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Abstract

The Asset Backed CDS contract was introduced in 2005 as an extension of the standard corporate CDS. It generally trades under the ISDA "pay-as-you-go" (PAUG) confirmation which handles the unique features of ABS – amortization, principal writedowns and interest shortfalls. The current market standard for pricing is a simple adaptation of the widely used intensity based model, where the amortization schedule of the security is deterministic.

Taking example from some European ABS, we establish stylized facts about their default. In particular, we show that principal writedowns often come along with an extension of the ABS' maturity and can also be preceded by interest shortfalls. This paper introduces adjustments to the classical framework to account for these specificities, with amortization profile becoming a default-dependent function. We show that the resulting duration becomes an increasing function of spread, capturing the fact that distressed ABS shift toward slower amortization.

Keywords: Asset-Backed Securities (ABS), credit default swap (CDS), ABCDS, "pay-as-you-go" (PAUG), securitization, valuation.

JEL Classification: G12, G13.

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1 Introduction

The European Structured Finance market has for long been quiet and preserved, not only from market turbulences, but also, and consequently, from rational pricing. The risk premium for senior bonds was negligible and attributed to non-credit risks, such as prepayment and liquidity. However the subprime crisis and the emergence of a domestic credit default swap market has pointed out the credit-risky nature of these securities. The same bonds nowadays trade with significant premium, materializing the rising fear of seeing defaults in the Asset Backed space. Yet the pricing assumptions have seen limited debates in the marketplace, with traders and asset managers relying on deterministic scenarios to price and risk manage their books, with little attention to ABS complexity.

Notwithstanding a large literature devoted to credit risk, ABS and CDS of ABS have been for now largely neglected. There has been for long extensive studies about the interest rate risk of mortgage backed securities (MBS), though, but these securities are knowledgeably not subject to credit risk. A recent paper from Brunel and Jribi [3] points out the inaccurateness of the market practice, but still targets interest rate risk and neglects credit. From our perspective, Fermanian [7] is the first to attempt to address credit risk and moreover simultaneously model prepayment, default and interest rate risk. However, this model intends more to be a relative value tool and is difficult to apply to actual transactions.

The purpose of this paper is to provide a practicable framework for the pricing of CDS of ABS (ABCDS) which accounts for ABS cash flow specificities. We will establish in this paper what we see as the "default paradox" of structured finance securities: how can one price a defaultable bond and simultaneously input standard assumptions for the computation of expected cash flows ? The dynamic of collateral losses together with the securities complex structure will generally make that, for example, a 5-years expected maturity bond is more likely to default in 10 or more years. But for the sake of simplicity it will still trade as a 5-year bond and one will generally ignore its extension risk. From our perspective, interest rate risk is less decisive that is this form of extension risk, in that ignoring it can particularly lead to an underestimation of the cost of protection.

The first part of this paper discusses extensively of ABS and introduces several stylized facts about ABS' default. We subsequently review the current framework for the pricing of ABCDS and propose some adjustments to account for extension risk and interest shortfalls. The last part compares the approaches and develops the model impact and sensitivities to underlying

parameters.

2 ABS and ABCDS, modeling and structures

Asset-backed securities are debt securities which cash flows come from an underlying pool of securitized assets such as mortgages or auto loans. This pool is purchased by an entity, the ABS trust, which funds by issuing a series of notes (generally between two and a dozen, depending on the complexity of the ABS structure). The redemption of these notes is made from the payments of the borrowers, following a specific waterfall so as to modulate their risk profile.

For example, BBVA RMBS 1 is a securitization of mortgages originated and serviced by BBVA, the Spanish bank. In this transaction, BBVA sold a portfolio of mortgage loans to the trust, that issued in turn five Series of notes to fund the purchase of the portfolio. At the issue date in February 2007, the collateral consisted in 17,184 loans granted to finance the purchase, building and renovation of residential homes located in Spain. As can be seen on the table below, 91.8% of the collateral is made of senior securities – the classes A1, A2 and A3, rated AAA. Symmetrically, note that the junior tranches are very thin; this implies that even limited loss can fully wipe out the bond. This was pointed out by Crouhy, Jarrow and Turnbull [4] as the "cliff" effect that characterizes junior bonds, with devastating effects in the ABS CDO space.

Overview of BBVA RMBS 1 (source : ABSNet, Prospectus)

Class Name	Balance (Eur million)	Balance (%)	Subord. (%)	Rating Fitch / Moody's	Coupon
A1	400	16.0%	8.2%	AAA/Aaa	3mE[1]+ 5 bps
A2	1,400	56.0%	8.2%	AAA/Aaa	3mE + 13 bps
A3	495	19.8%	8.2%	AAA/Aaa	3mE + 22 bps
B	120	4.8%	3.4%	A/Aa3	3mE + 30 bps
C	85	3.4%	-	BBB/Baa2	3mE + 54 bps

AAA rating of senior tranches is mainly achieved through the subordination of classes B and C, as well as a reserve fund (a cash reserve funded by a loan from BBVA) and excess spread. The excess spread is the difference between the weighted average interest rate received on the underlying collateral and the weighted average coupon paid on the issued security. In

[1] 3-months Euribor.

that case, it is structured through an interest rate swap which guarantees a spread of 65 bps. Roughly speaking, each year the first 0.65% of losses will be erased thanks to the excess spread, making the later the first layer of protection against collateral losses.

Such a sophistication is necessary so as to tailor the tranches and eventually suit to investor needs while minimizing funding costs. Investors generally look for a satisfying yield with respect to the tranche rating and expected maturity. From the originator point of view, the purpose of the securitization is to obtain liquidity and remove credit risk linked to mortgages on its balance sheet.

As shown by the example above, ABS' complexity comes from both sides of its balance sheet. We will discuss these largely in the below section, before drawing conclusions about what properties a desirable model should exhibit.

2.1 Asset side : Collateral modeling

Though collateral cash flows are uncertain, its is common in the European market – both cash and synthetic – to price on a standard forecasted scenario. We remind [2] below the typical parameters that define such a scenario:

- CPR : Conditional Prepayment Rate, is the percentage of outstanding mortgage loan principal that prepays in one year, based on the annualization of the Single Monthly Mortality (SMM), which reflects the outstanding mortgage loan principal that prepays in one month.
- CDR : Conditional Default Rate, the effective annual default rate applied to beginning collateral balance for that period (%).
- Recovery and arrears assumptions on the underlyings
- Call assumption (see below in section 2.2.2)

Note that the use of such aggregate parameters restrict this methodology to granular pools such as mortgages. Market practice for the pricing of ABS is akin to the pricing of cash bonds, e.g. the price P of an ABS tranche is simply the discounted PV of forecasted cash flows.

$$P = \sum_{t=0}^{T} \frac{\Delta N(t) + Coupon(t)}{(1 + r(t) + DM)^{(\frac{T-t}{Basis})}}$$

[2] Source : Intex Knowledge Base

Where $\Delta N(t)$ is the principal redeemed at time t, r (trivially) the risk-free interest rate curve, $Basis$ the day count adjustment (typically 365) and DM the extra-yield required by an investor, e.g. the discount margin. Hence – assuming a consensus on forecasted cash flows – quoting in price or spread is equivalent. Practically, ABS are generally quoted in discount margin together with the modeling assumption.

Proceeding that way allow market participants to trade ABS as vanilla bonds, once having agreed on the modeling assumptions. The same method is used to determine future cash flows for Credit Default Swaps (CDS). Though convenient, this approach suffers from its inconsistency; it is paradoxical to price a credit-risky security with standard assumptions that exclude the very event of default. More generally, the drawback of this approach is that it does not account for the specific risks of ABS, namely prepayment, interest rate and credit risk.

Note that those three are intimately correlated. High interest rate environments offer little incentive for borrower to refinance, while low interest rate is generally followed by flows of prepayments. This is a well-known topic to US mortgage backed securities (MBS) academics and professionals since the end of the 80's (see [16] and [17] as references)[3]. Meanwhile, bad economic condition will simultaneously cause credit losses and a decrease of prepayment speed. It is useless to point out that credit risk and interest are correlated as well.

Putting all pieces together, ABS cash flows are actually driven by prepayments and credit losses, both dependent of interest rate. To our knowledge the only attempt to describe the ABS collateral in such a fashion was performed by Fermanian [7]. However practical use of this model is made difficult by the complexity of ABS liabilities – amortization rules, optional redemption, performance-linked triggers, etc. – that we examine in the next section.

2.2 Liability side : Structures

Indeed to the complexity of the underlying cash flows adds the one from the ABS' liabilities. Each deal has a unique waterfall mechanism that determines cash flows allocation to the different tranches. Absent from any standardization of structures in the ABS market, professionals rely on third-party or proprietary cash flow models that replicate ABS waterfalls once

[3]Note that ABS differ from the later by the fact that they also bear credit risk – which is usually covered by government-sponsored agencies for MBS.

specified the forecasted scenario. We discuss below the main aspects of these structures from a modeling perspective.

2.2.1 Amortization

A security's principal is generally redeemed over its life from the underlying proceeds. This means that there is no such thing as a single maturity and that the stated maturity of a given bond is a "legal" maturity. The later generally has to be greater than the longest of the underlyings and will be much higher than the expected maturity date. Hence it generally provides little insight about actual amortization of the security. A third metric widely used by professionals, the weighted average life (WAL) is the average amount of time that a bond's principal is outstanding. It is computed as follow :

$$WAL = \sum_{t=0}^{T} \Delta N(t) \Delta t$$

For example the BBVA RMBS-1 A2 bond's legal maturity is June 2050 (43.4 years at issuance) but the expected maturity stated in the prospectus is march 2018 (11.1 years), assuming 10% CPR; under the same assumptions, the average life of the bond is 5.2 years.

Deals generally embed triggers and events that strongly modify the allocation of cash flows when losses and arrears appear in the pool – another form of protection to the senior notes. In the example of BBVA, the normal amortization of senior classes is sequential (that is, A2 notes amortization starts when A1 notes are fully redeemed, and A3 with respect to A2). However, if the the amount of performing loans is less or equal than the amount of the senior notes, principal due is to be allocated pro-rata between the senior notes (a *"Pro Rata Amortisation of Class A"* event). The junior classes normally amortize pro-rata once reached a satisfactory subordination level and once the creditworthiness of the pool is satisfactory in terms of delinquencies and losses (*"Conditions for Pro Rata Amortisation"*). This indeed implicitly defines triggers that defer the pro-rata amortization.

Then, when the quality of the underlying pools deteriorates – that is, when an ABS is expected to ultimately default – the cash flow waterfall is likely to change. In the case of BBVA, the senior tranches amortize pro rata while the junior ones stop receiving principal. At this stage we can therefore point out two factors modifying the amortization of ABS tranches in deteriorated environment. First, the redemption speed at the collateral level should decrease when the economy slows down; second, triggers may either

defer or accelerate tranche redemption as a consequence of deterioration of underlying collateral performance. In the next part we will examine a third one : optional redemption.

2.2.2 Optional Redemption

Deals generally embed options to call partially or fully outstanding liabilities, conditionally on some specific event. The call can actually aim at tailoring amortization, e.g. targeting a tranche's expected maturity date or WAL: this is extremely common as it allows to reduce amortization uncertainty and to exhibit reasonable expected maturity to investor, making the security easier to sell. In that case the ABS will be callable starting from a specific date, which is typically the expected maturity. It can also be designed to allow the originator to buy back outstanding classes when the collateral balance has been substantially amortized. Not calling would indeed leave a small amount of collateral to be serviced – which is generally uneconomical. The later are clean-up calls and are typically exercisable when the outstanding collateral falls below 10% of the original.

For the call option to be useful, e.g. so as to prove to market participants that the actual maturity of the ABS will be the expected maturity, one need to incentivize the call. In that objective there is usually a significant step-up in the coupon rate in the event the call is not exercised. As a result, ABS were commonly priced assuming the call is exercised, as the option is expected to be deeply in the money at the call date. With the subprime crisis going forward, this assumption has started to be stressed, however.

Back in 2005, Lehman Brothers [11] already pointed out the drivers of callability, stressing that even slightly stressed environment could lead to redemption deferral. In a study of a hypothetical UK non-conforming transaction, they show that the more stressed the economic scenario, the slower the ABS amortizes, in a conjunction of two effects: (i) because CPR typically diminishes when the economy slows down and (ii) because the *optimal* call time is postponed : "*If the pool performance is weak, the originator is likely to be better off earning just the excess spread for a while longer, until either the step-up coupon reduces the excess spread available or further sequential amortization increases the coupon costs from subordinate notes.*"

As securities' documentations generally stipulate the originator must call all the outstanding tranches at par[4], even more distressed scenarios simply exclude the possibility of the bonds being called. Not only because

[4]With some exceptions, however; some junior tranches can sometimes be called at par minus the amount of writedowns.

exercising the call would incur a loss as a result, but because *conditionally* on a given ABS bond being distressed, the originator will not necessarily have the financial strength to call.

For pricing purposes, market participants generally assume that the security is eventually called – as well as they do forecast standard prepayment, arrears, etc. It is the third pillar of our "default paradox", with little consequence for cash bonds though, as one eventually agrees on a price to par. However, it is more problematic for ABCDS as the survival probabilities and timing of default would be deceiving in such a framework. To our knowledge there has been little research on this contradiction from a pricing perspective. However before examining this issue quantitatively, we need to examine the specificities of ABCDS contracts.

2.3 Stylized facts about default

2.3.1 ABCDS and Pay-As-You-Go

The adaptation of the corporate CDS settlement to ABS is not straightforward, due to the securities' complexity. We remind that this contract allows two counterparties to swap the default risk of a reference entity; the buyer pays periodic payments to the seller in exchange for the right of a loss payment if a credit event occurs. This section sums up the main differences of ABS Pay-As-You-Go (PAUG) templates versus corporate ones, as they are stated by ISDA in the 2007 supplement [6]. Extensive studies have already be done by dealers, so one can refer to [9] and [12] for further reading.

Principal writedown and failure to pay It is notable that asset backed securities do not "default" – or more precisely do not *jump* to default as in the corporate world, where the credit event terminates the CDS contract. The default of an ABS is indeed almost predictable once the pool losses have accrued to a critical level. However even if the ultimate default of an ABS is unavoidable, it is likely that it will not be immediately effective. Practically, one notices a principal writedown when underlying losses allocated to a tranche eventually result in a principal reduction (*Principal writedown* event). It can also happen shall the security fails to pay principal (*Failure to pay principal* event).

These events can take a long time to occur, as the non-payment of *scheduled* principal is not a default – only the failure of *mandatory* non-payment is actually a default. This is mitigated when transactions embed a principal deficiency ledger (PDL) to be used as an *implicit* writedown. This

accounting ledger is debited when losses in a given tranche exceed available credit enhancement and is credited if and when such losses are reversed. Implicit writedown consequently ensures that the credit event would not be too back-loaded. It is particularly efficient for junior bonds (with limited subordination), yet it may still take some for senior tranches to be eventually hit – with the actual credit event generally occurring long after the initially expected maturity.

Both events can either trigger a floating or a "hard" credit event in a CDS, at the buyer's option. A "hard" credit event implies physical delivery of the underlying bond by the buyer to the seller and terminates the contract. A floating event does not terminate the contract and implies a loss payment of the protection seller that will be paid back whenever a reverse writedown occurs.

Interest shortfalls Another characteristic in distressed or defaulting securities is that they may miss or defer their interest payment. Standard European PAUG implies the payment of the shortfall by the protection seller, to the limit of the CDS premium (*Fixed Cap*) in a Floating Event. Shall the ABS eventually pay the shortfall, a reverse payment would occur from the buyer to the seller. In pricing terms, this means that one could account for interest shortfalls either in a separate "floating" leg, or as a reduction of the fee leg – we will adopt the latest in section 3.2.2.

Interest shortfalls are typical for subordinated notes when cash flows normally allocated to the payment of coupons are diverted to pay the senior classes' or to increase their credit enhancement, such as a reserve fund. This can also occur when the structure embed an Available Fund Cap[5]. Just as previously, these events can take time to occur as triggers often divert cash flows to ensure that all tranches have their coupon paid as long as possible, deferring the credit event.

2.3.2 Examples

We plot below a few examples of amortization of three ABS tranches (two seniors, one junior) realized thanks to ABSNet cash flow model. Each time

[5]AFC are mostly common in US Home Equity Loans ABS but are also prevalent in US RMBS and certain European CMBS. Such structure caps bond interest to available funds, which is the weighted average net coupon on the underlying collateral. Therefore, when bond coupon rates rise above the AFC rate, there will be a shortfall in the interest payments received by the holders of the reference obligation, exposing them to interest rate risk unless there is enough excess interest in the deal to cover it.

we plotted the forecasted factors under two scenarios[6]:

- a **base amortization**, where no significant losses occur in the underlying pool; the cash flows are run assuming the ABS is called at the earliest call date.
- a **stressed amortization**, where the tranche eventually defaults, where we assume a 5-10% CDR without exercice of the optional call.

As discussed beforehand, the amortization profile of BBVAR senior bonds is sequential; but as of summer 2008, the A1 class had already amortized, that's why the A2 class plotted in figure 1 starts amortizing immediately. The ABS structure is straightforwardly pass-through, e.g. all principal cash flows are used to redeemed the A2 class. In the stressed scenario, amortization is even faster as the tranche benefits from the recoveries of the defaulting loans. But from year 2.75 amortization simply stops and the ABS will never redeem anymore principal resulting in a final writedown of 53% of the principal at year 27.0 when the collateral is depleted. In fact at year 2.75 the funds become insufficient to fully pay the class C coupons and the proceeds that were reserved to pay senior classes' principal is diverted to pay as much coupon as possible. As losses accrue, interest shortfalls then occur on the class B (year 3.75) and to the senior tranches at year 11.25 (see figure 5, p.22).

Though belonging to a complex Master Trust structure, the amortization of Granite bond (figure 2) is quite standard as well. GRANM 2006-4 A7 is a AAA/Aaa/AAA UK prime RMBS issued by Northern Rock. Normal amortization is determined by an ad hoc schedule or *"Controlled Redemption"* which is defined in the ABS' prospectus [14]. The prospectus also stipulates that when senior tranches subordination become insufficient to cover portfolio losses (*"asset trigger event"*), all cash flows are to be passed through to the senior classes. This is what happens here in the stressed scenario where this mechanism proves to be efficient: the tranche eventually achieves to get almost all principal back with only 14.9% of the principal written down in year 21.0.

DELPH 2006-1 B (figure 3) is a Dutch RMBS junior note rated A1/A by Moody's and Fitch issued by Fortis Bank. The ABS structure stipulates that repayments received under the mortgage loans shall be used to purchase substitute mortgages to the originator up to the quarterly payment date

[6]The amortization profiles will be later respectively stated as the functions $N^{\overline{def}}$ and N^{def}.

preceding the first "*Optional Redemption Date*" (year 3.25). Fortis holds the option to call from this date (or on any payment date thereafter) all tranches, incentivized by the coupons step up after year 3.25 – and this is what occurs in the base case. The call price is par, except for junior notes that can be redeemed at their outstanding principal balance less any Principal Deficiency Ledger balance at the the call's date. We assumed here that the call is not exercised in the stressed scenario, hence the tranche remains current until exhaustion of the collateral in year 28.0, without redeeming any principal. However as the structure embed a PDL, the underlying losses result in the tranche being gradually written down between year 0.5 and 1.5. As a result this tranche does not suffer from any extension risk: whatever the outcome of the call, it actually endures front-loaded writedowns.

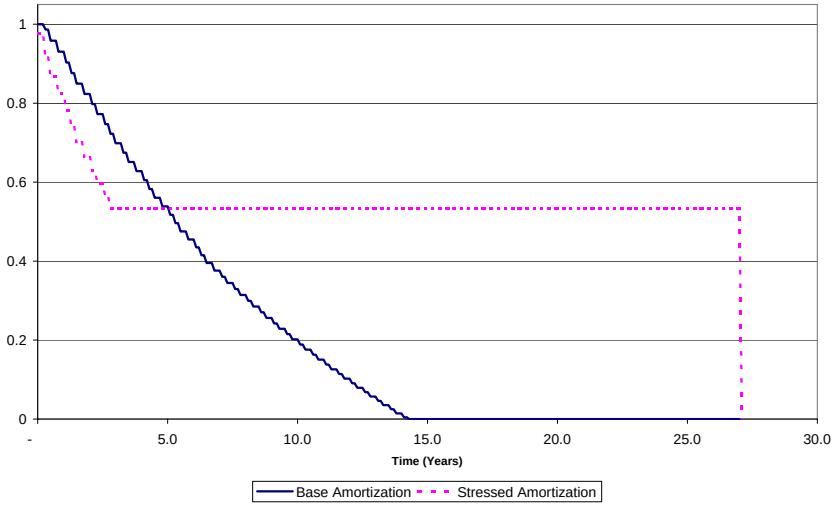


Figure 1: Amortization : BBVAR 2007-1 A2 (source : ABSNet).

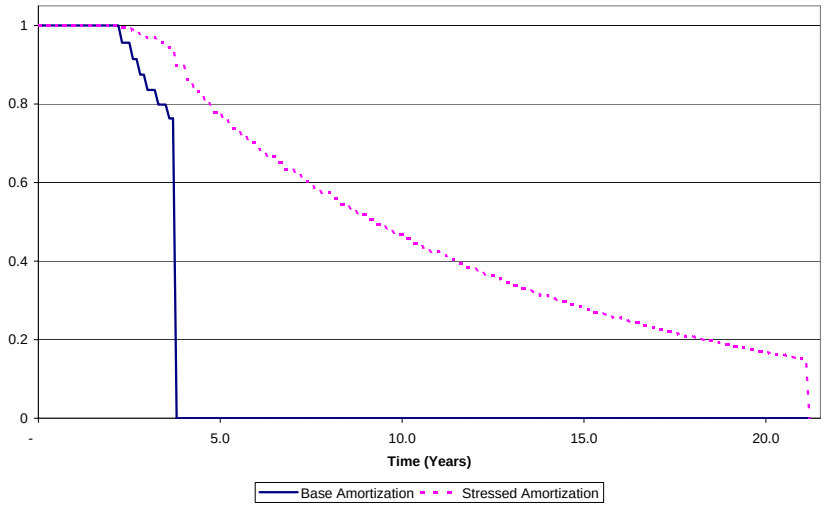


Figure 2: Amortization : GRANM 2006-4 A7 (source : ABSNet).

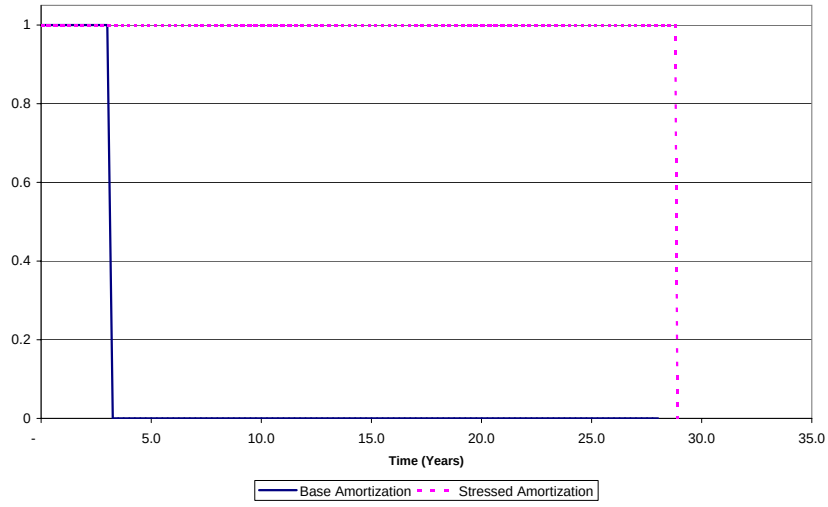


Figure 3: Amortization : DELPH 2006-1 B (source : ABSNet).

We sum up the above results in terms of WAL in table 1 below.

	Base Amortization	Stressed Amortization
BBVA RMBS-1 A2	5.7	14.8
GRANM 2006-4 A7	3.3	10.6
DELPH 2006-1 B	3.0	0.6

Table 1: Impact of default on ABS' WAL

Putting all pieces together, we can picture a "stylized" view of ABS defaults :

1. **Amortization** is strongly modified by the occurrence of default. While prepayment speed typically decreases when default rise, diversion of cash flows can either accelerate (figure 1, at least for a part of the amortization path) or decelerate (figures 2 and 3) the redemption speed of a security.

2. **Principal writedown** will occur after the expected maturity, unless documentation allows for implied writedown, yet still requiring some time for losses to exceed credit enhancement. As a result writedowns will typically be back-loaded for senior tranches and front-loaded for most junior tranches.

3. **Interest shortfalls** can precede a credit event, as in the example of BBVAR 2007-1 A2 bond here.

Items 1. and 2. illustrate the "default paradox" of structured finance securities that we somehow introduced throughout this part. Pricing ABS on standard assumptions with no care to the the event of default is contradictory. And, consequently, trading and risk managing an ABS only on the base of its expected maturity or WAL is deceiving, neglecting the fact that an ABS' WAL is likely to increase would it go to default. We will later see that this can lead to an underestimation of the ABCDS duration, with two consequences. First, the valuation of the cost of protection will be erroneous. Second, one willing to immune its book from spreads movements can end up misleaded. Indeed provided a security is characterized by the stylized facts 1. or 2., its duration should have the property of *increasing* with spread – as defaults come with a slow down of amortization. We will discuss further that one can easily capture the above stylized facts while keeping a simple model.

3 Pricing model

The current standard for the pricing of ABCDS is an extension of the corporate model for CDS such as Jarrow and Turnbull [8] (1995), Lando [10] (1998) and Hull & White [15] (2000), where the credit event occurs at the first jump time of some Poisson counting process. We will see that the main difference is that the underlying's notional is amortizing. This valuation method is legitimated by the need of pricing ABS consistently between bond and CDS, and by the fact that it is generally too expensive to adopt a more computer-intensive approach. However this model suffers from the same drawbacks as the pricing of cash bonds, failing at properly describing ABS' default. In particular, we will see that duration is always a decreasing function of spread – a contradiction with the "default paradox" that we exposed earlier.

3.1 Market practice

The classical framework is the reduced form model where default is a single random event defined by a Poisson process. Amortization is modeled through $N(t)$, a deterministic, non-increasing function equal to the current fraction of the notional at time t. By definition, $N(0) = 1$. We define T as the maturity of the ABS, e.g. $\forall t \geq T,\ N(t) = 0$ and $\forall t < T, N(t) > 0$. The amortization function is computed with the same underlying assumptions than the one used for the pricing of the ABS bond, hence T is the expected maturity, not the legal maturity.

Let R be the (deterministic) recovery fraction of the notional recovered in case of default. This means that for a given default time τ the loss is then equal to $(1 - R)N(\tau)$. Provided there is actually an amortization of the underlying security (e.g. the later is not a bullet bond), the recovery (in terms of the initial notional) is stochastic as a result.

As a CDS of ABS is referenced on a single bond, there is only one maturity to observe, which excludes the possibility of extracting term structure of spreads for a single ABS. Apart from this single spread, there is generally limited insight on the timing of losses. There may be quotes on notes of equivalent seniority on the same or on similar collateral, but of differing maturities. Information can also be extracted from the capital structure of a given ABS, e.g. the price of risk on bonds of different seniority backed by the same collateral. However such material is not enough systematically available to allow any robust method, unfortunately.

Assuming one wants to calibrate a model from the market spread only,

such calibration must then be determined by a single parameter. The simplest choice is to assume a constant value λ for the hazard rate. However, it is generally unlikely that an ABS would default in the first years following issuance. This pleads for the intensity to be be an increasing function of time $\lambda(t)$, which favors back-loaded timing of defaults. A common choice is to assume a step-up function, with the intensity being nil until a given time t_0 where the it jumps to a positive value. Formally $\lambda(t) = 0, \text{for } t < t_0,\ \lambda(t) = \lambda'$ otherwise. We plot an example of equivalent calibrations on figure 4 – note that in line with market practice, the amortization curve is the default-proof one plotted on figure 2 p.13.

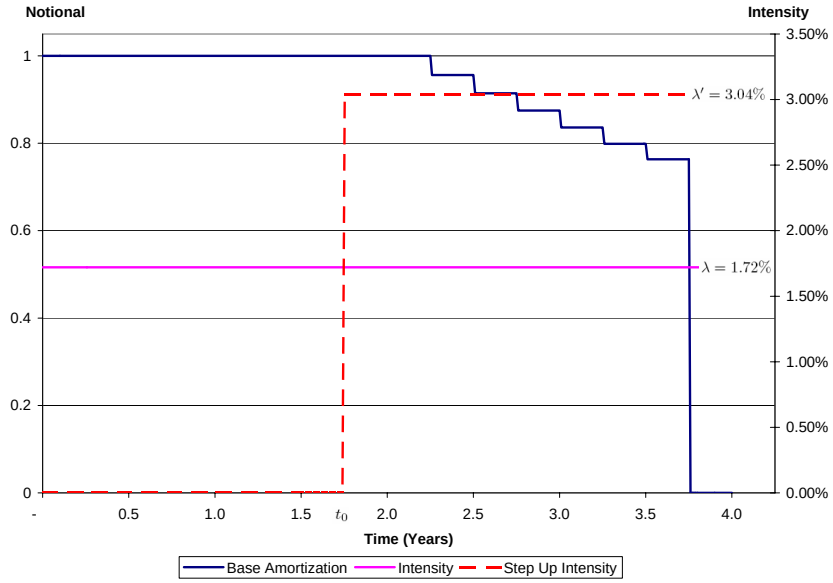


Figure 4: Step Up in the hazard rate : example form GRANM 2006-4 A7

Unsurprisingly, we assume deterministic interest rates as in most credit models, though the assumption is more challengeable for ABS where interest rate is a major driver of prepayment and portfolio losses. As a matter of simplification we will ignore accrued interest in case of default. We will also assume that protection is paid at the very time of default. The expectation of the default (or contingent) DL leg then writes :

$$\mathbb{E}(DL)(\lambda) = Notional \int_0^T N(t)(1-R)B(0,t)f(t)dt \qquad (1)$$

And the expectation of the fee (or premium) leg FL is :

$$\mathbb{E}(FL)(\lambda) = Notional \times Coupon \times dur(\lambda) + UF$$

with

$$dur(\lambda) = \int_0^T N(t)\mathbb{P}\left(\tau > t\right) B(0,t)dt \tag{2}$$

and where :

- $f(t)$ is the density of the default time's distribution, e.g. $f(t)dt = \mathbb{P}(\tau \in [\, t, t+dt])$. Survival probability can be defined as $\mathbb{P}(\tau > t)$ where τ is the default time of the ABS.
- $B(t,T)$ is the value at time t of the zero coupon bond maturing in T
- UF is the upfront fee (if any).

The model is calibrated by finding the λ for which the NPV of the CDS is zero.

3.2 Adjusted model

We can identify several issues raised by the above methodology, particularly the assumption of a deterministic amortization profile. One would expect to see a stochastic amortization, the latest being be a function of interest rates and intensity. From a valuation perspective, we cannot write a practical model with true random prepayments to address credit risk. The European market is too heterogeneous and not liquid enough to sustain this kind of approach – which can however be relevant for relative value and price discovery. Less ambitiously, we will solve the "default paradox" by allowing defaults to occur after the expected maturity. Luckily this approach implicitly correlates amortization and default so that duration will become an increasing function of spreads.

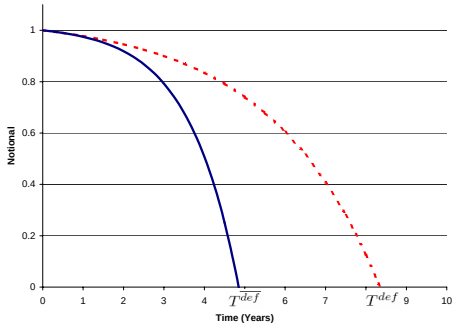
3.2.1 Extension-risk adjustment

Let's assume that, conditionally on the event of default, the ABS follows a different amortization path, that we will call N^{def}; otherwise it will stick to the same amortization profile as before, that we now name $N^{\overline{def}}$. Note that

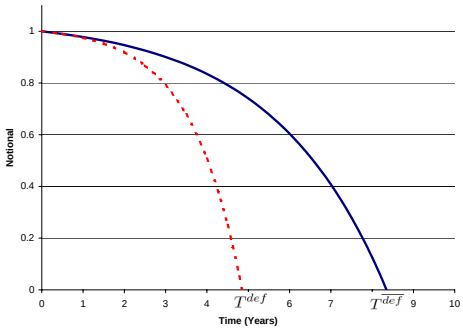
yet still being the commonly-agreed amortization curve, it is now risk-free. Practically, market participants need to agree on a second cash-flow scenario which generates the stressed curve, as in our examples in section 2.3.2 (p.10). Let $T^{\overline{def}}$ be the base-case maturity of the ABS, T^{def} the maximum maturity of the stressed amortization profile N^{def}, so that:

$$N(t) = \begin{cases} N(t)^{def} & \text{if } \tau \leq T^{def} \\ N(t)^{\overline{def}} & \text{if } \tau > T^{def} \end{cases}$$

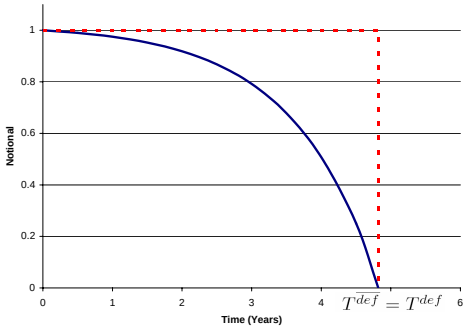
In this framework, defaults are characterized by $\tau \leq T^{def}$. This means that for all τ between 0 and T^{def}, the security defaults and the CDS contract terminates at this very time τ. Otherwise, the security survives and the ABS matures in $T^{\overline{def}}$. Hence, the choice of N^{def} is crucial in the characterizing of extension :



1. If $T^{\overline{def}} < T^{def}$, the ABS exhibits extension risk, because cash flows and defaults can occur after the standard expected maturity – e.g. we allow defaults to occur in the range $[T^{\overline{def}}, T^{def}]$.



2. If $T^{\overline{def}} > T^{def}$, defaults can only occur in the smaller range $[0, T^{def}]$: amortization *accelerates* in case of default.



3. Finally if $T^{\overline{def}} = T^{def}$, the expected maturity remains the same, but note that the ABS can still exhibit a distinct stressed amortization profile (in this example, it simply does not amortize).

The most likely case is 1., which will be relevant for most senior and mezzanine tranches. 2. is empirically true for junior bonds, but having in mind that the market model already allows front-loaded defaults, no adjustment to the curves is actually necessary for these cases. That's why we deliberately describe our model as extension-risk adjustment. We will discuss more thoroughly the choice of the amortization curves in section 4.1.1.

For the computation of each leg of the CDS, we will mathematically integrate on the range of all default times; the ABS will either follow the deterministic amortization path $N^{\overline{def}}$ or default in τ. We can now write the expression of the default leg:

$$\begin{aligned}
\mathbb{E}(DL_{adj})(\lambda) &= \mathbb{E}\big(DL \times (\ \mathbf{1}_{\tau \leq T^{def}} + \mathbf{1}_{\tau > T^{def}})\big) \\
&= \mathbb{E}\big(DL \times (\ \mathbf{1}_{\tau \leq T^{def}})\big) + \underbrace{\mathbb{E}\big(DL \times (\mathbf{1}_{\tau > T^{def}})\big)}_{=0} \\
&= Notional \int_0^{T^{def}} N(\tau)^{def}(1-R)B(0,\tau)f(\tau)d\tau \quad (3)
\end{aligned}$$

This is the same as equation 1 previously, except that the losses are recorded on the second amortization curve N^{def}. It actually writes as the losses scenarios weighted by their probability of occurrence $f(\tau)$. As the event of default is only driven by T^{def}, the expected maturity $T^{\overline{def}}$ do not figure in the equation.

In order to be able to compute the fee leg, we need to account for the new amortization in the duration computation. Let's have a look first at the security duration, conditional on the default time τ. This duration is the present value of 1 bp on notional $N(t)$ until the default event τ:

$$dur(\tau) = \int_0^\tau N(t)B(0,t)dt$$

Substituting $N(t)$ by its adjusted values, we can define $dur^{def}(\tau)$ and $dur^{\overline{def}}$, respectively the default and risk-free durations :

$$\begin{aligned} dur^{def}(\tau) &= \int_0^\tau N(t)^{def} B(0,t)dt \\ dur^{\overline{def}} &= \int_0^{T^{\overline{def}}} N(t)^{\overline{def}} B(0,t)dt \end{aligned}$$

Note that the duration is deterministic provided the bond survives; otherwise there are as many amortization paths as default times. We can now write the duration as an expectation under the distribution of τ (see the proof in the Annex). With $f(\tau)$ being the density of the default time distribution, we have :

$$\begin{aligned} dur_{adj}(\lambda) &= \int_0^\infty dur(\tau)f(\tau)d\tau \qquad (4) \\ &= \int_0^{T^{def}} dur^{def}(\tau)f(\tau)d\tau + \int_{T^{def}}^\infty dur^{\overline{def}} f(\tau)d\tau \\ &= \int_0^{T^{def}} dur^{def}(\tau)f(\tau)d\tau + \mathbb{P}(\tau > T^{def})\ dur^{\overline{def}} \qquad (5) \end{aligned}$$

The duration is the average of the default time-contingent durations defined before, weighted by the distribution of the default times.

The choice of a binary function might seem arbitrary at first glance; it would be tempting to write a more sophisticated amortization profile with a greater number of curves, each one associated to a range of default times so as to weight prepayment scenarios by their likelihood of occurrence. However one should have in mind that the amortization process of the ABS actually stops at the default time τ. Therefore our approach generates as many prepayment scenarios as default times.

More importantly, the models reaches its objective in allowing defaults to occur *after* their expected maturity. Yet being simple in its form, it

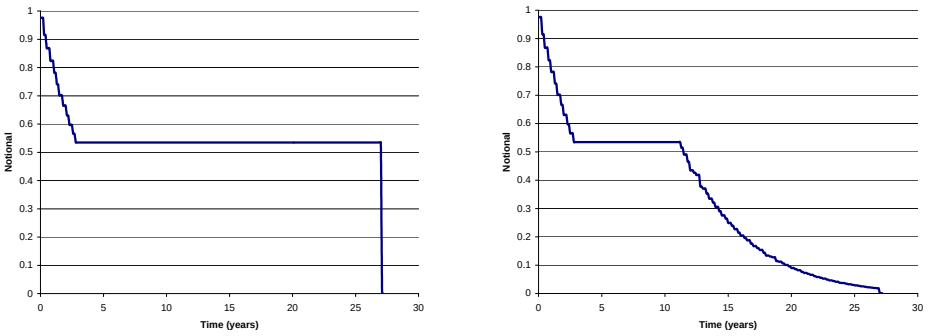
manages to describe the behavior of defaulting ABS. We will see later in 4.2.1 that the resulting duration, and the value of fee leg as a result, is higher than the market model's. This reflects the fact that default may take time to occur, and then that buying protection can be more expensive than expected. Besides the duration can become an increasing function of spread, representing the fact that the riskier the asset, the higher the probability of the amortization to slow down as the default will take time to materialize.

However, we have seen that interest shortfalls sometimes occur before the default of a security. As these shortfalls are a floating event, they do not terminate the CDS contract and are not accounted by the default leg. We will now see that our framework can address the occurrence of these events.

3.2.2 Interest-shortfall adjustment

PAUG settlement imply that shortfalls in the payment of coupons trigger a floating event where the corresponding loss is paid by the protection seller. Under the *Fixed Cap* documentation, these payments are capped to the amount of the CDS premium. Said otherwise, these shortfalls simply reduce the outstanding notional of the security on which the CDS coupon is paid. From a pricing perspective, while the extension effect was to *increase* the value of the duration, we can expect the occurrence of interest shortfalls to mitigate this effect by *reducing* it.

Figure 5 plots the impact of shortfalls on GRANM 2006-4 A7 bond in the same distressed scenario discussed earlier (figure 2). The left-hand plot (a) is the reproduction of figure 1 for comparison purposes. The right hand (b) is the same after accounting for the interest shortfalls that occur in the stressed case. Let's assume a scenario where the default's time occurs in $\tau = 20$. Ignoring for now the impact of interest shortfalls and assuming a flat interest rate curve of 4%, the duration will be the present value on the curve (a), e.g. $\int_0^{20} N(t)^{def} B(0,t)dt = 7.79$. Practically, the occurrence of interest shortfalls will reduce the value of the coupons actually paid by the protection buyer. The later will be paid on smaller notionals – plot (b) – reducing the duration value to 6.47.



(a) $N(t)^{def}$ (b) $N(t)^{def}$ after accounting for shortfalls

Figure 5: Interest shortfalls reduce the fee leg value, example from BBVAR 2007-1 A2 (source : ABSNet).

Hence we can simply model interest-shortfalls by assuming that, on average, a fraction of the fee leg won't be paid in default events. Let's call s the share of the coupon that will be canceled due to shortfalls. We will assume s to be a constant here, but one can easily set it to be a function of time, for instance. The default-contingent duration formula now becomes :

$$dur^{def}(\tau) = \int_0^\tau N_i^{def} B(0,t)(1-s)dt \tag{6}$$

Depending on the ABS' waterfall, s can vary from 0 (no interest shortfall is deemed conceivable, as in distressed scenarios the waterfall diverts underlying cash flows to pay the tranche's coupon) to 1 (a defaulting tranche will never pay any coupon). The choice of s is discussed in section 4.1.2.

As a result of this adjustment, the amortization profile used for the calculation of the duration differs from the one used for the computation of losses. Hence in the Granite example such modeling allow for a large, backloaded, loss [7] together with a decreasing coupon.

[7] which actually occurs in the stressed cash flow scenario run in ABSNet: the writedown of principal actually occur at final maturity.

4 Model calibration and results

4.1 Model calibration

As discussed ahead of time, we believe the strength of this model is its simplicity. While staying close to the current market practice – and hence to the corporate CDS pricer – it manages to account for stylized facts that are typical of the structured finance world. We discuss below on the choice of the parameters with this idea in mind.

4.1.1 Choice of amortization curves

Both amortization curves here are to be determined by market participants through the use of a cash flow model such as Bloomberg, ABSNet or Intex. We remind that the amortization curve represents the percentage of outstanding notional at time t. If computing such a value is quite straightforward in the base scenario, the stressed one requires a specific care.

Indeed, equation 3 states that the security's losses are actually computed on the factors N^{def}: they are endogenously modeled in the default process. This means that, when running the cash flow model, the $N^{def}(t)$ values have to be equal to the outstanding notional *before* losses that occur in this specific scenario. For example, if a given scenario generates writedowns at years 12 and 13 but that no redemption occur until final maturity, then the amortization profile should be bullet until maturity.

However, proceeding that way can lead to an underestimation of realistic amortization in two cases :

- **Front-loaded principal writedowns**, typically for junior tranches with PDL and limited subordination (see the example of DELPH 2006-1 B in section 2.3.2 p. 10)
- **Distressed rating**, a downgrade to CCC/Caa2/CCC or below, or a rating withdraw by one or more rating agency can trigger a credit event. Again, this is predominantly true for junior tranches.

For example, assuming no amortization for DELPH 2006-1 B as in figure 3 would mean that defaults can occur until final maturity (year 27), which is unrealistic. In this very example the ABS amortizes *faster* because of realized losses; this would not be captured without adjustment to N^{def}. These situations obviously require a case-by-case treatment. As a rule of thumb one could simply chose to use the market model in these cases (e.g. $N^{def} = N^{\overline{def}}$).

4.1.2 Recovery and interest shortfalls parameters

Recovery is a decisive parameter in CDS pricing, because of its impact on default probabilities. For a given spread, the higher the recovery assumption, the higher the default probability. Market practitioners usually set the recovery parameter for ABS at 40%, in line with the practice of the corporate world. But while the latter is well established and backed not only by statistical figures but also by market levels[8], there is no such proof for structured finance securities.

As a calibration method, we suggest to simply use the realized recovery in the benchmark default scenario. We find out that the 40% assumption almost never hold for ABS, having in mind that for pricing purposes, the recovery parameter is applied to the *outstanding* notional at the default time. While this is not problematic for bullet corporate bonds, for amortizing ABS the *face* recovery has nothing in common with the one observed on the notional at the default time. Numerical investigations on the sample ABS introduced in section 2.3.2 indicate that R should almost always be equal to 0. We believe this is almost always true for amortizing or junior bonds, provided they are remote from default – distressed bonds would obviously require a specific treatment. Senior bullet ABS should behave as senior corporate debt, however.

Again, the shortfall parameter should be computed through the same default scenario as for the amortization curve and the recovery. For computational reasons one would probably find more practical to use a single parameter. We suggest setting it to the value of the actual shortfall share in the later scenario. Calling $sf(t)$ the actual shortfalls that occur in the stressed scenario, then the proxy of interest shortfalls $\hat{s}$ writes as follow :

$$\hat{s} = \frac{\int_0^{T^{Def}} sf(t)dt}{\int_0^{T^{Def}} N^{Def}(t)dt} \tag{7}$$

Which is the WAL of interest shortfalls divided by the WAL of principal payments – the average share of interest shortfalls on figure 5. For the later ABS, we indeed have $\hat{s} = 0.397$.

[8] the recovery swaps market is barely liquid, though.

4.2 Model results

Applied to the above ABS, the model yields significant results and properly captures the cash flow "stylized facts" discussed earlier, e.g. duration are higher and are (mostly) an increasing function of spreads. This is crucial having in mind that duration not only indicates the cost of protection, but is a risk measure as well. In the section below, we first compare our approach to the current market practice and subsequently discuss the parameters sensitivities and impact on the fair spread of ABCDS.

4.2.1 Models comparison

Figure 6 plots the durations of the Granite bond introduced earlier computed with the current model (both *Market* and *Step Up Intensity* curves) as well as ours (*Extension Adjusted*). We assumed $R = 0$ ans $s = 0$, in line with the ABS' scheduled cash flows; for step-up intensity we arbitrarily assumed $t_0 = 2$, with the intensity being nil for $t < t_0$.

The two first curves are by definition capped by the ABS' WAL of 3.3[9] (as it is equal to the non-discounted duration assuming no default). Both durations are everywhere a decreasing function of spread, which means that the riskier the CDS, the shorter is his expected duration. It is a classical result for corporate CDS pricing : increasing spread increases hazard rate (table 2) and decreases the survival probability which weights the cash flows in the duration. Interestingly, step-up model exhibits lower durations than the one assuming constant hazard rate. It is surprising as the shape of the hazard rate curve should force back-loaded default. In fact the later is true holding intensity constant; but for a given spread the jump in hazard rate after t_0 actually more than compensates the effect of having a zero probability of default in the early years, and eventually reduces the duration.

On the contrary, the inherent extension risk of Granite bonds is well captured with the extension-adjusted model where the duration is greater everywhere. The effect of the adjustment is not negligible : for a $10 million trade, the difference in fee leg is worth $20,000. Moreover, the duration is here an increasing function of spread. When spreads widen, default probability increases and so does the likelihood of "shifting" to a slower amortization profile, increasing duration as a result. This is true provided the probability of default is not too high (we will see later that too high a recovery rate can induce such case).

[9] see table 1 p.14.

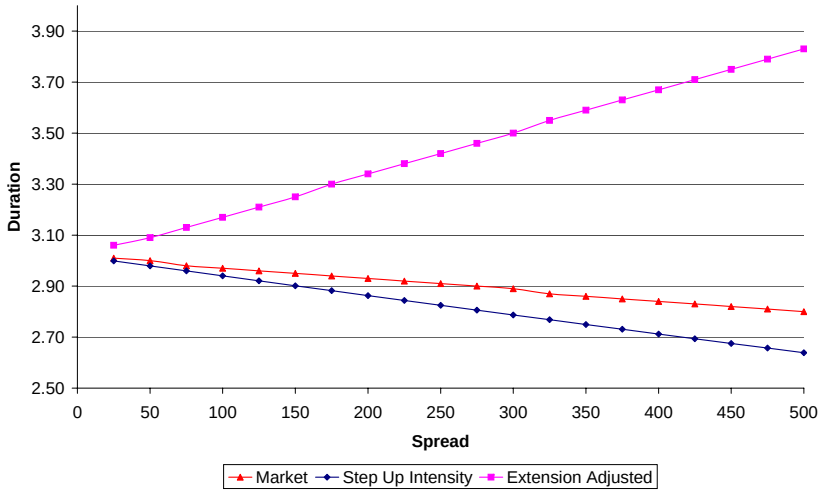


Figure 6: Models comparison : example from GRANM 2006-4 A7.

We believe the extension-adjusted model does not require a sophisticated shape for the hazard rate, as his flat intensity is already low, which makes the probability of immediate default negligible as compared to the market model (for example a premium of 100 bps will yield a hazard rate of 0.30% vs. 0.86% for the market model, see table 2, below).

		Market Model	Step Up	Extension-Adjuted
Spread = 25	Duration	3.01	3.00	3.06
	Intensity	0.21%	0.37%	0.07%
Spread = 100	Duration	2.97	2.94	3.17
	Intensity	0.86%	1.50%	0.30%
Spread = 200	Duration	2.93	2.86	3.34
	Intensity	1.72%	3.04%	0.65%
Spread = 500	Duration	2.80	2.64	3.83
	Intensity	4.34%	7.84%	2.05%

Table 2: Models comparison : numerical results (GRANM 2006-4 A7).

4.2.2 Sensitivity to underlying parameters

We reproduce below the extension-adjusted durations with various assumptions of recovery (figure 7) and interest shortfalls (figure 8). For the sake of simplicity we used the same bond as reference, but have in mind that the below results are true provided the ABS exhibits extension risk.

Interestingly the higher the recovery assumption, the greater is the slope of the curve in most cases. It is well known that holding spread constant, increasing the recovery assumption increases the probability of default. In our approach, this means putting more weight in the default amortization curve – which means increasing duration. This holds for "reasonable spreads", however; for too high a recovery rate (and too high an intensity as a result), the usual effect dominates, e.g. the increased intensity favors the probability of defaulting earlier.

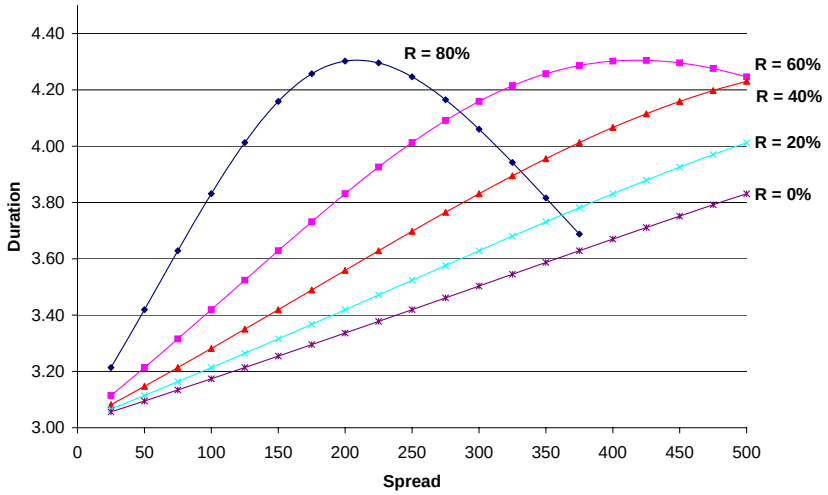


Figure 7: Model sensitivity : Recovery (GRANM 2006-4 A7).

The shortfall sensitivity is more straightforward, with s unequivocally driving the slope of the duration. The higher the value of the shortfall assumption, the more the ABS tends to be insensitive to extension risk. When the spread widens, so does the probability of default; but while the losses (writedowns) will be computed in the default leg, interest shortfalls are themselves accounted in the duration formula – preventing the coupon of the CDS from being paid. Hence the present value of the stream of coupons

may decrease with the occurrence of a default.

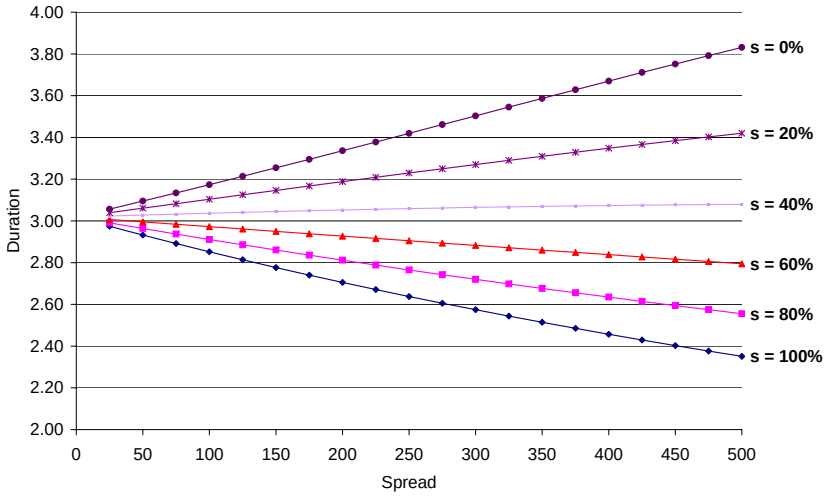


Figure 8: Model sensitivity : Interest Shortfalls (GRANM 2006-4 A7).

4.2.3 Impact on fair value

The model adjustment also modifies the no-arbitrage spread that one may infer from cash quotes[10]. Assuming no basis, one can compute the fair spread of a floating-coupon ABS. If a CDS where to pay a premium equal to the ABS' coupon, then it would require to trade with an upfront to avoid any arbitrage opportunity. The upfront is worth:

$$UF = Notional \times (Issue\ Price - Price)/100$$

Once known the duration of the CDS, the fair spread is trivially:

$$Spread = (UF + Premium \times duration)/duration$$

Table 3 depicts the impact of the adjustment on durations and breakeven spreads on DELPH 2006-1 bonds. All tranches have a non-default WAL of 3.01 (they are actually bullet bonds), with default WAL of A bond at 20.48. Please note that as discussed in section 4.1.1, we did not adjust for default

[10] A thorough study of no-arbitrage pricing can be found in [2].

the amortization curve of the junior bonds. Again we assumed $t_0 = 2$ for the step-up model.

Class	Price	Coupon	Model	Spread	Duration	RR	Shortfall
A	97.73	12.0	Market	94.7	2.74	11.4%	43.2%
			Step Up	95.8	2.71		
			Adjusted	91.5	2.85		
B	92.85	22.0	Market	289.4	2.68	0.0%	58.7%
			Step Up	300.2	2.57		
			Adjusted	295.6	2.61		
C	85.06	37.0	Market	621.3	2.56	0.0%	59.5%
			Step Up	687.5	2.30		
			Adjusted	650.7	2.43		
D	85.09	70.0	Market	656.0	2.54	0.0%	59.5%
			Step Up	726.9	2.27		
			Adjusted	685.0	2.42		

Table 3: Fair spread computation, DELPH 2006-1 (source : Markit, Bloomberg, 22/08/2008)

The adjusted model (with respect with the market one) allows a greater duration to the senior bond (though mitigated by the shortfall parameter) and lower durations for junior bonds that suffer from the loss of most of their coupon when the ABS defaults. We found the step-up approach to be deceiving here as it probably leads to an overestimation of the fair spread. In particular for the junior bonds, the duration appears to be lower than the adjusted model (which accounts for interest shortfalls) – which is the very fact that the step-up model wants to avoid.

5 Conclusion

We presented in this paper a way to account parsimoniously for the main "stylized" specificities of ABS cash flows. We believe this method to be more descriptive than the current market practice, while the later can yield some surprising pitfalls. It is also more intuitive and flexible, allowing to cope simultaneously with extension risk and interest shortfalls.

We also discussed the issue of the choice of the CDS parameters. In particular, we believe the choice of the same recovery assumption as corporate bonds to be generally deceiving.

Again, we stress this approach is satisfying for valuation purpose and does not intends to price ABS in a no-arbitrage way. In particular, the assumption of deterministic interest rate is critical and inconsistent with the well-known pricing of mortgage backed securities in the US. Extending our framework to deal with the latter would make a sensible step toward no-arbitrage pricing.

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A Annex : Proof of the duration calculation

We start from the "classical" form of the duration stated in 2 :

$$dur(\lambda) = \int_0^T N(t)\mathbb{P}\left(\tau > t\right) B(0,t)dt$$

Using the fact that $\forall x \geq T,\ N(x) = 0$ and replacing $\mathbb{P}\left(\tau > t\right)$ by its value, we have :

$$\begin{aligned} dur(\lambda) &= \int_0^\infty N(t) \int_t^\infty f(\tau)d\tau B(0,t)dt \\ &= \int_0^\infty \int_t^\infty N(t)f(\tau)B(0,t)d\tau dt \\ &= \int_0^\infty \int_0^\tau N(t)B(0,t)dt f(\tau)d\tau \\ &= \int_0^\infty dur(\tau)f(\tau)d\tau \end{aligned}$$

The later being equation 4.