# The role of information provision as a policy instrument to supplement environmental taxes: Empowering consumers to choose optimally

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# The role of information provision as a policy instrument to supplement environmental taxes: Empowering consumers to choose optimally

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**Abstract**

The present paper examines, within a dynamic framework, the use of information provision as a policy instrument to supplement environmental taxation. We assume that at least a fraction of consumers do not posses the required information to make the optimal choices, and that their behavior at each time period depends on the accumulated stock of information. We show that, as the accumulated stock of information provision increases, both the optimal level of information provided at each period of time and the optimal tax rate decline over time. Our results provide strong evidence in support of information campaigns as a policy instrument to supplement traditional environmental policies. Information provision can sift the demand towards environmentally friendly products over time and thus, reduce the required level of the tax rate.

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# 1 Introduction

Balancing human needs with the health of consumers and the natural environment may be the most pressing global concern of the twenty-first century. Consumers are becoming more and more concerned about products containing substances that are toxic, carcinogenic or in general harmful to them in their every day use and at the same time dangerous to the environment. Many products, including food, electric and electronic products, tools and toys have been proven to generate health damages to their users as well as environmental damages that might in turn generate health problems to the wider population.

One such example that has been discussed extensively over the past decade is the polyvinyl chloride (PVC), one of the largest-selling plastic in the world. It is widely used in building, packaging, consumer goods (including office supplies and toys), electronics industries and even in agriculture. During all phases of PVC production, as well as during its use and disposal, poisonous chemicals (dioxins) linked to cancer and birth defects are released. Therefore, PVC generate environmental damages[1] at the same time that poses health risks[2] to the users of products containing PVC as well as to certain groups of people such as the workers in the PVC industry, residents in the nearby areas and fire-fighters.

Similar problems are encountered with lead used in paints, asbestos used in buildings and many other elements used in the production of goods. Furthermore, many household goods, including electric and electronic devices contain toxic substances harmful to their users in the same time that their production and disposal generates environmental damages. In the food sector one could think of fruits and vegetables grown with the use of pesticides, which generate environmental externalities during production and also health problems to consumers from residues of pesticides.

Individuals have incentives to reduce the consumption of such products, by

[1] Such as groundwater contamination, air pollution and in addition they cannot be effectivelly recycled.

[2] Which include angiosarcoma of the liver, lung cancer, brain cancer, lymphomas, leukemia, and liver cirrhosis.

choosing, if available, less harmful substitutes. However, consumers are bewildered by - and often very skeptical about - the many health and environmental claims made by manufacturers and retailers for their products. Consumer associations and environmental groups could play a role in bridging the information gap. However, the effectiveness of these organizations is limited.[3] Thus, there is a clear need for government intervention, which over the years has taken different forms. In some cases governments have used direct policies banning the use of particular substances in products.[4] For example, in response to PVC's toxic threats, many governments around the world have passed policies to ban PVC from the use of certain product (with priority given to toys and food packaging) and switch to safer, healthier consumer products. Some governments have also used environmental taxes to create economic incentives for reducing the demand of such products. One such example is the Danish government's tax on PVCs.

The above examples indicate a general transition of environmental policy from the smokestacks and effluent pipes towards the process of production and finally to the consumption patterns, enriching in the same time, the policy instruments options with market-based approaches. However, due to the large number of the products that generate health and environmental damages and the complexity of their effects, it is difficult to address the problems only with direct policies and/or economic instruments. For example, in the process of switching to PVC-free products, the provision of information to consumers regarding the health risks of PVC has been proven extremely important. Information provision is still very important in countries that have not yet banned these products. Moreover, the importance of information provision has been established by many studies in the case of public antismoking campaigns.[5]

[3] Liston-Heyes (2001), Heijnen and Schoonbeek (2008) and Heyes and Maxwell (2004) examine the role and the effectiveness of environmental groups in informing consumers. These works examine also the interplay between the environmental group and the polluting firms.

[4] Such an example is the EU Directive on the Restriction on Use of Hazardous Substances (RoHS) which can be accessed at http://eur-lex.europa.eu/LexUriServ/LexUriServ.do?uri=CELEX:32002L0095:EN:HTML.

[5] For example, Choo and Clark (2006) find that information play an important role in encouraging particular groups of smokers to quit smoking. Their study is based on data from an antismoking campaign in US and Canada in the early 1990. Farrelly et.al. (2005), using data from an antismoking campaign in the US in early 2000, find that the campaign

The role of information provision in complementing traditional environmental policies has been recognized by Tietenberg and Wheeler (2001). They offer a number of examples of products and processes that generate damages to individual consumers and to the environment and they also review the empirical literature. They conclude that information provision can be an effective policy instrument. Since consumers have incomplete and inaccurate information regarding the health and environmental effects of particular products, the government could intervene and provide reliable information to consumers. The present paper addresses the issue of information provided by the government to consumers and specifically the question of choosing the optimal mix of environmental taxation and information provision. We use a dynamic framework in order to be able to take into account the lengthy process through which information affects consumers' habits and attitudes.

In particular we examine the case of a differentiated product offered in two types, produced by two firms competing in prices. During its lifetime this product generates environmental externalities (external damages) and at the same time imposes damages on each individual user (individual damages). The magnitude of both types of damages depends on the product type. We normalize by assuming that one type of the product does not generate damages (clean good), while the other type of the product (dirty good), generates both types of damages. We assume that consumers take into account the individual damages if they have relevant information. However, consumers' knowledge (perception) of individual damages is imperfect. For simplicity, we assume that there are two groups of consumers, those that have knowledge of the individual damages and those that they do not. Informed consumers substitute away from the dirty and towards the clean good.

Within this framework the government imposes at each time period a tax $\tau(t)$ and provides a *flow* of information $a(t)$. We assume that consumers' behavior at each time period depends on the accumulated stock of information $A(t)$

accounted for a significant portion of the decline in youth smoking in the period after the campaign. Pierce, Macaskill and Hill (1990) report similar results for an antismoking campaign in Sydney, Australia in 1983, and in Melbourne in 1984.

(state variable), rather than on currently provided information. More specifically, we assume that the stock of accumulated information $A(t)$ influences the composition of the two groups of consumers. The higher the stock of information is, the more consumers behave as informed. Following the main stream of the advertising literature we assume that consumers' response to the stock of information is S-shaped. At the initial stages of the information provision campaign, consumers are more responsive to the information they receive, while as the campaign develops consumers' responsiveness slows down.

We derive the optimal paths for the two policy instruments and the optimal steady states. We show that, under the assumptions of the model, two optimal steady states exist at the maximum, from which only one is stable. The level of the policy instruments depends on the rate at which information depreciates over time, the cost of information provision and behavioral parameters. The main result of our analysis is that the optimal tax rate declines over time as the accumulated stock of information increases. Therefore, if the government invests in informing people, shifting their consumption habits and attitudes, there is not need to regulate as strict as before. Taxation does not have long-lasting effects; the same tax level has to be imposed each time period in order to be effective. Furthermore, apart from the bureaucratic costs, taxation results in efficiency losses which are increasing at the tax level. On the contrary, information provision accumulates over time and does have long-lasting effects. As a result, the necessary level and the associated cost of information provision declines over time. Therefore, our analysis indicates that there are strong arguments for using information provision to support environmental taxation.

Information provision is one of the two ways in which advertisement has been analyzed in the economic literature. Some economists have emphasized the information disseminating role of advertisement (both direct provision of information or indirect in the form of signalling quality), assuming that consumers are not fully informed and they receive through advertisement complete, costless and instantly validated information (see for example Nelson (1974), Kotowitz and Mathewson (1979a), Kihlstrom and Riordan (1984) and Stigler (1961)).

Others have advocated that advertisement alters consumers' tastes resulting in higher demand or lower demand elasticity for the advertised product (see for example Galbraith (1958) and Dixit and Norman (1978)). Kotowitz and Mathewson (1979b) examine the "persuasive" aspect of advertisement without including advertisement into consumers' utility. This paper examines the use of environmental advertisement in the form of information provision as a public policy instrument.

In the environmental economics literature there are only a few papers addressing the issue of environmental advertisement following either of the above mentioned approaches. The most closely related work is Petrakis, et. al. (2005) which in a similar framework shows –using static analysis– that information provision could dominate, in some cases, environmental taxation in terms of welfare and that a combination of these two policies is welfare improving. They also examine the way in which each group of consumers is affected by information provision. The present paper differs considerably since it employs a richer structure of the way in which information provision affects consumers' behavior and uses a dynamic framework.

An earlier study by Kennedy et. al. (1994) examines also environmental information provision. The framework of their analysis differs substantially from the current paper. It is assumed that the polluting good generates environmental damages (there are no individual damages) and consumers cannot relate with certainty these damages to their utility. Information is provided to consumers at a cost by private firms. The informed consumers know the true marginal external damage and take into account the effect that their own consumption has on their utility. In the current paper we assume that each individual's consumption of the dirty good has a negligible effect on the total external damage affecting her utility, and for that reason we do not introduce environmental damages in individuals' utility. Instead, we assume that there are individual damages associated with the consumption of the dirty good. Furthermore Kennedy, Laplante and Maxwell assume that the uninformed consumers have a belief over the value of the marginal external damage which is positive and could exceed its true value. Under this assumption they show that even costless information provision could decrease welfare. Contrary to our results they find that environmental taxation alone can attain the social optimum since the only distortion is the environmental externality. The only case in which public information provision can play a role is when taxation induces private information provision. The structure of our model allows for a much richer role of information provided by the government.

The rest of the paper is organized as follows. Section 2 presents the model, Section 3 presents the policy options, while Section 4 defines the optimal mix. Section 5 presents the simulations using linear demand and Section 6 concludes the paper.

# 2 The model

Assume a horizontally differentiated product that generates individual and environmental damages. The product is offered in two types, and the magnitude of both individual and external damages differs between them. For simplicity, we normalize emission units so that the "clean" type of the product does not generate any damages, while the "dirty" type generates positive individual and external damages. We further assume that if consumers are informed, they take into account the individual damages in making their consumption choices. Utility derived from the dirty good shrinks due to the individual damages, while that from the clean good expands as consumers learn about the bad characteristics of the dirty good. The informed consumer derives higher utility from the consumption of the clean relative to the dirty good. We denote the utility differential per unit of consumed product by the parameter $\theta$.

The utility of the representative informed consumer is,[6]

$$U(q_c, q_d, \theta, \gamma) \ , \tag{1}$$

[6] We assume that the utility derived from the two types of the product is linearly separable from the utility of all other products. That is, total utility is $U(q_c, q_d, \theta, \gamma) + I$, where $I$ is the utility derived from the consumption of other goods. This assumption implies that there are no income effects and allows us to perform partial equilibrium analysis.

where $q_j$ $(j = c, d)$ are the quantities consumed of the clean and the dirty good respectively and $\gamma$ measures the degree of substitutability between the two types of the product.[7] Thus, we allow for two dimensions of product heterogeneity, vertical product differentiation based on individual damages[8] and horizontal product differentiation based on variety. We further assume that the utility function in (1) exhibits the standard properties to yield indifference curves that are negatively sloped and strictly convex.

We further assume that only a fraction of the consumers are informed about the individual damage that the dirty good generates. For simplicity, we assume two groups of consumers, those with perfect knowledge of the negative effect associated with the dirty good and those that have no knowledge at all. The informed consumers, which form $\mu$ fraction of the population, make their choices based on the correct values of the parameter $\theta$, while the uniformed consumers set $\theta = 0$. Thus, they are unable to distinguish between the two types of the product in terms of individual damages and they differentiate between them based only on $\gamma$. The total population of the consumers is normalized to unity.

Maximization of (1) subject to the budget constraint yields the direct demand function for each type of the product,

$$q_j(p_j, p_k, \gamma, \theta) \ , \tag{2}$$

where $j, k = c, d$, and $j \neq k$. The total demand for the clean and the dirty type of the product is, $Q_d = \mu q_{di} + (1-\mu)q_{dn}$ and $Q_c = \mu q_{ci} + (1-\mu)q_{cn}$ respectively, where $q_{di}$ and $q_{dn}$ ($q_{ci}$ and $q_{cn}$) are the quantities of the dirty (clean) good consumed by the informed and the uniformed consumer, respectively. That is, $q_{ji} \equiv q_j(p_j, p_k, \gamma, \theta)$ and $q_{jn} \equiv q_j(p_j, p_k, \gamma)$, where $j = c, d$.

The product is offered by two oligopolists, each offering only one of the two types of the product and they compete in prices. For simplicity we assume that

[7] The degree of substitutability between the two products reflects the fact that the product is non homogeneous and its level affects also the market structure. For example, in the linear model we employ in Section 5, we assume that the two products are substitute and thus, $\gamma$ is stricIty positive. If $\gamma = 0$, each firm has monopolistic market power, while if $\gamma = 1$, the products are perfect substitutes.

[8] We assume that the only measure of quality in a vertical sense that is different between the two types of the product is $\theta$.

they both produce with the same technology and the production cost is $C(q_j)$, with $C_q > 0$ and $C_{qq} \geq 0$. Firm $j$'s profit maximization problem is,

$$\max_{p_j} \pi_j \left(p_j, C(q_j(p_j, p_k, \gamma, \theta)), Q_j\left(p_j, p_k, \gamma, \theta, \mu\right)\right) \ .$$

The first order condition of the profit maximization problem, $\pi_j^{'}\left(p_j, p_k, \gamma, \theta\right) = 0$, yields oligopolist $j$'s reaction function $p_j = R_j(p_k, \gamma, \theta)$, assuming that each oligopolist's profit function is strictly concave on its own price everywhere, which implies that the second order condition $\pi_j^{''} < 0$ for a maximum is satisfied. We assume that both reaction functions exists, are interior solutions and given strict concavity of the profit function are unique. We further assume that $\frac{\partial \pi_j / \partial p_j}{\partial p_k} > 0$, which implies that the slope of the reaction function is positive, $\frac{\partial R_j}{\partial p_k} > 0$, making oligopolists' prices strategic complements. The two oligopolists' reaction functions are solved for the Nash equilibrium prices, $p_j\left(\mathbf{z}\right)$, where $\mathbf{z}$ is the vector of demand and cost parameters including $\theta$ and $\gamma$.

# 3 Policy options

In the absence of any regulatory intervention, we have two distortions related to the characteristics of the dirty good: first, an information asymmetry, since only a fraction of the consumers take into account the individual damage, and second an externality, one that cannot be eliminated even when all consumers are informed. Assuming that the government does not intervene separately to correct the market distortion arising from imperfect competition, any policy attempting to correct the two problems mentioned above, within a welfare maximizing framework, would have to take into account the existing market distortion. In what follows, we examine the case in which the regulator uses a combination of a tax on the dirty good, $\tau$, and information provision.

We model information provision as follows. The regulator provides a level of information $a(t)$ at each time period $t$. We assume that the provision of information increases the proportion of consumers that behave as informed consumers. However, it is not just the level of currently provided information that affects the fraction of informed consumers but rather the stock of information accumulated at time $t$. That is, we assume that while information provision directly persuades consumers that are not currently informed, those who are currently informed tend to forget and behave as uninformed.[9] We denote the stock of information at time $t$ by $A(t)$, which summarizes current and past information provision efforts. It is reasonable to assume that information provided in the past is less effective than currently provided information. We model this assumption by treating information provision as a capital good,

$$\dot{A} = a - \delta A \ , \tag{3}$$

assuming a constant rate of depreciation $\delta$.[10]

We assume that when the stock of effective information accumulated at time $t$ is $A(t)$, then a fraction $\phi(A)$ of the uninformed consumers become informed. The following properties for the informed consumers generating function (ICGF) $\phi(A)$ are assumed,[11]

$$\begin{aligned} &\phi(A) : \mathbb{R}_+ \rightarrow [0,1] \, , \phi_A(A) \geq 0 \ , \\ &\phi(0) = 0, \ \phi\left(\bar{A}\right) = 1, \ \bar{A} \leq \infty \ . \end{aligned} \tag{4}$$

These assumptions mean that an increase in the stock of accumulated information will never turn informed consumers to uniformed; zero information stock could not generate informed consumers, while there could exist a finite level of information stock at which all consumers become informed. We do not however restrict the ICGF to diminishing returns for $A \in (0, \infty)$, which implies

[9] We assume that this decay in the number of informed consumers does not apply to the initial fraction of consumers that behave as informed. These consumers have acquired their information through different channels and their behavior is not affected by the government's information provision policy.

[10] The classical paper by Nerlove and Arrow (1962) introduced the following model of the dynamic effects of advertising: $\dot{A} = a - \delta A$, where $A$ is the level of "goodwill" at time $t$, which affects consumers demand, $a$ is the level of advertising (in monetary terms) at time $t$ and $\delta$ is the depreciation rate of "goodwill". This model has been used extensively in the advertisement literature.

[11] To avoid misinterpretations we use, in many cases, subscripts to denote derivatives with respect to a certain variable.

that there could be intervals of increasing returns in the generation of informed consumers. The ICGF shares common characteristics with the sales response function to advertising which is commonly used in the advertisement literature. Thus, in order to characterize the shape of ICGF we turn to the advertisement literature. The view that advertisement exhibits some degree of economies of scale is widely acceptable by both theoreticians and practitioners.[12] Moreover, in the most recent literature an S-shaped response function to advertisement has been used extensively (see for example Feinberg (2001)). The S-shaped response function implies increasing marginal returns to advertising for low advertising levels followed, after an inflection point, by decreasing marginal returns. Despite the continuing debate on the shape of the advertising response function,[13] we adopt the view that consumers' response to the current stock of information is S-shaped. Therefore, the ICGF exhibits the following properties, in addition to those presented in (4),

$$\begin{aligned} \phi_A &\geq 0,\ \phi_A(0) \geq 0,\ \lim_{A\to\infty} \phi_A(A) = 0\ , \\ \exists A_{\text{inf}} &: \phi_{AA} \geq 0 \text{ for } A \in [0, A_{\text{inf}}] \text{ and } \phi_{AA} < 0 \text{ for } A \in (A_{\text{inf}}, \bar{A})\ , \end{aligned}$$

where $A_{\text{inf}}$ denotes the ICGF's inflection point.

Based on the above discussion, the fraction of the informed consumers $m(t)$ at each point of time is,

$$m(t) = \mu + (1 - \mu)\,\phi(A(t))\ ,$$

given an initial fraction of informed consumers $\mu$, $0 \leq \mu < 1$. The cost of providing information to consumers, $K(a)$, with $K(0) = 0$, is assumed to be increasing, $K_a > 0$ at an increasing rate, $K_{aa} > 0$.[14]

[12] It should be noted however, that there are some empirical studies showing little or no evidence of substantial returns to scale in advertisement (see for example Arndt and Simon (1983) and Seldona, Jewellb and O'Briena (2000)).

[13] See for example Cannon, Leckenby, and Abernethy (2002) and Dube, Hitsch and Manchanda (2005).

[14] Grossman and Shapiro (1984) use an advertisement cost function with the same properties in a model of product differentiation. In support of the assumption regarding the increasing rate at which the cost of advertisement rises, they argue that "..it becomes increasingly expensive to reach higher fractions of the population, either because preferred media become saturated, or because the target population is heterogeneous along a second dimension, namely, the tendency to view ads." (p. 66).

Within this policy framework, that is, a tax on the dirty good, $\tau$, and information provision, the two firms' profit maximization problem is,

$$\begin{aligned} \max_{p_c} \pi_c &= \pi_c \left(p_c, C(q_j(p_j, p_k, \gamma, \theta)), Q_c\left(p_{cj}, p_d, \gamma, \theta, m(\phi\left(A\right))\right)\right) , \\ \max_{p_d} \pi_d &= \pi_c \left(p_d, C(q_j(p_j, p_k, \gamma, \theta)), Q_d\left(p_d, p_c, \gamma, \theta, m(\phi\left(A\right)), \tau\right)\right) . \end{aligned}$$

Given that the assumptions made in the previous section regarding the profit and the resulting reaction functions hold, the oligopolists' reaction functions are solved for prices,

$$p_j\left(\tau, \phi\left(A\right); \mathbf{z}\right) . \tag{5}$$

Oligopolists' choice variables are now functions of the two policy instruments, $\tau$ and $A$, in addition to cost and demand parameters presented by the $\mathbf{z}$ vector.

# 4 Optimal policy mix

Substituting the values of $p_c$ and $p_d$ from equation (5) into the direct demand functions given in equation (2), yields $q_{ji}(\tau, \phi(A); \mathbf{z})$ and $q_{jn}(\tau, \phi(A); \mathbf{z})$, from which we obtain, $Q_j(t, \phi(A); \mathbf{z})$, $j = c, d$. Substituting the values of $q_{ji}(\tau, \phi(A); \mathbf{z})$ and $q_{jn}(\tau, \phi(A); \mathbf{z})$ into the individual's utility function yields the indirect utility of the informed $V_i(\tau, \phi(A); \mathbf{z})$, and the uniformed $V_n(\tau, \phi(A); \mathbf{z})$ consumer, both evaluated at the true values of $\theta$. That is, in deriving the optimal policy instruments, the regulator takes into account the full cost of the dirty type of product and thus, it uses the true value of the uninformed consumer's utility, even though the consumer does not take into account the individual damages when making her choices.

The regulator maximizes the present value of the sum of the consumer and producer surplus, that is, $v\left(\phi(A), \tau; \mathbf{z}\right) = mV_i(\tau, \phi(A); \mathbf{z}) + (1-m)V_n(\tau, \phi(A); \mathbf{z}) - c\left[Q_c(\tau, \phi(A); \mathbf{z}) + Q_d(\tau, \phi(A); \mathbf{z})\right] - D(Q_d(\tau, \phi(A); \mathbf{z}))$, where $D(Q_d)$ are the dirty good's external damages, which we assume to be convex on dirty goods' production, $D'$and $D'' > 0$. The government chooses the optimal time paths for the tax $\tau(t)$ and information provision $a(t)$ by solving the following problem,

$$\max_{\{a(t),\tau(t)\}} \int_0^\infty e^{-\rho t} \left[v\left(\phi(A),\tau;\mathbf{z}\right) - K(a)\right] dt$$

subject to:

$$\dot{A} = a - \delta A \quad A\left(0\right) = A_0 \geq 0$$

$$\phi\left(A\left(t\right)\right) \in \left[0,1\right], \forall t \geq 0$$

where $\rho$ is the discount rate and $K(a)$ is the cost of advertisement which we assume to be increasing at an increasing rate, $K_a$ and $K_{aa} > 0$. This is a formal optimal control problem with a pure state constraint $\phi\left(A(t)\right) \in \left[0,1\right], \forall t \geq 0$.

The current value Hamiltonian for the optimal control problem is,

$$\mathcal{H} = v\left(\phi(A),\tau;\mathbf{z}\right) - K(a) + \lambda\left(a - \delta A\right) + \xi_1\left(1 - \phi\left(A\right)\right) + \xi_2 \phi\left(A\right) \ . \tag{7}$$

where $\lambda$ is the costate variable reflecting the shadow price of the stock of accumulated information and $\xi_i, i = 1,2$ are multipliers associated with the pure state constraint $0 \leq \phi\left(A\right) \leq 1$. The necessary conditions for the choice of the optimal policy instruments $a$ and $\tau$ yield,

$$\frac{\partial \mathcal{H}}{\partial a} = 0 \ or \ K_a = \lambda \ , \text{ and } a^0 = a\left(\lambda\right) \ , \tag{8}$$

$$\frac{\partial \mathcal{H}}{\partial \tau} = 0 \ or \ \frac{\partial v\left(\phi(A),\tau;\mathbf{z}\right)}{\partial \tau} = v_\tau = 0 \ , \text{ and } \tau^0 = \tau\left(A\right) \ . \tag{9}$$

The paths for $\lambda$ and $A$ evaluated at the optimal choices $\left(a^0, \tau^0\right)$ should satisfy

$$\dot{\lambda} = \rho\lambda - \frac{\partial \mathcal{H}}{\partial A} \ , \ \dot{A} = \frac{\partial \mathcal{H}}{\partial \lambda} \ . \tag{10}$$

with $\xi_i = 0$, $i = 1,2$ if $0 < \phi\left(A\right) < 1$. If the state constraint is not effective, then the multipliers can be set to zero. This means that in this case we study interior solutions to the problem and the the optimal path for $A\left(t\right)$ remains in the interior of set $\left[0, \bar{A}\right]$ such that $\phi\left(A\left(t\right)\right) \in \left(0,1\right)$ for all $t$. For the rest of the paper we consider interior solutions by assuming $\phi\left(\infty\right) = 1$, $\phi'\left(0\right) > 0$. Using (8) and the fact that $K_{aa}\dot{a} = \dot{\lambda}$ to eliminate $\lambda$ and $\dot{\lambda}$ from (10), the modified

Hamiltonian dynamic system (MHDS) can be written in the control-state space as:

$$\dot{a} = (\rho+\delta)\,a - \frac{1}{K_{aa}} v_\phi \phi_A \ , \quad (11)$$

$$\dot{A} = a - \delta A,\ A(0) = A_0 \ . \quad (12)$$

An optimal steady state (OSS) in the stock $A$ and the flow $a$ of information provision is defined as $(A^*, a^*) : \dot{A} = 0,\ \dot{a} = 0$. To study the properties of the OSSs for interior solutions $A^* \in (0, \infty)$ we make the following additional assumptions:

- A1: The cost of advertisement $K(a)$ is, without loss of generality, quadratic, so that $K_{aa}$ is a constant.
- A2: The social welfare functional $v(\phi(A), \tau; \mathbf{z})$ which defines, for given parameters $(\tau, \mathbf{z})$, a map from the vector space containing $\phi$ to the real numbers, has the following properties:
  - The functional derivative (Fréchet derivative) $Dv = v_\phi \geq 0$ is non-negative and bounded, and $Dv > 0$ for $A = 0$.
  - It holds for the second derivative $D^2 v = v_{\phi\phi} \leq 0$

These assumptions imply that an increase in the fraction of the informed consumers does not reduce social welfare, an increase in the fraction of the informed consumer when the stock of effective accumulated information is close to zero increases social welfare and that the rate of increase in social welfare from an increase in the fraction of informed consumers is non increasing.[15]

**Proposition 1** *Under assumptions A1 and A2 and an S-shaped ICGF two at the most OSSs exist,* $0 < A_1^* < A_2^* < \infty$. *If two OSSs exist then the smaller one is unstable , while the larger one is a local saddle point. If one OSS exists then it is a saddle point.*

[15] It is verified that these assumptions are satisfied in our numerical simulations.

***Proof.*** *A steady state occurs at the intersection of the isocline* $\psi^a(A)|_{\dot{a}=0} = \frac{v_\phi \phi_A}{(\rho+\delta)K_{aa}}$, *defined through* $0 = (\rho+\delta)\,a - \frac{1}{K_{aa}} v_\phi \phi_A$ *with the isocline* $\psi^A(A)\Big|_{\dot{A}=0} = \delta A$ *defined through* $0 = a - \delta A$. *Using A1 and A2 and our assumptions about* $\phi_A$ *it follows that* $\lim_{A\to 0} \psi^a(A) > 0$ *and* $\lim_{A\to\infty} \psi^a(A) = 0$. *Furthermore since* $\phi$ *is S-shaped* $\phi_A(A)$ *and therefore* $\psi^a(A)|_{\dot{a}=0}$ *are single peaked, with* $\psi^a(A)$ *increasing around* $A = 0$ *and asymptotically approaching zero as* $A \to \infty$. *On the other hand* $\psi^A(A)$ *is a ray from the origin with positive slope* $\delta$. *Therefore* $\psi^a(A)|_{\dot{a}=0}$ *and* $\psi^A(A)\Big|_{\dot{A}=0}$ *will have in general zero, one, or two intersections.*[16]. *The Jacobian matrix of the MHDS defined by* (11) *and* (12) *evaluated at a steady state is:*

$$J = \begin{pmatrix} \rho+\delta & -\frac{1}{K_{aa}}\left(v_{\phi\phi}(\phi_A)^2 + v_\phi \phi_{AA}\right) \\ 1 & -\delta \end{pmatrix} = J(a^*, A^*)\ . \qquad (13)$$

*Since* $tr(J) = \rho > 0$ *and the eigenvalues of (13) are* $\beta_{1,2} = \frac{1}{2}\rho \pm \sqrt{\rho^2 - 4\det(J)}$, *where* $\det(J) = -\delta(\rho+\delta) + \frac{\sigma(A)}{K_{aa}}$, *and* $\sigma(A) = v_{\phi\phi}(\phi_A)^2 + v_\phi \phi_{AA}$, *the steady state will be either unstable or it will have the local saddle point property. If* $\sigma(A) < 0$, *then* $\det(J) < 0$, *the eigenvalues are real numbers, and thus the OSS is a local saddle point. Given the assumptions A2, for* $\sigma(A)$ *to be negative it suffices that* $\phi_{AA} < 0$. *Furthermore, since the slope of* $\psi^a(A)$ *is* $\frac{d\psi^a(A)}{dA} = \frac{\sigma(A)}{(\rho+\delta)K_{aa}}$, *then the saddle point OSS occurs at the declining part of the* $\psi^a(A)$ *curve. On the contrary, if* $\sigma(A) > 0$, *and* $\det(J) > 0$ *then the OSS is unstable. However, a necessary condition for* $\sigma(A) > 0$ *is that* $\phi_{AA} > 0$, *while the sufficient condition is* $v_\phi \phi_{AA} > -v_{\phi\phi}(\phi_A)^2$. *Therefore, if the sufficient condition for* $\sigma(A) > 0$ *holds, the unstable OSS occurs at the increasing part of the* $\psi^a(A)$ *curve. Since* $\psi^A(A)$ *is strictly increasing, if the pair* $(A_1^*, A_2^*)$ *denotes the unstable and the saddle point OSSs respectively, then* $0 < A_1^* < A_2^* < \infty$. *If the* $\psi^a(A)|_{\dot{a}=0}$ *and the* $\psi^A(A)\Big|_{\dot{A}=0}$ *intersect only once this can take place only at the declining part of the* $\psi^a(A)$ *isocline ant therefore the unique OSS is a saddle point. It should be noted that, the maximized Hamiltonian* $H^0(q,\lambda) = \max_a H(A,a,\lambda)$ *is defined as* $H^0(A,\lambda) = v(\phi(A)) + \frac{\lambda^2}{K_{aa}} - \lambda\delta\phi$. *If* $\sigma(A) < 0$ *the maximized Hamiltonian*

[16] We do not consider hairline cases where the two isoclines are tangent.

*is concave in the state variable $A$, so the Arrow-Kurz sufficiency condition for the Pontryagin's principle are satisfied at the saddle point OSS.* ■

The possibility of none, one or two OSSs , resulting from the intersections of the $\psi^a(A)|_{\dot{a}=0}$ and $\psi^A(A)\Big|_{\dot{A}=0}$ isoclines, is shown in figure 1. For saddle points, $A_2^*$ or $A^*$ convergence takes place along the stable manifold $MM$. For any initial value of the information stock $A$, in the neighborhood of the steady state, there exist an initial value of the flow of information provision $a$ such that there is convergence to the optimal steady state on the one dimensional manifolds $MM$. For example, if the initial stock of information is $A_0$ the converging path is along $ME$.

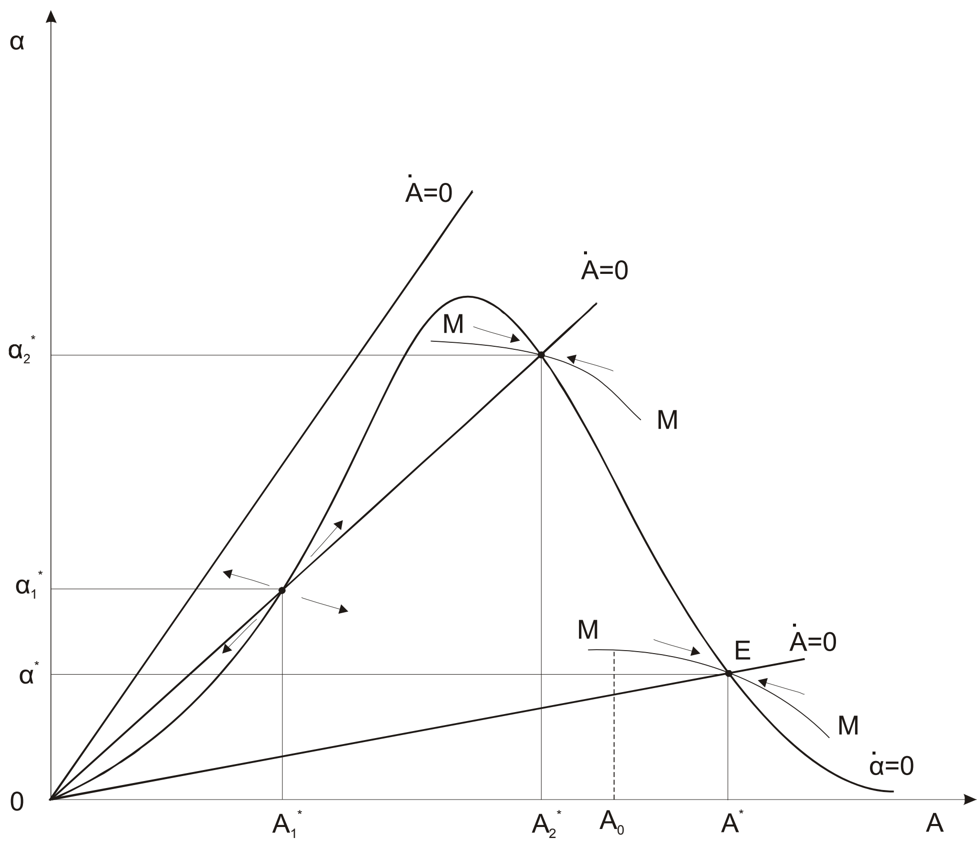


Figure 1. Optimal Steady State

We can further characterize the optimal policy by considering a linearization around an OSS $(A^*, a^*)$ with the saddle point property which can be written as

$$\begin{pmatrix} \dot{a} \\ \dot{A} \end{pmatrix} = J(A^*, a^*) \begin{pmatrix} a - a^* \\ A - A^* \end{pmatrix}$$

Let $\beta_1$ be the negative eigenvalue of the Jacobian $J(A^*, a^*)$, then the linear approximations of the optimal paths for the stock and flow of information in the neighborhood of the steady state are,

$$a^*(t) = C_1 \zeta_{11} e^{\beta_1 t} + a^* , \quad (14)$$

$$A^*(t) = C_1 \zeta_{21} e^{\beta_1 t} + A^* , \quad (15)$$

where $(\zeta_{11}, \zeta_{21})'$ is the eigenvector associated with $\beta_1$ and $C_1$ is a constant determined by initial conditions on $A$. Dividing the equations for the optimal paths we obtain

$$a^*(t) = \frac{\zeta_{11}}{\zeta_{21}} (A^*(t) - A^*) + a^* . \quad (16)$$

This is the linear approximation of the policy function and indicates the optimal amount of information provision at each point in time given the existing stock of information provision. This policy function is a linear approximation of the stable manifold in the neighborhood of the steady state.[17]

Having determined the optimal paths for information provision, the corresponding optimal path for the tax $\tau^*(t)$ can be obtained by solving the optimality condition (9) for $\tau(t)$, with $A$ replaced by the optimal path $A^*(t)$. Therefore,

$$\tau^*(t) : \frac{\partial v(\phi(A^*(t)), \tau^*(t); \mathbf{z})}{\partial \tau} = 0 . \quad (17)$$

The trade-off between the stock of information provision and taxation, at each point in time can be obtained by using the implicit function theorem in (17) to obtain

$$\frac{d\tau^*(t)}{dA^*(t)} = - \left. \frac{v_{\tau\phi}\phi_A}{v_{\tau\tau}} \right|_{A=A^*(t)} .$$

Let $a^*(t) = G(A^*(t))$ denote the optimal policy function in the neighborhood of a steady state with the saddle point property, which can be defined

[17] Solution for the nonlinear stable manifold which will be the nonlinear policy function can be obtained by using time elimination or multiple shooting methods.

either as the linear approximation (16) or as a nonlinear function corresponding to a stable manifold like $M_1M_1$ or $M_3M_3$ in figure 1. The monotonicity of the policy function implies that an inverse policy function $A^*(t) = g(a^*(t))$ exists. Substituting this function into (17) the trade-off between the two policy instruments, that is environmental taxation and the flow of information provision, along the optimal path can be defined as:

$$\frac{d\tau^*(t)}{da^*(t)} = - \left. \frac{v_{\tau\phi}\phi_A g_a}{v_{\tau\tau}} \right|_{a=a^*(t)} .$$

# 5 Numerical results using linear demand

In this Section we utilize specific functional forms for the utility and cost functions in order to obtain more tractable results for the model developed above. In order to incorporate both horizontal and vertical differentiation characteristics, the utility of the representative informed consumer is:[18]

$$U(q_c, q_d, \theta_d, \gamma) = (\alpha + \theta_c)\, q_c + (\alpha - \theta_d) q_d - \frac{1}{2}\left(q_c^2 + q_d^2 + 2\gamma q_c q_d\right) + I \ ,$$

where $I$ is the numeraire good produced by a competitive sector. Thus, utility is quadratic in the consumption of $q_j$ goods and linear in the consumption of other goods $I$. The parameter $\gamma \in [0, 1]$ measures the degree of substitutability between the types of the product. In this paper we assume that the two types of the product are less than perfect substitutes, that is, $\gamma < \gamma_k < 1$, where $\gamma_k$ is the critical value of the degree of substitutability guaranteeing that both informed and uninformed consumers purchase, in all cases examined, positive quantities of both types of the good.

The consumer's utility maximization problem yields direct demands for each good,

$$q_c = \frac{A_c + \Theta_c}{1 - \gamma^2}, \quad q_d = \frac{A_d - \Theta_d}{1 - \gamma^2}, \tag{18}$$

where $A_j = (1-\gamma)\alpha + \gamma p_k - p_j$, and $\Theta_j = \theta_j + \gamma\theta_k$, with $j, k = c, d$, and $j \neq k$.

[18] A similar type of utility function has been introduced by Dixit (1979) and used in many works such as Singh and Vives (1984). For a comprehensive and complete presentation, see Martin (2002).

For the ICGF we assume a simple algebraic sigmoid function of the following form,

$$\phi(A) = \frac{A^2}{1+A^2} \ ,$$

which is illustrated in figure2. This functional form satisfies all the properties we have assumed in the general model and has an inflection point at $A_{\text{inf}} = \frac{1}{\sqrt{3}}$.

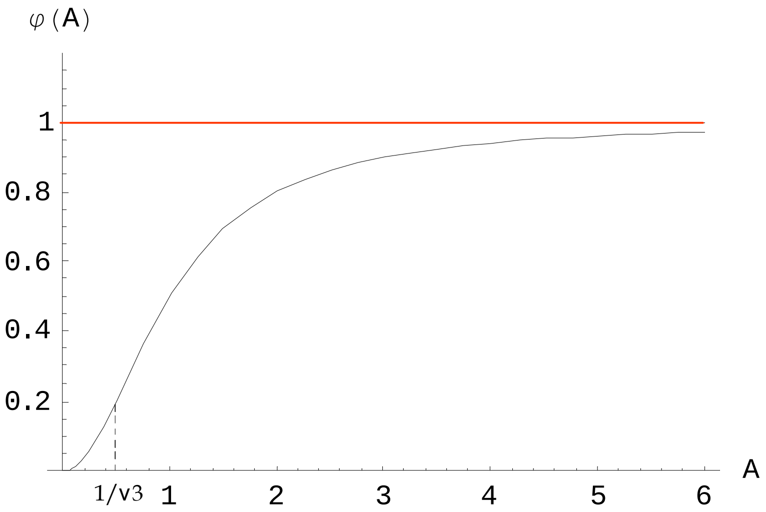


Figure 2. The ICGF sigmoid function

On the production side, in order to simplify the exposition, we assume that the two oligopolists produce with the same constant marginal cost $c$. Each firm's profit maximization problem is,

$$\begin{aligned} \max_{p_c} \pi_c &= (p_c - c)\left[mq_{ci} + (1-m)q_{cn}\right] \ , \\ \max_{p_d} \pi_d &= (p_d - \tau - c)\left[mq_{di} + (1-m)q_{dn}\right] \ . \end{aligned}$$

The reaction functions resulting from the duopolists' profit maximization problems are solved for the prices as functions of the two policy instruments,

$$p_c = \frac{B + m\Theta_c^p + \gamma\tau}{4-\gamma^2} \ , \text{and} \ \ p_d = \frac{B - m\Theta_d^p + 2\tau}{4-\gamma^2} \ . \tag{19}$$

where $B = (2+\gamma)[(1-\gamma)\alpha + c]$ and $\Theta_j^p = \left(2-\gamma^2\right)\theta_j + \gamma\theta_k$, with $j,k = c,d$ and $j \neq k$.

Substituting the values of $p_c$ and $p_d$ from equation (19) into the direct demand functions given in equation (18), yields $q_{ji}(\tau, \phi(A); \mathbf{z})$ and $q_{jn}(\tau, \phi(A); \mathbf{z})$, from which we obtain, $Q_j(t, \phi(A); \mathbf{z})$, $j = c, d$. Substituting the values of $q_{ji}(\tau, \phi(A); \mathbf{z})$ and $q_{jn}(\tau, \phi(A); \mathbf{z})$ into the individual's utility function yields the indirect utility of the informed $V_i(\tau, \phi(A); \mathbf{z})$, and the uniformed $V_n(\tau, \phi(A); \mathbf{z})$ consumer, both evaluated at the true values of $\theta$.

The regulator maximizes the present value of the sum of the consumer and producer surplus, that is, $v(\phi(A), \tau; \mathbf{z})$, as defined in the theoretical part of the paper. To further simplify the model we assume that the dirty good's external damages are linear in output, that is, $D(Q_d) = dQ_d$. The cost of advertisement is assumed to be quandratic $K(a) = \omega\frac{a^2}{2}$.[19] Each time period the government chooses the optimal level of the tax $\tau(t)$ and information provision $a(t)$ by solving the maximization problem stated in the theoretical part of the paper.

In order to define the optimal steady state in the stock $A$ and the flow $a$ of information provision $(A^*, a^*)$, we set the following values for the model's parameters: $a = 60$, $\theta = 15$, $\gamma = 0.4$, $\mu = 0.1$, $c = 15$, $\rho = 0.03$, $\delta = 0.01$, $\omega = 100$ and $d = 20$.

From the solution of the modified Hamiltonian dynamic system (MHDS), equations (11) and (12), for $A$ and $a$, we define the isoclines $\psi^a(A)|_{\dot{a}=0}$ and $\psi^A(A)\Big|_{\dot{A}=0}$ which are presented in figure 3. For expositional purposes figure 3 presents only part of the $\psi^a(A)|_{\dot{a}=0}$ isocline. The two isoclines intersect at two points. The coordinates of the first intersection, which corresponds to point X in the diagram, are $(0.0007929, 0.07929)$, and the coordinates of the second, corresponding to point Z, are $(0.128323, 12.8323)$. Recall that we have shown in the general case that the optimal steady state point corresponds to the intersection at the declining part of the $\psi^a(A)|_{\dot{a}=0}$ isocline. The simulations confirm this result since the eigenvalues corresponding to point X are complex with positive real parts, while the eigenvalues

[19]Tirole (1989), p. 293, employs this particular quadratic advertisement cost function.

corresponding to point Z are real $(-0.999846, 0.0175471)$. Therefore, in our example, the optimal steady state with the local saddle point property corresponds to a stock of advertisement $A^* = 12.8323$ and a flow of advertisement $a^* = 0.128323$. At the optimal steady state a fraction $\phi(A^*) = 0.993964$ of uninformed consumers become informed and the total fraction of informed consumers is $m = \mu + (1-\mu)\,\phi\,(A^*) = 0.994567$.

As in the general case, convergence to the optimal steady state occurs on the manifold MM. Starting, in the neighborhood of $Z$, from any level of the stock of information away from the optimal, the government can determine the path of the flow of information $a^*\,(t)$ leading to the optimal OSS $(A^*, a^*)$.

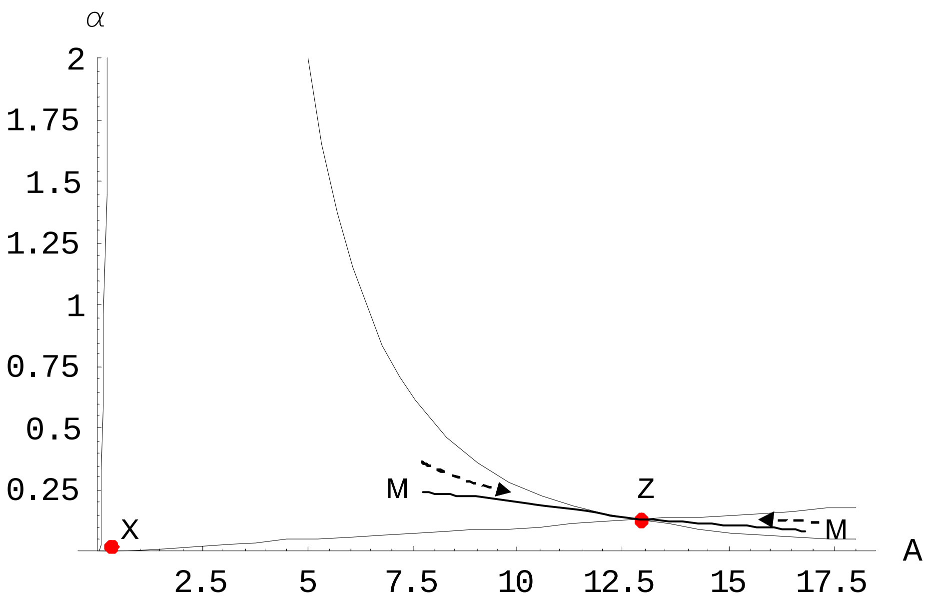


Figure 3. Steady states in the linear example

Figure 4 depicts the tax $\tau$ as a function of the stock of information $A$, derived from the optimality condition (9). When the stock of advertisement is zero, the tax rate is high, since taxation attempts to correct both the externality and the information asymmetry. As the stock of information is building up, the required tax rate decreases, approaching its lowest value as the stock of advertisement approached its optimal steady state value $A^*$. The curve depicting the optimal tax response has a horizontally inverted S-shape, that is, there is a fast decrease in the tax rate for low values of $A$ and a slower decrease for higher levels of A.

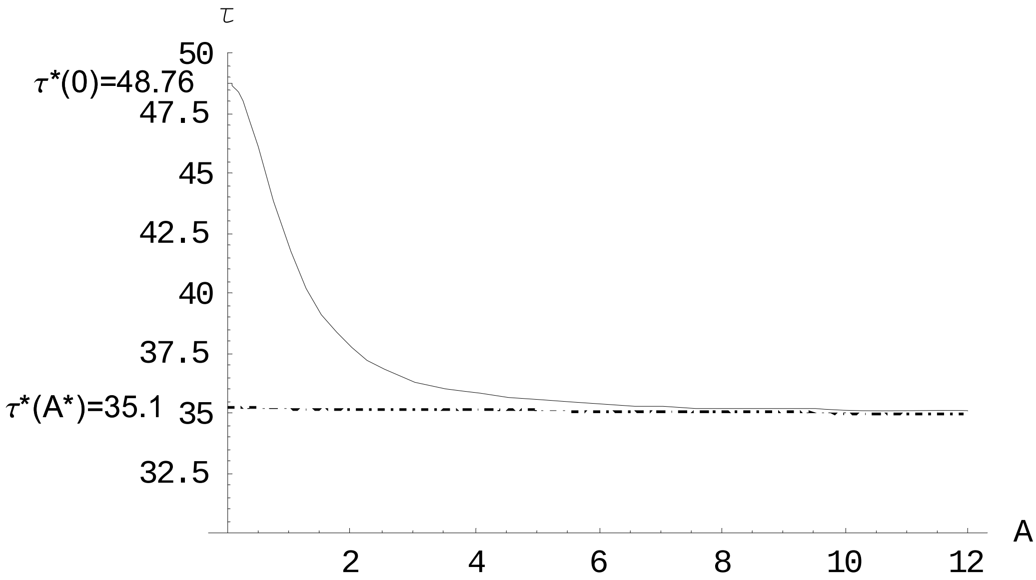


Figure 4. Tax response to changes in the stock of information

The numerical results of the linear demand model confirm the results of the general model. There exist one, under the assumptions of our model, optimal steady state with the saddle point property. As we approach the optimal steady state the optimal tax level decreases.

# 6 Conclusions

It is beyond any dispute that the public, either as consumers, workers or investors can play an important role in shifting production towards more healthy and environmentally safe products and processes if they have information about health and environmental risks. However, these risks are rarely common knowledge, and private firms that posses this information are unlikely to share them with the public voluntarily. Thus, the government has an incentive to provide reliable information in order to complement existing policies. The present paper examines the case of products that are responsible for both environmental and health damages. Given that consumers have incomplete information about health and environmental risks, we examine the role of information provision in supporting environmental taxation. We find that the combination of the two policy instruments is efficient since information provision results in lowering consumption of the good generating health and environmental damages and thus, it reduces the need for environmental taxation. Over time both the optimal tax rate and the degree of information provision decline resulting in declining costs, while the benefits are increasing.

Although in the present paper we do not introduce environmental damages in individuals' utility, a natural extension would be to examine cases in which consumers are willing to internalize part of the external damages they generate. In such cases, the government could provide the appropriate information in order to convince the public to, at least partially, internalize part of the environmental cost.

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