Predicting the Profit Potential of a Microeconomic Process: An Information Theoretic/Thermodynamic Approach

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A theory is the more impressive the greater the simplicity of its premises, the more different kinds of things it relates, the more extended its area of applicability. Therefore the deep impression that classical thermodynamics made upon me. It is the only theory of universal content which I am convinced will never be overthrown.

Albert Einstein

Abstract

It would be of great benefit if management could predict the huge profits that would result from modest investments in process improvement initiatives such as Lean, Six Sigma and Complexity reduction. While the application of these initiatives was initially restricted to manufacturing, they have been expanded to transactional processes such as product development, marketing, and indeed all microeconomic processes... This paper derives an equation that, subject to further testing, appears to make such a profit prediction possible allowing a rational investment in microeconomic process improvement.

That the profit of a company is greatly increased by the reduction of internal waste was originally demonstrated by Henry Ford, but has been greatly extended by Toyota. All waste in a process results in longer lead times, measured from the injection of work into the process until its delivery to the customer or user. Thus the increase in profit is principally driven by the reduction of lead time through process improvement. The lead time of any process is governed by the Queuing Theory formula known as Little's Law.

The central result of this paper is that the reduction lead time as expressed by Little's Law leads to an equation for the reduction of process Entropy. The expression is identical with the reduction of entropy and thermodynamic waste in a heat engine. Case studies are used to estimate the magnitude of Boltzmann's Constant for Microeconomic processes. The resulting Equation of Profit allows the prediction of the amount of waste cost elimination based on explicit Lean, Six Sigma and Complexity reduction process improvement parameters. More data is needed to more accurately estimate the magnitude of Boltzmann's constant for microeconomic processes.

Key Words: Profit Increase Prediction; Process Entropy; Information; Complexity; Waste; Equation of Profit; Little's Law; Business Analogies with Thermodynamics; Boltzmann's Constant of Business; Carnot; Shannon

Classification:C51,D2,D8,L6,L7,L8,L11,L15,M21
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Introduction:

The efficiency of the transformation of Revenue to Profit not only drives the share value of corporations but also the destiny of economies, nations, and the career opportunities available to their citizens. That the former "Big 3" automakers did not immediately and universally apply the Toyota Production and Toyota Design Development System when it was well understood in the 1980's contributed to the massive loss of market share and human opportunity of these firms. Had the leaders of the "Big 3" been able to project that > 10% cost advantage, higher quality, and faster time to market would result, they would more likely have taken immediate action. The goal of this paper is to propose an Equation of Profit that makes such projections possible...

In 1824 Sadi Carnot wrote that the economic supremacy of Britain depended as much on her invention of the steam engine as on her Navy and Empire. Carnot sought to understand the maximum useful Work output that an engine could deliver for a given heat energy input. We will first describe Carnot's investigations[2] and then apply them to the profit in a business. This application of thermodynamics to business is justified the the work of E. T. Jaynes and is dicussed herein.

Waste in an Engine: Carnot, followed by Clausius, reasoned that, in each cycle, an engine receives heat energy Q_H from a Hot combustion source at temperature T_H. With each power stroke of the piston, the engine transforms part of this input energy into useful work to drive a shaft. The rest of the input energy is expelled as waste energy Q_C to the environment at the Cold sink temperature of $T_C \approx 25°C$ at which point the cycle is complete and engine is ready to receive more heat energy. Carnot discovered that a quantity known as the Entropy $S=Q_H/T_H$ was drawn from the Hot source and at least that much Entropy was delivered to the Cold temperature sink. Even under ideal conditions:

$$\text{Entropy} = S = \frac{Q_H}{T_H} = \frac{Q_C}{T_C} \qquad (1)$$

thus the minimum waste energy Q_C delivered to the Cold temperature sink is

$$\text{Waste} = Q_C \geq T_C S \qquad (2)$$

Minimum waste in an engine is proportional to entropy. According to (1) entropy falls as T_H increases. This discovery helped inform the development of engines, from the atmospheric engines of the 18th Century which operated at 3% efficiency and about 100°C to the modern gas turbines which operate at 40% efficiency and 3000°C.

The explicit expression for the entropy of an ideal gas undergoing compression at a constant temperature is easily derived[2] and will be useful in studying business waste:

$$\text{Change in Entropy} = \Delta S = \int \frac{dQ}{T}\text{, but from the 1}^{st}\text{ Law of Thermodynamics}$$

dQ=dU+pdV where Q=heat,T=Temperature,U=internal energy,p=Pressure,V=Volume

$$\Delta S = \int \frac{(dU+pdV)}{T} = \int \frac{(c_v dT+pdV)}{T} = \int \frac{pdV}{T} \text{ for isothermal processes where } dT=0$$

for a mole of an ideal gas, pV=RT and c_v is the specific heat

$$\Delta S = \int \frac{RTdV}{VT} = R\int_{V_{Initial}}^{V_{Final}} \frac{dV}{V} = R\log(V_{Final} - V_{Initial}) \qquad (3)$$

Waste in a Business: Since the minimum waste in an engine is proportional to the entropy, we will inquire if comparable entropy exists in a microeconomic process and if its' equation can similarly inform the reduction of waste and increase in business profit. If the company has W units of Work In Process Inventory[3] and ships products which contain C units per year, then the company turns inventory Z=C/W times per year. Each turn of inventory is analogous to a power stroke of an engine. W units of Revenue are drawn in at revenue/unit r, processed, and under ideal condition W units of cost are expelled at cost/unit c. The input revenue per turn is $R_t = rW$, where r is the average revenue per unit and W is the number of units per turn. Likewise, a business expels cost per inventory turn of $C_t = cW$ where c is the average total cost per unit including indirect expenses such as G&A, R&D, Cost of Capital, etc. Notice that if we form the ratios:

$$\frac{R_t}{r} = \frac{rW}{r} = W = \frac{C_t}{c} = \frac{cW}{c} = W \quad (4)$$

Since under ideal conditions W units flow from Revenue to Cost, and we inquire if some function of W is related to the entropy and waste of a process.

Little's Law and Microeconomic Entropy

The major intuitive insight that waste elimination by process improvement drove faster lead time was due to Henry Ford. The Model T originally sold for $850 and took 14 days to produce. Process improvement eliminated nearly all the waste and the same car was produced in 33 hours and sold for $345 at higher total profit. Kiichiro Toyota essentially adapted this process to the production of a variety of cars using what is now known as Lean Six Sigma and Complexity reduction tools. This phenomenon has been observed in other transactional (non-manufacturing) microeconomic processes such as product development, marketing, planning, budgeting, etc[4]. Lead time is measured from the time of injection of raw material into the process to its completion as finished goods.[5]. The Lead Time of any process is governed by Little's Law[4], The time per cycle of production from injection of work into a process to its completion is:

$$\text{Lead Time of any Process} = \frac{\text{Number of Units of Work In Process}}{\text{Average Completion Rate}} = \frac{W}{D} = \tau = \text{time/cycle} \quad (5)$$

As an example of Little's Law, if a process has WIP of 50 units and has an average completion rate of 2 units per hour, then the average time for a unit of WIP to transit the process is:

$$\text{Lead Time of Process} = \frac{50 \text{ units}}{2 \text{ units/hour}} = 25 \text{ hours}$$

Thus a manufacturing cycle is completed every 25 hours. Even though the WIP may consist of a variety of different items having different completion times per task, only the average completion rate governs the lead time (often called cycle time) of the process. Moreover, Little's Law is distribution independent: whether the variety of task completion times follow a Gaussian distribution as in manufacturing, a Rayleigh distribution as in product development, or other is irrelevant to lead time. To discover if entropy exists in microeconomic processes, we transform Little's Law into a velocity equation by inversion:

$$\text{Process Velocity} = v = \frac{\text{Average Completion Rate}}{\text{No. of Units of Work In Process}} = \frac{1}{\tau} = \frac{D}{W} \text{ cycles/unit time} \quad (6)$$

This velocity is the number of manufacturing cycles completed per unit time. Clearly the velocity is inversely proportional to the Work In Process W and directly proportional to D. Assuming that Average completion Rate D= Market Demand, then D is a constant exogenous variable driven by the market during periods comparable to the lead time. The rate at which the velocity is accelerated is related to the rate at which W can be reduced. Thus -dW/dt is a factor in the force reducing the WIP and hence accelerating the process. Taking the first derivative of (6) we obtain:

$$\text{Acceleration} = a = \frac{dv}{dt} = -\frac{D}{W^2}\frac{dW}{dt} \text{ cycles/(units of time)}^2 \quad (7)$$

This is the acceleration of the velocity with which the WIP completes the cycle from Raw Material to Finished Goods. We recognize –dW/dt as a factor in the force reducing WIP

W.It is already clear from Little's Law that an increase in W impedes acceleration of the velocity which is the same impact as mass in a mechanical system. The W^2 factor is hence inversely related to Mass, and on a preliminary basis we assign Mass $=1/W^2$. Given that W is related to "mass", we can use the Variational Principle[6] known as the Principle of Least Action to determine if Newton's the Laws of Motion apply to a microeconomic process:

$$\text{Momentum} = p = Mv = W^2\left(\frac{D}{W}\right) = DW, \text{ and } v=\frac{D}{W} \text{ from (6) thus}$$

$$\text{Action}=\int_{ti}^{tf} pv\,dt = \int_{ti}^{tf} DW\left(\frac{D}{W}\right)dt = D^2\left(t_f - t_i\right) \text{ all of which are constant, hence:} \qquad (8)$$

$$\Delta\left(\text{Action}\right) = 0$$

Since the variation in Action is zero, the Euler-Lagrange criterion is satisfied, and Newton's Laws apply to the process[6]

Since W in Little's Law is a dimensionless number , W^2 and *Mass is a dimensionless number* in a process. To determine whether the D factor in (7) is part of force or mass, we will calculate the energy to accelerate the WIP and require that the resulting units of measure be in energy. Let us follow a unit of WIP down the process. Since process improvement is continually reducing setup time, batch size and hence WIP W, in time dt the unit of WIP will, on average, be slightly accelerated as it moves a distance ds down the process, reducing τ hence increasing the number of cycles per unit time. We will require that the 118

energy be measured in units of relevant kinetic energy, i.e., proportional to the square of a velocity since mass is dimensionless. The amount of energy done in accelerating the WIP due to process improvement is then:

$$\text{Energy} = \int_{Si}^{Sf} F\,ds \text{ , but if v is the velocity of the WIP, then ds=vdt, on a preliminary basis we select:}$$

$\text{Mass}=M=W^2$ which is dimensionless since W is a dimensionless number

and $F = -D\frac{dW}{dt}$. then with $v = \frac{D}{W}$ from (6) $ds = \frac{D}{W}dt$, therefore:

$$\text{Energy} = \int_{Si}^{Sf} F\,ds = \int\left(-D\frac{dW}{dt}\right)\left(\frac{D}{W}dt\right) = -D^2\int_{Wi}^{Wf}\frac{dW}{W} = -D^2\left(\log W_f - \log W_i\right) \qquad (9)$$

Mass is equal to W^2 but the WIP W is a dimensionless number. Therefore energy $\frac{1}{2}Mv^2$ relevant to a process is thus measured in units of a velocity squared, $(\text{units/ time})^2$.

The alternative parsing of between Force and Mass in (10) is:

$M=\frac{W^2}{D}$, $F = -\frac{dW}{dt}$, then

$$\text{Energy}=\int_{Si}^{Sf} F\,ds = \int\left(-\frac{dW}{dt}\right)\left(\frac{D}{W}dt\right) = -D\int_{Wi}^{Wf}\frac{dW}{W} = -D\left(\log W_f - \log W_i\right) \qquad (10)$$

And the units of measure are inconsistent with a Kinetic Energy since there is no velocity squared and must be rejected based on the criterion of units of measure of Kinetic energy.

Equation (9) is similar in form to the entropy of an ideal gas under isothermal compression as derived in (3).Notice that in (9) D^2 is a parameter, not a universal constant. Therefore , for a microeconomic process we set R=1 and we have

$$S = \text{Entropy} \rightarrow \log V \rightarrow \log W \qquad (10a).$$

Making the analogy that Volume→Amount of WIP W,(9) becomes:

$$(\text{Energy}) = D^2 \log W = D^2 (\text{Entropy}) \qquad (11)$$

$\dfrac{\partial(\text{Entropy})}{\partial(\text{Energy})} = \dfrac{1}{D^2} = \dfrac{1}{T}$ from thermodynamics,therefore the temperature of the process is:

$$T = D^2 , \qquad (12)$$

The average velocity of a piece of WIP is, from Kinetic Theory:

$$v = \sqrt{2.55\frac{kT}{M}} \rightarrow \sqrt{2.55\frac{kD^2}{W^2}} = \frac{D}{W}\sqrt{2.55k} \text{ but} \qquad (13)$$

$v = \dfrac{D}{W}$ according to Little's Law

Thus the derivation of the entropy of a process in (9) leads to a velocity (13)which is in form consistent with both kinetic theory and queuing theory is remarkable and lends credence to the result. While the value of the Boltzmann Constant of Business will have to be determined experimentally, had we chosen the form of (10), the equation of a velocity would have been of the form:

$$v = \sqrt{2.55\frac{kT}{M}} = \sqrt{2.55\frac{kD^2}{W^2/D}} = \sqrt{2.55\frac{kD^3}{W^2}} = \frac{D}{W}\sqrt{2.55kD}$$

Which form is not functionally consistent with Little's Law in (6). Thus our preliminary assignment of Mass=$1/W^2$ is internally consistent.
The equation (9) for the entropy of a business process is similar to that of Bryant (2007)[7]. The speed of WIP passing through a process is independent of the dollar value of the cost when the WIP enters the process or the revenue when it exits. Because the velocity is independent of dollars, so is the internal temperature of the process.
Discussion: That the mass of a process, with which it resists acceleration, is proportional to W^2 appears counterintuitive. However, each unit of WIP can only advance on average if all those ahead of it also advance, as well as all those behind it. Thus each unit of WIP is, on average, coupled to all the other units of WIP. This coupling is analogous to an inductor, in which each turn is coupled to all the other turns leading to self inductance proportional to the square of the number of turns. The mass of each unit of WIP is $W^2/W=W$. The energy needed to accelerate a process is proportional to the reduction of logW, and we will now discuss the nature of this "energy".

Information = Negative Entropy

To investigate this process "energy", let us compute logW to determine its' relationship to entropy and information. When, for example, we examine the total Work In Process W of a factory or transactional process, it consists of Q different types of items or tasksThen we can write:

$$W = w_1 + w_2 + \ldots w_Q = \sum_{i=1}^{Q} w_i$$ where w_i is the number of units of the i^{th} item or task type in WIP

We can write this for Q = 2:

$W = w_1 + w_2$

$\log_2 W = \frac{w_1+w_2}{W}\log_2 W = \frac{w_1}{W}\log_2 W + \frac{w_2}{W}\log_2 W = -\frac{w_1}{W}\log_2\left(\frac{1}{W}\right) - \frac{w_2}{W}\log_2\left(\frac{1}{W}\right)$, we will now add $0+0$

$\log_2 W = -\frac{w_1}{W}\log_2\left(\frac{1}{W}\right) - \frac{w_2}{W}\log_2\left(\frac{1}{W}\right) + \left(\frac{w_1}{W}\log_2 w_1 - \frac{w_1}{W}\log_2 w_1\right) + \left(\frac{w_2}{W}\log_2 w_2 - \frac{w_2}{W}\log_2 w_2\right)$

$\log_2 W = -\frac{w_1}{W}\log_2\left(\frac{w_1}{W}\right) - \frac{w_2}{W}\log_2\left(\frac{w_2}{W}\right) + \frac{w_1}{W}\log_2 w_1 + \frac{w_2}{W}\log_2 w_2$ which can be generalized for Q

different components by defining the Probability that a unit of WIP is the i^{th} product as $p_i = \frac{w_i}{W}$

$$\log_2 W = \sum_{i=1}^{Q} p_i \log_2 p_i + \sum_{i=1}^{Q} p_i \log_2 w_i \tag{14}$$

Note that the 2^{nd} term can be written:

$\sum_{i=1}^{Q} p_i \log_2 w_i$ = εexpectation of $\log_2 w_i = \varepsilon\log_2 w_i$ =average value of $\log_2 w_i$

We can therefore write (14) as:

$$\log_2 W = H_Q + \varepsilon_Q \log_2 w_i \tag{15}$$

$$W = 2^{H+\varepsilon\log_2 wi} = 2^H 2^{\varepsilon\log_2 w_i} \tag{16}$$

Information and Generalized Entropy: The first term of (15) is also known as the Shannon equation of Information in bits. It is also identical to the Boltzmann expression for thermodynamic entropy[8] with k=1. Thus the nature of the work required for the reduction of logW necessary to accelerate the process and eliminate waste is equivalent to the increase in information added to the process. Shannon's relation will be developed from first principles below. Since the first term in (15) is entropy bits, so must be the second term. Hence we refer to (15) as the Generalized Entropy of a Process.

Discussion of Terms in the Generalized Entropy, equation (15):

We can most easily explain the role of each term in (15) by considering limiting cases.

Complexity: Let us assume that each of the Q items of WIP W had about the same quantity of units $w_i = W/Q$. Then the probability of occurrence or the i^{th} item is $p_i = w_i/W = .1/Q$ and:

$$H_Q = -\sum_{i=1}^{Q} p_i \log_2 p_i = -\sum_{i=1}^{Q} \frac{1}{Q}\log_2\left(\frac{1}{Q}\right) = \frac{1}{Q}\log_2\left(\frac{1}{Q}\right) + \frac{1}{Q}\log_2\left(\frac{1}{Q}\right)\cdots Q \text{ terms} = \log_2 Q$$

Therefore, *H measures the variety of internal products in WIP needed to deliver m different end products to the customer*. The larger is H, the more setups will be required to meet demand, hence the greater the non value add cost of setup time, and accompanying scrap as well as the cost of tooling, dies, etc. As Q is reduced, more volume is driven through fewer part numbers leading to lower procurement costs, with similar impact on non-manufacturing processes.

*Lean:*The second term in (15) can similarly be understood. Assume i that $p_i = 1/Q$, $w_i = W/Q$, then:

$$\varepsilon\log_2 w_i = \sum_{i=1}^{Q} p_i \log_2 w_i = \sum_{i=1}^{Q}\left(\frac{1}{Q}\right)\log_2\left(\frac{W}{Q}\right) = \left(\frac{1}{Q}\right)\log_2\left(\frac{W}{Q}\right) + \left(\frac{1}{Q}\right)\log_2\left(\frac{W}{Q}\right) + \cdots Q\text{terms} = \log\left(\frac{W}{Q}\right)$$

Therefore the *second term* $\varepsilon\log_2 w_i$ *measures the average amount of WIP per part number*. Thus the larger is $\varepsilon\log_2 w_i$, the larger will be the waste due to scrap, rework, warehouses, distribution centers, transport, and IT systems, and all related indirect personnel to control and store all the material as well as expediting expense to compensate for long lead times. We will see in (17a) below that in manufacturing this $\varepsilon \log_2 w_i$ term is primarily driven by setup time, machine downtime, and quality defects.

WIP as a function of revenue demand and process parameters:

Two principal expressions for the calculation of WIP as a function of demand per unit time and process parameters have been derived:
Manufacturing: The minimum WIP in a factory has been derived by Patell and George[9], and a representative equation is:

$$\text{FactoryWIP} \geq \frac{QASD}{1\text{-}X\text{-}PD} + QA \qquad (17a)$$

Where S=setup time, A=number of workstations in the process, X=Defect rate, P=Processing time per unit. One can see that reducing the number of different internal part numbers Q by 50% reduces WIP by 50%, whereas reduction of setup time by 50% reduces WIP by very nearly 50% because $\frac{QASD}{1\text{-}X\text{-}PD} \gg QA$

In general, Q is directly proportional to the number of external part numbers m shipped to customers.
Transactional: (non manufacturing processes) such as Product Development, Marketing, Planning etc, generally to do not have the opportunity to batch identical items. The Work In Process is approximated by the Pollaczek-Khintchine equation[5]:
Fundamental Equation of Transactional Processses

$$\text{WIP} = \text{No.of Tasks In Process} \cong \left(\frac{1}{K+1}\right)\left(\frac{\rho^2\{1+Z\}^2}{1\text{-}\rho\{1+Z\}}\right)\left(\frac{C_S^2+C_A^2}{2}\right) \qquad (17b)$$

ρ = % of maximum capacity utilized

K= Number of resources cross trained

Z= % defectives that must be reworked

C_S=Coefficient of Variation of time to perform tasks

C_A=Coefficient of Variation of arrival of tasks

$$C = \frac{\sigma = 1 \text{ Standard Deviation}}{\mu = \text{Mean Time}}$$

If the setup time can be driven to zero, then according to the Patell-George equation below) w_i=1 and since log(1)=0, (16) becomes:

$$W = 2^{H+\varepsilon \log_2 w_i} = 2^H 2^{\varepsilon \log_2 w_i} = 2^H 2^0 = 2^H$$

In such an instance, there is only one unit per part number hence $p_i \equiv 1/Q$, $H \equiv \log_2 Q$ and W=Q as required. In non manufacturing processes, this $\varepsilon \log_2 w_i$ term is primarily driven by defects and non value add costs[10]. Thus adding information to the process to reduce setup time, defects, etc reduces generalized entropy and waste
Conclusion: Every practitioner of Lean Six Sigma[11] process improvement will agree that large WIP is due to a bad process and causes waste. Less well known is the impact of internal complexity[12] upon waste which is the subject of a case study below. Both forms of waste must be comprehended in any theory of microeconomic waste.

We have derived an equation (15) in which the two sources of waste appear co-equally important. We now have determined the internal entropy of a process. To determine the waste that can be eliminated and apply equation (2) we must determine T_C, the cold temperature, which when multiplied by the generalized entropy will yield the waste in a process.

Referring to equation (4)

$W=\frac{C_T}{c}$, where C_T is the cost per turn and c is the averge cost per unit

$\log W=(\text{Generalized Entropy})=\log C_T - \log c$

Now the waste cost C_T is analogous to the waste heat Energy Q_C.

Cost per unit c is not a function of C_T .Thus the temperature is:

$$\frac{\partial(\text{Entropy})}{\partial(\text{Energy})}=\frac{\partial(\text{Generalized Entropy})}{\partial C_T}=\frac{\partial(\log W)}{\partial C_T}=\frac{1}{C_T}+0=\frac{1}{T_{cost}}$$

$$T_{cost}=C_T \qquad (18)$$

Recall that the company turns inventory Z times per year. Now in a year there are Z turns, and ZC_T=Cost Of Goods Sold Therefore the waste per year is:

Waste in a Microeconomic Process

We are now in postion to use (18),(2),(3) and (10a) we can propose an equation for the waste in a microeconomic process :

Waste in a Thermodynamic Process= (Temperature)(Entropy)

Waste in a Microeconomic Process = (Cost of Goods Sold)(klogW)

Where k is the Boltzmann Constant of a microeconomic process. Therefore:

$$\$\text{Waste} \geq k(\$\text{Cost of Goods Sold})\log W = k(\$\text{Cost of Goods Sold})(H_N+\varepsilon \log w_i) \quad (19)$$

The value of k, measured in reciprocal bits, must be determined empirically.The guiding principle is that the reduction of generalized entropy is the key to the elimination of microeconomic waste and increase of profit just as increase in combustion temperature and reduction of entropy is the guiding principle of heat engine design.

Equation of Profit :

$$\text{Increased Profit} = \Delta\text{Profit} = (\text{Waste})_{\text{Initial}} - (\text{Waste})_{\text{Final}} \qquad (20)$$

As the Patell-George (17a) equation shows, if no lean initiative is launched and the volume and related revenue doubles the WIP per part will also double. However, if a lean initiative were launched, and setup times and batch sizes were cut in half, the total WIP and hence waste would remain constant.Thus the same amount of waste would be spread over twice as many parts. We therefore correct for changes in revenue by multiplying final WIP by a correction factor c_R

$c_R = \frac{(\text{Revenue})_{\text{initial}}}{(\text{Revenue})_{\text{final}}}$ and per (17a) the same factor applies to change in complexity

Cost of Goods Sold = Cost of Goods Sold$_{\text{initial}}$- ΔWaste

and from (19)

$$\Delta\text{Waste} = k\text{Cost of Goods Sold}_{\text{initial}}\left(\log W_i - \log\left(c_R W_f\right)\right)$$

$$(\text{Cost of Goods Sold})_{\text{final}} = (\text{Cost of Goods Sold})_{\text{initial}}\left(1 + k\log\left(\frac{c_R W_f}{W_i}\right)\right)$$

$$\%\text{ Reduction in Cost of Goods Sold} = Q = \frac{(\text{Cost of Goods Sold})_{\text{final}}}{(\text{Cost of Goods Sold})_{\text{initia}}} = 1 + k\log\left(\frac{c_R W_f}{W_i}\right) \qquad (21)$$

Empirical Estimation of Boltzmann Constants of Business, k

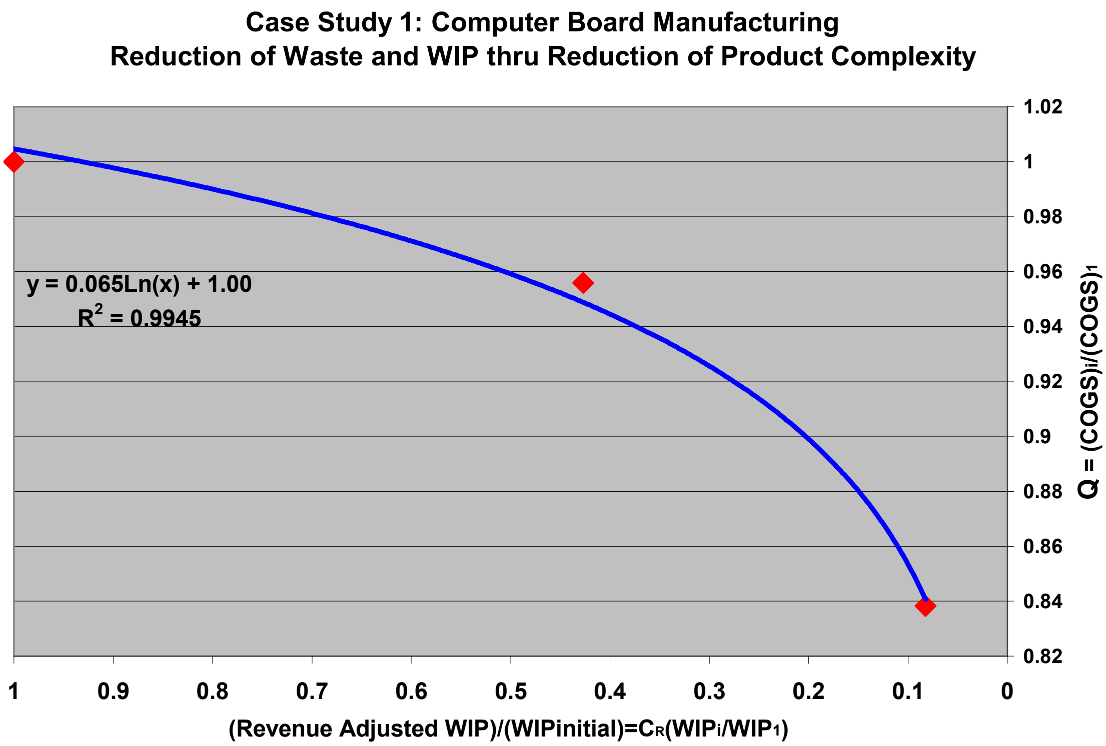
For equation (21) to be useful to predict potential profit increase due to process improvement, we must estimate the magnitude of k

Case Study 1:(Client name withheld) : External Complexity reduction

A $ 2.3 Billion revenue computer products company was losing money on a product line that consisted of m_{initial} =3500 different end items. We know from (17a) that the number of internal part numbers Q is proportional to the number of external part numbers m shipped to customers. Hence cutting m in half cuts Q and hence WIP in half. The new CEO reduced the number of part numbers offered to customers to m_{final} =499. The gross profit increased from 32% to 43%, due to a 32% reduction in labor and overhead cost. The relevant data is:

Year ,i=	1	2	3
1. Number of External Products, m	3500	2300	499
2. Revenue in $Millions	2300	3200	4000
3. c_R=Revenue$_1$/Revenuei	1	0.72	0.58
4. WIP_i/WIP_1	1	0.59	0.14
5. c_R (WIP_i/WIP_1)	1	0.43	0.08
6. Qi = (COGS% of Revenue)$_i$/(COGS% of Revenue)$_1$	1	0.96	0.83

We will graph Cost reduction vs WIP reduction (item 5 vs item 6 in the table)



Note that the equation of the graph has a coefficient of 0.065 for the natural log, Converting this to $\log_2$ using $\log_2 X = (\log_e X)(\log_2 e)$,and since $\log_2 e=1.44$ we have, in (21a):

$$k= (0.065)(1.44) = 0.093$$

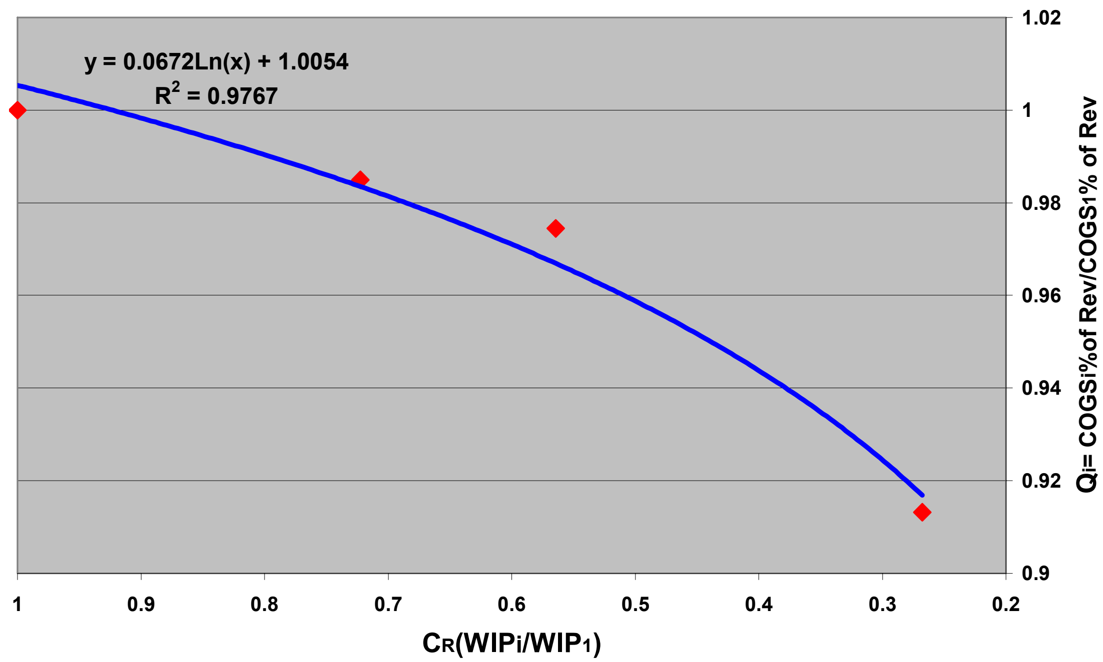
as our first estimate of the Boltzmann constant of a business process.
Return on Investment of the inititiative:>300% per year

Case Study 2: United Technologies Automotive, H&F div(PTG): Lean Initiative
The company produced m=168 different products with an average cost per part of $50 and operated at 10.5% GPM. Because internal components were qual tested and approved by clients such as Ford, GM etc negligible opportunity for internal complexity reduction existed. Rather that waste had to be eliminated via classical Toyota Production system lead time reduction. The setup time at key workstations was reduced from an average of 2 hours to approximately 10 minutes. The resulting gross margin increased from 12.0% to 19.5%. Operating margin grew from 5.4% to 13.8%. Sales grew from $144 million to $311million per year. Cost of Goods Sold rose from $127.4 Million to $250.6Million.Product complexity was essentially constant. The relevant data is:

Year ,i=	1	2	3	4
1) Number of External Products, m	168	155	170	175
2) Revenue in $Millions	144	191	246	311
3) c_R=Revenuei/$Revenue_1$	1	0.75	0.58	0.46
4) WIP_i/WIP_1	1	0.96	0.96	0.57
5) $c_R (WIP_i/WIP_1)$	1	0.72	0.56	0.26
6) Qi = $(\text{COGS\% of Revenue})_i/(\text{COGS\% of Revenue})_1$	1	0.98	0.97	0.91

We will again graph reduction of cost (item 6) vs reduction of WIP(item 5)

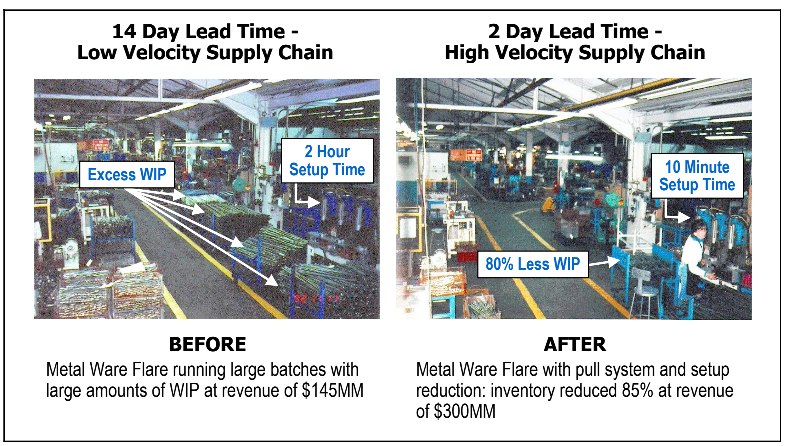
United Technologies Automotive Hose and Fittings div
Lean Six Sigma initiative reduces waste, WIP and cycle time



And obtain our second estimate of the Boltzmann Constant of business as:

k= (0.067)(1.44)=0.097

Elimination of Non Value add costs through 67% WIP reduction



The Equation of Profit asserts that waste is a function of logW. For large initial values of W, small changes in W remain in the flat area of the log curve. Only dramatic reductions toward the origin will drive the log function down. This was a result of Case Study 2, Notice that the actual data below showed that as W was initially reduced, the cost reduction was modest. As W approached 35% of its original value, the cost suddenly fell somewhat below the predicted logW curve. One of the major items of non value add cost that was eliminated was a warehouse comparable in size to the factory. The cost of the warehouse was fairly constant as WIP/part number fell. When WIP/part number and lead time fell to 35% of their original value, the lead time was such that the warehouse could be closed.

The cost reduction can only proceed until all waste is removed and only the value add cost remains. In a manufacturing process this sets W_f in (9) at Q. In a transactional process this sets W_f at the number of workstation in the process. Equation (21) predicts that complexity reduction which reduces Q is just as powerful as Lean initiatives which reduce w_i. This is also evident from the Patell-George[13] equation for factory WIP:

The Lean Six Sigma initiative cost approximately $2 million. The company was purchased for $64 million and sold 26 months later for $208 million for a return of 619% per year

Driver of Process Improvement: Information is Negative Entropy

But what are the connections with Entropy of initiatives such as Lean, Six Sigma and Complexity reduction? We will show that these initiatives inject Information into the process, and that Information is in fact Negative Entropy which reduces waste. Let's first define what we mean by Information. Information tells us something unexpected, i.e., there is a "surprise". The Ford Model T line held no surprise…every car coming off the line was an identical Black Model T, every flywheel magneto was the same with 100% probability, and hence no information was to be gained by looking at the next car or component coming off the line. But what if you were told that it was July 4th in Dallas and there was four feet of snow on the ground…this highly improbable event would be very surprising and hence convey huge information. Therefore we conclude that the *amount* of Information is inversely related to the probability of the event. It is also reasonable that, whatever the functional form of Information may be , if two independent events, 1 and 2 happen, the total information is the sum of their separate Information I_1 and I_2, i.e., $I_{1\&2}=I_1+I_2$. But the probability of independent events 1 and 2 both happening is the product of their probabilities $p_{1\&2}= p_1p_2$. So we need some function for Information I such that: $I_{1\&2}(p_1p_2) =I_1(p_1)+I_2(p_2)$ and the only function which satisfies this requirement is I = log(p) since $\log(p_1p_2) =\log(p_1)+\log(p_2)$. Therefore I(p)=log(p). But since we want the Information to be larger if the probability is smaller we will define I(p)=log(1/p) = -logp which is still OK since $\log(1/p_1p_2)=\log(1/p_1)+\log(1/p_2)$. The *average* amount of information among N choices is, like any other average, is just the sum of the probability of each choice times the value of each choice:

$$H= -\sum_{i=1}^{N} p_iI_i= -\sum_{i=1}^{N} p_i\log_2 p_i \qquad (24)$$

Equation (6e) is known as the Shannon equation of Information.

But how does Information relate to a company? Assume a company produces two products, product 1 in quantities n_1 per month, and product 2 in quantities n_2 per month, where $n_1 + n_2 = D$ total units produced per month. The actual demand of the market for the two products is random, and results in a variety of possible sequences such as :

1121221122212212
2211212211121221
2122122111211212
, etc.

The market makes N Choices monthly (in this case, the unit of time is a month) of either 1 or 2. Each sequence is a state of the market in the sense of Gibbs .The number of *distinct* sequences or "messages" sent by the market, to be satisfied by the company, is calculated by the usual combinatorial formula[14]:

$$\text{Number of Distinct Messages}=M=\frac{D!}{n_1!n_2!}=\binom{D}{n_1}=\frac{D!}{n_1!(D-n_1)!} \qquad (25)$$

We will follow the trail Boltzmann's has already blazed by taking the logarithm of the number of states, which in the business case is the number of distinct messages from the market: According to Stirling's formula, to first order[15]:

$$\log_2 D! \cong D\log_2 D - D, \text{ note that } D=(D-n_1)+n_1=n_2+n_1$$

$$\log_2 M = \left(D\log_2 D - n_1\log_2 n_1 - (D-n_1)\log_2(D-n_1)\right)$$

$$\log_2 M = \left(((D-n_1)+n_1)\log_2 D - n_1\log_2 n_1 - (D-n_1)\log_2(D-n_1)\right)$$

$$\log_2 M = -\left((D-n_1)\log_2\left(\frac{D-n_1}{D}\right) + n_1\log_2\left(\frac{n_1}{D}\right)\right), \text{multiplying by } \frac{D}{D}, \text{obtain:}$$

$$\log_2 M = -D\left(\left(\frac{D-n_1}{D}\right)\log_2\left(\frac{D-n_1}{D}\right) + \left(\frac{n_1}{D}\right)\log_2\left(\frac{n_1}{D}\right)\right), \text{let } p_1=\frac{n_1}{D}, p_2=\frac{N-n_1}{D}$$

$$\log_2 M = D\left(-\{p_1\log_2 p_1 + p_2\log_2 p_2\}\right) \rightarrow D\left\{-\sum_{i=1}^{m} p_i\log_2 p_i\right\} = DH_m \text{ for m products,} \qquad (26)$$

$$M=2^{DH_m} = \text{Number of Distinct Messages M due to m different products}$$

Notice that Shannon's equation for Information popped up naturally. The market is making D variety choices per month, selected from one of the m products , each with information H_m. The M messages per month corresponds to the number of states per month .

$$H_m = -\sum_{i=1}^{m} p_i\log p_i = \text{Shannon Information in Bits per Choice}$$

$$\text{Transmission Rate of Market} \rightarrow DH_m \rightarrow \left(\frac{\text{Choices}}{\text{Month}}\right)\left(\frac{\text{Bits}}{\text{Choice}}\right) \rightarrow \text{Variety Bits per Month} \quad (27)$$

Thus the market is acting like a communication system, transmitting D_H bits of information per month about the variety of products it wants to buy which the company presently offers. Referring to the early automotive market, initially the market demanded utility transportation and Ford responded with m=1 in the form of the Model T. As the technology of cars improved from 1908 to 1925, Ford continued on with m=1 whereas the market demanded variety as brilliantly offered by Sloan of GM where m>5, and the seemingly impregnable Model T was quickly destroyed[16]. Thus the market began sending more complex messages, which will be discussed in the next section.

How does Lean Six Sigma and Complexity Reduction add information to a process?

When processing in batches of quantity B, how much information is added by selecting a given product to setup and run? Let us assume that a factory consists of A workstations, each of which processes Q/A = N part numbers. Clearly if there are N products produced at a given workstation, the decision to select one creates H_N bits of information.

However, the probability is 1 of running that product for the rest of B-1 units in the batch. Therefore, the B-1 units add zero information. As the setup time is cut in half, the batch size can be cut in half and still maintain the same production rate according to (22). Now however we add information twice as often because we select the particular product of the N possibilities twice as often. In general, the information supplied to the process is thus:

$$I_N\text{=Information in production of N Products per month=}\frac{N}{B}H_N$$

$B \geq \frac{SD}{1\text{-}X\text{-}PD}+1$ according to Patell-George, where S=Setup Time, X=scrap rate, P=Processing time/Unit, D=total demand in units/unit time, hence

$$I_N=\frac{N}{\left(\frac{SD}{1\text{-}X\text{-}PD}+1\right)}H_N \rightarrow NH_N \text{ as } S\rightarrow 0 \qquad (28)$$

and for A workstations, AN=Q which is necessary to produce m external products for customers

$$ANH_N \rightarrow QH_N=H_m \qquad (28a)$$

Thus the goal of the Toyota Production system to respond "Just In Time" and produce only what is needed when it is needed is equivalent to an information flow within the factory which matches market demand. In regard to entropy due to average WIP, Lean Six Sigma process improvement results in: :

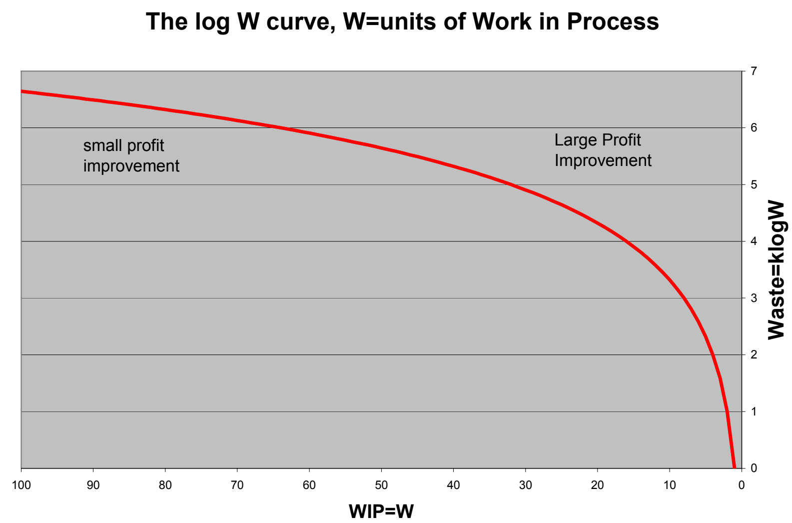
$$S_{initial}-S_{final}=c_0kw\varepsilon_Q\left(\log_2 w_{jinitial}-\log_2 w_{jfinal}\right) \qquad (29)$$

$$S_{initial}-S_{final}=c_0kw\varepsilon_Q\left(\log_2\left(\frac{S_{initial}D}{1\text{-}X_{iniital}-P_{initial}D}+1\right)-\log_2\left(\frac{S_{final}D}{1\text{-}X_{final}-P_{final}D}+1\right)\right) \qquad (30)$$

Applying *Lean* initiatives such as driving S→0 drives entropy related to WIP→0 and leaves only the entropy related to H_m due to the *Complexity* of parts .The addition of information by lean as a means of reducing entropy is merely one example of a general theory propounded by the Physicist Leon Brillouin in which he coined the term *Negentropy* for Information since it is Negative Entropy as is seen in (30) as the amount of entropy subtracted by addition of process information. Although the specific tools change, the same conclusion applies to transactional processes[5].

Conclusion: The Equation of Profit predicts that waste follows a logW curve, and management has the following opportunities and guides for profit improvement:

1. **Complexity Reduction:** The impact of the Cost of Complexity must viewed as yet another source of profit improvement of equal magnitude to Lean Six Sigma initiatives. and must drive Product Portfolio (Case 1) as well as internal standardization initiatives
2. **Return on Investment:** Based on the case studies presented the ROIC exceeds 300% per year compared with an ROIC of 10.2% for the S&P 500.
3. **Great Gains are from High, not Low Hanging Fruit:** The Equation of Profit predicts that waste will follow a logW curve. Hence the gains from modest reductions of WIP are negligible but a reduction of WIP of greater than 70% will yield very significant returns per the Equation of Profit . Thus the "high hanging fruit" are biggest as is depicted by the log curve below and limited by W_{final} in (9).



Thus the goal of Toyota to drive WIP to Q, that is one unit of each item, which has been so puzzling to many executives, can be understood.

4. **The Corporation as an Information System:** The market is transmitting DHm bits per month per (27). The company receives information at this rate, and processes it per (28) for example. If the company can apply process improvement such that the rate at which the company internally processes information matches the rate of transmission from the market, all related waste is eliminated. The Toyota Production system is therefore a method of maximizing the external entropy with which the company(28) so that it exactly responds to the market (27) while minimizing the internal entropy (15)and waste (21) by reductions of logW thru reductions of W via process improvement

5. **Boltzmann Constant of Business Processes:** The case studies indicate that the generalized entropy $H+\varepsilon \log w_i = \log W$ drives the waste in a process through $1+k\log\left(\frac{c_R W_f}{W_i}\right)$ where k is estimated at $\cong 0.95$ per bit of entropy reduced from the process. More data is needed to estimate the error in this value.

Next Steps: Additional data will be collected on properly instrumented companies to refine the value of Boltzmann's constant of Business and confirm the Equation of Profit. Those who wish to cooperate in the study should contact the author at entropy3141@yahoo.com).

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[1] Director, Institute of Business Entropy, Dallas, TX; Founder, The George Group
[2] Fermi, Enrico 1956 "Thermodynamics", Dover

[3] Work In Process are the units of work that have been released into production from Raw Material but have not yet become Finished Goods, and appears in a footnote in the 10K of public corporations.
[4] *Lean Six Sigma for Service* and *Fast Innovation* by George et al
[5] MIT LAI paper 2007:George, Works, Maaseidvaag, "The Application of Lean to Knowledge Processes"
[6] Goldstein, Herbert *Classical Mechanics*, Chapter 7, Addison-Wesley 1950
[7] Bryant, John *A Thermodynamic Theory of Economics*, page 11, equation (3.26) the International Journal of Exergy, published by Interscience Publishers *Int. J. Exergy, Vol 4, No. 3, pp.302-337.*

[8] Feynmann, R. P., Statistical Mechanics, p. 6, equation (1.3)
[9] US Patent 6,993,492 issued Jan 31, 2006

[10] Op cit
[11]George 2002, Lean Six Sigma, McGraw Hill
[12] George and Wilson (2004),Conquering Complexity in Your Business,McGraw Hill
[13] US Patent 6,993,492 issued Jan 31, 2006.
[14] Walpole,Ronald et al 2002 "Probability and Statistics for Engineers and Scientists" p.37
[15] Stirling's formula is only in error by 1% when the number of products shipped per month is D=10, and of course is entirely negligible for most companies when D>>10. See Reif, F 1965, *Fundamentals of Statistical and Thermal Physics*, pp 613-614 for an investigation of the accuracy of Stirling's formula.
[16] LAI paper 2007 op cit

Appendix 1

Demonstration that Shannon Entropy is an approximation of Boltzmann Entropy

Boltzmann Entropy=$S=k\log\Omega$, where Ω=Number of distinct States

Take the simplest case where there are N total products shipped in a given month consisting of M= 2 types, where n_i = number of the i^{th} product shipped in a month.

$D = n_1+n_2$, then

$$\Omega=\frac{D!}{n_1!n_2!}$$

$\log\Omega = \log D!-\log n_1!-\log n_2!$

Stirling's approximation can be derived to 2nd order from Poisson dist* as:

$$\log D!=D\log D-D+\frac{1}{2}\log 2\pi D$$

$$\log\Omega=D\log D-D+\frac{1}{2}\log 2\pi D-\left(n_1\log n_1-n_1+\frac{1}{2}\log 2\pi n_1\right)-\left(n_2\log n_2-n_2+\frac{1}{2}\log 2\pi n_2\right)$$

but since $D=n_1+n_2$

$$\log\Omega=D\log D+\frac{1}{2}\log 2\pi D-\left(n_1\log n_1+\frac{1}{2}\log 2\pi n_1\right)-\left(n_2\log n_2+\frac{1}{2}\log 2\pi n_2\right),\therefore,$$

$$\log\Omega=D\log D+\frac{1}{2}\log 2\pi D-n_1\log n_1-n_2\log n_2-\frac{1}{2}\log\left(2\pi n_2 n_1\right) \text{ now } n_2=D-n_1 \therefore$$

$$\log\Omega=D\log D+\frac{1}{2}\log 2\pi D-n_1\log n_1-\left(D-n_1\right)\log\left(D-n_1\right)-\frac{1}{2}\log\left(2\pi\left\{D-n_1\right\}n_1\right) \text{ now add}$$

$0 = -n_1\log D+n_1\log D$ and obtain:

$$\log\Omega=\left(D-n_1\right)\log D-n_1\log\left(\frac{n_1}{D}\right)-\left(D-n_1\right)\log\left(D-n_1\right)-\frac{1}{2}\log\left(2\pi\left\{D-n_1\right\}n_1\right)+\frac{1}{2}\log 2\pi D$$

$$\log\Omega=-n_1\log\left(\frac{n_1}{D}\right)-\left(D-n_1\right)\log\left(\frac{D-n_1}{D}\right)-\frac{1}{2}\log\left(2\pi\left\{D-n_1\right\}n_1\right)+\frac{1}{2}\log 2\pi D\text{, now mult by }\frac{D}{D}:$$

$$\log\Omega=D\left(-\left(\frac{n_1}{D}\right)\log\left(\frac{n_1}{D}\right)-\left(\frac{D-n_1}{D}\right)\log\left(\frac{D-n_1}{D}\right)+\frac{1}{2D}\log 2\pi D-\frac{1}{2D}\log\left(2\pi\left\{D-n_1\right\}n_1\right)\right)=\frac{S}{k}$$

define $p_1=\left(\frac{n_1}{D}\right)$ etc.

$$\log\Omega=D\left(H-\frac{1}{2D}\log\left(\frac{\left\{D-n_1\right\}n_1}{D}\right)\right)=\frac{S}{k},\ \Omega=2^{D\left(H-\frac{1}{2D}\log\left(\frac{\left\{D-n_1\right\}n_1}{D}\right)\right)} \text{ to second order,also}$$

$$H=\frac{S}{Dk}+\frac{1}{2D}\log\left(\frac{\left\{D-n_1\right\}n_1}{D}\right)\rightarrow\frac{S}{Dk}+\frac{1}{2D}\log\left(\frac{1}{D}\prod_{i=1}^{M}n_i\right) \text{ for M types}$$

However, as noted above, the Stirling approximation is only in error by 1% when D=10.

- MacKay, Information Theory, Inference, and Learning Algorithms, page 2