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Models of Equilibrium Pricing with Internalized Powers of Independent Judgment Based on Autonomy

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Abstract

In this paper I want to describe one way of thinking about information. In effect, this paper relates to the notion that organizational or societal models of economic equilibrium incorporate a decision-making mechanism that is influenced by the independent judgment of each individual.

This decision-making mechanism has nothing to do with the notion of acting to maximize one's expected utility under a set of exogenous prices or predictions, nor are the consequences of the decisions based on this mechanism related in any way to exogenous prices, predictions, or other parameters. In terms of information theory, the goal is to generate information and make more accurate predictions about a particular condition. However, the information itself can be fundamentally flawed. Another point is that information generated for this purpose has the feature of being irreversible. If communication is defined as a process of engaging in an economic activity while observing and analyzing information generated on the basis of the above mechanism, then it is worth considering whether communication so defined has any utility from an economic standpoint. To pursue this question, I will apply the concept postulated above to theories of economic organization and pricing, and also discuss the externality which information brings into an economy.

JEL classification: A1, B4, C0, D0, D6, D7, D8
Key words: Autonomy, Information, Interaction, Communication, Externality of Information, Herd behavior

Contents

1. Introduction

In this paper I want to describe one way of thinking about information. In effect, this paper relates to the notion that organizational or societal models of economic equilibrium incorporate a decision-making mechanism that is influenced by the independent judgment of each individual.

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If communication is defined as a process of engaging in an economic activity while observing and analyzing information generated on the basis of the above mechanism, then it is worth considering whether communication so defined has any utility from an economic standpoint. To pursue this question, I will apply the concept postulated above to theories of economic organization and pricing, and also discuss the externality which information brings into an economy.

2. Central Thesis

First, an attempt is made, from the perspective of maximizing expected utility, to identify the conditions under which each autonomous economic agent in an economic organization or community is able to retain its independent judgmental capacity. This entails introducing parameters that describe the independence (judgmental power) of each agent, and utilizing a two-dimensional model to derive all conditions from conditional probabilities based on the information sets that each agent is able to utilize. In other words, an effort is made to derive conditions for the existence of situations (stages) under which it is considered better to exercise one's own, independent judgmental power on the basis of one's autonomy, in addition to the conventional notion of maximizing one's expected utility after scrutinizing parameters indicative of external conditions (e.g., prices and / or the outcomes of judgments made by other agents). In such situations, it is possible for the outcome of the agent's judgment and the parameters of external conditions to lose their correlations and be transformed into factors conducive to wild price fluctuations or other unstable situations. It is accordingly plausible to conceive of two scenarios: one in which autonomy is the priority of each agent, and efforts to maintain equilibrium are dynamic and independent; and another in which priority is placed on acting in accordance with external parameters, and the efforts to maintain equilibrium are based on a static or fixed strategy. Although the (dynamic) scenario of autonomy is marked by a certain degree of probabilistic instability, from an information theory standpoint it is assumed that it also generates information that allows for a more accurate prediction of trends in the real world.

First, let us consider the influences that communication or interaction by agents within an economic organization can have on the decision-making mechanism. One role of an economic organization is to collect information under conditions of uncertainty, screen that information and utilize it to make more accurate predictions about the world, and ultimately improve the organization's expected utility. The decision by an individual as to whether or not to exercise independent judgment is based on individual utility maximization. However, as information, the results of that independent judgment will be of value to the organization to which that individual belongs irrespective of whether it was accurate or not. More specifically, independent judgments can lead to more accurate predictions of a situation in the real world, in turn improving the expected utility of the organization as a whole. Conversely, the results of strategy-driven judgments have no value as information to the organization; moreover, if individuals (economic agents) within the organization interact or communicate with each other, they are likely to learn of the results of (independent) decisions made by one or more other individuals and inevitably lose some or all incentive to make independent decisions of their own. Although the independent judgmental power of the individual is crucial to the organization at large, the processes of communication within the organization inform individuals of the independent decisions made by others and undermine their incentive to make independent decisions of their own. It is within this paradoxical framework that the decisions of the organization (committees, hierarchies, etc.) are made.

Next, let us consider the economic implications of communication in terms of the theory of pricing. It is not always apparent that information generated during a stage of equilibrium will be linearly related to the effectiveness it has in boosting expected utility (welfare) on a general scale within an organization or community that is risk-neutral or risk-averse. Furthermore, it is to be expected that an organization or community comprising a heterogeneous mixture of individuals and an incomplete information set will fall into a quasi-steady state whether it is risk-neutral or not, and be confronted by disparities in its actual and predicted equilibrium prices. Consequently, this is a problem difficult to fully appreciate in analytical terms.

It is assumed in this paper that speculation is an economic behavior that involves acting in accord with new predictions that are based on the invocation of autonomy (independent judgmental power) and discontinuous jumps in the prediction equation $[P(M=1 \mid \Phi_i(T-1)) = \beta_i]$. Speculative behavior was once thought to be the province of certain special types of people: i.e., risk-loving individuals characterized by convex utility functions. However, as will be demonstrated in Chapters 6 and 7, even people who adhere to a risk-neutral or risk-averse economic rationale are capable of exercising their autonomy and engaging in speculation. Accordingly, in this context it is also shown that autonomy (independent judgmental power) can have a disruptive (negative) impact on community welfare.

3. Basic model of independent Judgement

First of all, I model what the independence is essentially like. There exist two states in the world, Mode 1 and Mode 0. Mode 1 is a "good" state and Mode 0 is a "bad" state. These states are represented as a binary variable, M. (M=1 for Mode 1 and M=0 for Mode 0.) An agent i, when he or she receives the input of this variable, M, judges by its own criteria whether this variable is in a "good" or "bad" state, and outputs its own decision whether to approve or reject it. I denote this binary output variable by D_i. (D_i=1 for approval, D_i=0 for rejection.) This independent decision making process easily proves to be equivalent to the binary communication channel as shown in Figure 1.

Considering that every agent is fallible, and that it is natural to think that the probability of approval when a "good" state is input, $P(D_i=1|M=1)\equiv P_1^i$, is greater than the probability of approval when a "bad" state is input, $P(D_i=1|M=0)\equiv P_0^i$, I assume $0< P_0^i< P_1^i<1$. According to the information theory, the mutual information of this binary channel, $I(M; D_i)$ is positive because $P_0\neq P_i$. This means that one time independent decision making produces positive bits of information which makes the prediction regarding the value of M more sure.

I also define the initial probabilistic portion of a "good" state as α. That is, $P(M=1)=\alpha$. I assume that α is a public information known to all agents. However, in the course of the "interaction", each agent can observe all or a part of decisions which were made in the past by him or herself or by other agents.

In general, defining the information set of independent decisions which agent i can observe at time t=T, as $\Phi_i(T-1)$, the conditional probability of M=1, held by agent i, is denoted as $P(M=1|\Phi_i(T-1))$. Needless to say, $P(M=1|\Phi_i(0))=P(M=1)=\alpha$.

4. Communication (Interaction)

Now let us assume that an agent can observe the consequences of other agents' judgment (decision making) in an economic organization or a society. Then how would each agent behave him or herself interactively with others agents?

I assume two ways of decision making. One way is to output its "independent" decision, D_i^T, through the model of independence explained in Chapter 3. (i denotes i's agent, and T denotes the round when communication is held.) The other is to output the decision so that it might simply maximize the agent's own expected payoff, given an available information set and exogenous variables like prices. Superficially this output is the same as that of an independent decision in the form of a signal, but different in the process of decision making. Therefore I call this kind of output "a strategic decision" and define it as S_i^T. As I already explain in Chapter 3, the consequence of an independent decision holds a positive bit of information in terms of information theory, and influences the next round's prediction regarding the state of the world. But it is not related in any way to an existing information set nor exogenous parameters like prices. On the other hand, the consequence of a strategic decision itself does not hold any information since simply it is the result from a utility maximization process. I also define the cost of an independent decision making as C_I, which is considered to be the cost needed to think and judge by him or herself and to make his or her own decision, therefore which should be nonnegative.

Let us also define the consequence of a final judgment of an organization or a society as D_o^T. These variables (D_i^T, S_i^T, D_o^T) are all binary ones.

Each agent has its own payoff function, whose general form is represented by $\Pi_i^T(D_i^T=k,\ M=j,\ D_0^T=m)$ in an independent decision, and $\Pi_i^T(S_i^T=k,\ M=j,\ D_0^T=m)$ in a strategic decision. k, j, m are respectively 1 or 0. Let us also define the available information set for agent i at round T as $\Phi_i(T-1)$, the total information set an organization or a society holds, as $\Phi(T-1)$, assuming $\Phi_i(T-1)\in\Phi(T-1)\equiv\{D_i^t\}_{t=0,1,\ \ldots,T-1,\ \forall i}$. For simplicity, we assume that the payoff function does not depend on any final judgment of an organization or a society, therefore has a form represented by $\Pi_i^T(D_i^T=k,\ M=j)$ or $\Pi_i^T(S_i^T=k,\ M=j)$.

At round T, agent i thus chooses the one option which maximizes its own expected payoff among the followings;

- I: to take an independent decision, D_i^T.
- S^1: to take a strategic decision, $S_i^T=1$.
- S^0: to take a strategic decision, $S_i^T=0$.

Thus the expected payoff of agent i for taking an independent decision, D_i, is;

$$
\begin{aligned}
&E(\Pi_i^T(D_i^T,\ M)\,|\,\Phi_i(T-1))-C_i\\
&= P(D_i^T=1,\ M=1\,|\,\Phi_i(T-1))\Pi_i^T(D_i^T=1,\ M=1)+P(D_i^T=1,\ M=0\,|\,\Phi_i(T-1))\Pi_i^T(D_i^T=1,\ M=0)\\
&\quad +P(D_i^T=0,M=1\,|\,\Phi_i(T-1))\Pi_i^T(D_i^T=0,\ M=1)+P(D_i^T=0,\ M=0\,|\,\Phi_i(T-1))\Pi_i^T(D_i^T=0,\ M=0)-C_i\\
&= P(M=1\,|\,\Phi_i(T-1))\{P(D_i^T=1\,|\,M=1)\Pi_i^T(D_i^T=1,\ M=1)+P(D_i^T=0\,|\,M=1)\Pi_i^T(D_i^T=0,\ M=1)\}\\
&\quad +P(M=0\,|\,\Phi_i(T-1))\{P(D_i^T=1\,|\,M=0)\Pi_i^T(D_i^T=1,M=0)+P(D_i^T=0\,|\,M=0)\Pi_i^T(D_i^T=0,\ M=0)\}-C_i
\end{aligned}
$$

On the other hand, the expected payoff of agent i for taking a strategic decision, S_i, is;

$$E(\Pi_i^T(S_i^T=k,\ M)\,|\,\Phi_i(T-1))$$
$$= P(M=1|\Phi_i(T-1))\Pi_i^T(S_i^T=k,\ M=1) + P(M=0|\Phi_i(T-1))\Pi_i^T(S_i^T=k,\ M=0)$$

Here I used the following formulas;

$$P(B\cap C|A)=P(B|A)P(C|A\cap B)$$
$$P(D_i^T=k|M=j,\ \Phi_i(T-1))=P(D_i^T=k|M=j)$$

Next, I define the difference of the expected payoff among I, S^1 and S^0 as;

$$\Delta_i(I,\ S^1)\equiv \{E(\Pi_i^T(D_i^T,\ M)|\Phi_i(T-1))-C_i\}-E(\Pi_i^T(S_i^T=1,\ M)|\Phi_i(T-1))$$
$$\Delta_i(I,\ S^0)\equiv \{E(\Pi_i^T(D_i^T,\ M)|\Phi_i(T-1))-C_i\}-E(\Pi_i^T(S_i^T=0,\ M)|\Phi_i(T-1))$$
$$\Delta_i(S^1,\ S^0)\equiv E(\Pi_i^T(S_i^T=1,\ M)|\Phi_i(T-1))-E(\Pi_i^T(S_i^T=0,\ M)|\Phi_i(T-1))$$

Here I put the following assumption.

[Assumption]
The payoff function of each agent does not explicitly include D_o^T, the consequence of a decision making as a total organization or a society. It also has a common form for all agents and all rounds, i.e., $\Pi_i^T(D_i^T=k,\ M=j)=\Pi_i^T(S_i^T=k,\ M=j)=\Pi_{kj}$ $(\forall k,\ j)$. The cost of an independent decision is zero, i.e., $C_i=0$.

Then assuming $A\equiv\Pi_{11}-\Pi_{01}>0$, $B\equiv\Pi_{00}-\Pi_{10}>0$, $\beta\equiv P(M=1|\Phi(T-1)) =P(M=1|\Phi_i(T-1))\equiv\beta_i$, we can rewrite them as following;

$$\Delta_i(I,\ S^1)= -\beta(1-P_1^i)A+(1-\beta)(1-P_0^i)B$$
$$\Delta_i(I,\ S^0)= \beta P_1^iA-(1-\beta)P_0^iB$$
$$\Delta_i(S^1,\ S^0)= \beta A-(1-\beta)B$$

Thus the condition that agent i chooses I (taking an independent decision, D_i^T) at round T is;

$$\Delta_i(I,\ S^1)= -\beta(1-P_1^i)A+(1-\beta)(1-P_0^i)B>0$$
$$\Delta_i(I,\ S^0)= \beta P_1^iA-(1-\beta)P_0^iB>0$$

Rewriting this, we get;

$$P_1^i-1>X(P_0^i-1)$$
$$P_1^i>XP_0^i$$
where $X\equiv\{(1-\beta)B\}/\{\beta A\}>0$ $(X_i\equiv\{(1-\beta_i)B\}/\{\beta_iA\})$

This condition is shown in Figure 2.

5. Examples of communication (Interaction)

Let us examine how each agent's judgmental process would be influenced by other agents' results within an homogeneous organization or a society. I assume by the term "homogeneous" that every agent shares the same payoff function and the same parameters for an independent decision. That is, $\Pi_i(D_i=k, M=j)=\Pi_i(S_i=k, M=j)=\Pi_{kj}$ ($\forall$i, k, j), $P_1^i=P_1$, $P_0^i=P_0$ ($\forall$i). Additionally assume that the cost for an independent decision is zero, i.e., $C_i=0$, so that the incentive for an independent decision (taking I) might be the biggest possible.

Let us also assume that an agent can observe another (just one) agent's independent decision output, D_{i-1}. Then the simple calculation gives area maps which tells the option to take among I, S^1 and S^0 respectively for D_{i-1}= 1, D_{i-1}=0 and α=1, α=0.5, α=0.

Under these conditions, a "chain reaction" can be observed within an organization. For example, assume α=0.5, P_1=0.8 and P_0=0.1. Also assume that an agent takes an independent decision and outputs D=1. Then all other agents who observe this judgment would subsequently take S^1 (taking a strategic decision, S=1), which gives them the best expected payoff. This means that all other agents abandon to produce their own independent decision making function, and results from these strategic decision making are worthless information even though they are right judgments. I call this phenomenon a positive chain reaction. For another example, assume α=0.5, P_1=0.9 and P_0=0.2. Also assume that agent 1 takes an independent decision and outputs D=0. Then all other agents who observe this judgment would subsequently take S^0 (taking a strategic decision, S=0). This means that all other agents abandon to produce their own independent decision making function, and results from these strategic decision making are worthless information even though they are right judgments. I call this phenomenon a negative chain reaction. A chain reaction defined here is considered to be sort of a herd behavior in decision making mechanism.

[Proposition]
Under the assumptions described above, there cannot exist any condition in which the strictly best choice for an agent is always to take I (taking an independent decision, D) regardless of the outputs of other agents' independent decisions.

(Proof)
The condition in which the strictly best choice for an agent is always to take I (taking an independent decision, D) is;

$$\Delta_i(I, S^1) = -\beta_i\ (1-P_1^i)A+(1-\beta_i)(1-P_0^i)B>0 \quad (1)$$

$$\Delta_i(I, S^0) = \beta_i\ P_1^i A-(1-\beta_i)P_0^i B>0 \quad (2)$$

where

$$\beta^1 \equiv P(M=1|D_{i-1}=1) = \frac{\alpha P_1}{\alpha P_1 + (1-\alpha)P_0}$$

$$\beta^0 \equiv P(M=1|D_{i-1}=0) = \frac{\alpha(1-P_1)}{\alpha(1-P_1) + (1-\alpha)(1-P_0)}$$

Since either of (1) and (2) must hold for both cases, $\beta_i=\beta^1$ and $\beta_i=\beta^0$,

$\beta_i=\beta^1$: (1) $\Leftrightarrow -\alpha P_1(1-P_1)A+(1-\alpha)\,P_0(1-P_0)B>0$ (a)

(2) $\Leftrightarrow \alpha P_1^2A-(1-\alpha)\,P_0^2B>0$ (b)

$\beta_i=\beta^0$: (1) $\Leftrightarrow -\alpha(1-P_1)^2A+(1-\alpha)\,(1-P_0)^2B>0$ (c)

(2) $\Leftrightarrow \alpha P_1(1-P_1)A-(1-\alpha)\,P_0(1-P_0)B>0$ (d)

Clearly (a) and (d) does not hold at the same time. Then proof is done.

This proposition holds also in the case that an agent can observe the outputs of more than one agents' independent decisions.

Conventional theories assume that each individual in an organization always makes its decisions in an independent manner. On this understanding, it is assumed that unless it is primarily concerned about its management costs, an organization can expect to reduce its probability of making erroneous decisions (usually expressed as type-1 or type-2 errors) and increase its expected profits the larger the number of individuals who belong to it. Conversely, the variation (dispersion) of profits will be reduced. In reality, though, the ideal size will be determined by the tradeoffs attributable to climbing management costs as the number of individuals in an organization increases.

However, as observed earlier, it is clear that each of the active agents within an organization will be inclined to readily relinquish their autonomy (independent judgment) and adopt strategy-based decisions if they communicate with their peers within the organization. Consequently, in this case, it may be concluded that in contrast to conventional theory, the probability of judgmental error will not diminish as the organization grows larger, nor will the organization experience an increase in expected profits or a decrease in profit variation.

Let us consider the situation in a committee that does not rely on communication, and whose final decisions are based on consensus by a majority of their individual members. If we let decision-making by the committee be D_T, and the payoff for the committee as a whole be defined as $\Pi_T(D_T=k,\ M=j)$, then D_T will be 1 when a majority of the committee's individual members are in consensus, and zero in all other cases. If interaction (communication) is not a matter for consideration, then, in keeping with the law of large numbers, the probability of making an erroneous decision will approach a limit of zero the larger the number of individuals, N, on the committee. Now, for comparison, consider a committee that does value interaction (communication) and whose members have opportunities to weigh the independent views or decisions of their peers. Also, let us assume that $\alpha=0.5$ and $P_1=1-P_0$ in the equation described above. For this scenario, it seems clear that the outcome of the first judgment independently made by an individual on the committee will be relayed in a chain reaction to all other committee members, and ultimately be adopted as a decision by the entire committee. In this case, the probability of making an erroneous decision will not be any smaller than that for a committee of one individual; consequently, there will be no increase in expected profits.

As the above example demonstrates, in an age of uncertainty where corporations and other organizations or communities are compelled to make decisions about the orientation of their future management strategy, decision-making mechanisms that place value on interaction (communication) can lose their effectiveness depending on the structure of the decision-making committee or hierarchy.

6. Model of Equilibrium Pricing Incorporating Autonomy in a Risk-neutral World

An economic agent with autonomy continuously chooses the best option in terms of an expected payoff among an independent decision and strategic decisions, while always observing prices and predictions regarding the state of the world. Therefore it is definitely necessary to construct models of equilibrium pricing under such courses.[1] [2]

Thus, in this chapter I construct the interactive pricing model at the risk neutral world on the basis of the interactive decision making process mainly explained in Chapter 4. The algorithm is shown in Appendix-C.

First of all, we assume that there exist the binary states of the world, a "good" state (M=1) and a "bad" state (M=0), and also assume that there exists a portfolio which would produce one unit of payoff at a "good" state and zero unit of payoff at a "bad" state. Since the real state of the world is not revealed for the moment, each agent just predicts the state of the world from the probabilistic point of view, based on its available information set.

Based on this model, I made Monte-Carlo simulation. Results are following.

[Properties]

7. The average of an equilibrium prediction is strongly and positively correlated with an equilibrium price, but is not necessarily the same.
8. The variance of an equilibrium price is larger than that of the average of an equilibrium prediction.
9. Figure 4 shows the relationship between the degree of interaction and the average increase in an individual "subjective" welfare at an homogeneous system. At a system whose individual judgmental capacity is relatively low (P_1=0.55) , the average increase in an individual "subjective" welfare is an increasing function with the degree of interaction. On the other hand, at a system whose individual judgmental capacity is relatively high (P_1=0.65), the average increase in an individual "subjective" welfare is a decreasing function with the degree of

[1] This pricing process can be regarded as a discontinuous, instantaneous and microscopic shift (jump) in a competitive (Walrasian) equilibrium on account of the production of information.

[2] The point we must be very careful of is that the price of a portfolio itself is described as the function of all agents' predictions regarding the state of the world. Therefore, an agent may get a positive bit of information regarding the state of the world through the price as well as through other agents' independent decisions. However, to make our arguments simpler, we set an assumption that in all the microscopic competitive (Walrasian) pricing processes used in this paper, the agent may get information only through other agents' independent decisions, if observable. This simplification, for sure, does not distort the essence of our arguments.

interaction. In either cases, the average increase in an individual "subjective" welfare decreases with the degree of interaction where the degree of interaction is very strong ($L \fallingdotseq N-1$).

Since the average increase in an individual objective welfare is always zero, as explained in 4., this increase is an irrational result from an incompleteness of individual's information, and it remains as an incorrect expected payoff until the state of the world is truly revealed.

10. When the prediction regarding the state of the world is equal to the current price, then this agent is the most likely to take I (an independent decision). Especially, assuming zero cost of communication, the agent always takes I.
11. The average of an individual "objective" welfare is always 0 at the equilibrium. (This is a natural result from the assumption of risk neutrality.)
12. The larger the degree of "interaction", the longer the time to an equilibrium becomes.
13. At an economic system which shares a complete information, there does not exist an equilibrium. That is, even though the prediction regarding the state of the world is almost sure ($\beta_i^T \equiv P(M=1|\Phi_i(T-1))=1$ or 0), the system can never arrive at an equilibrium and a pricing process fluctuates for ever. This is a totally different point from a risk averse world.

7. Model of Equilibrium Pricing Incorporating Autonomy in a risk-averse world

In Appendix B, in order to examine the effect on society of exercising autonomy (independent judgement) in a risk-averse world, we consider the case with 2 agents and 2 types (a type which always makes strategic judgements, and another type which always makes independent judgements). In actuality, an autonomous economic agent would be expected to make subjective forecasts regarding price and state, observe her own endowment, and then take the economic action of making an autonomous selection between two choices: where strategic judgement would result in higher expected utility, the agent would make a strategic judgement, while the agent would make an independent judgement in the event that this would result in higher expected utility. Therefore it is necessary to construct a price equilibrium model which incorporates a mechanism for reaching equilibrium given such a decision-making process. In other words, we require a price equilibrium model under uncertainty which incorporates autonomous judgement functions based on economic rationality.[3] [4]

Thus, in this chapter we construct a pricing theory model for a risk averse world. We define a binary variable to represent the state, with M=1 representing a 'good' state, and M=0 representing a 'bad' state. We also consider 2 portfolios. The first is a risk-free portfolio. The second is a risky portfolio where the pay-off is influenced by the prevailing state. All (N) agents are risk-averse and share common utility functions of the following forms.[6]

$u_i(x_1) \equiv \gamma_1 (x_1)^{1-a}$ (Portfolio 1)
$u_i^j(x_2) \equiv \gamma_2^j (x_2)^{1-a}$ (Portfolio 2, State M=j (j=0,1))
'a' represents the coefficient of relative risk aversion.

An algorithm for a price equilibrium model incorporating autonomy in a risk-averse world is presented in Appendix C.

Simulations were conducted based on the above model. The following is a summary of properties of this model.

Properties of Equilibrium Model in Risk-Averse World

1. In a homogeneous system where the ability to exercise independent judgement is low, independent judgement will not be exercised in the equilibrium outcome. This is the same as for traditional equilibrium models.
2. Once the ability to exercise independent judgement exceeds a certain threshold value, independent judgement will be exercised (see Figure 5).
3. The higher the coefficient of relative risk aversion, the higher the threshold

[3] Same as the footnote 2.
[4] Same as the footnote 3.

[6] We adopt a state dependent utility function.

value beyond which independent judgement will be exercised.
4. In a homogenous system, if we assume identical initial forecasts and identical initial endowments, objective welfare will decrease in equilibrium as a result of the exercising of independent judgement (see Figure 6).
In a homogenous system with identical initial forecasts and identical initial endowments, the initial state is Pareto optimal, and the system is initially at its competitive equilibrium. Therefore, the invocation of autonomy (independent judgment) will lead to variations in prediction (disruptive effect) and, regardless of the favorable or unfavorable nature of the situation, ultimately distort the portfolio balance under conditions of equilibrium, and reduce the amount of objective welfare. However, if the initial conditions are not identical and already marked by significant variation, the invocation of autonomy (independent judgment) will effectively consolidate the predictions of each individual in the system, ultimately eliminating disruptions in the portfolio balance, identifying the correct price range, and contributing to an increase in objective welfare. To rephrase, the invocation of autonomy has the effect of disrupting (reducing) objective welfare during the transition from a stage of predictive ambiguity to a stage in which actual conditions are clearly known. However, this process eventually leads to a quasi-steady state of competitive equilibrium that is not necessarily Pareto optimal nor even preferable.
5. In a homogenous system where the coefficient of relative risk aversion is low, a non-linear region exists where the ability to exercise independent judgement is low and where subjective welfare exceeds actual welfare. Such a region does not exist in homogenous systems where the coefficient of relative risk aversion is high (see Figure 6).
As can be seen in Figure 6, in the case where the true state is bad (M=0), in the region in which the ability to exercise independent judgement is low, that is, where P_1=0.65-0.85, despite the fact that objective welfare (R_o) is actually decreasing as a result of the exercising of autonomy (independent judgement), subjective welfare (R_S) is increasing significantly. This phenomenon is observed when the coefficient of relative risk aversion a is low (in Figure 5, a=0.1). This average increase in subjective welfare (R_S) arises from the fact that each agent has incomplete information, and is an irrational increase in expected utility which vanishes when the true state is revealed, but if the true state is not revealed after the equilibrium is reached, this expectation may remain for a considerable time in the mental image held by each agent.
6. In a homogenous system where agents share perfect information, an equilibrium exists, and both prices and subjective welfare reflect the true state.

In order to examine the effect on society of exercising this type of autonomy (independent judgement) in a risk-averse world, we consider the case with 2 agents. The two agents i (i=1,2) share common utility functions of the following forms.

$u_i(x_1) \equiv \gamma_1 (x_1)^{1-a}$ (Portfolio 1)
$u_i^j(x_2) \equiv \gamma_2^j (x_2)^{1-a}$ (Portfolio 2, State M=j (j=0,1))
'a' represents the coefficient of relative risk aversion.

There are two possible types for each agent:

Type 1: Agent has no autonomy and *always* makes a strategic judgement.
Type 2^*: Agent is autonomous, and under given prices, always compares the strategic judgement with the independent judgement, choosing the alternative which results in the highest expected utility.

For simplicity, assume γ_1=1.0, γ_2^1=1.5, γ_2^0=0.5, α=0.5.
Also, if initial endowments are set to $E_1^1=E_2^1=1.0+s$ and $E_1^2=E_2^2=1.0-s$ and the prices of each portfolio are set to $q_1=q_2=1.0$, then the initial endowments satisfy the Pareto optimality conditions irrespective of whether M=1 (good state) or M=0 (bad state), and the competitive equilibrium conditions are satisfied if an initial forecast of α is assumed (i.e. for case where autonomy is not exercised). (Only the price of the risky portfolio, q_2, does not reflect the true state (q_2=1.5 if M=1, q_2=0.5 if M=0).) This indicates the exercising of autonomy (independent judgement) by a Type 2^* individual acts to distort the optimal allocation of resources for society as a whole, and thus the exercising of autonomy (independent judgement) has negative economic value for society as a whole. While the case where the initial allocations are already Pareto optimal may be viewed as a somewhat special case, it is achieved as a competitive equilibrium in a situation where there is no imbalance in information (both agents use an initial forecast of α), and is chosen in order to focus on the disruptive effect of autonomy (independent judgement).

We now consider 2 cases.

Case A: where Individual 1 is of Type 1 and Individual 2 is of Type 2^* with independent judgement parameter P_1 (= $1-P_0$).

Assume initial endowments of $E_1^1=E_2^1=1.0+s$ and $E_1^2=E_2^2=1.0-s$.
Figure 7 shows the objective welfare ratio (R_o), as a function of P_1, for each individual and for society as a whole when s=0, that is, when Individuals 1 and 2 have identical initial endowments. As the initial state is already Pareto optimal, it can be seen that there exists a P_1 such that objective welfare decreases for both Individual 1 and Individual 2. Individual 1's objective welfare decreases as a result of the exercising of autonomy by Individual 2, and the objective welfare of society as a whole also decreases. This may be interpreted as Pareto inefficiency stemming from the asymmetry of information, but the problem is that in the region where P_1 is low, Individual 2's objective welfare also decreases. This region provides a dramatic illustration of disruptive effect on welfare of a mistaken independent judgement. As P_1 increases, Individual 2's objective welfare also increases, and in the region where P_1 exceeds a certain level, Individual 2's objective welfare is higher than in the initial state. Individual 1's objective welfare is a decreasing function of P_1. That is, Individual 2 is able to use the information gained from exercising her autonomy (independent judgement) to decrease Individual 1's welfare and capture it for herself. Other qualitative properties are similar to those of Case A in Appendix B.

Case B^*: where both Individual 1 and Individual 2 are of Type 2^* with identical independent judgement parameter P_1 (= $1-P_0$).

Assume initial endowments of $E_1^1=E_2^1=1.0+s$ and $E_1^2=E_2^2=1.0-s$. Qualitative properties are similar to those of Case B in Appendix B.

Why is it that the exercise of autonomy (powers of independent judgment) fails to achieve optimal benefits for the community (usually explained in terms of Pareto optimality), or at least lead to a preferable state of affairs? The reason is explained in simple terms below.

As shown by the model of independent judgment in Chapter 3, autonomy (independent judgmental power) can be statistically expected to function as a type of information mechanism that increases the volume of information. However, this mechanism does not contain any information indicative of how predictions of a given state may be altered in the future. That is, as an information mechanism, it contributes to discontinuous alterations in predictions of a given condition only if and when a decision is actually produced (thus generating information of either positive or negative value). However, until that decision is made, it will be completely impossible to make any prediction whatsoever regarding future changes in a prediction. The information mechanism itself may be described as a new, additional stochastic variable associated with predictions of a state. However, from the standpoint of maximizing the statistically expected utility (welfare) to a community, the (competitive) equilibrium prices derived from models of price equilibrium under conditions of uncertainty will already reflect the current prediction about the state in question. Although a Pareto optimum based on that prediction will be achieved, it will not contain any information about predictions after the information mechanism has been invoked. To put it another way, it cannot be marginalized until subsequent predictions are made. Accordingly, each new equilibrium price and allocation of inputs will be achieved on the basis of predictions about fresh states that derive from decisions made through the invocation of autonomy (independent judgmental power). Additionally, the utility (welfare) expected prior to and after invocation will be calculated as a linear combination (weighted by expression probabilities) of the actual utility (welfare) achieved in response to the decision outcomes capable of being generated. However, in view of the fact that only one Pareto optimum (under standard conditions) can be premised on a true state (based on complete information), it is impossible for all actual benefits (welfare) achieved in response to possible decision outcomes to be representative of a Pareto optimum. Likewise, it follows that the utility (welfare) expected after invocation and expressed as a linear relationship will not always result in a Pareto optimum, nor even in a preferable state of affairs.

8. Final Remarks

In this paper, the key concepts are "speculation" and "interaction." I sought to illustrate these concepts using the simplest of models. In particular, the models in Chapter 7 were essentially derived from the pricing mechanism in an interacting society or economy of two groups or two players. In its most basic form, this mechanism is known as the "Edgeworth box," and does not include any corporate activity. Additionally, I treated speculative behavior as the invocation of a type of independent judgmental power (information mechanism), and drew attention to its instantaneous and irreversible nature. I assumed that the other, more disruptive side of speculative behavior has already been adequately portrayed. Also, in view of the fact that efforts in fund management by hedge funds and other speculative organizations have now reached a scale too significant to be ignored by the real economy at large, I sought to identify the potential nonlinear "shocks" from a variety of risk-aversion coefficients affecting economic agents that do not possess independent judgmental power (information mechanisms).

It is assumed in this paper that speculation is an economic behavior that involves acting in accord with new predictions that are based on the invocation of autonomy (independent judgmental power) and discontinuous jumps in the prediction equation. Speculative behavior was once thought to be the province of certain special types of people: i.e., risk-loving individuals characterized by convex utility functions. However, we demonstrated in Chapters 6 and 7 that even people who adhere to a risk-neutral or risk-averse economic rationale are capable of exercising their autonomy and engaging in speculation. Accordingly, in this context it is also shown that autonomy (independent judgmental power) can have a disruptive (negative) impact on community welfare.

The models enlisted in Chapters 6 and 7 may be described as models of price equilibrium in an uncertain world that internalize a degree of "sensitivity" to information. The findings from these models demonstrate that (overall) objective welfare is not altered by the invocation of autonomy (independent judgment) in a risk-neutral world, and that homogeneous systems with complete information do not attain equilibrium. It was also shown that in a risk-averse world, (overall) objective welfare may actually be reduced by the invocation of autonomy (independent judgment) in certain cases. In other words, within imperfectly homogeneous systems of individuals (marked by identical initial predictions and input allocations), the invocation of autonomy (independent judgment) will lead to variations in prediction (disruptive effect) and, regardless of the favorable or unfavorable nature of the situation, ultimately distort the portfolio balance under conditions of equilibrium, and reduce the amount of objective welfare. However, if the initial conditions are not identical and already marked by significant variation, the invocation of autonomy (independent judgment) will effectively consolidate the predictions of each individual in the system, ultimately eliminating disruptions in the portfolio balance, identifying the correct price range, and contributing to an increase in objective welfare. To rephrase, the invocation of autonomy has the effect of disrupting (reducing) objective welfare during the transition from a stage of predictive ambiguity to a stage in which actual conditions are clearly known. However, this process eventually leads to a quasi-steady state of competitive equilibrium that is not necessarily Pareto optimal or even preferable. Furthermore, it should be noted that this process of equilibrium is instantaneous and microscopically irreversible in nature.

From these conclusions, the implication may appear to be that the invocation of autonomy (independent judgment) can have negative economic value in terms of community welfare. However, this can happen only when within an economic system the uncertainty, in which an economic agent is involved, also involves another economic agent. Furthermore, this is because no attention has been devoted to the positive or negative feedback to the individual (economic agent) deriving from the accumulation of information by the system (community) in the course of exercising its autonomy. For example, that feedback may include the creation of new industrial societies based on widespread public recognition and acceptance of innovations in technology; the formation of new social rules based on a majority consensus by individuals in the community; factors capable of increasing the benefits to the individual in an entirely new dimension; or the internalization of information as a marketable commodity with a tangible trading price. The point, in short, is that the surplus value (if any) created by the accumulation of information has not been allocated among agents (individuals) within the system. This is an issue that will conceivably deserve further study and debate in terms of the (positive or negative) externality of information. However, as it happens, the international financial market crisis of recent years has been marked by repeated cycles of speculative trading in areas rather far removed from trends in the actual economy. Such speculation has been fed by the increasing liberalization of capital flows, the utilization of breakthroughs in computer technology for financial trading purposes, and pure competition for control over information itself. In any case, this reality appears to be closely consistent with the model described above.

For this paper, I relied on two hypothetical scenarios for the decision-making mechanism as invoked by individuals: an "independent" scenario on the one hand, and a "strategy-based" scenario on the other. Further, every effort was made to devise pure models reflective of considerations for the (dynamic) activation and generation of information by the independent system, as well as the frozen (static) strategy-based system, under which no information is generated. Finally, I treated the "predictions" deriving from each of these state models as a conceptualized "magnetic field," and in my analysis, sought to identify the processes of decision-making and price equilibrium that these models would generate.

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Appendix A: The Economic Value of Independent Judgement (where pay-off function is exogenous)

We now calculate the economic value of Agent i's independent judgement capabilities for the case where the pay-off function is given exogenously.

Definition 1: Agent i's available pre-generated information set: $\Phi_i=\{D_1, D_2, \ldots, D_{i-1}\}$

Definition 2: Agent i's ungenerated result of independent judgement: D_i

Definition 3: Cost of Agent i's independent judgement: zero ($C_i=0$)

Definition 4: Pay-off function: $\Pi_i(D_i=k, M=j)=\Pi_i(S_i=k, M=j)=\Pi_{kj}$ ($\forall k, j$)

Also, assume $A\equiv\Pi_{11}-\Pi_{01}>0$ and $B\equiv\Pi_{00}-\Pi_{10}>0$.

Definition 5: Difference between expected pay-offs of independent judgement (D_i) and strategic judgement ($S_i=0,1$):

$$\Delta_i(I, S^1)\equiv E(\Pi_i(D_i, M)|\Phi_i)-E(\Pi_i(S_i=1, M)|\Phi_i)$$
$$\Delta_i(I, S^0)\equiv E(\Pi_i(D_i, M)|\Phi_i)-E(\Pi_i(S_i=0, M)|\Phi_i)$$
$$\Delta_i(S^1, S^0)\equiv E(\Pi_i(S_i=1, M)|\Phi_i)-E(\Pi_i(S_i=0, M)|\Phi_i)$$

Conditions: D_i and $D_{i'}$ ($i\neq i'$) are conditionally independent. That is,

$$P(D_i=k, D_{i'}=k'|M=j)=P(D_i=k|M=j)P(D_{i'}=k'|M=j) \quad (\forall k, k', j, i\neq i')$$

The following is clear from this condition:

$$\beta_i\equiv P(M=1|\Phi_i)= P(M=1| D_i, \Phi_i)$$

That is, the expected value of Agent i's forecast regarding the state, β_i, is unchanged by Agent i's exercising of independent judgement.

The economic value of Agent i's independent judgement capabilities, V_i, can be expressed by the following formula.

$$V_i=\min\{\max\{\Delta_i(I, S^1), 0\}, \max\{\Delta_i(I, S^0), 0\}\}$$

Figure A-1 shows the graph of V_i as a function of β_i.

As can be seen from the graph, independent judgement has positive economic value only when $\beta_i\equiv P(M=1|\Phi_i)$ lies in the interval

$$\left(\frac{P_0^i B}{P_1^i A+P_0^i B}, \frac{(1-P_0^i)B}{(1-P_1^i)A+(1-P_0^i)B}\right)$$

and is zero otherwise. The economic interpretation of this graph is as follows.

Φ_i and D_i are conditionally independent with respect to the value of M. That is, the values of Φ_i and D_i are independent and uncorrelated irrespective of whether M=1 (good state) or M=0 (bad state). (To express it another way, the result of Agent i's judgement is influenced by the state M, but is not influenced by her information set Φ_i.) Therefore, when the information set Φ_i is pre-generated and Agent i's independent judgement D_i is yet to be generated, generating D_i would have no effect on β_i, the forecast regarding the state M. That is, $\beta_i \equiv P(M=1|\Phi_i) = P(M=1|\Phi_i, D_i)$. (NOTE) Therefore, independent judgement only has positive economic value in the case where generating the independent judgement D_i would affect (influence) the actions taken by oneself, or a society or organization; in other cases, independent judgement would not be exercised, or even if it were, the expected payoff would not increase, and so the economic value of the independent judgement is zero.

For example, consider a committee comprised of 11 members with the same judgement capabilities, where majority rule applies. Suppose that each member of the committee makes a conditionally independent judgement regarding the state M.

Suppose that 10 out of the 11 members have already made their judgements, with all of them judging that a certain project should be approved ($D_i=1$, $i=1,2,\ldots,10$). In such a case, the conclusion of the committee, that the project should be approved, is unlikely to change irrespective of whether the 11^{th} member decides that the project should ($D_{11}=1$) or should not ($D_{11}=0$) be approved. Thus, the economic value of this member's independent judgement capability is zero.

Now suppose that of the first 10 members, 5 have judged that the project should be approved ($D_i=1$, $i=1,2,\ldots,5$), while the remaining five have concluded that it should not be approved ($D_i=0$, $i=6,2,\ldots,10$). In this case, if the final member ($i=11$) concludes that the project should be approved ($D_{11}=1$), then the committee as a whole will approve the project, while the committee will reject the project if the final member concludes that the project should be rejected ($D_{11}=0$). That is, the final member ($i=11$) holds the casting vote regarding the committee's conclusion, and only in such a case does the independent judgement capability have any economic value. The interval shown in Figure A-1,

$$\left(\frac{P_0^i B}{P_1^i A + P_0^i B}, \frac{(1-P_0^i)B}{(1-P_1^i)A + (1-P_0^i)B}\right)$$

can be viewed as the range of the forecast β_i for which the result of Agent i's independent judgement is the casting vote determining the committee's final decision. [A1]

[A1] It is necessary to consider a separate case where the independent judgement (D_i) has already been generated. For example, if $D_i=1$, β_i shifts towards 1, while if $D_i=0$, β_i shifts towards 0. Therefore, the independent judgement holds different economic value than in the case where it is yet to be generated, and a dilemma arises due to the irreversibility of information, in that the generated (announced) result of an independent judgement (D_i) cannot be 'unannounced' or 'ungenerated', and thus cannot be reflected in the price.

Appendix B: The Economic Value of Independent Judgement in Equilibrium Pricing Model

In Appendix A, we showed that independent judgement can have non-negative economic value if an agent is able to autonomously decide whether or not to exercise that judgement. To show this, we assumed that the pay-off function was exogenously given as a function of the state and the results of a (strategic or independent) judgement. Let us now consider what happens in a price equilibrium model with multiple economic agents.

In order to examine the effect on society of exercising autonomy (independent judgement) in a risk-averse world, we consider the case with 2 agents. We define a binary variable to represent the state, with M=1 representing a 'good' state, and M=0 representing a 'bad' state. We also consider 2 portfolios. The first is a risk-free portfolio. The second is a risky portfolio where the pay-off is influenced by the prevailing state.

The two agents i (i=1,2) share common utility functions of the following forms.

$u_i(x_1) \equiv \gamma_1 (x_1)^{1-a}$ (Portfolio 1)

$u_i^j(x_2) \equiv \gamma_2^j (x_2)^{1-a}$ (Portfolio 2, State M=j (j=0,1))

'a' represents the coefficient of relative risk aversion.

There are two possible types for each agent. This model differs from those of Chapters 6 & 7 in that one type always makes a strategic judgement, while the other type always makes an independent judgement.

Type 1: Agent has no autonomy and *always* makes a strategic judgement.
Type 2: Agent is autonomous and *always* makes an independent judgement.

Let Agent i's (i=1,2) initial endowment of portfolio k (k=1,2) and desired amount be denoted by E_k^i and x_k^i, respectively, and define q_k to be the price of portfolio k.

An individual of Type 1 will solve the following utility maximization problem.

$$\max_{x_1, x_2} \; u_i(x_1^i) + \{\alpha u_i^1(x_2^i) + (1-\alpha) u_i^0(x_2^i)\} \quad \text{s.t} \quad q_1 x_1^i + q_2 x_2^i \le q_1 E_1^i + q_2 E_2^i$$

An individual of Type 2 who has generated D_i=1 will solve the following utility maximization problem.

$$\max_{x_1, x_2} \; u_i(x_1^i) + u_i^1(x_2^i) \quad \text{s.t} \quad q_1 x_1^i + q_2 x_2^i \le q_1 E_1^i + q_2 E_2^i$$

An individual of Type 2 who has generated D_i=0 will solve the following utility maximization problem.

$$\max_{x_1, x_2} \; u_i(x_1^i) + u_i^0(x_2^i) \quad \text{s.t} \quad q_1 x_1^i + q_2 x_2^i \le q_1 E_1^i + q_2 E_2^i$$

The following equations must be satisfied in order to meet the condition of demand=supply.

$x_1^1 + x_1^2 = E_1^1 + E_1^2$
$x_2^1 + x_2^2 = E_2^1 + E_2^2$

Normalizing q_1 to 1 (that is, assuming that the price of the riskless portfolio is always 1), final portfolio holdings x_k^{ei} and the equilibrium price for portfolio k, q_k^e(i, k=1,2), can be determined by solving the above 4 equations.

Each agent's objective expected welfare at an equilibrium is calculated as;

$OWF_e \equiv u_i(x_1^i) + u_i^1(x_2^i)$ if M=1
$OWF_e \equiv u_i(x_1^i) + u_i^0(x_2^i)$ if M=0

Each agent's initial expected welfare at an equilibrium is calculated as;

$OWF_I \equiv u_i(E_1^i) + u_i^1(E_2^i)$ if M=1
$OWF_I \equiv u_i(E_1^i) + u_i^0(E_2^i)$ if M=0

We also degfine a next ratio;

Objective welfare ratio; $R_o \equiv OWF_e/OWF_I$

When M=1 (good state), an individual i (i=1,2) of Type 2 generates D_i=1 with probability P_1 and D_i=0 with probability (1-P_1). When M=0 (bad state), an individual i (i=0,1) of Type 2 generates D_i=1 with probability P_0 and D_i=0 with probability (1-P_0). For simplicity, assume γ_1=1.0, γ_2^1=1.5, γ_2^0=0.5, α=0.5.
Also, if initial endowments are set to $E_1^1=E_2^1$=1.0+s and $E_1^2=E_2^2$=1.0-s and the prices of each portfolio are set to $q_1=q_2$=1.0, then the initial endowments satisfy the Pareto optimality conditions irrespective of whether M=1 (good state) or M=0 (bad state), and the competitive equilibrium conditions are satisfied if an initial forecast of α is assumed (i.e. for case where autonomy is not exercised). (Only the price of the risky portfolio, q_2, does not reflect the true state (q_2=1.5 if M=1, q_2=0.5 if M=0).) This indicates the exercising of autonomy (independent judgement) by a Type 2 individual acts to distort the optimal allocation of resources for society as a whole, and thus the exercising of autonomy (independent judgement) has negative economic value for society as a whole. While the case where the initial allocations are already Pareto optimal may be viewed as a somewhat special case, it is achieved as a competitive equilibrium in a situation where there is no imbalance in information (both agents use an initial forecast of α), and is chosen in order to focus on the disruptive effect of autonomy (independent judgement). (This is the same case considered in Chapter 7.)

We now consider 2 cases.

Case A: where Individual 1 is of Type 1 and Individual 2 is of Type 2 with independent judgement parameter P_1 (= 1-P_0).

Assume initial endowments of $E_1^1=E_2^1$=1.0+s and $E_1^2=E_2^2$=1.0-s.
Figure B-1 shows the objective welfare ratio (R_o), as a function of P_1, for each individual and for society as a whole when s=0, that is, when Individuals 1 and 2 have identical initial endowments. As the initial state is already Pareto optimal, it can be seen that there exists a P_1 such that objective welfare decreases for both

Individual 1 and Individual 2.

Now assume that P_1 is given with P_1=1.0, that is, that Individual 2 has perfect independent judgement capabilities. Figure B-2 shows the objective welfare ratio (R_o), as a function of s, for each individual and for society as a whole (for the bad state, where M=0).

Property 1: The objective welfare ratio (R_o) is a monotone increasing function of s for Individual 2 (Type 2). That is, the higher Individual 2's initial endowment, the lower her objective welfare ratio (R_o).
Irrespective of whether the state is good or bad, in the region where the coefficient of relative risk aversion (a) is large, the objective welfare ratio (R_o) is a monotone increasing function of s for Individual 1 (Type 1). That is, the higher Individual 1's initial endowment, the higher her objective welfare ratio (R_o). In the region where the coefficient of relative risk aversion (a) is small, a minimum value, s exists for Individual 1's (Type 1) objective welfare ratio (R_o).
Individual 2's (Type 2) objective welfare ratio (R_o) is larger than 1, while that of Individual 1 (Type 1) is less than 1.

Case B*: where both Individual 1 and Individual 2 are of Type 2 with identical independent judgement parameter P_1 (= 1-P_0).

Assume initial endowments of $E_1^1=E_2^1=1.0+s$ and $E_1^2=E_2^2=1.0-s$.
Figure B-3 shows the objective welfare ratio (R_o), as a function of s, for each individual and for society as a whole, assuming that P_1 is given with P_1=0.8 and that the good state applies (M=1). The following is a summary of the properties.

Property 1: Irrespective of whether the state is good or bad, and also irrespective of the value of the coefficient of relative risk aversion, the objective welfare ratio (R_o) for the individual with the lower initial endowment is higher than that for the individual with the higher initial endowment. (In other words, in a society consisting of 2 people with the same judgement capabilities, the poorer individual is 'better off' than the richer individual.)

Property 2: Irrespective of whether the state is good or bad, and also irrespective of the value of the coefficient of relative risk aversion, the objective welfare ratio (R_o) for the individual with the lower initial endowment ($E_1^i=E_2^i<1$) is a monotone decreasing function of the initial endowment. For the individual with the higher initial endowment ($E_1^i=E_2^i>1$), a minimum value, s exists for the objective welfare ratio (R_o).

In Case B, the two individuals possess the same independent judgement parameter, and in terms of information theory, produce the same amount of information. The world is symmetric. However, it is clear that the exercising of autonomy (independent judgement) may have a disruptive effect on utility (welfare).

In these cases, it is not a simple matter to define the economic value of the information resulting from autonomy (independent judgement). It could be defined as the increase in objective welfare for the individual who created the information, or as the increase in objective welfare for society as a whole, or as the difference between the increase in objective welfare for the individual who created the information and the increase in objective welfare for the other individual. In any of these cases, it is clear that the economic value of the information may be negative.

Appendix C: Algorithm for Equilibrium Pricing Model Incorporating Autonomy

[Model of the interactive pricing process at the risk neutral world]

(Definition)

1. Binary modes of the world; M=j (j=0,1)
2. N agents; i=1,2, ... , N
3. Discrete rounds of the pricing process; t=1,2,3, ...
4. There exist a portfolio which would produce one unit of payoff at a "good" state (M=1), and zero unit of payoff at a "bad" state (M=0).
5. Price of the portfolio at t=T; q^T
6. Each agent has a right to declare in public to sell or buy one unit of the portfolio at each round, T.
7. Binary independent decision making of agent i at round t; $D_i^t=k$ (k=0,1), where $D_i^t=1$ is the decision to sell one unit of portfolio and $D_i^t=0$ is the decision to buy one unit of portfolio.
8. Entry cost of independent decision making of agent i; C_i for one independent decision making
9. Binary strategic decision making of agent i at round t; $S_i^t=k$ (k=0,1), where $S_i^t=1$ is the decision to sell one unit of portfolio and $S_i^t=0$ is the decision to buy one unit of portfolio.
10. Past independent decision history (information set), which is available for agent i at round T; $\Phi_i(T-1)\in\Phi(T-1)\equiv\{D_i^t\}_{t=1,\ldots,T-1,\ \forall i}$, where if agent i' takes a strategic decision $S_{i'}^{t'}=k'$ at round t', then $D_{i'}^{t'}=k'$ cannot be defined nor $S_{i'}^{t'}$ cannot be included in $\Phi(T-1)$.
11. Expected payoff of agent i for taking an independent decision, D_i^T, conditional on $\Phi_i(T-1)$ is;
$$E(\Pi_i^T(D_i^T,\ M)\,|\,\Phi_i(T-1))\equiv E\Pi_i^T(I)$$
where Π_i^T is the payoff of agent i at round T as a function of D_i^T and M.
12. Expected payoff of agent i for taking a strategic decision, $S_i^T=k$, conditional on $\Phi_i(T-1)$ is;
$$E(\Pi_i^T(S_i^T=k,\ M)\,|\,\Phi_i(T-1))\equiv E\Pi_i^T(S^k)$$
where Π_i^T is the payoff of agent i at round T as a function of $S_i^T=k$ and M.

(Pricing process)

1. At round 1, the pricing process begins.
2. There are three choices for agent i at round T;
   - I: to take an independent decision, D_i^T.
   - S^1: to take a strategic decision, $S_i^T=1$.
   - S^0: to take a strategic decision, $S_i^T=0$.

Assume that at round T-1 agent i does not take I. Agent i chooses the one choice among $E\Pi_i^T(I)-C_i$, and $E\Pi_i^T(S^k)$ (k=0,1), and declares (outputs) his or her corresponding decision, $D_i^T=k$ or $S_i^T=k$ (k=0,1), at round, T.

Assume that at round T-1 agent i does take I. Among $E\Pi_i^T(I)$ and $E\Pi_i^T(S^k)$ (k=0,1), if $E\Pi_i^T(I)$ is the highest payoff, then agent i takes I, keeps and declares the same decision as as t=T-1, i.e., $D_i^T=D_i^{T-1}=k$ at round T. If $E\Pi_i^T(S^k)$ is the highest payoff, then agent i takes S^k, and declares S^k at round T.

3. At each round, T, if there is an excess demand for the portfolio, i.e., the number of agents who would decide to buy the portfolio ($D_i^T=1$ or $S_i^T=1$) exceeds the number of agents who would decide to sell the portfolio ($D_i^T=0$ or $S_i^T=0$), then the price goes up at round T+1, i.e., $q^{T+1}=q^T+\varepsilon$. If there is an excess supply for the portfolio, i.e., the number of agents who would decide to sell the portfolio ($D_i^T=0$ or $S_i^T=0$) exceeds the number of agents who would decide to buy the portfolio ($D_i^T=1$ or $S_i^T=1$), then the price goes down at round T+1, i.e., $q^{T+1}=q^T-\varepsilon$.

4. At round T=Te, if there are the equal supply and demand, and any agents would not change his or her decision making status any more, then the pricing process ends at round Te, and q_e is the equilibrium price, i.e., $q_e\equiv q^{Te}$.

Just the calculation derives;

$$\Pi_i^T(D_i^T=k,\ M=j)=\Pi_i^T(S_i^T=k,\ M=j)\equiv\Pi_{kj}^{\ T}(\forall k,\ j,\ T),$$

where $\Pi_{11}^T=1-q^T$, $\Pi_{10}^T=-q^T$, $\Pi_{01}^T=-(1-q^T)$, $\Pi_{00}^T=q^T$

Furthermore, for simplicity, we make the following definition;

$$A^T\equiv\Pi_{11}^T-\Pi_{01}^T>0,\ B^T\equiv\Pi_{00}^T-\Pi_{10}^T>0,$$

$$\beta^T\equiv P(M=1|\Phi(T-1)),\ \beta_i^T\equiv P(M=1|\Phi_i(T-1)),$$

$$X^T\equiv\{(1-\beta^T)B^T\}/\{\beta^TA^T\}),\ X_i^T\equiv\{(1-\beta_i^T)B^T\}/\{\beta_i^TA^T\})$$

Just the calculation derives;

$$E(\Pi_i^T(D_i^T,\ M)|\Phi_i(T-1))\equiv E\Pi_i^T(I)$$

$$=\beta_i^T\{P_1^i(1-q^T)-(1-P_1^i)(1-q^T)\}+(1-\beta_i^T)\{P_0^i(-q^T)+(1-P_0^i)q^T\}$$

$$=\beta_i^T(1-q^T)(2P_1^i-1)+(1-\beta_i^T)q^T(1-2P_0^i)$$

$$E(\Pi_i^T(S_i^T=1,\ M)|\Phi_i(T-1))\equiv E\Pi_i^T(S^1)$$

$$=\beta_i^T(1-q^T)+(1-\beta_i^T)(-q^T)$$

$$E(\Pi_i^T(S_i^T=0,\ M)|\Phi_i(T-1))\equiv E\Pi_i^T(S^0)$$

$$=-\beta_i^T(1-q^T)+(1-\beta_i^T)q^T=-E\Pi_i^T(S^1)$$

we define the difference of the expected payoff among I, S^1 and S^0 as;

$$\Delta_i^T(I,\ S^1)\equiv\{E(\Pi_i^T(D_i^T,\ M)|\Phi_i(T-1))-C_i\}-E(\Pi_i^T(S_i^T=1,\ M)|\Phi_i(T-1))$$

$$\Delta_i^T(I,\ S^0)\equiv\{E(\Pi_i^T(D_i^T,\ M)|\Phi_i(T-1))-C_i\}-E(\Pi_i^T(S_i^T=0,\ M)|\Phi_i(T-1))$$

$$\Delta_i^T(S^1,\ S^0)\equiv E(\Pi_i^T(S_i^T=1,\ M)|\Phi_i(T-1))-E(\Pi_i^T(S_i^T=0,\ M)|\Phi_i(T-1))$$

Assuming $C_i=0$, the same argument as in Chapter 4 derives the following conditions that each agent should take I or S^k (k=0,1).

The condition to take I: $P_1^i-1>X_i^T(P_0^i-1)$ and $P_1^i>X_i^TP_0^i$

The condition to take S^1: $P_1^i-1\le X_i^T(P_0^i-1)$ and $1\ge X_i^T$

The condition to take S^0: $P_1^i\le X_i^TP_0^i$ and $1<X_i^T$

We make the following definitions.

[Definition 1]

The society with a complete information basis is such that:

$$\Phi_i(T-1)=\Phi(T-1)\equiv\{D_i^t\}_{t=1,\ ..,T-1,\ \forall i},\ (\forall i,\ \forall T\ge 2)$$

The society with an incomplete information basis is such that:

$$\Phi_i(T-1) \in \Phi(T-1) \equiv \{D_i^{\,t}\}_{t=1,\ ..,T-1,\ \forall i},\ \ (\forall i,\ \forall T \geq 2)$$

[Definition 2]
In the society with N agents, when each agent can observe all of the past independent decisions of L adjacent other agents plus his or herself (L+1 agents in total), then we define L as the degree of interaction in the society.

[Definition 3]
Homogeneous society is such that all agents are identical, i.e., $P_1^{\,i}=P_1$, $P_0^{\,i}=P_0$, $C_i=C$ ($\forall$i).
Heterogeneous society is such that the independent decision making parameters of all agents are different, i.e., $P_1^{\,i} \neq P_1^{\,j}$, $P_0^{\,i} \neq P_0^{\,j}$ ($\forall i \neq j$).

Each agent's subjective expected welfare at round T is calculated as;

$$SWF(t=T) \equiv \max\ \{E\Pi_i^{\,T}(I),\ \{E\Pi_i^{\,T}(S^k)\}_{k=0,1}\}$$

[Model of the interactive pricing process at the risk averse exchange economy]

(Definition)
1. N agents; i=1,2, ... , N
2. There exist two portfolios, k=1 and 2. The one is riskless, and the other is risky and is subject to the binary states of the world. (M=1 is a good state and M=0 is a bad state.) Agent i has separable utilities for these two portfolios in the following form:

$u_i(x_1) \equiv \gamma_1 (x_1)^{1-a}$ for portfolio 1

$u_i^{\,j}(x_2) \equiv \gamma_2^{\,j} (x_2)^{1-a}$ for portfolio 2, state M=j (j=0,1)

3. Initial endowment of agent i for portfolio k; $E_k^{\,i}$
4. Discrete rounds of the pricing process; t=1,2,3, ...
5. Price of portfolio k at t=T; $q_k^{\,T}$, where the price of portfolio 1 is always normalized at 1 at each round T. ($q_1^{\,T}=1$ ($\forall$T))
6. Each agent has to declare in public the amounts of two portfolios which he or she would like to hold at hand under his or her budget constraint, given the current prices of portfolios, $q_k^{\,T}$ (k=1,2), at each round T, and given his initial endowmwnts for portfolio k, $E_k^{\,i}$ (k=1,2).
7. Binary independent decision making status of agent i for portfolio k=2 at round T; $D_i^{\,T}=h$ (h=0,1), where $D_i^{\,T}=1$ is his or her conviction that portfolio 2 is to be in a good state (M=1) and $D_i^{\,T}=0$ is his or her conviction that portfolio 2 is to be in a bad state (M=0). At $D_i^{\,T}=h$, agent i declares the holdings of these two portfolios so that they maximize his or her utility under his or her budget constraint, given the current prices of portfolios, $q_k^{\,T}$ (k=1,2), and given his initial endowmwnts for portfolio k, $E_k^{\,i}$ (k=1,2), assuming the state of the world is h (M=h).
8. Entry cost of independent decision making of agent i; C_i for one independent decision making.
9. Past independent decision history (information set), which is available for agent

i at round T; $\Phi_i(T-1)\in\Phi(T-1)\equiv\{D_i^{\,t}\}_{t=1,\ldots,T-1,\ \forall i}$, where if agent i' takes a strategic decision $S_{i'}^{t'}$ at round t', then $D_{i'}^{t'}$=h' cannot be defined nor $S_{i'}^{t'}$ cannot be included in $\Phi(T-1)$.

10. Strategic decision making status of agent i at round t; S_i^T, where S_i^T is the status that he or she declares the holdings of these two portfolios so that they maximize his or her expected utility conditionally on $\Phi_i(T-1)$ under his or her budget constraint, given the current prices of portfolios, q_k^T (k=1,2), and given his initial endowmwnts for portfolio k, E_k^i (k=1,2).

11. Expected utility of agent i for taking an independent decision, D_i^T, conditional on $\Phi_i(T-1)$ is;

$$E(U_i^T(D_i^T,\ M)\,|\,\Phi_i(T-1))\equiv EU_i^T(I)$$

where U_i^T is the utility of agent i at round T as a function of D_i^T and M.

12. Expected utility of agent i for taking a strategic decision, S_i^T, conditional on $\Phi_i(T-1)$ is;

$$E(U_i^T(S_i^T,\ M)\,|\,\Phi_i(T-1))\equiv EU_i^T(S)$$

where U_i^T is the payoff of agent i at round T as a function of S_i^T and M.

(Pricing process)

1. At round 1, the pricing process begins.
2. There are two choices for agent i at round T;
   I: to take an independent decision, D_i^T.
   S: to take a strategic decision, S_i^T.

Assume that at round T-1 agent i does not take I. Agent i chooses the one choice among $EU_i^T(I)-C_i$, and $EU_i^T(S)$, and declares (outputs) his or her corresponding decision, D_i^T=h or S_i^T, and his or her corresponding preferable holdings, $x_k^{i,T+1}$ at each round T.

Assume that at round T-1 agent i does take I. Among $EU_i^T(I)$ and $EU_i^T(S)$, if $EU_i^T(I)$ is the highest utility, then agent i takes I, keeps and declares the same independent decision as t=T-1, i.e., $D_i^T=D_i^{T-1}$=h, and his or her corresponding holdings, $x_k^{i,T+1}$ at round T. If $EU_i^T(S)$ is the highest utility, then agent i declares S and his or her corresponding preferable holdings at round T.

3. At each round T, if there is an excess demand for the portfolio, i.e., the sum of portofolio 2 which each agent would like to hold exceeds the sum of each agent's initial endowment, then the price of portofolio 2 goes up at round T+1, i.e., $q_2^{T+1}=q_2^T+\varepsilon$, where ε is a very small value. If there is an excess supply for portfolio 2, i.e., the sum of portofolio 2 which each agent would like to hold is smaller than the sum of each agent's initial endowment, then the price of portofolio 2 goes down at round T+1, i.e., $q_2^{T+1}=q_2^T-\varepsilon$.
4. At round T=Te, if there are the equal supply and demand, and any agents would not change his or her decision making status nor the corresponding declared holdings any more, then the pricing process ends at round Te, and the deal comes into existence where q_e is the equilibrium price, i.e., $q_e\equiv q_2^{Te}$.

In case that agent i takes S at round T, he or she solves the following maximization problem;

$$\max_{x_1,x_2} u_i(x_1)+\{\beta_i^T u_i^1(x_2)+(1-\beta_i^T)u_i^0(x_2)\} \quad s.t \quad q_1^T x_1+q_2^T x_2 \le q_1^T E_1^i+q_2^T E_2^i$$

Using $u_i(x_1)\equiv\gamma_1(x_1)^{1-a}$, $u_i^j(x_2)\equiv\gamma_2^j(x_2)^{1-a}$, $\beta_i^T\equiv P(M=1|\Phi_i(T-1))$, and assuming $\gamma_1>0$ and $\gamma_2^j>0$ (j=1,2), the declared holdings of agent i at the next round T+1 in case that he or she takes S are;

$$x_1^{i,T+1}(S)\equiv(q_1^T+q_2^T K_S^{i,T})^{-1}(q_1^T E_1^i+q_2^T E_2^i)$$
$$x_2^{i,T+1}(S)\equiv K_S^{i,T}(q_1^T+q_2^T K_S^{i,T})^{-1}(q_1^T E_1^i+q_2^T E_2^i)$$
$$\text{where } K_S^{i,T}\equiv((\gamma_1/(\gamma_2^1\beta_i^T+\gamma_2^0(1-\beta_i^T)))(q_2^T/q_1^T))^{-1/a}$$

In case that agent i takes I and outputs $D_i^T=1$ at round T, he or she solves the following maximization problem;

$$\max_{x_1,x_2} u_i(x_1)+u_i^1(x_2) \quad s.t \quad q_1^T x_1+q_2^T x_2 \le q_1^T E_1^i+q_2^T E_2^i$$

In this case, the declared holdings of agent i at the next round T+1 are;

$$x_1^{i,T+1}(I^1)\equiv(q_1^T+q_2^T K_1^{i,T})^{-1}(q_1^T E_1^i+q_2^T E_2^i)$$
$$x_2^{i,T+1}(I^1)\equiv K_1^{i,T}(q_1^T+q_2^T K_1^{i,T})^{-1}(q_1^T E_1^i+q_2^T E_2^i)$$
$$\text{where } K_1^{i,T}\equiv((\gamma_1/\gamma_2^1)(q_2^T/q_1^T))^{-1/a}$$

In case that agent i takes I and outputs $D_i^T=0$ at round T, he or she solves the following maximization problem;

$$\max_{x_1,x_2} u_i(x_1)+u_i^0(x_2) \quad s.t \quad q_1^T x_1+q_2^T x_2 \le q_1^T E_1^i+q_2^T E_2^i$$

In this case, the declared holdings of agent i at the next round T+1 are;

$$x_1^{i,T+1}(I^0)\equiv(q_1^T+q_2^T K_1^{i,T})^{-1}(q_1^T E_1^i+q_2^T E_2^i)$$
$$x_2^{i,T+1}(I^0)\equiv K_0^{i,T}(q_1^T+q_2^T K_0^{i,T})^{-1}(q_1^T E_1^i+q_2^T E_2^i)$$
$$\text{where } K_0^{i,T}\equiv((\gamma_1/\gamma_2^0)(q_2^T/q_1^T))^{-1/a}$$

Therefore, the expected utility from taking S is;

$$EU_i^T(S)=\gamma_1(x_1^{i,T+1}(S))^{1-a}+(\gamma_2^1\beta_i^T+\gamma_2^0(1-\beta_i^T))(x_2^{i,T+1}(S))^{1-a}$$

The expected utility from taking I is;

$$EU_i^T(I)=\gamma_1(\beta_i^T P_1^i+(1-\beta_i^T)P_0^i)(x_1^{i,T+1}(I^1))^{1-a}+\gamma_1(\beta_i^T(1-P_1^i)+(1-\beta_i^T)(1-P_0^i))(x_1^{i,T+1}(I^0))^{1-a}+\gamma_2^1\beta_i^T P_1^i(x_2^{i,T+1}(I^1))^{1-a}+\gamma_2^1\beta_i^T(1-P_1^i)(x_2^{i,T+1}(I^0))^{1-a}+\gamma_2^0(1-\beta_i^T)P_0^i(x_2^{i,T+1}(I^1))^{1-a}+\gamma_2^0(1-\beta_i^T)(1-P_0^i)(x_2^{i,T+1}(I^0))^{1-a}$$

Each agent's subjective expected welfare at round T is calculated as;

$$SWF(t=T)\equiv\max\{EU_i^T(I),\ EU_i^T(S)\}$$

Each agent's objective expected welfare at round T is calculated as;

$$OWF(t=T)\equiv u_i(x_1^{i,T+1})+u_i^1(x_2^{i,T+1}) \quad \text{if } M=1$$
$$OWF(t=T)\equiv u_i(x_1^{i,T+1})+u_i^0(x_2^{i,T+1}) \quad \text{if } M=0$$

Each agent's subjective initial welfare is calculated as;

$$SWF(t=0)\equiv u_i(E_1^i)+\beta_i^1 u_i^1(E_2^i)+(1-\beta_i^1)u_i^0(E_2^i)$$
$$=\gamma_1(E_1^i)^{1-a}+(\gamma_2^1\beta_i^1+\gamma_2^0(1-\beta_i^1))(E_2^i)^{1-a}$$

Each agent's objective initial welfare at round T is calculated as;

$OWF(t=0) \equiv u_i(E_1^i)+u_i^1(E_2^i)$ if $M=1$

$OWF(t=0) \equiv u_i(E_1^i)+u_i^0(E_2^i)$ if $M=0$

We define following ratios.

Subjective welfare ratio; $R_s \equiv SWF(t=Te)/ SWF(t=0)$

Objective welfare ratio; $R_o \equiv OWF(t=Te)/ OWF(t=0)$

Appendix D: Amount of Information from Independent Judgement

We calculate the information produced from the independence. Assume the binary communication channel as shown in Figure 1. Also assume $P_1^i = P_1$, $P_0^i = P_0$ $(\forall i)$.

Then the enthropy of each variable is;

$$H(M) = -\{\alpha \log_2 \alpha + (1-\alpha)\log_2(1-\alpha)\} \equiv \eta(\alpha)$$

$$H(D_i) = -\{(\alpha P_1 + (1-\alpha)P_0)\log_2(\alpha P_1 + (1-\alpha)P_0) + (\alpha(1-P_1) + (1-\alpha)(1-P_0))\log_2(\alpha(1-P_1) + (1-\alpha)(1-P_0))\}$$

$$= \eta(\alpha P_1 + (1-\alpha)P_0)$$

The equivocation of each variable is;

$$H(D_i|M) = P(M=1)H(D_i|M=1) + P(M=0)H(D_i|M=0)$$

$$= \alpha\eta(P_1) + (1-\alpha)\eta(P_0)$$

$$H(M|D_i) = P(D_i=1)H(M|D_i=1) + P(D_i=0)H(M|D_i=0)$$

$$= (\alpha P_1 + (1-\alpha)P_0)\eta\left(\frac{\alpha P_1}{\alpha P_1 + (1-\alpha)P_0}\right)$$

$$+ (\alpha(1-P_1) + (1-\alpha)(1-P_0))\eta\left(\frac{\alpha(1-P_1)}{\alpha(1-P_1) + (1-\alpha)(1-P_0)}\right)$$

The amount of information produced by one independent decision making is represented as the mutual information, $I(M; D_i)$.

$$I(M; D_i) = H(D_i) - H(D_i|M) = \eta(\alpha P_1 + (1-\alpha)P_0) - \{\alpha\eta(P_1) + (1-\alpha)\eta(P_0)\}$$

Consider that there exist N identical agents. N agents are independent conditionally on M in the sense that;

$$P(D_i=k, D_{i'}=k' | M=j) = P(D_i=k|M=j)P(D_{i'}=k'|M=j) \quad (\forall k, k', j, i \neq i')$$

Then, defining $D \equiv \{D_1, D_2, \ldots, D_N\}$, we get;

$$H(D) = -\sum_{k=0}^{N}\{\alpha B(N,k,P_1) + (1-\alpha)B(N,k,P_0)\}\log_2\{\alpha P_1^k(1-P_1)^{N-k} + (1-\alpha)P_0^k(1-P_0)^{N-k}\}$$

where $B(N,k,P) \equiv {}_N C_k P^k (1-P)^{N-k}$

$$H(D|M) = NH(D_i|M) = N\{\alpha\eta(P_1) + (1-\alpha)\eta(P_0)\}$$

Now we can define the channel capacity as;

$$C \equiv \max_{\alpha}\{H(D) - H(D|M)\}$$

For simplicity, assume the binary symmetric channel, $P_1 = 1 - P_0$.

Then

$$H(D|M) = N\eta(P_0) = \text{const.}, \quad C \equiv \max_{\alpha}\{H(D)\} - N\eta(P_0)$$

Just the calculation shows that H(D) is maximazed at $\alpha = 0.5$.

Then at $\alpha = 0.5$;

$$C = -\sum_{k=0}^{N} {}_N C_k \frac{\{P_1^k(1-P)^{N-k} + P_0^k(1-P_0)^{N-k}\}}{2}\log_2\left\{\frac{P_1^k(1-P_1)^{N-k} + P_0^k(1-P_0)^{N-k}}{2}\right\}$$

$$-N\eta(P_0)$$

$$\equiv C_N$$

Using $P_1=1-P_0$, we get for N=1 and 2;

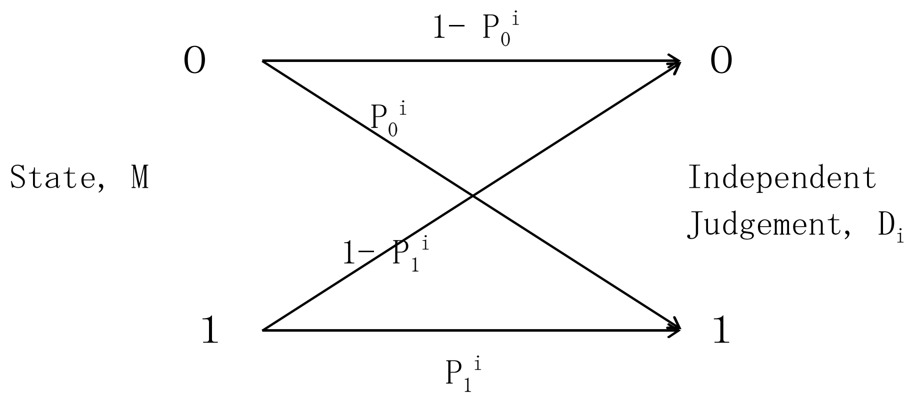
$$C_1=1-\eta(P_0)$$

$$C_2=1+\eta(\{P_0\}^2+\{P_1\}^2)-2\eta(P_0)$$

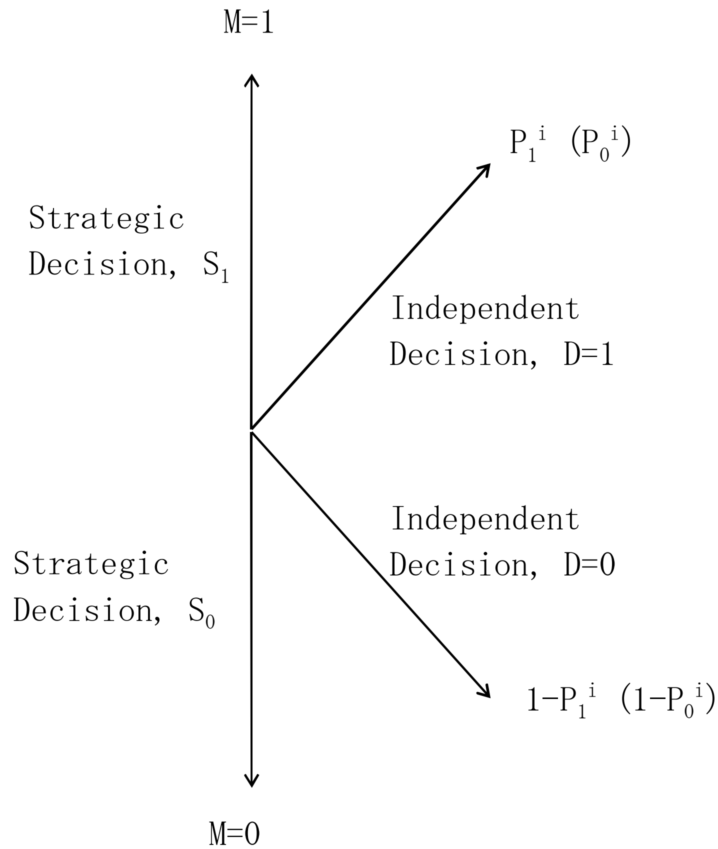
Shannon's channel coding theorem says that if $H(M)=\eta(\alpha)<C_N$, then there exists a coding system such that in the infinitely sequential input of the variable, M, M can be transmitted over the channel with an arbitrarily small frequency of errors.
For N=1 and 2, this condition is equivalent to;

$$\eta(\alpha)<1-\eta(P_0) \quad (N=1)$$

$$\eta(\alpha)<1+\eta(\{P_0\}^2+\{P_1\}^2)-2\eta(P_0) \quad (N=2)$$



Binary Channel for Independent Judgement



Strategic Decision and Independent Decision

Figure 1

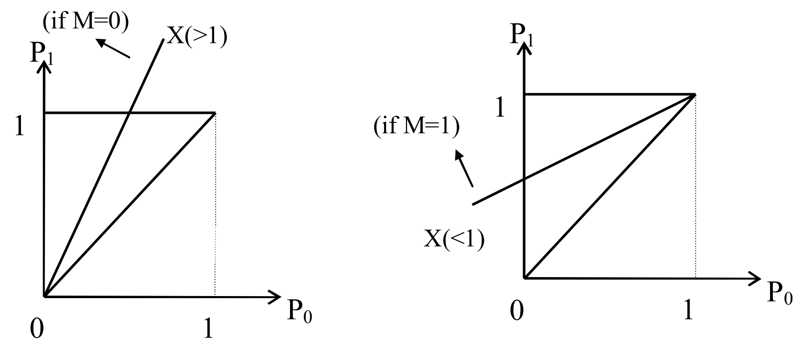


Figure 2

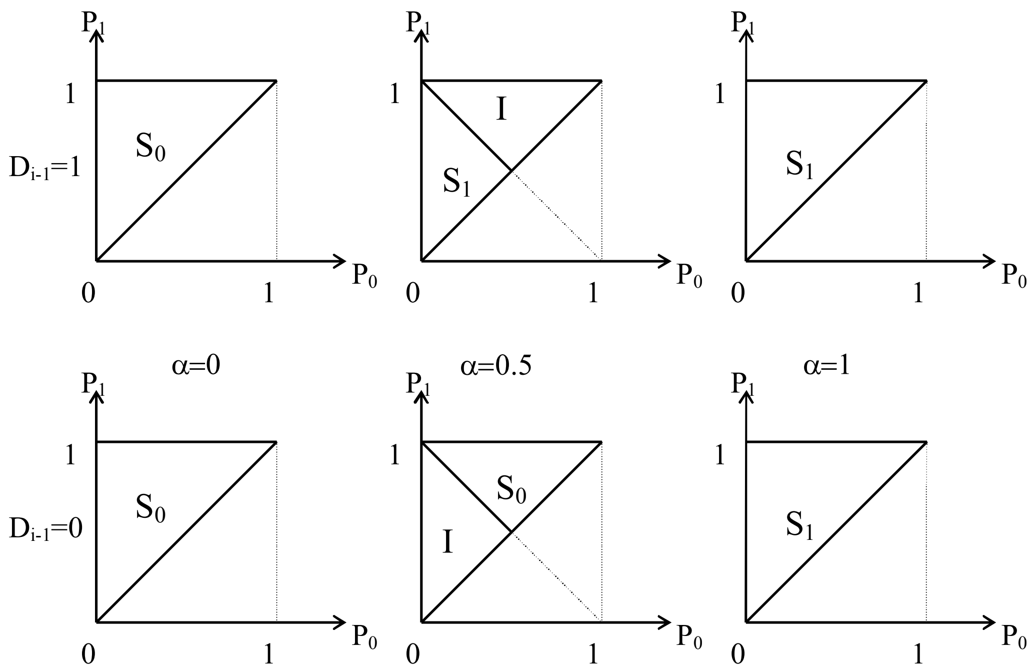
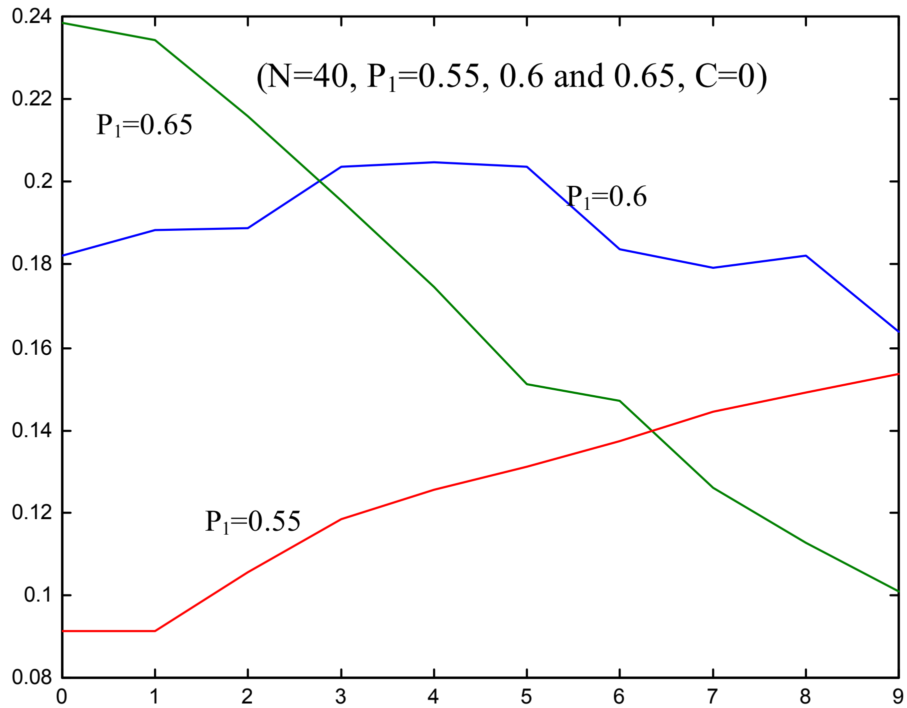

P1
1
S0
Di-1=1
0
1
P0
P1
1
I
S1
0
1
P0
P1
1
S1
0
1
P0
α=0
α=0.5
α=1
P1
1
S0
Di-1=0
0
1
P0
P1
1
S0
I
0
1
P0
P1
1
S1
0
1
P0

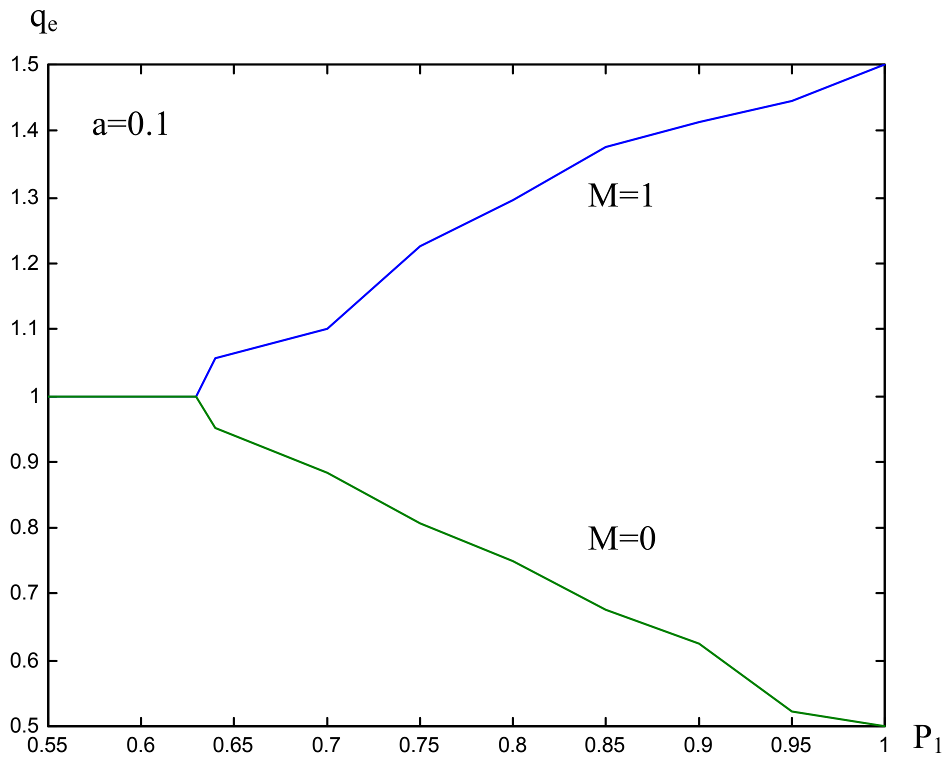
Figure 3



L (Degree of interaction)

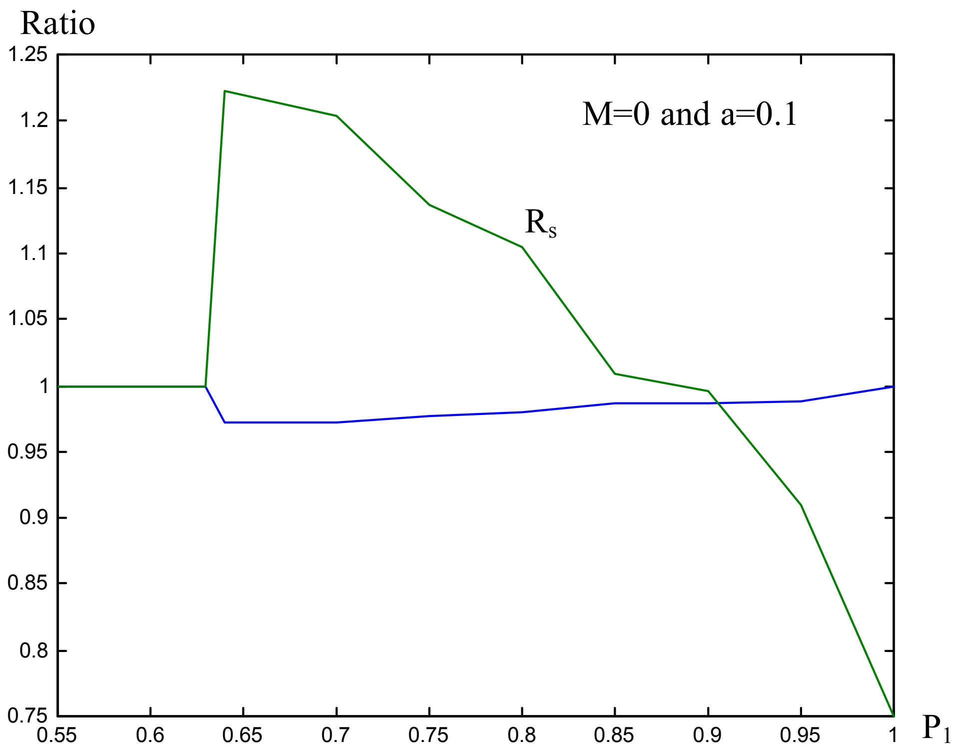
Average Increase in Subjective Welfare

Figure 4



Equilibrium Price of Portfolio 2 (M=1 and M=0)

Figure 5



Average Subjective Welfare Ratio (R_s) and Average Objective Welfare Ratio (R_o)

Figure 6

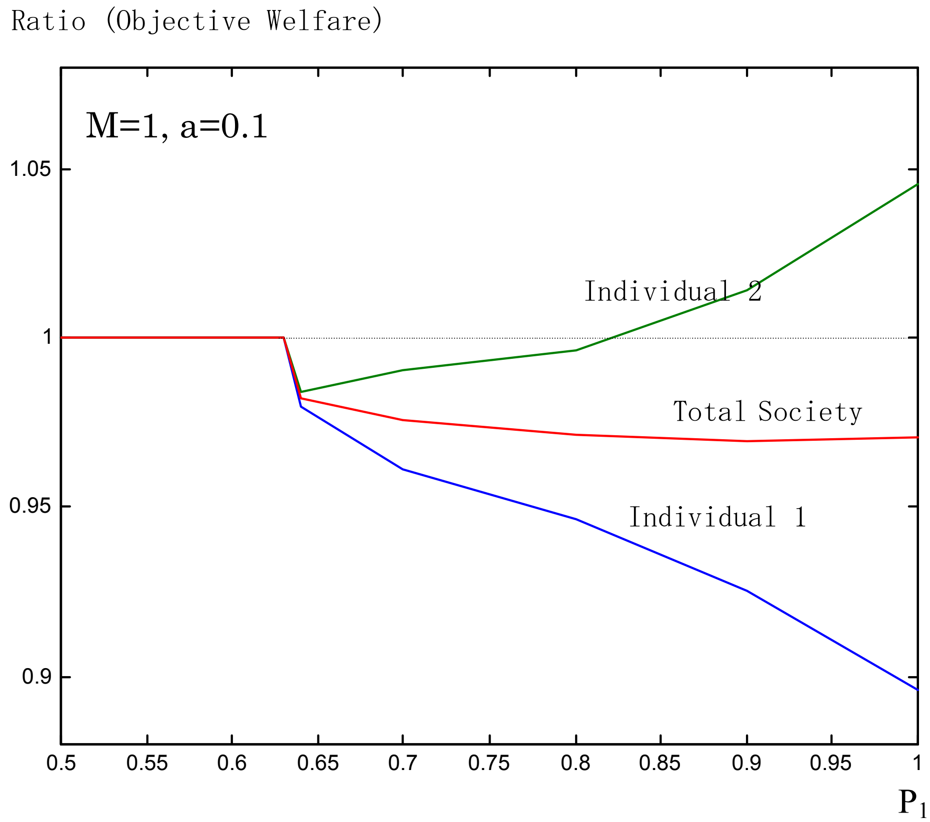

Ratio (Objective Welfare)
M=1, a=0.1
1.05
1
0.95
0.9
Individual 2
Total Society
Individual 1
0.5
0.55
0.6
0.65
0.7
0.75
0.8
0.85
0.9
0.95
1
P_1

Figure 7

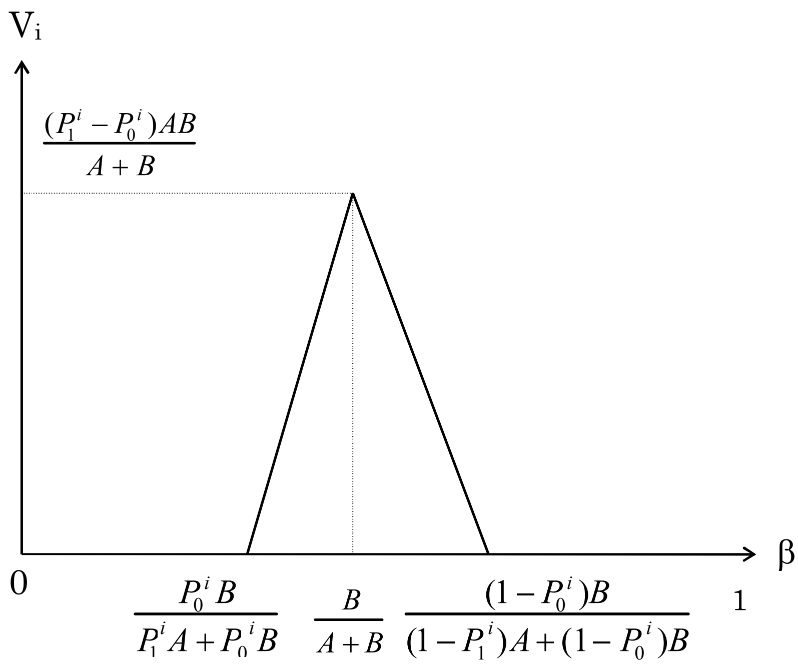

V_i
$\frac{(P_1^i - P_0^i)AB}{A+B}$
0
$\frac{P_0^i B}{P_1^i A + P_0^i B}$
$\frac{B}{A+B}$
$\frac{(1-P_0^i)B}{(1-P_1^i)A+(1-P_0^i)B}$
1
β_i

Figure A-1

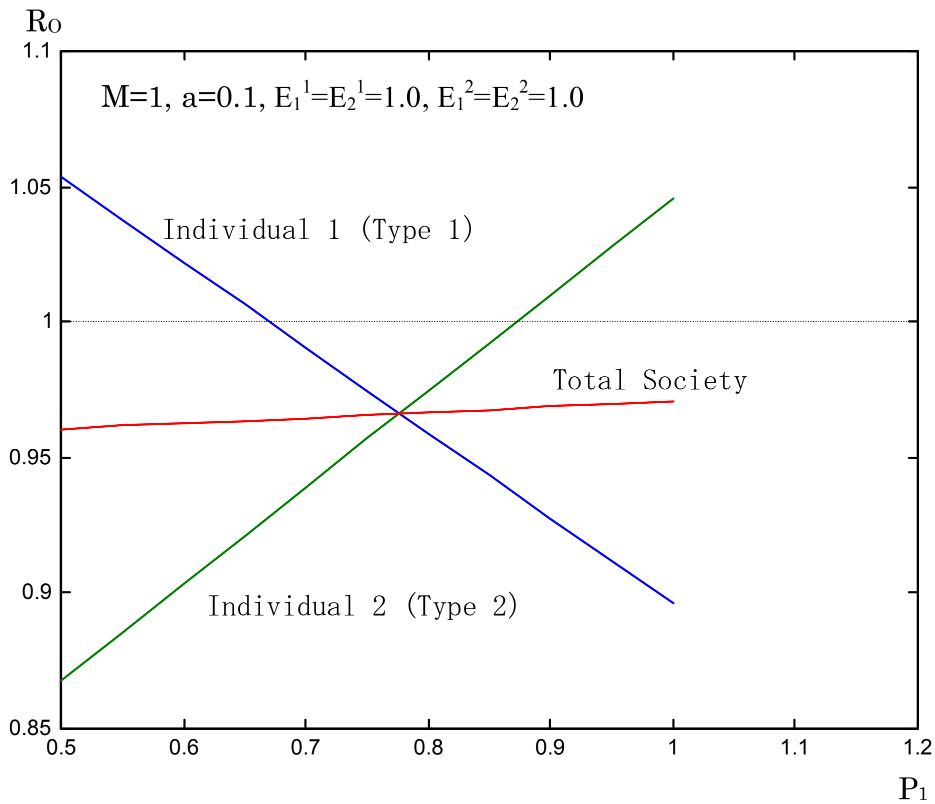
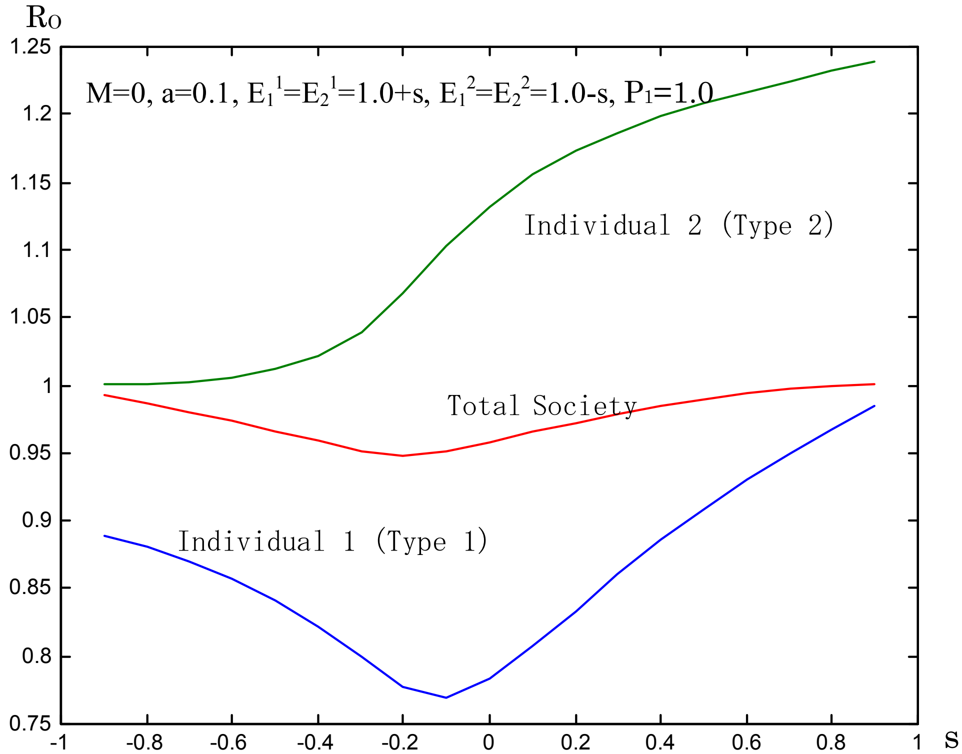
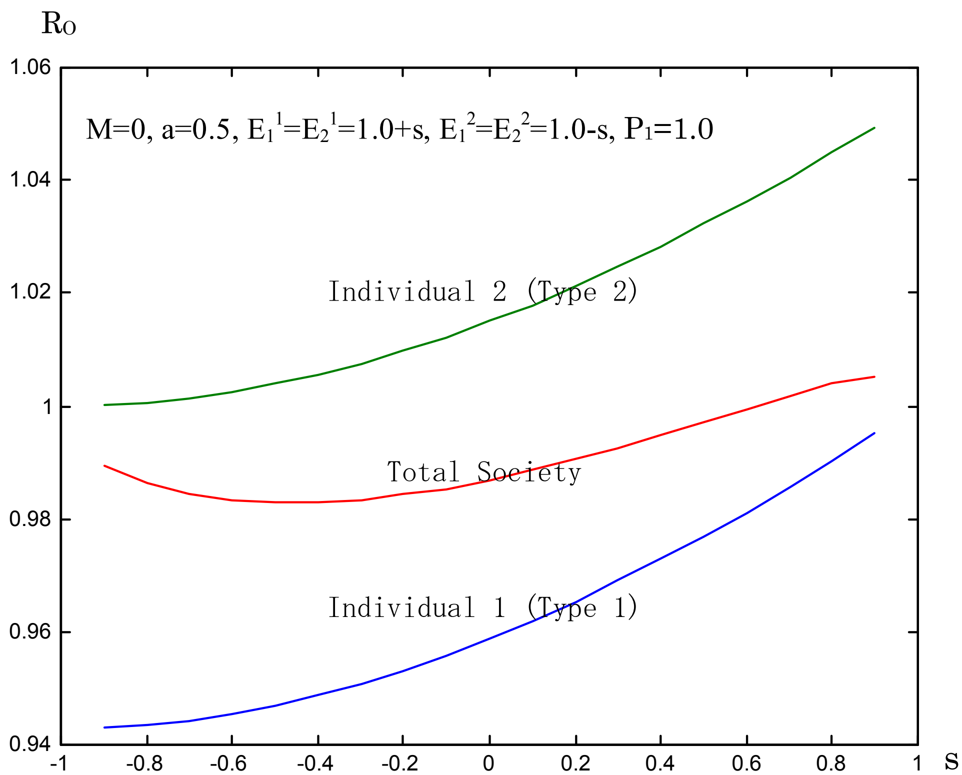

Ro
1.1
1.05
1
0.95
0.9
0.85
M=1, a=0.1, E1 1=E2 1=1.0, E1 2=E2 2=1.0
Individual 1 (Type 1)
Total Society
Individual 2 (Type 2)
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
P1

Figure B-1



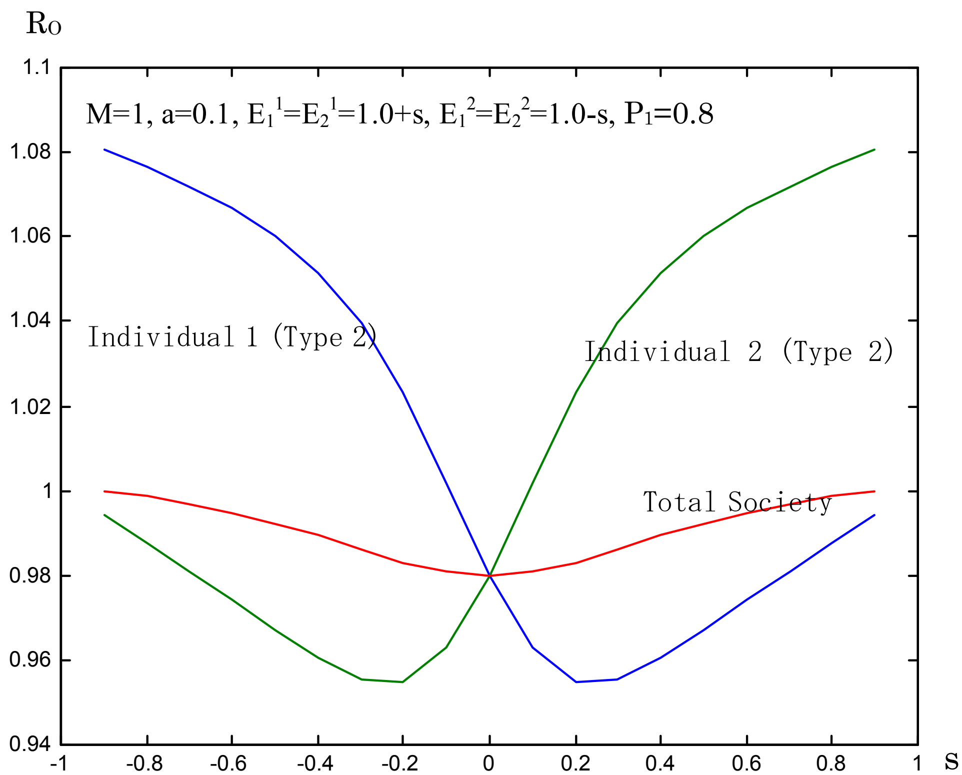
Ro
1.25
1.2
1.15
1.1
1.05
1
0.95
0.9
0.85
0.8
0.75
M=0, a=0.1, $E_1^1=E_2^1=1.0+s$, $E_1^2=E_2^2=1.0-s$, $P_1=1.0$
Individual 2 (Type 2)
Total Society
Individual 1 (Type 1)
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
s

Figure B-2-1



Ro
1.06
1.04
1.02
1
0.98
0.96
0.94
M=0, a=0.5, $E_1^1=E_2^1=1.0+s$, $E_1^2=E_2^2=1.0-s$, $P_1=1.0$
Individual 2 (Type 2)
Total Society
Individual 1 (Type 1)
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
s

Figure B-2-2



Ro
M=1, a=0.1, E1^1=E2^1=1.0+s, E1^2=E2^2=1.0-s, P1=0.8
Individual 1 (Type 2)
Individual 2 (Type 2)
Total Society
1.1
1.08
1.06
1.04
1.02
1
0.98
0.96
0.94
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
s

Figure B-3