Political Bad Reputation

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Abstract

The goal of this paper is to explore how the connection between political ideology and voters’ preferences is able to generate different equilibria in a yardstick competition game, where good incumbents are forced to create a bad reputation or, in other words, to mimic the bad incumbents’ behavior in order to win the elections in a two-candidate political competition.
1 Introduction

The literature about the yardstick competition is becoming one of the main topic of research in the political economy, since the seminal paper by Besley & Case ([6]). The typical assumption which characterizes a yardstick competition game is that voters are assumed to make comparisons between jurisdictions to overcome political agency problems, even though they are not able neither to distinguish bad incumbents (rent-seekers) from good incumbents (welfaristic) nor to evaluate the real state of economy. The voters’ ability to observe tax-setting in different jurisdictions generates two effects: a selection effects, which separates good governments from bad governments and a discipline effect, which forces bad governments to act as if they were good. In the presence of a political competition, where elections take place, bad incumbents realize that if voters are able to compare domestic level of taxation with level of taxation of other jurisdictions, they have less possibility to distort resources from society’s welfare. Indeed, they are forced to build a good reputation, adopting good incumbent’s policies in order to be re-elected. Nevertheless, there is no a common agreement among researchers about the ability of the yardstick competition to reach both the effects. Furthermore, Besley & Smart ([3]) remind as economists who believe that government is benevolent are prone to see inter-governmental competition as a source of negative externalities which lower welfare, whilst the public choice perspective which assumes the existence of Leviathan governments sees yardstick competition as potentially beneficial for welfare. The authors underline the importance of information advantages of yardstick competition since, in equilibrium, voters are more likely to retain good domestic incumbents, once they see that governments in other jurisdictions extract rents; secondly, the initial reputation of a government is fundamental to determine the existence of pooling equilibria; a low reputation of the foreign incumbent reduces the probability that voters re-elect a bad incumbent. Bordignon et al. ([7]) underline the role of degrees of correlation between economies and the importance of initial reputation of incumbents which may lead toward more pooling or separating equilibria. Nevertheless, the classical approach to the yardstick competition let some questions open. First of all, the assumption that voters perfectly know the other jurisdictions’ policy is a strong one. Hardly one may imagine that a typical citizen is acknowledged about tax rates, incomes and expenses of all the other local governments; still, when a citizen has to evaluate an incumbent may link Taxes with the usage the incumbent has made of tax revenues (i.e. building new parkings, parks, metro lines and so forth); from this point of view, if a citizen evaluate that the incumbent has made a good usage of taxes collected with respect to other local jurisdictions, it would be prone to accept an higher level of taxation. Even supposing that citizens would be perfectly informed about all the other jurisdictions’ economic performance and they
would exclusively vote according to the relative level of taxation they still would not know which policy would be suitable for the local economy. If we assume that economic performances are a private information of candidates, that means that also the degree of correlation between economies is a private information of candidates. As a consequence, if a voter does not know what his local economy needs, he does not know what another local needs, either. At this point, who may convince him that the bording jurisdiction needs a lower (higher) taxation than his jurisdictions? Suppose two jurisdiction A and B, whose economies are perfectly correlated and affected by a negative shock, which entails the need for a raise in taxation. Suppose also that they only differ because in jurisdiction A there is a bad incumbent who is willing to win elections. Finally, suppose that the good incumbent in jurisdiction B acts as the good type and raises taxes as the economy needs, but that jurisdiction A’s bad incumbent accuses the former to be bad and raising taxes only for private interests, assuring the electorate that the state of economy is good. If voters are neither able to evaluate the state of the economy nor the degree of correlation between jurisdictions which candidate do they believe is good? In this case, there would be more than an incentive for the candidate to play populist strategy in order to attract voters and playing his favourite strategy during the second term. At this point, the good government find itself in a critical situation because if voters believe in bad candidate, he runs into risk to be taken as a bad type and not to be re-elected. Thus, it is rational to think that, in this case, he could mimic the bad incumbent in playing a populistic strategy as to win elections in the first term and, at least in the second term, playing a welfarist policy. In a nut-shell the good government is creating a bad reputation, since he is acting as if he was a bad candidate. This papers analysis just this situation. I argue that, especially when the economy is affected by negative shocks, pooling equilibria where good governments mimic bad governments arise. This result may be considered as negative, since it weakens the previous results in literature, which affirmed that the yardstick competition acts both as a selection and a discipline device. According to my results, bad reputation pooling equilibria reduce the role of yardstick competition to act as a discipline device; otherwise, yardstick competition may become even harmful when voters simply compare economic results among jurisdictions without knowing what single economies really need.

2 The model

2.1 Candidates

I consider a two-period model with three political candidates D, R and W, where D stands for Democratic, R for Republican and W for Welfaristic. On each party I attached a political label, which I assume is exogenously
taken at the beginning of the electoral campaign. For instance, we may think about the most familiar labeling system in the U.S., where candidates are located in a left-right or liberal-conservative scale. In my model I assume that candidate R is labeled as “conservative” and that it is supported by voters who get higher utility from labor taxation, rather than capital. Otherwise, candidate D is labeled as “liberal” and it is supported by voters who get higher utility from capital taxation, rather than labor. Thus, the space of candidates is given by $\Theta^C = \{D, R, W\}$. Furthermore, candidate D may be a good or a bad type and so may be candidate R. Thus, the space of type is $\Theta^G = \{\theta^D_1, \theta^D_2, \theta^R_1, \theta^R_2\}$. Each candidate may play only four policies: two populistic policies (the right-wing policy $a^R$ and the left-wing policy $a^L$), which provide more welfare to oriented voters, a welfaristic policy $a^W$, which is neutral and a bad policy $a^{bp}$ which enables the government to subtract rents to the society’s welfare. Furthermore, I allow for the possibility that some shocks occur in the economy; these shocks may be seen as all of those exogenous events which may increase (or decrease) efficiency in the production of public goods. Shocks may be either positive (P) or negative (N); if a shock is positive, then the efficiency in the production increases whilst if the shock is negative the efficiency decreases. An increase in efficiency may be seen as the possibility to produce the same amount of good at a lower cost, or to produce an higher amount of that good at the same cost. The cost of production is borne by tax-payers and thus, citizens are better off when a shock is positive, since they pay less taxes. An important assumption here is that the sign and the magnitude of the shock is a private information of candidates, which perfectly observe whether these are positive or negative. Otherwise, voters only perceive the existence of shocks but are not able to measure neither the magnitude nor the sign. Thus, at the beginning of the game, Nature chooses both the type of candidates and the type of shock. Candidates observe Nature’s choice and then announce (and commit) to a policy which depends on the type of shock, that is $a(\epsilon)$; voters observe candidate’s policies and vote for the candidate which have chosen the nearest policy to their IP.

The role of ideology

I have assumed that voters attach an ideological label to each candidate, which denotes its position on a predictive scale. As a consequence, the ideology directly enters into the candidates’ utility function. I suppose that if a candidate chooses a policy which stands over the bisector $\tau_L = \tau_K + 1$ it gets an utility deriving from the ideology equal to zero. Otherwise, it gets a positive utility equal to $i$ if it chooses a policy which stands in the triangle generated by the bisector and its preferred IP and a negative utility equal to $-i$ if the policy stands in the triangle generated by the bisector and the other candidate’s IP. I study a case where the incumbent may be either a good or a bad type and it faces a good challenger which may be either a
“super-welfarist” in a sense that it is committed to play strategy $a^W$ or a bad Government which is committed to play strategy $a^{bp}$. Thus, the game can be formalized in the following structure:

$$\Theta^C = \{D, R, W\}$$

$$\Theta^G = \{\theta^D, \theta^D, \theta^R, \theta^W\}$$

$$A^I = A^D = \{a^D, a^R, a^W, a^{bp}\} \subseteq E_2$$

$$A^{Gg} = A^{Rg} = \{a^R\} \subseteq E_2$$

$$A^{Gb} = A^{Rb} = \{a^{bp}\} \subseteq E_2$$

$$\Pr(\theta^R = \theta^R_g) = q$$

$$\Pr(\theta^R = \theta^R_b) = 1 - q$$

### 2.2 Voters

I suppose the existence of a population of voters, portioned in three equal groups (i.e. one voter per group): the welfarist voters, who are those who have not any particular preference for a candidate, candidate D-oriented voters, who support the Democratic Party and candidate R-oriented voters, who support the Republican Party.

### 2.3 Equilibria of the game

#### 2.3.1 Shock is positive

**Proposition 1** If the incumbent is a good type, he chooses the populistic policy $a^D(\epsilon^P)$ if $q \in \left[0, \frac{2i}{(1+i)^2}\right)$, whilst he chooses the welfaristic policy $a^W(\epsilon^P)$ if $q \in \left(\frac{2i}{(1+i)^2}, 1\right]$.

**Proof**: $EU(a^D(\epsilon^P)) = 1 + i + \frac{1}{2}(1 + i)\beta q + (1 + i)\beta(1 - q)$ strictly dominates $EU(a^R(\epsilon^P)) = 1 - i + \frac{1}{2}(1 + i)\beta q + (1 + i)\beta(1 - q)$, since $i > 0$. It also dominates $EU(a^D(\epsilon^N)) = -1 + i + (1 + i)\beta(1 - q)$ and according the transitive property also $EU(a^W(\epsilon^N)) = -1 + (1 + i)\beta(1 - q)$ and $EU(a^R(\epsilon^N)) = -1 - i + (1 + i)\beta(1 - q)$, since $-1 + i > -1 > -1 - i$. $EU(a^D(\epsilon^P)) > EU(a^W(\epsilon^P))$ when $q \in \left[0, \frac{2i}{(1+i)^2}\right)$. This result can be interpreted as follows: if the probability to face a good challenger is sufficiently small, the incumbent has the opportunity to play a populistic policy which stays on his IP and favor its electorate. Otherwise,
if the probability to face a good challenger is high, the incumbent realizes that playing his preferred policy would not be sufficiently safe to assure the re-election and then he prefers to play a welfaristic policy. Notice that the probability is a function of the ideological parameter \( i \), suggesting that the higher is \( i \), the broader is the interval to play the populistic policy and that if the the political environment is characterized by an electorate ideologically oriented, an equilibrium where politicians deviate from welfaristic policies is more likely.

**Proposition 2** If the incumbent is a bad type different equilibrium policy arise, depending on the value of ideology parameter.

**Proof:** see Appendix B.

In the case the equilibrium strategies depend on the both the ideology parameter and the probability to challenge a good type. First of all, notice that the bad policy is played when the probability to face a good challenger is high, meaning that when this probability is high it becomes too costly for the bad to mimic the good type. Otherwise, when this probability is sufficiently low, pooling equilibria arise, since the bad incumbent plays populistic or welfaristic strategy, realizing that in doing so it has a good probability to be re-elected and it play its preferred strategy in the second period. Secondly, notice the importance of the role represented by the ideology parameter and the discount factor. These two parameters entail the existence of mimicking strategies (i.e. look at the the fact that the bad is forced to play a very good policy (welfaristic policy) in intervals \( q^3 - q^4 \) and \( q^4 - q^3 \) when \( i \in [0, 2] \) and \( \beta \in (\frac{2}{3}, 1) \) or when \( i \in [2, \frac{2}{3} - 1] \) and \( \beta \in (0, \frac{2}{3}) \).

2.3.2 Shock is negative

**Proposition 3** If the incumbent is a good type, he chooses the populistic policy \( a^D(e^P) \) if \( q \in [0, \frac{4}{\beta(1+i)}] \), whilst he chooses the welfaristic policy \( a^W(e^N) \) if \( q \in (\frac{4}{\beta(1+i)}, 1) \), if \( i > 2 \). Otherwise, if \( i < 2 \) the incumbent plays \( a^D(e^N) \) if \( q \in [0, \frac{2i}{\beta(1+i)}] \), whilst it plays \( a^W(e^N) \) if \( q \in (\frac{2i}{\beta(1+i)}, 1) \).

**Proof:** \( EU(a^D(e^N)) = 1 + i + \frac{1}{2}(1 + i)\beta q + \beta(1 + i)(1 - q) \) strictly dominates \( EU(a^R(e^N)) = 1 - i + \frac{1}{2}(1 + i)\beta q + \beta(1 + i)(1 - q) \) and thus also \( EU(a^W(e^P)) = -1 + (1+i)\beta, EU(a^R(e^P)) = -1 - i + (1+i)\beta \) and \( EU(a^R) = i \). \( EU(a^D(e^N)) > EU(a^W(e^N)) \) when \( q \in [0, \frac{2i}{\beta(1+i)}] \) and \( EU(a^D(e^N)) > EU(a^P(e^N)) \) when \( q \in [0, \frac{4}{\beta(1+i)}] \). \( EU(a^W(e^N)) > EU(a^D(e^P)) \) when \( i < 2 \). This case is very interesting. Notice that we are analysing a good incumbent facing a negative shock, so that it would be always preferable, from its and society point of view it plays a policy function of the negative shock (i.e. we may imagine a restrictive fiscal policy). Nevertheless, it can be seen that in some intervals it plays a policy which is a function of the good shock. This
happens when the probability to face a bad incumbent is high and when the ideological parameter is high likewise. This is a bad reputation case. Indeed, the good incumbent is forced to play a very populistic strategy and mimic the bad type, because behaving like a good type is not attractive from a political perspective. Even though the economy needs a tough fiscal policy, this policy is very far from voters’ IP and thus, selecting such a policy would mean losing elections. Concluding, the good type is forced to play a non-optimal policy, because in doing so it aims to the re-election to have a possibility to play the optimal policy at least during the second period.

**Proposition 4** If the incumbent is a bad type, he chooses the populistic policy $a^D(\epsilon^N)$ if $i > \frac{2}{\beta} - 1$, whilst he chooses the bad policy $a^{bp}$ if $i > \frac{2}{\beta} - 1$.

**Proof:** $EU(a^D(\epsilon^P)) = -2 + i + \beta(1 + i)$ is greater than $EU(a^R(\epsilon^P)) = -2 - i + \beta(1 + i)$ and than $EU(a^W(\epsilon^P)) = -2 + \beta(1 + i)$. $EU(a^D(\epsilon^N)) = i + \frac{1}{2}(1+i)\beta q + \beta(1+i)(1-q)$ is greater than $EU(a^R(\epsilon^N)) = -i + \frac{1}{2}(1+i)\beta q + \beta(1+i)(1-q)$. $EU(a^{bp}) = 1 + i$ strictly dominates $EU(a^W(\epsilon^N)) = (1 + i)\beta$. $EU(a^{bp})$ is greater than $EU(a^D(\epsilon^N))$ when $i < \frac{2}{\beta}$, whilst $EU(a^D(\epsilon^N))$ is greater $EU(a^W(\epsilon^N))$ when $i > \frac{2}{\beta}$.

3 A yardstick competition case

3.1 Shock is positive

Suppose that the foreigner country government is always a good type who is committed to play $a^F(\epsilon^P)$.

**Proposition 5** If the incumbent is a good type, he chooses the populistic policy $a^D(\epsilon^P)$ if $q \in [0, \frac{2i}{\beta(1+i)}]$, whilst it chooses the welfaristic strategy $a^W(\epsilon^P)$ if $q \in (\frac{2i}{\beta(1+i)}, 1]$ and $i < \frac{\beta}{2-\beta}$.

**Proof:** $EU(a^D(\epsilon^N)) = -1 + i$ is greater than $EU(a^{bp}) = -2 + i$, $EU(a^R(\epsilon^N)) = -1 - i$ and $EU(a^W(\epsilon^N)) = -1$. $EU(a^D(\epsilon^P)) = 1 + i + \frac{1}{2}(1+i)\beta q + (1+i)\beta(1-q)$ is greater than $EU(a^D(\epsilon^N))$. $EU(a^W(\epsilon^N))$ is greater than $EU(a^D(\epsilon^P))$ when $q \in (\frac{2i}{\beta(1+i)}, 1]$. 

**Proposition 6** If the incumbent is a bad type, it chooses the populistic policy $a^D(\epsilon^P)$ if $q \in [0, 2 - \frac{6}{\beta(1+i)}]$, whilst it chooses the bad strategy $a^{bp}$ if $q \in (2 - \frac{6}{\beta(1+i)}, 1]$ and $i < \frac{2}{\beta} - 1$.

**Proof:** $EU(a^{bp}) = 1 + i$ is greater than $EU(a^D(\epsilon^N)) = i$, $EU(a^W(\epsilon^N)) = 0$, $EU(a^R(\epsilon^N)) = -i$, $EU(a^W(\epsilon^P)) = -2 + (1 + i)\beta$. $EU(a^D(\epsilon^P)) = -2 + i + \frac{1}{2}\beta(1+i)q + \beta(1+i)(1-q)$ is greater than $EU(a^R(\epsilon^P)) = -2 - i + \frac{1}{2}\beta(1+i)q + \beta(1+i)(1-q)$. $EU(a^D(\epsilon^P))$ is greater than $EU(a^{bp})$ when $q \in [0, 2 - \frac{6}{\beta(1+i)}]$. 

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3.2 Shock is negative

**Proposition 7** If the incumbent is a good type, it chooses the populistic policy $a^D(e^N)$ if $q \in [0, 2\frac{i}{\beta(1+i)}]$, whilst it chooses the welfaristic policy $a^W(e^N)$ if $q \in (\frac{2i}{\beta(1+i)}, 1]$ if $i < 2$. Otherwise, when $i > 2$, it chooses the populistic policy $a^D(e^P)$ if $q \in [0, 4\frac{1}{\beta(1+i)}]$, whilst it chooses the populistic policy $a^D(e^N)$, if $q \in (\frac{4}{\beta(1+i)}, 1]$. 

Proof: $EU(a^D(e^N)) = 1 + i + \frac{1}{2}(1 + i)\beta q + \beta(1 + i)(1 - q)$ is always greater than $EU(a^R(e^N)) = 1 - i + \frac{1}{2}(1 + i)\beta q + \beta(1 + i)(1 - q)$ and $EU(a^\psi) = i$; $EU(a^D(e^P)) = -1 + i + \beta(1 + i)$ is always greater than $EU(a^R(e^P)) = -1 - i + \beta(1 + i)$ and $EU(a^W(e^N)) = 1 + (1 + i)\beta$ strictly dominates $EU(a^W(e^P)) = -1 + (1 + i)\beta$. $EU(a^D(e^N))$ is greater than $EU(a^R(e^N))$ when $q < \frac{1}{\beta(1+i)}$, $EU(a^D(e^P))$ is greater than $EU(a^W(e^N))$ when $i > 2$ and $EU(a^D(e^N))$ is greater than $EU(a^W(e^N))$ when $q < \frac{2i}{\beta(1+i)}$. This is another case of bad reputation, since again we are facing a good incumbent which is forced to play a populistic strategy which is not optimal given the state of the shock which affects the economy.

**Proposition 8** If the incumbent is a bad type, it chooses the populistic policy $a^D(e^N)$ if $q \in [0, 2(1 - \frac{1}{\beta(1+i)})]$, whilst it chooses the bad policy $a^\psi$ if $q \in (2(1 - \frac{1}{\beta(1+i)}), 1]$ if $i < \frac{3}{\beta} - 1$. Otherwise, when $i > \frac{3}{\beta} - 1$, it chooses the populistic policy $a^D(e^N)$ if $q \in [0, \frac{4}{\beta(1+i)}]$, whilst it chooses the populistic policy $a^D(e^P)$, if $q \in (\frac{4}{\beta(1+i)}, 1]$. 

Proof: $EU(a^D(e^N)) = i + \frac{1}{2}(1 + i)\beta q + \beta(1 + i)(1 - q)$ is always greater than $EU(a^D(e^N)) = -i + \frac{1}{2}(1 + i)\beta q + \beta(1 + i)(1 - q)$; $EU(a^D(e^P)) = -2 + i + \beta(1 + i)$ is always greater than $EU(a^D(e^P)) = -2 - i + \beta(1 + i)$ and $EU(a^W(e^P)) = -2 + \beta(1 + i)$; $EU(a^\psi) = i + 1$ is always greater than $EU(a^W(e^N)) = (i + 1)\beta$. $EU(a^\psi)$ is greater than $EU(a^D(e^N))$ when $q < 2(1 - \frac{1}{\beta(1+i)})$. $EU(a^\psi)$ is greater than $EU(a^D(e^P))$ when $i > \frac{3}{\beta} - 1$. $EU(a^D(e^N))$ is greater than $EU(a^D(e^P))$ when $q < \frac{4}{\beta(1+i)}$.

4 Conclusion

In this paper I analysed the impact of the ideology in a yardstick competition game, using a spatial voting model approach. I demonstrated that, when the economy is characterised by the presence of shocks, different pooling or separating equilibria may arise. In particular, an important result refers to the bad reputation (pooling) equilibria, where a good incumbent is forced to mimic the bad incumbent, adopting populistic strategies which are not optimal given the state of the economy, to win the elections. This equilibria arise only when the shock is negative, suggesting that when the economy is
affected by a negative environment is more difficult for the good government to promote optimal policies, since they are more difficult to be accepted by the electorate.
5 Appendix A

Suppose $Z_i$ be a positive definite matrix of parameters, where every $z_i \in Z_i$ represents the salience that voter $i$ attaches to an issue, $x$ the preferred position (or ideal point, IP) of voter $i$ and $a_j$ the policy vector chosen by a candidate $j$ and $k_{ij} > 0$ the candidate $i$’s ideology evaluation of candidate $j$. Then $u_i(a_j) = k_{ij} - ||a_j - x_i||$ represents a quadratic metric loss function or, in other words, the loss which a voter suffers for not to stand on his IP. Notice that, since the function is additive, $k_{ij}$ and $a_j$ are independent, meaning that the policy chosen by a candidate do not influence the ideology of a voter.

The difference in utility between the two candidates is $u_i(a_D) - u_i(a_R) = k_{iD} - k_{iR} + z_i[(a_D - x_i)^2 - (a_R - x_i)^2] = k_{iD} - k_{iR} + 2z_i(a_D - a_R)(x_i - \frac{a_D + a_R}{2})$.

If $u_i(a_D) > u_i(a_R)$ then voter $i$ votes for the Democratic Party, if $u_i(a_D) = u_i(a_R)$ he is indifferent between the Democratic and the Republican Party. Notice that, even though the two candidates selected the same policies (i.e. $a_D = a_R$), the election outcome would not be a tie, since voters still would strictly prefer the candidate he is more ideologically oriented to and the difference $k_{iD} - k_{iR}$ would be the only driver to voter’s decision. Suppose $k_{iD} = k_{iR}$ for all $i = 1, ..., N$. In this case voters are equally ideologically oriented toward the two candidates and they prefer Democratic or Republican Party if and only if $a_D$ is closer to their ID than is $a_R$. Obviously, if the IP is equidistant from the two policy, voters would be indifferent to the two candidates. Thus, both candidates would selects the same policy which coincides with median voter’s one. Suppose now that $k_{iD} \neq k_{iR}$, for some $i$ and that $\Delta k_{ij}$ is a random variable. I make a further assumption. Indeed, suppose to portion the total group of voters $N$ in two groups ($N_0$ and $N_f$); the former have an IP equal to zero, whilst the latter have an IP equal to $f$. The $i$’th voter in the former $N_0$ will vote for the Democratic candidate if and only if

$$\frac{y_i}{k_{iD} - k_{iR}} < \frac{a_D^2}{a_R^2}$$

Suppose $y_j \sim \mathcal{N}(0, \sigma^2)$. Thus, the proportion of the $N_0$ who votes for the Democratic Party is

$$\Pr_0(a_D, a_R) = \int_{-\infty}^{+\infty} \Delta a^2 \exp\left(-\frac{y^2}{2\sigma^2}\right)dy$$

The vote for the Democratic Party form this group is then $N_0 \Pr_0(a_D, a_R)$. Suppose now to analyse the case of $N_f$ voters, the $i$ – th voter in this group will vote for the Democratic Party if and only if

$$\frac{y_i}{k_{iD} - k_{iR}} < (a_D - f)^2 - (a_R - f)^2$$
Again, the proportion of the $N_f$ who votes for the Democratic Party is

$$\Pr_f(a_D, a_R) = \int_{-\infty}^{\infty} \Delta(a_j - f)^2 \left(\frac{1}{\sqrt{2\pi}\sigma_f}\exp(-\frac{y^2}{2\sigma_f^2})\right)dy$$

The vote for the Democratic Party from this group is then $N_f \Pr_f(a_D, a_R)$. Thus, the total vote for the Democratic Party is $V(a_D, a_R) = N_0 \Pr_0(a_D, a_R) + N_f \Pr_f(a_D, a_R)$. Likewise, the total vote for the Republican Party is $1 - V(a_D, a_R)$.

6 Appendix B

$$EU(a^{bp}) = 1 + i + \frac{1}{2}(1 + i)\beta(1 - q)$$

$$EU(a^D(e^N)) = 1 + (1 + i)\beta(1 - q)$$

$$EU(a^D(e^P)) = -2 + i + \frac{1}{2}(1 + i)\beta q + (1 + i)\beta(1 - q)$$

$$EU(a^R(e^N)) = -i + (1 + i)\beta(1 - q)$$

$$EU(a^R(e^P)) = -2 - i + \frac{1}{2}(1 + i)\beta q + (1 + i)\beta(1 - q)$$

$$EU(a^W(e^N)) = (1 + i)\beta(1 - q)$$

$$EU(a^W(e^P)) = -2 + (1 + i)\beta$$

$EU(a^D(e^N)) > EU(a^D(e^P))$ if $q \in [0, \frac{4}{5(1+i)}]$; $EU(a^D(e^N)) > EU(a^W(e^P))$ if $q \in [0, \frac{2}{5(1+i)}]$; $EU(a^D(e^P)) > EU(a^W(e^P))$ if $q \in [0, \frac{2(3+i)}{5(1+i)}]$; $EU(a^{bp}) > EU(a^D(e^N))$ if $q \in [0, 1 - \frac{2}{5(1+i)}]$; $EU(a^{bp}) > EU(a^D(e^P))$ if $i < \frac{2}{3} - 1$;

$EU(a^{bp}) > EU(a^W(e^P))$ if $q \in [0, \frac{2(3+i)}{5(1+i)} - 1]$. Recapitulating, we have five probabilities: $q^1 = \frac{4}{5(1+i)}$; $q^2 = \frac{2+i}{5(1+i)}$; $q^3 = \frac{2i}{5(1+i)}$; $q^4 = 1 - \frac{2}{5(1+i)}$; $q^5 = \frac{2(3+i)}{5(1+i)} - 1$.

Case A: $i < \frac{2}{3} - 1$

Notice that $q^4$ is always greater than $q^5$, $q^1$ and $q^2$ are always greater than $q^4$; $q^3$ is always greater than $q^4$, $q^5$ is always greater than $q^3$. $q^5$ is greater than $q^1$ if $\beta < 2$, $q^5$ is always greater than $q^2$. $q^4$ is always greater than $q^2$ if $i < 2$ and $q^2$ is always greater than $q^3$ if $i < 2$.

Thus the equilibrium strategies change depending on the parameters of the model.

$i \in [0, 2]$ and $\beta \in (0, \frac{2}{3})$.
\( i \in [2, \frac{2}{β} - 1] \) and \( β \in (0, \frac{2}{3}) \).

\[
\begin{array}{cccccccc}
0 - q^1 & q^1 - q^2 & q^2 - q^3 & q^3 - q^4 & q^4 - q^5 & q^5 - q^6 & q^6 - 1 \\
\alpha^D(ε^N) & \alpha^D(ε^N) & \alpha^W(ε^N) & \alpha^W(ε^N) & \alpha^{bp} & \alpha^{bp} & \alpha^{bp} \\
\end{array}
\]

\( i \in [0, 2] \) and \( β \in \left(\frac{2}{3}, 1\right) \).

\[
\begin{array}{cccccccc}
0 - q^1 & q^1 - q^2 & q^2 - q^3 & q^3 - q^4 & q^4 - q^5 & q^5 - q^6 & q^6 - 1 \\
\alpha^D(ε^N) & \alpha^D(ε^N) & \alpha^W(ε^N) & \alpha^W(ε^N) & \alpha^{bp} & \alpha^{bp} & \alpha^{bp} \\
\end{array}
\]

\( i \in [0, 2] \) and \( β \in \left(\frac{2}{3}, 1\right) \) is impossible.

**case B:** \( i > \frac{2}{β} - 1 \)

\( i \in [0, 2] \) and \( β \in (0, \frac{2}{3}) \).

\[
\begin{array}{cccccccc}
0 - q^1 & q^1 - q^2 & q^2 - q^3 & q^3 - q^4 & q^4 - q^5 & q^5 - q^6 & q^6 - 1 \\
\alpha^D(ε^N) & \alpha^D(ε^N) & \alpha^D(ε^N) & \alpha^{bp} & \alpha^{bp} & \alpha^{bp} & \alpha^{bp} \\
\end{array}
\]

\( i \in [2, \frac{2}{β} - 1] \) and \( β \in (0, \frac{2}{3}) \) is impossible.

\( i \in [0, 2] \) and \( β \in \left(\frac{2}{3}, 1\right) \).

\[
\begin{array}{cccccccc}
0 - q^1 & q^1 - q^2 & q^2 - q^3 & q^3 - q^4 & q^4 - q^5 & q^5 - q^6 & q^6 - 1 \\
\alpha^D(ε^N) & \alpha^D(ε^N) & \alpha^D(ε^N) & \alpha^{bp} & \alpha^{bp} & \alpha^{bp} & \alpha^{bp} \\
\end{array}
\]

\( i \in [2, \frac{2}{β} - 1] \) and \( β \in \left(\frac{2}{3}, 1\right) \).

\[
\begin{array}{cccccccc}
0 - q^1 & q^1 - q^2 & q^2 - q^3 & q^3 - q^4 & q^4 - q^5 & q^5 - q^6 & q^6 - 1 \\
\alpha^D(ε^N) & \alpha^D(ε^N) & \alpha^W(ε^N) & \alpha^W(ε^N) & \alpha^{bp} & \alpha^{bp} & \alpha^{bp} \\
\end{array}
\]
References


