Storage and Security of Supply in the Medium Run

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Abstract

This paper analyzes the role of private storage in a market for a commodity (e.g. natural gas) whose supply is subject to the threat of an irreversible disruption. We focus on the medium term in which seasonality of demand and exhaustibility can be neglected. We characterize the price and inventory dynamics (accumulation, drainage and limit stocks) in a competitive equilibrium with rational expectations. We show the robustness of our results to alternative scenarios in which either a disruption has finite duration or the crisis is foreseen. During the crisis consumers may put pressure on the Government to intervene, but too severe antispeculative measures would inefficiently discourage storage. Practical solutions to this dilemma cause welfare losses that we characterize and quantify.

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1 Introduction

Natural gas consumption has grown fast in the European Union over the last decades. In 2005, about one-quarter of the EU primary energy consumption was based on natural gas, and imports from neighboring producers, mainly Russia, accounted for 35% of the total EU25 demand (DG TREN, 2006). Dependence on external supplies is going to increase in the next years, as gas consumption in Europe is expected to grow whereas indigenous sources are forecasted to slow down. This prospect raises serious concerns about security of supply.

What can be done? Pipelines and a diversified portfolio of long-term contracts with producers are the primary insurance against supply interruptions. Security of supply targets can also be met by increasing system flexibility (fuel switching, interruptible contracts and liquid spot markets). However, these mechanisms have a limited capacity to absorb shocks such as extreme weather, technical breakdown, terrorism, which would endanger all the European countries at the same time and trigger a crisis (Weisser, 2007). In the short-medium term, precautionary gas storage is indispensable to ensure uninterrupted services in face of events of "low probability but high potential market impact" (Stern, 2004).[1]

The issue is a very complex one, so simplification is essential if any progress is to be made. We focus on the medium term in which both the seasonality of demand and the exhaustibility of gas can be practically neglected. All the agents know that there is a probability of an irreversible crisis. Storers are assumed to be risk-neutral and price-takers; they keep a stock of gas if expected price gains balance storage and interest cost. When the crisis occurs, the supply price jumps and the economy enters in a competitive Hotelling regime: storers gradually sell their stock and the price rises towards a ceiling, at which a stationary equilibrium is reached.

During the abundance phase, the anticipation of the crisis dynamics determines precautionary actions. The expected gains with respect to current prices provide a rationale for stockpiling. This stage is not trivial as we have to characterize a dynamic equilibrium consisting of both price and stocks trajectories. As long as the crisis has not hit the economy, accumulation starts fast and declines smoothly to approach but never reach limit stocks. Indeed, as time passes, storers become gradually cautious in their purchases and relieve pressure on the current price.

[1] After the liberalization of the gas market, the European Union has addressed security of supply in Directive 2004/67/EC. It obliges member countries to define the roles and responsibilities of all market players in ensuring gas availability and set minimum targets for gas storage, at national or industry level.

The basic insights of our analysis hold if we relax the irreversibility hypothesis, by analyzing crises of finite duration and foreseen supply disruptions. Thus the idea of a permanent supply shock, besides allowing explicit resolution of the model, is less restrictive than it would appear to be.[2]

Security of supply has an inevitable political dimension: consumers are reluctant to pay high prices in the crisis period and they put pressure on the Government to intervene. Stockholders may fear that if the consumers' lobby prevails, antispeculative measures would dramatically decrease the capital gains they could expect. This might discourage storage completely. The Government has to find a practical solution to the trade-off between the costs of controlling prices and the benefits of strategic reserves. The resulting second best equilibrium inevitably creates welfare losses that we characterize and quantify.

Our approach is a useful complement to the models on oil supply security. We focus on the medium-term horizon analysis of the supply disruption, an issue that has been largely neglected in the works inspired by the theory of exhaustible resources.[3] It is our opinion that this strand of the literature does not adequately represent the actual EU gas security of supply problem. Due to the high import dependence and the fast decline of internal gas production, it is unlikely that European countries are willing to further slow down the gas extraction rate in order to ensure future supply. Moreover, gas producers like Russia, Norway, Algeria do not seem to form a proper gas cartel.

Our work is related to Teisberg (1981), who developed a macroeconomic dynamic programming model. Our model shares with this paper the stochastic specification of the supply disruption. However, we put forth a rather different perspective, since our model focuses on a microeconomic foundations (in particular arbitrage) to explain stocks formation and drawdown. This is also a noticeable difference with respect to Bergström *et al.* (1985) in which stocks are built up at the exogenous world price as they analyze the case of a "small country" that does not influence the international trade of the commodity exposed to an embargo threat.

Our analysis resembles the Wright and Williams (1982)'s approach in

[2]Creti and Villeneuve (2007) develop an algorithm for solving a Markovian version of the present model in which the economy alternates between abundance and crisis. Though this latter approach may be deemed more realistic, its drawback is that most results are based on simulations.

[3]There are two sets of models: works that consider the extraction rate of one country when foreign import, though needed to complement national production, can suddenly default (for example Stiglitz, 1977; Sweeney, 1977; Tolley and Wilman, 1977; Hillman and Van Long, 1983; Hugues Hallet, 1984); those that introduce strategic behavior of consuming countries confronting oligopolistic or cartelized supply (Nichols and Zeckhauser, 1977; Crawford, Sobel and Takahashi, 1984; Devarajan and Weiner, 1987; Hogan, 1983).

that we derive the equilibrium dynamics of accumulation and drawdown in a continuous time context. Furthermore, like Williams and Wright (1991), we analyze welfare effect of public interventions. However, the assumption of i.i.d. shocks used by those authors cannot capture the persistence that supply crises are likely to exhibit. Moreover, in Williams and Wright, the complexity of the dynamic models makes the policy evaluation impossible to solve analytically. Our method, which uses only information on supply and demand fundamentals and the stochastic process, allows full description and evaluation of the trajectories of the economy.

The paper is organized as follows. Section 2 presents the dynamic model. Section 3 provides the characterization of the competitive equilibrium. We find equilibrium prices, limit stocks and drainage time. In Section 4, we show the robustness of our results to alternative scenarios in which either a disruption has finite duration or the crisis is foreseen. Section 5 is devoted to policy issues. To give useful orders of magnitude, we illustrate our method with parameters roughly calibrated on the UK gas market. We suggest in Section 6 two important extensions of the basic model: non negligible injection and release costs, and limited storage capacity. We conclude by pointing out that the methodology can be adapted to other commodities or regions. Proofs and technical results are relegated to the Appendix.

2 The model

The economy starts at date 0 in a state of abundance A and passes irreversibly at a random date in a state of crisis C. Time is continuous. The probability that the economy switches from A to C in a time interval dt is λdt, where λ is the publicly known parameter of this survival process. Thus, if the economy is in state A at some date, the economy will still be this state t periods later with probability $e^{-\lambda t}$. This simple modeling has three properties: (1) irreversibility ; (2) the crisis is certain only when it happens (no warning); (3) λ is independent of the state of inventories. The first two properties are relaxed in Section 4 whereas the third is kept throughout the paper.[4] In any case, this structure represents the notion of low probability/high impact event (Stern, 2004).

We assume that consumers and producers only respond to the current price and the state $\sigma = A, C$. These responses are summarized by the "excess

[4]Since disruption risk linked with terrorist attack, civil war or pipeline breakdown can reasonably be seen as independent of accumulated reserves. Teisberg (1981) considers the deterrence effect of having sufficient reserves. However, the specification is given *a priori* and not founded on an explicit game between producer countries and the US.

supply functions" $\Delta_\sigma[\cdot]$ defined over $\mathbb{R}_+^*$, where $\Delta_\sigma[p]$ is the difference in state σ and for price p between current *primary production* and current *final consumption*. For example, $\Delta_C[\cdot]$ incorporates the supply shock and the adaptation of demand to the new state (e.g. use of interruptible contracts, fuel switching).[5]

Excess supply function $\Delta_\sigma[\cdot]$ is increasing and has a unique finite positive zero in R_+^*, denoted by p_σ^*; this is the price at which the spot market would be balanced without recourse to storage. Therefore, if the current price p is above p_σ^*, then the economy stores ($\Delta_\sigma[p] > 0$); if p is below p_σ^*, then the economy draws on gas inventories ($\Delta_\sigma[p] < 0$). Naturally, we assume that the abundance static equilibrium price p_A^* is strictly smaller than the crisis static equilibrium price p_C^*. See Figure 1 for an illustrative example.

$\Delta_\sigma[\cdot]$ is a flow in the sense that if price p is sustained for the interval dt, then the quantity that is stored is $\Delta_\sigma[p]dt$. Thus, if we denote the total inventories in the economy by $S \geq 0$, conservation of matter imposes the following conditions

$$\begin{cases} \frac{dS}{dt} = \Delta_\sigma[p] & \text{if } S > 0 \text{ or } \Delta_\sigma[p] > 0, \\ \frac{dS}{dt} = 0 & \text{if } S = 0 \text{ and } \Delta_\sigma[p] \leq 0. \end{cases} \tag{1}$$

Storers are assumed to be risk-neutral price-takers, so that the price dynamics will be driven by arbitrage.[6] Storage exhibits constant returns to scale. Carrying costs consist of the opportunity cost of capital (r being the interest rate) and a cost c (per unit of commodity and per unit of time).[7]

We define the equilibrium as follows.

Definition 1 *A competitive equilibrium starts at date* 0*, in state* A*, with some initial stocks* S_0*; it consists of contingent prices and stocks trajectories*

$$\{p_A[t], p_C[t,\tau]\}_{t\geq 0, \tau \geq 0} \ \textit{and} \ \{S_A[t], S_C[t,\tau]\}_{t\geq 0, \tau \geq 0} \tag{2}$$

where t *is the current date and* τ *the (random) date at which the crisis breaks out.*

Three conditions must hold: (1) price-taking behavior by all agents (consumers, producers, storers); (2) rational expectations; (3) conservation of matter.

[5]This modeling is rationalizable with agents maximizing intertemporal utility or profit, provided objectives are time separable and quasi-linear. For a full justification, see Appendix A.4 where surpluses are calculated.

[6]We should rather write "quasi arbitrage", since speculators break even in expectation only.

[7]A more general structure with injection and withdrawal costs and limited storage capacity is discussed in Section 6.

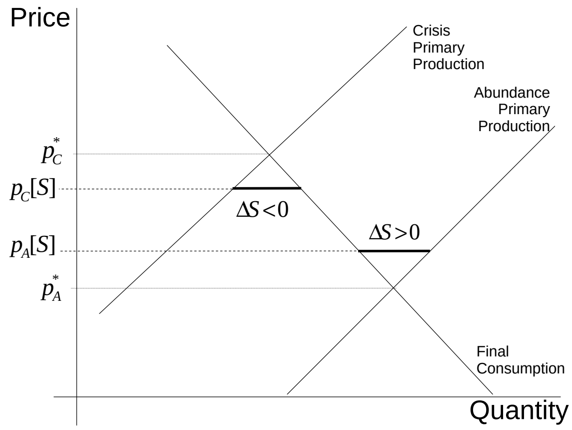


Figure 1: Supply disruption in the linear case.

The date of the crisis τ has no impact on $p_A[\cdot]$ nor $S_A[\cdot]$; moreover $p_C[\cdot]$ and $S_C[\cdot]$ are defined only for dates posterior to the disruption. Non-strategic behavior of the agents, strictly increasing excess supply functions, linearity of the storage technology, risk-neutrality, all these hypotheses suffice to ensure that the competitive equilibrium is Pareto optimal.

3 Price and stock dynamics

Storers keep a stock of gas if expected price gains balance storage and interest cost. Whenever storages are non-empty, for a time increment dt, the no-arbitrage equations read

$$p_C[t,\tau] + cdt = (1 - rdt)p_C[t + dt, \tau], \qquad t \geq \tau, \tag{3}$$
$$p_A[t] + cdt = (1 - rdt)\left((1 - \lambda dt)\, p_A[t + dt] + (\lambda dt) p_C[t + dt, t]\right). \tag{4}$$

In the above equations, the LHS is the unit price plus stockholding cost in states of crisis C and abundance A respectively. The RHS is the expected present unit value of the stocks after dt has elapsed. Equation (4) incorporates the risk of a regime switch. After elimination of second order terms, we get

$$\frac{\partial p_C[t,\tau]}{\partial t} = r p_C[t,\tau] + c, \qquad t \geq \tau, \tag{5}$$
$$\frac{dp_A[t]}{dt} = (r + \lambda) p_A[t] - \lambda p_C[t,t] + c. \tag{6}$$

We solve the model backwards. Once the crisis has broken out, the economy follows the Hotelling (competitive) dynamics; the gas price increases and the stocks shrink. Equation (5) is integrated for a fixed date τ and gives for all $t \geq \tau$:

$$p_C[t,\tau] = \min\left\{(p_C[\tau,\tau] + \frac{c}{r})\exp[r(t-\tau)] - \frac{c}{r}, p_C^*\right\}. \tag{7}$$

The price stops at p_C^* when the precautionary reserves are exhausted. Indeed, if the price were to overpass p_C^*, the economy would start accumulating gas without bound or time limit, which cannot be an equilibrium.

The economy drains the stocks that were in place at date τ, thus conservation of matter implies:

$$S_C[t,\tau] = -\int_t^{+\infty} \Delta_C[p_C[s,\tau]]ds, \tag{8}$$

$$S_A[t] = S_0 + \int_0^t \Delta_A[p_A[s]]ds, \tag{9}$$

$$S_A[t] = S_C[t,t] \text{ for all } t. \tag{10}$$

None of the model's parameters—interest rate, costs, crisis probability—depend on time. This simplifies the representation of the equilibrium, as the following proposition shows.

Proposition 1 *The equilibrium prices are only functions of current stocks. Functions $p_A[S]$ and $p_C[S]$ are continuous and decreasing for all $S \geq 0$; $p_C[S]$ has a simple implicit expression*

$$S = -\int_{p_C[S]}^{p_C^*} \frac{\Delta_C[p]}{rp+c}dp. \tag{11}$$

By using the results of Proposition 1 and equation (7), we obtain drainage duration for stocks S:

$$D[S] = \frac{1}{r}\ln\left[\frac{rp_C^* + c}{rp_C[S] + c}\right]. \tag{12}$$

This confirms that larger stocks always need more time to be drained. Drainage duration is necessarily finite: once the price has reached p_C^*, it would be uneconomical to keep costly stocks whose value will never increase.

The following proposition contains the fundamental properties of the equilibrium trajectories.

Proposition 2

1. *The maximum inventories during abundance* S^* *is*

$$S^* = -\int_{\overline{p}_C}^{p_C^*} \frac{\Delta_C[p]}{rp+c} dp, \tag{13}$$

where

$$\overline{p}_C \equiv \left(\frac{r+\lambda}{\lambda}\right) p_A^* + \frac{c}{\lambda}. \tag{14}$$

S^* *is positive if and only if* $p_C^* > \overline{p}_C$*. Moreover,* S^* *verifies* $p_C[S^*] = \overline{p}_C$ *and* $p_A[S^*] = p_A^*$.

2. *The protection offered to the economy by the stocks has a maximum duration*

$$D^* = D[S^*] = \frac{1}{r} \ln \left[\frac{\lambda}{r+\lambda} \frac{rp_C^* + c}{rp_A^* + c}\right]. \tag{15}$$

3. *When* $S^* > 0$*, the economy approaches* S^* *without reaching it.*

The price threshold and the limit stocks are remarkably useful to describe the behavior of the economy. During the state of abundance, storers are willing to pay a premium proportional to the expected capital gains. As stocks approach S^*, these gains are progressively eroded and storers relax their pressure on prices. Accumulation slows down so much that the limit stock is never attained.

The time length D^* is positive if and only if S^* is positive. Maximum duration of drainage in equation (15) only depends on the boundary prices p_C^* and p_A^*, the interest rate and the unit cost. As a purely illustrative example, let's take c negligible with respect to the opportunity cost of the stock (price times interest rate). Limit stock and drainage time are non null if:

$$\frac{p_C^*}{p_A^*} > \frac{r+\lambda}{\lambda}. \tag{16}$$

For instance, with an interest rate of 5% and a "one-in-twenty-years" crisis (λ = 5% approximately), equation (16) implies that some precautionary storage takes place if the ratio p_C^*/p_A^* is larger than 2.

The impacts of parameters c, r, λ are unambiguous. With a higher unit storage cost or interest rate, the integrand in (13) decreases (the denominator increases) and the lower bound of integration $\overline{p}_C$ increases, thus S^* decreases. With a higher crisis probability, $\overline{p}_C$ is smaller, which gives a larger S^*. The effects on D^* are similar.

4 Weakening the irreversibility hypothesis

In this section, we somewhat relax the irreversibility hypothesis by extending the model in two directions: first, we consider finite duration of the crisis, and second, we study the impact of "alerts" in the management of stocks.

Crisis of finite duration. The notion of excess supply function $\Delta_C[\cdot]$ incorporates the short-term reactivity of the economy to the shock via demand curtailment or fuel switching. Liberalized gas market also offer interesting possibilities to overcome disruption problems: the supply crisis can be solved by negotiating new contracts with gas producers and developing *ad hoc* transport infrastructure. Since these solutions entail long and complex procedures, a crisis may be of long but finite duration.

Assume that agents know that the crisis will last a period of length L, after which the economy returns to abundance. When $L > D^*$, the accumulation and drainage dynamics behave *as if* the crisis were irreversible. If $L < D^*$, the limit stock, denoted by S^L, is smaller than S^* and it increases with the crisis duration L; in fact when the crisis duration approaches the threshold D^* from below, the limit stock S^L goes to S^*. If the shock occurs early, the accumulated stocks might be insufficient to last the whole duration of the crisis. If, in contrast, the economy has approached S^L sufficiently, the price will pass from $\overline{p}_C$ at the beginning of the crisis, as we saw earlier, to a maximum value $(\overline{p}_C + \frac{c}{r}) \exp[rL] - \frac{c}{r} < p_C^*$ at the end of the crisis which coincides with complete stockout. Quite intuitively, storage is more effective at keeping moderate prices for short crises.

Alert and crisis. Assume that the crisis is announced (the "alert") *before* it happens. In the abundance state, an alert occurs with probability λdt in a time interval dt; after a delay of T time units, T being perfectly known, the disruption itself takes place. We could think of T as being a few weeks or months (up to now, everything was as if we had assumed $T = 0$).

There are two finite thresholds $\underline{T}$ and $\overline{T}$ with $\underline{T} < \overline{T}$ separating the three different regimes that we are going to describe (see Appendix A.3 for calculations).

Assume that the date of the crisis τ has always been known. As stockpiling too early is not profitable, there is a unique $\overline{T}$ such that at date $\tau - \overline{T}$, the economy starts storing and does so until date τ; from then on, the stock is drained.

Clearly, if $T > \overline{T}$, accumulation starts after $T - \overline{T}$ time units spent in the alert state and continues until the crisis actually occurs. Thus, the price stays at p_A^* until $\overline{T}$ time units before the crisis, and then grows continuously up to p_C^*. In the transition, stocks are piled up during the alert and drained once the crisis has hit the economy. A consequence of the increasing price is that accumulation accelerates as the date of disruption approaches, a remarkable difference with the basic model.

If T is slightly below $\overline{T}$, the price of gas jumps as soon as the crisis is announced and storers accumulate until the crisis occurs. Quite intuitively, the jump increases as T shortens.

If T is small enough, the price just after the alert could jump to $\overline{p}_C$ or more if inventories were quasi empty. The threshold is denoted by $\underline{T}$. Thus, if $T < \underline{T}$ (with $0 < \underline{T} < \overline{T}$), some accumulation takes place *before* the alert. In that case, accumulation can be broken down into two phases. Before announcement of the disruption, the stock converges towards a limit; during this phase, the price decreases smoothly towards p_A^*. The alert starts a new phase in which the price jumps and increases until stocks are exhausted.

5 Dynamic welfare costs of antispeculative policy

In theory, governments should not interfere with security of supply, as competitive markets realize efficient solutions (Bohi *et al.*, 1996). However, the Government might pursue short term political goals, supported by the consumers' pressure groups demanding stable supply of energy at an *affordable* price, no matter what the circumstances are (Mulder and Zwart, 2006). In view of this, storers would anticipate strict price controls.[8] Given the discouraging effects of this threat, the Government may wish to mitigate in advance its own foreseeable antispeculative intervention.[9] Our objective is to quantify the welfare loss of such second best policies.

The result of this political process can be summarized in terms of our model as follows. The policy consists of an "antispeculative" price p_C^G which

[8] As Wright and Williams (1982) put it: "the oil industry has abundant reason to believe that there is some oil price at which Government will intervene to control the realizations of oil drawn down from private storage in times of shortage, when profit-maximizing private storers and importers may well be branded as "speculators" or "price gougers". In fact, it may well be impossible for any administration credibly to guarantee against such action by itself or its successors."

[9] An alternative view is the following, described, for trade policy, in the lobby model of Grossman and Helpman (1994). Storers, requiring protection of their industry interest, can make implicit offers to the Government. The Government maximizes the sum of voters' welfare and total contributions from storers. A full-fledged version of this approach is beyond our scope.

is smaller than p_C^* and independent of S for clarity. It is the price at which gas is sold and purchased as long as there are stocks to be drained. Since p_C^G induces a fixed drainage rate $\Delta_C[p_C^G]$, the price p_C^G is guaranteed for a fixed period only. From then on, stocks stay empty and the price is p_C^*. Since storing during crisis yields negative returns (the price cap prevents capital gains), storers sell all they have as soon as the crisis starts. To accommodate this, the Government can establish a public stabilization fund, which may either directly manage storage, or remunerate owners of storage facilities for their services, or pay stockholders their opportunity cost. All these schemes are equivalent as they engender the same surplus in total, though they differ as for how it is distributed across actors.

There are two cases, leading to very different equilibrium outcomes. If the crisis controlled price p_C^G is expected to be below $\overline{p}_C$, the smallest price that makes stockholding profitable, storage is totally discouraged in the abundance phase. If, on the contrary, p_C^G is above $\overline{p}_C$, storers see it as a price floor and they will not stop accumulation on their own in the abundance phase. Any inventory level can be attained if the crisis occurrence lags. To avoid this distortion, the Government has to put an upper bound on gas inventories, denoted by S^G. Here two variations are possible: either the abundance price is endogenous or it is also controlled by the Government.

We take the second option. Indeed, if p_A^G were determined by the market, arbitrage would make it equal to $\frac{\lambda}{r+\lambda}p_C^G - \frac{c}{r+\lambda}$ all along the accumulation phase. The stabilization fund established by the Government can replicate this price, hence our approach may deemed rather general. Moreover, the theory of the second best says that p_C^G being distorted by political pressure, p_A^G may be voluntarily distorted by the Government: along with S^G, p_A^G serve to mitigate post crisis inefficiencies generated by the price cap.

To evaluate the antispeculative policy, we calculate the expected present surplus based on generated price and stocks trajectories. This yields a function of S, the stocks at the date the value is computed. Welfare being determined up to some arbitrary constant, we normalize our comparisons by setting at zero the value of the counterfactual no-storage policy (as if storage were impossible or too costly).

We denote the value of the optimal policy by $V_A^*[S]$ and the value of the antispeculative policy by $V_A^G[S]$. The following index measures welfare performance:

$$v = \frac{V_A^G[S]}{V_A^*[S]}. \tag{17}$$

The maximum possible index is 1. A negative v would indicate a clear failure as the evaluated policy would do worse that no storage at all: the policy spoils resources by, e.g., building exaggerated stocks too fast and by using them too timidly. Such examples are (unfortunately for the society) quite easy to find as we shall see.

The detailed calculations of the total expected present surplus and of the index in equation (17) are relegated to Appendix A.4.

Linear model. The application assumes linear excess supply functions:

$$\Delta_C[p_C] = bp_C - a \quad ; \quad \Delta_A[p_A] = \beta p_A - \alpha. \tag{18}$$

The reference prices are $p_C^* = a/b > p_A^* = \alpha/\beta$. Figure 1 illustrates the supply disruption in the linear case.

We compare now the two scenarios:

1. Competitive/surplus maximizing scenario;
2. Antispeculative policy summarized by $\left(p_A^G, p_C^G, S^G\right)$.

The surplus maximizing limit stock S^* and drainage time D^* have explicit formulas that are calculated by using equations (13) and (15) respectively.[10] Moreover, $p_C[S]$ can also be calculated, whereas $p_A[S]$ and $V_A^*[S]$ are solved numerically.

To give realistic orders of magnitude, we take parameters as roughly calibrated on the 2006 UK gas market. The UK having recently moved from a position of relative self-sufficiency to one of import-dependence, the need to implement precautionary gas stocks has been debated and much data have been released (see Appendix A.5 for details).

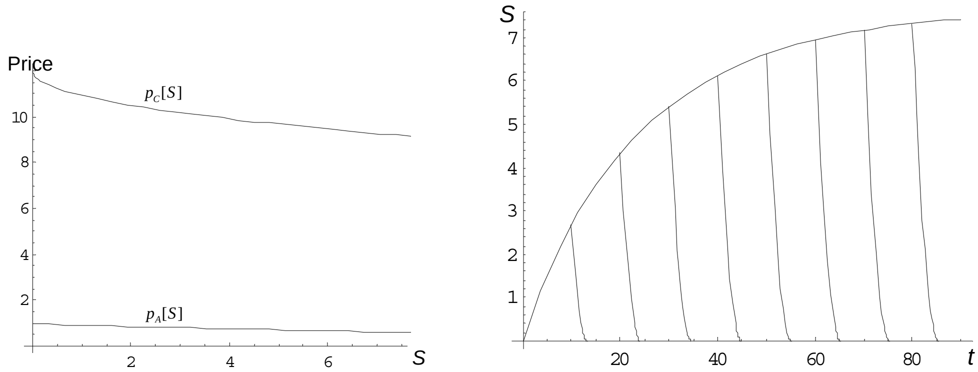
In Figure 2(a), we show prices as a function of the stocks. Figure 2(b) depicts accumulation and drainage for alternative scenarios.[11] Accumulation starts at date $t = 0$ with $S = 0$ and the shock occurs at dates $t = 10, 20, \cdots, 80$. During the abundance phase, stocks are gradually piled up to approach $S^* = 7.7$ and the price decreases toward $p_A^* = .6$. When the crisis hits the economy, the price jumps to $p_C[S]$ and increases toward $p_C^* = 12$. Though it can take as long as $D^* = 5.4$, drainage appears as much faster than accumulation.

[10] The limit stock is

$$S^* = \frac{bc + ar}{r^2} \ln \left[\frac{\lambda}{r + \lambda} \frac{rp_C^* + c}{rp_A^* + c} \right] + \frac{b}{r} \left(\left(\frac{r + \lambda}{\lambda} p_A^* + c \right) - p_C^* \right). \tag{19}$$

The expression for D^* involves Lambert's W function, the inverse of $f(w) = we^w$.

[11] Time unit is the year, prices are in £/therm and quantities in billion therm.



(a) Equilibrium price functions. (b) An array of stocks trajectories.

Figure 2: The equilibrium prices and trajectories.

As for the second best policy, we numerically calculated the surplus maximizing antispeculative storage.[12] We found $p_A^G = .84, p_C^G = 10.5$ and $S^G = 4.9$. Accumulation takes 21.4 years, if no crisis breaks out before; drainage itself takes a maximum of 3.4 years.

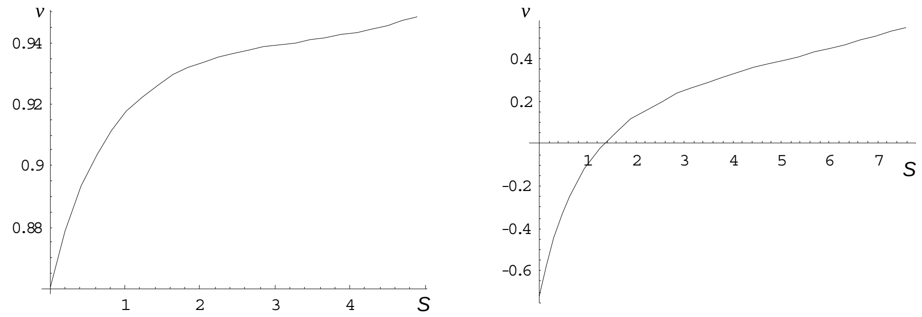
Welfare comparison. Figure 3(a) displays the relative value v of the antispeculative policy. Over the interval $[0, 4.9)$ where both surplus are defined, the index approaches 1 as inventories S increase:

- At $S = 0$, the suboptimal policy achieves 86% of the potential surplus;
- Gains increase very fast at the beginning of accumulation: at $S = 1$ (that is 20% of S^G), 64% of the initial efficiency loss are recouped;
- At S^G, 95% of the maximum surplus are captured by the suboptimal policy.

The latter effect is easily explained: as storage increases, the inefficiency of the *accumulation* strategy is sunk and thus disappears from the welfare comparison.

The expected present surplus is quite sensitive to the chosen policy. An simple example of a policy that dramatically underperforms the no storage option is proposed. Assume that the Government keeps S^G as a target but imposes a twice larger accumulation rate and a twice slower drainage rate than those obtained under the surplus maximizing antispeculative policy.

[12] $V_A^G[0]$ can be expressed as an explicit function of constrained prices and target stock (p_A^G, p_C^G, S^G).



(a) Constrained optimum. (b) Alternative.

Figure 3: Relative value of suboptimal policies.

As Figure 3(b) shows, at zero stocks and up to $S = 1.4$ approximately, the policy imposes huge welfare costs (the index starts at $-.72$). This means that the economy would be better off if storage were impossible. Due to fast accumulation, the price is very high during the accumulation phase, which penalizes consumers; in addition, the economy sustains the cost of excessive reserves. This effect becomes attenuated as storage expenditures get sunk, but to a much lesser extent than with the constrained optimum.

6 Extensions

Injection and release costs. The analysis can be easily extended to the case where the costs of injecting and releasing gas are non negligible. Denote unit injection cost by i and unit release cost by s. With perfect competition, gas *outside* and *inside* the reservoir state can be traded at prices that we denote respectively by $p_\sigma[S]$ and $p^I_\sigma[S]$ (with $\sigma = A, C$ and $S \geq 0$). The market equilibrium between outside and inside gases implies that, whenever $S > 0$,

$$p_A[S] + i = p^I_A[S] \text{ and } p_C[S] = p^I_C[S] + s. \tag{20}$$

The structure of the system of equations is preserved, with p^I_σ replacing p_σ. Arbitrage conditions (27) and (28) become

$$\Delta_C[p^I_C + s] \cdot \frac{dp^I_C}{dS} = rp^I_C + c, \tag{21}$$

$$\Delta_A[p^I_A - i] \cdot \frac{dp^I_A}{dS} = (r + \lambda)p^I_A - \lambda p^I_C + c. \tag{22}$$

Remark that the excess supply functions are shifted, thus boundary conditions are

$$p_C^I[0] = p_C^* - s, \tag{23}$$
$$p_A^I[S^*] = p_A^* + i. \tag{24}$$

The range of p_σ^I is narrower than that of p_σ: the minimum is higher, the maximum is lower. As a result, the condition ensuring positivity of the limit stock is more restrictive, i.e.

$$p_C^* - s > \left(\frac{r+\lambda}{\lambda}\right)(p_A^* + i) + \frac{c}{\lambda}. \tag{25}$$

Expressions of optimal limit stock and drainage time are now based on shifted excess supply functions and shifted boundary prices.

Limited storage capacity. Gas is mostly stored in depleted fields and aquifers; the development of such facilities is naturally limited. If the capacity devoted to precautionary storage K exceeds S^* previously calculated, then the unconstrained solution remains valid; otherwise, the maximum stock is constrained to equal K, which in turn affects price trajectories and the value of storage facilities.

During the crisis, $p_C[S]$ is unchanged compared to the unconstrained case. Reserves are gradually drained, meaning that the storage price, under competitive assumption, remains fixed at the marginal cost c. In the abundance state, the price function $p_A^K[S]$ depends on K: the accumulation process must stop when capacity is saturated, therefore $p_A^K[K] = p_A^*$. The storage price is also c as long as some capacity remains vacant; when K is attained, it jumps to $\pi_A^K > c$, with

$$\pi_A^K = \lambda(p_C[K] - p_A^*) - rp_A^*. \tag{26}$$

The net rent $\pi_A^K - c$, captured by the owners of the storage capacity, balances the carrying costs of a fixed stock with its expected benefits. Storage capacity units gain value as K diminishes. This combines two effects: the smaller K becomes, the larger π_A, and also the shorter the time before saturation will be. The first effect (the monotonicity of π_A) derives directly from the monotonicity of $p_C[K]$. The second effect is shown in Appendix A.6.

7 Conclusion

We developed a model of optimal stockpiling and reserve duration to face up to a potential irreversible supply shock. Our key ingredient is that price trajectories, accumulation and drainage behavior are interdependent in equilibrium. This differentiates the approach from inventory management models in which prices are given, or precautionary reserve studies in which the welfare costs of building the stocks are ignored.

We found results that might prove useful not only in the context of the gas industry, but also for all those primary commodities whose market is exposed to a supply disruption threat. A simple condition determines whether precautionary stocks should be accumulated. General cost structures, in particular limited storage capacity, or a richer timing of the crisis occurrence are shown to have intuitive and calculable effects on the main properties of the equilibrium. The impact of expropriation threats that discourage storage can be dramatic. Our insights into politically sustainable solutions could be easily transposed to other examples or market structures beyond the specific application we have analyzed.

A Appendix

A.1 Proof of Proposition 1

The RHS of (8) is strictly increasing in the value of $p_C[\tau, \tau]$, thus it gives a unique strictly decreasing relationship between p_C and $S \geq 0$, denoted by $p_C[S]$. We can then define $p_A[S]$ by backwards induction.

We can now replace the price dynamics in (5) and (6) by

$$\Delta_C[p_C[S]] \cdot \frac{dp_C[S]}{dS} = rp_C[S] + c, \quad (27)$$

$$\Delta_A[p_A[S]] \cdot \frac{dp_A[S]}{dS} = (r + \lambda)p_A[S] - \lambda p_C[S] + c, \quad (28)$$

for $S > 0$.

Equation (27) can be integrated directly to get equation (11). The RHS of (28) cannot be positive (otherwise storers would liquidate inventories at once) implying that $\frac{dp_A[S]}{dS} < 0$.

A.2 Proof of Proposition 2

1. Remark that $\overline{p}_C$ is the minimum value $p_C[\cdot]$ can take: storers are just indifferent between keeping or selling their stocks if the abundance price is as low as p_A^*, since the carrying costs ($rp_A^* + c$ per unit) equals the *expected* earning ($\lambda(p_C[S^*] - p_A^*)$ per unit). The corresponding stocks are denoted by S^*; S^* being the maximum stocks, it verifies $p_C[S^*] = \overline{p}_C$ and $p_A[S^*] = p_A^*$.

This reasoning implies in particular that if $\overline{p}_C \geq p_C^*$, then $S^* = 0$: holding inventories cannot be profitable and the crisis will simply cause a price jump from p_A^* to p_C^*.

2. By plugging $\overline{p}_C$ into (11), we obtain the expression in the text.

3. The price p_A must converge continuously towards p_A^* before the occurrence of the gas disruption. As p_A covers half its difference with the limit p_A^*, the variation rate of the stock per unit of time Δ_A is approximately halved (the derivative of excess demand at p_A^* is not zero), meaning that the convergence speed dS/dt is approximately halved. This implies that, whatever the proximity of the limit, the duration to cover half the distance to the limit is approximately constant, thus the limit is not attained in finite time.

A.3 Alert duration thresholds

Derivation of $\overline{T}$. Assume that T is large. Once the economy is in alert, uncertainty vanishes and the price passes continuously from p_A^* to p_C^* following the differential equation

$$\frac{dp}{dt} = rp + c. \tag{29}$$

Let $\overline{p} \in (p_A^*, p_C^*)$ be the price reached when the crisis occurs. Using the same change of variable as in the text, we know that conservation of matter implies

$$\int_{p_A^*}^{\overline{p}} \frac{\Delta_A[p]}{rp + c} dp + \int_{\overline{p}}^{p_C^*} \frac{\Delta_C[p]}{rp + c} dp = 0. \tag{30}$$

$\overline{p}$ is unique since both terms increase as $\overline{p}$ increases, whereas the LHS is negative for $\overline{p} = p_A^*$ and positive for $\overline{p} = p_C^*$.

$\overline{T}$ is the time required for the price to pass from p_A^* to $\overline{p}$, i.e.

$$\overline{T} = \frac{1}{r} \ln \left[\frac{r\overline{p} + c}{rp_A^* + c} \right]. \tag{31}$$

Derivation of $\underline{T}$. For all T between $\underline{T}$ and $\overline{T}$, the immediate post-alert price p_A^T must be such that, prior to alert, storage is not profitable, i.e.

$$p_A^T < \overline{p}_C. \tag{32}$$

Let $\widetilde{p}$ be the price of gas at the instant the crisis occurs; it is uniquely defined by the conservation of matter equation

$$\int_{p_A^T}^{\widetilde{p}} \frac{\Delta_A[p]}{rp + c} dp + \int_{\widetilde{p}}^{p_C^*} \frac{\Delta_C[p]}{rp + c} dp = 0. \tag{33}$$

Given the price dynamics, the time $\widetilde{T}$ required for the price to pass from $\widetilde{p}$ to p_C^* is

$$\widetilde{T} = \frac{1}{r} \ln \left[\frac{r p_C^* + c}{r\widetilde{p} + c} \right]. \tag{34}$$

Thus $\widetilde{T}$ decreases when $\widetilde{p}$ increases, which implies in turn that the quantity to be drained is decreasing w.r.t. $\widetilde{p}$ (drainage time is an increasing function of the inventories).

To accumulate these smaller stocks, the price finishes higher in the abundance phase (last accumulation price is $\widetilde{p}$, meaning that this phase of duration T has to be shorter when $\widetilde{p}$ increases). Given that T is the time to pass from p_A^T to $\widetilde{p}$, p_A^T has to increase.

We conclude that T decreases as p_A^T increases in the interval $[p_A^*, \overline{p}_C]$. In particular that $p_A^{\overline{T}} = p_A^*$ and $p_A^{\underline{T}} = \overline{p}_C$, thus $\underline{T} < \overline{T}$.

A.4 Expected present surplus

Consider a representative consumer whose intertemporal utility function valorizes gas consumption and a separable numéraire. Leaving aside uncertainty at this stage, the consumer's objective can be written as

$$\int_0^{+\infty} \left(u_\sigma[q_t] - p_t q_t\right) e^{-rt} dt, \;\; \sigma = A, C, \tag{35}$$

where u_σ is a state dependent, increasing and concave utility, q_t is date t gas consumption and $p_t q_t$ is date t expenditure. Consider also a representative producer whose technology can by aggregated at t by a state dependent convex cost function $C_\sigma[q_t]$.

For a given price p, final demand is $u_\sigma'^{-1}[p]$ and primary production is $C_\sigma'^{-1}[p]$, thus excess supply functions as we defined them can be expressed

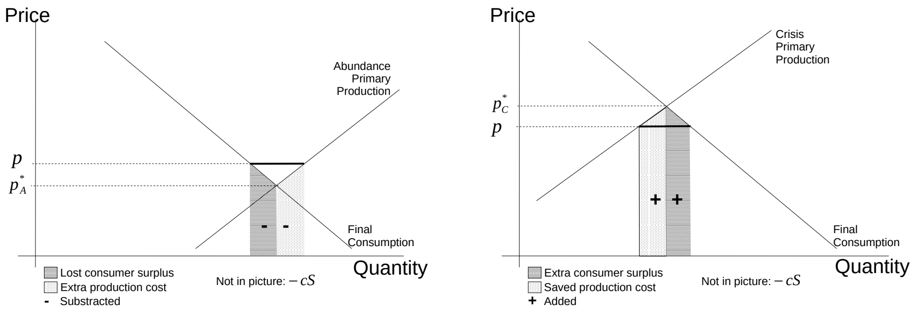
$$\Delta_\sigma[p] = C_\sigma'^{-1}[p] - u_\sigma'^{-1}[p]. \tag{36}$$

The instantaneous surplus depends only on the state σ, S and the current price p

$$W_\sigma^0 + W_\sigma[p] - cS \tag{37}$$

where W_σ^0 denotes the *reference surplus*, i.e. calculated at price p_σ^*, and where cS is the cost of keeping the inventories. The key point is that $W_\sigma[p]$ can be derived from the excess supply function $\Delta_\sigma[p]$:

$$W_\sigma[p] = \int_{p_\sigma^*}^{p} \Delta_\sigma[x] dx - p\Delta_\sigma[p], \tag{38}$$



(a) Abundance state.

(b) Crisis state.

Figure 4: Instantaneous surpluses in abundance and crisis states.

as we can directly see in Figure 4.

The final step consists of calculating the expected present surplus, by discounting all future instantaneous surpluses and then taking the expectation. Given the initial state A, stock S_0 at date 0 and the stockholding dynamics, the expected intertemporal surplus of the *optimal policy* is denoted:

$$V_A^0 + V_A^*[S_0], \tag{39}$$

where V_A^0 is defined as

$$V_A^0 = \frac{W_A^0}{r+\lambda} + \frac{\lambda W_C^0}{r(r+\lambda)}, \tag{40}$$

and

$$V_A^*[S_0] = E\left\{ \int_0^{+\infty} \left(W_{\sigma_t}[p^*_{\sigma_t}[S_t]] - cS_t\right) e^{-rt} dt \right\}, \tag{41}$$

σ_t being the (random) state at date t.

The antispeculative policy (summarized by p_σ^G and constrained accumulation and drainage functions $\Delta_\sigma[p_\sigma^G]$) will generate the instantaneous surplus $W_\sigma^* + W_\sigma^G[p_\sigma^G] - cS$. Total expected present surplus is:

$$V_A^0 + V_A^G[S_0], \tag{42}$$

where V_A^0 is defined by (40) and

$$V_A^G[S_0] = E\left\{ \int_0^{+\infty} \left(W_{\sigma_t}[p^G_{\sigma_t}] - cS_t\right) e^{-rt} dt \right\}. \tag{43}$$

The Bernoulli process driving the evolution of σ_t being exogenous and time independent, the terms comprising W_A^0 and W_C^0 are identical whatever the policy evaluated and therefore V_A^0 can be normalized at zero. This is why we state that the no storage policy (a useful reference) can be given null value. The relative value of a given policy with respect to the optimum is therefore correctly captured by the index:

$$v = \frac{V_A^G[S]}{V_A^*[S]}. \tag{44}$$

A.5 Calibration on UK data

To calibrate our model, we take the following parameters values:

Table 1: Parameters			
Excess supply in C	$b = .95$	$a = 11.48$	$p_C^* = 12$
Excess supply in A	$\beta = .95$	$\alpha = .57$	$p_A^* = .6$
Costs	$r = .035$	$c = .15$	
Crisis arrival	$\lambda = .02$		

Time unit is the year, prices are in £/therm (1 therm = 2.76 m^3), quantities are expressed in billion therm. As for the probability of crisis ($\lambda = .02$), we consider the value that DTI (2006, p. 90) estimates as a "realistic chance of a significant supply interruption", based on ILEX (2006), JESS (2006), Oxera (2006) reports. The interest rate r and the maximal crisis price ($a/b = 12$) is taken from ILEX (2006, p. 106).[13] The average 2006 price ($\alpha/\beta = .6$) and annual consumption (about 36 billion therm) is documented by DTI (2007). The marginal cost of storage c is evaluated from available information, released by *Centrica Storage Ltd*, on the largest UK storage facility. Missing parameters are calculated with identifying assumptions: in case of major crisis, consumption could be reduced by 30% (price 12, inventories release notwithstanding). Finally we adopt a last (arbitrary) condition: $b = \beta$.

A.6 Monotonicity of the scarcity rent

The function p_A^K follows ODE (28), with boundary condition $p_A^K[K] = p_A^*$. As the function p_C is independent of K, the Cauchy-Lipschitz theorem implies that the price functions for two different capacities below S^* never cross.

[13]This value, corresponding to an emergency cash out price, is assumed to reflect the damages to the economy of a sudden supply interruption.

Thus for all $S \in [0, K]$ and $K < K'$, $p_A^K[S] < p_A^{K'}[S]$ with both functions decreasing. We now show that the time T_K needed for the price to pass from $p_A^K[0]$ to p_A^* is longer the larger is the capacity K. Using equation (28), we have

$$T_K = -\int_{p_A^*}^{p_A^K[0]} \frac{dp_A}{(r+\lambda)p_A - \lambda p_C[p_A^{K(-1)}[p_A]] + c}. \tag{45}$$

Given the monotonicity of p_A^K with respect to K, the above sum with a larger K integrates a function of higher absolute value over a longer interval. This gives us the announced result.

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