence on the interrelation in factor adjustment in episodes of considerable adjustment in capital and labor for the U.S. Letterie et al. (2004) analyze this adjustment for Danish firms and Polder and Verick (2004) compare the adjustment dynamics in Denmark and Germany. Eslava et al. (2004) use the same data set used in this paper and employ the methodology of Caballero et al. (1995, 1997) to analyze the nonlinear interrelation between capital and labor adjustment. None of these studies, however, contain the structural approach and comprehensive analysis that this paper presents.

all episodes of adjustment.

While the main focus of this paper is the adjustment of capital and labor, I also consider energy and materials adjustment in order to motivate the assumption below that these factors are adjusted at no cost. This initial exploration of the facts is the basis for the factor adjustment model presented later.

## 2.1 Data

The data come from the Colombian Annual Manufacturing Census (AMS) during the period 1982-1998. The AMS is an annual unbalanced panel of around 13,000 firms with more than 10 employees or sales above a certain amount. It contains the values of production, materials, and energy consumption; physical quantities of energy; prices for each product and material used; production and nonproduction workers and payroll; and book values of equipment and structures. I use the panel of pairwise continuing firms constructed by Eslava et al. (2004), which accounts for a total of 2,167 firms in the period 1982-1998. I choose to work with a balanced panel because I do not analyze the effects of firm entry or exit. In this section I describe how the variables were constructed. For more information about the construction of the variables, see Eslava et al. (2004).

Price-level indices are constructed for output and materials using Tornqvist indices, which are the weighted average of the growth in prices for all individual products (or materials) generated (or used) by the plant. The weights are the average of the shares in the total value of production (or materials used). Formally, the index for each plant  $j$  producing outputs (or using materials)  $h$  in year  $t$  is:  $\ln P_{jt} = \ln P_{jt-1} + \Delta P_{jt}$  with  $P_{j1982}$  as the base year,  $\Delta P_{jt} = \sum_{h=1}^H \bar{s}_{hjt} \Delta \ln(P_{hjt})$  as the weighted average of growth in prices for all products  $h$ , and  $\bar{s}_{hjt} = \frac{s_{hjt} + s_{hjt-1}}{2}$  as the simple average of the share of product (material)  $h$  in plant  $j$ 's total value of production (materials usage).

Quantities of materials and output are constructed by dividing the reported value by the prices. Energy quantities and number of workers are reported by each plant. Investment represents gross investment and is generated from the information on fixed assets reported by the plants. Specifically, gross investment is calculated recursively with the formula  $I_{jt} = K_{jt}^{NF} - K_{jt}^{NI} + d_{jt} - \pi_{jt}^A$  where  $K_{jt}^{NF} - K_{jt}^{NI}$  is the difference in the value of the fixed assets reported by plant  $j$  at the end and beginning of year  $t$

and  $d_{jt} - \pi_{jt}^A$  is the depreciation minus the inflation adjustment reported by plant  $j$  at the end of year  $t$ . In the rest of the paper, all the variables are in logs unless otherwise indicated. The reported growth is the log difference, and the observations are considered outliers if they are greater than 10 (adjustment of 1,000%) or lower than -1 (adjustment of -100%). Finally, an observation is defined as the adjustment of firm  $j$  in year  $t$ .

## 2.2 Factor Adjustment: Distributions, Correlations, and Dynamic Interrelation

*Distributions of Factor Adjustments:* Table 1 and figure 1 illustrate the distribution of factor adjustments (capital, employment (number of workers), energy, and materials) for all continuing Colombian plants from 1982 to 1998. I will analyze these distributions in terms of the frequency, symmetry, and size of the adjustments.

With respect to the size of adjustments, there is no standard definition for large or small adjustments, but a visual inspection of the capital and labor adjustment distributions shows a combination of small and large values with a mass point around the inaction region (i.e., zero adjustment). The existence of these mass points around zero adjustment and the fat tails of the distributions can be interpreted as lumpy adjustment.<sup>18</sup> For example, in table 1, the lumpy pattern can be observed in the proportion of adjustments above 20%, considering this number as a large adjustment, compared to the proportion of adjustments lower than 1% in absolute value, considering this as the inaction region; on the other hand, smooth adjustment can be observed in the proportion of adjustments below 20% and above the inaction region. Materials and energy adjustment distributions are not considered lumpy under this criterion since the percentage of observations in the inaction region is not very high and the medium-size adjustments (those between 1% and 20%) represent an important number of observations, meaning more continuous adjustment.

Perhaps a more standard measure of how many extreme observations are observed in a distribution is the excess kurtosis. In the case of input factor adjustments, all distributions have large excess kurtosis, indicating that many observations are far from

---

<sup>18</sup>Recall that lumpy adjustment is defined here as the coexistence of inaction and large adjustments with little in between, and a smooth adjustment is defined as a continuous one; see footnote (3).

the mean.<sup>19</sup> Specifically, even excluding the outliers greater than 2 (adjustment of 200%), the excess kurtosis measures are large and equal to 7.65, 7.96, 2.2, and 3.85, respectively.

These numbers indicate fat tails in the distributions of adjustment and a clear lumpy adjustment pattern for capital and labor because of the existence of a mass point around the inaction region; in the case of materials and energy, as explained above, it indicates the existence of large adjustments but not of a lumpy pattern as there is no mass point around the inaction region.

In the capital adjustment distribution, large adjustments may, for example, reflect large investments in machinery installed in one year or less. The inaction region may be due to small capital investments or to real inaction periods, like the period it takes to make the decision to build a plant, or the time a big machine lasts without replacement. The smooth adjustment in capital can come from continuous medium-size investments or from continuous investment in a project, like the time it takes to build a plant. For example, a firm may take 5 years to decide to build a new plant (inaction region) and start building it (lumpy adjustment) during a 3-year period (smooth adjustment). In the case of employment, lumpy adjustment may reflect a massive firing period.

Figure 1 shows a very asymmetric investment distribution, with very little negative investment, suggesting high irreversibility of capital in Colombia. The labor adjustment distribution is much more symmetric. Putting together those two distributions, we can conclude that it is easy for firms to reduce employment but not capital, perhaps because of differences in the adjustment costs involved for each input factor (i.e., the selling price of capital is lower than the buying price). Materials and energy distributions are also symmetric, though not as much so as the employment distribution.

Firms often leave capital and employment essentially fixed but adjust materials and energy more frequently. This is the justification for the assumption in the model that materials and energy are not subject to adjustment costs. The evidence for Colombia is in line with the evidence for the U.S. using the Longitudinal Research Database, where lumpiness in capital and employment is also present together with significant periods of smooth adjustment. This evidence alone suggests the presence of adjustment costs and,

---

<sup>19</sup>The normal distribution has an excess kurtosis of zero. The fatter the tails of a distribution, the bigger the excess kurtosis.

as many others have pointed out, may indicate (S,s) behavior in capital and employment adjustment.

Summarizing the information observed in the distributions, we conclude that: (i) the distributions of input factor adjustment have fat tails; (ii) the investment distribution is asymmetric, showing a large degree of irreversibility, while the labor adjustment distribution is highly symmetric; (iii) capital and employment adjustments are infrequent (large mass point around zero adjustment) but materials and labor adjustment are much more frequent (no mass points around zero); and (iv) small and large values coexist in the distributions of adjustment for all the factors. Points (i) and (iii) signal the existence of lumpiness in the adjustment of capital and labor due to non-convex costs in capital and labor adjustment

***Correlations of Factor Adjustments:*** Table 2 shows the contemporaneous correlations among factor adjustments,<sup>20</sup> which are all statistically different from zero and give a sense that capital and employment adjustment periods are interrelated.

It is worth analyzing the patterns of adjustment during inaction periods, during spike episodes, and during a combination of both to further explore the correlations among factor adjustment.<sup>21</sup> The question is whether inaction or spikes in one factor increase the probability that firms adjust the other factor. Equivalently, we could ask if firms stagger capital and labor adjustments; if they do, we will observe an increase in the probability of adjustment conditional on inaction in the other factor.

Table 3 explores the probabilities of inaction/spike in one factor conditional on inaction/spike in the other factor; it shows the results of estimating four different logit models, one each for investment and employment and for inaction and spikes, where the dependent variable is the probability of adjustment or inaction in one factor and the

---

<sup>20</sup>The correlations are calculated regressing adjustment on time dummies to take out business cycle effects, and I consider only the residuals, which reflect firm-level shocks. Another interesting possibility would be to consider the business cycle effects as well, but this is left for future work.

<sup>21</sup>The inaction and spike episodes are defined as above: less than 1% in absolute value for inaction and more than 20% in absolute value for the spikes. Even though the definition of inaction and spikes given here may be somewhat arbitrary and may capture adjustments that are not small or large for certain types of capital (for example, many firms need small tools important for production that signal a positive investment but lower than 1% of their capital stock), it is useful to characterize the behavior of capital and employment around “small” and “large” episodes of adjustment.

independent variables include the adjustment and inaction<sup>22</sup> status in the other factor. The logit estimation includes controls for firm-specific variables such as total factor productivity (TFP) and demand shocks<sup>23</sup> and the adjustment of energy and materials. It is important to control for the shocks because they lessen the chances that the comovement between employment and investment is merely due to an omitted third factor. Moreover, controlling for the TFP and demand shocks identifies the movements in employment and capital as dependent not only from shocks but also from other sources, in this case interpreted later as adjustment costs.

From table 3 it can be observed that the probability of inaction in investment increases if there is inaction in employment. The reverse is also true, but the effect is not statistically significant. At the same time, the probability of an investment spike increases if there is an employment spike, and the probability of an employment spike increases when there is an investment spike. These numbers show that capital and labor tend to move together. The probability of inaction in employment decreases when there is an investment spike. The other effects are not statistically significant. All these numbers suggest that, on the one hand, when there is a large adjustment in either capital or employment, it is more likely that firms make large adjustments in both factors; on the other hand, when firms do not adjust employment, it is more likely that they do not adjust capital, but when firms do not adjust capital, it does not necessarily mean that they do not adjust labor.

The analysis in table 3 examines the case of large changes in capital and labor. A natural question that follows is whether this analysis extends to all adjustments in a dynamic context. I discuss the basic empirical approach for this problem next.

***Factor Adjustment Interrelation: Dynamic Dependence:*** Table 4 presents the coefficients of a simple VAR with one lag, to show how firms' factor adjustments are dynamically interrelated. The coefficients and their statistical significance are quite robust to several controls and for year effects.<sup>24</sup> Table 4 shows that an increase in labor demand signals a posterior investment episode (the coefficient of the effect of lagged labor growth on investment is positive and statistically significant). Moreover, the coefficient

---

<sup>22</sup>Both dummies are included at the same time.

<sup>23</sup>The estimation of these shocks is explained later in the paper in the calibration section.

<sup>24</sup>Controls include shocks in demand and productivity estimated as explained in section 4.1.

of lagged investment in the labor equation is negative and significant. F-tests for the cross coefficients of labor and capital in this VAR with 4 variables and in a simpler version with just capital and employment were run to verify Granger causality. Both coefficients are different from zero, which does not give much more information since it is not conclusive about which one causes the other.

The VAR indicates that the firms use several margins of adjustment in capital, labor, materials, and energy. Moreover, the diagonal elements (the autocorrelation) for all factors are negative, suggesting that if firms adjust in one period it is very likely that either they will not do so the next period or they will adjust in the opposite direction. The negative autocorrelation in the VAR is a reflection of the patterns from the distribution of adjustments. Capital is the factor with smaller negative autocorrelation in adjustment, which may signal inaction in the following period.<sup>25</sup> Materials and energy are the factors with higher negative autocorrelation in adjustment, which may signal instead free adjustment in the opposite direction the following period.

The empirical evidence presented above shows the infrequent nature and the mix of smooth and lumpy adjustment in capital and labor, and indicates a dynamic dependence in these adjustments. Notice that the analysis is carried over the whole population of firms, and not just for a subset as in most previous studies. In the following section, I set up a model that aims to explain the patterns observed in the data, in particular the infrequent and lumpy adjustments and the interrelation among factor adjustments.

### 3 A Dynamic Model of Firms' Factor Adjustment

The main features of the model are the presence of convex and nonconvex adjustment costs in capital and labor (but not in the other factors) and the possibility of mutual interaction effects in the form of congestion (if more costly) or complementarities (if cheaper) in the adjustment process through different adjustment costs if the firms

---

<sup>25</sup>This follows because the distribution show a mass around zero and the negative coefficient signals inaction or adjustment in the opposite direction. It is worth mention also that the investment rate autocorrelation coefficient changes sign when controlling for individual firm characteristics through fixed effects (it is positive when fixed effects are not present, showing a similar coefficient to that of the simple contemporaneous correlations). This suggests that unobservable characteristics are important and that the simple autocorrelation observed before in table 2 may be a result of aggregation effects more than firm-level effects.

adjust capital and labor independently or at the same time. As stated previously, non-convexities in the decision problem coming from the adjustment costs cause jumps or infrequent movements in firms' factors, and interaction effects are defined as changes in cost due to joint adjustment.

The driving forces of factor adjustment in the model are technology and demand shocks, but not factor price shocks; this is assumed for simplicity. Simplicity also leads me to assume symmetry in the adjustment costs. Another important assumption is the lack of inventories as an adjustment variable. Factor price effects, symmetry of the adjustment costs, and inventory adjustments are interesting by themselves and left for future work. For notation purposes, the subscript *it* is dropped.

### 3.1 Basics

***Demand and Production Function:*** There is imperfect competition and firms face a downward sloping demand curve  $Q^d = \left(\frac{P}{X}\right)^{-\psi}$ , where  $X$  is a stochastic shock to demand,  $\psi$  is the price elasticity of demand, and  $P$  is the price level.

The production function incorporates capital, labor, materials, and energy. Capital and labor are costly to adjust, while materials, energy, and hours<sup>26</sup> per worker can be adjusted at no cost. In this context, hours can be thought of as a form of labor utilization and, while not explored in the model, energy is likely to be correlated with capital utilization. The assumption here is that all firms have the same Cobb-Douglas production function and that elasticities and factor shares do not vary by sector.<sup>27</sup> This function does not represent an aggregate production function but instead the production function of each firm. If we were to assume heterogeneity in the production function, it would introduce too much complexity in the problem and a separate analysis would have to be done for each firm or industry. Since I am interested here in the average behavior of firms, this functional form seems to be the one that can characterize the greatest number of firms. Formally,  $Q^s = Bk^\chi (lh)^\alpha e^\xi m^\nu = Ak^\chi l^\alpha$ , where  $\chi, \alpha, \xi$ , and  $\nu$  are the input factor elasticities for capital, labor, energy, and materials,  $A = Be^\xi m^\nu h^\alpha$ ,  $B$  is a productivity shock,  $k$  is the capital level,  $l$  is the stock of workers,  $h$  represents hours

---

<sup>26</sup>When firms adjust employment instead of hours, it is because of the wage premia for overtime.

<sup>27</sup>In future work I plan to explore the role of heterogeneity in the elasticities.



per worker,  $e$  is energy consumption and  $m$  is the materials level. It is convenient for notation to cluster the terms for hours, energy, materials and productivity in the term  $A$ , since they are optimally chosen period by period in a static maximization problem and there are no intertemporal links in their case.

**Revenue Function and Profit:** Putting together the demand equation and the production function where  $Q^s = Q^d$ , we get the revenue function:

$$R(\bar{z}, k, l) = \bar{z}k^{\hat{\theta}}l^{\hat{\mu}} \quad (1)$$

where  $\bar{z} = XA^{(1-\frac{1}{\psi})}$ ,  $\hat{\theta} = \chi \left(1 - \frac{1}{\psi}\right)$ , and  $\hat{\mu} = \alpha \left(1 - \frac{1}{\psi}\right)$ . This type of revenue function is used in most of the literature because of the lack of firm-level information on prices. It has a demand component present in the terms  $X$  and  $\psi$ , making it hard to identify separate from the technology shock in  $A$ , even if estimated at the firm level. For the Colombian Annual Census of Manufacturing, however, firm-level prices are observed. In equation (1), firms implicitly account for the effects of their input choices on output prices when maximizing profit. The availability of firm-level prices allowed Eslava et al. (2004) to obtain arguably unbiased estimates of the input elasticities and the elasticity of demand, which I utilize in this paper.

Profit incorporates the cost of all the inputs, including the adjustment cost which will be defined more formally later in this section. For now, profit is given by  $\Pi(z, l_{-1}, l, k, k') = zk^{\theta}l^{\mu} - w(l) - C(z, l_{-1}, l, k, k')$ , where  $w(l)$  is the payment to employment,  $C(\bullet)$  is the adjustment cost,  $z$  is a term that incorporates technology and demand shocks together with price shocks in materials and energy. Note that the input factor elasticities are given by  $\theta = \hat{\theta} * M$  and  $\mu = \hat{\mu} * N$ , where  $M$  and  $N$  are terms that incorporate the elasticities of materials and energy coming from the static optimal firm choice for hours, materials and energy.<sup>28</sup>

---

<sup>28</sup>Without this simplifying notation, the complete expression including materials and energy would be  $\bar{\Pi}(\bullet) = \bar{z}k^{\hat{\theta}}l^{\hat{\mu}} - l(w_0 + w_1h^{\zeta}) - C(z, l_{-1}, l, k, k') - p_e e - p_m m$ . Note the functional form assumed in the wage equation representing a base payment plus a payment for the hours, where  $\zeta$  is the hours-wage elasticity. The main difference in both equations is the term for the shocks,  $\bar{z}$ , and the explicitness of the energy and material prices,  $p_e e - p_m m$ . In the main text, the term  $z$  captures both the shocks  $\bar{z}$ , and the prices and levels in materials and energy coming from the term  $p_e e - p_m m$ . This notation is possible since the firm solves a static optimization problem in energy and materials every period that can be characterized in the term  $z$ .

**Firms' Decision Problem:**<sup>29</sup> Firms determine new employment levels into account the employment level in the previous period. Newly hired workers become productive in the same period. However, new investment becomes productive only in the next period. In this sense there is a “time to build” for capital but not a “time to hire” for employment<sup>30</sup>. Firms also choose hours, energy, and materials and can adjust them at no cost. Formally, the firms' problem is:<sup>31</sup>

$$\begin{aligned}
 V(z, k, l_{-1}) &= \max_{k', l} \left\{ \Pi(z, l_{-1}, l, k, k') + \beta \int V(z', k', l) f(z'/z) dz' \right\} \\
 &= \max_{k', l} \{ z k^\theta l^\mu - w(l) - C(z, l_{-1}, l, k, k') \\
 &\quad + \beta \int V(z', k', l) f(z'/z) dz' \} \tag{2}
 \end{aligned}$$

where  $(\cdot)$  means next period value. The investment is implicitly defined in this equation and given by  $I = k' - (1 - \delta)k$ , where  $\delta$  represents depreciation.  $C(\bullet)$  is the cost of adjustment, which takes different parameter values depending on whether the firm adjusts employment, capital, or both.  $\beta$  is the discount factor, and the integral term represents the expected value of the firm subject to shocks  $z$  which include demand and technology shocks (which I assume are independent).<sup>32</sup>

The cost of adjustment  $C(\bullet)$  potentially includes disruption costs, fixed costs, and convex costs. The existence of a disruption cost taking the form of lower productivity in adjustment periods is justified by the findings of Contreras (2006), Power (1998) and Sakellaris (2004). The disruption cost can also be associated with the stochastic adjustment cost seen in Caballero and Engel (1999), and may be caused by the reallocation of resources while adjusting.

The convex cost term does not have a clear micro foundation but has been assumed

---

<sup>29</sup>The model proposed is similar in several dimensions to the ones used by Abel and Eberly (1998), Cooper et al. (1999), Cooper and Haltiwanger (2005), and Cooper et al. (2004).

<sup>30</sup>This reflects that it is easier for firms to adjust employment, while adjusting capital may take more planning time (for example to buy and install a new machine or build a plant).

<sup>31</sup>The notation implicitly states that firms optimally choose energy, materials and hours in a static maximization problem. These values are embedded in the term  $z$ .

<sup>32</sup>This assumption would not hold if the productivity shocks were not idiosyncratic but instead common, and TFP would be correlated with demand. Since the estimations of the shocks in this paper take out the aggregate effects, this effect is mitigated. This assumption has been used before by Syverson (2005) to estimate production functions and in the recent discussion about the relevance of VAR to business cycle analysis.

to exist in previous literature because of the smooth adjustment observed at the macro level. The distributions shown in section 2 also present regions of smooth and lumpy adjustments as discussed previously. One of the main advantages of the modeling mechanism used in this paper is the ability to identify the importance of such a cost at the firm level. The fixed adjustment cost can be seen as representing installation costs (in both time and resources) in the case of capital and firing and hiring costs in the case of labor.<sup>33</sup>

Specifically, the functional form assumed for the adjustment costs when firms adjust  $l$  (labor),  $k$  (capital), or  $lk$  (labor and capital), is the following:

$$C(z, k, I, l, l_{-1}) = \begin{cases} C^l = \lambda_l R(\bullet) + F_l l_{-1} + \frac{\gamma_l}{2} \left(\frac{\Delta l}{l_{-1}}\right)^2 l_{-1} & \text{if } \Delta l \neq 0 \\ C^k = \lambda_k R(\bullet) + F_k k + \frac{\gamma_k}{2} \left(\frac{I}{k}\right)^2 k + p_I * I & \text{if } I \neq 0 \\ C^{kl} = C^l + C^k + \lambda_{kl} R(\bullet) + F_{kl} \sqrt{l_{-1} k} \\ \quad + \frac{\gamma_{kl}}{2} \left(\frac{I}{k}\right) \left(\frac{\Delta l}{l_{-1}}\right) \sqrt{l_{-1} k} & \text{if } \Delta l * I \neq 0 \end{cases} \quad (3)$$

If the firm adjust either capital or labor (at a cost  $C^k$  and  $C^l$  respectively) the first term in the cost equations,  $\lambda_j R(\bullet)$ , represents the disruption cost, the second term involving  $F_j$  represents the fixed cost and the third term involving  $\gamma_j$  represents the convex cost, where  $j = k(\text{capital})$  or  $l(\text{labor})$ . In the case of capital adjustment, there is an extra cost that represents the investment price and it can take values of  $p_I = \{p_{buy}, p_{sell}\}$  depending on whether the firm buys or sells capital. The asymmetry in the price for buying and selling capital implies that capital is not fully reversible, which, as noted above, is consistent with the observed distribution of capital adjustments.

If the firm adjusts capital and employment at the same time, the adjustment cost is the sum of the cost of adjusting capital ( $C^k$ ) plus the cost of adjusting employment independently ( $C^l$ ) plus a collection of terms that represent the extra cost of the joint adjustment. This means that the cost function for joint capital and employment adjustment is assumed to have the form  $C^{kl} = C^k + C^l + C^{joint \text{ adjustment}}$ . The parameters are then  $\{\lambda_k, \lambda_l, \lambda_{kl}\}$  for the disruption cost,  $\{F_k, F_l, F_{kl}\}$  for the fixed cost,  $\{\gamma_k, \gamma_l, \gamma_{kl}\}$  for the convex cost and  $p_I$  for the investment price. If interaction effects exist, the terms in

---

<sup>33</sup>The setup of the adjustment cost model will allow the estimation procedure below to distinguish which component is more important in the factor adjustment process.

(3) reflecting joint adjustment ( $\lambda_{kl}, \gamma_{kl}$  and  $F_{kl}$ ) will be different from zero. This interaction will be in the form of a congestion effect if the cost is positive (a loss) or in the form of a complementarity if the cost is negative (a benefit).

Intuitively, interaction effects in the disruption cost mean that forgone profits due to interruption in the production process, or decreases in productivity while adjusting capital and labor, may be higher (congestion) or lower (complementarities) than when adjusting only one factor. For example, the learning process for new workers operating new machines may induce a congestion effect because it is more expensive than the cost of training new workers and of buying new machines separately. Congestion effects in the fixed cost may be due to higher installation costs for workers specific to certain machines, and congestion effects in the convex costs may be due to a longer adjustment period in the plants.

### 3.2 Analysis of the Model

The economics of joint adjustment can be summarized as “adjust if the marginal benefit is bigger than the marginal cost of adjustment.” In this sense, the relative values of the adjustment costs play a key role, since they determine which factor to adjust. The subsections that follow will highlight the main differences between this model and the conventional models, and the new implications of considering joint adjustment of capital and labor when they are costly to adjust and there is an interrelation in their adjustment.<sup>34</sup>

#### Case 1. No Adjustment Costs

In this case, the firm faces a static optimization problem for capital and labor, as well as for energy and materials. This problem is captured by the term  $z$  in the case of energy and materials. The first-order conditions (FOCs)<sup>35</sup> are:

$$k' : \quad \beta \int V_{k'}(z', k', l) f(z'|z) dz' = p_I \quad (4)$$

$$l : \quad \mu z k^\theta l^{\mu-1} = w_l(l) \quad (5)$$

---

<sup>34</sup>The analysis will consider that firms have already optimally chosen hours, energy, and materials as a function of the state space composed of the shocks in demand and technology, the capital stock, and number of workers.

<sup>35</sup>The FOCs for labor, energy, materials, and hours under the full specification of the model (i.e., opening up the term  $z$ ) can be derived as well using the profit function in footnote (28).

Without adjustment costs, shocks directly affect capital and employment decisions and the correlation among all is high every time a shock is realized. It is clear that in order to reproduce the main features of the data some nominal or real rigidity is needed. The initial candidate in the literature was the convex cost component, later adding nonconvex costs to account for inaction and lumpiness. This topic is analyzed next.

## Case 2. Adjustment Costs for Capital and Employment

If firms face any type of adjustment costs, the analysis changes. The FOCs and envelope conditions of the problem in the general case are:

$$k' : \quad C_{k'}(z, l_{-1}, l, k, k') + \beta \int V_{k'}(z', k', l) f(z'|z) dz' \leq 0 \quad (6)$$

$$l : \mu z k^\theta l^{\mu-1} - w_l(l) - C_l(z, l_{-1}, l, k, k') + \beta \int V_l(z', k', l) f(z'|z) dz' \leq 0 \quad (7)$$

$$C_{l_{-1}}(z, l_{-1}, l, k, k') - V_{l_{-1}}(z, k, l_{-1}) \leq 0 \quad (8)$$

$$\theta z k^{\theta-1} l^\mu + (1 - \delta) * C_{k'}(z, l_{-1}, l, k, k') - C_k(z, l_{-1}, l, k, k') - V_k(z, k, l_{-1}) \leq 0 \quad (9)$$

These FOCs and envelope conditions reveal that the functional form of the adjustment costs is crucial to understanding firms' factor adjustment. These expressions are inequality conditions because of the possibility of corner solutions. Equations (6), (7), (8), and (9) hold with equality only when the adjustment in both factors is nonzero. The firm adjusts one factor if the net gain of adjustment is higher than if it adjusts the other factor or both employment and capital together. This opens the possibility of staggered adjustment, even if the adjustment cost is convex: in previous models, convex adjustment costs imply continuous adjustment. Another point to notice is that the discounted marginal value of labor adjustment depends on investment, and the discounted marginal value of investment depends on labor adjustment, whenever firms decide to adjust both factors or whenever the adjustment cost reflects interaction effects.

**Convex Adjustment Costs:** Equation (6) is the general case of Tobin's  $q$ , obtained with a quadratic functional form considering only capital. If firms face convex costs in both factors, given the inequalities in the FOC and depending on the parameter values and the functional forms, there is a possibility of inaction if adjusting one factor gives a higher net marginal benefit than adjusting both factors at the same time. If equations (6) and (7) hold with equality (i.e., when adjustment is nonzero in both factors), an

interesting result can be seen, assuming a convex cost only for the interrelation term:  $\gamma_{kl}(l - l_{-1}) \left(\frac{l}{k}\right)$ . Updating (8) and plugging the result into (7), we find:

$$E \left( \frac{I'}{k'} \right) = \frac{1}{\beta \gamma_{kl}} \left\{ w_0 + \gamma_{kl} \left( \frac{I}{k} \right) - \mu z k^\theta l^{\mu-1} \right\} \quad (10)$$

From (10) we can see that investment rates are positively correlated in time, which is a common feature of convex cost models of investment. Moreover, since  $\mu < 1$ , it can be shown that the investment rate in period  $t + 1$  is positively correlated with the change in labor in period  $t$ .

**Convex and Nonconvex Adjustment Costs:** When hit by a demand or productivity shock, firms decide whether to adjust capital, labor, or both, and at the same time decides the optimal level of materials, energy, and hours.<sup>36</sup> The firms' problem is given by (2) and the optimality conditions for capital and labor are given by equations (6), (7), (8), and (9). If we define  $V^k$  as the value of adjusting only capital,  $V^l$  as the value of adjusting only labor,  $V^{kl}$  as the value of adjusting both capital and labor, and  $V^n$  as the value of nonadjustment, we can redefine the problem as a continuous choice problem nested in a discrete choice framework. Given the firms' decision problem in (2), we can express it as  $V(\bullet) = \max[V^N, V^K, V^L, V^{LK}]$ , where firms choose the action that gives them the highest  $V$ . That is: (i) firms do not adjust if  $V^n > [V^k, V^l, V^{kl}]$ ; (ii) firms adjust labor if  $V^l > [V^n, V^k, V^{kl}]$ ; (iii) firms adjust capital if  $V^k > [V^n, V^l, V^{kl}]$ ; and (iv) firms adjust capital and labor if  $V^{kl} > [V^n, V^l, V^k]$ . Given these options, and as it will become clear later in the numerical analysis, firms will follow an (S,s) policy in both capital and labor.

The model presented cannot be solved analytically because of the nonconvex nature of the decision rules. Therefore, a numerical solution is needed. In the next section I explain the numerical procedure used to solve the model, analyze the decision rules, and

---

<sup>36</sup>The FOCs for the static optimization problem are conditional on  $\lambda$  and, under the full specification of the model, given by:

$$h : \hat{\mu}(1 - \lambda_j) \bar{z} k^{\hat{\theta}} (lh)^{\hat{\mu}} h^{-1} e^\tau m^\varphi - \zeta w_1 l h^{\zeta-1} = 0 \quad (11)$$

$$e : \tau(1 - \lambda_j) \bar{z} k^{\hat{\theta}} (lh)^{\hat{\mu}} e^{\tau-1} m^\varphi = p_e \quad (12)$$

$$m : \phi(1 - \lambda_j) \bar{z} k^{\hat{\theta}} (lh)^{\hat{\mu}} e^\tau m^{\varphi-1} = p_m \quad (13)$$

$\forall(\bar{z}, k, l)$ ;  $j = \text{capital, labor, capital and labor (adjustment type)}$ , and  $\lambda_j = 0$  if there is no adjustment.

explore the qualitative implications emerging from this model.

## 4 Capital and Labor Adjustment Dynamics: A Numerical Analysis

The primary purpose of this paper is to analyze how firms make dynamic joint decisions about capital and employment. Given the evidence and the model presented above, this section describes firms' decision rules with respect to capital and employment adjustment and numerically analyzes whether these decision rules are able to generate simulated economies that reproduce the facts observed in the data. First, I explain the computational methods used to solve the model numerically. Second, I parametrize the model with adjustment cost parameters that appear to be ex-ante reasonable, analyzing the decision rules that the model implies and presenting time series realizations under several configurations of adjustment costs, in order to give an idea of the ergodic distribution of the state variables and which decision rules the firms visit more often.

### 4.1 Numerical Methods

The equation to solve is the Bellman equation given in (2). The solution must give the firms' optimal choices for hours, energy, materials, investment, and employment given the vector of shocks  $z$ , capital  $k$ , and previous employment level  $l_{-1}$ . The optimal levels of hours, energy, and materials are a function of the state space  $(z, k, l_{-1})$ , and this problem can be solved analytically for every period as a function of this state space conditional on the disruption cost  $\lambda_j$  as shown in footnote (36). The variables left to solve are capital and labor, for which a numerical procedure should be used given the dynamic links and non convexities that they exhibit in the model.

In the solution of this equation, three basic choices must be made: (i) the procedure for maximization over the state space, (ii) the procedure to solve the unknown value function  $V(z, k, l_{-1})$  and (iii) the procedure to solve for the integral over the shocks that represents firms' expectations about the future value  $V(\bullet)$ .

For (i), I choose a small number of points for the state space (3 for the productivity shock, 2 for the demand shock, 70 for the capital stock, and 30 for the number of workers)

and use a golden section search method to determine the maximum over the entire state space, bracketing first the optimal region and then using linear interpolation of the optimal values to the values in the grid. For (ii) I use value function iteration, since the existence of kinks in the value function did not ensure reliable results for the full range of parameters using other methods like policy iteration or polynomial parametrization. For (iii), I integrate using quadrature methods as in Tauchen and Hussey (1991). This quadrature is solved with Hermite polynomials, which are the best for this situation since the shocks are assumed (and estimated from the data) as AR(1) log-processes with lognormal error terms. I assume independence of both shocks such that the shock calibrated in the model is the result of the multiplication of those two. The code is written in Matlab and C, linking the programs through MEX-files.

## 4.2 About the Parameters

There are two sets of parameters: those that can be directly estimated without imposing an economic model (reduced-form parameters) and those that need to be calibrated or estimated with some simulation procedure (structural parameters). In the first group we have: the production function coefficients (i.e, the input factor elasticities)  $\chi$ ,  $\alpha$ ,  $\xi$ , and  $\nu$  for capital, labor, energy and materials respectively; the demand shock process  $X_{it}$ ; the technology shock process  $A_{it}$ ; the elasticity of demand  $\psi$ ; the depreciation rate  $\delta$ ; the hours wage elasticity  $\zeta$ ; and the discount factor  $\beta$ . In the second group we have the adjustment cost parameters:  $\gamma_j$  for the convex cost,  $F_j$  for the fixed cost,  $\lambda_j$  for the disruption cost, and  $\{p_I, p_{sell}\}$  for the price of capital. Because the data set does not contain capital prices and although I can calculate the mean input prices from the data in the case of employment, energy, and materials, I choose to calibrate them using the theoretical model because of the lack of capital prices in order to put all the factor prices on equal footing. The calibration based on the model come very close to the relative prices calculated from the data for energy and materials. I cannot compare the wage calibration with the data because hourly wages and capital prices are not available.

The estimates for production function coefficients, the demand and technology shock processes, and the elasticity of demand are taken from Eslava et al. (2004).<sup>37</sup> The

---

<sup>37</sup>They use information on prices to estimate an output-based KLEM production function with demand shift instruments, taking advantage of Syverson (2005)'s insight that using demand as an instru-



coefficients are 0.213 for capital, 0.303 for labor, 0.176 for materials and 0.275 for energy. The demand elasticity is  $\psi = 2.28$ . With respect to other reduced-form parameters, I assume that the demand shocks and the technology shocks follow an AR(1) lognormal process such that (with lowercase letters meaning logs and dropping the subindex  $i$ )  $a_t = \rho_a a_{t-1} + u_t$  and  $x_t = \rho_x x_{t-1} + \varepsilon_t$ . These equations are estimated as in Eslava et al. (2004) using year effects. The coefficients obtained are  $\rho_a = 0.922$ ,  $\rho_x = 0.985$ ,  $\sigma_a = 0.77$ , and  $\sigma_x = 0.89$ . Given that  $\sigma_a^2 = \frac{\sigma_u^2}{1-\rho_a^2}$  and  $\sigma_x^2 = \frac{\sigma_\varepsilon^2}{1-\rho_x^2}$ , we get  $\sigma_u = 0.297$  and  $\sigma_\varepsilon = 0.151$ . I normalize the mean values of the shocks to 1 (affecting also the input prices). The shocks can be seen as deviations from an aggregate trend in technological progress and/or demand. The discount factor  $\beta$  is set as 0.95, the depreciation rate  $\delta$  as 0.1, and the hours-wage elasticity as 1.1.

Prices for energy and materials are calculated by solving the FOCs of the problem without adjustment costs.<sup>38</sup> The values for capital, labor, energy, and materials plugged into these equations are the means of the actual values. It is interesting to note that the implied prices are very close to those obtained by dividing energy and materials expenditures by physical amounts. The labor payment parameters  $w_0$  and  $w_1$  are obtained by solving the system composed of the FOCs for hours and labor in the problem without adjustment costs<sup>39</sup> and again using mean values. The investment price is obtained by solving the dynamic problem in the case of no uncertainty and no adjustment costs.<sup>40</sup>

With respect to the structural parameters, they are estimated in section 5. However,

---

ment for input factors in production function estimation can get rid of the endogeneity problems that are well known in such situations. They implement this idea creating downstream demand shift instruments selected with Shea (1997)'s relevance and exogeneity criteria, in order to find the production function coefficients and the technology shock process. An advantage of their methodology is that the price information allows them to isolate a firm-specific price deflator, so that TFP estimations do not use a common price deflator that, as Klette and Griliches (1996) and Foster et al. (2005) illustrate, would bias the estimates of the production function coefficients. Next, Eslava et al. (2004) take advantage of the price information and estimate a downward sloping demand curve similar to the one assumed here, instrumenting output using the calculated TFP and getting the demand shocks as the residuals of this demand regression

<sup>38</sup>That is, equations (12) and (13) respectively without the  $(1 - \lambda_j)$  term. Even though those prices can be an important source of fluctuations and relative changes in input adjustment, this paper focuses on the firms' responses to demand and productivity shocks. This is the reason to look for an "average" input price in each case and guides the calculation of the prices.

<sup>39</sup>Equation (11) without the  $(1 - \lambda_j)$  term in the case of hours and  $\mu \bar{z} k^{\hat{\theta}} l^{\hat{\mu}-1} e^{\tau} m^{\varphi} = w_0 + w_1 h^{\zeta}$  in the case of labor.

<sup>40</sup>The implicit assumption for the calculation of the prices is that the frictionless FOCs hold "on average."

in order to numerically analyze the decision rules generated by the model, I chose for the moment in this section the adjustment cost parameters in an arbitrary manner such that the adjustment costs are the same for all types of costs and collectively account for either 4% or 20% of average profit. From this point on, I focus on the congestion effect because it is the one that is later supported by the data. Table 5 shows the chosen parameters for the adjustment costs and the average adjustment cost as a proportion of the contemporaneous profit. These costs do not increase proportionally from one column to the next because they were obtained with simulations and reflect an approximate rather than exact value. Finally, in this section I take the capital resale price equal to 70% of the price of buying capital (partially reversible investment).

### 4.3 Decision Rules for Capital and Employment Adjustment

I now present the numerical results for the decision rules (or invariant policy functions) and their implications for firm behavior, given by the chosen initial parametrization for the adjustment costs. Before I start analyzing the numerical results, it is important to clarify the firms' decision problem. There are three states defining the situation of a firm: an idiosyncratic deviation from an aggregate level of demand or technology (i.e., a shock), a capital level, and an employment level. If a firm found itself in another state of demand or technology in the next period (i.e., because of a higher seasonal demand level or a change in management, that results in a demand or a technology shock), then the firm has the option of not adjusting any factor, adjusting only capital, adjusting only employment, or adjusting both capital and employment. The policy rules solve the question: if the firm is hit by a shock  $z$ , what is the optimal decision for the next period's capital and labor given the current period's capital and labor?

The decision rules are presented in a graphical analysis over the space determined by the values of capital (y-axis) and labor(x-axis) for each value of the shock. In this context, a shock represents a combination of a demand and a productivity shock, discretized in 2 and 3 states respectively (6 shocks total).<sup>41</sup> As expected, there is a region of inaction whose size depends on the value of the shocks, the type of adjustment costs, and the

---

<sup>41</sup>Specifically, shock 1 in the figures represents low demand and low productivity; shock 2 represents low demand and average productivity; shock 3 represents low demand and high productivity; shock 4 represent high demand low productivity; shock 5 represents high demand and average productivity; and shock 6 represents high demand and high productivity

existence or not of congestion effects in adjustment. This region of inaction determines a bidimensional (S,s) policy in capital and labor depending not only on the shocks and the other state variables but also on the other choice variables. However, some regions are more likely to affect firm behavior than others since the ergodic distribution of the state space is an important point to take into account. To better illustrate this point, it would be optimal to plot the probability distribution over each adjustment/inaction region to know how likely is that a firm will find itself in that specific region. However, such a plot is graphically confusing; instead I present several time series realizations of firms' behavior under different configurations of adjustment costs.

Figure 2 shows the policy functions for the case where all costs are present for capital and labor (and equal to 4%) and there are congestion effects in adjustment. The first thing to note is that there is an inaction region whose shape and size depend on the shock and the value of both factors. This inaction region defines a bidimensional (S,s) policy for capital and labor. The optimal (S,s) policy depends on the shock and the choice of the other variable (either capital or labor).

Even though some adjustment regions for capital and labor are not convex sets, there are defined zones in which it is optimal to adjust only labor and others in which it is optimal to adjust only capital, especially for low demand shocks (i.e, shocks 1 to 3; see figure 2). In some of the policy rules there exist disjoint sets in the capital-labor (kl) space for a given shock. For example, for the same capital to labor ratio there may exist inaction or adjustment depending on the level of capital and labor, opening the possibility of multiple optimal regimes. According to the optimal rule, firms adjust capital and labor together only if the firm has a high demand shock (in this case, shocks 4 to 6). This implies that there is an implicit target for a relationship between capital and labor, and that target changes with the nature and size of the shock.

With just one factor, a standard (S,s) rule would hold, with the shock being the only determinant of the optimal adjustment policy. With the possibility of adjusting capital and labor, firms make decisions depending on where they are with respect to the optimal target of not just one isolated factor but a composite of capital and labor. This result is similar to that of Eslava et al. (2005), when implementing the gap approach by Caballero et al. (1995, 1997); their empirical results in these two studies suggest that firms adjust

labor and capital depending on the gap between desired and actual employment and labor. The structural analysis here implies that this gap is implicitly affected by the gap in the other factor, resulting in the bivariate (S,s) policy.

Figures 3, 4 and 5 compare the decision rules implied by the different types of adjustment costs considered one by one in the presence (or absence) of congestion effects in the case of a bad shock (low demand and low productivity shocks), an intermediate shock (high demand and low productivity shocks) and a good shock (high demand and high productivity shocks). With respect to the adjustment cost types, the convex cost implies a larger region of adjustment in both capital and employment than either the fixed cost and the disruption cost. It is important to note, as the theoretical analysis of the model revealed above, that even with convex costs there is a region of inaction, though much smaller than the one present under nonconvex costs. The biggest inaction zone corresponds to the disruption cost. The fixed cost implies more staggered adjustment than the disruption cost, under which simultaneous adjustment of both factors is more frequent. The behavior implied by the disruption cost looks more in line with the empirical evidence, which shows a positive comovement between capital and employment.

For low values of the shock (i.e., low demand and low productivity shocks) as shown in figure 3, the behavior under convex or disruption costs does not depend much on the existence of congestion effects, and the only difference between the behavior under convex costs and under disruption costs is the adjustment of only labor in a small region under convex costs. For the low shock case, fixed costs generate a more differentiated behavior, as there is no joint adjustment of capital and employment in the case of congestion effects. In the intermediate shock case (i.e., high demand and low productivity shocks) as shown in figure 4, we observe that without congestion effects, there is more joint adjustment of capital and employment. This does not necessarily imply a higher correlation of adjustments under this regime, however, because capital and labor can move in opposite directions. The inaction zones are defined as double (S,s) bands. Interestingly, there is more joint adjustment in the convex cost case with congestion effects than without them. The pattern for fixed costs observed in the low shock case repeats itself here: joint adjustment is rare in the presence of congestion effects. If firms face a high shock (i.e.,

high demand and high productivity shocks) as shown in figure 5, there is much more joint adjustment and the behavior under convex and disruption costs is very similar, with a larger inaction zone in the disruption case as expected. Again, fixed costs present the most different pattern: in the presence of congestion effects, firms will adjust only labor under certain circumstances.

The decision rules as presented above do not say anything about the direction or size of the adjustments. For example, capital and labor can move at the same time but in different directions. There exist enormous nonlinearities in the decision rules. Figures 6 and 7 show the decision rules for labor adjustment and investment for several values for capital and labor and in the case of an intermediate shock (i.e., high demand and low productivity shocks) with low adjustment costs. From Figure 6, we observe that at low values of capital and labor the adjustments are in the same direction (positive), but for higher values, firms stagger depending on the state of  $(k, l)$ ; that is, under some combinations of capital and labor, firms reduce capital and do not adjust labor, and under other values they do the opposite. Figure 7 shows that factors can adjust in opposite directions when a firm has high values of capital and low values of employment or vice versa (higher or lower relative to their frictionless optimum). This means that at low values for both capital and labor the correlation between their adjustments is likely to be positive, while at higher values this correlation is likely to be negative.

Thus we have different possibilities of firm behavior depending on whether firms face good or bad shocks, whether they face congestion effects in the adjustment and depending on the type of dominant adjustment cost. In summary, the main features of these decision rules are as follows:

(i) The decision rules for capital and employment adjustment exhibit a nonlinear pattern with inaction zones, zones of joint adjustment, and zones of single factor adjustment. That is, the decision rules present a non linear (S,s) rule in both capital and labor. Adjustment or inaction in capital and labor depend not only on the states, such as the productivity and demand shocks and the capital to labor ratio, but also on the choices of capital and labor.

(ii) Persistent bad shocks (low demand and low productivity shocks) and fixed costs lead to a less frequent joint adjustment in capital and employment. Persistent good

shocks (high demand and high productivity shocks) increase the joint employment and capital adjustment.

(iii) In general, the presence of convex costs tends to increase the likelihood of joint adjustment for capital and labor while the presence of fixed costs tends to reduce it. Disruption costs can accommodate a richer type of adjustment, depending on a firm's capital to labor ratio. This does not translate directly into measures of correlation since the adjustment of capital and labor can occur in opposite directions.

(iv) Under intermediate shocks (high demand and low productivity shocks) or high shocks (high demand and high productivity shocks), congestion effects in the adjustment increase the likelihood of joint adjustment. Under low shocks (low demand and low productivity shocks), this is true only if fixed costs are dominant.

(v) Inaction regions are present with all types of costs, but larger when firms face disruption or fixed costs. However, in the case of fixed costs, inaction zones can present disjoint sets, which open the possibility for multiple optimal regimes of capital and labor for the same capital to labor ratio. Inaction zones are more important in the case of intermediate shocks, being almost nonexistent for bad shocks and smaller for good shocks.

(vi) Congestion effects increase the inaction in both capital and labor adjustments in all the cases. When congestion effects are not present, fixed costs increase inaction and volatility in both capital and labor with respect to the convex and disruption cases; when congestion effects are present, convex costs increase the volatility of the investment rate and decrease the volatility in labor growth compared with the case when either fixed or disruption costs are present. However, when congestion effects are present and firms face all types of adjustment costs, the volatility of labor growth increases and the volatility of the investment rate decreases.

(vii) The decision rules presented show a model that can accommodate at the same time smooth, lumpy, and infrequent adjustment, depending on the value of the shocks, the level of capital and labor, the type of adjustment costs, and the presence or not of congestion effects.

Other important implications of different configurations of the adjustment costs can be seen by plotting the time series realizations that the policy rules imply as a way of visualizing the regions where the firms spend more time. For example, we could try

to find out whether bigger inaction regions imply less variability of investment or are compensated by larger adjustments. Figure 8 shows the time series realization of one shock series when a firm faces all types of adjustment costs comparing the effects of congestion vs. no congestion, figure 9 shows the labor growth when the firm faces different types of adjustment costs with and without congestion effects, and figure 10 shows the investment responses of the same.

These time series realizations show that the presence of congestion effects increases the inaction in both capital and labor adjustments in all the cases. On the other hand, fixed costs and disruption costs have the same effect on investment rates when there are congestion effects, even if the decision rules are different. When congestion effects are not present, fixed costs increase inaction and volatility in both capital and labor with respect to the convex and disruption cases. When congestion effects are present and firms face all types of adjustment costs, the volatility of labor growth increases and the volatility of the investment rate decreases. Also when congestion effects are present, convex costs increase the volatility of the investment rate and decrease the volatility in labor growth compared with the cases when either fixed or disruption costs are present.

## 5 Adjustment Cost Parameters: Estimation

In this section, I use a minimum distance algorithm to find the adjustment cost parameters that allow me to match the moments from the data to the moments from a simulated panel generated with the model. The main assumption is that the model is a good approximation of the way firms make decisions about labor and capital.

The methodology to apply is the method of simulated moments, in the spirit of McFadden (1989) and Hall and Rust (2003), among others. The choice of this estimation method is made for computational feasibility. The parameter set to be estimated is composed of fixed, convex, and disruption costs for capital, labor, and joint capital-labor adjustment, and the resale price of capital. These are represented by a vector  $\Theta = [(F_k, F_l, F_{kl}), (\gamma_k, \gamma_l, \gamma_{kl}), (\lambda_k, \lambda_l, \lambda_{kl}), p_i]$ .

The algorithm consists of solving the dynamic programming problem (DPP) given a set of parameters  $\Theta$ , getting the policy functions for that specific parametrization, simu-

lating a panel of plants, calculating the chosen moments from that panel and comparing them with the moments from the data. The function that depends on the parameters and must be minimized is given by:  $\min_{\Theta} J(\Theta) = \{\mathbf{M}_{data} - \mathbf{M}_{simulated}(\Theta)\}' W \{\mathbf{M}_{data} - \mathbf{M}_{simulated}(\Theta)\}$ , where  $\mathbf{M}$  is a vector of moments,  $W$  is a weighting matrix and  $\Theta$  is a vector of parameters to be estimated.

To find standard errors, there are two options. The first is to conduct a Monte Carlo simulation, repeating the procedure under different realizations of the stochastic shock. The second is by obtaining the asymptotic distribution of the estimator, as in Hall and Rust (2003); this method (used in this paper) is simpler and much faster. I construct the simulated panels with 1000 firms over 500 periods.<sup>42</sup> To solve for parameters I use the Nelder-Mead simplex algorithm.

Regarding the moments to match, the main questions are which moments reflect variation in the parameters and which are worth matching given their relevance in understanding firm behavior. The empirical evidence in the second section presented characteristic distributions for capital and labor. In particular, the distributions showed highly irreversible capital and lumpy adjustment and inaction zones for both capital and labor. However, given the arbitrary definition of inaction and the implications for different types of capital,<sup>43</sup> I choose not to match this feature. Instead, I focus on the lumpiness of the distributions and match the adjustments above the 90<sup>th</sup> and below the 10<sup>th</sup> percentiles in the distribution of capital and labor adjustment (i.e., the fraction of positive and negative spikes). The VAR(1) illustrates the dynamic interrelations of capital and labor and is an important feature to consider. On the other hand, the model highlights the importance of shocks in the movements of capital and labor; the correlation between adjustments and shocks is therefore the other important moment. Finally, the correlation between capital and labor adjustment is of prime interest in this paper and completes the set of moments I attempt to match.

---

<sup>42</sup>At the moment, I set  $W$  to the identity matrix in order to estimate the parameters, which gives consistent but not efficient estimates; however, in a second stage, I recalculate  $W$  as in Hall and Rust (2003) in order to calculate the standard errors. In future work, I plan to reestimate the parameters with this optimal weighting matrix to get efficient estimates. See Hall and Rust (2003) for more details.

<sup>43</sup>For example, a hammer, some special cutting tools, and a milling machine are capital goods, and defining an investment rate lower than 1% as inaction would not account for the investment in small but important units of capital.



## 5.1 Adjustment Cost Parameters: Fitting the Data

In order to analyze whether congestion effects in the adjustment for capital and labor are important, I first match the moments assuming that all parameters are present, including the congestion effect parameters  $(F_{kl}, \gamma_{kl}, \lambda_{kl})$ , and compare the results with those obtained not including the congestion effect terms. In order to analyze which adjustment costs are important, I compare the results of the full specification model with the results of models that shut down a particular type of adjustment cost. The calculated moments are presented in table 6, and table 7 presents the calculated parameters with the standard errors for the full specification.

Two important facts emerge from table 6:<sup>44</sup> First, the model that considers congestion effects does better in several dimensions than the model that does not. In particular, one of the fitted VAR coefficients in the case without congestion effects has the wrong sign relative to the data, and the contemporaneous correlation between capital and employment adjustment is too high in the case without congestion effects. This suggests that when firms adjust both factors they pay a price instead of benefiting from adjusting them together. It is important to emphasize that even if the firms pay an extra cost of adjusting capital and labor together, the discounted expected net benefit can still be higher than if firms adjust one factor each period. The second important fact from table 6 is that the model that considers convex costs as the only cost faced by firms when adjusting does the worse job in explaining the data moments.

Table 7 has important information about the size of the adjustment costs and about the statistical significance of the estimates. With respect to the size of the estimates, the high degree of irreversibility of capital in Colombia is observed in the lower selling price of capital for all the models except in the benchmark convex case (by assumption  $p_{sell} = p_{buy}$  in this case). This irreversibility is much bigger than the one found by Cooper and Haltiwanger (2005) for the U.S. This degree of irreversibility is consistent with the asymmetric distribution for the gross investment observed in the Colombian census. The high costs of adjusting both factors at the same time are somewhat surprising. However, the functional form for the congestion effects case does not allow for a direct comparison

---

<sup>44</sup>As in any nonlinear numerical optimization these results should be taken with care, because the minimization routine is susceptible to getting local minima as solutions.

between the individual costs and the congestion cost. The adjustment costs are not large, but ignoring them does not allow a good match of the moments in the data.

With respect to the statistical significance of the estimates, the procedure makes it possible to accept statistically the disruption costs for capital and labor, the convex costs for capital but not for labor, and the congestion effect present in the convex and disruption cost joint parameters. The low statistical significance of the fixed costs may be due to the fact that the disruption cost absorbs all its effects. Also, I cannot reject the possibility that the convex costs are present in the Colombian firms when they adjust capital or capital and labor at the same time due to the statistical significance.

## 5.2 Nonformal Tests of Goodness of Fit

After determining the parameters, the following exercises look to determine how well the model can fit the distributions of adjustment and other moments not considered in the estimation procedure. Figure 11 replicates the distributions of adjustments generated using the model and the estimated parameters. The simulated distributions are lumpier, but the main characteristics observed in the data are present here. In particular, the continuous adjustment in materials and energy contrasts with the lumpy pattern observed in the distributions of adjustments of capital and labor. It is interesting to notice how the model mimics the asymmetry in the capital adjustment distribution and the symmetry in the labor adjustment distribution observed in the data.

Table 8 tries to replicate the logit estimates for the probability of spikes in adjustment or inaction given such spikes in the other factor. In order to do so, I generated a panel with 1000 firms and 500 periods using the estimated parameters, dropping the first twenty observations in each simulated series. The logit estimated using the simulated panel does a mixed job matching the empirical coefficients, but note that the definition of small or large adjustment is arbitrary and given the lumpier nature of the simulated panel these definitions can affect the results. In particular, there is a significant and negative probability of adjusting capital and labor when there is inaction in the other factor, which is not observed in the data.

## 6 Conclusions

In this paper I have used the Colombian Annual Census of Manufacturing to analyze the interrelationship of labor and capital adjustments, whether there are congestion effects in the adjustment process, and the nature of the adjustment costs.

Empirically, firms adjust employment and capital in an interrelated way, using several margins of adjustment in the process. As is the case for U.S. firms, there is a distribution of adjustment that is lumpy and infrequent for capital and labor, and more frequent in the case of materials and energy. These patterns suggest that to understand the effect of policies such as tax investment incentives or reductions in firing/hiring costs, a model of joint capital and labor adjustment is needed. I argue that these patterns can be explained with a dynamic model in which labor and capital are costly to adjust. The adjustment cost structure is chosen to match key facts observed in the data, such as the decrease in output after adjustment, the cost of hiring and firing workers, the cost of installing capital, a convex component to capture the mix of smooth and lumpy adjustment, and an interaction term in the adjustment cost.

The firms' decision rules implied by the proposed model show highly nonlinear adjustment patterns that can be characterized as a bidimensional (S,s) policy, where adjustment depends not just on the states of the system but also on the choices, meaning that the firms decide to adjust either capital or labor or both depending of the shocks and the initial capital to labor ratio. I estimate the parameters of the model using a minimum distance algorithm. This method reveals that a model incorporating congestion effects fits the data best. Also, based on the estimation procedure, I am able to statistically reject the existence of fixed costs and to accept the existence of disruption costs for capital and labor, the existence of convex costs for capital but not for labor, and the existence of congestion effects.

The main conclusion is that labor and capital adjustment should be analyzed together. This is supported both by theory and by the facts. The data show an interrelated adjustment pattern. Moreover, a model that incorporates adjustments for both capital and labor generates sharply different predictions if adjustment costs are assumed for one factor alone. The main advantage of the methodology proposed in this paper is that

several policy experiments can be analyzed. The effects of taxes and related policies on capital and employment and the aggregate effects of these policies are among the main ones. Finally, it may be worth exploring sectoral differences in firm behavior, especially as the parameters and functional forms may not be the same for all types of industries.

Table 1: Distribution of Factor Adjustment (%)

	$\frac{I}{K}$	$\frac{\Delta L}{L}$	$\frac{\Delta m}{m}$	$\frac{\Delta e}{e}$
Inaction ( $abs(y) < 1\%$ )	18.9	13.4	3.2	5.6
Positive Spike ( $y > 20\%$ )	29.8	11.6	28.4	23.7
Negative Spike ( $y < 20\%$ )	1.8	11.1	18.8	15.8
$\rho(y, y_{-1})$	0.025	-0.057	0.0	-0.298
Number of Observations	24,467	34,243	31,977	34,597

Table 2: Factor Adjustment Contemporaneous Correlation

	$\frac{I}{K}$	$\frac{\Delta L}{L}$	$\frac{\Delta m}{m}$	$\frac{\Delta e}{e}$
$\frac{I}{K}$	1			
$\frac{\Delta L}{L}$	0.057	1		
$\frac{\Delta m}{m}$	0.026	0.175	1	
$\frac{\Delta e}{e}$	0.041	0.107	0.147	1

All correlations are statistically different from zero at 1% significance.

Table 3: Probability of Inaction/Adjustment Conditional on Inaction/Adjustment of the Other Factor

Variable x		Investment		Employment Growth	
		P(Inaction/x)	P(Spike/x)	P(Inaction/x)	P(Spike/x)
Investment	Inaction			0.057 (0.071)	0.012 (0.061)
	Spike			-0.11† (-0.06)	0.205** (0.048)
Employment Growth	Inaction	0.143* (0.067)	-0.033 (-0.058)		
	Spike	-0.033 (-0.059)	0.208** (0.046)		
Observations		12,864	17,055	12,410	14,459

†/\*/\*\* significant at 10%, 5%, and 1%. TFP, demand shocks, and year effects in regression. Dummies for inaction are defined as 1 if  $abs(x) < 0.01$  and dummies for spikes are defined as 1 if  $x > 0.2$ .

Table 4: Dynamic Relations in Factor Adjustment

	$\frac{I}{K}$	$\frac{\Delta L}{L}$	$\frac{\Delta m}{m}$	$\frac{\Delta e}{e}$
$(\frac{I}{K})_{-1}$	-0.008 0.006	-0.008 (0.003)**	0.023 (0.004)**	-0.038 (0.006)**
$(\frac{\Delta L}{L})_{-1}$	0.037 (0.017)*	-0.147 (0.007)**	0.049 (0.012)**	0.091 (0.017)**
$(\frac{\Delta m}{m})_{-1}$	-0.007 0.01	0.024 (0.004)**	-0.247 (0.007)**	0.036 (0.010)**
$(\frac{\Delta e}{e})_{-1}$	0.013 -0.007	0.009 (0.003)**	0.006 (0.005)*	-0.345 (0.007)**
Observations	17,653	17,653	17,653	17,653
R-squared	0.03	0.07	0.26	0.23

Standard errors in parentheses; \*\*/\* significant at 1% / 5%;  
year effects and shocks in regression.

Table 5: Parameter Adjustment Costs

Adjusted Factor	Adjustment Cost Type	Cost/ $\Pi$	
		4%	20%
Labor	Fixed ( $F_l$ )	0.22	1
	Convex ( $\gamma_l$ )	0.05	0.15
	Disrupt ( $\lambda_l$ )	0.01	0.035
Capital	Fixed ( $F_k$ )	0.01	0.044
	Convex ( $\gamma_k$ )	0.0007	0.006
	Disrupt ( $\lambda_k$ )	0.01	0.045
Joint Adjustment (Congestion)	Interaction Effects: Congestion/ $\Pi = 30\%$		
	Fixed ( $F_{kl}$ )	5	5
	Convex ( $\gamma_{kl}$ )	0.11	0.11
	Disrupt ( $\lambda_{kl}$ )	0.07	0.07

Table 6: Simulated Moments

Moments		Data	Simulated Moments		
			Convex+Fixed+Disruption Comple.	No comple.	Convex
Positive Spikes	$0.61 < (I/K)$	0.1	0.181	0.3	0.38
(90 <sup>th</sup> percentile)	$0.23 < (\Delta L/L)$	0.1	0.133	0.27	0.25
Negative Spikes	$0 > (I/K)$	0.1	0.152	0.21	0.26
(10 <sup>th</sup> percentile)	$-0.22 > (\Delta L/L)$	0.1	0.105	0.19	0.21
VAR Coefficients	$bkk$	-0.008	-0.009	-0.014	-0.003
	$bkl$	-0.008	-0.016	0.21	0.065
	$blk$	0.037	0.068	0.045	0.005
	$bll$	-0.147	-0.203	-0.073	-0.866
Correlations	$\rho(\frac{I}{K}, \frac{\Delta L}{L})$	0.057	0.049	0.139	0.203
	$\rho(\frac{I}{K}, z)$	0.1	0.089	0.03	0.074
	$\rho(\frac{\Delta L}{L}, z)$	0.05	0.135	0.154	0.297
$J(\theta)$		NA	0.022	0.152	0.745

NA = not applicable.

Table 7: Calculated Adjustment Cost Parameters

	Convex+Fixed+Disruption		Convex
	Congestion Effects	No Congestion Effects	
$P_{sell}$	0.42*pi (0.7307)	0.57*pi	pi
$F_k$	0.0002 (0.0021014)	0.0065	NA
$F_l$	0.007 (0.090392)	0.013	NA
$F_{kl}$	0.14 (0.40376)	NA	NA
$\gamma_k$	0.000006 (0.0000004)**	0.000016	0.092
$\gamma_l$	0.0009 (0.0045079)	0.0000571	0.0134
$\gamma_{kl}$	0.016 (0.0065368)*	NA	0.024
$\lambda_k$	0.0002 (0.0000182)**	0.00065	NA
$\lambda_l$	0.0215 (0.0043362)**	0.092	NA
$\lambda_{kl}$	0.046 (0.0199720)*	NA	NA

Standard errors in parentheses; \*\*/\* significant at 1% and 5%.

NA = not applicable.

Table 8: Probability of Inaction/Adjustment Conditional on Inaction/Adjustment of the Other Factor: Simulated Data

Variable x		Investment		Employment Growth	
		P(Inaction/x)	P(Spike/x)	P(Inaction/x)	P(Spike/x)
Investment	Inaction			0.026 ** (0.007)	-0.021** (0.007)
	Spike			-0.017* (-0.008)	-0.001 (-0.008)
Employment Growth	Inaction	0.031** (0.008)	-0.035** (0.008)		
	Spike	-0.003 (-0.008)	-0.008 (0.008)		
Observations		49,000	49,000	49,000	49,000

†/\*/\*\* significant at 10%, 5% and 1%. TFP and demand shocks in regression  
Dummies for inaction are defined as 1 if  $abs(x) < 0.01$  and dummies for spikes are defined as 1 if  $x > 0.2$



Figure 1. Distributions of Factor Adjustment.  
Percentage of Observations (y-axis) in a Range of Adjustment (x-axis)

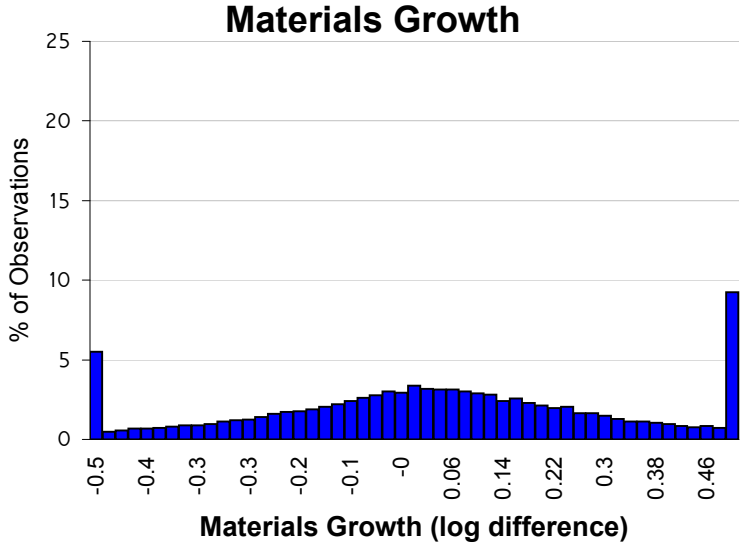
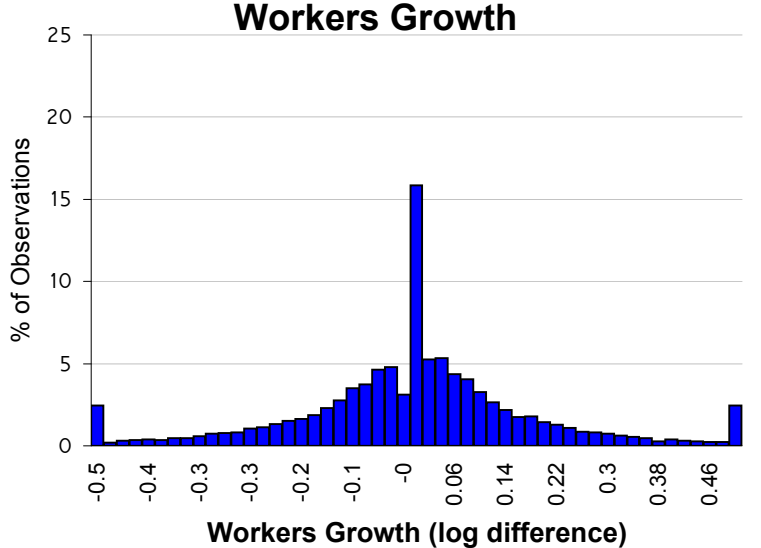
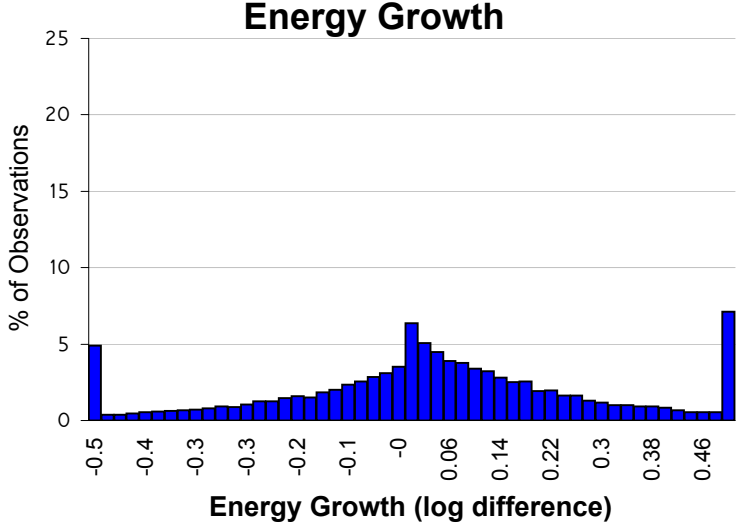
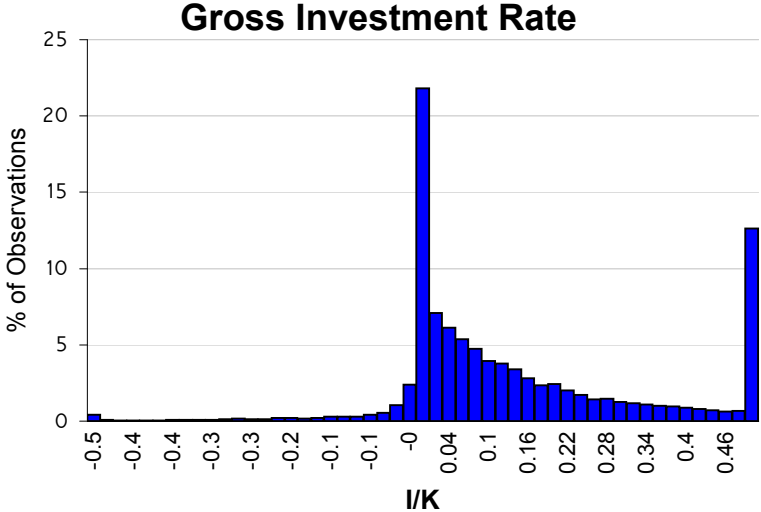


Figure 2. Decision Rules: All Costs in Capital and Labor.  
 Shock 1 to 3: low demand shock with low, average and high productivity respectively.  
 Shock 4 to 6: high demand shock with low, average and high productivity respectively

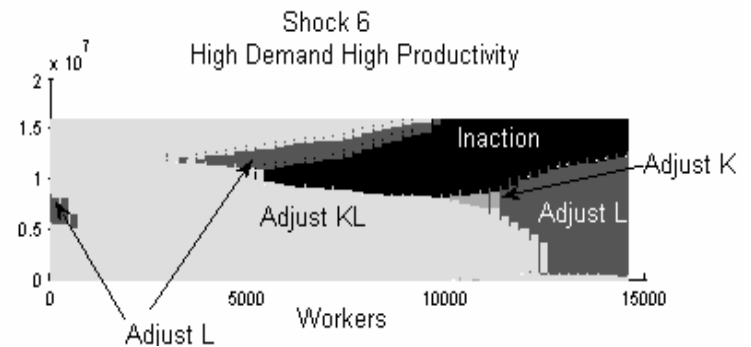
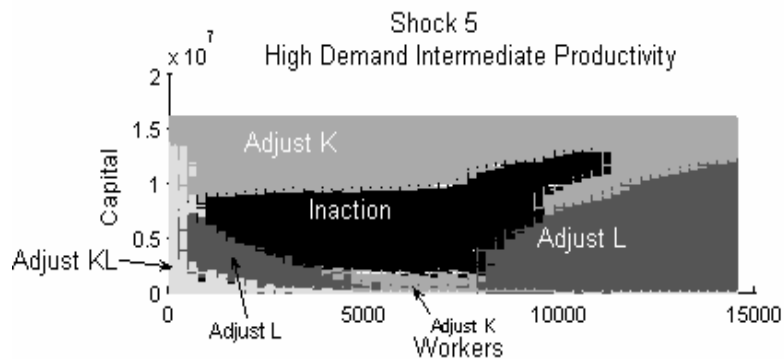
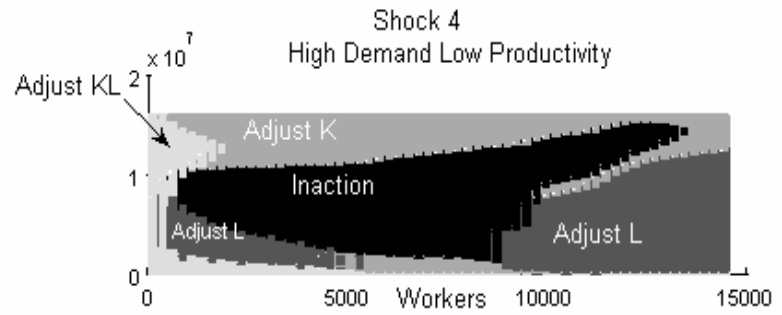
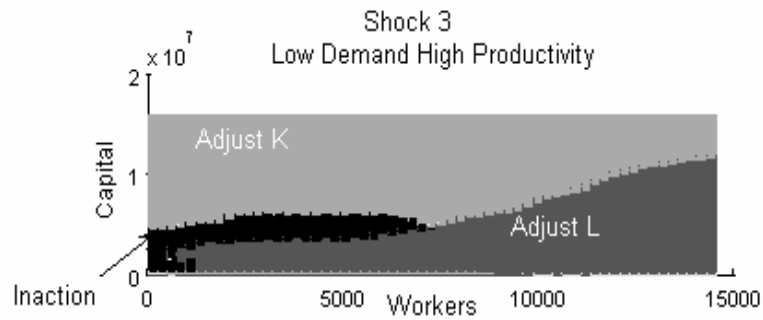
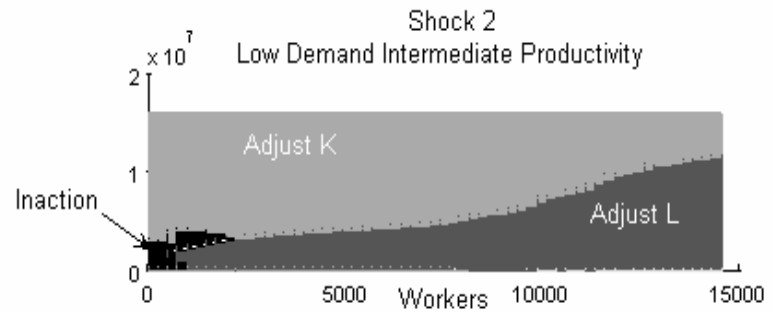
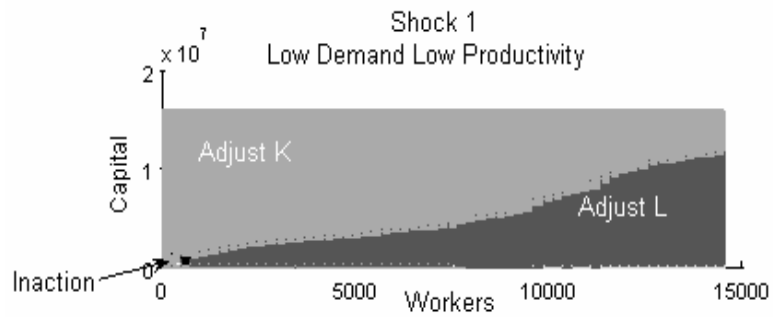


Figure 3. Decision Rules: Comparison Among Adjustment Costs, Low Shock  
(Low Demand Shock with Low Productivity Shock)

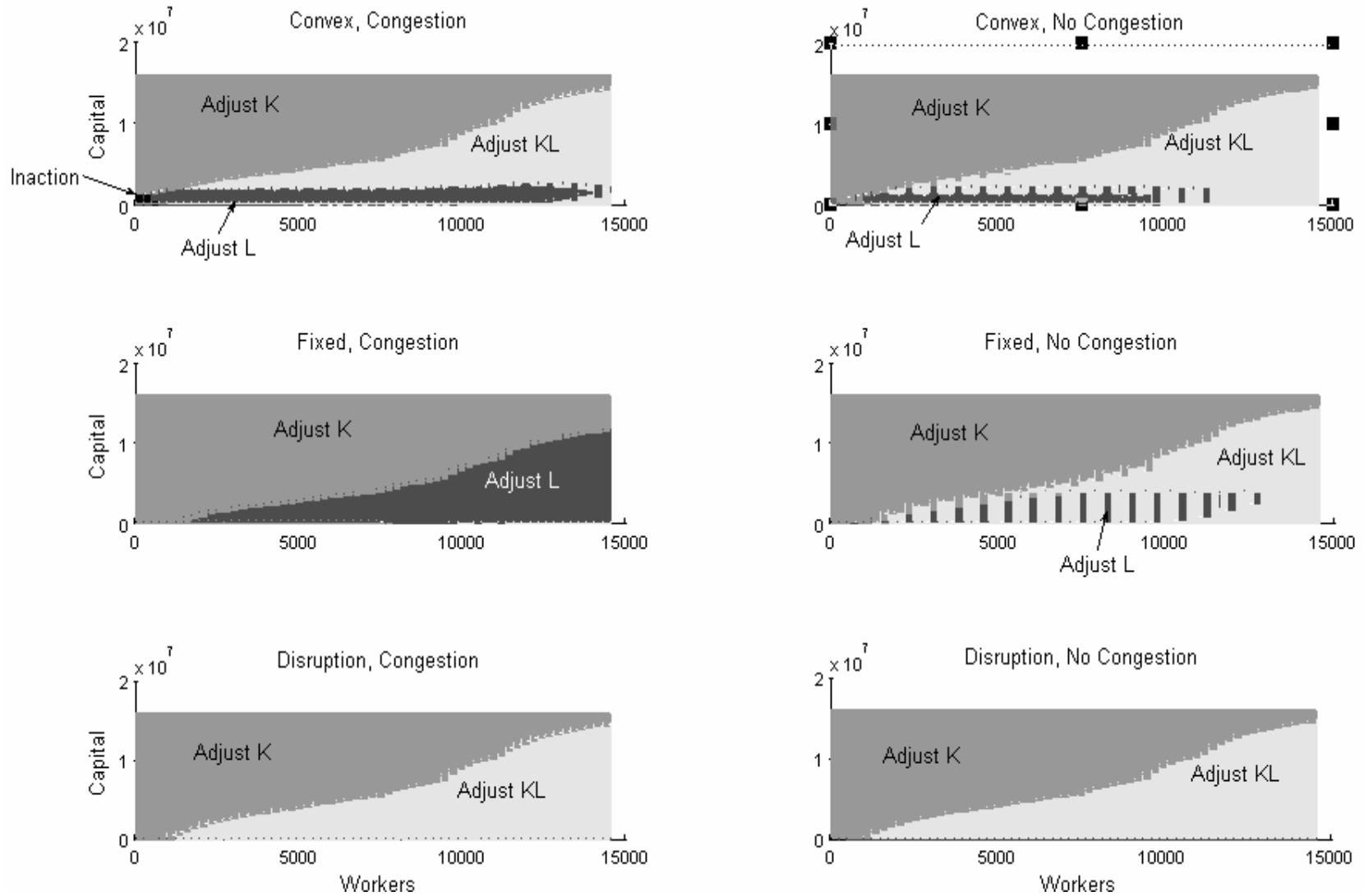


Figure 4. Decision Rules: Comparison Among Adjustment Costs, Intermediate Shock  
(High Demand Shock with Low Productivity Shock)

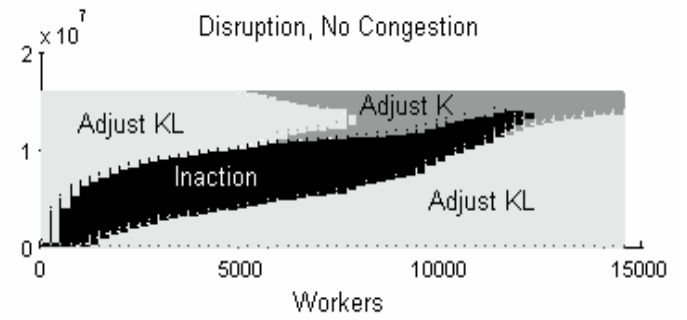
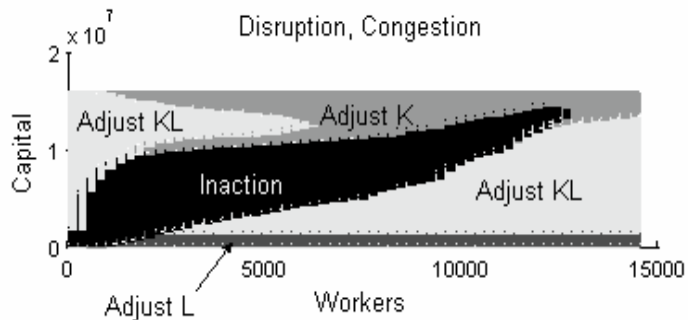
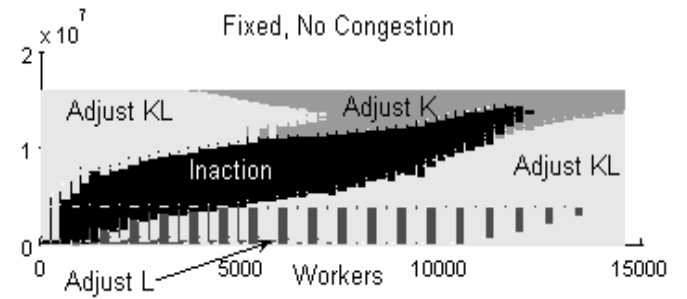
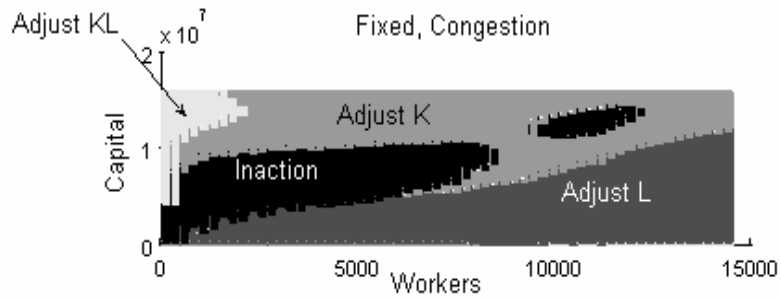
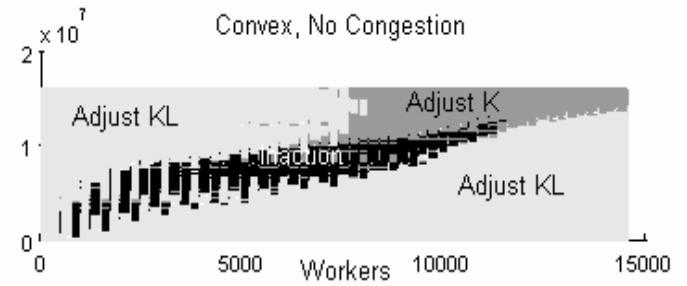
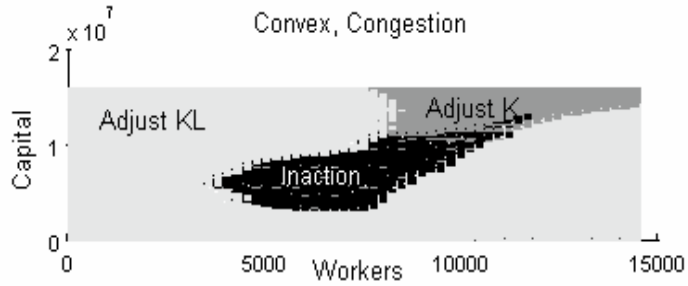


Figure 5. Decision Rules: Comparison Among Adjustment Costs, High Shock  
(High Demand Shock with High Productivity Shock)

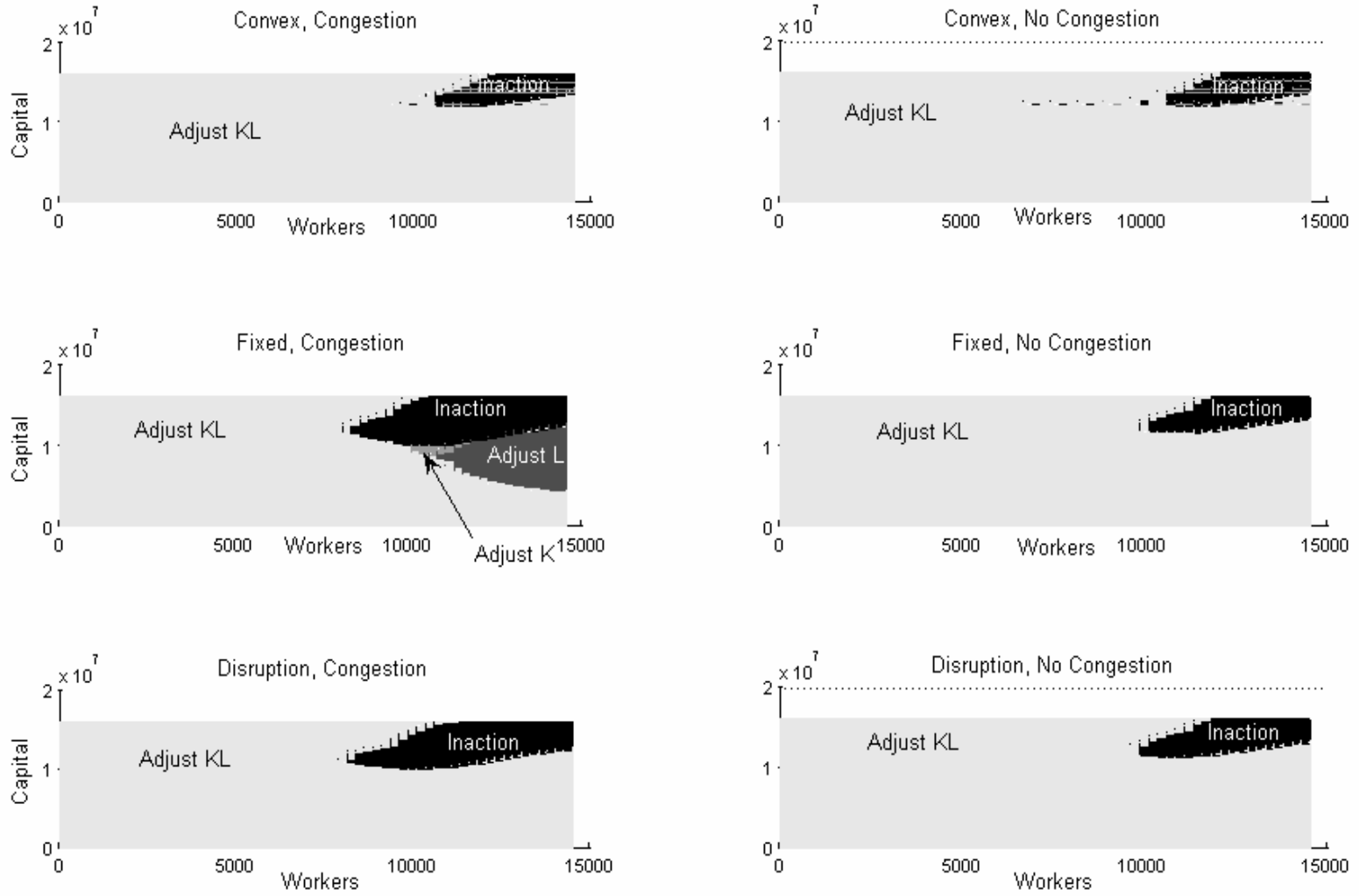


Figure 6. Labor and Capital Adjustment Levels.  
 Intermediate Shock, (High Demand Shock with Low Productivity Shock).  
 Decision Rules at the Extreme Values for Capital and Labor, Both High or Both Low

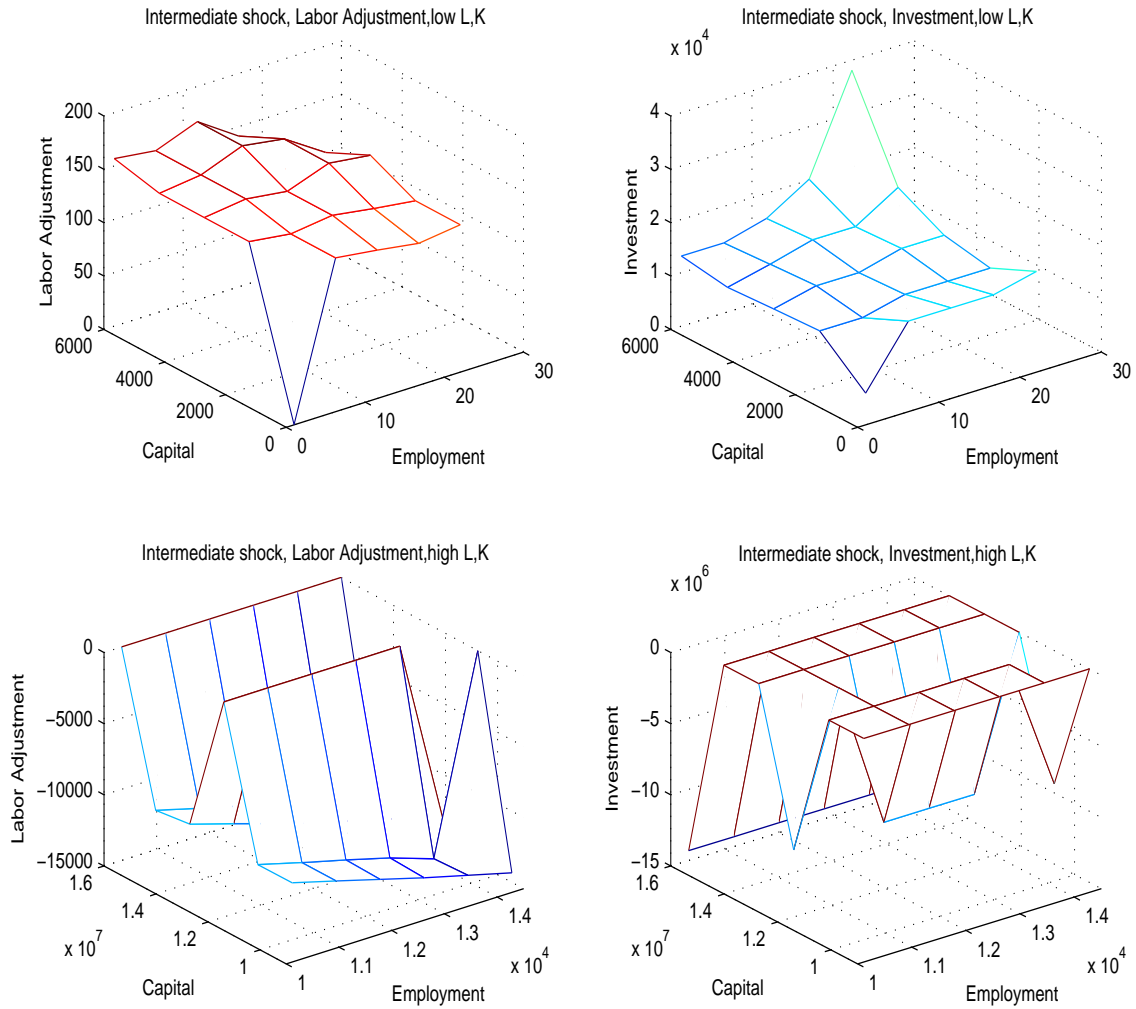


Figure 7. Labor and Capital Adjustment Levels.  
 Intermediate Shock, (High Demand Shock with Low Productivity Shock).  
 Decision Rules at the Extreme Values, Mix of High and Low values for Capital and Labor

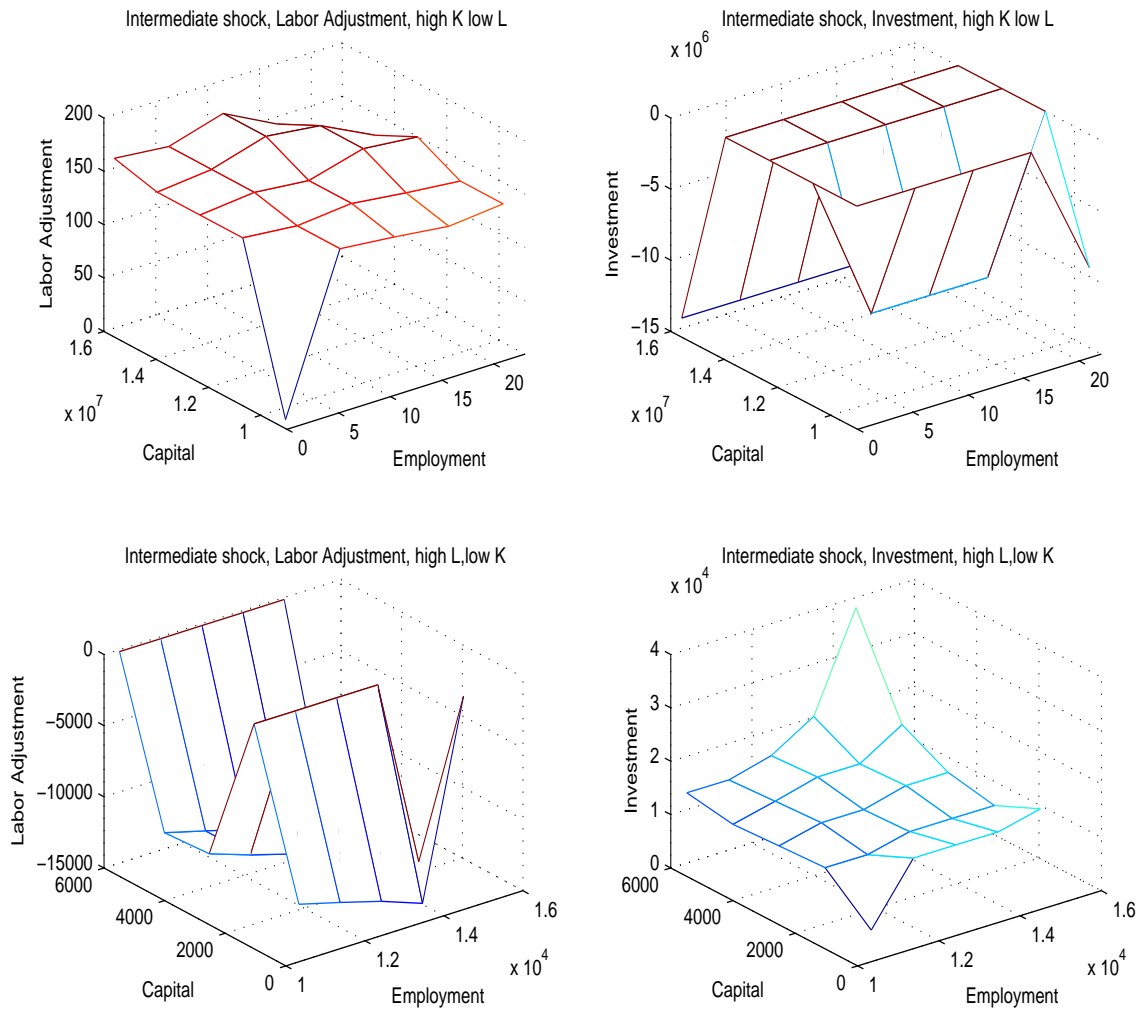
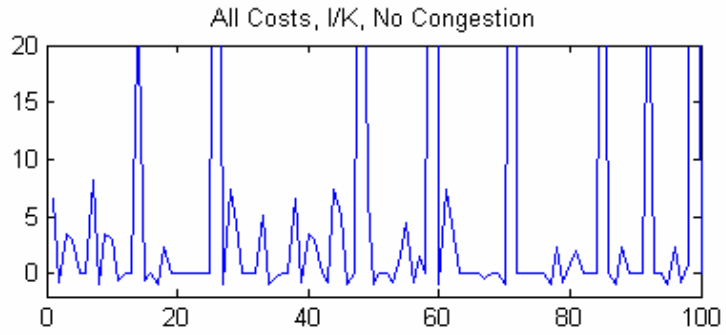
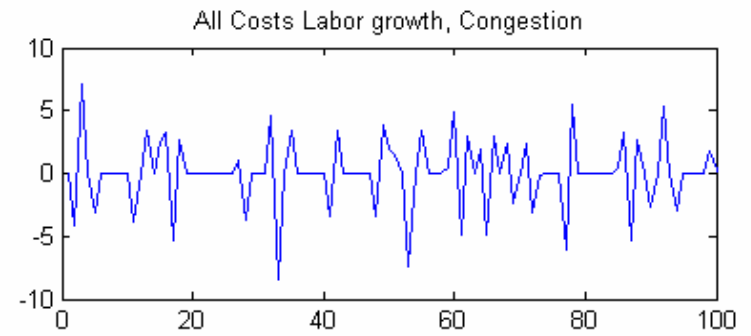
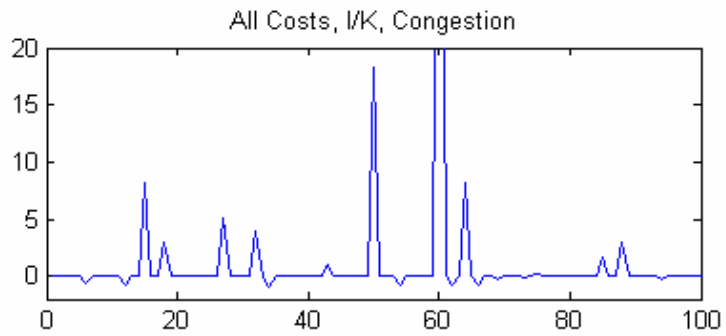
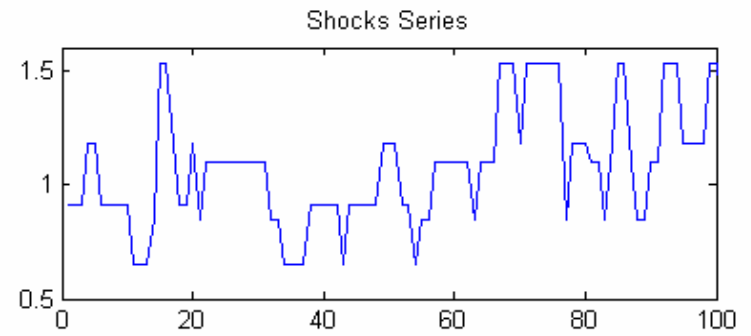
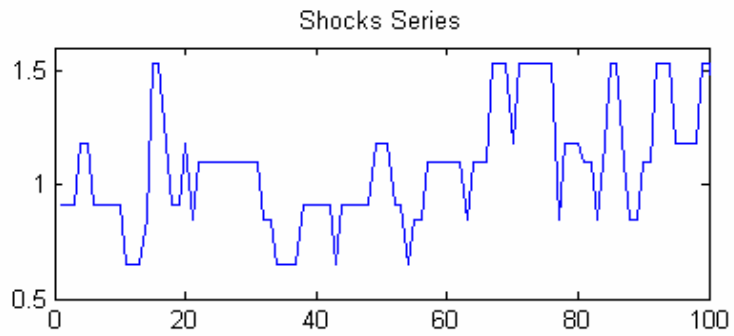


Figure 8. Time Series. All Costs Present



45



Figure 9. Time Series. Investment Rate, Adjustment Costs Comparison

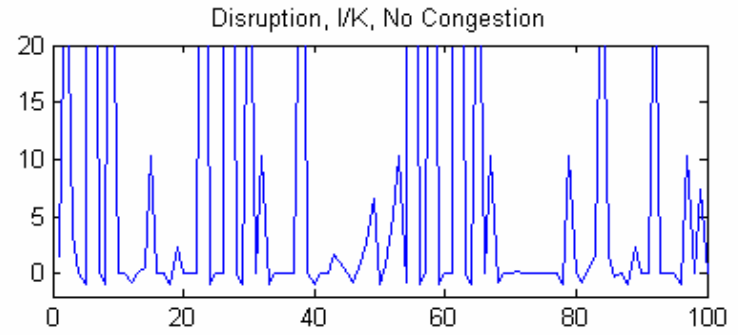
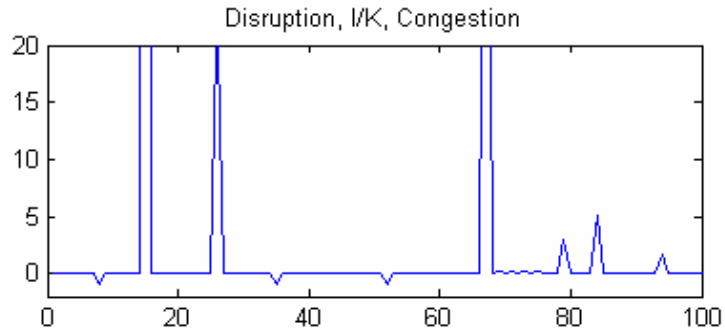
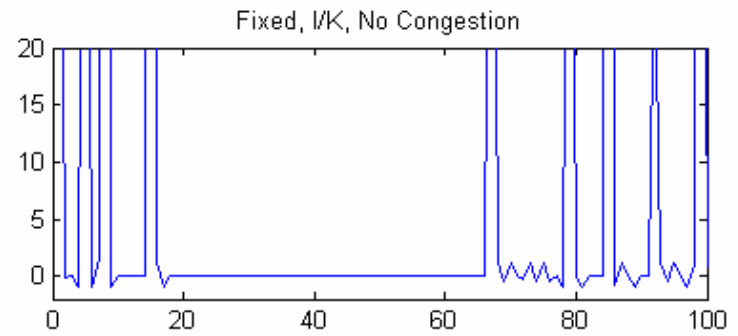
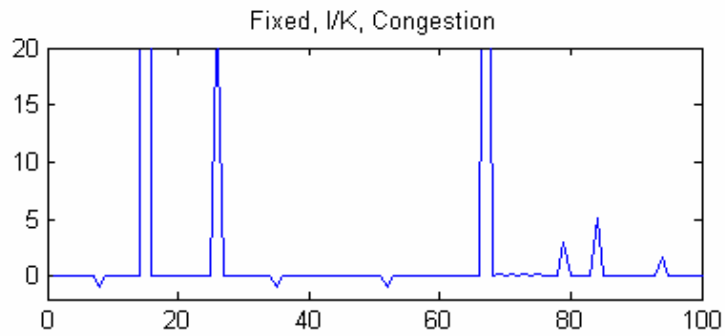
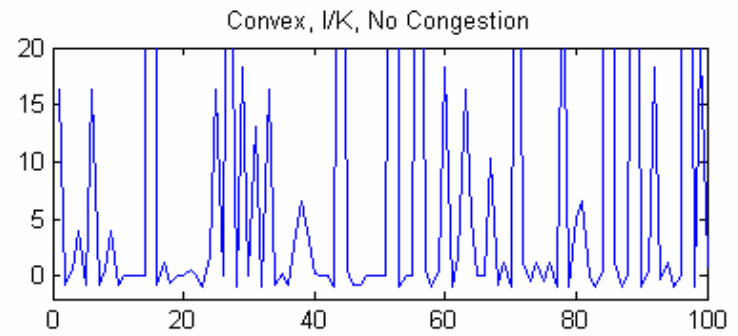
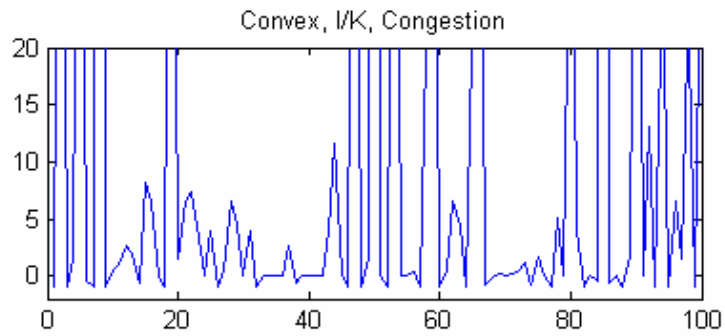


Figure 10. Time Series. Labor Growth, Adjustment Costs Comparison

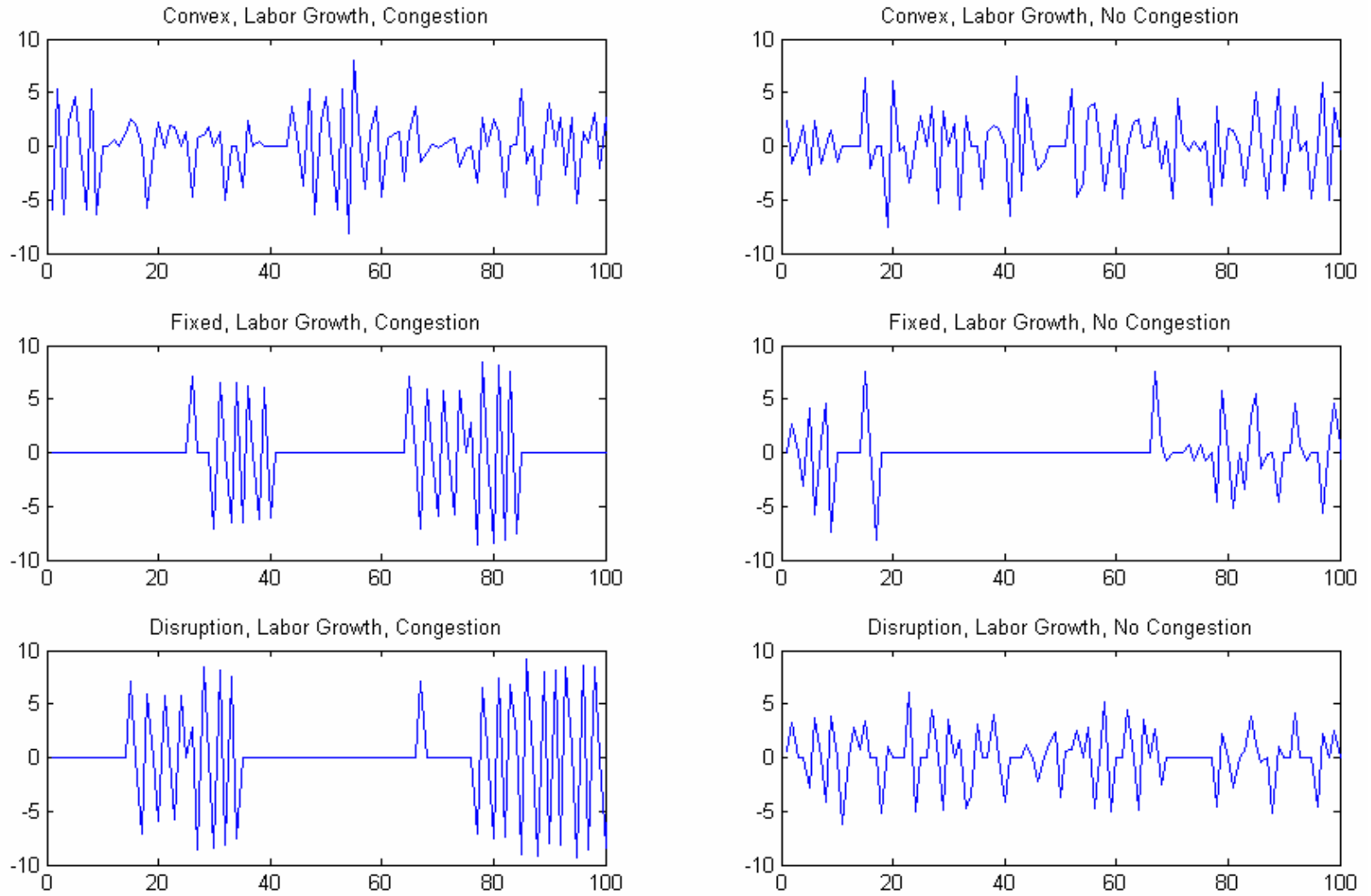
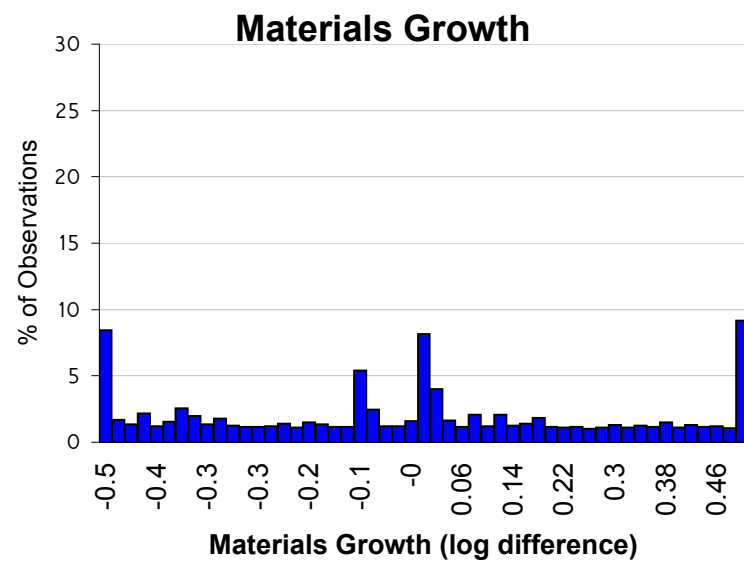
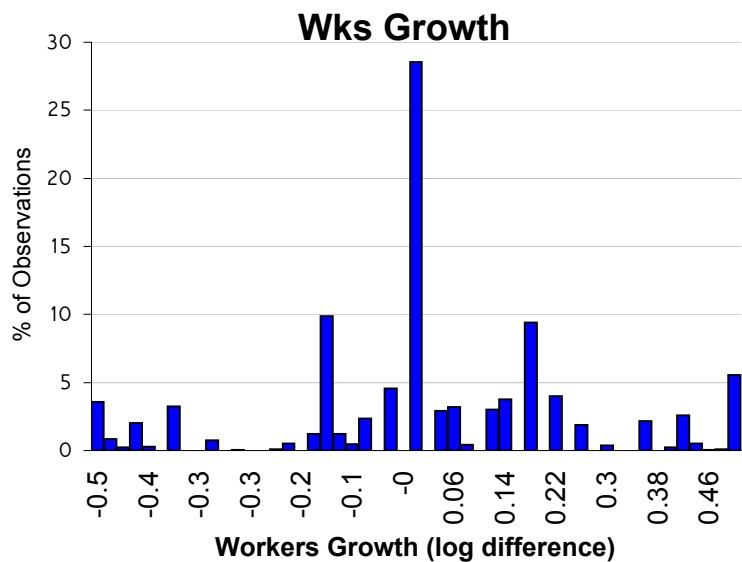
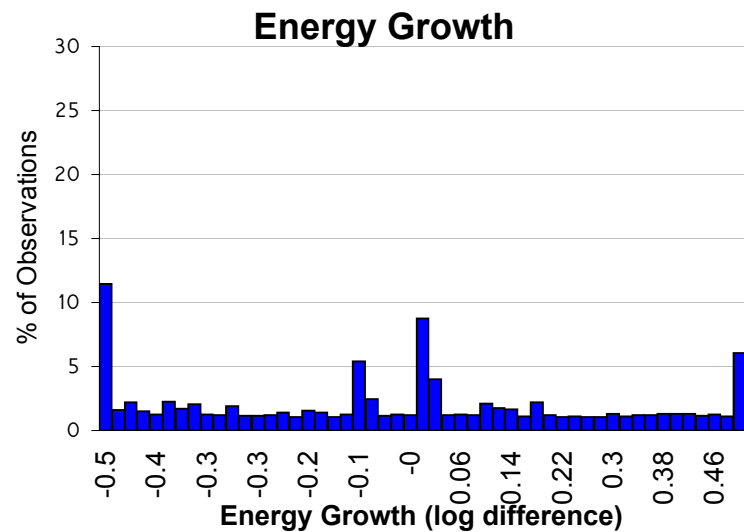
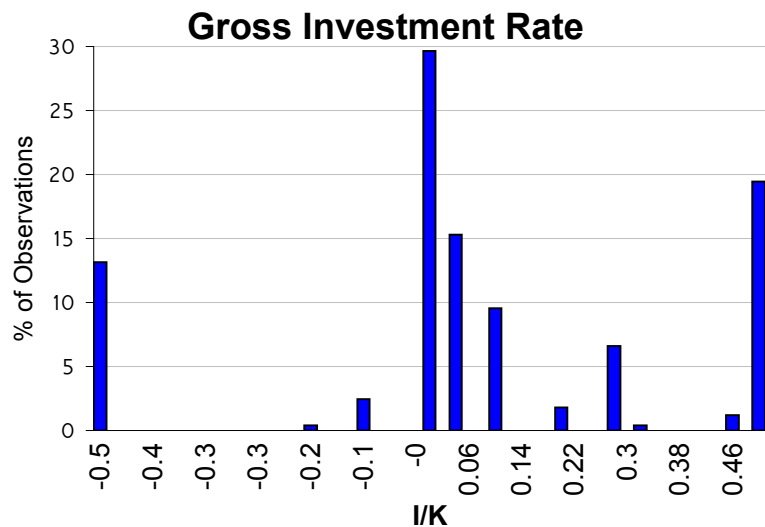


Figure 11. Distributions of Factor Adjustments for the Simulated Series.  
 Percentage of Observations (y-axis) in a Range of Adjustment (x-axis)



## References

- Abel, A. (1979). *Investment and the Value of Capital*. Garland, New York.
- Abel, A. and Eberly, J. (1994). A Unified Model of Investment Under Uncertainty. *American Economic Review*, 84:1369–1384.
- Abel, A. and Eberly, J. (1998). The Mix and Scale of Factors with Irreversibility and Fixed Costs of Investment. *Carnegie-Rochester Conferences in Public Policy*, 48:101–135.
- Alexopoulos, M. (2004). Read All about It!! What Happens Following a Technology Shock? Working paper, University of Toronto.
- Bloom, N. (2006). The Impact of Uncertainty Shocks: Firm Level Estimation and a 9/11 Simulation. Discussion Paper 718.
- Caballero, R. J. and Engel, E. (1999). Explaining Investment Dynamics in US Manufacturing: A Generalized (S,s) Approach. *Econometrica*, 67:783–826.
- Caballero, R. J., Engel, E., and Haltiwanger, J. C. (1995). Plant Level Adjustment and Aggregate Investment Dynamics. *Brookings Papers on Economic Activity*, 2:1–54.
- Caballero, R. J., Engel, E., and Haltiwanger, J. C. (1997). Aggregate Employment Dynamics: Building from Microeconomic Evidence. *American Economic Review*, 87:115–137.
- Contreras, J. M. (2006). Costs of Adjusting Production Factors: The Case of a Glass Mould Company. Chapter 2 Ph.D. Thesis, University of Maryland.
- Cooper, R. and Haltiwanger, J. C. (2005). On the Nature of Capital Adjustment Costs, forthcoming. *Review of Economic Studies*.
- Cooper, R., Haltiwanger, J. C., and Power, L. (1999). Machine replacement and the business cycle: Lumps and bumps. *American Economic Review*, 89(4):921–946.
- Cooper, R., Haltiwanger, J. C., and Willis, J. (2004). Dynamics of Labor Demand: Evidence from Plant Level Observations and Aggregate Implications. Working Paper 10297, NBER.

- Davis, S. and Haltiwanger, J. C. (1992). Gross Job Creation, Gross Job Destruction and Employment Reallocation. *Quarterly Journal of Economics*, 107(3):819–863.
- Davis, S., Haltiwanger, J. C., and Shuh, S. (1996). *Job Creation and Destruction*. The MIT Press.
- Dixit, A. and Pindyck, R. (1994). *Investment Under Uncertainty*. Princeton University Press.
- Eslava, M., Haltiwanger, J. C., Kugler, A., and Kugler, M. (2004). The Effects of Structural Reforms on Productivity and Profitability Enhancing Reallocation: Evidence from Colombia. *Journal of Development Economics*, 75(2):333–371. Special Issue.
- Eslava, M., Haltiwanger, J. C., Kugler, A., and Kugler, M. (2005). Employment and Capital Adjustment after Factor Market Deregulation: Panel Evidence from Colombian Plants. Working paper.
- Foster, L., Haltiwanger, J. C., and Syverson, C. (2005). Reallocation, Firm Turnover and Efficiency: Selection on Productivity or Profitability? Working Paper 11555, NBER.
- Hall, G. and Rust, J. (2003). Simulated Minimum Distance Estimation of a Model of Optimal Commodity Price Especulation with Endogenously Sampled Prices. Working paper.
- Hall, R. (2004). Measuring Factor Adjustment Costs. *Quarterly Journal of Economics*, 119:899–927.
- Hall, R. and Jorgenson, D. W. (1967). Tax Policy and Investment Behavior. *American Economic Review*, pages 391–414.
- Hamermesh, D. (1989). Labor Demand and the Structure of Adjustment Costs. *American Economic Review*, 79(4):674–689.
- Hamermesh, D. and Pfann, G. (1996). Adjustment Costs in Factor Demand. *Journal of Economic Literature*, 34:1264–1292.

- Klette, T. J. and Griliches, Z. (1996). The Inconsistency of Common Scale Estimators When Output Prices Are Unobserved and Endogenous. *Journal of Applied Econometrics*, 11(4):343–361.
- Letterie, W., Pfann, G., and Polder, M. (2004). Factor Adjustment Spikes and Interrelation: An Empirical Investigation. *Economic Letters*, 85:145–150.
- Nadiri, I. and Rosen, S. (1969). Interrelated Factor Demand Functions. *American Economic Review*, 59:457–471.
- Narazani, E. (2004). Interrelationship Between Labor and Capital Adjustment Decisions. Working paper.
- Polder, M. and Verick, S. (2004). Dynamics of Labour and Capital Adjustment-A Comparison of Germany and the Netherlands. Discussion Paper 1212, IZA.
- Rendon, S. (2005). Job Creation and Investment in Imperfect Capital and Labor Markets. Working paper, ITAM.
- Rust, J. (1987). Optimal Replacement of GMC Bus Engines: An Empirical Model of Harold Zurcher. *Econometrica*, 55:999–1033.
- Sakellaris, P. (2004). Patterns of Plant Adjustment. *Journal of Monetary Economics*, 51:425–450.
- Shapiro, M. (1986). The Dynamic Demand for Capital and Labor. *Quarterly Journal of Economics*, 101:513–542.
- Shea, J. (1997). The Input Output Approach to Instrument Selection. *Journal of Business and Economic Statistics*, 11:145–155.
- Syverson, C. (2005). Market Structure and Productivity: A Concrete Example. *Journal of Political Economy*, 112(6):1181–1222.
- Tauchen, G. and Hussey, R. (1991). Quadrature-Based Methods for Obtaining Approximate Solutions to Nonlinear Asset Pricing Models. *Econometrica*, 59:371–396.
- Tobin, J. (1969). A General Equilibrium Approach to Monetary Theory. *Journal of Money, Credit and Banking*, 1:15–29.