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The Performance and Robustness of Interest-Rate Rules in Models of the Euro Area^{*}

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In this paper, we examine the performance and robustness of optimized interest-rate rules in four models of the euro area that differ considerably in terms of size, degree of aggregation, relevance of forward-looking behavioral elements, and adherence to microfoundations. Our findings are broadly consistent with results documented for models of the U.S. economy: backward-looking models require relatively more aggressive policies with, at most, moderate inertia; rules that are optimized for such models tend to perform reasonably well in forward-looking models, while the reverse is not necessarily true; and, hence, the operating characteristics of robust rules (i.e., rules that perform satisfactorily in all models) are heavily weighted towards those required by backward-looking models. JEL Codes: E31, E52, E58, E61

^{*}The paper summarizes the results of a project aimed at evaluating the performance of monetary policy rules in a number of small- to medium-scale models developed by Eurosystem staff that have been regularly used for policy analysis. Helpful comments by Keith Küster, Sergio Nicoletti Altimari, Athanasios Orphanides, Glenn Rudebusch, Frank Smets, Volker Wieland, John Williams, Raf Wouters; by participants at the ECB conference on "Monetary Policy and Imperfect Knowledge"; and by an anonymous referee are greatly appreciated. The opinions expressed are those of the authors and do not necessarily reflect views of the European Central Bank or the Banca d'Italia. Any remaining errors are the sole responsibility of the authors. Correspondence: Coenen: Directorate General Research, European Central Bank, e-mail: gunter.coenen@ecb.int, homepage: www.guentercoenen.com; Adalid: Directorate General Research, European Central Bank, e-mail: ramon.adalid_lozano@ecb.int; McAdam: Directorate General Research, European Central Bank, e-mail: peter.mcadam@ecb.int; Siviero: Economic Research Department, Banca d'Italia, e-mail: stefano.siviero@bancaditalia.it.

The design of robust monetary policy rules has attracted considerable interest in recent macroeconomic research. This interest largely reflects the increased awareness on the part of policymakers and academics alike that, when evaluating the possible consequences of alternative monetary policies, the existing uncertainties about the structure of the economy need to be given due account. As a result, a large body of research has been oriented toward identifying the characteristics of monetary policy rules that perform "reasonably well" across a range of potentially nonnested and competing macroeconomic models.¹ While this line of research has primarily focused on models of the U.S. economy, no systematic study has yet been conducted for the euro area.²

In an attempt to fill this gap, we concentrate on the euro area and examine the performance and robustness of monetary policy rules using a rather diverse set of macroeconomic models of the euro area economy. Such an examination seems particularly relevant as the single monetary policy of the European Central Bank (ECB) has to focus on the euro area as a whole, being a new and still relatively unexplored economic entity, and with models of the euro area having been developed only very recently. Specifically, our analysis relies on four models of the euro area economy that have been built by staff of the Eurosystem in recent years. These models differ considerably in terms of size, degree of aggregation, relevance of forward-looking behavioral elements, and adherence to microfoundations. They thus cover a wide range of features that are *a priori* considered to be of high relevance in the context of evaluating the robustness of monetary policy rules.³ One of our main goals is to ascertain which of the various features with respect to which these models differ are of importance when designing rules suitable for model-based evaluations of monetary policy, and which features are, by contrast, arguably less important.

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¹For an early claim in this respect see McCallum (1988).

²Earlier studies of the performance of interest-rate rules across alternative models of the U.S. economy are provided in Bryant, Hooper, and Mann (1993) and Taylor (1999). More recent studies have been undertaken by Levin, Wieland, and Williams (2003) and Levin and Williams (2003). The study of Côté et al. (2002) focuses on models of the Canadian economy.

³Indeed, Svensson (2003) argues that models used to analyze simple monetary policy rules, such as the Taylor rule, were too similar in structure (i.e., of the New Keynesian variety) to constitute a true test of their robustness.

In terms of methodology, our analysis builds on recent work by Levin, Wieland, and Williams (1999, 2003) and Levin and Williams (2003).⁴ This methodology involves implementing simple reaction functions describing the response of the short-term nominal interest rate to inflation and the output gap, either observed or forecast, and then optimizing over the respective response coefficients. The performance of these optimized interest-rate rules is then evaluated across alternative, potentially competing models with regard to their ability to stabilize inflation and output around their targets, while avoiding undue fluctuations in the nominal interest rate itself.

Of course, variables other than inflation and the output gap may enter the interest-rate rule, such as the exchange rate or monetary and financial-market indicators. However, the literature tends to suggest that including information variables of this kind in the policy rule yields relatively modest gains in model-based evaluations because they are typically highly correlated with the interest rate itself or closely related with the measures of inflation and the output gap entering the rule (although this result may not hold in sufficiently complex models). Furthermore, simple rules with a feedback to a small set of variables are arguably more robust to model misspecification and uncertainty than rules based on a larger set of variables, which might overfit specific model characteristics.

From an institutional viewpoint, the advantage of simple interestrate rules is clearly their transparency and, thus, the ease with which they may be communicated to and monitored by the outside world. While it is unlikely that monetary policymakers will follow their literal execution, such rules may nonetheless be a useful benchmark for assessing the actual conduct of monetary policy. Likewise, from an empirical point of view, simple interest-rate rules seem to match the data well for the United States as well as a number of European countries (see Clarida, Galí, and Gertler 1998). More recently, evidence for the euro area as a whole has been provided by Gerdesmeier and Roffia (2004).

⁴There are alternative approaches to analyzing the consequences of uncertainty about the structure of the economy (see, e.g., Giannoni 2002; Giannoni and Woodford 2002; Hansen and Sargent 2002; Onatski and Stock 2002; Onatski and Williams 2003; and Tetlow and von zur Muehlen 2001). This coexistence of alternative approaches reflects the fact that there has not yet emerged a consensus on how to address the issue of model uncertainty.

The findings of our analysis are broadly consistent with the results documented for models of the U.S. economy: backward-looking models require relatively more aggressive policies with, at most, moderate inertia; rules that are optimized for such models tend to perform reasonably well in forward-looking models, while the reverse is not necessarily true; and, hence, the operating characteristics of robust rules (i.e., rules that perform satisfactorily in all models) are heavily weighted toward those required by backward-looking models. In the course of collecting these results, we highlight a number of model features (notably, the degree of forward-lookingness) that play a crucial role in shaping the operating characteristics of optimized interest-rate rules. This in turn suggests that future empirical research that aims at casting additional light on these features may enhance the reliability and usefulness of interest-rate rules for modelbased evaluations of monetary policy.

The remainder of this paper is organized as follows. Section 1 outlines the set of euro area models used in our analysis and illustrates the implied differences in inflation and output-gap dynamics in response to a monetary policy shock. Section 2 briefly describes the methodology used for evaluating the performance of interest-rate rules and provides a set of optimized benchmark rules for each of the euro area models under examination. Section 3 evaluates the robustness of these benchmark rules when there is uncertainty about the true structure of the euro area economy, as represented by the coexistence of possibly competing models, while section 4 identifies the operating characteristics of rules that attain satisfactory outcomes across the set of models used. Section 5 reports additional sensitivity analysis, and section 6 concludes.

1. The Models of the Euro Area

To examine the performance of monetary policy rules across a range of potentially competing models of the euro area economy, we utilize a set of four macroeconomic models that differ considerably in terms of variable and country coverage, relevance of forward-looking behavioral elements, and adherence to microfoundations. All models are estimated with aggregate euro area-wide data except one, which separately models the three largest euro area economies. Two of the models are entirely or at least predominantly backward looking, while forward-looking elements abound in the other two. All models are consistent with basic economic principles; however, there is only one in our set of models that is strictly based on the assumption of optimizing agents. Hence, the models under examination cover a fairly broad range of different modeling strategies.

In the remainder of this section, we briefly present the four models of the euro area and illustrate the implied differences in inflation and output-gap dynamics in response to a monetary policy shock.

1.1 An Outline of the Euro Area Models

1.1.1 The Coenen-Wieland Model

The Coenen-Wieland (CW) model (see Coenen and Wieland 2000) is a small-scale model of aggregate supply and aggregate demand that is designed to capture the broad characteristics of inflation and output dynamics in the euro area. Since its development, the model has been mainly used as a laboratory for evaluating the performance of alternative monetary policy strategies.

The supply side of the model incorporates price and wage staggering, with wage setters negotiating long-term nominal wage contracts with reference to past contracts that are still in effect and future contracts that will be negotiated over the life of the current contract. If wage setters expect the output gap to be positive, they adjust the current contract wages upward, and vice versa. Consequently, inflation depends on its own leads and lags, excess-demand conditions, and transitory contract wage shocks—the latter representing "cost-push" shocks. There are two versions of the supply side that feature distinct types of staggered wage contracts: the nominal wage contracting specification due to Taylor (1980) and the relative real wage contracting specification by Fuhrer and Moore (1995).⁵ The two specifications differ with respect to the degree of inflation persistence that they induce, with Fuhrer-Moore-type contracts giving

⁵While the two types of specifications represent rules for price and wage setting that are not explicitly derived from a framework with optimizing agents, they need not be necessarily inconsistent with such a framework. More recently, Taylor-type contracts have been analyzed within more fully fleshed-out dynamic general equilibrium models (see, e.g., Chari, Kehoe, and McGrattan 2000 or King and Wolman 1999). Fuhrer-Moore-type contracts, however, have typically been criticized for lacking microfoundations.

more weight to past inflation developments. In this paper, the version with Taylor-type contracts is used.⁶

A simple aggregate demand relationship relates the output gap (measured as the deviation of actual output from a smooth trend) to several lags of itself, the ex-ante long-term real interest rate, and a transitory demand shock. The long-term real rate is determined jointly by a term-structure relationship and the Fisher equation. The short-term nominal interest rate is the instrument of monetary policy, and changes in the latter affect aggregate demand through the impact on the ex-ante long-term real interest rate.

1.1.2 The Smets-Wouters Model

The Smets-Wouters (SW) model (see Smets and Wouters 2003) is an extended version of the standard New Keynesian dynamic stochastic general equilibrium (DSGE) model with sticky prices and wages. The model is estimated by Bayesian techniques using seven euro area macroeconomic time series: real gross domestic product (GDP), consumption, investment, employment, real wages, inflation, and the nominal short-term interest rate.

The model features three types of economic agents: households, firms, and the monetary policy authority. Households maximize a utility function with two arguments (goods and leisure) over an infinite life horizon. Consumption appears in the utility function relative to a time-varying external habit-formation variable. Labor is differentiated over households, so that there is some monopoly power over wages, which results in an explicit wage equation and allows for the introduction of sticky nominal wages à la Calvo (1983). Households also rent capital services to firms and decide how much capital to accumulate given certain capital adjustment costs. As the rental price of capital goes up, the capital stock can be used more intensively according to some cost schedule. Firms produce differentiated goods, decide on labor and capital inputs, and set prices, again à la Calvo (1983). The Calvo model in both wage and price setting is augmented

⁶See Coenen (2003) for an evaluation of the performance of monetary policy rules across the two variants with Taylor- and Fuhrer-Moore-type contracts, respectively. This paper shows that, when faced with uncertainty about the prevailing degree of inflation persistence, a cautious monetary policymaker is welladvised to design interest-rate policies under the assumption that the inflation process is characterized by a high degree of persistence—as induced by Fuhrer-Moore-type contracts.

by the assumption that those prices and wages that cannot be freely set are partially indexed to past inflation. Prices are therefore set as a function of current and expected real marginal cost, but are also influenced by past inflation. Real marginal cost depends on wages and the rental rate of capital. The short-term nominal interest rate is the instrument of monetary policy.⁷

The stochastic behavior of the model is driven by ten exogenous shocks: five shocks arising from technology and preferences, three cost-push shocks, and two monetary policy shocks. The first set of shocks is assumed to follow first-order autoregressive processes, whereas the second set is assumed to follow serially uncorrelated processes. Consistent with the DSGE setup, potential output is defined as the level of output that would prevail under flexible prices and wages in the absence of cost-push shocks.

1.1.3 The Area-Wide Model

The Area-Wide Model (AWM: see Fagan, Henry, and Mestre 2001) is a medium-size structural macroeconomic model that treats the euro area as a single economy.⁸ It has a long-run neoclassical equilibrium with a vertical Phillips curve, but with some short-run frictions in price and wage setting and factor demands. Consequently, activity is demand-determined in the short run but supply-determined in the long run. In the latter, employment converges to a level consistent with the exogenously given level of equilibrium unemployment, and factor demands are consistent with the solution of the firms' profit-maximization problem. Stock-flow adjustments are accounted for by, for example, the inclusion of a wealth term in consumption. At present, the treatment of expectations in the model is quite limited. With the exception of the exchange rate (determined by uncovered interest parity) and the long-term nominal interest rate (modeled by a term-structure relationship), the model embodies backwardlooking expectations.

⁷Extending the study of Coenen (2003), Angeloni, Coenen, and Smets (2003) utilize the SW model to analyze the design of monetary policy when the monetary policymaker is uncertain about the degree of nominal as well as real persistence. The results of this analysis confirm the conclusions of the earlier study.

⁸For an examination of optimal monetary policy in the context of the AWM alone, see Dieppe, Küster, and McAdam (2004).

As to the mechanisms through which monetary policy affects the economy, aggregate demand in the AWM is presently influenced only by short-term real interest rates. Long-term nominal rates determine the government's debt service but do not explicitly enter investment decisions. The expectations channel in principle allows monetary policy to influence inflation via wage- and price-setting behavior. In addition to these influences, further effects enter through the exchange rate. Apart from an indirect effect of exchange rates on domestic demand, there is also a direct exchange-rate effect on consumer price inflation through the price for imported goods. The output gap is defined as the ratio of actual output to potential output, which is based on an aggregate Cobb-Douglas production function with constant returns to scale and Hicks-neutral technical progress. For this, trend total factor productivity has been estimated within-sample by applying the Hodrick-Prescott filter to the Solow residual derived from the production function.

1.1.4 A Disaggregate Model of the Euro Area

The Disaggregate Model (DM) of the euro area used in Angelini et al. (2002) and Monteforte and Siviero (2002) is a multicountry version of the simple backward-looking two-equation model in Rudebusch and Svensson (1999). It consists of an aggregate supply equation and an aggregate demand equation for each of the three largest economies in the euro area: Germany, France, and Italy, which jointly account for over 70 percent of euro area GDP. The first equation, interpretable as a Phillips curve, determines inflation in each country as a function of lagged inflation and the output gap, as well as inflation "imported" from the other two countries. The sum of the coefficients on lagged and imported inflation is constrained to equal unity (a restriction accepted by the data), so that a long-run vertical Phillips curve exists for all countries. The second equation represents an IS curve and relates the output gap of each country (modeled as the deviation of actual output from a smooth trend) to its own lagged values, the short-term real interest rate, and the output gaps in the other two countries (a design meant to capture the trade linkages among euro area economies). Euro area inflation and the aggregate output gap are determined via identities as weighted averages of the corresponding individual country variables. Finally,

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the area-wide short-term nominal interest rate is the instrument of monetary policy.

As the model allows for simultaneous cross-country linkages, it was estimated with the three-stage least-squares (3SLS) method using data on inflation, output, and nominal interest rates. Not all cross-country terms are significant in all equations. This results in a clear causal pattern, with the German economy affecting the other two comparatively more, and with the Italian economy being essentially recursive. Furthermore, the estimation results indicate that there exists a significant degree of heterogeneity among the three economies included in the model.⁹ Accordingly, the results in Angelini et al. (2002) suggest that monetary policy effectiveness may be considerably enhanced if country-specific information is used.

1.2 Differences in Monetary Policy Transmission

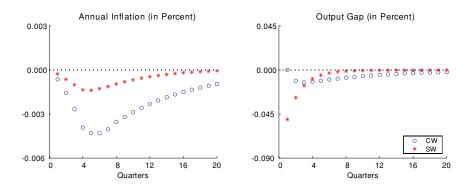
Figure 1 provides a comparison of the effects of an unexpected onequarter tightening of monetary policy by 100 basis points in the four different models of the euro area, with the monetary policymaker following the interest-rate rule proposed by Taylor (1993) thereafter. Panel A of the figure depicts the dynamic responses of annual inflation and the output gap for the largely forward-looking models (CW and SW), while panel B shows the responses for the predominantly backward-looking models (AWM and DM).

Qualitatively, the tightening of policy has the same consequences in the four models. As the short-term nominal interest rate rises unexpectedly, demand falls short of potential and inflation falls below the monetary policymaker's target, with the dynamic adjustment being drawn out lastingly. Quantitatively, however, the responses exhibit some noticeable differences. Most importantly, the disinflation effect is considerably larger and more persistent in the backwardlooking models with the timing of the peak effect on inflation noticeably delayed relative to that on the output gap. By contrast, in the

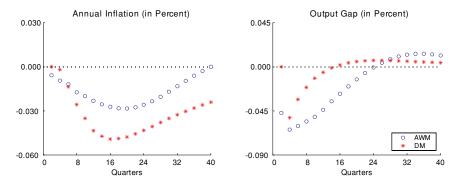
 $^{^{9}}$ Monteforte and Siviero (2002) also report estimates of an aggregate companion of the DM (referred to as Aggregate Model, AM, of the euro area), consisting of just two equations. Both statistical and economic criteria indicate that the DM is to be preferred to the AM.

Figure 1. Responses to an Interest-Rate Shock (100 Basis Points) under Taylor's Rule

A. The Forward-Looking Euro Area Models



B. The Backward-Looking Euro Area Models



forward-looking models, the timing of the peak effect on inflation is much closer to that on the output gap, with the reversion to base of both inflation and the output gap being more rapid and smoother. Differences in the responses of the output gap, notably in the initial periods, largely reflect differences in the employed output-gap concepts. Overall, this suggests that the degree of forward-lookingness is of utmost relevance for explaining the differences in inflation and output-gap dynamics across models.

Based on the documented patterns of the dynamic responses to the monetary policy shock under Taylor's rule, we summarize that a given interest-rate rule may perform quite differently in terms of inflation and output-gap stabilization, depending on the characteristics of the particular euro area model used. Hence, it is evident why monetary policymakers should be concerned about the design of the interest-rate rule to be used in model-based analyses.

2. Evaluating the Performance of Interest-Rate Rules

We now proceed to describe the methodology that we will use to evaluate the stabilization performance of alternative monetary policies across our set of models of the euro area economy. Our starting point is an evaluation of simple interest-rate rules that respond to outcomes or forecasts of annual inflation and the output gap and allow for inertia due to dependence on the lagged short-term nominal interest rate.

2.1 The Methodology

Following the approach in Levin, Wieland, and Williams (2003), we consider a three-parameter family of simple interest-rate rules,

$$i_{t} = \rho i_{t-1} + (1-\rho) \left(r^{*} + \mathbf{E}_{t} \left[\tilde{\pi}_{t+\theta} \right] \right) + \alpha \mathbf{E}_{t} \left[\tilde{\pi}_{t+\theta} - \pi^{*} \right] + \beta \mathbf{E}_{t} \left[y_{t+\kappa} \right],$$

where i_t denotes the short-term nominal interest rate, r^* is the equilibrium real interest rate, $\tilde{\pi}_t = p_t - p_{t-4}$ is the annual inflation rate, π^* denotes the monetary policymaker's inflation target, and y_t is the output gap.¹⁰ Under rational expectations, the operator $E_t[\cdot]$ indicates the model-consistent forecast of a particular variable, using information available in period t.¹¹ The integer parameters θ and κ denote the length of the forecast horizons for inflation and the output gap, respectively. This specification accommodates forecastbased rules (with forecast horizons $\theta, \kappa > 0$) as well as outcome-based

 $^{^{10}\}mathrm{Even}$ in the case where the multicountry model DM is used, both inflation and the output gap are to be interpreted as area-wide variables, reflecting the assumption that the policymaker is concerned with area-wide developments.

 $^{^{11}}$ We employ the AIM algorithm of Anderson and Moore (1985), which uses the Blanchard and Kahn (1980) method for solving linear rational expectations models, to compute model-consistent expectations.

rules ($\theta = \kappa = 0$) and simplifies to the one proposed by Taylor (1993) if $\theta = \kappa = 0$, $\rho = 0$ and $\alpha = \beta = 0.5$.¹²

For fixed inflation and output-gap forecast horizons θ and κ , the above family of interest-rate rules is defined by the response coefficients ρ , α , and β . The coefficients α and β represent the policymaker's short-term reaction to inflation in deviation from target and the output gap, respectively, while ρ determines the inertia of the interest-rate response, commonly interpreted as the desired degree of policy "smoothing." The latter plays an important role for modelbased evaluations of interest-rate rules. In particular, the degree to which the optimal policy in any given model embodies smoothing tends to depend largely on the expectation formation mechanism embedded in that model. Typically, if expectations are backward looking, values of ρ at or above unity can perform poorly since they may engender undampened oscillations in the model economy due to instrument instability (see, e.g., Rudebusch and Svensson 1999 and Batini and Nelson 2001). By contrast, models with largely forwardlooking expectations, notably small optimizing New Keynesian models, tend to favor a comparatively high degree of smoothing because the inertial adjustment of the short-term interest rate enables the policymaker to steer expectations and thereby to stabilize the economy more effectively.

In our evaluation of the stabilization performance of variants of the above family of interest-rate rules, we assume that the monetary policymaker has a standard loss function equal to the weighted sum of the unconditional variances of inflation, the output gap, and changes in the short-term nominal interest rate,

$$\mathcal{L} = \operatorname{Var}[\pi_t] + \lambda \operatorname{Var}[y_t] + \mu \operatorname{Var}[\Delta i_t].$$

Here, inflation is measured by the annualized one-quarter inflation rate, $\pi_t = 4 (p_t - p_{t-1})$. The weight $\lambda \ge 0$ refers to the policymaker's preference for reducing output variability relative to inflation variability, and the weight $\mu \ge 0$ on the variability of changes in the

¹²In the special case with $\rho = 1$, the rule represents a first-difference rule, a class of rules that Levin, Wieland, and Williams (2003) advocate as being robust when examining the performance of simple interest-rate rules across a set of distinct models of the U.S. economy—none of them, however, being fully backward-looking. Orphanides and Williams (2002) emphasize that first-difference rules are also robust to misperceptions about the equilibrium real interest rate r^* .

short-term nominal interest rate, $\Delta i_t = i_t - i_{t-1}$, reflects a desire to avoid undue fluctuations in the nominal interest rate itself.¹³ Establishing this loss function is consistent with the assumption that the policymaker aims at stabilizing inflation around the inflation target π^* and actual output around potential, with the concern regarding excessive interest-rate variability typically justified by financial stability considerations.¹⁴

For fixed inflation and output-gap forecast horizons θ and κ , the three-parameter family of interest-rate rules defined above is optimized by minimizing the policymaker's loss function \mathcal{L} with respect to the coefficients ρ , α , and β . In this context, we repeatedly need to compute the unconditional variances of the models' endogenous variables for a given interest-rate rule. In preparation for these computations, we first identify the series of historical shocks that would be consistent with the alternative models under rational expectations. Based on the covariance matrix of the historical shocks, it is then possible to calculate the unconditional covariance matrix of the endogenous variables for any given interest-rate rule by applying standard methods to the reduced-form solution of the model including that rule.¹⁵

In the subsequent analysis, we will consider four alternative values for the relative weight on output-gap variability, namely $\lambda = 0, 1/3, 1, \text{ and } 3$. Regarding the weight on the variability of interest-rate changes, we will concentrate the analysis on a fixed value of $\mu = 0.10$. We shall briefly discuss additional results for a lower weight of $\mu = 0.01$ in the sensitivity analysis in section 5. There, we will also report on results for interest-rate rules that only

¹³For an explicit derivation of the policymaker's loss function \mathcal{L} from quadratic intertemporal preferences, the reader is referred to Rudebusch and Svensson (1999). In Svensson's terminology, the case of $\lambda = \mu = 0$ corresponds to "strict" inflation targeting, while "flexible" inflation targeting is characterized by $\lambda, \mu > 0$ (see Svensson 1999).

¹⁴It is recognized that it would be beneficial to use a welfare criterion derived as an approximation of the representative agent's utility function (see, e.g., Rotemberg and Woodford 1997). The weights in this approximate welfare criterion would be functions of the parameters of the structural model itself. However, to the extent that the models used in this paper, with the exception of the SW model, are lacking microfoundations, a well-defined welfare criterion does not exist.

¹⁵The exception is the SW model, for which the covariance matrix of shocks is estimated jointly with the model's structural parameters.

allow for a response to lagged inflation and the lagged output gap; that is, with $\theta = \kappa = -1$. Such rules have been proposed as a proxy for actual policymaking in "real time" (see McCallum 1988).

2.2 The Performance of Optimized Benchmark Rules

As benchmarks for evaluating the performance of simple interestrate rules across our set of models, we focus on two types of rules: outcome-based rules with $\theta = \kappa = 0$, and forecast-based rules that relate the interest rate to the one-year-ahead forecast of inflation and the contemporaneous output gap with $\theta = 4$ and $\kappa = 0$.

Table 1 reports the optimized response coefficients of the two types of rules for the four different models of the euro area economy and for alternative values of the preference parameter λ , together with an indication of their stabilization performance. The latter is measured by the value of the policymaker's loss function yielded under the rule optimized for a particular model \mathcal{L} and alternatively, in relative terms, as the percentage-point difference of the loss under the optimized rule from the loss under the fully optimal policy for that model, $\%\Delta\mathcal{L}$.¹⁶ Panel A in table 1 indicates the results for the largely forward-looking models (CW and SW), while panel B shows the results for the predominantly backward-looking models (AWM and DM).

We start with the results for the forward-looking models in panel A and observe that, regardless of the policymaker's preference for output stabilization, the optimized rules are characterized by a substantial degree of interest-rate smoothing, as captured by the high coefficient on the lagged interest rate, ρ . Interestingly, with a low weight on output stabilization and, in particular, with the inflation forecast horizon extending one year into the future, the magnitude of ρ tends to exceed unity, notably for the SW model. This feature is commonly referred to as "super-inertia" in the interest rate. Not surprisingly, as the weight on output stabilization increases, the coefficient on the output gap, β , rises while the coefficient on inflation,

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¹⁶For the CW and SW models, as well as the AWM, the fully optimal policy corresponds to the optimal policy under commitment. See Finan and Tetlow (1999) for details on computing the optimal policy under commitment for large rational expectations models using AIM. Details regarding the unconditional variability of the individual target variables underlying the calculation of the loss function, \mathcal{L} , can be found in table A of the appendix.

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|---|----------|----------|-----------|--------|----------|---------|---------------|------------------------|------|----------|---------|---------------|------------------------|
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | | | | | | CW | | | | | SW | | |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | θ | κ | λ | ρ | α | β | \mathcal{L} | $\%\Delta \mathcal{L}$ | ρ | α | β | \mathcal{L} | $\%\Delta \mathcal{L}$ |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | 0 | 0 | 0 | 0.97 | 0.81 | 0.10 | 1.8 | 2 | 1.03 | 0.81 | 0.08 | 1.0 | 3 |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | | | 1/3 | 0.81 | 0.49 | 0.86 | 2.5 | 5 | 1.00 | 0.64 | 1.53 | 1.3 | 8 |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | | | 1 | 0.79 | 0.30 | 1.55 | 3.2 | 4 | 0.99 | 0.43 | 3.05 | 1.5 | 6 |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | | | 3 | 0.77 | 0.23 | 2.50 | 4.7 | 3 | 0.92 | 0.11 | 5.87 | 1.7 | 5 |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | 4 | 0 | 0 | 1.59 | 4.36 | -0.25 | 1.9 | 5 | 1.96 | 6.59 | -0.05 | 1.0 | 6 |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | | | 1/3 | 0.83 | 1.18 | 0.84 | 2.6 | 6 | 1.01 | 1.19 | 1.50 | 1.3 | 9 |
| B. The Backward-Looking Euro Area Models $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | | | 1 | 0.79 | 0.76 | 1.55 | 3.2 | 4 | 1.00 | 0.79 | 2.99 | 1.5 | 7 |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | | | 3 | 0.77 | 0.59 | 2.49 | 4.6 | 3 | 0.99 | 0.53 | 5.64 | 1.7 | 5 |
| $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$ | В. | The | Backw | ard-Lo | oking | Euro Ar | ea M | odels | | | | | |
| $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$ | | | | | | AWM | | | | | DM | | |
| 1/3 0.39 0.93 2.37 1.9 20 0.28 5.78 3.89 6 1 0.44 0.65 3.69 2.6 18 0.25 5.71 4.26 8 3 0.50 0.40 5.79 3.8 15 0.19 5.59 5.01 12 4 0 0.37 2.47 1.67 1.8 13 0.51 4.06 0.74 2.22 | θ | κ | λ | ρ | α | β | \mathcal{L} | $\%\Delta \mathcal{L}$ | ρ | α | β | \mathcal{L} | $\%\Delta \mathcal{L}$ |
| $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$ | 0 | 0 | 0 | 0.44 | 1.14 | 1.13 | 1.3 | 26 | 0.31 | 5.81 | 3.65 | 6.0 | 54 |
| 3 0.50 0.40 5.79 3.8 15 0.19 5.59 5.01 12 4 0 0.47 2.22 0.55 1.2 15 0.55 3.99 0.56 4 1/3 0.37 2.47 1.67 1.8 13 0.51 4.06 0.74 4 | | | 1/3 | 0.39 | 0.93 | 2.37 | 1.9 | 20 | 0.28 | 5.78 | 3.89 | 6.9 | 53 |
| 4 0 0 0.47 2.22 0.55 1.2 15 0.55 3.99 0.56 1/3 0.37 2.47 1.67 1.8 13 0.51 4.06 0.74 | | | 1 | 0.44 | 0.65 | 3.69 | 2.6 | 18 | 0.25 | 5.71 | 4.26 | 8.5 | 51 |
| 1/3 0.37 2.47 1.67 1.8 13 0.51 4.06 0.74 | | | 3 | 0.50 | 0.40 | 5.79 | 3.8 | 15 | 0.19 | 5.59 | 5.01 | 12.5 | 47 |
| | 4 | 0 | 0 | 0.47 | 2.22 | 0.55 | 1.2 | 15 | 0.55 | 3.99 | 0.56 | 4.6 | 20 |
| $1 0.39 2.49 3.00 2.5 12 \qquad 0.46 4.18 1.05 0.46 1.05 $ | | | 1/3 | 0.37 | 2.47 | 1.67 | 1.8 | 13 | 0.51 | 4.06 | 0.74 | 5.4 | 21 |
| | | | 1 | 0.39 | 2.49 | 3.00 | 2.5 | 12 | 0.46 | 4.18 | 1.05 | 6.9 | 23 |
| 3 0.42 2.38 5.12 3.7 11 0.36 4.39 1.75 10 | | | 3 | 0.42 | 2.38 | 5.12 | 3.7 | 11 | 0.36 | 4.39 | 1.75 | 10.6 | 24 |

 Table 1. The Stabilization Performance of Optimized

 Interest-Rate Rules

Note: For each choice of the inflation and output-gap forecast horizons (θ and κ), for each preference parameter (λ) and for each model, this table indicates the optimized interest-rate response coefficients (ρ , α , and β), the value of the policymaker's loss function (\mathcal{L}), and the percentage-point difference of the latter from the loss under the fully optimal policy ($\%\Delta\mathcal{L}$).

 α , falls. As regards the stabilization performance of the optimized interest-rate rules, we observe that the implied values of the policymaker's loss function are typically somewhat larger in the CW model, possibly reflecting a less dominant role for the expectations channel in transmitting monetary policy to aggregate demand. At the same time, for both models the performance of monetary policy deteriorates fairly little when the policymaker follows a relatively simple optimized interest-rate rule rather than the fully optimal policy. The value of the policymaker's loss function never rises by more than 9 percentage points. We also observe that there is no discernible stabilization gain from following forecast-based as opposed to outcome-based rules.

Turning to the backward-looking models in panel B, a number of differences are noteworthy. First, the optimized interest-rate rules embody only a mild degree of smoothing, with ρ varying between 0.4 and 0.5 for the AWM and between 0.2 and 0.6 for the DM. This discrepancy largely reflects the fact that expectations, notably those determining long-term interest rates, do not play a dominant role in either model. Thus, there is no scope for increasing the effectiveness of monetary policy by an inertial adjustment of the short-term rate. Second, the optimized response coefficients α and β are typically quite a bit larger than for the forward-looking models, in particular for the DM. This mirrors the fact that, in backward-looking models, inflation and the output gap are much harder to stabilize and, as a result, the policymaker has to respond more aggressively to any signs of rising inflation and cumulating output gaps. Third, when compared with the outcomes under the fully optimal policies, the stabilization performance of simple interest-rate rules deteriorates quite substantially, albeit less strongly for the AWM. In the extreme case when the policymaker does not attach any weight to output stabilization, the loss under the optimized outcome-based rule exceeds the loss associated with the fully optimal policy by more than 50 percentage points for the DM, while the loss differs by about 25 percentage points for the AWM.¹⁷ In the case of the AWM, the relatively poor performance of simple optimized rules is likely due to the model's fairly high degree of structural detail and its rather complex dynamics (see Finan and Tetlow [1999] for a similar observation based on the Federal Reserve Board's FRB/US model).¹⁸ Similarly,

 $^{^{17}}$ Evidently, while the values of the loss function obtained for the AWM are comparable with those yielded in the forward-looking models, the losses for the DM are quite a bit larger.

¹⁸Giannoni and Woodford (2002) show that the fully optimal policy for a model, regardless of its degree of structural detail, can be represented as a generalized Taylor rule, in which the relation between the interest-rate instrument and the other target variables includes leads and lags of the target variables. Therefore, the relatively poor performance of the simple interest-rate rules examined here ought to be mitigated by the inclusion of additional leads and lags of the

the DM departs from the rest of the models to the extent that it is the only model in which the three largest euro area economies are modeled separately. Fourth, contrary to the results for the forwardlooking models, there are substantial gains in performance from using forecast-based rather than outcome-based rules, with the maximum gain equal to 11 percentage points for the AWM and 34 percentage points for the DM (in the case of $\lambda = 0$). We attribute these improvements in performance to the information-encompassing nature of forecast-based rules, with the inflation forecast incorporating information about incipient risks to future inflation arising from a possibly larger set of underlying determinants (see Batini and Haldane 1999) and the transmission lag of monetary policy (see Batini and Nelson 2001).

3. Evaluating the Robustness of Interest-Rate Rules

In the previous section, we implicitly assumed that the policymaker knows the "true" model of the euro area economy when optimizing variants of the three-parameter family of interest-rate rules for any given model. While the optimized rules typically succeeded in stabilizing inflation and output satisfactorily for that given model, we now proceed to analyze to what extent the optimized rules are robust to model uncertainty, in the sense of also performing reasonably well across the other, possibly competing models of the euro area.

3.1 The Performance of Optimized Rules Across Models

Table 2 summarizes our findings regarding the stabilization performance of the optimized benchmark rules documented in table 1 above across our set of models. Here, performance is measured by the value of the policymaker's loss function, \mathcal{L} , and by the percentage-point difference of the latter from the loss under the fully optimal policy, $\%\Delta\mathcal{L}$, when the rule optimized for any particular model m is evaluated in the possibly competing model $n \neq m$. Panel A in table 2 reports the findings for the rules optimized for

inflation rate, the output gap, and the interest rate. In fact, Williams (2003) shows for the FRB/US model that moderately more complicated Taylor-type interest-rate rules that respond to a slightly enhanced number of lags of the target variables yield noticeable, albeit fairly small, stabilization gains over optimized three-parameter rules.

 Table 2. The Robustness of Optimized Interest-Rate Rules in Models of the Euro Area

| | | | R | ules Op | timize | d for | CW | | R | ules Op | timize | d for | SW | |
|---|----------|-----|-----|---------------|----------|-------|------------------------|----------|-----|---------------|----------|-------|------------------------|----------|
| | | | | \mathcal{L} | | | $\%\Delta \mathcal{L}$ | | | \mathcal{L} | | | $\%\Delta \mathcal{L}$ | |
| θ | κ | λ | SW | AWM | DM | SW | AWM | DM | CW | AWM | DM | CW | AWM | DM |
| 0 | 0 | 0 | 1.0 | ∞ | ∞ | 6 | ∞ | ∞ | 1.9 | ∞ | ∞ | 3 | ∞ | ∞ |
| | | 1/3 | 1.4 | 2.5 | 35.1 | 15 | 56 | 684 | 2.7 | 4.8 | 23.2 | 13 | 202 | 419 |
| | | 1 | 1.6 | 3.4 | 465.6 | 18 | 54 | 8228 | 3.7 | 4.6 | 59.8 | 23 | 110 | 969 |
| | | 3 | 2.0 | 5.2 | ∞ | 25 | 58 | ∞ | 6.2 | 6.1 | ∞ | 38 | 85 | ∞ |
| 4 | 0 | 0 | 1.0 | ∞ | ∞ | 9 | ∞ | ∞ | 1.9 | ∞ | ∞ | 6 | ∞ | ∞ |
| | | 1/3 | 1.4 | 2.4 | 11.4 | 15 | 52 | 155 | 2.8 | 5.2 | ∞ | 14 | 232 | ∞ |
| | | 1 | 1.6 | 3.2 | 23.3 | 17 | 45 | 318 | 3.7 | 4.9 | 67.4 | 23 | 122 | 1106 |
| | | 3 | 2.0 | 5.0 | 73.3 | 25 | 51 | 763 | 6.4 | 5.7 | ∞ | 43 | 72 | ∞ |

A. Rules Optimized for the Forward-Looking Euro Area Models

B. Rules Optimized for the Backward-Looking Euro Area Models

| | | | Rul | es Op | timized | for A | WM | | R | ules O | ptimize | d for I | DМ | |
|----------|----------|-----------|-----|---------------|----------|-------|------------------------|----------|-----|---------------|---------|---------|--------------|-----|
| | | | | \mathcal{L} | | | $\%\Delta \mathcal{L}$ | | | \mathcal{L} | | | $\%\Delta t$ | c . |
| θ | κ | λ | CW | SW | DM | CW | SW | DM | CW | SW | AWM | CW | SW | AWM |
| 0 | 0 | 0 | 2.2 | 1.3 | 48.6 | 20 | 36 | 1157 | 3.4 | 1.4 | 2.0 | 87 | 56 | 94 |
| | | 1/3 | 2.7 | 1.5 | 222.2 | 11 | 24 | 4859 | 3.8 | 1.7 | 2.3 | 58 | 39 | 43 |
| | | 1 | 3.5 | 1.7 | ∞ | 14 | 23 | ∞ | 4.7 | 2.2 | 2.7 | 53 | 55 | 24 |
| | | 3 | ME | 2.0 | ∞ | ME | 23 | ∞ | 6.8 | 3.2 | 3.9 | 52 | 97 | 17 |
| 4 | 0 | 0 | 2.3 | 1.3 | 11.8 | 25 | 40 | 205 | 2.5 | 1.3 | 1.3 | 40 | 37 | 27 |
| | | 1/3 | 2.7 | 1.6 | 15.2 | 13 | 30 | 238 | 2.8 | 1.7 | 2.1 | 16 | 35 | 30 |
| | | 1 | 3.4 | 1.8 | 20.2 | 13 | 31 | 261 | 3.7 | 2.3 | 3.1 | 20 | 64 | 41 |
| | | 3 | 5.2 | 2.1 | 34.3 | 16 | 31 | 303 | 5.8 | 3.6 | 5.2 | 29 | 120 | 56 |

Note: For each choice of the inflation and output-gap forecast horizons (θ and κ), for each preference parameter (λ) and for each model, this table indicates the value of the policymaker's loss function (\mathcal{L}) and the percentage-point difference of the latter from the loss under the fully optimal policy ($\%\Delta\mathcal{L}$), when the rule optimized for model m is evaluated in model $n \neq m$. The notation "ME" indicates that the implemented rule yields multiple equilibria; the notation " ∞ " indicates that the implemented rule results in instability.

the forward-looking models, while panel B documents those for the backward-looking ones.

Beginning with the rules optimized for the forward-looking models in panel A, we observe that rules that are optimized for the CW model also perform satisfactorily when evaluated in the SW model,

and vice versa. For example, with $\theta = \kappa = 0$ and $\lambda = 1/3$, implementing the CW-based rule in the SW model leads to an increase in the loss function of about 15 percentage points, relative to the loss associated with the fully optimal policy for the SW model. When compared to the performance of the interest-rate rule optimized for the SW model itself (see table 1), the loss increases by about 7 percentage points. Similarly, when implementing the SW-based rule in the CW model, the increase in the loss function amounts to 13 and 8 percentage points, depending on the benchmark for comparison. By contrast, when evaluating the CW- and SW-based rules in the backward-looking models, the performance of monetary policy tends to deteriorate quite substantially. For the AWM, the loss increases by about 50 percentage points relative to the fully optimal policy when CW-based rules are implemented and by more than 100 percentage points on average for SW-based rules. The deterioration is found to be particularly dramatic if the policymaker puts zero weight on output stabilization since the CW- and SW-based rules do not succeed in stabilizing the AWM any longer. Finally, when evaluated in the DM, the performance of CW- and SW-based rules is even worse, generating even higher increases in relative losses and resulting in instability more often.

Turning to the rules optimized for the backward-looking models in panel B, we observe that the AWM-based rules typically result in reasonable stabilization outcomes when evaluated in the CW or the SW model. The exception is the outcome-based rule for $\lambda = 3$, which yields multiple equilibria when implemented in the CW model. By contrast, the AWM-based rules do not perform satisfactorily in the DM and may occasionally even generate instability. Finally, the DM-based rules result in a substantial deterioration in the performance of monetary policy when evaluated in any of the three other models, although the deterioration exceeds 100 percentage points only in exceptional cases. Interestingly, for forecast-based rules, the deterioration is found to be somewhat more benign.

Based on these results, we conclude that simple interest-rate rules that are designed for the predominantly backward-looking euro area models tend to perform reasonably well in the largely forwardlooking models, while the reverse is not necessarily true.¹⁹ The

¹⁹It is interesting to note that the aggregate companion of the DM (AM; see footnote 9) demands policies that are closer to those obtained for the AWM (the

fault-tolerance analysis undertaken in the next section will cast some further light on the reasons underlying these findings.

3.2 A Fault-Tolerance Analysis of Optimized Rules

Fault-tolerance analysis of optimized interest-rate rules, as proposed by Levin and Williams (2003), is deemed to provide useful insights into the reasons that underlie our earlier findings. Fault-tolerance analysis is a concept borrowed from engineering and involves, in the present context, appraising the increase in the loss function that results when a single parameter of an optimized interest-rate rule is varied, holding the other parameters constant at their optimized values. A highly *fault tolerant* model is one for which the parameters of the rule may vary over a relatively broad range of values without resulting in a large deterioration of its performance. By contrast, an intolerant model would be a model whose performance deteriorates dramatically as soon as one deviates even modestly from some optimized parameter value (i.e., the loss function exhibits strong curvature with respect to suboptimal variations in some parameter). Clearly, if one is dealing with a set of relatively tolerant models, there is a fair chance to find a robust policy rule. If all models are intolerant, then a robust rule may not exist.

Figure 2 depicts the fault tolerances of our four models for the case $\theta = \kappa = 0$ and $\lambda = 1/3$ (i.e., for the outcome-based rules obtained with a moderate weight on output stabilization).²⁰ Each curve shows the percentage-point change in the policymaker's loss function under the optimized rule as a single parameter is varied, with its minimum of zero attained at the optimized value itself. As can be seen in the figure, while any single parameter may be varied over a relatively broad range of values without deteriorating dramatically the performance of the individual model concerned, there are no obvious overlapping regions of high *mutual* fault tolerance for all four models under examination and for all three policy-rule parameters at the same time.

other predominantly backward-looking model), suggesting that the choice of the degree of aggregation is also a relevant factor in shaping the results.

 $^{^{20} \}mathrm{For}$ other values of the preference parameter $\lambda,$ the results are qualitatively similar.

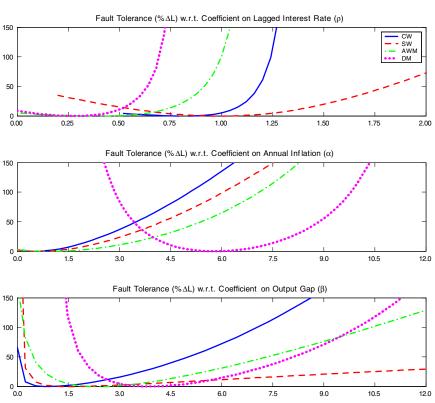


Figure 2. Fault-Tolerance Analysis of Outcome-Based Interest-Rate Rules

Note: For $\lambda = 1/3$ and for each model, the figure indicates the percentage-point change in the policymaker's loss function ($\%\Delta\mathcal{L}$) under the optimized outcomebased interest-rate rule ($\theta = \kappa = 0$) as a single parameter (ρ , α , or β) of the optimized rule is varied, holding the other two parameters fixed at their respective optimized values.

Regarding the fault tolerances with respect to the smoothing coefficient ρ (displayed in the upper panel of the figure), the two forward-looking models perform best when ρ is close to unity, while the performance for both the AWM and DM deteriorates quite markedly in this region, eventually resulting in instrument instability for the DM. In contrast, the AWM prefers a ρ coefficient of somewhat below 0.5 and the DM an even lower coefficient of about 0.25. Values

of ρ in this region, however, tend to yield indeterminate equilibria for the forward-looking models, notably for the CW model. With respect to the response coefficient α (depicted in the middle panel), the forward-looking models and the AWM seem mutually tolerant to variations in α in the region of close to 0 to 2.5. The DM, however, behaves very differently in its optimal prescription for α , demanding a significantly higher value of about 6, but with the curvature of the loss function in that region being modest. By contrast, as shown in the lower panel, a comfortable region of relatively high mutual tolerance seems to exist with respect to variations in the response coefficient β in the range of 2 to 4. While the DM favors a strong response to the output gap in the range of 2 to 7.5, the three other models perform satisfactorily as a group for coefficients in the range of 0.5 to 4, with the CW model constraining the upper bound of this range.

Interestingly, as shown in figure 3, the four models exhibit considerably larger regions of mutual fault tolerance when the policymaker follows a forecast-based rule with $\theta = 4$ and $\kappa = 0$. In this case, variations in ρ in the range of 0.4 to 0.7 result in a reasonable performance of all four models. Similarly, there are regions of mutual tolerance with respect to variations in α and β , the regions being centered at 3.5 and 1, respectively.

4. Designing Robust Interest-Rate Rules

The fault-tolerance analysis in the previous section has provided an indication under which circumstances a robust interest-rate rule might exist for our set of euro area models. However, to the extent that fault-tolerance analysis rests on suboptimal variations in a single policy-rule parameter, holding fixed the other parameters at their optimized values, we finally proceed to use a more formal approach that allows taking into account the interaction among all policyrule parameters to identify the operating characteristics of interestrate rules that are likely to yield satisfactory outcomes across our models.

In search of such robust rules we follow the Bayesian approach outlined in Levin, Wieland, and Williams (1999, 2003) and optimize the response coefficients of the three-parameter family of interest-rate rules across our four models simultaneously by

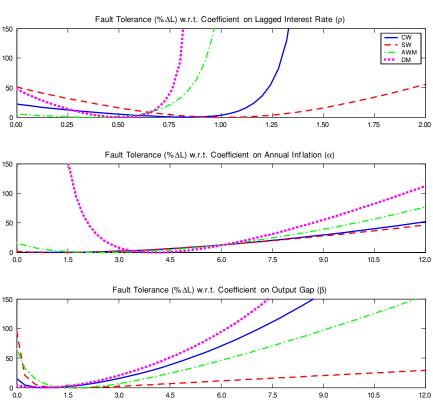


Figure 3. Fault-Tolerance Analysis of Forecast-Based Interest-Rate Rules

Note: For $\lambda = 1/3$ and for each model, the figure indicates the percentage-point change in the policymaker's loss function ($\%\Delta\mathcal{L}$) under the optimized forecast-based interest-rate rule ($\theta = 4$, $\kappa = 0$) as a single parameter (ρ , α , or β) of the optimized rule is varied, holding the other two parameters fixed at their respective optimized values.

minimizing a weighted average of the loss functions associated with the individual models,

$$\bar{\mathcal{L}} = \sum_{m \in \mathcal{M}} \, \omega_m \, \mathcal{L}_m,$$

where ω_m denotes the weight attached to any given model $m \in \mathcal{M} \subseteq \{\text{CW, SW, AWM, DM}\}$ with $\omega_m > 0$ and $\sum \omega_m = 1$. For $\omega_m = 1/|\mathcal{M}|$, the average loss function corresponds to the expected

| | | | | | | Ĺ | \mathcal{L}_m ger | nerated i | n | $\%\Delta$ | \mathcal{L}_m ge | enerated | in |
|----|----------|--------|--------|---------|---------|---------|---------------------|-----------|------|------------|--------------------|----------|----|
| θ | κ | λ | ρ | α | β | CW | SW | AWM | DM | CW | SW | AWM | DM |
| Α. | Opti | imizat | ion Ac | ross Al | ll Mode | els | | | | | | | |
| 0 | 0 | 0 | 0.54 | 3.38 | 2.28 | 2.6 | 1.3 | 1.6 | 6.4 | 46 | 38 | 58 | 64 |
| | | 1/3 | 0.58 | 3.22 | 2.71 | 3.1 | 1.5 | 2.3 | 7.2 | 28 | 22 | 46 | 60 |
| | | 1 | 0.58 | 3.21 | 3.33 | 3.9 | 1.8 | 3.2 | 8.8 | 28 | 30 | 45 | 58 |
| | | 3 | 0.54 | 3.33 | 4.48 | 5.8 | 2.4 | 4.8 | 13.2 | 29 | 50 | 47 | 56 |
| 4 | 0 | 0 | 0.60 | 3.28 | 0.49 | 2.3 | 1.2 | 1.3 | 4.7 | 26 | 33 | 25 | 22 |
| | | 1/3 | 0.62 | 3.27 | 1.16 | 2.7 | 1.5 | 2.0 | 5.7 | 11 | 22 | 25 | 28 |
| | | 1 | 0.60 | 3.46 | 1.90 | 3.4 | 1.8 | 2.7 | 7.5 | 10 | 29 | 25 | 34 |
| | | 3 | 0.54 | 3.76 | 3.16 | 5.0 | 2.3 | 4.1 | 12.1 | 11 | 42 | 24 | 43 |
| В. | Opti | imizat | ion Ac | ross Al | l Aggr | egate I | Models | 3 | | | | | |
| 0 | 0 | 0 | 0.71 | 0.76 | 0.55 | 2.1 | 1.1 | 1.4 | | 14 | 24 | 33 | |
| | | 1/3 | 0.63 | 0.64 | 1.72 | 2.6 | 1.4 | 2.0 | | 8 | 17 | 24 | |
| | | 1 | 0.64 | 0.42 | 2.78 | 3.3 | 1.6 | 2.7 | | 8 | 17 | 22 | |
| | | 3 | 0.64 | 0.37 | 4.37 | 5.0 | 1.9 | 4.0 | | 10 | 18 | 20 | |
| 4 | 0 | 0 | 0.72 | 1.26 | 0.39 | 2.2 | 1.2 | 1.2 | | 20 | 29 | 20 | |
| | | 1/3 | 0.64 | 1.40 | 1.46 | 2.6 | 1.5 | 1.9 | | 8 | 19 | 17 | |
| | | 1 | 0.62 | 1.38 | 2.56 | 3.3 | 1.7 | 2.6 | | 8 | 19 | 17 | |
| | | 3 | 0.62 | 1.42 | 4.17 | 4.9 | 2.0 | 3.8 | | 9 | 20 | 16 | |

 Table 3. The Stabilization Performance of Bayesian Robust

 Interest-Rate Rules

Note: For each choice of the inflation and output-gap forecast horizons (θ and κ) and for each preference parameter (λ), this table indicates the jointly optimized interest-rate response coefficients (ρ , α , and β), the contribution of the individual model m to the policymaker's loss function (\mathcal{L}_m), and the percentage-point difference of this contribution from the loss under the fully optimal policy ($(\Delta \mathcal{L}_m)$).

loss function when the policymaker has uniform prior beliefs as to which model in \mathcal{M} is a plausible representation of the euro area economy.

Panel A of table 3 reports the response coefficients of outcomeand forecast-based rules that are optimized across all four models simultaneously (assuming uniform prior beliefs) and indicates the

performance of these rules yielded in the individual models. Here, performance is measured as the contributions of the individual models to the value of the policymaker's overall loss function, \mathcal{L}_m , and as the percentage-point difference of these contributions from the losses under the fully optimal policies, $\%\Delta \mathcal{L}_m$.

Starting with the outcome-based rules with $\theta = \kappa = 0$, it turns out that the jointly optimized (that is, *Bayesian robust*) policies are heavily weighted toward those demanded by the DM, with the optimized interest-rate response coefficients close to those implied by the DM alone (see table 1). Specifically, the Bayesian robust policies prescribe a degree of interest-rate smoothing in the range of 0.5 to 0.6, while the responses to inflation and the output gap are rather aggressive. The response to inflation is relatively stable at around 3, while the response to the output gap varies quite a bit, namely in the range of 2 to 4, depending on the weight given to output stabilization. In light of the fault-tolerance analysis reported above, this outcome is not really surprising, since, in the absence of regions of high mutual tolerance with respect to some parameter, the least tolerant model is supposed to be most influential in shaping the operating characteristics of the robust policy with respect to that parameter.²¹ Nevertheless, the robust policies perform reasonably well in all four models, notably if a sensible weight is given to output-gap stabilization, as can be seen when comparing the outcomes under the robust policies with the performance measures reported in table 1. In fact, the increase in the reported losses never exceeds 50 percentage points, even if the sole policy objective is to stabilize inflation (i.e., for $\lambda = 0$), with the deterioration in performance obviously smallest for the DM due to its influential role in shaping the operating characteristics of the robust policies. This finding is broadly consistent with the results documented in Levin and Williams (2003) for a set of models of the U.S. economy including the backward-looking model of Rudebusch and Svensson (1999). Also in this study, the contours of the robust policies are found to be

²¹In fact, the DM is rather intolerant to nontrivial deviations from the optimized coefficients implied by the DM alone, as can be seen in figure 2. Furthermore, the decisive role of the DM in our Bayesian analysis, after all, relates to the fact that the baseline level of the loss for the DM is relatively high when compared with the baseline losses for the other models.

heavily weighted towards those demanded by the backward-looking model.

Turning to the forecast-based rules with $\theta = 4$ and $\kappa = 0$, the performance of Bayesian robust policies is found to be even more satisfactory across models. Yet again, this finding is not surprising in light of the fault-tolerance analysis above indicating the existence of regions of relatively high mutual tolerance for all three parameters. However, the optimized parameters of the robust rules appear to be largely influenced by the DM again.

To the extent that the robust policies for the full set of models are heavily weighted toward the policies demanded by the DM, panel B of table 3 also reports results obtained when optimizing across all aggregate models, but excluding the DM. In this case, the robust policies are characterized by a uniformly higher degree of interest-rate smoothing (in a range of 0.6 to 0.7) and, overall, by less aggressive responses to inflation and the output gap. As expected, the performance of the robust policies designed for the two forward-looking models (CW and SW) and the AWM alone appears more favorable across these models when compared to the performance under the robust rules designed for the full set of models.

5. Sensitivity Analysis

Now we briefly summarize some additional sensitivity analysis regarding the results presented above. First, we consider the implications of changing the weight μ on the variability of interest-rate changes in the policymaker's loss function. For the preceding analysis we have chosen a weight of $\mu = 0.1$, which has been widely employed in policy evaluation exercises like ours. As shown in table B-1 in the appendix, with a weight of $\mu = 0.01$ on interest-rate variability, the relative stabilization performance of optimized rules is not significantly affected when compared to the baseline results reported in table 1 above. However, with the variability in interestrate changes penalized less, there are a number of noticeable differences with regard to the pattern of the optimized coefficients. First, the response coefficients to inflation and the output gap are a good bit larger; second, for the backward-looking models, the optimal degree of interest-rate smoothing is found to be even lower. However, despite these changes in the operating characteristics of the optimized rules, we observe—by comparing the relative performance measures reported in table B-2 with those in table 2—that the robustness properties of the optimized rules are broadly unaffected if the weight on interest-rate variability is lowered to $\mu = 0.01$. Finally, as documented in table B-3, jointly optimized interest-rate rules are yet again found to be weighted toward the DM, but with the relatively favorable performance of robust forecast-based rules being less clear-cut.

Regarding the sensitivity of our results with respect to using a real-time information set with $\theta = \kappa = -1$, table C-1 shows that the performance of real-time-based rules tends to deteriorate relative to that of outcome-based rules. For the more backward-looking models, this deterioration is rather pronounced (up to 69 and 34 percentage points for the AWM and the DM, respectively, depending on the weight given to output stabilization). Similarly, as shown in table C-2, the robustness of real-time-based rules tends to be inferior to that of outcome-based ones, resulting in a somewhat larger deterioration of performance across models. Finally, as indicated in table C-3, the performance of robust real-time-based rules is slightly inferior across models as well.

6. Conclusions

In this paper, we examined the performance and robustness of optimized interest-rate rules in four models of the euro area that differ considerably in terms of size, degree of aggregation, relevance of forward-looking behavioral elements, and adherence to microfoundations. Based on our examination, we conclude that simple interest-rate rules that are optimized for a given model may perform satisfactorily in models with a rather similar structure but may result in a dramatic deterioration of performance in models with markedly different features. In particular, rules that are designed for largely forward-looking models tend to perform poorly in predominantly backward-looking models, although the reverse is not necessarily true. In view of this asymmetry, we find that the operating characteristics of interest-rate rules that are optimized across our set of models simultaneously are heavily weighted toward those required by the backward-looking models. Nevertheless, the Bayesian robust policies identified in such a way perform reasonably well in all four models, notably if a sensible weight is given to output stabilization. Especially, we find that a forecast-based rule that relates the short-term interest rate to the one-year-ahead forecast of inflation and the contemporaneous output gap and (importantly) that allows for only a moderate degree of inertia, attains reasonable outcomes.

While other model features such as variable and country coverage and adherence to microfoundations are apparently of relevance as well, the nature of the expectation formation mechanism embedded in the various models seems to be of key importance for explaining our results. This in turn suggests that future research that aims at casting light on the empirical relevance of forward-looking behavioral elements in macroeconomic models may enhance the reliability and usefulness of interest-rate rules for model-based evaluations of monetary policy. Of course, in the case that a policy rule prescribed to set the interest rate in response to forecasts of future inflation, we assumed in our analysis that these forecasts happened to be consistent with the structure of the particular model in which the performance of the forecast-based rule was evaluated. Similarly, the measure of the output gap used when evaluating the rule was consistent with the output-gap concept employed in that particular model. To the extent that the monetary policymaker faces uncertainty regarding the reliability of the inflation forecast itself or the correct measurement of the output gap, these additional sources of uncertainty may heighten the risks associated with relying too heavily on a rule optimized for any particular model.²² Extensions of our study along these directions are left for future research.

 $^{^{22}}$ Indeed, the analysis in Coenen (2003) shows that erroneously relying on the false forecasting model may result in a significant deterioration of the stabilization performance of a forecast-based rule, even when the rule itself is designed under the assumption that the structure of the economy is known. The consequences of relying on alternative, possibly inconsistent output-gap concepts is analyzed in Smets and Wouters (2002).

Appendix

3

1.04

0.69

| Table A. Detailed Results f | for the Optimized Interest-Rate |
|-----------------------------|---------------------------------|
| I | Rules |

| А. | The Fo | orward-L | ooking E | luro Area | a Models | | | | | |
|----|----------|-----------|----------------|------------|---------------------|------|----------------|------------|---------------------|---------------|
| | | | | С | W | | | S | W | |
| θ | κ | λ | σ_{π} | σ_y | $\sigma_{\Delta i}$ | L | σ_{π} | σ_y | $\sigma_{\Delta i}$ | \mathcal{L} |
| 0 | 0 | 0 | 1.33 | 2.30 | 0.80 | 1.83 | 0.93 | 2.47 | 0.47 | 0.95 |
| | | 1/3 | 1.45 | 1.07 | 0.82 | 2.54 | 1.06 | 0.60 | 0.87 | 1.33 |
| | | 1 | 1.48 | 0.91 | 1.26 | 3.17 | 1.08 | 0.40 | 1.21 | 1.48 |
| | | 3 | 1.48 | 0.83 | 1.93 | 4.65 | 1.09 | 0.28 | 1.68 | 1.71 |
| 4 | 0 | 0 | 1.35 | 2.09 | 0.79 | 1.89 | 0.94 | 2.44 | 0.51 | 0.98 |
| | | 1/3 | 1.47 | 1.02 | 0.83 | 2.58 | 1.07 | 0.58 | 0.87 | 1.34 |
| | | 1 | 1.48 | 0.90 | 1.26 | 3.18 | 1.08 | 0.40 | 1.21 | 1.48 |
| | | 3 | 1.49 | 0.83 | 1.91 | 4.65 | 1.09 | 0.29 | 1.64 | 1.70 |
| В. | The Ba | ackward- | Looking | Euro Ar | ea Model | s | | | | |
| | | | | AV | VM | | | Γ | DМ | |
| θ | κ | λ | σ_{π} | σ_y | $\sigma_{\Delta i}$ | L | σ_{π} | σ_y | $\sigma_{\Delta i}$ | L |
| 0 | 0 | 0 | 1.09 | 1.57 | 1.04 | 1.30 | 2.15 | 1.68 | 3.69 | 5.97 |
| | | 1/3 | 1.07 | 1.17 | 1.74 | 1.90 | 2.17 | 1.60 | 3.62 | 6.86 |
| | | 1 | 1.07 | 0.91 | 2.50 | 2.59 | 2.22 | 1.50 | 3.59 | 8.45 |
| | | 3 | 1.07 | 0.69 | 3.52 | 3.79 | 2.36 | 1.36 | 3.68 | 12.48 |
| 4 | 0 | 0 | 1.04 | 1.59 | 1.01 | 1.19 | 1.89 | 1.58 | 3.29 | 4.64 |
| | | 1/3 | 1.02 | 1.16 | 1.69 | 1.78 | 1.90 | 1.51 | 3.24 | 5.44 |
| | | 1 | 1.03 | 0.90 | 2.43 | 2.46 | 1.95 | 1.43 | 3.23 | 6.87 |
| | | | | | | | | | | |

Note: For each choice of the inflation and output-gap forecast horizons (θ and κ), for each preference parameter (λ), and for each model, this table indicates the unconditional standard deviations of the target variables (σ_{π} , σ_{y} , and $\sigma_{\Delta i}$) and the value of the policymaker's loss function (\mathcal{L}).

3.65

3.42

2.07

1.31

3.34

10.56

| Δ | Th | - Forw | ard-Lo | oking E | uro Area | Mod | ale | | | | | |
|-----|----------|-----------|--------|---------|----------|-------|------------------------|------|-------|-------|-----|------------------------|
| 11. | 1 110 | . 101 W | aru-Lo | oking L | CW | , mou | 010 | | | SW | | |
| θ | κ | λ | ρ | α | β | L | $\%\Delta \mathcal{L}$ | ρ | α | β | L | $\%\Delta \mathcal{L}$ |
| 0 | 0 | 0 | 0.98 | 2.98 | 0.25 | 1.7 | 1 | 1.03 | 1.71 | 0.16 | 0.9 | 2 |
| | | 1/3 | 0.75 | 1.26 | 2.29 | 2.4 | 5 | 0.99 | 1.92 | 5.25 | 1.2 | 9 |
| | | 1 | 0.73 | 0.74 | 4.09 | 2.9 | 4 | 1.02 | 0.83 | 11.22 | 1.3 | 6 |
| | | 3 | 0.69 | 0.44 | 6.54 | 4.0 | 3 | 1.00 | 0.84 | 20.93 | 1.3 | 4 |
| 4 | 0 | 0 | 2.75 | 29.26 | -0.24 | 1.7 | 5 | 2.32 | 18.26 | -0.06 | 0.9 | 4 |
| | | 1/3 | 0.79 | 3.30 | 2.14 | 2.5 | 7 | 1.01 | 3.40 | 4.73 | 1.2 | 9 |
| | | 1 | 0.73 | 1.96 | 4.14 | 2.9 | 4 | 1.00 | 2.49 | 10.32 | 1.3 | 7 |
| | | 3 | 0.68 | 1.33 | 6.62 | 4.0 | 3 | 1.00 | 1.72 | 20.42 | 1.3 | 4 |

Table B-1. The Stabilization Performance of Optimized Interest-Rate Rules Generated with a Lower Weight of $\mu = 0.01$ on Interest-Rate Variability

B. The Backward-Looking Euro Area Models

| | | | | | AWM | | | | | DM | | |
|----------|----------|-----------|------|----------|---------|---------------|------------------------|------|----------|---------|---------------|------------------------|
| θ | κ | λ | ρ | α | β | \mathcal{L} | $\%\Delta \mathcal{L}$ | ρ | α | β | \mathcal{L} | $\%\Delta \mathcal{L}$ |
| 0 | 0 | 0 | 0.17 | 3.05 | 3.00 | 1.1 | 24 | 0.11 | 13.83 | 8.33 | 3.8 | 41 |
| | | 1/3 | 0.42 | 2.03 | 6.57 | 1.4 | 12 | 0.02 | 13.24 | 9.11 | 4.8 | 42 |
| | | 1 | 0.51 | 1.37 | 9.71 | 1.6 | 9 | 0.00 | 11.88 | 9.65 | 6.4 | 42 |
| | | 3 | 0.55 | 0.84 | 14.55 | 2.0 | 6 | 0.00 | 9.94 | 10.22 | 10.4 | 41 |
| 4 | 0 | 0 | 0.23 | 6.76 | 1.16 | 1.0 | 11 | 0.39 | 10.58 | 1.53 | 2.9 | 9 |
| | | 1/3 | 0.30 | 5.57 | 4.69 | 1.3 | 5 | 0.27 | 10.43 | 2.30 | 3.8 | 12 |
| | | 1 | 0.37 | 4.53 | 8.08 | 1.6 | 4 | 0.15 | 10.28 | 3.33 | 5.2 | 16 |
| | | 3 | 0.44 | 3.53 | 13.10 | 2.0 | 4 | 0.00 | 9.83 | 5.12 | 8.8 | 19 |

Note: For each choice of the inflation and output-gap forecast horizons (θ and κ), for each preference parameter (λ), and for each model, this table indicates the optimized interest-rate response coefficients (ρ , α , and β), the value of the policymaker's loss function (\mathcal{L}), and the percentage-point difference of the latter from the loss under the fully optimal policy ($\%\Delta\mathcal{L}$).

| A | . R | ules | Optin | nized for | r the F | Forwa | rd-Look | ing Eu | iro Ai | rea Mod | els | | | |
|---|----------|------|-------|-----------|----------|--------|------------------------|----------|--------|----------|----------|---------|------------------------|----------|
| | | | | Rules | Optim | ized f | for CW | | | Rules | Optin | nized f | for SW | |
| | | | | L | | | $\%\Delta \mathcal{L}$ | | | L | | | $\%\Delta \mathcal{L}$ | |
| θ | κ | λ | SW | AWM | DM | SW | AWM | DM | CW | AWM | DM | CW | AWM | DM |
| 0 | 0 | 0 | 0.9 | ∞ | ∞ | 3 | ∞ | ∞ | 1.7 | ∞ | ∞ | 4 | ∞ | ∞ |
| | | 1/3 | 1.3 | 1.7 | 17.6 | 12 | 36 | 420 | 2.6 | 1.7 | ∞ | 13 | 34 | ∞ |
| | | 1 | 1.4 | 1.9 | 97.4 | 13 | 29 | 2046 | 3.6 | 1.9 | ∞ | 31 | 29 | ∞ |
| | | 3 | 1.5 | 2.5 | ∞ | 17 | 30 | ∞ | 6.5 | 2.2 | ∞ | 67 | 16 | ∞ |
| 4 | 0 | 0 | 0.9 | ∞ | ∞ | 4 | ∞ | ∞ | 1.7 | ∞ | ∞ | 6 | ∞ | ∞ |
| | | 1/3 | 1.3 | 1.6 | 27.3 | 12 | 30 | 704 | 2.6 | 1.8 | ∞ | 13 | 43 | ∞ |
| | | 1 | 1.3 | 1.8 | 30.3 | 12 | 21 | 567 | 3.5 | 1.8 | ∞ | 25 | 23 | ∞ |
| | | 3 | 1.5 | 2.4 | 90.2 | 16 | 23 | 1125 | 6.3 | 2.3 | ∞ | 63 | 18 | ∞ |

Table B-2. The Robustness of Optimized Interest-Rate Rules Generated with a Lower Weight of $\mu = 0.01$ on Interest-Rate Variability

| | | | | Rules | Optimi | zed for | r AWN | 1 | | Rule | s Optim | ized fo | or DM | |
|---|----------|-----|-----|---------------|----------|---------|------------------------|----------|-----|---------------|---------|---------|------------------------|-----|
| | | | | \mathcal{L} | | | $\%\Delta \mathcal{L}$ | | | \mathcal{L} | | | $\%\Delta \mathcal{L}$ | |
| θ | κ | λ | CW | SW | DM | CW | SW | DM | CW | SW | AWM | CW | SW | AWM |
| 0 | 0 | 0 | 2.1 | 1.2 | 17.6 | 28 | 30 | 556 | 2.6 | 1.2 | 1.3 | 57 | 29 | 44 |
| | | 1/3 | 2.5 | 1.3 | 52.7 | 10 | 12 | 1454 | 3.0 | 1.4 | 1.5 | 31 | 26 | 19 |
| | | 1 | 3.1 | 1.3 | 199.4 | 11 | 10 | 4291 | 3.7 | 2.8 | 1.7 | 33 | 47 | 14 |
| | | 3 | 4.5 | 1.4 | ∞ | 15 | 9 | ∞ | 5.4 | 2.5 | 2.2 | 39 | 92 | 17 |
| 4 | 0 | 0 | 1.9 | 1.2 | 24.8 | 18 | 33 | 824 | 2.1 | 1.2 | 1.1 | 27 | 30 | 15 |
| | | 1/3 | 2.6 | 1.3 | 32.0 | 12 | 16 | 842 | 3.3 | 1.4 | 1.5 | 41 | 26 | 21 |
| | | 1 | 3.0 | 1.4 | 60.7 | 9 | 13 | 1237 | 3.3 | 1.7 | 2.0 | 17 | 45 | 33 |
| | | 3 | 4.3 | 1.4 | 342.5 | 10 | 12 | 4551 | 4.9 | 2.4 | 3.0 | 26 | 88 | 54 |

B. Rules Optimized for the Backward-Looking Euro Area Models

Note: For each choice of the inflation and output-gap forecast horizons (θ and κ), for each preference parameter (λ), and for each model, this table indicates the value of the policymaker's loss function (\mathcal{L}) and the percentage-point difference of the latter from the loss under the fully optimal policy ($\%\Delta\mathcal{L}$), when the rule optimized for model m is evaluated in model $n \neq m$. The notation " ∞ " indicates that the implemented rule results in instability.

| | | | | | | Ĺ | \mathcal{L}_m ger | nerated i | n | $\%\Delta$ | \mathcal{L}_m ge | enerated | in |
|----|----------|-----------|---------|----------|----------|---------|---------------------|-----------|------|------------|--------------------|----------|----|
| θ | κ | λ | ρ | α | β | CW | SW | AWM | DM | CW | SW | AWM | DM |
| A. | Op | timiza | ation A | cross A | All Mod | els | | | | | | | |
| 0 | 0 | 0 | 0.42 | 8.54 | 5.81 | 2.3 | 1.1 | 1.2 | 3.9 | 39 | 25 | 32 | 46 |
| | | 1/3 | 0.39 | 7.86 | 6.69 | 2.7 | 1.3 | 1.6 | 5.0 | 17 | 18 | 27 | 47 |
| | | 1 | 0.33 | 7.63 | 8.02 | 3.3 | 1.6 | 2.0 | 6.7 | 20 | 31 | 36 | 49 |
| | | 3 | 0.21 | 7.54 | 10.31 | 4.9 | 2.0 | 2.9 | 11.0 | 26 | 60 | 53 | 49 |
| 4 | 0 | 0 | 0.47 | 8.62 | 1.19 | 2.2 | 1.2 | 1.1 | 3.9 | 33 | 28 | 14 | 45 |
| | | 1/3 | 0.45 | 8.14 | 3.17 | 2.5 | 1.3 | 1.4 | 4.7 | 10 | 17 | 13 | 38 |
| | | 1 | 0.38 | 8.31 | 4.93 | 3.0 | 1.5 | 1.7 | 6.2 | 10 | 23 | 16 | 37 |
| | | 3 | 0.26 | 8.56 | 7.56 | 4.3 | 1.8 | 2.3 | 10.3 | 12 | 39 | 22 | 39 |
| В. | Op | timiza | ation A | cross A | All Aggr | egate 1 | Models | 3 | | | | | |
| 0 | 0 | 0 | 0.56 | 2.73 | 1.53 | 2.0 | 1.1 | 1.2 | | 21 | 21 | 32 | |
| | | 1/3 | 0.59 | 1.74 | 4.78 | 2.5 | 1.2 | 1.4 | | 7 | 11 | 14 | |
| | | 1 | 0.63 | 1.04 | 7.03 | 3.0 | 1.3 | 1.7 | | 6 | 10 | 12 | |
| | | 3 | 0.61 | 0.65 | 10.37 | 4.1 | 1.4 | 2.1 | | 6 | 11 | 10 | |
| 4 | 0 | 0 | 0.64 | 4.78 | 0.81 | 2.1 | 1.1 | 1.1 | | 29 | 25 | 16 | |
| | | 1/3 | 0.57 | 3.77 | 4.07 | 2.5 | 1.3 | 1.3 | | 8 | 13 | 7 | |
| | | 1 | 0.59 | 3.09 | 6.63 | 3.0 | 1.3 | 1.6 | | 6 | 11 | 7 | |
| | | 3 | 0.58 | 2.66 | 10.17 | 4.1 | 1.4 | 2.1 | | 6 | 12 | 7 | |

Table B-3. The Stabilization Performance of BayesianRobust Interest-Rate Rules Generated with a LowerWeight of $\mu = 0.01$ on Interest-Rate Variability

Note: For each choice of the inflation and output-gap forecast horizons (θ and κ) and for each preference parameter (λ), this table indicates the jointly optimized interest-rate response coefficients (ρ , α , and β), the contribution of the individual model m to the policymaker's loss function (\mathcal{L}_m), and the percentage-point difference of this contribution from the loss under the fully optimal policy ($(\mathcal{A} \mathcal{L}_m)$).

| | | | CW | | | | | SW | | |
|-----|------|----------|---------|-----|------------------------|------|----------|---------|---------------|------------------------|
| λ | ρ | α | β | L | $\%\Delta \mathcal{L}$ | ρ | α | β | \mathcal{L} | $\%\Delta \mathcal{L}$ |
| 0 | 0.95 | 0.79 | 0.11 | 1.8 | 2 | 1.02 | 0.79 | 0.08 | 1.0 | 3 |
| 1/3 | 0.76 | 0.48 | 0.91 | 2.6 | 6 | 0.98 | 0.56 | 1.38 | 1.4 | 11 |
| 1 | 0.74 | 0.29 | 1.58 | 3.2 | 5 | 0.96 | 0.35 | 2.62 | 1.6 | 12 |
| 3 | 0.72 | 0.28 | 2.43 | 4.8 | 6 | 0.98 | 0.00 | 3.44 | 1.9 | 20 |

Table C-1. The Stabilization Performance of OptimizedInterest-Rate Rules Based on a Real-Time InformationSet with $\theta = \kappa = -1$

| B. The Backward-Looking Euro Area Models | | | | | | | | | | | | | |
|--|------|----------|---------|---------------|------------------------|------|----------|---------|---------------|------------------------|--|--|--|
| | | | AWM | | | | DM | | | | | | |
| λ | ρ | α | β | \mathcal{L} | $\%\Delta \mathcal{L}$ | ρ | α | β | \mathcal{L} | $\%\Delta \mathcal{L}$ | | | |
| 0 | 0.42 | 0.99 | 1.11 | 1.4 | 34 | 0.17 | 6.07 | 4.26 | 7.3 | 88 | | | |
| 1/3 | 0.27 | 0.82 | 2.16 | 2.2 | 40 | 0.13 | 6.06 | 4.46 | 8.3 | 85 | | | |
| 1 | 0.26 | 0.54 | 3.19 | 3.4 | 55 | 0.07 | 6.02 | 4.77 | 10.1 | 81 | | | |
| 3 | 0.25 | 0.34 | 4.60 | 6.1 | 84 | 0.00 | 5.79 | 5.30 | 14.8 | 74 | | | |

Note: For each preference parameter (λ) and for each model, this table indicates the optimized interest-rate response coefficients (ρ , α , and β), the value of the policymaker's loss function (\mathcal{L}), and the percentage-point difference of the latter from the loss under the fully optimal policy ($\%\Delta\mathcal{L}$).

| Α. | A. Rules Optimized for the Forward-Looking Euro Area Models | | | | | | | | | | | | | | |
|--|---|----------|----------|------------------------|----------|----------|-----|------------------------|----------|------------------------|----------|----------|--|--|--|
| | Rules Optimized for CW | | | | | | | Rules Optimized for SW | | | | | | | |
| | | L | | $\%\Delta \mathcal{L}$ | | | | $\% \mathcal{L}$ | | $\%\Delta \mathcal{L}$ | | | | | |
| λ | SW | AWM | DM | SW | AWM | DM | CW | AWM | DM | CW | AWM | DM | | | |
| 0 | 1.0 | ∞ | ∞ | 6 | ∞ | ∞ | 1.9 | ∞ | ∞ | 4 | ∞ | ∞ | | | |
| 1/3 | 1.4 | 2.8 | 66.7 | 18 | 80 | 1388 | 2.8 | 13.7 | 68.0 | 14 | 766 | 1417 | | | |
| 1 | 1.7 | 4.5 | ∞ | 24 | 103 | ∞ | 3.8 | 21.4 | ∞ | 23 | 872 | ∞ | | | |
| 3 | 2.2 | 8.5 | ∞ | 38 | 157 | ∞ | ME | ∞ | ∞ | ME | ∞ | ∞ | | | |
| B. Rules Optimized for the Backward-Looking Euro Area Models | | | | | | | | | | | | | | | |
| | Rules Optimized for AWM | | | | | | | Rules Optimized for DM | | | | | | | |

Table C-2. The Robustness of Optimized Interest-Rate Rules Based on a Real-Time Information Set with $\theta = \kappa = -1$

| | | ituleb | optim | 1200 10 | | | | | | | | | | |
|-----------|-----|--------|----------|------------------------|----|----------|-----|-----|-----|------------------------|-----|-----|--|--|
| | L | | | $\%\Delta \mathcal{L}$ | | | L | | | $\%\Delta \mathcal{L}$ | | | | |
| λ | CW | SW | DM | CW | SW | DM | CW | SW | AWM | CW | SW | AWM | | |
| 0 | 2.2 | 1.3 | 94.5 | 20 | 36 | 2345 | 3.5 | 1.6 | 2.5 | 93 | 71 | 140 | | |
| 1/3 | 2.7 | 1.6 | ∞ | 11 | 29 | ∞ | 3.9 | 1.9 | 3.2 | 61 | 50 | 106 | | |
| 1 | ME | 1.9 | ∞ | ME | 38 | ∞ | 4.8 | 2.3 | 4.6 | 59 | 67 | 110 | | |
| 3 | ME | 2.6 | ∞ | ME | 58 | ∞ | 7.1 | 3.5 | 8.0 | 57 | 115 | 143 | | |

Note: For each preference parameter (λ) and for each model, this table indicates the value of the policymaker's loss function (\mathcal{L}) and the percentage-point difference of the latter from the loss under the fully optimal policy $(\%\Delta\mathcal{L})$, when the rule optimized for model m is evaluated in model $n \neq m$. The notation "ME" indicates that the implemented rule yields multiple equilibria; the notation " ∞ " indicates that the implemented rule results in instability.

| | | | | | \mathcal{L}_m ger | nerated in | L | $\% \Delta \mathcal{L}_m$ generated in | | | | |
|---|------|----------|---------|-----|---------------------|------------|------|--|----|-----|-----|--|
| λ | ho | α | β | CW | SW | AWM | DM | CW | SW | AWM | DM | |
| A. Optimization Across All Models | | | | | | | | | | | | |
| 0 | 0.41 | 3.62 | 2.52 | 2.7 | 1.3 | 1.9 | 7.8 | 48 | 44 | 81 | 103 | |
| 1/3 | 0.41 | 3.57 | 2.88 | 3.2 | 1.6 | 2.8 | 8.7 | 31 | 30 | 80 | 95 | |
| 1 | 0.39 | 3.62 | 3.46 | 4.1 | 2.0 | 4.3 | 10.6 | 33 | 44 | 95 | 89 | |
| 3 | 0.32 | 3.75 | 4.54 | 6.2 | 2.9 | 7.4 | 15.5 | 37 | 76 | 125 | 82 | |
| B. Optimization Across All Aggregate Models | | | | | | | | | | | | |
| 0 | 0.65 | 0.78 | 0.61 | 2.1 | 1.2 | 1.4 | | 14 | 26 | 40 | | |
| 1/3 | 0.51 | 0.64 | 1.70 | 2.6 | 1.5 | 2.3 | | 8 | 22 | 44 | | |
| 1 | 0.47 | 0.57 | 2.68 | 3.3 | 1.8 | 3.5 | | 9 | 29 | 60 | | |
| 3 | 0.41 | 0.59 | 4.10 | 5.1 | 2.4 | 6.2 | | 13 | 45 | 88 | | |

Table C-3. The Stabilization Performance of BayesianRobust Interest-Rate Rules Based on a Real-TimeInformation Set with $\theta = \kappa = -1$

Note: For each preference parameter (λ) , this table indicates the jointly optimized interest-rate response coefficients $(\rho, \alpha, \text{ and } \beta)$, the contribution of the individual model m to the policymaker's loss function (\mathcal{L}_m) , and the percentagepoint difference of this contribution from the loss under the fully optimal policy $(\% \Delta \mathcal{L}_m)$.

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