On the (Sub)optimality of Multi-tier Hierarchies: Coordination versus Motivation

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On the (Sub)optimality of Multi-tier Hierarchies: Coordination versus Motivation*

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Abstract

This paper studies internal organization of a firm using an incomplete contracting approach à la Grossman-Hart-Moore and Aghion-Tirole. The two key ingredients of our model are externalities among tasks that require coordination, and investment in task-specific human capital. We compare three types of organizational structures: centralization where the decision authority for all tasks is given to the party without task-specific human capital, decentralization where the decision authority for each task is given to the party with necessary human capital, and hierarchical delegation where the decision authority is allocated in a hierarchical fashion. Centralization is optimal when externalities and the requisite coordination are the main issue in organization design. Decentralization is optimal if the investment in human capital is more important. Hierarchical delegation is optimal in the intermediate case. We also discuss the optimal pattern of hierarchical delegation as well as several directions extending the basic model.

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1 Introduction

Hierarchies are ubiquitous in modern corporations. Alfred Chandler's account of American corporate history takes us back to the mid-19th century for the genesis of hierarchical organizations. The victory of the railway over the waterway transformed the organizational form of American businesses: "It meant the employment of a set of managers to supervise [these] functional activities over an extensive geographical area; and the appointment of an administrative command of middle and top executives to monitor, evaluate, and coordinate the work of managers responsible for the day-to-day operations. [...] Hence, the operational requirements of the railroads demanded the creation of the first administrative hierarchies in American business" (Chandler, 1977, p. 87). As the organizational operation expanded and the problem of managerial overload became apparent, the hierarchical structure evolved further into what has become known as a multi-divisional, or M-form, organization (Williamson, 1981). The distinctive advantage of an M-form organization is its scale and scope (Chandler, 1990), which M-form organizations could achieve by creating steep organizational hierarchies (Rajan and Zingales, 2000). In M-form organizations, corporate headquarters make key strategic decisions while operating decisions are delegated to profit centers. Within each profit center, there is further delegation of some decision rights to middle-level managers, who delegate part of their decision authorities further down the hierarchy. Despite Peter Drucker's prediction that future businesses would be modeled on a symphony like Mahler's Eighth, where a single conductor leads more than 1,000 participants without any intermediaries or assistants, hierarchical organizations still dominate modern businesses (Leavitt, 2003).

Why are there hierarchies? Under what circumstances do hierarchies perform better than alternative forms of organizations? This paper offers an economic rationale for hierarchies from an incentive perspective.[1] More specifically, our focus is on multi-tier hierarchies rather than two-tier hierarchies that have been studied extensively in the principal-agent literature. A general conclusion from the existing studies is that centralized organizations dominate hierarchies unless there are elements that prevent the well-functioning of centralized organizations.[2] That is, we need to look at an environment in which centralized contracting is costly. In line with the incomplete contracting approach, this paper focuses on an environment where the costs of writing 'complete' contracts are prohibitively large. Our model builds upon the property rights literature of Grossman-Hart-Moore (Grossman and Hart, 1986; Hart and Moore, 1990; Hart, 1995), and the control rights literature initiated by Aghion and Tirole (1997).

[1]The management literature adds that hierarchies fulfill our psychological needs for order and security (Leavitt, 2003).

[2]This literature is reviewed later in this section.

The key insight from Grossman and Hart (1986) is that ownership of assets, or the residual control over assets, leads to improved incentives to make specific investments. The cost of ownership held by one party is the reduced investment incentives from the other party whose ownership is taken away. Hart and Moore (1990) extend the Grossman-Hart model into a multiple-agent, multiple-asset setting where the ownership of physical capital can be extended to a control over human capital. The focus in Aghion and Tirole (1997) is the allocation of authority, or decision rights, within the firm. Granting formal authority to employees not only improves their incentives for information gathering necessary for decision making, but also encourages their participation. The cost of giving away formal authority is the principal's loss of control. Likewise, we identify the costs and benefits of different organizational structures when contracts specify only the allocation of decision rights, and subsequent bargaining over the organization's outcome. There are two main ingredients in our model that we draw from Grossman-Hart-Moore and Aghion-Tirole. As in Aghion-Tirole, we focus on the allocation of authority within the organization. The authority, as in Grossman-Hart-Moore, leads to larger ex post bargaining power, which in turn improves the party's incentive for specific investment. However, neither authority nor specific investment is sufficient for an explanation of hierarchies. An organizational answer would be either complete centralization or complete delegation, not an intermediate form such as hierarchy. We thus add a third ingredient: externalities in decisions that necessitate coordination within the organization.

To illustrate our main point, consider an organization with three agents, whom we call the manager, agent A and agent B. There are two tasks called task A and B. The outcome from each task depends on task-specific decisions and the relevant agent's input such as, say, human capital. Only agent A (resp. B) has the necessary input for task A (resp. B). Moreover there are externalities between the tasks. For example, task A can be product design for which agent A has expertise while task B can be sales of the product for which agent B has expertise. Agent A can design a single standard product at lower costs than several differentiated, more marketable products. The latter choice will make it easier for agent B to market the products: given the same level of human capital, agent B's performance will be better when products are more marketable. Externalities can run in both directions as well. For example, the two agents can choose the degree of complementarities between the products they make.[3] The manager plays a potential role of coordinating decisions for the two tasks. We say 'potential' since, in some organizational structure, the manager's coordination role may not be called for. Since the level of human capital investment is each agent's own

[3] Although not directly concerned with internal organization of a firm, Che and Hausch (1999) provide various examples of 'cooperative' investments.

decision, an organizational structure depends on who makes a decision for each task, which is identified with the authority over the task.

We consider three types of organizational structures. In *centralization*, the manager makes decisions for both tasks. In *decentralization*, each agent makes a decision for the task he has expertise in, which makes the manager's input unnecessary. In *hierarchical delegation*, the manager makes a decision for one task while one of the agents plays the role as a 'middleman' making a decision for the other task. As in Grossman-Hart-Moore, the party that makes a task-specific decision bargains over the ex post outcome from the task with the agent whose human capital is necessary to complete the task.[4] Thus the agent with decision-making authority has bargaining power over and above what is conferred upon him through his human capital. Then the costs and benefits of alternative organizational structures become apparent. Centralization can internalize externalities but reduces the agents' incentives to invest in human capital since the marginal return from their investment would be shared with the manager. In decentralization, each agent's incentive to invest in human capital is higher, but externalities cannot be internalized since each agent cares only about the outcome from his own task. An optimal organizational structure is the one that optimally balances these costs and benefits.

Our main conclusions can be summarized as follows. Centralization becomes an optimal organizational structure when externalities and the requisite coordination are the main issue in organization design. On the other hand, if externalities are not significant relative to the importance of human capital, decentralization becomes an optimal organizational structure. Hierarchical delegation emerges as an optimal organizational structure in the intermediate case. It 'empowers' the agent at the middle tier, who has stronger incentives to invest in his human capital compared to centralization, and internalizes externalities to some extent, although the agent at the bottom tier has the least incentives to invest in human capital. Although it is intuitive why the intermediate range of coordination benefits is a necessary condition for hierarchy to be optimal, it is by no means trivial to show that it is also a sufficient condition.

Additional conclusions are drawn regarding the optimal pattern of hierarchical delegation. If the agents are asymmetric in their ability, then the agent who is more efficient *and* better motivated through empowerment should be at the middle tier of the hierarchy. The latter condition stresses the importance of *marginal* incentives to invest in human capital. We show, by an example, that the more efficient agent should not necessarily be at the middle tier of a hierarchy if his marginal incentives are lower than the

[4] We focus on the symmetric Nash bargaining solution as in Grossman and Hart (1986) for the main part of the analysis. Our conclusions are shown to remain intact when we adopt solution concepts such as the Shapley value as in Hart and Moore (1990).

other agent. We discuss further the optimal pattern of hierarchical delegation when the externalities are one-sided. In this case, the beneficiary rather than the benefactor has to be at the middle tier of the hierarchy since the beneficiary can make a decision that can internalize the externality. In further extensions of our basic model, we show that our results are robust to an alternative bargaining solution such as the Shapley value. Finally, the case for hierarchical delegation is shown to be weakened if either of the agents has unlimited expertise or there is a possibility of renegotiation before decisions are made.

There is ample evidence on how organizational structures change to tackle the tradeoff between coordination and motivation. As human capital becomes more and more important relative to inanimate physical capital in modern corporations, corporate hierarchies tend to become flatter (Rajan and Zingales, 2000, 2001). Such flattening of hierarchies is more pronounced in industries with more intensive human capital than those with more intensive physical capital (Rajan and Wulf, 2003).[5] The recent move towards empowerment of employees at lower tiers of the corporate hierarchy is also more visible in industries such as banking and finance, and high technology. On the other hand, as corporations face increasingly serious coordination problems, they tend to switch to a more centralized structure. For example, Hewlett Packard underwent substantial changes in its organizational structure through the 1980s (Milgrom and Roberts, 1992, p. 547). It had more or less a multi-divisional structure with each division given considerable autonomy so that innovation and creativity could be encouraged. With introduction of its first personal computer in 1980, the problem of coordination among divisions became the central organizational issue for Hewlett Packard. To cope with this problem, Hewlett Packard experimented with increasing centralization through the 1980s.[6]

We close this section with a brief review of the relevant literature. There is an extensive literature on incentive-based explanations of hierarchy.[7] For concreteness, it is useful to call the optimal centralized mechanism without collusion the second best, and that with the possibility of collusion the third best.[8] Baron and Besanko (1992), and Gilbert and Riordan (1995) establish

[5]Rajan and Wulf (2003) show that the intensity of physical capital as measured by the real value of fixed assets per employee is positively and significantly correlated with the depth of hierarchy in an organization.

[6]An alternative to centralization in achieving coordination is to foster communication between divisions as adopted by General Motors and PepsiCo (Alonso, Dessein and Matouschek, 2008).

[7]Other strands of literature on multi-tier hierarchy are concerned with the issue of monitoring and loss of control (Williamson, 1967; Calvo and Wellisz, 1978; Qian, 1994), or the information-processing capacity of hierarchy (Radner, 1993; Van Zandt, 1999). Hart and Moore (2005) focus on the allocation of authority within hierarchies but without the element of incentives.

[8]They can be also called the collusion-free mechanism and the collusion-proof mecha-

the equivalence of the second best and a hierarchy when risk-neutral agents provide complementary inputs to production. McAfee and McMillan (1995) consider a hierarchy subject to limited liability constraints, showing losses involved in a hierarchy relative to the second best. In an adverse selection environment but without limits on communication, Melumad, Mookherjee and Reichelstein (1995) show that the second-best outcome can be achieved using a hierarchy and proper sequencing of the hierarchical contracts. Laffont and Martimort (1998) further establish the equivalence of the second best and a hierarchy even when there are limits on communication. An additional conclusion from Laffont and Martimort is that, with limits on communication (anonymity of centralized contracts in their model), a hierarchy can dominate the third best. Mookherjee and Tsumagari (2004) generalize the model in Laffont and Martimort to show that hierarchical delegation is in general strictly dominated by the third best due to double marginalization of rents. The equivalence of a hierarchy and the third best is established in a moral hazard environment by Baliga and Sjöström (1998), and Macho-Stadler and Pérez-Castrillo (1998), and in a principal-supervisor-agent setup by Faure-Grimaud, Laffont and Martimort (2003). The general conclusion from these studies is that, to have a hierarchy as an efficient organizational form, there need to be some transactions costs in using the centralized mechanism.

Although not directly concerned with hierarchies, several studies focus on the benefits of delegation. Itoh (1992, 1993) shows that, in a multiple-agent moral hazard environment, the principal can benefit by allowing coalition of agents, when agents can monitor each other. Dessein (2002) shows that delegation of authority can dominate communication of local information when communication is strategic and the goals of the principal and the agent are not too disparate. Alonso, Dessein and Matouschek (2008) and Rantakari (2008) address how organizations can achieve coordination through decentralization and lateral communication of local information. In these studies, cheap talk communication between division managers is the vehicle through which coordination is achieved, the benefits of which improve with decentralization. Our paper complements these studies by showing that, in the absence of such communication, hierarchical decentralization can be another way to improve coordination relative to complete decentralization.

The rest of the paper is organized as follows. Section 2 introduces the basic model. Section 3 describes three organizational structures and solves for equilibrium in each of them. Section 4 compares the three organizational structures and provides our main dominance results. In Section 5, we discuss various ways our basic model can be extended. Section 6 concludes the paper. All proofs are relegated to the appendix.

nism, respectively.

2 The Model

We consider a simple organization that consists of three risk-neutral parties, called manager and agents A and B, whose reservations payoffs are normalized to zero. There are two projects, called project A and B. Implementation of project i ($i = A, B$) requires a decision and agent i's human capital. The decision for project A (B, resp.) is denoted by $a \in \mathcal{A}$ ($b \in \mathcal{B}$, resp.) where $\mathcal{A} = \mathcal{B} \equiv \{S, C\}$. Decision S is *selfish* in that, compared to decision C, it leads to a higher return for one project, but a smaller external benefit for the other project. Decision C is *cooperative* in that it results in a lower return for one project, but a larger external benefit for the other project. Examples of these decisions have been discussed in the previous section.

Agent i chooses the level of human capital denoted by $e_i \in [0, E]$, which incurs cost $c(e_i)$. We assume that $c' > 0$, $c'' > 0$, $c'(0) = c(0) = 0$, $c'(E) = \infty$ and $c(E) < \infty$. In our basic model, the two agents are assumed symmetric. We discuss the case of asymmetric agents in Section 5.1. The outcome from each project is binary and is called success or failure. The success probability of project n ($n = A, B$) depends on the level of human capital chosen by agent n and is denoted by $P(e_n)$, which satisfies $P' > 0$, $P'' < 0$ and $P(e) \in (0, 1)$ for all $e \in [0, E]$. When project A succeeds, its realized return is given by

$$h(a) + q(b). \tag{1}$$

Thus the return from project A is affected not only by decision a but also by decision b for project B. Similarly, when project B succeeds, its realized return is given by $h(b) + q(a)$.[9] If a project fails, its return is zero.

The choice of human capital is each agent's inalienable decision. However, the decision right for each project can be allocated. A party is said to have the *authority* over project A (B, resp.) when the party has the right to choose a (b, resp.) and to implement the project if it is successful. Note that our definition of authority goes beyond the usual definition as, for example, in Grossman-Hart-Moore.[10] The usual definition involves only the right to make a decision whereas, in our model, authority combines the right to make a decision *and* the right to implement it. We believe that separating decision making and implementation processes describes more

[9]Externalities are thus two-sided and symmetric. Symmetry is not essential for our main results but simplifies notation and analysis. In Section 5.2, we discuss the case of one-sided externality.

[10]In the Grossman-Hart-Moore setting, only the investment in human capital is made at the ex ante stage while renegotiation over the surplus proceeds before the decision is made. This gives bargaining power to the party who is initially allocated ownership even though ownership entails only the right to make a decision. It is a matter of interpretation but we are inclined to favor our interpretation as the one that describes better the processes of project choice and implementation within an organization. In Section 5.5, we discuss how our results change when we adopt the time-line as in Grossman-Hart-Moore.

accurately how business organizations operate in reality.[11] The party that makes a decision learns valuable knowledge about the project during the decision-making process, which is indispensable in the subsequent implementation of the project. Then it is clear why it is reasonable to assign the right to make a decision and the right to implement a project to the same party. The knowledge about and the right to implement a project is the source of bargaining power for the party with authority. This is described in more detail below. We follow the incomplete contracting literature and assume that none of the variables (a, b) and (e_A, e_B) are contractible. The initial contract thus specifies the allocation of authorities only. The timing of the game is as follows:

1. Date 0 (Organization Choice): An organizational structure is chosen by determining the allocation of authorities. We focus on the optimal structure that maximizes the total payoff for the organization as a whole.[12]

2. Date 1 (Decisions and Human Capital Investment): All parties simultaneously choose a, b, e_A and e_B. The parties with relevant authorities make decisions over the projects, and each agent chooses the level of human capital in addition to the decision over the project, should he be allocated the authority. The realized values of a, b, e_A and e_B are observed by all parties at the end of date 1.

3. Date 2 (Bargaining and Implementation): The relevant parties bargain over the return from each successful project. We assume that the bargaining takes place bilaterally and leads to a symmetric Nash bargaining outcome. For each project, there are at most two relevant parties: the party with authority and the party with necessary human capital.[13] We further assume that the bargaining proceeds sequentially: a negotiation over the return from project A occurs first and, given this bargaining outcome, a negotiation over the return from project B takes place.[14]

[11] See, for example, Khalil et al. (2006).

[12] For example, if all the parties have unlimited wealth, then the organization that maximizes the total payoff is optimal due to the Coase theorem. Alternatively one can imagine the design problem faced by outside stakeholders, say, shareholders.

[13] If one party has the authority as well as necessary human capital, then that party has the full claim over the return.

[14] As an example, consider hierarchical delegation where agent A is at the middle tier of the hierarchy. Then, the manager (who has the authority over project A) first bargains with agent A (who has necessary human capital for project A) over the return from project A. Next, agent A (who has the authority over project B) bargains over the return from project B with agent B (who has necessary human capital for project B). The parties with authorities implement successful projects and the return is realized.

We offer some elaboration on the above time-line. First, for each pair bargaining over the return from the project, the bargaining power is derived from the authority and/or necessary human capital. The party with authority may refuse to implement a successful project while the party with necessary human capital may threaten to withdraw his human capital. Thus the authority and human capital are both indispensable in realizing the return from the successful project.[15] Second, we assume that decisions (a, b) and the levels of human capital (e_A, e_B) are simultaneously determined at date 1. This is equivalent to assuming that the agents invest in (e_A, e_B) before decisions (a, b) are made, but (e_A, e_B) remain private information when the decisions are made. In this scenario, there is no room for renegotiation of the initial allocation of authorities. However, if the levels of human capital are chosen and *observed* before the decisions are made, then a mutually beneficial renegotiation of allocating authorities may occur in the interim stage, given the levels of human capital. Section 5.5 discusses this case. Third, we adopt the specific bargaining protocol whereby bargaining over the return from each project takes place bilaterally and sequentially, and each bilateral bargaining leads to a symmetric Nash bargaining outcome. This is for the sake of clarity in our analysis. In Section 5.3, we show that our results do not change if we use the Shapley value to calculate the bargaining payoff for each party.

In what follows, we will adopt the shorthand notation: $h_s \equiv h(S)$, $h_c \equiv h(C)$, $q_s \equiv q(S)$ and $q_c \equiv q(C)$. Also let $\Delta h \equiv h_s - h_c$ and $\Delta q \equiv q_c - q_s$. We maintain the following assumption throughout the paper.

Assumption 1. *(i)* $h_s > h_c \geq 0$, *(ii)* $\Delta q > \Delta h$, *and (iii)* $q_s \geq 0$.

Assumption 1 says that the selfish decision maximizes the return from each project (Assumption 1(i)) but the cooperative decision maximizes the overall return from the two projects, provided the two agents choose the same level of human capital (Assumption 1(ii)). To see this, note that the ex ante total surplus is given by $P(e_A)[h(a) + q(b)] + P(e_B)[h(b) + q(a)] - c(e_A) - c(e_B)$. Then Assumption 1(ii) implies that, given $e_A = e_B$, the decision that maximizes the above surplus is $a = b = C$. It is easy to see that $a = b = C$ along with $e_A = e_B = e^* \equiv \phi(h_c + q_c)$ constitute the first-best outcome where ϕ denotes the inverse function of c'/P'.

Since there are three parties and two decisions, there are nine possible

[15] Of course, the party with authority will always implement the project as long as his/her bargaining payoff is nonnegative. Similarly, the party with necessary human capital will refrain from withdrawing his human capital as long as his bargaining payoff is nonnegative. One might ask whether the authority to implement the project can be transferred to another party at date 2. We assume this is not possible since the party with authority learns the details about the project through his/her decision making, which cannot be transferred to another party.

organizational structures depending on the allocation of initial authorities. This is shown in Table 1 where the column corresponding to each party lists various cases in which the party has the authority over relevant decision(s). In *centralization*, the manager has the authority over both projects. In *partial decentralization*, the manager has the authority over project n ($n = A, B$) while agent k ($k \neq n$) has the authority over project k. Intuitively such partial decentralization is unable to achieve coordination while depriving motivation from agent n. In Section 3.2, we show that partial decentralization is indeed dominated by decentralization, which is defined next. In *decentralization*, each agent is delegated the authority over one project. Two cases arise. First, agent n is allocated the authority over project n ($n = A, B$). Second, agent n is allocated the authority over project k where $n \neq k$ ($n, k = A, B$), which we call cross-authority decentralization. In Section 3.2, we show that cross-authority decentralization turns out to be equivalent to centralization.

In *hierarchical delegation*, the authority over one project, say project A, is allocated to the manager, while the authority over the other project, i.e., project B, is allocated to agent A. In this case, a three-tier hierarchy is characterized by successive allocation of authorities where agent A plays the role as a 'middleman' while agent B is at the bottom tier of the hierarchy. The three-tier hierarchy can be best understood as a chain of command where the party at the lower tier should 'report' to the immediate superior. For example, consider hierarchical delegation where agent A is the middleman. Then agent B needs to report to agent A, who makes a decision that governs agent B's task. Agent A in turn needs to report to the manager, who makes a decision that governs agent A's task. Finally, in *concentrated delegation*, one agent is given authority over both projects. Clearly this should be superior to hierarchical delegation because the coordination benefits can be secured without loss in the incentives to invest in human capital; in both types of delegation, the agent without authority suffers from the same lack of incentives to invest in human capital. This is shown in Section 5.4.

— Table 1 goes about here. —

In our basic model, we assume that each agent has limited expertise, implying that each agent can deal with at most one project. Thus we do not consider the case of concentrated delegation except in Section 5.4. The limited expertise is partly due to the time constraint each agent faces in learning various aspects about a project in order to make a decision. Moreover, agents need to commit sufficient resources to investment in human capital. Indeed, limited expertise is one of the reasons why decision-making is delegated in a hierarchical fashion despite possible coordination problems. The manager, on the other hand, can make decisions for both projects since she may have some prior knowledge about the projects from past management experience, nor does she need to make investment in project-specific human capital. That the manager does not have limited expertise is a necessary condition for centralization to be feasible in the first place.

Assumption 2. *Each agent can deal with the decision for at most one project.*

Assumption 2, along with afore-mentioned discussions, allows us to focus our analyses on three types of organizational structures: centralization, decentralization, and hierarchical delegation.

3 Organizational Structures and Equilibria

3.1 Centralization

In centralization, the manager has the authority over both projects. Due to our assumption on the bargaining protocol, the manager first bargains with agent A over the return from project A. Then the bargaining over the return from project B proceeds between the manager and agent B.

Suppose that project A succeeds. Then, given the realization of the return $h(a) + q(b)$, the bargaining between the manager and agent A leads to the following payoff for the manager:

$$\frac{1}{2}\{h(a) + q(b)\}. \tag{2}$$

Similarly, when project B succeeds, the manager obtains the following payoff from the negotiation with agent B over the return from project B:

$$\frac{1}{2}\{h(b) + q(a)\}. \tag{3}$$

Thus the manager's total expected payoff is given by

$$P(e_A)\frac{1}{2}\{h(a) + q(b)\} + P(e_B)\frac{1}{2}\{h(b) + q(a)\}. \tag{4}$$

Agent A's expected payoff is

$$\frac{1}{2}P(e_A)\{h(a) + q(b)\} - c(e_A), \tag{5}$$

while agent B's expected payoff is

$$\frac{1}{2}P(e_B)\{h(b) + q(a)\} - c(e_B). \tag{6}$$

At date 1, the manager chooses a and b while the agents choose e_A and e_B to maximize respective expected payoff. Let us denote

$$\overline{e} \equiv \phi\left(\frac{1}{2}(h_s + q_c)\right), \quad \underline{e} \equiv \phi\left(\frac{1}{2}(h_c + q_s)\right) \tag{7}$$

where ϕ was defined as $\phi \equiv (c'/P')^{-1}$.

In the above, $\overline{e}$ ($\underline{e}$, resp.) is the level of human capital that agent n ($n = A, B$) will choose if he bargains over the return from project n with one other party, and if the decision for project n is a selfish (cooperative, resp.) one while the decision for project k ($k \neq n$) is a cooperative (selfish, resp.) one. Since ϕ is monotone increasing, we must have $\overline{e} > \underline{e}$. Then we obtain the following.

Proposition 1. *Suppose the following condition holds:*

$$P(\overline{e})\Delta h < P(\underline{e})\Delta q. \tag{*}$$

Then, in centralization, there exists a unique equilibrium in which the manager chooses $a = b = C$ and both agents choose the same level of human capital given by

$$e^c \equiv \phi\left(\frac{1}{2}(h_c + q_c)\right). \tag{8}$$

Proof. See Appendix.

Condition (*) states that the external benefit from cooperative decision (Δq) is large enough relative to the importance of human capital. Since the manager has the authority over both projects, she would fully internalize the externality in this case. On the other hand, each agent's incentive to invest in human capital is distorted downward as he obtains only a half of the return, i.e., $(1/2)(h_c + q_c)$, by bargaining with the manager. Note that $e^c < e^* \equiv \phi(h_c + q_c)$, the level of investment in the first-best outcome. Note also that, given $a = b = C$, the agents choose the same level of human capital.

If condition (*) is violated, then it can be shown that an asymmetric equilibrium exists in which the manager chooses $a = S$ and $b = C$, agent A chooses $e_A = \overline{e}$, and agent B chooses $e_B = \underline{e}$. In Section 3.3, we show that this asymmetric equilibrium is equivalent to the one under hierarchical delegation. To make the welfare comparison of different organizational structures nontrivial, we maintain condition (*) throughout the paper. Then the total equilibrium expected payoffs under centralization are

$$V_C \equiv 2\{P(e^c)(h_c + q_c) - c(e^c).\} \tag{9}$$

3.2 Decentralization

There are two possible cases of decentralization. Consider first the case of *cross-authority decentralization* in which agent A (B, resp.) is allocated the authority over project B (A, resp.). In this case, agent n ($n = A, B$) obtains the following expected payoff

$$(1/2)P(e_A)(h(a) + q(b)) + (1/2)P(e_B)(h(b) + q(a)) - c(e_n). \tag{10}$$

The following lemma shows that, under condition (*), cross-authority decentralization leads to the same equilibrium outcome as in centralization.

Lemma. *Suppose that agent n is assigned the authority over project k ($n, k = A, B$, $n \neq k$). Then, if condition (*) is satisfied, there exists a unique equilibrium in which both agents choose the same level of human capital e^c and $a = b = C$.*

Proof. See Appendix.

The above lemma shows that decentralization leads to an outcome different from centralization only when agent n is allocated the authority over project n, $n = A, B$. In what follows, we will refer to this as decentralization: each agent is allocated the authority over the project for which his human capital is indispensable. This implies that each agent enjoys the full return from the relevant project.

Then agent A chooses a and e_A to maximize the expected payoff:

$$P(e_A)\{h(a) + q(b)\} - c(e_A), \tag{11}$$

given agent B choosing b and e_B. By Assumption 1 (i), agent A then chooses $a = S$ irrespective of e_A. Similarly, agent B chooses b and e_B to maximize

$$P(e_B)\{h(b) + q(a)\} - c(e_B), \tag{12}$$

given a and e_A. Thus agent B chooses $b = S$.

Given $a = b = S$, agent n's expected payoff is

$$P(e_n)(h_s + q_s) - c(e_n). \tag{13}$$

Maximizing this with respect to e_n leads to

$$e_n = e^d \equiv \phi(h_s + q_s). \tag{14}$$

Then the total equilibrium expected payoffs under decentralization are

$$V_D \equiv 2\{P(e^d)(h_s + q_s) - c(e^d)\}. \tag{15}$$

Before turning to hierarchical delegation, we need to comment on partial decentralization: the manager has the authority over one project, say project A, and delegates the authority over project B to agent B. It is easy to see that such an organizational structure is dominated by decentralization. Observe first that neither the manager nor agent B cares about the externality. The manager's payoff depends only on the return from project A, hence she will choose $a = S$ to maximize her expected payoff. Agent B will also choose $b = S$ since his payoff depends only on the return from project B. Then agent B's expected payoff is $P(e_B)(h_s + q_s) - c(e_B)$, the same as that in decentralization. Thus agent B chooses the same level of human capital, e^d, as in decentralization. However, agent A's expected payoff is $(1/2)P(e_A)(h_s + q_s) - c(e_A)$, hence his equilibrium level of human capital, denoted by $\hat{e}$, is strictly less than that in decentralization. Since e^d maximizes $P(e)(h_s + q_s) - c(e)$ over $e \in [0, E]$ and $e^d \neq \hat{e}$, we must have $V_D > P(\hat{e})(h_s + q_s) - c(\hat{e}) + P(e^d)(h_s + q_s) - c(e^d)$. Therefore partial decentralization is dominated by decentralization.

3.3 Hierarchical delegation

We now study hierarchical delegation where the manager has the authority over one project, say project A, and delegates to agent A the authority over project B. In this case, agent A plays the role of a "middle agent", who bargains with the manager over the return from project A since his human capital is needed to complete project A, and with agent B over the return from project B since he makes the decision for project B.

Then the manager's expected payoff from bargaining with agent A is

$$\frac{1}{2}P(e_A)\{h(a) + q(b)\}, \tag{16}$$

and agent B's expected payoff from bargaining with agent A is

$$\frac{1}{2}P(e_B)\{h(b) + q(a)\} - c(e_B). \tag{17}$$

On the other hand, agent A obtains a half of the return from each successful project. Thus his expected payoff is

$$\frac{1}{2}P(e_A)\{h(a) + q(b)\} + \frac{1}{2}P(e_B)\{h(b) + q(a)\} - c(e_A). \tag{18}$$

At date 1, the manager chooses a to maximize her expected payoff in (16), agent B chooses e_B to maximize his expected payoff in (17), and agent A chooses b and e_A to maximize his expected payoff in (18). Since the manager does not care about the return from project B, it is clear that she will make a selfish decision $a = S$ by Assumption 1 (i), irrespective of b, e_A and e_B. Agent A, however, cares about the return from both projects, implying that he will internalize the externality to some extent by making a cooperative decision $b = C$. Moreover his incentive to invest in human capital is also higher compared to centralization. Insofar as the externality is not fully internalized, agent B's incentive to invest in human capital is lower than in centralization. More formally, we can establish the following result.

Proposition 2. *In hierarchical delegation, there exists a unique equilibrium in which the manager chooses $a = S$, the agent at the middle tier of the hierarchy chooses $b = C$ and $\overline{e} \equiv \phi((1/2)(h_s + q_c))$, and the agent at the bottom tier chooses $\underline{e} \equiv \phi((1/2)(h_c + q_s))$.*

Proof. See Appendix.

It is easy to check $\underline{e} < e^c < \overline{e}$. That is, in hierarchical delegation, the middle agent chooses a higher level of human capital than in centralization although the bottom agent chooses a lower level of human capital. Two effects arise under hierarchal delegation. First, since the manager has the authority over project A only, she will choose the selfish strategy $a = S$ to maximize only the return from project A. This can raise agent A's bargaining payoff from project A because his human capital is necessary in implementing project A. Second, since agent A has the authority over project B, he has, due to Assumption 1, the incentive to internalize the externality by choosing the cooperative strategy $b = C$ for project B, although it will reduce agent B's incentive to invest in human capital.

Let V_H denote the total expected payoffs under hierarchal delegation:

$$V_H \equiv P(\overline{e})(h_s + q_c) - c(\overline{e}) + P(\underline{e})(h_c + q_s) - c(\underline{e}). \tag{19}$$

In Table 2, we summarize the results we obtained so far.

— Table 2 goes about here. —

4 Comparison of Organizational Structures

The analyses of the previous section highlight the costs and benefits of different organizational structures. Centralization can coordinate decisions for the projects, thereby internalizing the externalities. But the agents' incentives to invest in human capital are weaker than in decentralization. The cost of decentralization is the lack of coordination and the accompanied loss of external benefits. In hierarchical delegation, the 'empowered' middle agent has the strongest incentive to invest in human capital, at the same time internalizing the externalities to some extent through his decision-making authority over one project. The downside is that the 'disempowered' bottom agent has the weakest incentive to invest in human capital. This section offers a more detailed welfare comparison of the three organizational structures.

The comparison of centralization and decentralization is rather straightforward. For this, let us call Δq the coordination benefit, which is the increase in the return from one project regardless of its decision when the decision for the other project changes from S to C. Similarly, let us call Δh the deviation benefit, which is the increase in the return from one project when its own decision changes from C to S regardless of the decision for the other project. We may then call $\Delta q - \Delta h$ the relative coordination benefit. Then we can show the following.

Proposition 3. *(i) If Δq is sufficiently large, then centralization dominates decentralization. (ii) If $\Delta q - \Delta h$ converges to zero, then decentralization dominates centralization. (iii) If both Δq and Δh converge to zero, then decentralization becomes optimal, i.e., it dominates both centralization and hierarchical delegation.*

Proof. See Appendix.

The intuition behind Proposition 3 can be offered as follows. If the coordination benefit is large, then the two decisions should be made by a single party, i.e., the manager. Thus centralization becomes optimal. On the other hand, if the relative coordination benefit is small, the main issue is how to motivate the agents to invest in their human capital. Since such incentives are stronger in decentralization than in centralization, the former dominates the latter when $\Delta q - \Delta h$ becomes small. However, this does not necessarily mean that decentralization also dominates hierarchical delegation. It is because, in hierarchical delegation, the middle agent still has the strongest incentive even in the limiting case where $\Delta q - \Delta h$ converges to zero: his choice of human capital is given by $\phi((1/2)(h_s + q_c)) = \phi((1/2)(h_c + q_s + \Delta h + \Delta q))$, which converges to $\phi((1/2)(h_c + q_s + 2\Delta h))$ when $\Delta q \to \Delta h$. Thus the middle agent is motivated to invest in human capital not only through the coordination benefit but also through the deviation benefit. Insofar as these benefits are significant, hierarchical delegation provides strong incentives to the middle agent. Therefore, it is not clear whether decentralization becomes optimal even in the limiting case of $\Delta q \to \Delta h$. However, if both Δq and Δh become sufficiently small, the level of human capital chosen by the middle agent in hierarchical delegation converges to that chosen by the bottom agent. In this case, not only does the coordination benefit of hierarchical delegation disappear, but also 'empowerment' fails to provide strong incentives to invest in human capital. In such a limiting case, decentralization becomes optimal.

Then when does hierarchical delegation emerge as an optimal organizational structure? Our discussions so far suggest it would be the case in the intermediate range of the coordination benefit. To show this as clearly as possible, we will simplify the basic model and notation as follows: $h \equiv h_s > 0$,

$h_c = 0$, $q \equiv q_c > 0$ and $q_s > 0$ where $q \geq q_s + h$.

Then condition (*) can be written as

$$P(\phi((1/2)(h+q)))h < P(\phi((1/2)q_s)(q - q_s). \tag{*}$$

Since $P(\cdot) < 1$, one sufficient condition for condition (*) to hold is

$$h/P(\phi((1/2)q_s) + q_s \leq q.$$

Given the above simplification and condition (*), our results so far can be summarized again. In centralization, both agents choose the same level of human capital $e^c \equiv \phi((1/2)q)$ and the total expected payoffs are given by

$$V_C \equiv 2\{P(e^c)q - c(e^c)\}. \tag{20}$$

In decentralization, both agents choose the same level of human capital $e^d \equiv \phi(h + q_s)$ and the total expected payoffs are given by

$$V_D \equiv 2\{P(e^d)(h + q_s) - c(e^d)\}. \tag{21}$$

In hierarchical delegation, the middle agent chooses the level of human capital $\overline{e} \equiv \phi((1/2)(h+q))$ and the bottom agent chooses $\underline{e} \equiv \phi((1/2)q_s)$, and the total expected payoffs are given by

$$V_H \equiv P(\overline{e})(h+q) - c(\overline{e}) + P(\underline{e})q_s - c(\underline{e}). \tag{22}$$

The following proposition shows that hierarchical delegation is indeed an optimal organizational structure in the intermediate range of the coordination benefit.

Proposition 4. *If $h > 0$, $q_s > 0$ are small and $h_c = 0$, then there exists an interval $(\underline{q}, \overline{q})$ such that, for all $q \in (\underline{q}, \overline{q})$, hierarchical delegation is an optimal organizational structure.*

Proof. See Appendix.

Figure 1 shows the total expected payoffs in different organizational structures as the functions of $q \equiv q_c$ where $h \equiv h_s > 0$ and q_s are sufficiently small and $h_c = 0$. Given the simplification of parameter values, q approximates the coordination benefit and h represents the deviation benefit. As shown in the figure, when the deviation benefit is sufficiently small, decentralization is never optimal for any value of q. On the other hand, centralization becomes optimal when q becomes large. Hierarchical delegation dominates the other two in the intermediate range of q such that condition (*) is not violated.

— Figure 1 goes about here. —

5 Extensions of the Basic Model

This section discusses several extensions of our basic model. First, we introduce some asymmetries: the agents may differ either in how their human capital contributes to the success of projects, or in their cost of investment in human capital. Given the asymmetries, we study the optimal pattern of hierarchical delegation. That is, who should be at the middle tier of the hierarchy? Second and related to the first extension, we consider the case of one-sided externality: the external benefit flows from one project to the other, but not the other way around. We again study the optimal pattern of hierarchical delegation. Third, our results so far have been derived using the symmetric Nash bargaining solution to determine the outcome from each bilateral bargaining. We show that they are robust to alternative bargaining solutions such as the Shapley value. Fourth, we allow the agents to have sufficient expertise so that both decisions can be made by a single agent, if necessary. We show in this case that hierarchical delegation is never optimal, which confirms the folklore that limited expertise is one of the reasons for hierarchical organizations. Finally, we look at alternative timing of the game in which the project-specific decisions are made after the agents choose to invest in their human capital and observe each other's choice. This opens up the possibility of renegotiation over the allocation of decision authority. Hierarchical delegation is again shown to be suboptimal in this case.

5.1 The case of asymmetric agents

The agents were assumed symmetric in our basic model. Therefore either of the two agents can be at the middle tier of a hierarchy. We now relax the symmetry assumption to discuss the optimal pattern of hierarchical delegation. That is, we ask who should be the middle agent in optimal hierarchical delegation when the agents are asymmetric. Asymmetry can be introduced either through the success probability of each project or through the agents' cost of investment in human capital. Since the main benefit of hierarchical delegation stems from stronger incentives provided to the middle agent, one could conjecture that the agent who can be better motivated through empowerment should be at the middle tier of the hierarchy. In what follows, we confirm this conjecture.

Let us denote by $P_n(e) \in (0,1)$ and $c_n(e)$ the success probability of project n and agent n's cost of investing in human capital, respectively. Denote the inverse function of c_n'/P_n' by $\phi_n \equiv (c_n'/P_n')^{-1}$. Assume, as before, that ϕ_n is increasing. Recall that ϕ_n' measures agent n's marginal incentive to invest in human capital. Then we can show the following.

Proposition 5. *If* $2P_n(\phi_n(x)) + c_n'(\phi_n(x))\phi_n'(x) > 2P_j(\phi_j(x)) + c_j'(\phi_j(x))\phi_j'(x)$ *for all* $x \in [(1/2)(h_c+q_s), (1/2)(h_s+q_c)]$ *for* $n \neq j$*, then* *hierarchical delegation with agent n at the middle tier dominates hierarchical delegation with agent $j \neq n$ at the middle tier.*

Proof. See Appendix.

Although the condition in Proposition 5 may seem rather complicated, we can verify that it is satisfied in the following case: $c(e) \equiv c_A(e) = c_B(e)$ but $P_n(e) > P_j(e)$, $P'_n(e) > P'_j(e)$ for all $e \in [0, E]$, and $\phi'_n(x) > \phi'_j(x)$ for all $x \in [(1/2)(h_c + q_s), (1/2)(h_s + q_c)]$. In other words, the agents have the same cost function but their success probabilities and marginal incentives defined by ϕ'_n are different. In this case, since $\phi_n(x) > \phi_j(x)$ holds, one can verify the condition in Proposition 5 using $c'' > 0$ and $P'_i > 0$. It seems reasonable to interpret the condition to imply that agent n is more efficient than agent j in terms of success probabilities - both total and marginal - and better motivated in terms of marginal incentives. Then the above proposition says that the more efficient agent who can be better motivated should be assigned to the middle tier of a hierarchy. This intuition continues to hold when the agents differ in their costs of investment in human capital. Consider a quadratic example: $c_n(e) = (\gamma_n/2)e^2$ and $P_n(e) = \eta_n e$ where $\gamma_n > 0$ and $\eta_n > 0$. Then $\phi_n(x) = (\eta_n/\gamma_n)x$, hence $2P_n(\phi_n(x)) + c'_n(\phi_n(x))\phi'_n(x) = 3(\eta_n^2/\gamma_n)x$. Thus, if agent n is more efficient than agent j in the sense that $\gamma_n < \gamma_j$ and $\eta_n > \eta_j$, then agent n should be assigned to the middle tier of a hierarchy. It is straightforward to check $\phi'_n > \phi'_j$ in this case, hence agent n also has a higher marginal incentive.

However, it is not necessarily the case that the more efficient agent, e.g., whose cost is lower than the other, should always be at the middle tier of a hierarchy. An additional condition is that the middle agent should also have a higher marginal incentive than the other. To illustrate this, we take an extreme example: $c_A(e) \equiv 0$ for all $e \in [0, E]$ but $c_B(e)$ has the same property as in the previous example. We also assume that $P_A(e) > P_B(e)$ for all $e \in [0, E]$. Thus agent A is more efficient than agent B in the sense that his cost is lower and his success probability is higher. However, agent A will always choose the highest level of human capital E in any organizational structures since $c_A(e) \equiv 0$. Thus his marginal incentive is zero. On the other hand, agent B will choose $\underline{e}_B \equiv \phi_B((1/2)(h_c + q_s))$ if he is at the bottom tier, and $\overline{e}_B \equiv \phi_B((1/2)(h_s + q_c))$ if he is at the middle tier.[16]

The total expected payoffs under hierarchical delegation are then

$$V_H^A \equiv P_A(E)(h_s + q_c) + P_B(\underline{e}_B)(h_c + q_s) - c_B(\underline{e}_B), \tag{23}$$

[16]Here we are assuming that the middle agent makes the cooperative decision C regardless of who is at the middle tier of the hierarchy. This is true when agent A is at the middle tier because $P_A(E)\Delta q > P_B(e_B)\Delta h$ always holds for all $e_B \in [0, E]$. When agent B is at the middle tier, he would choose $b = C$ if $P_B(\hat{e}_B)\Delta q \geq P_A(E)\Delta h$ where $\hat{e}_B \equiv \phi_B((1/2)(h_s + q_s))$. We assume this condition is satisfied.

when agent A is at the middle tier of a hierarchy, and

$$V_H^B \equiv P_B(\bar{e}_B)(h_s + q_c) + P_A(E)(h_c + q_s) - c_B(\bar{e}_B). \tag{24}$$

when agent B is at the middle tier of a hierarchy.

Taking the difference between the two expected payoffs and using $\phi \equiv (c'/P')^{-1}$, we have

$$V_H^A - V_H^B = \int_{(1/2)(h_c+q_s)}^{(1/2)(h_s+q_c)} \{2P_A(E) - [2P_B(\phi_B(x)) + c'_B(\phi_B(x))\phi'_B(x)]\}dx \tag{25}$$

which is negative if

$$2P_A(E) < 2P_B(\phi_B(x)) + c'_B(\phi_B(x))\phi'_B(x) \tag{26}$$

for all $x \in [(1/2)(h_c + q_s), (1/2)(h_s + q_c)]$. To see when condition (26) holds, consider the quadratic example again: $P_B(e) = \eta_B e$, $P_A(e) = \eta_A e$, where $\eta_A > \eta_B$, and $c_B(e) = (\gamma_B/2)e^2$ where $\gamma_B > 0$. We also have $c_A(e) \equiv 0$. Since $2P_B(\phi_B(x)) + c'_B(\phi_B(x))\phi'_B(x) = 3(\eta_B^2/\gamma_B)x$, the above inequality will hold if $2P_A(E) < 3(\eta_B^2/\gamma_B)[(1/2)(h_c + q_s)]$. In this case, hierarchical delegation with agent A in the middle performs worse than that with agent B in the middle, even if agent A is more efficient than agent B.

We summarize the discussion in this subsection. When the agents are asymmetric, the optimal pattern of hierarchical delegation depends not only on the relative efficiency in terms of success probabilities and costs, but also on the marginal incentive that empowerment provides to each agent. If one agent is more efficient as well as better motivated, then he should be at the middle tier of a hierarchy. However, the more efficient agent should not necessarily be at the middle tier of a hierarchy if his marginal incentive is lower than the other.

5.2 One-sided externality

In many organizations, externalities may run only in one direction. For example, the production division can choose to produce a single standard product at lower costs, or several differentiated, more marketable products at higher costs. The latter choice will make it easier for the sales division to market the products: given the same level of human capital invested in the sales division, its performance will be better when products are more marketable. The relevant question is then which division should be at the middle tier of a hierarchical organization. Intuitively, the beneficiary of the externality should make the decision for the benefactor division. The main reason is that, given the decision authority over the benefactor division, the beneficiary will make a decision that internalizes the externality. This further strengthens the beneficiary's incentive to invest in human capital.

To formalize this, suppose the externality runs from project B to project A only. The return from project B is now given by $h(b)$ while the return from project A is the same as before, $h(a) + q(b)$. The rest of the model remains the same as our basic model.

Suppose first that agent B is at the middle tier of the hierarchy. Then the manager makes the decision for project B while agent B makes the decision for project A. Clearly, the manager's optimal decision is $b = S$. Let us denote the equilibrium levels of human capital by $\tilde{e}_A$ and $\tilde{e}_B$. It is easy to see that agent B also chooses $a = S$ and, given $\tilde{e}_A$, he chooses e_B to maximize the following expected payoff:

$$\frac{1}{2}P(e_B)h_s + \frac{1}{2}P(\tilde{e}_A)(h_s + q_s) - c(e_B). \tag{27}$$

This leads to

$$\tilde{e}_B = \phi((1/2)h_s). \tag{28}$$

Given $a = b = S$ and $\tilde{e}_B$, agent A chooses e_A to maximize his expected payoff,

$$\frac{1}{2}P(e_A)(h_s + q_s) - c(e_A) \tag{29}$$

leading to

$$\tilde{e}_A = \phi((1/2)(h_s + q_s)). \tag{30}$$

Then the total expected payoffs are

$$V_H^B \equiv P(\tilde{e}_A)(h_s + q_s) + P(\tilde{e}_B)h_s - c(\tilde{e}_A) - c(\tilde{e}_B). \tag{31}$$

Suppose now that agent A is at the middle tier of the hierarchy. The manager's optimal choice is again $a = S$. Let us denote the equilibrium levels of human capital by e'_A and e'_B. Given $a = S$ and e'_B, agent A chooses b and e_A to maximize his expected payoff,

$$\frac{1}{2}P(e_A)(h_s + q(b)) + \frac{1}{2}P(e'_B)h(b) - c(e_A). \tag{32}$$

Agent A chooses $b = C$ if and only if $P(e'_A)\Delta q \geq P(e'_B)\Delta h$. It is easy to see that, for any $b \in \{S, C\}$, we have $e'_A > e'_B$. Thus agent A's optimal decision is $b = C$ and his equilibrium level of human capital is given by

$$e'_A = \phi((1/2)(h_s + q_c)). \tag{33}$$

Given $a = S, b = C$ and e'_A, agent B's equilibrium level of human capital is given by

$$e'_B = \phi((1/2)h_c). \tag{34}$$

The total expected payoffs are then

$$V_H^A \equiv P(e'_A)(h_s + q_c) + P(e'_B)h_c - c(e'_A) - c(e'_B). \tag{35}$$

Summarizing the above discussions, hierarchical delegation with agent B – the benefactor of the externality – at the middle tier leads to $a = b = S$, $e_A = \tilde{e}_A$, $e_B = \tilde{e}_B$, while hierarchical delegation with agent A – the beneficiary of the externality – at the middle tier leads to $a = S$, $b = C$, $e_A = e'_A$, $e_B = e'_B$. Comparing these equilibria, the benefits of putting the beneficiary at the middle tier of the hierarchy become clear. First, the beneficiary makes a decision that internalizes the externality. Second, this further strengthens the beneficiary's own incentives to invest in human capital. As can be seen from (30) and (33), agent A's equilibrium level of human capital is higher when he is at the middle tier of the hierarchy: $e'_A > \tilde{e}_A$. We are now ready to establish the following proposition.

Proposition 6. *Suppose the externality runs from project B to project A only. Then hierarchical delegation where agent A is at the middle tier dominates hierarchical delegation where agent B is at the middle tier.*

Proof. See Appendix.

5.3 Alternative bargaining solution: Shapley value

We have so far assumed a simple bargaining structure whereby bargaining over the return from each project takes place bilaterally and sequentially, and each bilateral bargaining leads to a symmetric Nash bargaining outcome. In this subsection we show that our results remain robust if we use the Shapley value instead to determine the ex post bargaining payoff for all three parties.[17] Let us denote the return from project A by $f(a, b) \equiv h(a) + q(b)$. Then the return from project B is $f(b, a)$.

Consider first centralization. Suppose project A succeeds. Since the manager has the decision authority over project A, agent A has the necessary human capital, and agent B plays no role in project A, $f(a, b)$ is shared equally between the manager and agent A. Similarly, if project B succeeds, its return $f(b, a)$ is split equally between the manager and agent B. Thus the manager's expected payoff determined by the Shapley value is given by

$$\begin{aligned} & P(e_A)P(e_B)\{(1/2)f(a,b) + (1/2)f(b,a)\} + P(e_A)(1 - P(e_B))(1/2)f(a,b) \\ & + (1 - P(e_A))P(e_B)(1/2)f(b,a) \\ & = P(e_A)(1/2)f(a,b) + P(e_B)(1/2)f(b,a) \end{aligned} \tag{36}$$

which is the same as the manager's expected payoff in our basic model. It is easy to check that each agent's expected payoff determined by the Shapley value is also the same as the one in our basic model. Consequently, replacing

[17] See, for example, Gul (1989) for a non-cooperative game theoretic foundation of the Shapley value.

the bargaining outcomes by those based on the Shapley value does not alter any of the results.

Consider next hierarchical delegation where agent A is at the middle tier. If project A succeeds, then the manager and agent A equally split the return $f(a, b)$. If both projects succeed, then agent A obtains his Shapley value payoff $(1/2)f(a, b) + (1/2)f(b, a)$ since he has the necessary human capital for project A and the decision authority over project B. Finally, if only project B succeeds, then agent A will obtain $(1/2)f(b, a)$. Thus, agent A's expected payoff determined by the Shapley value can be written as

$$\begin{aligned}
&P(e_A)(1 - P(e_B))(1/2)f(a, b) + P(e_A)P(e_B)\{(1/2)f(a, b) + (1/2)f(b, a)\} \\
&+ (1 - P(e_A))P(e_B)(1/2)f(b, a) - c(e_A) \\
&= (1/2)P(e_A)f(a, b) + (1/2)P(e_B)f(b, a) - c(e_A) \qquad (37)
\end{aligned}$$

which is again the same as the one in our basic model. It is also straightforward to check that the expected payoffs of the manager and agent B do not change either.

Finally, in decentralization, each agent obtains the full return from his project, implying that their expected payoffs remain the same even if we replace the bargaining outcomes in our basic model by those based on the Shapley value.

5.4 The case of unlimited expertise

One of the reasons for hierarchical delegation is the agents' limited expertise. Expertise in our model is identified with the capability of exercising the authority over a given project. With limited expertise, feasible organizational structures are limited to decentralization or hierarchical delegation. Suppose now one of the agents has unlimited expertise, capable of exercising the authority over both projects. Intuitively, then, allocating the authority over both projects to that agent should dominate hierarchical delegation. There are two primary benefits. First, the agent with authority will internalize externalities. Second, this will motivate the agent without authority to invest in his human capital beyond the level chosen in hierarchical delegation. Indeed the second benefit is realized regardless of the decisions made since, in hierarchical delegation, the "disempowered" bottom agent's incentive to invest in human capital is the lowest among all organizational structures.

To formalize this, suppose that agent A is allocated the authority over both projects. We will call this organizational structure *concentrated delegation.* In this case, agent A obtains the full return from project A and a half of the return from project B. Thus agent A chooses $a \in \{S, C\}$, $b \in \{S, C\}$ and $e_A \in [0, E]$, given agent B's level of human capital e_B, to maximize his expected payoff:

$$P(e_A)(h(a) + q(b)) + \frac{1}{2}P(e_B)(h(b) + q(a)) - c(e_A). \tag{38}$$

Agent B chooses e_B to maximize his expected payoff:

$$\frac{1}{2}P(e_B)(h(b) + q(a)) - c(e_B). \tag{39}$$

Let $(\tilde{a}, \tilde{b}, \tilde{e}_A, \tilde{e}_B)$ be the equilibrium outcome in concentrated delegation. Note that agent B's equilibrium level of human capital $(\tilde{e}_B)$ is higher than that $(\underline{e})$ in hierarchical delegation with agent B at the bottom tier. This is because $\underline{e} = \phi((1/2)(h_c + q_s)) \leq \tilde{e}_B = \phi((1/2)(h(\tilde{b}) + q(\tilde{a})))$ since $h(b) \geq h_c$ and $q(a) \geq q_s$ for any a and b. Then the total expected payoffs under concentrated delegation are

$$V_{CD} \equiv P(\tilde{e}_A)(h(\tilde{a}) + q(\tilde{b})) - c(\tilde{e}_A) + P(\tilde{e}_B)(h(\tilde{b}) + q(\tilde{a})) - c(\tilde{e}_B). \tag{40}$$

The following proposition shows that concentrated delegation dominates hierarchical delegation.

Proposition 7. *Suppose that one agent has sufficient expertise to exercise the authority over both projects. Then hierarchical delegation is dominated by concentrated delegation where the authority over both projects is delegated to the agent with sufficient expertise.*

Proof. See Appendix.

5.5 Alternative timing and the possibility of renegotiation

In our basic model, the investment in human capital and the project-specific decisions are made simultaneously. A rationale for this modeling assumption was that the investment in human capital may not be observed even if the project-specific decisions are made after the investment. In this case, there is no room for renegotiating the initial allocation of authorities before the decisions are made. However, renegotiation becomes an issue if the investment in human capital is observed, albeit not verifiable, before the project-specific decisions are made.

This subsection considers alternative timing and discusses how the possibility of renegotiation changes the comparison of different organizational structures. Specifically we consider the following scenario. At date 0, an organizational structure is chosen by determining the allocation of decision authorities.[18] At date 1, the agents choose simultaneously the levels of human capital, which are then observed by all parties. Between date 1 and date

[18] Assumption 2 is retained here. That is, both agents have limited expertise.

2, called the interim stage, the parties renegotiate over the re-allocation of authorities. At date 2, the parties with authorities after renegotiation make the project-specific decisions.

To simply the argument, we will assume that $h \equiv h_s > h_c = 0$ and $q \equiv q_c > q_s = 0$. We further make the following assumption:

$$P(E)h \leq P(0)q, \tag{**}$$

which implies $P(e_i)h \leq P(e_j)q$ for all $e_i \in [0, E]$ and $e_j \in [0, E]$, $i \neq j$. In this case, the benefit from cooperative decision is large enough so that it becomes optimal to re-allocate the decision authorities at the interim stage, should that induce the cooperative decisions for both projects *whatever* levels of human capital were chosen at date 1. From our previous analysis, we know that the cooperative decisions for both projects are supported in equilibrium if and only if centralization was chosen at date 0. Therefore renegotiation at the interim stage has potential benefits if either decentralization or hierarchical delegation was chosen at date 0.

We assume the following renegotiation process. At the interim stage, the manager (agent n, resp.) can make a take-it-or-leave-it renegotiation offer to the other players with probability $\alpha_P \in (0, 1)$ ($\alpha_n \in (0, 1)$, resp.), where $\alpha_P + \alpha_A + \alpha_B = 1$. We further impose the symmetry condition: $\alpha \equiv \alpha_A = \alpha_B$ where $\alpha \in (0, 1/2)$. If the renegotiation offer is rejected by any player, then the subsequent game is played according to the initial allocation of authorities.

First, suppose decentralization was chosen at date 0. Then, after e_A and e_B are chosen at date 1, any player can make the renegotiation offer with some side-payments to the other players such that the authorities over both projects are re-allocated to the manager.[19] Given the authorities over both projects, the manager will choose $a = b = C$ after the renegotiation. This leads to the following interim total surplus:

$$P(e_A)q + P(e_B)q - [P(e_A)h + P(e_B)h]. \tag{41}$$

Agent n ($n = A, B$) then obtains the following renegotiation payoff:

$$U_n^{DR}(e_n, e_k) \equiv \alpha[P(e_A) + P(e_B)](q - h) + P(e_n)h, \tag{42}$$

which is the sum of α fraction of the renegotiation surplus and his status quo payoff $P(e_n)h$ from decentralization. Anticipating this renegotiation outcome, agent n will choose e_n to maximize $U_n^R(e_n, e_k) - c(e_n)$. The equilibrium level of human capital is thus given by

$$e^{DR} \equiv \phi(\alpha q + (1 - \alpha)h). \tag{43}$$

[19]Since lump-sum side payments represent redistribution of the return among the three parties without any incentive effects, we ignore side-payments in our analyses.

Thus the total ex ante expected payoffs are

$$V_D^R \equiv 2\{P(e^{DR})q - c(e^{DR})\}. \tag{44}$$

Next, suppose hierarchical delegation was chosen at date 0 where the manager has the authority over project A and agent A has the authority over project B. If renegotiation breaks down at the interim stage, the manager obtains the expected payoff $(1/2)P(e_A)(h+q)$, agent A, $(1/2)P(e_A)(h+q)$, and agent B, nothing.[20] As in decentralization, potential benefits of renegotiation can be realized if the authorities over both projects are reallocated to the manager. Then agent A's renegotiation payoff is

$$U_A^{HR}(e_A, e_B) \equiv \alpha\{P(e_A)q + P(e_B)q - P(e_A)(h+q)\} + \frac{1}{2}P(e_A)(h+q), \tag{45}$$

which is the sum of α fraction of the renegotiation surplus $[P(e_A)+P(e_A)]q - P(e_A)(h+q)]$ and his status quo payoff $(1/2)P(e_A)(h+q)$ from hierarchical delegation. On the other hand, agent B obtains the renegotiation payoff:

$$U_B^{HR}(e_B, e_A) \equiv \alpha\{[P(e_A) + P(e_B)]q - P(e_A)(h+q)\}. \tag{46}$$

Thus agents choose e_A and e_B to maximize $U_A^{HR}(e_A, e_B) - c(e_A)$ and $U_B^{HR}(e_B, e_A) - c(e_B)$, respectively. The equilibrium levels of human capital are then given by

$$e_A = \overline{e}^{HR} \equiv \phi\left(\frac{1}{2}q + \left(\frac{1}{2} - \alpha\right)h\right), \tag{47}$$

$$e_B = \underline{e}^{HR} \equiv \phi(\alpha q). \tag{48}$$

Thus the total ex ante expected payoffs are

$$V_H^R \equiv P(\overline{e}^{HR})q - c(\overline{e}^{HR}) + P(\underline{e}^{HR})q - c(\underline{e}^{HR}). \tag{49}$$

Since there are no benefits from renegotiation when centralization was chosen at date 0, the total payoffs remain the same as before: $V_C = 2\{P(e^c)q - c(e^c)\}$ where $e^c = \phi((1/2)q)$. Then we can show the following.

Proposition 8. *Suppose $\phi'' \leq 0$[21] and that renegotiation is possible after the investments in human capital are made but before the project-specific decisions are made. Then hierarchical delegation is never optimal. If $h > 0$ is small enough, then centralization becomes optimal; otherwise, decentralization becomes optimal.*

Proof. See Appendix.

[20] Note that we are assuming $h_c = q_s = 0$.

[21] The quadratic example we studied before satisfies $\phi'' \leq 0$. With $P(e) = \eta e$ and $c(e) = (\gamma/2)e^2$, we have $\phi'' = 0$.

6 Conclusion

This paper has studied internal organization of a firm using an incomplete contracting approach à la Grossman-Hart-Moore and Aghion-Tirole. The two key ingredients of our model are externalities among tasks that require coordination, and investment in task-specific human capital. The return from each task is shared between the relevant parties through ex post bargaining. This is due to our assumption on contracting technology: in the absence of complete, binding contracts, the return is shared between the party with decision authority and the party with necessary human capital. Depending on how the decision authority over each task is allocated, we have compared three types of organizational structures: centralization where the decision authority for all tasks is given to the party without task-specific human capital, decentralization where the decision authority for each task is given to the party with necessary human capital, and hierarchical delegation where the decision authority is allocated in a hierarchical fashion.

Our main findings can be summarized as follows. Centralization can coordinate the externalities, hence becomes optimal when the externalities and the requisite coordination are the main issue in organization design. The downside is that, compared to decentralization, centralization leads to reduced incentives to invest in human capital. Decentralization becomes optimal if the provision of incentives for investment in human capital is the central issue. However, decentralization fails to coordinate the externalities. Hierarchical delegation is optimal in the intermediate case. It empowers the agent at the middle tier of the hierarchy, who is given stronger incentives to invest in human capital than in centralization and to coordinate the externalities to some extent. On the other hand, the incentive to invest in human capital is the weakest for the agent at the bottom tier of the hierarchy.

We have also discussed several directions where the basic model is extended. The first is related to the optimal pattern of hierarchical delegation, which concerns who should be at the middle tier of the hierarchy. If the agents are asymmetric in their ability, then the agent who is more efficient *and* better motivated through empowerment should be at the middle tier of the hierarchy. If the externalities are one-sided, then the beneficiary rather than the benefactor has to be at the middle tier of the hierarchy. Second, our results are shown to be robust to an alternative bargaining solution such as the Shapley value. Third, we have confirmed the folklore that managers' limited expertise is a necessary condition for hierarchical organizations. With unlimited expertise, a hierarchical organization is dominated by the one where the authority is concentrated. Finally, we have considered alternative timing of the game and introduced the possibility of renegotiation. It was shown that hierarchical delegation is again dominated by other organizational structures. Thus the case for hierarchical delegation is weakened if either of the agents has unlimited expertise or there is a possibility of renegotiation before decisions are made.

7 Appendix

7.1 Proof of Proposition 1

First, suppose the manager chooses $a = b = C$ in equilibrium. Then agent n ($n = A, B$) chooses e_n to maximize his expected payoff:

$$\frac{1}{2}P(e_n)(h_c + q_c) - c(e_n).$$

By the definition of e^c, the above expected payoff is maximized when $e_n = e^c$. Given $e_A = e_B = e^c$, the manager optimally chooses $a = b = C$ to maximize her expected payoff:

$$\frac{1}{2}P(e^c)(h(a) + q(b)) + \frac{1}{2}P(e^c)(h(b) + q(a)).$$

This is due to Assumption 1 (ii). Thus, $a = b = C$ and $e_A = e_B = e^c$ constitute an equilibrium in centralization.

Next, we will show that no other equilibria exist. Suppose that there exists an equilibrium in which $e_A \neq e_B$. For this to be possible in equilibrium, it must be that $a \neq b$. Otherwise, $a = b$ leads to $e_A = e_B$. Without loss of generality, suppose that $a = S$ and $b = C$. Then agent A chooses e_A to maximize

$$\frac{1}{2}P(e_A)(h_s + q_c) - c(e_A),$$

which leads to:

$$e_A = \overline{e} \equiv \phi\left(\frac{1}{2}(h_s + q_c)\right).$$

Similarly, agent B chooses e_B to maximize

$$\frac{1}{2}P(e_B)(h_c + q_s) - c(e_B),$$

which leads to:

$$e_B = \underline{e} \equiv \phi\left(\frac{1}{2}(h_c + q_s)\right).$$

Given $e_A = \overline{e}$ and $e_B = \underline{e}$, the manager chooses a and b to maximize

$$\frac{1}{2}P(\overline{e})[h(a) + q(b)] + \frac{1}{2}P(\underline{e})[h(b) + q(a)].$$

The assumed choice of $a = S$, rather than $a = C$, is optimal for the manager if and only if

$$P(\overline{e})h_s + P(\underline{e})q_s \geq P(\overline{e})h_c + P(\underline{e})q_c,$$

which is equivalent to $P(\overline{e})\Delta h \geq P(\underline{e})\Delta q$. But this contradicts condition (*). Thus, no asymmetric equilibria exist.

Finally, at any symmetric equilibrium with $e_A = e_B$, it must be that $a = b = C$ by Assumption 1 (ii). Thus the equilibrium we derived is a unique equilibrium under centralization. Q.E.D.

7.2 Proof of Lemma

We will first show that there exists an equilibrium in which both agents choose e^c and $a = b = C$. Given $e_B = e^c$ and $a = C$, agent A chooses e_A and b to maximize his expected payoff:

$$(1/2)P(e_A)(h_c + q(b)) + (1/2)P(e^c)(h(b) + q_c) - c(e_A).$$

Thus $b = C$ is optimal for agent A if and only if $P(e_A)\Delta q \geq P(e^c)\Delta h$. Agent A's expected payoff is then

$$\begin{aligned}
&(1/2)\{P(e_A)(h_c + q_c) + P(e^c)(h_c + q_c)\} - c(e_A) \text{ if } P(e_A)\Delta q \geq P(e^c)\Delta h, \\
&(1/2)\{P(e_A)(h_c + q_s) + P(e^c)(h_s + q_c)\} - c(e_A) \text{ if } P(e_A)\Delta q < P(e^c)\Delta h.
\end{aligned}$$

Under condition (*), we have $P(\underline{e})\Delta q > P(\overline{e})\Delta h > P(e^c)\Delta h$ since $\overline{e} \equiv \phi((1/2)(h_s + q_c)) > e^c \equiv \phi((1/2)(h_c + q_c))$. Note also that $\underline{e}$ maximizes $(1/2)P(e)(h_c + q_s) - c(e)$. Thus, $e_A = e^c$ and $b = C$ are optimal for agent A given $e_B = e^c$ and $a = C$.

Next, we will show that no other equilibria exist under condition (*). Suppose first that there exists an equilibrium in which agent A chooses e'_A and $b = S$ while agent B chooses e'_B and $a = C$. Then agent A obtains the equilibrium expected payoff:

$$(1/2)\{P(e'_A)(h_c + q_s) + P(e'_B)(h_s + q_c)\} - c(e'_A),$$

and agent B obtains

$$(1/2)\{P(e'_A)(h_c + q_s) + P(e'_B)(h_s + q_c)\} - c(e'_B).$$

Since agent B could have chosen $e_B = \overline{e}$ and $a = C$ given e'_A and $b = S$, we must have $e'_B = \overline{e}$ because $\overline{e}$ maximizes $(1/2)P(e)(h_s + q_c) - c(e)$. Also, for agent A to choose $b = S$, we must have $P(e'_A)\Delta q \leq P(e'_B)\Delta h$. Combining these with condition (*), we must have

$$e'_A < \underline{e} \equiv \phi((1/2)(h_c + q_s)).$$

Consider now agent A's deviation such that, given $e_B = \overline{e}$ and $a = C$, he chooses $e_A = \underline{e}$ and $b = C$. Such a deviation gives agent A a higher expected payoff than in the assumed equilibrium since

$$\begin{aligned}
&(1/2)\{P(\underline{e})(h_c + q_c) + P(\overline{e})(h_c + q_c)\} - c(\underline{e}) \\
> \ &(1/2)\{P(\underline{e})(h_c + q_s) + P(\overline{e})(h_s + q_c)\} - c(\underline{e}) \\
> \ &(1/2)\{P(e'_A)(h_c + q_s) + P(\overline{e})(h_s + q_c)\} - c(e'_A),
\end{aligned}$$

where the first inequality follows from condition (*), and the second inequality is due to $e'_A < \underline{e}$ and the fact that $\underline{e}$ maximizes $(1/2)P(e)(h_c + q_s) - c(e)$.

Suppose now that there exists an equilibrium with $a = b = S$ and (e''_A, e''_B). For this to be optimal for each agent, we must have $P(e''_A)\Delta q \leq P(e''_B)\Delta h$ and $P(e''_B)\Delta q \leq P(e''_A)\Delta h$. The first inequality implies $e''_A < e''_B$ since $\Delta q > \Delta h$. Similarly, the second inequality implies $e''_B < e''_A$, a contradiction. Combining all the above, we can conclude that the equilibrium we derived is a unique one. Q.E.D.

7.3 Proof of Proposition 2

Since the two agents are symmetric, we will, without loss of generality, assume that agent A is the middle agent. Clearly it is optimal for the manager to choose $a = S$.

Next, given $a = S$ and e_B, agent A chooses b and e_A, to maximize

$$\frac{1}{2}P(e_A)\{h_s + q(b)\} + \frac{1}{2}P(e_B)\{h(b) + q_s\} - c(e_A).$$

The optimal choice of e_A is thus given by

$$\frac{1}{2}P'(e_A)\{h_s + q(b)\} = c'(e_A).$$

When $b = C$, agent A will choose $e_A = \bar{e}$. The optimal decision b is given by $b = C$ if and only if

$$P(e_A)q_c + P(e_B)h_c \geq P(e_A)q_s + P(e_B)h_s,$$

which is equivalent to $P(e_A)\Delta q \geq P(e_B)\Delta h$.

We now show that there exists an equilibrium in which agent A chooses $b = C$ and $e_A = \bar{e}$ while agent B chooses $\underline{e}$. Given $e_A = \bar{e}$ and $b = C$, agent B's expected payoff is

$$\frac{1}{2}P(e_B)\{h_c + q_s\} - c(e_B).$$

Thus agent B optimally chooses $\underline{e}$.

Let e^* be the level of human capital satisfying $P(e^*)\Delta q = P(\underline{e})\Delta h$. Then, given $e_B = \underline{e}$, agent A will choose $b = C$ if and only if $e_A \geq e^*$ because $b \in \{S, C\}$ should be chosen to maximize the relevant part of his expected payoff, $P(e_A)q(b) + P(\underline{e})h(b)$.

Thus, given $e_B = \underline{e}$ and $a = S$, agent A's expected payoff can be written as $\max\{U^c(e_A), U^s(e_A)\}$ where

$$U^c(e_A) \equiv \frac{1}{2}P(e_A)(h_s + q_c) - c(e_A) + \frac{1}{2}P(\underline{e})(h_c + q_s) \quad \forall\ e_A \geq e^*$$

and

$$U^s(e_A) \equiv \frac{1}{2}P(e_A)(h_s+q_s) - c(e_A) + \frac{1}{2}P(\underline{e})(h_s+q_s) \;\; \forall\, e_A < e^*.$$

Note that $U^c(e^*) = U^s(e^*)$ by the definition of e^* and $U^c(e_A) \geq (<)\; U^s(e_A)$ for $e_A \geq (<)\; e^*$. Note also that $e^* < \underline{e}$ due to $\Delta q > \Delta h$ and $P' > 0$. Moreover we have $e^* < \underline{e} < \hat{e}$ where $\hat{e} \equiv \phi((1/2)(h_s+q_s)) = \text{argmax}\; U^s(e_A)$. Since $\overline{e} \equiv \phi((1/2)(h_s+q_c))$ maximizes $U^c(e_A)$ and $\overline{e} > \hat{e} > \underline{e} > e^*$, we have

$$\begin{aligned} U^c(\overline{e}) &> U^c(\hat{e}) \\ &> U^s(\hat{e}) \\ &> U^s(e_A) \quad \forall\, e_A < e^*. \end{aligned}$$

Since $e_A < e^*$ must hold if it is optimal for agent A to choose $b = S$, the above inequality shows that agent A optimally chooses $b = C$ and $e_A = \overline{e}$.

Next we will show that no other equilibria exist. Suppose that there exists an equilibrium in which agent A chooses $b = S$ and e'_A while agent B chooses e'_B. For $b = S$ to be optimal for agent A, it must be that $P(e'_A)\Delta q \leq P(e'_B)\Delta h$. Also, given $a = b = S$, agent B maximizes his expected payoff $(1/2)P(e_B)(h_s+q_s) - c(e_B)$ by choosing $e'_B = \hat{e} \equiv \phi((1/2)(h_s+q_s))$. However, given $e'_B = \hat{e}$, agent A can increase his expected payoff by choosing $b = C$ and $e_A = \hat{e}$ rather than the assumed $b = S$ and e'_A. This is because

$$\begin{aligned} &\frac{1}{2}P(\hat{e})(h_s+q_c) - c(\hat{e}) + \frac{1}{2}P(\hat{e})(h_c+q_s) \\ &> \frac{1}{2}P(\hat{e})(h_s+q_s) - c(\hat{e}) + \frac{1}{2}P(\hat{e})(h_s+q_s) \\ &> \frac{1}{2}P(e'_A)(h_s+q_s) - c(e'_A) + \frac{1}{2}P(\hat{e})(h_s+q_s) \end{aligned}$$

for all e'_A satisfying $P(e'_A)\Delta q \leq P(\hat{e})\Delta h$, where the first inequality is due to Assumption 1 (i) and the second inequality follows from the definition of $\hat{e}$. Thus there does not exist an equilibrium with $b = S$. Q.E.D.

7.4 Proof of Proposition 3

(i) Fix all other parameters except q_c. First, note that V_D is independent of q_c. Second, V_C is strictly increasing in q_c since

$$\begin{aligned} \frac{dV_C}{dq_c} &= 2\left\{P(e^c) + [P'(e^c)(h_c+q_c) - c'(e^c)]\frac{de^c}{dq_c}\right\} \\ &= 2\left\{P(e^c) + c'(e^c)\frac{de^c}{dq_c}\right\} \\ &> 0 \end{aligned}$$

due to $P'(e^c)(h_c + q_c) = 2c'(e^c)$ and $de^c/dq_c = (1/2)\phi'((1/2)(h_c + q_c)) > 0$. Moreover we have

$$\lim_{q_c \to \infty} V_C = 2\left[2P(E)\left(\frac{c'(E)}{P'(E)}\right) - c(E)\right] = +\infty$$

because $c'(E) = +\infty$, $P'(E) < +\infty$, and $\lim_{q_c \to \infty} e^c = E$. Thus we have $V_C > V_D$ for large enough q_c. Note also that condition (*) is still satisfied for large q_c because $\lim_{q_c \to \infty} P(\phi((1/2)(h_s + q_c)))\Delta h < \Delta h < \lim_{q_c \to \infty} P(\phi((1/2)(h_c + q_s)))\Delta q = +\infty$.

(ii) Since $h_c + q_c = h_c + q_s + \Delta q$ and $h_s + q_s = h_c + q_s + \Delta h$, we have $e^c \to \phi((1/2)(h_c + q_s + \Delta h))$ and $e^d \to \phi(h_c + q_s + \Delta h)$ if $\Delta q \to \Delta h$. Thus $e^d > e^c$ holds in the limit as $\Delta q \to \Delta h$. Also we have $V_C \to 2\{P(e^c)(h_c + q_s + \Delta h) - c(e^c)\}$ and $V_D \to 2\{P(e^d)(h_c + q_s + \Delta h) - c(e^d)\}$ as $\Delta q \to \Delta h$. Thus we have $V_D > V_C$ when $\Delta q \to \Delta h$ because $e^d \neq e^c$ and e^d maximizes $P(e)(h_c + q_s + \Delta h) - c(e)$ over e.

(iii) Take the limit as $\Delta q \to 0$ and $\Delta h \to 0$. Fixing $h_s + q_s = K$ for some $K > 0$, all e^c, $\overline{e}$ and $\underline{e}$ converge to $\phi((1/2)K)$. However, we have $e^d \to \phi(K)$ even when $\Delta q \to 0$ and $\Delta h \to 0$. Thus we have $V_D = 2\{P(\phi(K))K - c(\phi(K))\} > V_H = V_C = 2\{P(\phi((1/2)K))K - c(\phi((1/2)K))\}$ when $\Delta q \to 0$ and $\Delta h \to 0$. Q.E.D.

7.5 Proof of Proposition 4

We will vary the value of $q > 0$ while fixing all other parameter values.

First, at $q = h + 2q_s$, we have $\overline{e} = \phi((1/2)(h+q)) = \phi((1/2)(2h+2q_s)) = \phi(h + q_s) = e^d$. Thus, at $q = h + 2q_s$,

$$\begin{aligned}
V_H &= P(\overline{e})(h+q) - c(\overline{e}) + P(\underline{e})q_s - c(\underline{e}) \\
&= P(e^d)2(h+q_s) - c(e^d) + P(\underline{e})q_s - c(\underline{e}) \\
&> 2\{P(e^d)(h+q_s) - c(e^d)\} + P(\underline{e})q_s - c(\underline{e}) \\
&= V_D + \{P(\underline{e})q_s - c(\underline{e})\} \\
&> V_D.
\end{aligned}$$

In the above, the last inequality follows from:

$$\begin{aligned}
P(\underline{e})q_s - c(\underline{e}) &> P(\underline{e})(1/2)q_s - c(\underline{e}) \\
&= \max_e P(e)(1/2)q_s - c(e) \\
&\geq P(0)(1/2)q_s \\
&> 0.
\end{aligned}$$

Second, at $q = h + 2q_s$, we have $e^c = \phi((1/2)q) = \phi((1/2)(h + 2q_s)) = \phi((1/2)h + q_s) < \phi(h + q_s) = e^d$ for all $q_s > 0$, and $V_C = 2\{P(e^c)(h +$

$2q_s) - c(e^c)\}$. Since e^d uniquely maximizes $P(e)(h+q_s) - c(e)$ and $e^d \neq e^c$ at $q = h + 2q_s$, we obtain, at $q = h + 2q_s$,

$$V_D \simeq 2\{P(e^d)h - c(e^d)\} > V_C \simeq 2\{P(e^c)h - c(e^c)\}$$

when $q_s > 0$ is small enough.

Third, since $P'(\overline{e})(1/2)(h+q) = c'(\overline{e})$ and $d\overline{e}/dq = (1/2)\phi'((1/2)(h+q)) > 0$, we have

$$\frac{dV_H}{dq} = P(\overline{e}) + c'(\overline{e})\frac{d\overline{e}}{dq} > 0.$$

Similarly, we have

$$\frac{dV_C}{dq} = 2\left\{P(e^c) + c'(e^c)\frac{de^c}{dq}\right\} > 0.$$

Finally, since $h/P(\phi((1/2)q_s) + q_s \leq h + 2q_s$ when $h > 0$ is sufficiently small, condition (*) holds for all $q \geq h + 2q_s$ if $h > 0$ is small enough.

Summarizing the above argument, we can conclude that, for $h > 0$ and q_s small enough, there exists an interval $(\underline{q}, \overline{q})$ such that, for all $q \in (\underline{q}, \overline{q})$, we have $V_H > \max\{V_C, V_D\}$ while satisfying condition (*).[22] Q.E.D.

7.6 Proof of Proposition 5

Let $\overline{e}_n$ and $\underline{e}_n$ denote the levels of human capital defined as: $\overline{e}_n \equiv \phi_n((1/2)(h_s + q_c))$, $\underline{e}_n \equiv \phi_n((1/2)(h_c + q_s))$. Since ϕ_n is increasing, we have $\overline{e}_n > \underline{e}_n$. Let ψ_n be a function of e, defined as: $\psi_n(e) \equiv 2P_n(e)(c'_n(e)/P'_n(e)) - c_n(e)$. Then, by using $h_s + q_c = 2c'_n(\overline{e}_n)/P'_n(\overline{e}_n)$, $h_c + q_s = 2c'_n(\underline{e}_n)/P'_n(\underline{e}_n)$, the total expected payoffs in hierarchical delegation with agent n at the middle tier (thus agent $j \neq n$ is at the bottom tier) can be written as

$$\begin{aligned} V_H^n &\equiv P(\overline{e}_n)(h_s + q_c) + P(\underline{e}_k)(h_c + q_s) - c(\overline{e}_n) - c(\underline{e}_k) \\ &= \psi_n(\overline{e}_n) + \psi_j(\underline{e}_j) \\ &= \psi_n(\phi_n((1/2)(h_s + q_c))) + \psi_j(\phi_j((1/2)(h_c + q_s))). \end{aligned}$$

Using the above and $\psi'_n = 2P_n(c'_n/P'_n)' + c'_n$, $\phi'_n = 1/(c'_n/P'_n)'$, we then have

$$\begin{aligned} V_H^n - V_H^j &= \int_{(1/2)(h_c+q_s)}^{(1/2)(h_s+q_c)} \left\{\psi'_n\phi'_n(x) - \psi'_j\phi'_j(x)\right\} dx \\ &= \int_{(1/2)(h_c+q_s)}^{(1/2)(h_s+q_c)} \{2P_n(\phi_n(x)) + c'_n(\phi_n(x))\phi'_n(x) \\ &\qquad - 2P_j(\phi_j(x)) - c'_j(\phi_j(x))\phi'_j(x)\} dx. \end{aligned}$$

From the above follows the proposition. Q.E.D.

[22]Here $\overline{q} = +\infty$ can be the case if $V_H > V_C$ holds for all $q \geq h + 2q_s$.

7.7 Proof of Proposition 6

We will show $V_H^A > V_H^B$ for all $h_c \in [0, h_s]$. Let $F(h_c)$ denote V_H^A as a function of h_c. First we will show $F(0) > V_H^B$. Next we will show $F'(h_c) > 0$. This proves our claim since V_H^B is independent of h_c.

To show $F(0) > V_H^B$, we can derive the following inequalities:

$$\begin{aligned}
V_H^A &= \left\{\frac{1}{2}P(e'_A)(h_s+q_c)+\frac{1}{2}P(e'_B)h_c - c(e'_A)\right\} \\
&\quad + \left\{\frac{1}{2}P(e'_B)h_c - c(e'_B)\right\} + \frac{1}{2}P(e'_A)(h_s+q_c) \\
&= \max_{e_A}\left\{\frac{1}{2}P(e_A)(h_s+q_c)+\frac{1}{2}P(e'_B)h_c - c(e_A)\right\} \\
&\quad + \max_{e_B}\left\{\frac{1}{2}P(e_B)h_c - c(e_B)\right\} + \frac{1}{2}P(e'_A)(h_s+q_c) \\
&\geq \left\{\frac{1}{2}P(\tilde{e}_A)(h_s+q_c)+\frac{1}{2}P(e'_B)h_c - c(\tilde{e}_A)\right\} \\
&\quad + \left\{\frac{1}{2}P(\tilde{e}_B)h_c - c(\tilde{e}_B)\right\} + \frac{1}{2}P(e'_A)(h_s+q_c) \\
&\geq \left\{\frac{1}{2}P(\tilde{e}_A)(h_s+q_c)+\frac{1}{2}P(e'_B)h_c - c(\tilde{e}_A)\right\} \\
&\quad + \left\{\frac{1}{2}P(\tilde{e}_B)h_c - c(\tilde{e}_B)\right\} + \frac{1}{2}P(\tilde{e}_A)(h_s+q_c) \\
&\equiv \hat{V}
\end{aligned}$$

where the first inequality follows from the definition of e'_A and e'_B, and the second inequality is due to $e'_A \geq \tilde{e}_A$. Then straightforward algebra shows

$$\hat{V} - V_H^B = P(\tilde{e}_A)\Delta q - P(\tilde{e}_B)\Delta h - \frac{1}{2}\left[P(\tilde{e}_B) - P(e'_B)\right]h_c.$$

which is positive if $h_c = 0$ since $\tilde{e}_A > \tilde{e}_B$ and $\Delta q > \Delta h$. Thus $F(0) > V_H^B$.

Next, differentiating $F(h_c)$ yields $F'(h_c) = [P'(e'_B)h_c - c'(e'_B)]\frac{de'_B}{dh_c} + P(e'_B) > 0$ since $(1/2)P'(e'_B)h_c - c'(e'_B) = 0$ by the definition of e'_B, hence $P'(e'_B)h_c - c'(e'_B) = c'(e'_B) > 0$. Q.E.D.

7.8 Proof of Proposition 7

We can derive the following inequalities:

$$\begin{aligned}
V_{CD} &= \left\{P(\tilde{e}_A)(h(\tilde{a})+q(\tilde{b})) - c(\tilde{e}_A) + \frac{1}{2}P(\tilde{e}_B)(h(\tilde{b})+q(\tilde{a}))\right\} \\
&\quad + \left\{\frac{1}{2}P(\tilde{e}_B)(h(\tilde{b})+q(\tilde{a})) - c(\tilde{e}_B)\right\} \\
&= \max_{a,b,e_A}\left\{P(e_A)(h(a)+q(b)) - c(e_A) + \frac{1}{2}P(\tilde{e}_B)(h(b)+q(a))\right\}
\end{aligned}$$

$$
\begin{aligned}
& + \left\{\frac{1}{2}P(\tilde{e}_B)(h(\tilde{b}) + q(\tilde{a})) - c(\tilde{e}_B)\right\} \\
\geq \; & P(\overline{e})(h(S) + q(C)) - c(\overline{e}) + \frac{1}{2}P(\tilde{e}_B)(h(C) + q(S)) \\
& + \left\{\frac{1}{2}P(\tilde{e}_B)(h(\tilde{b}) + q(\tilde{a})) - c(\tilde{e}_B)\right\} \\
\geq \; & P(\overline{e})(h_s + q_c) - c(\overline{e}) + \frac{1}{2}P(\underline{e})(h_c + q_s) \\
& + \left\{\frac{1}{2}P(\tilde{e}_B)(h(\tilde{b}) + q(\tilde{a})) - c(\tilde{e}_B)\right\} \\
= \; & P(\overline{e})(h_s + q_c) - c(\overline{e}) + \frac{1}{2}P(\underline{e})(h_c + q_s) \\
& + \max_{e_B}\left\{\frac{1}{2}P(e_B)(h(\tilde{b}) + q(\tilde{a})) - c(e_B)\right\} \\
\geq \; & P(\overline{e})(h_s + q_c) - c(\overline{e}) + \frac{1}{2}P(\underline{e})(h_c + q_s) \\
& + \left\{\frac{1}{2}P(\underline{e})(h(\tilde{b}) + q(\tilde{a})) - c(\underline{e})\right\} \\
\geq \; & P(\overline{e})(h_s + q_c) - c(\overline{e}) + \frac{1}{2}P(\underline{e})(h_c + q_s) \\
& + \left\{\frac{1}{2}P(\underline{e})(h_c + q_s) - c(\underline{e})\right\} \\
= \; & V_H
\end{aligned}
$$

where the second inequality follows from $\tilde{e}_B \geq \underline{e}$ and $P' > 0$, and the last inequality from $h(\tilde{b}) \geq h_c$ and $q(\tilde{a}) \geq q_s$. Thus concentrated delegation dominates hierarchical delegation for any parameter values. Q.E.D.

7.9 Proof of Proposition 8

Fix all other parameter values but h. Let $V_H^R(h)$ and $V_D^R(h)$ denote V_H^R and V_D^R as functions of h.

First, at $h = 0$, we have

$$
\begin{aligned}
V_C &= 2\{P(\phi((1/2)q))q - c(\phi((1/2)q)) \\
&> 2\{P(\phi(\alpha q)) - c(\phi(\alpha q))\} \\
&= V_D^R(0)
\end{aligned}
$$

since $\alpha < 1/2$ and $P(e)q - c(q)$ is increasing in $e \in [0, \phi(q))$. Also, at $h = 0$, we have

$$
\begin{aligned}
V_C &= 2\{P(\phi((1/2)q))q - c(\phi((1/2)q))\} \\
&> P(\phi((1/2)q))q - c(\phi((1/2)q)) + P(\phi(\alpha q))q - c(\phi(\alpha q)) \\
&= V_H^R(0) \\
&> 2\{P(\phi(\alpha q))q - c(\phi(\alpha q))\}
\end{aligned}
$$

$$= \quad V_D^R(0).$$

Moreover we have

$$\frac{\partial V_D^R}{\partial h} = 2\{P'(e^{DR})q - c'(e^{DR})\}\frac{de^{DR}}{dh} > 0,$$

$$\frac{\partial V_H^R}{\partial h} = \{P'(\overline{e}^{HR})q - c'(\overline{e}^{HR})\}\frac{d\overline{e}^{HR}}{dh} > 0,$$

where we used the fact that $P'(e)q - c'(e) > 0$ for all $e \in [0, \phi(q))$. Finally, at $h = q$, we have

$$V_H^R(q) = H(\alpha) \equiv P(\phi((1-\alpha)q))q - c(\phi((1-\alpha)q)) + P(\phi(\alpha q))q - c(\phi(\alpha q)).$$

Note that $H(1/2) = V_C$. Letting $\Gamma(x) \equiv P(\phi(x))q - c(\phi(x))$, we can show

$$\frac{dH}{d\alpha} = q\{\Gamma'(\alpha q) - \Gamma'((1-\alpha)q)\},$$

which is positive if $\phi'' \leq 0$ because $\alpha < 1/2$ and $\Gamma''(x) = [P''(\phi(x))q - c''(\phi(x))](\phi'(x))^2 + [P'(\phi(x))q - c'(\phi(x))]\phi''(x) < 0$ for any $x < q$. This shows that $H(\alpha) < H(1/2)$ for all $\alpha \in (0, 1/2)$. Since $H(1/2) = V_C$, we then have $V_H^R(q) = H(\alpha) < V_C$.

Summarizing the above argument, we first obtained $V_H^R(h) < V_C$ for all $h \in (0, q)$. Thus hierarchical delegation is dominated by centralization. Second, since V_C is constant over h and V_D^R is increasing in h, there exists a unique cutoff value h^* such that $V_C > V_D^R$ for all $h < h^*$, and $V_D^R > V_C$ for all $h > h^*$.[23] In addition, we need to satisfy assumption (**), which can be rewritten as $h \leq (P(0)/P(E))q$. Thus decentralization is optimal for all $h \in (h^*, (P(0)/P(E))q)$ if $h^* \leq (P(0)/P(E))q$ holds. Otherwise, centralization becomes optimal. Q.E.D.

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Table 1: Possible Organizational Structures

Organizational structure	Manager	Agent A	Agent B	Note
Centralization	a, b			
Partial decentralization (A)	b	a		Dominated by decentralization (Section 3.2)
Partial decentralization (B)	a		b	Dominated by decentralization (Section 3.2)
Decentralization		a	b	
Cross-authority decentralization		b	a	Equivalent to centralization (Section 3.2)
Hierarchical delegation (A)	a	b		
Hierarchical delegation (B)	b		a	
Concentrated delegation (A)		a, b		Dominates hierarchical delegation (Section 5.4)
Concentrated delegation (B)			a, b	Dominates hierarchical delegation (Section 5.4)

Table 2: Equilibria in Different Organizational Structures

	Decision a	Decision b	Human capital e_A	Human capital e_B
Centralization	C	C	$e^c = \phi\left(\frac{1}{2}(h_c + q_c)\right)$	$e^c = \phi\left(\frac{1}{2}(h_c + q_c)\right)$
Decentralization	S	S	$e^d = \phi(h_s + q_s)$	$e^d = \phi(h_s + q_s)$
Hierarchical Delegation*	S	C	$\bar{e} = \phi\left(\frac{1}{2}(h_s + q_c)\right)$	$\underline{e} = \phi\left(\frac{1}{2}(h_c + q_s)\right)$

* In hierarchical delegation, agent A is at the middle tier of the hierarchy.

Figure 1: Comparison of Different Organizational Structures