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Communication by Interest Groups and The Organization of Lobbying[1]

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This Version

Abstract: This paper uses a mechanism design approach to characterize the optimal organization of lobbying groups in a political context where those groups have private information on their ideal points in a one-dimensional policy space. First, we derive the optimal mechanism for one single group and show that it depends on the conflict of interests between his own preferences and those of the policy-maker but also on how informative the distribution of the interest group's ideal point is. We then extend the analysis to the case of multiple interest groups. Although dealing with a coalition of those groups allows the policy-maker to benefit from a more precise information (an *informativeness effect*), the optimal organization may nevertheless call for a decentralized mechanism where groups compete because this is the only way to transmit information on the relative strength of their preferences (a *screening effect*). A coalition of interest groups dominates for small values of the conflict of interests whereas competing interest groups emerge for greater values.

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1 Introduction

The number of interest groups active in the U.S. polity has been dramatically increased over the last four decades, going from 5000 in 1955 to over 33000 at the end of the twentieth century.[1] Major political scientists like Huntington (1975), Salisbury (1990) and Wilson (1979) among others have viewed the proliferation of interests as indicative of a more fragmented and atomistic political system. This tremendous increase in the number of interest groups may not only reflect a huge diversity in their objectives, in their audiences and in the related economic issues at stake but it may also be more surely related to the structure of the fundamental transaction costs which shape interactions in the political arena. Even though understanding the general macro-organization of interest groups is of a tantamount importance to explain the design and implementation of economic policies, still very little is known on the fundamental incentives which induce those groups to merge or to stay split apart. Indeed, even if the number of interest groups has raised, the pattern of behavior and the mix of competition and cooperation among those groups is more complex. This important fact of modern politics has been noticed by several political scientists, most noticeably Hula (1999) who argued that *"the macroeconomic view of the interest groups community often overlooks a number of institutional links between interest groups, most notably the increasing use of long-term, recurrent, and institutionalized coalitions in many policy arenas"*.

In this paper, we present a theoretical framework to understand this macro organization of the lobbying activity. This framework is based on the by-now well admitted fact, both among political scientists and economists, that lobbying has an important *informational role*.[2] Lobbyists spend time and resources to convey information on their preferences to uninformed political decision-makers and help them making choices which may please their own interests. That lobbying groups have private information puts them in a unique position to influence policy-making at least on the very issues on which their private interests are at stake. Because there may exist a conflict between the preferences of policy-makers and those of interest groups, the latters have incentives to manipulate information. The implemented policies result thus from a trade-off between choosing a rigid policy independent on the interest group preferences and communicating with the interest group to implement a more flexible policy but at the cost of letting the preferences of this group drive policy choice.

Informational asymmetries are thus crucial to understand the relationships between lobbying groups and policy-makers. Clearly, those asymmetries not only explain much of

[1] As a further example, the Encyclopedia of Associations (2002) counts more than 115,000 U.S. non-profit organizations with State, city and local scope and interests.

[2] See Milbrath (1963), Wright (1990), Hansen (1991) and more recently Grossman and Helpman (2001).

the policies implemented in practice but also generate the transaction costs that plague the relationships between policy-makers and interest groups. The overall organization of lobbying should then certainly be driven by the desire to minimize these transaction costs. Asymmetric information clearly provides thus a valuable perspective on the scope and size of interest groups, an issue which has been of great interest since at least the seminal work of Olson (1965).

To understand first how informational asymmetries shapes policy design, we start with a simple relationship involving a single interest group and a policy-maker both having different ideal points in a one-dimensional space of policy. The interest group has private information on his ideal point which is relevant to determine the optimal policy from the decision-maker's viewpoint. In that simple context, we analyze the determinants of the policy choice. As the conflict of interests between the interest group and the policy-maker is more pronounced, a rigid policy involving pooling across a larger set of preference profiles becomes more attractive. Communication is less valuable when the interest group has stronger incentives to manipulate information to influence decision-making. The same is true when the decision-maker has already a relatively precise idea of the interest group's preferences. Indeed, as the distribution of these preferences puts more weight around its mean, the decision-maker relies less on active communication. The optimal policy rule comes closer to the ex ante optimal rigid policy that the policy-maker chooses without relying on the interest group's information.

This *informativeness effect* is particularly crucial to understand one first force which could justify the formation of large interest groups. Consider indeed two interest groups with independently and identically distributed shocks on their ideal points who share their information in a credible way and merge as one active player. That coalition only needs to communicate the average preferences of the two groups to the decision-maker. This piece of information is less noisy than the type of each individual interest group taken separately. Dealing with such a coalition, the policy-maker gets a more precise idea of the relevant preferences profile and, by the same token, does not need to rely so much on communication.[3] There exist thus strong *scope economies* due to communicating private information.

Those scope economies favor the emergence of large coalitional groups compared with smaller ones holding more diffuse private information which is harder to communicate. Although intuitive, this insight turns out to be incomplete. When all interest groups merge as one, the statistics used by this coalition to communicate with the decision-maker may indeed be too rough to describe with enough precision the exact preferences

[3]In the limit, the Law of Large Numbers would apply and a coalition of a large number of potential interest groups with preferences independently distributed would provide very accurate information to the decision-maker.

profile. The information aggregation which already takes place within such a large group smooths out large discrepancies in the preferences of his individual members and cannot account for the diversity of opinions. Indeed, because less communication is needed with a coalition, a rigid policy is chosen for a larger set of preference profiles and, most noticeably, when the preferences of the groups are sufficiently far away from each other. A better communication pattern can then be implemented with a decentralized mechanism such that either one or the other group's ideal point is ultimately chosen by the policy-maker. Either group becomes then residual claimant for the policy implemented depending on how close he is to the decision-maker. When the conflict of interests between the principal and the interest groups is sufficiently pronounced, it is more efficient to deal separately with each of them than with a coalition.The corresponding decentralized mechanism improves both the decision-maker and the interest groups' expected payoffs. This *screening effect* justifies that it can be preferable to keep interest groups apart from each other even though this decentralized organization may not enjoy the scope economies of merging highlighted above. For lower levels of conflict instead, the informativeness effect dominates and justifies that a coalition of interest groups emerges.

To derive those insights on the organizational pattern of lobbying activity, we depart significantly from the existing literature on the informational role of interest groups. Starting with the seminal paper of Crawford and Sobel (1982), this literature has viewed lobbying groups as informed Stackelberg leaders in the communication game played with policy-maker.[4] Although attractive on positive grounds, that approach faces some difficulties when it comes to discuss the overall organization of lobbying. Cheap-talk signaling games are generally plagued with a multiplicity of inefficient partition equilibria[5] and in the absence of a convincing equilibrium refinement, the comparison of outcomes between alternative scenarios and organizational forms is only indicative of the forces shaping the overall organization of lobbying groups. Instead, we adopt in this paper a mechanism design perspective.[6] We revert the timing of standard lobbying games and assume that the policy-maker commits ex ante to a mechanism describing policy choices as a function of the various messages that might be sent by lobbyists on their preference profiles at the communication stage. This approach is in lines with Melumad and Shibano (1991) who earlier adopted that perspective although in the context of a single informed agent and Baron and Meirowitz (2001) who more recently offered some further links between the signaling and the mechanism design approach.[7] The mechanism design approach is useful on normative grounds because the Revelation Principle gives us a complete char-

[4]See Austen-Smith (1997) and Grossman and Helpman (2001) for synthetic views of this approach.

[5]A noticeable exception is the multidimensional world described in Battaglini (2002).

[6]Laffont (2000) offers a more general defense of this perspective to explain constitutional choices.

[7]More specifically, these authors argue that with a convenient modeling of out-of equilibrium beliefs capturing the lack of commitment assumption included in the signaling perspective, the two approaches are equivalent.

acterization of the whole set of implementable allocations achieved as equilibria[8] of the communication game played by lobbies. The comparison between the costs and benefits of different organizational forms of lobbying and the corresponding optimal mechanisms is thus facilitated.

Our normative approach comes also very close to the social choice literature which, following the seminal work of Moulin (1980),[9] has analyzed incentive compatible mechanisms in political contexts with single-peaked preferences as we do here. Contrary to most of this literature, we restrict the domain of preferences to quadratic ones. This of course leads us to describe a slightly larger set of dominant strategy mechanisms than without this restriction[10] and to characterize those mechanisms with a differentiable approach. More importantly, this restriction on preferences allows us to compare the expected payoffs that interest groups and the policy-maker obtain under various organizational forms. This is an important step of the analysis to get some normative insights on why groups coalesce or not.

Even though our mechanism design approach can be viewed as an alternative to the signaling literature in order to address the organization of lobbying, this literature has already provided valuable insights on those issues by comparing equilibria outcomes. Austen-Smith (1990, 1993a, 1993b) has analyzed communication patterns when groups report either sequentially or simultaneously. Krishna and Morgan (2000) have studied a lobbying game with two informed lobbyists who share the same information on the state of nature but may have conflicting or congruent views on what should be the optimal policy.[11] They showed that conflicting views help the policy-maker to extract information. Our mechanism design perspective would predict in such a context the existence of a costless fully communicative equilibria *irrespectively* of the bias.[12] This leads us to focus on the case where each interest group has private information. Grossman and Helpman (2001, Chapter 4) have also compared the informativeness of various equilibria depending on whether the lobbyists messages are public or private, an issue which is also put aside by the mechanism design approach.[13] Finally, Battaglini and Benabou (2003) have recently analyzed the incentives of interest groups to coalesce or not using the signaling approach. They study a common value environment where the extend of one group's lob-

[8]Modulo an implementation concept and here we will focus on dominant strategy for simplicity.

[9]See also Sprumont (1995) for a survey of this literature.

[10]Roughly speaking, discontinuous mechanisms are possible in our framework whereas they are not if the set of preferences is larger.

[11]See Lipman and Seppi (1995) also.

[12]This would be obtained by using a *revelation mechanism* à la Maskin. The issue may nevertheless be the existence of other non-truthful equilibria.

[13]Indeed, the fact that communication with each interest group is publicly observable is part of the mechanism. Relaxing that assumption would move us towards a third-best world where bilateral communication between the decision-maker and each interest group may create some scope for vertical collusion. On this issue in a related model, see Gromb and Martimort (2004).

bying depends on whether other groups show up in the lobbying process and on whether the decision-maker finds those activities trustworthy or not. Even though their methodology and the informational structure they use both differ from ours, we share with them results which are qualitatively similar. For small levels of conflict, they also show that a coalition of interest groups is preferred whereas a non-coordinated behavior dominates otherwise.

Section 2 presents the model. In Section 3, we derive the optimal mechanism which maximizes the expected payoff of the policy-maker when there is a single interest group. We give sufficient conditions for that mechanism to be continuous and highlight then its simple structure. Continuity is important to get a simple characterization of optimal mechanisms which is more easily amenable to comparisons between different organizational forms. We provide there also some comparative statics analyzing the impact of the level of conflict between the policy-maker and the interest groups and the role of the informativeness of the distribution of the latter's ideal point. This section is thus of independent value for readers interested in the mechanism design approach for lobbying games. In Section 4, we analyze the scope economies which arise when two groups coalesce to influence a policy-maker. Finally, in Section 5, we compare two organizations of the playing field for lobbying: one with a coalition of two groups, the other with two competing groups. We highlight there the better possibilities for screening which arise in the decentralized organization where groups remain split apart, at least when the conflict of interests is large enough. Section 6 concludes. Proofs are relegated to an Appendix.

2 The Model

In the economy, there are n special interest groups (or agents) indexed by $1, \dots, n$ and a single policy-maker (or principal). The policy q chosen by the principal is one-dimensional.[14] Interest groups have single-peaked quadratic preferences with ideal points θ_i:

$$U_i(q, \theta_i) = -\frac{1}{2}(q - \theta_i)^2.$$

The principal aggregates the preferences of those interest groups but may also take into account the preferences of the rest of society. We capture this effect in assuming that the principal also a quadratic utility function given by:

$$V(q, \theta_1, \dots, \theta_n) = -\frac{1}{2}\left(q - \frac{1}{n}\left(\sum_{i=1}^{n} \theta_i\right) - \delta\right)^2.$$

[14]This can be a tax or an import tariff for instance.

The parameter $\delta > 0$ represents the bias in the preferences of the policy-maker compared to the benchmark policy which would consist in averaging the interest groups' ideal points. That the policy-maker cares also on the interests of the general public even though it is not organized as an active lobby might be justified if, for instance, he has reelection concerns.

Importantly, the principal and the agents have non-monotonic preferences, a condition which is necessary for communication between the principal and the agents to take place in a mechanism design framework without monetary transfers.[15]

Interest group i has private information on his ideal point θ_i. The preference parameters $\theta_1, \dots, \theta_n$ are drawn identically and independently on the same interval $\Theta = [\underline{\theta}, \bar{\theta}]$ according to the cumulative distribution $F(\cdot)$ which has an atomless and everywhere positive density $f = F'$. We will sometimes assume that $F(\cdot)$ is log-concave (i.e. $\frac{d}{d\theta}\left(\frac{F(\theta)}{f(\theta)}\right) > 0$). We denote by $E_f(\theta)$ the expectation of θ according to $f(\cdot)$. Note that our formulation is such that all groups have a priori the same conflict with the principal even though ex post their ideal points may differ. They both tend to prefer on average a more conservative policy than the principal's ideal point.[16]

We follow a mechanism design approach where the principal commits to a policy rule stipulating which decision should be made as a function of the lobbyists' reports on their preferences. From the Revelation Principle, there is indeed no loss of generality in restricting the principal to offer a direct revelation mechanism $\{q(\hat{\theta}_1, \dots, \hat{\theta}_n)\}_{\hat{\theta}_i \in \Theta}$ which is truthful. When we deal with multiple interest groups, we use dominant strategies as the implementation concept.

The motivation for the commitment assumption is two fold. From a practical viewpoint first, this assumption seems reasonable. The principal may have already promised to the interest groups which have earlier contributed to his electoral campaign a particular policy platform to respond to their needs. He must thus stick to this scheme when in office to maintain his reputation in view of raising future campaign contributions. More importantly maybe, the commitment assumption is also attractive because it solves the equilibrium indeterminacy that arises in the signaling environment where lobbyists would move first. The mechanism design approach allows also a full characterization of communication patterns achievable at any equilibrium of a communication game among the agents. This property seems also quite attractive as far as one is concerned with the normative comparison between various organizational forms of lobbying.

The timing of the lobbying game is as follows. First, each lobbyist observes only his own preferences. Second, the policy-maker offers a mechanism $\{q(\hat{\theta}_1, \dots, \hat{\theta}_n)\}_{\hat{\theta}_i \in \Theta}$. Third,

[15] It is well known that this is also true for the signaling version of the model.

[16] This assumption is mainly done to simplify. We could account for differences among interest groups in the directions in which they want the policy to be pushed by having preferences being drawn from different supports.

the lobbyists report their preferences to the policy-maker. Fourth, the corresponding policy is implemented.

For further references, let us determine the policy chosen when the policy-maker knows the whole vector of preferences $(\theta_1, \dots, \theta_n)$. The first-best policy chosen by the principal is then:

$$q^*(\theta_1, \dots, \theta_n) = \frac{1}{n}\left(\sum_{i=1}^{n} \theta_i\right) + \delta. \tag{1}$$

Instead, the principal could remain uninformed on the preferences profile and commit ex ante to a policy without communicating with interest groups. He would then choose a rigid policy q^p to maximize an ex ante criterion, namely $-\frac{1}{2}\int_{\underline{\theta}}^{\bar{\theta}}(q-\theta-\delta)^2 f(\theta)d\theta$, and thus

$$q^p = \delta + E_f(\theta).$$

Remark: By the Law of Large Numbers, the complete information policy converges towards the rigid policy when the number of agents is large. This effect is of a particular importance in view of what follows. It already suggests that communication is in fact less needed as the principal faces more interest groups. ■

3 One Interest Group

Let us suppose that the principal faces a single privately informed interest group.

Incentive compatibility constraints for that group write as:

$$-\frac{1}{2}(q(\theta)-\theta)^2 \geq -\frac{1}{2}(q(\hat{\theta})-\theta)^2 \qquad \forall(\theta, \hat{\theta}) \in \Theta^2. \tag{2}$$

The principal's mechanism design problem can thus be expressed as:

$$\max_{\{q(\cdot)\}} -\frac{1}{2}\int_{\underline{\theta}}^{\bar{\theta}} (q(\theta)-\theta-\delta)^2 f(\theta)d\theta$$

subject to (2).

3.1 Incentive Compatibility

The general structure of the set of incentive compatible mechanisms with quadratic preferences can be easily derived following the work of Melumad and Shibano (1991).

Lemma 1 *: (Melumad and Shibano (1991)*[17]*) An incentive compatible scheme $q(\cdot)$ must satisfy the following conditions:*

- *$q(\theta)$ is weakly increasing and thus almost everywhere differentiable;*
- *if $q(\theta)$ is strictly increasing, $q(\theta) = \theta$;*
- *if $q(\theta)$ is discontinuous at a point θ_1 then:*

$$\begin{aligned} &- \quad q(\theta_1^+) + q(\theta_1^-) = 2\theta_1, \qquad (3)\\ &- \quad q(\theta) \textit{ is flat on the right and the left of } \theta_1,\\ &- \quad q(\theta_1) \textit{ belongs to the pair } \{q(\theta_1^-), q(\theta_1^+)\}. \end{aligned}$$

The proof of this lemma is instructive because it yields some insights on the nature of incentive compatible schemes. At any differentiability point of $q(\cdot)$, we must indeed have:

$$(q(\theta) - \theta)\dot{q}(\theta) = 0 \tag{4}$$

This incentive constraint is satisfied by two interesting classes of schemes: the pooling ones where $q(\theta) = q$ on all Θ, and the fully separating one corresponding to the interest group's ideal point, $q(\theta) = \theta$ on all Θ. The optimal mechanism will be a compromise between such schemes.

In what follows and for tractability, we focus on continuous mechanisms by first providing a general condition which guarantees that the optimal mechanism is indeed continuous and, second, by giving examples of distributions which satisfy this required property.

Continuous mechanisms have a simple form in our environment. It can be easily seen that they have at most one strictly increasing part. Typically, let us denote $\hat{\theta}_1$ and $\hat{\theta}_2$ the boundary of the segment where $q(\theta) = \theta$, a continuous scheme writes as:

$$q(\theta) = \min\{\hat{\theta}_2, \max\{\theta, \hat{\theta}_1\}\}. \tag{5}$$

The reader accustomed with the social choice literature will have recognized the *min-max rule* (at least in the case of one agent only) due to Moulin (1980). Moulin's characterization of dominant strategy incentive mechanisms was obtained by assuming that the domain of single-peaked preferences is much larger than the specific quadratic preferences we use throughout. The cost of such an enlargement is that the differentiable approach we

[17] Melumad and Shibano (1991) have proved this lemma for general single-peaked utility functions satisfying the single-crossing property. We get a shorter proof by specializing to the quadratic case.

rely on is lost. Incidentally, the comparison of Lemma 1 and Moulin's result shows that enlarging the preferences domain only rules out discontinuous schemes. This robustness of continuous schemes when preferences go beyond quadratic might be viewed as another justification to focus on schemes given in (5). Even if one is not ready to focus a priori on those continuous schemes, the next Lemma provides a sufficient condition on the types distribution to ensure that the optimal mechanism is actually continuous with quadratic preferences.

Lemma 2 *: The optimal mechanism is continuous at any point $x \in \Theta$ if for any $\Delta \geq 0$:*[18]

$$\delta\left(2F(x) - F(x-\Delta) - F(x+\Delta)\right) - \int_{x-\Delta}^{x} F(y)dy + \int_{x}^{x+\Delta} F(y)dy \geq 0. \tag{6}$$

Condition (6) is somewhat weaker than requiring the concavity of the cumulative distribution $F(\cdot)$. However, for any decreasing density $f(\cdot)$, the optimal mechanism is in fact continuous. Lemma 2 shows that, under condition (6), any scheme with a discontinuity at a point x is dominated by a continuous scheme which follows the most preferred policy of the interest group on an interval around x.

Several quite standard distributions satisfy condition (6). Let us give two examples:

Example 1: Uniform distribution on $[\underline{\theta}, \bar{\theta}]$, $f(\theta) = \frac{1}{\bar{\theta}-\underline{\theta}}$.[19]

Example 2: Folded normal on $[0, +\infty[$, $f(\theta) = \frac{2}{\sqrt{2\pi}} e^{-\frac{\theta^2}{2}}$.

3.2 Optimal Mechanism

We are now ready to give a simple characterization of the optimal mechanism when continuity is ensured.

Proposition 1 *: Assume that condition (6) holds, that the distribution $F(\cdot)$ is strictly log-concave, and that the degree of conflict between the policy-maker and the interest group is not too severe, i.e., $\delta < \int_{\underline{\theta}}^{\bar{\theta}} F(y)dy = \bar{\theta} - E_f(\theta)$. The optimal mechanism is then unique, continuous and has the following features:*

$$q(\theta) = \max\{\theta, \hat{\theta}\}$$

where the cut-off $\hat{\theta}$ is uniquely defined by the condition

$$\delta = \frac{1}{F(\hat{\theta})} \int_{\underline{\theta}}^{\hat{\theta}} F(\theta)d\theta = \hat{\theta} - E_f(\theta | \theta \leq \hat{\theta}). \tag{7}$$

[18] If $x+\Delta > \bar{\theta}$ or $x-\Delta < \underline{\theta}$ then in (6) $x+\Delta$ is replaced by $\bar{\theta}$ and $x-\Delta$ is replaced by $\underline{\theta}$ correspondingly.

[19] For that case, Melumad and Shibano (1991) proved directly the continuity of the optimal scheme.

That proposition highlights the trade-off faced by the policy-maker in designing an optimal communication mechanism. On the one-hand, the principal would like to learn the true ideal point θ of the interest group to implement his own most preferred policy $q^*(\theta) = \theta + \delta$. However, this choice is not incentive compatible from (4). If the principal wants to follow a flexible type-dependent policy, it has to be necessarily the interest group's ideal point and thus $q(\theta) = \theta$. This latter policy differs from the policy-maker's own ideal point by an amount δ and is thus "too" low compared to $q^*(\theta)$. Instead, the principal might want to avoid any communication and implement a more rigid policy. This policy entails much pooling but is closer on average of his own ex ante optimal policy.

The key insight provided by Proposition 2 is that the principal always benefits from some communication on the upper tail of the types distribution whereas pooling is instead preferred on the lower tail. The intuition behind this result is the following. First, we know that a continuous mechanism has at most one interval where effective communication takes place between the principal and the agent (see (5)). Clearly, having a pooling mechanism on an upper tail $\theta \geq \hat{\theta}_2$ cannot be optimal. Indeed, the principal can offer an incentive scheme with effective communication inducing a policy $q(\theta) = \theta$ for $\theta \geq \hat{\theta}_2$ which is closer to his own ideal point $q^*(\theta) = \theta + \delta$ than any pooling scheme $q(\theta) = \hat{\theta}_2$ when θ is large enough. Hence, communication is valuable on the upper tail.

Second, consider now the lower tail of the type distribution. The policy-maker prefers to offer a pooling policy on such an interval. The cost of doing so is of course that the policy is not responsive to the preference parameter θ. The benefit is that, for the highest values of θ on that interval, this pooling policy is closer *on average* to the principal's ideal point. The benefits of having a near-by policy for these intermediate values may exceed the cost borne on the very lowest values with a pooling scheme even though this pooling may be quite further away from the principal's ideal policy. This is so when relatively little weight is left to that lower tail. Log-concavity ensures this latter property.

Proposition 1 highlights a fundamental trade-off between rigidity on the lower tail and flexibility on the upper tail of the types distribution. The cut-off value $\hat{\theta}$ summarizes how those two forces compensate each other. The alternative expression in (7) reinforces our understanding of this trade-off. On the lower tail, the principal chooses a policy $\delta + E_f(\theta|\theta \leq \hat{\theta})$ whereas on the upper tail, the interest group is residual claimant for the policy which fits his own preferences. The cut-off is determined by making the principal indifferent between these two options.

Corollary 1 : *Assume that the conditions of Proposition 1 hold and that the conflict of interests between the principal and the agent is large enough, i.e.,* $\delta \geq \int_{\underline{\theta}}^{\bar{\theta}} F(\theta)d\theta$ *then there is no communication at the optimal mechanism. When* $\delta < \int_{\underline{\theta}}^{\bar{\theta}} F(\theta)d\theta$*, there is* *always some partial communication.*

It is striking to see that a similar upper bound on the level of conflict is needed to ensure no-communication in the signaling environment where the interest group moves first.[20] Large conflicts of interest destroy any possibility for communicating preferences in an incentive compatible way even when the principal recovers some commitment power in the mechanism design environment. In the sequel, we will focus on levels of the conflict of interests small enough to still justify some communication even if it is quite partial.

Coming back to our examples where continuity of the optimal mechanism is ensured, we can easily compute the values of the cut-off $\hat{\theta}$:

Example 1 (continued):

$$\delta = \frac{1}{\hat{\theta} - \underline{\theta}} \int_{\underline{\theta}}^{\hat{\theta}} (\theta - \underline{\theta}) d\theta = \frac{\hat{\theta} - \underline{\theta}}{2},$$

so that communication never takes place when $\delta > \frac{\Delta\theta}{2}$.

Of course, the continuity of the optimal mechanism is not ensured for any distribution. For normal distributions, discontinuities appear when δ is not too large. Condition (6) is indeed harder to satisfy when distributions put enough mass on a hump. This loss of tractability will force us to limit the scope of our comparison between a coalition and separated groups in some of the Propositions below by specifying distributions and/or conditions on δ ensuring continuity.

Next proposition gives more conditions to justify the focus on continuous schemes.

Proposition 2 *: Take any single-peaked, symmetric and log-concave cumulative distribution $F(\cdot)$, the optimal mechanism is continuous if the optimal continuous mechanism*

$$q(\theta) = \max\{\theta, \hat{\theta}\}$$

with $\delta = \frac{1}{F(\hat{\theta})} \int_{\underline{\theta}}^{\hat{\theta}} F(\theta) d\theta$ is such that $\hat{\theta} \geq E_f(\theta)$.[21]

This proposition is extremely useful to check continuity without having to deal directly with condition (6). Take for instance the case of a normal distribution with zero mean From Proposition 2, it is enough to find δ so that $\hat{\theta} > 0$ to make sure that the optimal mechanism with this distribution is continuous.

[20] See Grossman and Helpman (2001, Section 4.1.4) for instance.

[21] For distributions with unbounded support, we need in addition the technical assumption that $\lim_{x \to -\infty} \int_{-\infty}^{x} \theta^2 dF(\theta) = 0$.

3.3 Comparative Statics

3.3.1 Conflict of Interests

In this subsection, we analyze how the policy-maker and the interest group's expected utility vary with the degree of conflict δ.

First, note that the expected utilities of the policy-maker and the interest group can be written respectively as

$$V(\delta) = -\frac{1}{2}\left[\int_{\underline{\theta}}^{\hat{\theta}(\delta)} (\hat{\theta}(\delta) - \theta - \delta)^2 f(\theta) d\theta + \delta^2 (1 - F(\hat{\theta}(\delta)))\right]$$

and

$$U(\delta) = -\frac{1}{2}\int_{\underline{\theta}}^{\hat{\theta}(\delta)} (\hat{\theta}(\delta) - \theta)^2 f(\theta) d\theta$$

where $\hat{\theta}(\delta)$ is defined implicitly from (7).

Proposition 3 *: Assume that the optimal mechanism is continuous, then the following properties hold:*

- *There is less communication as the degree of conflict increases ($\hat{\theta}(\delta)$ increases with δ) when $F(\cdot)$ is log-concave.*
- *Both the principal and the agent's expected utilities decrease with the degree of conflict ($\dot{V}(\delta) < 0, \dot{U}(\delta) < 0$).*

The intuition behind these results is straightforward. As the conflict of interests between the policy-maker and the interest group is more pronounced, communication is less attractive to the principal who prefers a rigid policy for a greater set of values of the preference parameter.

Of course, enlarging that pooling region hurts the interest group who prefers a flexible policy since the latter fits his own ideal point. Moreover, more conflict hurts also the principal because it increases the wedge between his ideal point and that of the interest group over the range of values of θ where communication indeed takes place.

3.3.2 Informativeness of the Distribution

Let us now keep the bias δ as fixed and consider some comparative statics with respect to the informativeness of the distribution.

For simplicity, we will now assume that $f(\cdot)$ is distributed on the whole real line, centered at zero so that $E_f(\theta) = \int_{-\infty}^{+\infty} \theta f(\theta) d\theta = 0$ and $\text{Var}_f(\theta) = \int_{-\infty}^{+\infty} \theta^2 f(\theta) d\theta < +\infty$.

Consider now the distribution $G(\theta, \sigma)$ with density $g(\theta, \sigma) = \sigma f(\sigma\theta)$ where σ is a positive parameter. It is clear that

$$E_g(\theta) = \int_{-\infty}^{+\infty} \sigma\theta f(\sigma\theta) d\theta = 0$$

and

$$\text{Var}_g(\theta) = \int_{-\infty}^{+\infty} \sigma\theta^2 f(\sigma\theta) d\theta = \frac{1}{\sigma^2} \int_{-\infty}^{+\infty} u^2 f(u) du = \frac{\text{Var}_f(\theta)}{\sigma^2}.$$

An increase in σ means an increase in the informativeness of the distribution since $G(\cdot)$ second-order stochastically dominates $F(\cdot)$ by sweeping more weight around the same mean $E_f(\theta) = E_g(\theta) = 0$.

As a benchmark, note that a policy-maker who never communicates with the interest group would choose a rigid policy $q^p = \delta$ but his expected payoff would always obviously be greater when the distribution of types is more centered around its mean.

Let us turn to the case where communication is possible. We define now the principal's expected welfare $V(\sigma)$ as a function of the parameter σ:

$$V(\sigma) = -\frac{1}{2} \left[\int_{-\infty}^{\hat{\theta}(\sigma)} (\hat{\theta}(\sigma) - \theta - \delta)^2 g(\theta, \sigma) d\theta + \delta^2 (1 - G(\hat{\theta}(\sigma), \sigma)) \right]$$

where the optimal cut-off $\hat{\theta}(\sigma)$ is defined implicitly by:

$$\delta = \frac{1}{G(\hat{\theta}(\sigma), \sigma)} \int_{-\infty}^{\hat{\theta}(\sigma)} G(\theta, \sigma) d\theta = \frac{1}{\sigma F(\sigma\hat{\theta}(\sigma))} \int_{-\infty}^{\sigma\hat{\theta}(\sigma)} F(\theta) d\theta. \qquad (8)$$

If we denote $\tilde{\theta}(1, \delta)$ the solution to (8) when $\sigma = 1$, we observe that $\hat{\theta}(\sigma) = \frac{1}{\sigma}\tilde{\theta}(1, \delta\sigma)$.

From (8), we obtain after integrating by parts

$$\delta\sigma = \tilde{\theta}(1, \delta\sigma) - \frac{1}{F(\tilde{\theta}(1, \delta\sigma))} \int_{-\infty}^{\tilde{\theta}(1,\delta\sigma)} \theta f(\theta) d\theta$$

and thus

$$\delta\sigma \underset{\sigma\to\infty}{\sim} \tilde{\theta}(1, \delta\sigma) \qquad \text{so that} \qquad \hat{\theta}(\sigma) \underset{\sigma\to\infty}{\longrightarrow} \delta.$$

This first result shows that, as the distribution on the interest group's ideal point becomes more informative, the principal chooses a policy which comes closer to what he would do with a rigid policy based on no communication at all. Indeed, such policy is almost first-best when the distribution puts increasingly more weight around the mean.

Proposition 4 *: The principal's expected payoff increases with the informativeness of the distribution:* $\dot{V}(\sigma) > 0$*. The interest group's expected payoff decreases with the informativeness of the distribution:* $\dot{U}(\sigma) < 0$*.*

This result is of a particular importance since it gives us some insights on why a policy-maker might want to face "larger" groups to facilitate communication. The logic behind this proposition is straightforward. As the distribution of the ideal point θ becomes more centered around the mean, the principal does not really care about the policy out of this particular value and communication has less value for him than in the case of a less informative distribution. Relying on the interest group's report is of a lesser importance.

Example 3: Assume that $\theta \sim N(0, \frac{1}{\sigma^2})$. Then, as σ converges towards infinity, the distribution converges towards a Dirac at zero and the principal's expected payoff with a continuous mechanism increases. We will give below some condition ensuring that the optimal mechanism with such a normal distribution is indeed continuous. ■

4 Coalition

The previous section has shown the nature of the transaction costs associated to asymmetric information between an interest group and the policy-maker. Communication is needed for distributions which are not too informative on the preferences of the interest group but the cost of this communication is that policies depart from the principal's ideal point. In this section, we are interested in the properties of those transaction costs when groups coalesce.

Let us now consider two groups with preference parameters θ_1 and θ_2 respectively. Those parameters are identically and independently distributed in the same distribution $F(\cdot)$ with density $f(\cdot)$. Those two groups merge as one by credibly sharing their information on preferences and have an equal bargaining weight of one half in the coalition. We follow the social choice literature in skipping the issue of how information sharing within a coalition is incentive compatible.[22] That interest groups share credibly information in a coalition has been documented in the political science literature. Laumann and Knoke (1987) and Heinz and al. (1990) examine information exchanges between group dyads as a key to intergroup coordination. Hula (1999, Chapter 4) reported that interest groups are linked by the career path of their staff members and that a phenomenon akin to the "revolving door" takes place between groups. This phenomenon certainly facilitates information flows between distinct organizations.

[22]This is a standard assumption going back to at least Green and Laffont (1979).

The utility of the merged group can thus be written as:

$$U_M(q,\theta_1,\theta_2) = -\frac{1}{2}\left[\frac{1}{2}(q-\theta_1)^2 + \frac{1}{2}(q-\theta_2)^2\right] = -\frac{1}{2}\left(q - \frac{\theta_1+\theta_2}{2}\right)^2 - \frac{(\theta_2-\theta_1)^2}{8}. \quad (9)$$

Up to some terms which are independent from the policy chosen and thus cannot be screened by the policy-maker, the utility of that coalition only depends on the average ideal point between group members.

The game of communication between that merged group and the uninformed policy-maker is thus similar to that we solved in Section 3. The only difference comes now from the fact that the initial distribution of types $F(\cdot)$ is replaced by the distribution $G(\cdot)$ of the "average" between two independent draws of the initial distribution. Of course, $G(\cdot)$ is more informative than $F(\cdot)$ since it shifts more weight around the same mean. The logic behind Proposition 4 should again apply and that increase in the informativeness of the distribution should definitively improve the principal's expected payoff. Below, we show this result for two examples of distribution.

4.1 Uniform Distribution

Consider a uniform distribution on $[0,1]$. From Lemma 2, we know that the optimal mechanism for a single group is necessarily continuous and that the cut-off $\hat{\theta}_f$ between the pooling and separating regions satisfies $\hat{\theta}_f = 2\delta$.[23]

Consider now the average between two independent draws $\frac{\theta_1+\theta_2}{2}$. It has a density $g(\cdot)$ given by:

$$g(\theta) = \begin{cases} 4\theta & \text{if } 0 \leq \theta \leq \frac{1}{2} \\ 4-4\theta & \text{if } 1 \geq \theta \geq \frac{1}{2}. \end{cases}$$

The optimal continuous mechanism for such a distribution gives a cut-off satisfying (7) which can also be written as:

$$\int_0^{\hat{\theta}_g} (\hat{\theta}_g - \theta - \delta) g(\theta) d\theta = 0.$$

For $\delta \leq \frac{1}{6}$, the solution is $\hat{\theta}_g = 3\delta > \hat{\theta}_f$ meaning that the pooling region increases for small conflicts of interests. For $\delta \geq \frac{1}{6}$, the cut-off $\hat{\theta}_g$ is determined by

$$(\hat{\theta}_g - \delta)(12\hat{\theta}_g^2 - 24\hat{\theta}_g + 6) + 12\hat{\theta}_g^2 - 8\hat{\theta}_g^3 - 1 = 0. \quad (10)$$

[23] Remember that partial communication takes only place if $2\delta \leq 1$.

The solution $\hat{\theta}_g(\delta)$ of (10) is increasing in the size of the conflict of interests δ and it is also concave in δ. Moreover, $\hat{\theta}_f(\delta) = 2\delta$ is such that the left-hand side of (10) is negative so that $\hat{\theta}_g(\delta) > \hat{\theta}_f(\delta) = 2\delta$.

We summarize these findings as:

Proposition 5 *: Assume that the ideal points θ_i $(i = 1, 2)$ are uniformly and independently distributed on* $[0, 1]$.

- *The policy-maker uses an optimal mechanism which is continuous with a coalition . It entails less communication than what occurs with each individual interest group taken separately.*
- *The principal's expected payoff when dealing with a coalition is greater than what he obtains with a single interest group for any δ in $\left[\frac{4}{27}, \frac{1}{2}\right]$. For $\delta \in [0, \frac{4}{27}]$, the principal is better off when dealing with a single interest group.*
- *Each interest group loses from merging compared with the case where he deals alone with the policy-maker whatever δ.*

When dealing with a coalition, the principal benefits from more precise information. As in Proposition 4, a more precise signal makes it less necessary to rely on communication and a rigid policy over a greater pooling region performs relatively well. This pooling, of course, hurts the interest groups since the implemented policy does not fit their own preferences. Interest groups benefit from more screening since then the policy is better aligned with their ideal points. They always prefer being alone to communicate with the principal.

For small values of δ, the most important thing is the range of values of θ where there is indeed communication since communication gets easier. Indeed, the principal and the interest groups' ideal points are close to each other and, since $\hat{\theta}_g > \hat{\theta}_f$, there is too much pooling with a coalition. In other words, the density $g(\theta)$ being not "too concentrated" around the mean, the gains from communicating to the right of cut-off value $\hat{\theta}_g$ are less with the density $g(\cdot)$ than with the uniform density $f(\cdot)$. Technically, the relative gains from communicating with a coalition depend on a term $\frac{\delta^2}{2} G\left(\hat{\theta}_g\right) = 9\delta^4$ which is of order four in δ. At the same time these gains with a single group are represented by a term $\frac{\delta^2}{2} F\left(\hat{\theta}_f\right) = \delta^3$ which is only of order three. For a small δ, the single-agent structure certainly dominates.

4.2 Normal Distribution

Normal distributions are interesting because they provide an example of a closed class such that the average $\frac{\theta_1+\theta_2}{2}$ is still normally distributed $N(0, \frac{1}{2\sigma^2})$ with the same mean and half the variance when the θ_i's are normally distributed $N(0, \frac{1}{\sigma^2})$.

To make easier the comparison between individual lobbying and the coalition, we impose in the next proposition a condition on the variance which ensures continuity of the optimal mechanism. Continuity is ensured when the conflict of interests is large enough compared with the variance of the distribution. Improving the informativeness of the distribution by increasing σ increases the cut-off $\hat{\theta}(\sigma)$ since communication is then less valuable to the principal as he gets a more precise idea of the interest groups' preferences. Under these conditions we can now prove

Proposition 6 *: Assume that the interest groups' ideal points θ_i $(i = 1, 2)$ are normally and independently distributed ($\theta_i \sim N(0, \frac{1}{\sigma^2})$) and that $\delta\sigma > \sqrt{\frac{2}{\pi}}$, then, the following holds:*

- *The policy-maker uses a continuous mechanism with a merged group. There is less communication than what occurs with each individual interest group.*
- *The principal's expected payoff from dealing with a coalition is greater than what he obtains with a single interest group.*
- *Each interest group loses from merging compared with the case where he stands alone with the policy-maker.*

Propositions 5 and 6 altogether show the basic tension that drives the formation of large interest groups. On the one hand, by dealing with a coalition, the policy-maker gets a more precise information on preferences and communication becomes less necessary. On the other hand, individual interest groups may be reluctant to form such a coalition because their respective ideal points are never chosen as communication is less necessary and, when it takes place, this is only an aggregate of the preferences of both groups which ends up being used by the principal.

To sharpen intuition, let consider the case of n groups merging as one. Using (7) in the case of a normal distribution we get, after integrating by parts, the simple expression of the cut-off $\hat{\theta}(n)$

$$\delta = \hat{\theta}(n) + \frac{1}{n\sigma^2}\frac{f(\hat{\theta}(n), n)}{F(\hat{\theta}(n), n)}, \tag{11}$$

where $F(\theta, n) = \sqrt{\frac{n\sigma^2}{2\pi}} \int_{-\infty}^{\theta} e^{-\frac{n\sigma^2 x^2}{2}} dx$ is the cumulative distribution function for the law $N\left(0, \frac{1}{n\sigma^2}\right)$ of the average $\frac{\sum_{i=1}^{n} \theta_i}{n}$. It is easy to check on (11) that $\hat{\theta}(n)$ converges towards δ as n gets large. The rigid policy $q^p = \delta$ is thus almost the optimal mechanism offered by the policy-maker.

An alternative interpretation of this result is worth to be stressed. As n increases, the ideal point of each individual group becomes less relevant for the chosen policy. The individual incentives to lie of each individual interest group get pooled altogether in a coalition and the collective incentives to lie are weak. Indeed, by the Law of Large Numbers, uncertainty on the average type $\frac{\sum_{i=1}^{n} \theta_i}{n}$ is washed out in the aggregate. Because a larger group better communicates a piece of information which is known for sure by the policy-maker, the incentive problem is easier to solve. Having a grand-coalition of interest groups allows to benefit from some *economies of scope* in the transaction costs associated to asymmetric information. Those economies of scope take a spectacular form with a large number of interest groups.[24]

5 Coalition versus Multiple Interest Groups

In the previous section, we limited ourselves to comparing what happens if the principal deals with a coalition of interest groups and what happens if he deals with each of them *separately*. In this section, we undertake a less biased comparison by considering the case where the policy-maker deals simultaneously with both groups.

To model the groups' non-cooperative behavior, we use dominant strategy as the relevant implementation concept. Although Bayesian implementation would relax incentive constraints, dominant strategy implementation is amenable to a more direct comparison with the case on a single interest group.[25] Another motivation for this implementation concept is that it is less sensitive to the beliefs that interest groups may have on each other. This seems an attractive property in a constitutional perspective which would insist on finding mechanisms and organizations of the lobbying activity which are robust to those details of the environments. Finally, focusing on dominant strategy is already enough to obtain the result that dealing separately with the different groups may improve on the outcome achieved with a coalition.

[24]In a signaling models, Lohman (1993, 1994) showed that agents may still have an incentives to signal their types even though they are informationally small. The mechanism design approach that we use in this paper shows that dealing with a single large coalition weakens indeed the cost of credible communication.

[25]It is well known from Mookherjee and Reichelstein (1992) that dominant strategy entails no loss compared to Bayesian implementation in some models with monetary transfers and quasi-linearity. This equivalence of course fails in our context without monetary transfer.

With dominant strategy, incentive compatibility for interest group i can be written as

$$-\frac{1}{2}(q(\theta_i,\theta_{-i})-\theta_i)^2 \geq -\frac{1}{2}(q(\hat{\theta}_i,\theta_{-i})-\theta_i)^2, \ \ \forall i=1,2, \ (\theta_i,\hat{\theta}_i,\theta_{-i}) \in \Theta^3.$$

Proceeding with the same differentiable approach as in Section 3,[26] we can easily prove that $q(\cdot)$ is monotonically increasing in each of its arguments and thus almost everywhere differentiable in (θ_1, θ_2). At any point of differentiability, incentive constraints can be written as:

$$\frac{\partial q}{\partial \theta_i}(\theta_i,\theta_{-i})(q(\theta_i,\theta_{-i})-\theta_i)=0, \text{ for } i=1,2, \ (\theta_i,\theta_{-i}) \in \Theta^2. \tag{12}$$

Hence, $q(\cdot)$ is either constant along θ_i or equal to the ideal point of group i and thus independent on θ_{-i}. This leads to the following characterization of dominant strategy *continuous* schemes.

Lemma 3 *: For any symmetric dominant strategy and continuous mechanism $q(\cdot)$, there exist cut-offs θ^*, θ^{**} and θ^{***} such that $\theta^* \leq \theta^{**} \leq \theta^{***}$ and :*

$$q(\theta) = \min\{\theta^{***}, \max\{\theta_1,\theta^{**}\}, \max\{\theta_2,\theta^{**}\}, \max\{\theta_1,\theta_2,\theta^*\}\}. \tag{13}$$

These continuous mechanisms generalize those found with a single interest groups. Indeed, along each argument θ_i, the trace of the mechanism $q(\theta_i,\cdot)$ looks like a one-dimensional continuous mechanism. Again, we recover with our differentiable approach the *minmax* mechanisms of Moulin (1980), now in a two-agent framework.

Those dominant strategy mechanisms can be given an interesting interpretation. For the region where the policy is flexible, the ideal point of either of the interest groups is implemented by the policy process. Everything happens as if this group becomes then residual claimant for the policy.

Generally, it is hard to find out the optimal cut-offs for any distribution of types. We will contend ourselves with analyzing explicitly the case of a uniform distribution on $[0,1]$ and will give numerical simulations for the case of a normal distribution.

[26]See Laffont and Maskin (1980) for the differentiable approach of dominant strategy mechanisms in settings with monetary transfers.

5.1 Uniform Distribution

Of particular importance in the analysis below are the following mechanisms:[27]

$$q(\theta_1,\theta_2) = \begin{cases} \theta^* & \text{if } \max(\theta_1,\theta_2) \leq \theta^* \\ \max(\theta_1,\theta_2) & \text{if } \theta^* \leq \max(\theta_1,\theta_2) \leq \theta^{**} \\ \theta^{**} & \text{if } \min(\theta_1,\theta_2) \leq \theta^{**}, \max(\theta_1,\theta_2) \geq \theta^{**} \\ \min(\theta_1,\theta_2) & \text{if } \theta^{**} \leq \min(\theta_1,\theta_2) \end{cases} \tag{14}$$

Those schemes implement thus a policy which fits with the highest ideal point of the interest groups when there is screening in the intermediate region and with the smallest ideal point when there is screening in the upper-right corner of the square $[0,1] \times [0,1]$.

Denoting then by $V(\theta^*,\theta^{**})$ the principal's expected payoff with such a dominant strategy continuous scheme, we have, using symmetry and the uniform distribution:

$$\begin{aligned} V(\theta^*,\theta^{**}) \quad = \quad & -\int_0^{\theta^*}\left(\int_0^{\theta_1}\left(\theta^* - \frac{\theta_1+\theta_2}{2} - \delta\right)^2 d\theta_2\right) d\theta_1 \\ & -\int_{\theta^*}^{\theta^{**}}\left(\int_0^{\theta_1}\left(\theta_1 - \frac{\theta_1+\theta_2}{2} - \delta\right)^2 d\theta_2\right) d\theta_1 \\ & -\int_{\theta^{**}}^{1}\left(\int_0^{\theta^{**}}\left(\theta^{**} - \frac{\theta_1+\theta_2}{2} - \delta\right)^2 d\theta_2\right) d\theta_1 \\ & +\int_{\theta^{**}}^{1}\left(\int_{\theta^{**}}^{\theta_1}\left(\theta_2 - \frac{\theta_1+\theta_2}{2} - \delta\right)^2 d\theta_2\right) d\theta_1 \end{aligned}$$

Optimizing with respect to θ^* and θ^{**} we obtain

$$\theta^*(\delta) = 2\delta \quad \text{and} \quad \theta^{**}(\delta) = \frac{1}{2} + 2\delta, \tag{15}$$

and $\theta^{**}(\delta) < 1$ if and only if $\delta \leq \frac{1}{4}$. For $\frac{1}{2} \geq \delta \geq \frac{1}{4}$, the scheme has only one cut-off $\theta^*(\delta) = 2\delta$.

We are now ready to state the main result of this section.

Proposition 7 *: Assume that θ_1 and θ_2 are independently and uniformly distributed on $[0,1]$. Then, the optimal mechanism with a coalition is continuous. Moreover, there exist two critical levels of the conflict of interests δ^* and δ^{**} such that $\delta^{**} > \delta^*$ and the following ranking holds:*

- *When $\delta < \delta^*$ (resp. $\delta \geq \delta^*$), the policy-maker is better (resp. worse) off with a coalition than with separated groups.*

[27] It will be shown in Appendix that these mechanisms are in fact optimal ones.

- *When $\delta < \delta^{**}$ (resp. $\delta \geq \delta^{**}$), interest groups are better (resp. worse) off if they merge than if they remain separated.*

The main idea of the proof is to compare the optimal mechanism with a merged group with a dominant strategy mechanism of type (14) dealing with both groups. To do that, the following figure is useful.

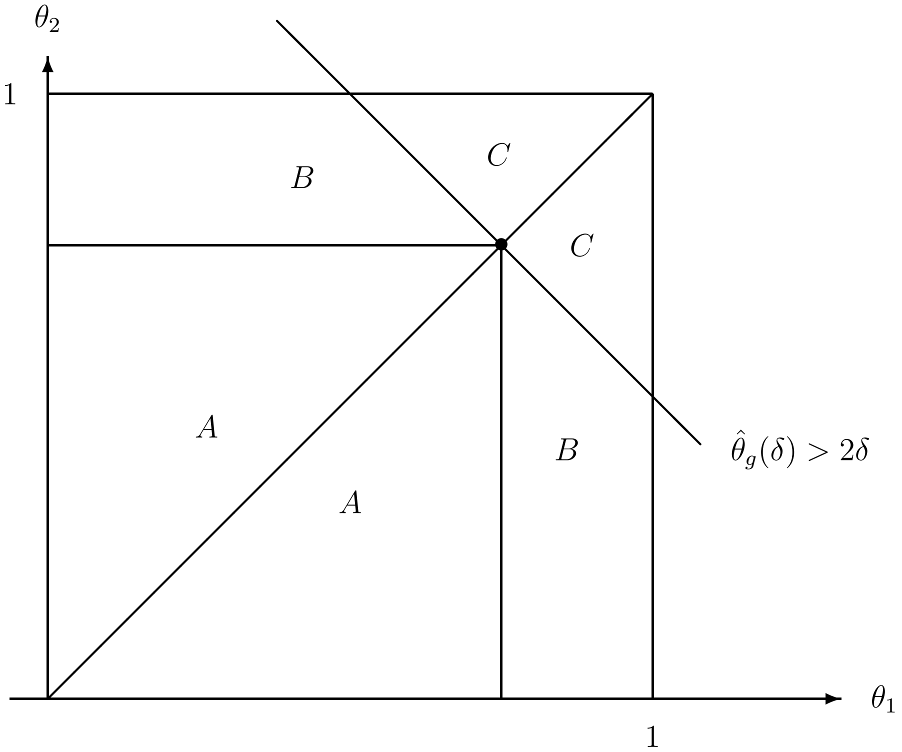

Figure 1: Comparing dominant strategy mechanisms and a coalition. The case $\delta > \frac{1}{6}$.

Decision-Maker: On Figure 1, we have drawn the line $\frac{\theta_1+\theta_2}{2} = \hat{\theta}_g > 2\delta$ which separates the areas $A + B$ where a coalition of both groups receives a rigid policy from the area C where the ideal policy of this coalition is chosen. On C, isopolicy lines are 45^0 downward slopping lines which thus correspond to none of the ideal points of either group except on the measure zero set of events where both groups have the same preferences.

Consider now the dominant strategy mechanism of type (14) having a cut-off $\theta^* = \hat{\theta}_g(\delta)$. On area A, this mechanism is pooling and yields to the principal the same expected payoff as the mechanism given to a coalition since the rigid policy chosen is the same in both cases.

On area B, dealing separately with each group allows to make a policy contingent on the preferences of the group who has the highest ideal point. Screening although imperfect and biased in one direction is possible whereas dealing with a coalition still entails a rigid policy. This *screening effect* captures the benefits of keeping groups split apart. When the interest group's preferences are sufficiently apart one from the other, the dominant strategy mechanism elicits the preferences of the group with the highest ideal point whereas communicating with a coalition does not allow this. To understand this effect, it is useful to come back on the comparative statics made in Section 3.3.1. We showed there that the optimal policy with a single group requires more communication as the conflict of interests between this group and the policy-maker is less pronounced. With multiple interest groups, the same logic applies. The "virtual conflict of interests" between group 1 and the policy-maker depends now on the realized preferences of group 2 and will typically be equal to

$$\delta' = \delta + \frac{\theta_1 + \theta_2}{2} - \theta_1 = \delta - \frac{(\theta_1 - \theta_2)}{2}.$$

This virtual conflict is a decreasing function of the "distance" between the ideal points of both groups. As the distance between the groups' ideal points increases, the principal's preferences are more aligned with the group having the highest ideal point given that δ is large enough to ensure that the virtual conflict of interests remain positive. Everything happens as if an endogenous bias towards that group appears. Instead, the mechanism dealing with a coalition smooths out this discrepancy between preferences and is thus unable to account for this endogenous bias.

On area C, dealing with a coalition allows a more efficient communication with the principal since only the statistics $\frac{\theta_1+\theta_2}{2}$ is relevant for decision-making and isopolicy lines cannot depend on this statistics with dominant strategy. However, since the density of $\frac{\theta_1+\theta_2}{2}$ decreases over $\left[\hat{\theta}_g(\delta), 1\right]$, this potential benefit of a coalition is not big enough to offset the cost of an excessive bunching when the interest groups' preferences are sufficiently far apart.

To give further intuition on the benefits of a decentralized mechanism it is useful to think about the case of a sufficiently large conflict of interests. Typically, we will set $\delta = \frac{1}{2} - \varepsilon$ for ε small enough so that, with a coalition, $\hat{\theta}_g(\delta)$ is close to 1 and there is almost no communication with a coalition. The screening area C is then a small triangle having an area of order ε^2. The gain of communicating with a coalition with respect to full pooling everywhere is thus of order ε^2. Instead, by dealing separately with each interesting group with a dominant strategy mechanism, the principal can screen preferences on areas $B + C$ and obtains a gain of communicating of ε everywhere. This dimensionality argument underscores the benefits of a dominant strategy mechanism at least for large conflicts of interests.

For small values of the conflict of interests, the 45^0 degree line $\frac{\theta_1+\theta_2}{2} = \hat{\theta}_g$ lies on the very south-west of the $[0, 1] \times [0, 1]$ square. There is communication almost everywhere with a coalition and the communicated statistics perfectly fits what is needed by the principal for deciding which policy to implement. Clearly, dealing with a merger dominates in that case. In the extreme case where δ is close to zero and thus there is almost no pooling, using a decentralized mechanism is clearly suboptimal whatever the values of the cut-offs θ^*, θ^{**} and θ^{***}. Indeed, such mechanisms have isopolicy lines which are never aligned with those of the principal contrary to what happens with a coalition.

Interest Groups: Proposition 7 shows that interest groups also gain from being split apart for sufficiently large δ. Indeed, when they remain separated, there is a positive likelihood that the principal's policy is exactly the ideal points of each of those groups. This beneficial force is strong enough to dominate the cost of having the preferences of the other group being exactly implemented sometimes. Instead, the more average policy that would be chosen with a coalition never fits the ideal points of either group except, with probability zero, when these ideal points are the same and above $\hat{\theta}_g(\delta)$.

Our model suggests therefore that the multiplication of interest groups found in practice may find its logic in the fact that each of these groups is, over some range of preferences profiles, residual claimant for the decision made. A coalition of interest groups does not reflect as easily individual preferences.

5.2 Normal Distribution

That case is significantly less tractable than the case of a uniform distribution. We will content ourselves with giving a few numerical examples showing the generality of the insights gleaned in Section 5.1.

Below we present the values of the payoffs of the principals in both settings (coalition and decentralization) for the mechanisms described on Figure 1. We denote by δ_M^+ the limiting value of the bias where the solution of the equation (8) is positive which guarantees from Proposition 2 that the optimal mechanism for a coalition is continuous. Since the average of normal distributions with variance $\frac{1}{\sigma^2}$ has the variance $\frac{1}{2\sigma^2}$, we can see from Lemma 4 in the Appendix that $\delta_M^+ = \frac{1}{\sigma}\sqrt[2]{\frac{1}{\pi}}$ Then, for $\sigma = 1$ we have $\delta_M^+ = 0.56$. The next table summarizes, for various values of δ, the optimal cut-off with a coalition and the principal's payoff V_M and V_D under either a coalition or a decentralized organization with $\theta^* = \theta_M(\delta)$.

δ	$\widehat{\theta}_M(\delta)$	V_M	V_D	Result
$\delta_M^+ = 0.56$	0	-0.125	-0.104	Decentralized
0.75	0.43	-0.170	-0.121	Decentralized
1	0.84	-0.210	-0.152	Decentralized
2	1.995	-0.249	-0.233	Decentralized

For $\delta \geq 2$, the difference between the two organizational modes becomes negligibly small. The difference $V_D(\delta) - V_M(\delta)$ is increasing and then decreasing but remains always positive. So, the decentralized case overperforms the merged case. Similar calculations show the same result for different values of σ. We have to point out that for small values of σ the difference between the payoffs that the principal obtains in both cases becomes very small, still the principal prefers decentralization for $\delta > \delta_M^+$. Further simulations show that, for $\delta < \delta_M^+$, the coalition is preferred by the policy-maker, exactly as in the uniform case and the intuition is the same.[28]

Let us turn to the interest groups. To compare their payoffs under both organizational forms, we have now to consider the optimal mechanisms in both cases.[29] It is clear that the limiting value δ_D^+ for which the optimal decentralized mechanism is necessarily continuous is bigger than the corresponding value δ_M^+. Therefore, to ensure that the optimal mechanisms in both cases are continuous we have to take values of δ starting at δ_D^+. The following Table shows that for $\sigma = 1$ the decentralized scheme is preferable from the groups' point of view. Interests groups are aligned with the principal and prefer a more decentralized organization of the lobbying activity.

δ	$\widehat{\theta}_M(\delta)$	θ^*, θ^{**}	U_M	U_D	Result
$\delta_D^+ = 0.798$	0.51	$0, 1.95$	-0.596	-0.514	Decentralized
1	0.84	$0.48, 2.33$	-0.840	-0.697	Decentralized
2	1.995	$1.93, 4.20$	-2.489	-2.382	Decentralized

6 Conclusion

In this paper,we have proposed a mechanism design approach useful to analyze the determinants of group formation in an environment where interest groups communicate information which is relevant for optimal policy-making.

We have first characterized the optimal mechanism and communication pattern in a framework with only one interest group. More communication takes place as the conflict between the interest group and the decision-maker is less pronounced and as the distribution of the interest group's preference parameter becomes less informative.

This last point shows that a policy-maker could have strong incentives to facilitate the formation of large coalition of interest groups. Indeed, the "average" ideal points of such coalitions are better known and communication becomes less of an issue. This

[28]This result is obtained by considering the optimal continuous mechanism with a coalition. We do not need to prove it for the optimal mechanism.

[29]For a normal distribution, the optimal mechanism in a decentralized case within class (14) has two kinks at θ^* and θ^{**}.

information effect is however not enough in itself to explain alone the organization of lobbies. A coalition of interest groups, although it transmits a more precise information to the decision-maker, is unable to account for the possible large discrepancies in the preferences of the colluding interest groups. The policy-maker and the interest groups as well may sometimes prefer a more decentralized way of communication information where each interest group is approached separately. Communication is then easier, more specifically when the interest groups preferences are sufficiently far apart and the conflict of interests with the principal significant. A playing field where several competing interest groups independently communicate with the decision-maker emerges. Our model based on the precise analysis of the transaction costs due to asymmetric information in lobbying games offers thus a justification for the coexistence of multiple interest groups on the policy arena.

Of course, those results are partial and sometimes obtained by making restrictive assumptions on distributions of preference parameters. A more thorough analysis which would generalize our findings to other distributions is called for but is likely to face strong technical difficulties coming from the fact that optimal mechanisms cannot be easily characterized and compared without specifying those distributions.

An alternative path would be to relax incentive constraints and the constraints on communication by allowing monetary transfers between the policy-maker and the interest groups. Although, those transfers can be motivated in practice,[30] such an extension of the model might lead to qualitatively different insights.[31]

It would also be worth to generalize our analysis to the cases where interest groups have different biases and to other information structures involving for instance correlation between the various interest groups' ideal points.

Finally, the organization of lobbying which emerges in our model depends of course of the decision-maker's objective function. Here, we have assumed that this principal is somewhat a social welfare maximizer, ideally choosing an average between the interest groups' ideal points and that of the general public. That decision-maker could follow another objective, for instance, in a political contest environment, he would certainly prefer to choose a policy corresponding to the median of the public and the interest groups' ideal points. The whole pattern of communication and transaction costs would be modified accordingly leading potentially to an interesting feed-back on the incentives of groups to coalesce or not.

We hope to investigate some of these issues in future research.

[30] They may for instance stand for bribes, campaign contributions, or even official redistributive policies.

[31] For instance, in the I.O. literature, Baron and Besanko (1992), Gilbert and Riordan (1995), Dana (1993), and others have found conditions under which dealing with a coalition of suppliers Pareto dominates decentralized mechanisms in a context where agents have quasi-linear utility functions.

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Appendix

• **Proof of Lemma 1:** From standard revealed preferences arguments, we have indeed for any $(\theta, \hat{\theta}) \in \Theta^2$ such that $\theta > \hat{\theta}$:

$$-\frac{1}{2}(q(\theta) - \theta)^2 \geq -\frac{1}{2}(q(\hat{\theta}) - \theta)^2$$

and

$$-\frac{1}{2}(q(\hat{\theta}) - \hat{\theta})^2 \geq -\frac{1}{2}(q(\theta) - \hat{\theta})^2.$$

Summing those two constraints yields

$$(\theta - \hat{\theta})(q(\theta) - q(\hat{\theta})) \geq 0.$$

Therefore, $q(\cdot)$ is weakly increasing and thus almost everywhere differentiable.

At any point of differentiability, we must have (4) and thus $q(\theta)$ is either flat or $q(\theta) = \theta$, i.e., corresponds to the most-preferred choice of the lobbying group.

If $q(\cdot)$ is discontinuous at θ_1, $q(\theta_1^+) \neq q(\theta_1^-)$, but type θ_1 must be indifferent between choosing the policies which are respectively on the left and on the right of θ_1

$$-\frac{1}{2}(q(\theta_1^-) - \theta_1)^2 = -\frac{1}{2}(q(\theta_1) - \theta_1)^2 = -\frac{1}{2}(q(\theta_1^+) - \theta_1)^2. \tag{A1}$$

Using the right- and left-hand sides, we then get (3).

Because $q(\theta_1^+) \neq q(\theta_1^-)$ at a discontinuity and $q(\cdot)$ is differentiable on both sides, it cannot be that $q(\theta_1)$ is not flat on those sides.

Finally, (A1) shows that either $q(\theta_1) = q(\theta_1^+)$ or $q(\theta_1) = q(\theta_1^-)$. ■

• **Proof of Lemma 2:** Suppose to the contrary that the optimal mechanism $q(\theta)$ is discontinuous. Then using the structure of incentive compatible mechanisms given in Lemma 1, there exists an interval $[\theta_1, \theta_2] \subseteq [\underline{\theta}, \bar{\theta}]$ such that on this interval:

$$q(\theta) = \begin{cases} \theta_1 & \text{if } \theta < \hat{\theta} \\ 2\hat{\theta} - \theta_1 & \text{if } \theta > \hat{\theta} \end{cases}$$

where $\hat{\theta} = \frac{\theta_1+\theta_2}{2}$. We can consider the new mechanism $q(\theta, \tilde{\theta})$, which differs from $q(\theta)$ only in $[\theta_1, \theta_2]$, and which is parameterized by $\tilde{\theta} \in [\theta_1, \hat{\theta}]$:

$$q(\theta, \tilde{\theta}) = \begin{cases} \theta & \text{if } \theta \in [\theta_1, \tilde{\theta}) \\ \tilde{\theta} & \text{if } \theta \in [\tilde{\theta}, \hat{\theta}] \\ 2\hat{\theta} - \tilde{\theta} & \text{if } \theta \in (\hat{\theta}, 2\hat{\theta} - \tilde{\theta}] \\ \theta & \text{if } \theta \in [2\hat{\theta} - \tilde{\theta}, 2\hat{\theta} - \theta_1]. \end{cases}$$

This new mechanism preserves incentive compatibility. The derivative of the utility of the principal on this interval with respect to $\tilde{\theta}$ under condition (6) is positive, which contradicts the optimality of $q(\theta)$. ■

• **Proof of Proposition 1:** Note that the principal's expected payoff with a continuous scheme characterized by the cut-offs $\hat{\theta}_1$ and $\hat{\theta}_2$ can be written as

$$V(\hat{\theta}_1, \hat{\theta}_2) = -\frac{1}{2}\left\{\int_{\underline{\theta}}^{\hat{\theta}_1} (\hat{\theta}_1 - \theta - \delta)^2 f(\theta)d\theta + \delta^2 \int_{\hat{\theta}_1}^{\hat{\theta}_2} f(\theta)d\theta + \int_{\hat{\theta}_2}^{\bar{\theta}} (\hat{\theta}_2 - \theta - \delta)^2 f(\theta)d\theta\right\}.$$

Optimizing with respect to $\hat{\theta}_2$ with the constraint $\hat{\theta}_2 \geq \hat{\theta}_1$ yields

$$\frac{\partial V}{\partial \hat{\theta}_2}(\hat{\theta}_1, \hat{\theta}_2) = -\int_{\hat{\theta}_2}^{\bar{\theta}} (\hat{\theta}_2 - \theta - \delta) f(\theta)d\theta < 0$$

so that $\hat{\theta}_2 = \hat{\theta}_1$ and we can rewrite $V(\cdot)$ as a function of $\hat{\theta}_1$ only. Abusing notations, we have:

$$V(\hat{\theta}_1) = -\frac{1}{2}\left\{\int_{\underline{\theta}}^{\hat{\theta}_1} (\hat{\theta}_1 - \theta - \delta)^2 f(\theta)d\theta + \delta^2(1 - F(\hat{\theta}_1))\right\}.$$

The first-order condition $\frac{d}{d\hat{\theta}_1} V(\hat{\theta}) = 0$ yields the expression of $\hat{\theta}$ given by (7).

Moreover, we have

$$\frac{d^2}{d\hat{\theta}_1} V(\hat{\theta}_1) = \delta f(\hat{\theta}_1) - F(\hat{\theta}_1)$$

and thus the second-order condition

$$\left.\frac{d^2}{d\hat{\theta}_1} V(\hat{\theta}_1)\right|_{\hat{\theta}_1 = \hat{\theta}} = \delta f(\hat{\theta}) - F(\hat{\theta}) \leq 0$$

is satisfied if and only if

$$\frac{f(\hat{\theta})}{F^2(\hat{\theta})} \int_{\underline{\theta}}^{\hat{\theta}} F(\theta)d\theta \leq 1. \tag{A2}$$

However, consider the function $\varphi(\theta) = \frac{1}{F(\theta)} \int_{\underline{\theta}}^{\theta} F(x)dx$. We have

$$\frac{d}{d\theta}(\varphi(\theta)) = 1 - \frac{f(\theta)}{F^2(\theta)} \int_{\underline{\theta}}^{\theta} F(x)dx = 1 - \frac{f(\theta)}{F(\theta)}\varphi(\theta).$$

But when $F(\cdot)$ is log-concave, $\frac{d}{d\theta}\left(\frac{F(\theta)}{f(\theta)}\right) > 0$ and

$$\varphi(\theta) = \frac{1}{F(\theta)} \int_{\underline{\theta}}^{\theta} \frac{F(x)}{f(x)} f(x)dx \leq \frac{F(\theta)}{f(\theta)},$$

so that $\frac{d}{d\theta}(\varphi(\theta)) \geq 0$. Finally, (A2) holds. ■

- **Proof of Corollary 1:** Since $\varphi(\theta)$ is increasing, $\hat{\theta} \leq \bar{\theta}$ if and only if $\delta \leq \int_{\underline{\theta}}^{\bar{\theta}} F(\theta)d\theta$. ■

- **Proof of Proposition 2:**

The case of finite support with a finite number of discontinuities.

a) Suppose that the optimal mechanism $q(\cdot)$ has n jumps at points $a_1 > a_2 > \cdots > a_n$. First we establish that in $(\underline{\theta}, \hat{\theta})$ there are no interval where the mechanism coincides with the agent's ideal point $q(\theta) = \theta$. Since the support of the distribution is finite we can consider the first interval from the left (x_2, x_1) where the optimal mechanism is separating. Then in the interval $(\underline{\theta}, x_1)$ the mechanism has a form:[32]

$$q(\theta) = \begin{cases} x_2 & \text{if } \theta < x_2 \\ \theta & \text{if } x_2 < \theta < x_1. \end{cases}$$

Consider the mechanism $q^{\varepsilon}(\theta)$ which is the same as $q(\theta)$ except on the interval $(\underline{\theta}, x_2)$ where it is equal to

$$q^{\varepsilon}(\theta) = \begin{cases} x_2 + \varepsilon & \text{if } \theta < x_2 + \varepsilon \\ \theta & \text{if } x_2 + \varepsilon < \theta < x_1. \end{cases}$$

If we differentiate the principal's expected payoff achieved with the mechanism $q^{\varepsilon}(\theta)$ with respect to ε we have:

$$V'(q^{\varepsilon}(\theta)) = \delta F(x_2 + \varepsilon) - \int_{\underline{\theta}}^{x_2+\varepsilon} F(\theta)\, d\theta > 0.$$

The inequality follows from (8) and $x_2 + \varepsilon < \hat{\theta}$.[33] Therefore we can always improve the mechanism $q(\theta)$. A contradiction with optimality.

[32] We do not take into account the values of the mechanism on the borders of intervals since this is a set of measure zero.

[33] It is easy to see that the case $x_2 + \varepsilon > \hat{\theta}$ is impossible if there is at least one discontinuity.

b) It is easy to see that in the decreasing part of the support $\left(E_f(\theta), \bar{\theta}\right)$ there might be only one point of discontinuity, namely a_1. Summarizing we can conclude that, for the optimal mechanism, we must have

$$q(\theta) = \begin{cases} x_{n+1} & \text{if } \theta \in \left(\underline{\theta}, \frac{x_{n+1}+x_n}{2}\right) \\ x_i & \text{if } \theta \in \left(\frac{x_i+x_{i+1}}{2}, \frac{x_{i-1}+x_i}{2}\right), n \geq i \geq 2 \\ x_1 & \text{if } \theta \in \left(\frac{x_1+x_2}{2}, x_1\right) \\ \theta & \text{if } \theta \in \left(x_1, \bar{\theta}\right). \end{cases} \tag{A3}$$

The mechanism has jumps at points $a_i = \frac{x_i+x_{i+1}}{2}$ and $a_2 = \frac{x_2+x_3}{2} \leq \hat{\theta}$. Suppose that $n \geq 2$. We show that the mechanism (A3) is dominated by the mechanism:

$$\tilde{q}(\theta) = \begin{cases} x_n & \text{if } \theta \in \left(\underline{\theta}, \frac{x_n+x_{n-1}}{2}\right) \\ x_i & \text{if } \theta \in \left(\frac{x_i+x_{i+1}}{2}, \frac{x_{i-1}+x_i}{2}\right), n-1 \geq i \geq 2 \\ x_1 & \text{if } \theta \in \left(\frac{x_1+x_2}{2}, x_1\right) \\ \theta & \text{if } \theta \in \left(x_1, \bar{\theta}\right). \end{cases} \tag{A4}$$

The mechanisms $q(\cdot)$ and $\tilde{q}(\cdot)$ are the same except for the discontinuity at a_n of $q(\cdot)$ which is smoothed in $\tilde{q}(\cdot)$.

Taking the difference $d = E_\theta(V(\tilde{q}(\theta), \theta) - V(q(\theta), \theta))$ and performing calculations we have

$$\begin{aligned} d &= \frac{1}{2}\int_{\underline{\theta}}^{a_n} (x_{n+1} - \theta - \delta)^2 f(\theta)\, d\theta - \frac{1}{2}\int_{\underline{\theta}}^{a_n} (x_n - \theta - \delta)^2 f(\theta)\, d\theta \\ &= \frac{1}{2}\int_{\underline{\theta}}^{a_n} (x_{n+1} - x_n)(x_n + x_{n+1} - 2\theta - 2\delta) f(\theta)\, d\theta \\ &= \frac{1}{2}\int_{\underline{\theta}}^{a_n} (x_n - x_{n+1})(2\theta + 2\delta - (x_n + x_{n-1})) f(\theta)\, d\theta. \end{aligned}$$

Since $x_n - x_{n+1} > 0$, the sign of d is the same as the sign of:

$$\begin{aligned} \frac{1}{2}\int_{\underline{\theta}}^{a_n} (2\theta + 2\delta - (x_n + x_{n+1})) f(\theta)\, d\theta &= \delta F(a_n) - \frac{(x_n + x_{n+1})}{2} F(a_n) + \int_{\underline{\theta}}^{a_n} \theta f(\theta)\, d\theta \\ &= \delta F(a_n) - \int_{\underline{\theta}}^{a_n} F(\theta)\, d\theta > 0. \end{aligned}$$

The last inequality follows from log-concavity of $F(\cdot)$, the form of δ in (8) and $a_n < \widehat{\theta}$. Therefore $E_f(V(\tilde{q}(\theta), \theta)) > E_f(V(q(\theta), \theta))$ which contradicts the optimality of $q(\cdot)$. So, the mechanism must have at most one point of discontinuity $a_1 \geq \hat{\theta} > x_2$:

$$q(\theta) = \begin{cases} x_2 & \text{if } \theta < a_1 \\ x_1 & \text{if } a_1 < \theta < x_1 \\ \theta & \text{if } \theta > x_1. \end{cases} \tag{A5}$$

c) We have established that the optimal mechanism with a discontinuity at a_1 has the form given by (A5). We prove now that this mechanism is not optimal in the case of a symmetric, single-peaked distribution. Namely it is dominated by the mechanism:

$$q^*(\theta) = \begin{cases} x_2 & \text{if} \quad \theta < x_2 \\ \theta & \text{if} \quad \theta > x_2. \end{cases}$$

Consider the difference $d^* = E_f\left(V\left(q^*(\theta),\theta\right) - V\left(q(\theta),\theta\right)\right)$ then we have

$$\begin{aligned} d^* &= \frac{1}{2}\int_{x_2}^{a_1} (x_2-\theta-\delta)^2 f(\theta)\, d\theta + \frac{1}{2}\int_{a_1}^{x_1} (x_1-\theta-\delta)^2 f(\theta)\, d\theta \\ &\quad -\frac{1}{2}\int_{x_2}^{a_1} \delta^2 f(\theta)\, d\theta - \frac{1}{2}\int_{a_1}^{x_1} \delta^2 f(\theta)\, d\theta \\ &= \frac{1}{2}\int_{x_2}^{a_1} (x_2-\theta-2\delta)(x_2-\theta) f(\theta)\, d\theta + \frac{1}{2}\int_{a_1}^{x_1} (x_1-\theta-2\delta)(x_1-\theta) f(\theta)\, d\theta. \end{aligned}$$

In the first integral in the expression above, we change variables and introduce $\tilde{\theta} = x_1 + x_2 - \theta$. Then we have

$$\begin{aligned} \int_{x_2}^{a_1} (x_2-\theta-2\delta)(x_2-\theta) f(\theta)\, d\theta &= \int_{a_1}^{x_1} \left(\tilde{\theta}-2\delta-x_1\right)\left(\tilde{\theta}-x_1\right) f\left(x_1+x_2-\tilde{\theta}\right) d\tilde{\theta} \quad \text{(A6)} \\ &= \int_{a_1}^{x_1} \left(2\delta+x_1-\tilde{\theta}\right)\left(x_1-\tilde{\theta}\right) f\left(x_1+x_2-\tilde{\theta}\right) d\tilde{\theta} \\ &> \int_{a_1}^{x_1} \left(2\delta+x_1-\tilde{\theta}\right)\left(x_1-\tilde{\theta}\right) f\left(\tilde{\theta}\right) d\tilde{\theta}. \end{aligned}$$

For the last inequality, we use two facts. First, $\left(2\delta+x_1-\tilde{\theta}\right)\left(x_1-\tilde{\theta}\right) > 0$ for $\tilde{\theta} \in (a_1, x_1)$. Second, $f\left(x_1+x_2-\tilde{\theta}\right) \geq f\left(\tilde{\theta}\right)$. Indeed, since $f(\cdot)$ is symmetric around $E_f(\theta)$, $f\left(x_1+x_2-\tilde{\theta}\right) = f\left(2E_f(\theta)+\theta-(x_1+x_2)\right)$. Now, since $f(\cdot)$ is single-peaked and $2E_f(\theta)+\theta-(x_1+x_2) \leq \tilde{\theta}$ we have the second inequality.

Using (A6) we can find a lower bound on the difference d^* and we get

$$\begin{aligned} d^* &> \frac{1}{2}\int_{a_1}^{x_1} \left(2\delta+x_1-\tilde{\theta}\right)\left(x_1-\tilde{\theta}\right) f\left(\tilde{\theta}\right) d\tilde{\theta} - \frac{1}{2}\int_{a_1}^{x_1} (\theta+2\delta-x_1)(x_1-\theta) f(\theta)\, d\theta \\ &= \frac{1}{2}\int_{a_1}^{x_1} (2x_1-2\theta)(x_1-\theta) f(\theta)\, d\theta > 0. \end{aligned}$$

Therefore $d^* > 0$ which contradicts with optimality of $q(\theta)$.

The case of an infinite support with a countable non-dense set of discontinuities.

d) Suppose now that the optimal mechanism $q(\theta)$has an infinite but countable number of discontinuities at points $a_1 > a_2 > ... > a_n > ...$ Also suppose for a while that, to the left of the expected value, there are no intervals where the agent's ideal point is chosen. Then we must have

$$q(\theta) = \begin{cases} x_i & \text{if } \theta \in \left(\frac{x_i+x_{i+1}}{2}, \frac{x_{i-1}+x_i}{2}\right), n-1 \geq i \geq 2 \\ x_1 & \text{if } \theta \in \left(\frac{x_1+x_2}{2}, x_1\right) \\ \theta & \text{if } \theta \in (x_1, \infty). \end{cases}$$

Consider the system of mechanisms $\{q_k(\theta)\}_{k=2}^{\infty}$ defined by:

$$q_k(\theta) = \begin{cases} x_k & \text{if } \theta \in \left(-\infty, \frac{x_k+x_{n-1}}{2}\right) \\ x_i & \text{if } \theta \in \left(\frac{x_i+x_{i+1}}{2}, \frac{x_{i-1}+x_i}{2}\right), k-1 \geq i \geq 2 \\ x_1 & \text{if } \theta \in \left(\frac{x_1+x_2}{2}, x_1\right) \\ \theta & \text{if } \theta \in (x_1, \infty). \end{cases}$$

From *b)* we know that $E_f(V(q_2(\theta), \theta)) > E_f(V(q_3(\theta), \theta)) > \cdots > E_f(V(q_k(\theta), \theta)) > \cdots$. Suppose $E_f(V(q_2(\theta), \theta)) - E_f(V(q_3(\theta), \theta)) = \varepsilon$. Then there exists n such that:

$$E_f(V(q(\theta), \theta)) - E_f(V(q_n(\theta), \theta)) < \frac{1}{2} \int_{-\infty}^{a_n} (x_n - \theta - \delta)^2 f(\theta)\, d\theta < \varepsilon.$$

Indeed

$$\int_{-\infty}^{a_n} \theta^2 f(\theta)\, d\theta - \int_{-\infty}^{a_n} (x_n - \theta - \delta)^2 f(\theta)\, d\theta = \int_{-\infty}^{a_n} (x_n - \delta)(2\theta + \delta - x_n) f(\theta)\, d\theta.$$

We have $x_n < \delta$ and

$$\int_{-\infty}^{a_n} (2\theta + \delta - x_n) f(\theta)\, d\theta = 2x_{n-1} F(a_n) - 2\int_{-\infty}^{a_n} F(\theta)\, d\theta + \delta F(a_n) < 0$$

for x_{n-1} sufficiently negative. That means that

$$\lim_{x_n \to -\infty} \int_{-\infty}^{a_n} (x_n - \theta - \delta)^2 f(\theta)\, d\theta = 0.$$

Therefore, we have

$$E_f(V(q(\theta), \theta)) < E_f(V(q_n(\theta), \theta) = \varepsilon.$$

From this inequality, we deduce that replacing the initial mechanism $q(\theta)$ by the one-jump mechanism $q_2(\theta)$ strictly increases the expected utility of the principal. Now we just have to use steps of *c)* to prove that $q_2(\theta)$ is dominated by a continuous mechanism.

e) Using *a)* and *d)* we can easily prove that there are no interval to the left of the expected value where the mechanism coincides with the agent's ideal point.

The case of the dense set of discontinuities.

Since for the finite support we can always start from the beginning of the interval the previous techniques is applicable. If the condensation point is $\underline{\theta}$ then we always can replace the mechanism on small interval $(\underline{\theta}, \delta)$ by the agent's ideal point and then proceed as in *a)*. The loss of optimality from replacing the initial mechanism by the agent's ideal point can be made arbitrarily small. Then, since the finite interval is a compact set we can focus on finite sequence of mechanisms each of whom dominates the previous one and the last mechanism in this sequence is continuous.

For infinite support the same argument applies. We just have to apply techniques of section *d)* - we can "cut" the mechanism from the left and replace it by the rigid policy there. Doing this we can arbitrarily closely approach the initial mechanism. Then the previous considerations for the finite support are applicable since we do not have to bother about the mechanism to the right of the expected value. ■

• **Proof of Proposition 3:** Note first that $\delta = \varphi(\hat{\theta}(\delta))$. Because $\varphi(\cdot)$ is increasing when $F(\cdot)$ is log-concave, $\hat{\theta}(\delta)$ increases with δ. Using the Envelope Theorem:

$$\dot{V}(\delta) = \int_{\underline{\theta}}^{\hat{\theta}(\delta)} (\hat{\theta}(\delta) - \theta - \delta) f(\theta) d\theta - \delta(1 - F(\hat{\theta}(\delta))).$$

Because (7) holds, the first term on the r.h.s. is zero and $\dot{V}(\delta) = -\delta(1 - F(\hat{\theta}(\delta))) < 0$. Also, we have:

$$\dot{U}(\delta) = -\left(\int_{\underline{\theta}}^{\hat{\theta}(\delta)} (\hat{\theta}(\delta) - \theta) f(\theta) d\theta \right) \frac{d}{d\delta}\hat{\theta}(\delta) = -\delta F(\hat{\theta}(\delta)) \frac{d}{d\delta}\hat{\theta}(\delta) < 0. \quad \blacksquare$$

• **Proof of Proposition 4:** Using the Envelope Theorem, we get:

$$2\dot{V}(\sigma) = \delta^2 \frac{\partial G}{\partial \sigma}(\hat{\theta}(\sigma), \sigma) - \int_{-\infty}^{\hat{\theta}(\sigma)} (\hat{\theta}(\sigma) - \theta - \delta)^2 \frac{\partial g}{\partial \sigma}(\theta, \sigma) d\theta.$$

Integrating by parts,

$$\begin{aligned} \dot{V}(\sigma) &= -\int_{-\infty}^{\hat{\theta}(\sigma)} (\hat{\theta}(\sigma) - \theta - \delta) \frac{\partial G}{\partial \sigma}(\theta, \sigma) d\theta \\ &= -\int_{-\infty}^{\hat{\theta}(\sigma)} \frac{\frac{\partial G}{\partial \sigma}(\theta, \sigma)}{g(\theta, \sigma)} \Big(g(\theta, \sigma)(\hat{\theta}(\sigma) - \theta - \delta) \Big) d\theta. \end{aligned}$$

Integrating by parts again,

$$\dot{V}(\sigma) = \int_{-\infty}^{\hat{\theta}(\sigma)} \left(\int_{-\infty}^{\theta} (\hat{\theta}(\sigma) - \tilde{\theta} - \delta) g(\tilde{\theta}, \sigma) d\tilde{\theta} \right) \frac{\partial}{\partial \theta} \left(\frac{\frac{\partial G}{\partial \sigma}(\theta, \sigma)}{g(\theta, \sigma)} \right) d\theta.$$

Note that $\frac{\frac{\partial G}{\partial \sigma}(\theta,\sigma)}{g(\theta,\sigma)} = \frac{\theta}{\sigma}$ and that, by definition of $\hat{\theta}(\sigma)$,

$$\int_{-\infty}^{\theta} (\hat{\theta}(\sigma) - \tilde{\theta} - \delta) g(\tilde{\theta}, \sigma) d\tilde{\theta} \geq 0 \quad \text{for } \theta \leq \hat{\theta}(\sigma).$$

Therefore, $\dot{V}(\sigma) \geq 0$.

For the interest group, we have:

$$U(\sigma) = -\frac{1}{2}\left(\int_{-\infty}^{\hat{\theta}(\sigma)} (\hat{\theta}(\sigma) - \theta)^2 g(\theta, \sigma) d\theta\right).$$

Thus,

$$\dot{U}(\sigma) = -\frac{1}{2}\left(2\left(\int_{-\infty}^{\hat{\theta}(\sigma)} (\hat{\theta}(\sigma) - \theta) g(\theta, \sigma) d\theta\right) \frac{d}{d\sigma}\hat{\theta}(\sigma) + \int_{-\infty}^{\hat{\theta}(\sigma)} (\hat{\theta}(\sigma) - \theta)^2 \frac{\partial}{\partial \sigma} g(\theta, \sigma) d\theta\right)$$

and

$$\dot{U}(\sigma) = -\frac{1}{2}\left(\int_{-\infty}^{\hat{\theta}(\sigma)} (\hat{\theta}(\sigma) - \theta)^2 f(\theta, \sigma) d\theta + 2\left(\int_{-\infty}^{\hat{\theta}(\sigma)} (\hat{\theta}(\sigma) - \theta) f(\theta, \sigma) d\theta\right)\left(1 + \sigma \frac{d}{d\sigma}\hat{\theta}(\sigma)\right)\right).$$

Finally, since $\frac{d}{d\sigma}\hat{\theta}(\sigma) > 0$ we get that $\dot{U}(\sigma) < 0$. ■

• **Proof of Proposition 5:** First, we prove the following Lemma:

Lemma 4 *: The optimal mechanism for the distribution $G(\cdot)$ corresponding to the coalition is necessarily continuous.*

Proof: First, consider the following property of the optimal mechanism $q(\theta)$ which is a direct consequence of Lemma 2. If at a point x for Δ such that both $x + \Delta$ and $x - \Delta$ belong to Θ, the following is true: for all $t \in (0, \Delta)$

$$f(x - t) - \delta f'(x - t) + f(x + t) - \delta f'(x + t) > 0 \tag{A7}$$

then there is no discontinuity of the optimal mechanism on $[x - \Delta, x + \Delta]$ of the type

$$q(\theta) = \begin{cases} x - \Delta & \text{if } x - \Delta \leq \theta < x, \\ x + \Delta & \text{if } x < \theta \leq x + \Delta. \end{cases} \tag{A8}$$

Indeed, consider the function $\varphi(t)$, defined on $[0, \Delta]$ by

$$\begin{aligned}\varphi(t) &= \delta(2F(x) - F(x-t) - F(x+t)) - \\ &\quad - \int_{x-t}^{x} F(y)\,dy + \int_{x}^{x+t} F(y)\,dy.\end{aligned}$$

Then, if the condition (A7) is satisfied we have $\varphi(0) = 0$ and $\varphi'(t) > 0$. Therefore $\varphi(\Delta) > 0$ and acting as in the proof of Lemma 2 we get that the optimal mechanism cannot have discontinuity of type (A8).

From the proof of Proposition 2, we know that, if there is a discontinuity of the optimal mechanism then it is unique. Suppose, we have an optimal mechanism with a discontinuity at x of the form (A8). Five cases should be considered depending on the locations of $x - \Delta, x$ and $x + \Delta$.

1. $1 > x + \Delta > x - \Delta > \frac{1}{2} = E_g(\theta)$. Then, since the whole interval $[x - \Delta, x + \Delta]$ lies on the interval where the density is decreasing, that mechanism is thus dominated by the mechanism which is identical to $q(\theta)$ except on the interval $[x - \Delta, x + \Delta]$ where $q(\theta)$ is replaced by θ.

2. $0 < x - \Delta < \frac{1}{2} < x + \Delta < 1$. Taking into account the expression of $g(\cdot)$, (A7) can be rewritten as:

$$x - \Delta - \delta + 1 - x - \Delta + \delta = 1 - 2\Delta > 0,$$

which holds for $\Delta < \frac{1}{2}$.

3. $x + \Delta < \frac{1}{2}$. We have

$$x - \Delta - \delta + x + \Delta + \delta = 2(x - \delta).$$

So (A7) holds when $x > \delta$. If $x < \delta < \widehat{\theta}$, then it is easy to see (as in the proof of Proposition 2) that $q(\theta)$ is dominated by

$$\tilde{q}(\theta) = \begin{cases} x + \Delta & \text{if } \theta < x + \Delta, \\ \theta & \text{if } x + \Delta < \theta, \end{cases}$$

4. $x - \Delta < 0$. If $\hat{\theta} > x + \Delta$, then, as in the proof of Proposition 2 we can show that the mechanism $q(\theta)$ is dominated. Therefore we can consider the case $\hat{\theta} < x + \Delta$, then $q(\theta)$ is dominated if[34]

$$f(x - \Delta) - \delta f'(x - \Delta) > 0. \tag{A9}$$

[34]If $x - \Delta < \underline{\theta}$ then condition (A7) becomes

$$f(x + \Delta) - \delta f'(x + \Delta) > 0$$

if $\hat{\theta} \leq x + \Delta$.

There are two cases to consider; If $x + \Delta < \frac{1}{2}$ then (A8) becomes

$$x + \Delta - \delta > 0$$

which is true, since otherwise, we have $x + \Delta < \delta < \hat{\theta}$, a contradiction with the assumption.

If $x + \Delta > \frac{1}{2}$ then (A8) holds since the inequality $1 - x - \Delta + \delta > 0$ is always true.

5. $x + \Delta > 1$. In this case, it is easy to see that: if $x > \frac{1}{2}$, then the gains of smoothing the mechanism are bigger than the losses, if $x < \frac{1}{2}$, then since $x + \Delta > 1$, we have $x - \Delta < 0$ and the mechanism is dominated by the rigid mechanism $x + \Delta$.

Putting everything together, we can conclude that the optimal mechanism for the coalition with uniform priors is continuous. ■

Our purpose is now to compare the principal's expected payoffs in the one-agent setting and with a coalition. For the case of one agent, the expected utility of the principal as a function of the level of the conflict δ is written as

$$V(\delta) = -\frac{1}{2}\left\{\int_0^{2\delta} (\delta - \theta)^2 d\theta + \delta^2(1 - 2\delta)\right\}.$$

In front of a coalition, his expected payoff for $\delta > \frac{1}{6}$ is written as

$$\begin{aligned} V_M(\delta) &= -2\int_0^{1/2} (\hat{\theta}_g - \theta - \delta)^2 \theta d\theta \\ &\quad -2\int_{1/2}^{\hat{\theta}_g} (\hat{\theta}_g - \theta - \delta)^2 (1 - \theta) d\theta - \frac{1}{2}\delta^2(1 - G(\hat{\theta}_g))). \end{aligned}$$

Where $\hat{\theta}_g$ is determined by (10) and $G(\hat{\theta}_g) = 1 - 2(1 - \hat{\theta}_g)^2$ is c.d.f. for the convolution.

Taking the difference $d_V(\hat{\theta}_g(\delta)) = V_M(\delta) - V(\delta)$ and using (10) for $\delta = \delta(\hat{\theta}_g)$ we get the following expression which is positive for all $\hat{\theta}_g \in [1/2, 1]$

$$\begin{aligned} d_V(\hat{\theta}_g)) &= \frac{1}{1296\left(1 - 4\hat{\theta}_g + 2\hat{\theta}_g^2\right)^3}\Big[-5 + 108\hat{\theta}_g - 1062\hat{\theta}_g^2 + 6000\hat{\theta}_g^3 - 20772\hat{\theta}_g^4 \\ &\quad +44784\hat{\theta}_g^5 - 59640\hat{\theta}_g^6 + 47808\hat{\theta}_g^7 - 22176\hat{\theta}_g^8 + 5504\hat{\theta}_g^9 - 576\hat{\theta}_g^{10}\Big]. \end{aligned}$$

It can be shown that $d_V\left(\frac{1}{2}\right) = 0.00038 > 0$ and $d_V(\hat{\theta}_g)$ is increasing on $[1/2, 1]$. It suggests to us that for smaller values of δ, $d_V(\hat{\theta}_g)$ is less than zero and the one-agent structure dominates a coalition. Indeed, for $\delta < \frac{1}{6}$ from Lemma 4, the optimal mechanism is continuous with $\hat{\theta}_g(\delta) = 3\delta > \hat{\theta}_f(\delta) = 2\delta$.

$$V_M(\delta) - V(\delta) = -\frac{1}{2}\int_0^{\hat{\theta}_g(\delta)} (\hat{\theta}_g(\delta) - \theta - \delta)^2 g(\theta) d\theta - \frac{1}{2}\delta^2(1 - G(\hat{\theta}_g(\delta)))$$

$$+\frac{1}{2}\int_0^{\hat{\theta}_f(\delta)} (\hat{\theta}_f(\delta) - \theta - \delta)^2 f(\theta) d\theta + \frac{1}{2}\delta^2(1 - F(\hat{\theta}_f(\delta)))$$

$$= -2\int_0^{3\delta} (2\delta - \theta)^2 \theta d\theta - \frac{1}{2}\delta^2(1 - 18\delta^2) + \frac{1}{2}\int_0^{2\delta} (\delta - \theta)^2 d\theta + \frac{1}{2}\delta^2(1 - 2\delta)$$

$$= \frac{\delta^2}{6}(27\delta - 4).$$

We obtain that, for $\delta \geq \frac{4}{27}$, the payoff of the principal for coalition $V_M(\delta)$ is greater that his payoff in the one-agent setting $V(\delta)$. For $\delta \leq \frac{4}{27}$, the reverse is true.

For the interest groups, we compare the expected utility of one group $U(\delta) = -\frac{1}{2}\int_0^{2\delta} (2\delta - \theta)^2 \, d\theta$ with half the expected utility of the merged group which can be rewritten for simplicity as $U_M(\delta) = -\frac{1}{2}\int_0^{\hat{\theta}_g} (\hat{\theta}_g - \theta)^2 g(\theta) d\theta - \frac{1}{48}$ (here we use the equality $E_f\left(\frac{(\theta_2 - \theta_1)^2}{8}\right) = \frac{1}{48}$).

Then the difference $d'(\hat{\theta}_g(\delta)) = U_M(\hat{\theta}_g) - U(\delta)$ is equal to

$$d'(\hat{\theta}_g) = \frac{(1 - \hat{\theta}_g)\left(-1 - 10\hat{\theta}_g + 251\hat{\theta}_g^2 - 1627\hat{\theta}_g^3 + 5222\hat{\theta}_g^4 - 9610\hat{\theta}_g^5 + 10484\hat{\theta}_g^6 - 6292\hat{\theta}_g^7 + 1880\hat{\theta}_g^8 - 216\hat{\theta}_g^9\right)}{162(1 - 4\hat{\theta}_g + 2\hat{\theta}_g^2)^3}$$

and for $\delta \leq \frac{1}{6}$, $d'(\hat{\theta}_g(\delta)) = \frac{1}{48}(1 - 64\delta^3 + 648\delta^4)$. It can be seen that $d'(\hat{\theta}_g) > 0$ for $\hat{\theta}_g \in [0, 1]$. Intuitively $U(\delta) > U_M(\delta)$ because $2\delta < \hat{\theta}_g$ i.e. the kink of one-agent scheme is located to the left of the kink of the optimal mechanism for a coalition and therefore, for the one-agent scheme the region, where the optimal mechanism coincides with the agent's first best is bigger than for coalition. ■

• **Proof of Proposition 6:** We first prove the following Lemma:

Lemma 5 *: For the normal distribution $N(0, \frac{1}{\sigma^2})$, the optimal mechanism is continuous and unique when $\delta\sigma > \sqrt{\frac{2}{\pi}}$. The cut-off $\hat{\theta}(\sigma)$ is then an increasing function of σ.*

Proof: The normal distribution is log-concave. We know from Proposition 2 that the optimal mechanism if it is continuous is necessarily unique. From (7), we have

$$(\hat{\theta}(\sigma) - \delta)F(\hat{\theta}(\sigma), \sigma) = \int_{-\infty}^{\hat{\theta}(\sigma)} \theta f(\theta, \sigma) d\theta = \sigma \int_{-\infty}^{\hat{\theta}(\sigma)} \frac{\theta e^{-\frac{\theta^2\sigma^2}{2}} d\theta}{\sqrt{2\pi}}$$

where $F(\theta, \sigma)$ is the c.d.f. for $N(0, \frac{1}{\sigma^2})$. Thus we find

$$\delta = \hat{\theta}(\sigma) + \frac{f(\hat{\theta}(\sigma), \sigma)}{\sigma^2 F(\hat{\theta}(\sigma), \sigma)}. \tag{A10}$$

The right-hand side of (A10), viewed as a function of $\hat{\theta}$, is increasing in $\hat{\theta}$ and goes to infinity and for $\delta\sigma > \sqrt{\frac{2}{\pi}}, \hat{\theta}(\eta)$ always exists and is necessarily positive. Proposition 2 then applies to ensure continuity of the optimal mechanism. ■

To prove Proposition 6, note then that, as two groups merge, $z = \frac{\theta_1+\theta_2}{2}$ follows $N(0, \frac{1}{2\sigma^2})$. The result follows from Proposition 4. ■

• **Proof of Lemma 3:** First, consider the sets $P_i = P_i^l \cup P_i^r,\ i = 1, 2$ where

$$P_1^l = \left\{(\theta_1, \theta_2)|\forall\hat{\theta}, \hat{\theta} < \theta_1, q(\hat{\theta}, \theta_2) = q(\theta_1, \theta_2)\right\},$$

and

$$P_1^r = \left\{(\theta_1, \theta_2)|\forall\hat{\theta}, \hat{\theta} > \theta_1, q(\theta_1, \theta_2) = q(\hat{\theta}, \theta_2)\right\}.$$

The sets P_1 and P_2 are sets of pooling on θ_1 and θ_2 respectively and, for example, P_1^l is a pooling part (on θ_1) of the continuous mechanisms for each value of θ_2 started from the left of the area $\Theta \times \Theta$.

Using dominant strategy incentive compatibility we observe that the sets P_i^l and P_i^u have continuous graphs, i.e., for example the function $s(\theta_2) = \sup_{(\theta_1,\theta_2)\in P_1^l} \theta_1$ is continuous. So, both sets P_1 and P_2 are closed sets.

Since a dominant strategy incentive compatible scheme along lines which are parallel to axes has to be either flat or coincides with the first best of the agents it is straightforward to see that the boundaries of sets P_i^l are straight lines which are either vertical (horizontal) for P_1^j (P_2^j) or belong to the diagonal of $\Theta \times \Theta$.

By symmetry we restrict ourselves to the area $\{(\theta_1, \theta_2)\ \ s.t.\ \theta_1 > \theta_2\}$ and we construct the dominant strategy incentive compatible scheme moving from $(\underline{\theta}, \underline{\theta})$ to the right.

Define θ^* from $P_1^l \cap P_2^l = \{(\theta_1, \theta_2)\ \ s.t.\ \theta_1 \leq \theta^*,\ \theta_2 \leq \theta^*\}$.[35] There are possible three situations: a) the mechanism $q(\theta_1, \theta_2)$ coincides with θ_1 on $S = \{(\theta_1, \theta_2)\ \ s.t.\ \theta_1 \leq \theta^* + \varepsilon,\ \theta_2 \leq \theta^* + \varepsilon\}$; b) we have $\sup_{(\theta_1,\theta_2)\in P_1^l \cap P_2^l} \theta_1 = \overline{\theta}$; c) the mechanism $q(\theta_1, \theta_2)$ coincides with θ_2 on $S' = \{(\theta_1, \theta_2)\ \ s.t.\ \theta_1 \leq \varepsilon,\ \theta_2 \leq \varepsilon\}$.[36] In the cases b) and c) we set up $\theta^* = \theta^{**} = \overline{\theta}$. Since the continuous mechanism cannot be interrupted by the region of pooling we have that the next to the right pooling area $P_1^r \cap P_2^l$ ends up in the right frontier of $\Theta \times \Theta$ and

[35]In the case $P_1^l \cap P_2^l = \varnothing$ we set up θ^* to be equal to $\underline{\theta}$.

[36]This is possible only in case $P_1^l \cap P_2^l = \varnothing$.

constitutes a rectangle $\{(\theta_1, \theta_2) \ \ s.t. \ \theta_1 \geq \theta^{**}, \ \ \theta_2 \leq \theta^{**}\}$.[37] In the area up to $P_1^r \cap P_2^l$ for $\{(\theta_1, \theta_2) \ \ s.t. \ \theta_1 > \theta_2\}$ there are possible two cases: either $b = \overline{\theta}$ or $q(\theta_1, \theta_2)$ coincides with θ_2. Finally the mechanism will reach the area $P_1^r \cap P_2^r = \{(\theta_1, \theta_2) \ \ s.t. \ \theta_1 \geq \theta^{***}, \ \ \theta_2 \geq \theta^{***}\}$.[38]■

• **Proof of Proposition 7:** Let us provide a Lemma which shows the conditions under which continuity of that dominant strategy mechanism arises.

Lemma 6 : *For distributions satisfying condition (7), the optimal dominant strategy incentive compatible mechanism $q(\cdot)$ is necessarily continuous.*

Proof: Suppose that, at point $a = (a_1, a_2)$ we have a discontinuity of the optimal mechanism. There are two possible cases: either $a_1 > a_2$ or $a_1 < a_2$. Suppose, without loss of generality, that $a_1 > a_2$. Then it has to be that for one of the uni-dimensional mechanisms determined by $\theta_1 = a_1$ or $\theta_2 = a_2$ and $q(\theta_1, \theta_2)$ there is a discontinuity at either point a_2 or a_1 correspondingly. Note that it cannot be the case that there are discontinuities in both cases simultaneously. Now suppose first that, for the uni-dimensional mechanism $\theta_2 = a_2$, there is a discontinuity at point a_1.

Then the points of discontinuity close to $a = (a_1, a_2)$ constitute a straight line up to the line $\theta_1 = \theta_2$ By symmetry we must have a symmetric line of points of discontinuity at $\theta_2 = a_1$.

Replacing the mechanism by θ_1 on flat regions in area $\theta_1 > \theta_2$ and by θ_2 on $\theta_1 < \theta_2$ and using the argument of Lemma 2 we conclude that the original mechanism can be improved.

The second case is possible, when at point $a = (a_1, a_2)$, there is a uni-dimensional discontinuity for $\theta_1 = a_1$. Here we replace the mechanism by θ_2 on area $\theta_1 > \theta_2$ and by θ_1 on $\theta_1 < \theta_2$. ■

Remark 1: First we prove that the optimal mechanism has indeed the form (14). If we denote by $V(\theta^*, \theta^{**}, \theta^{***})$ the payoff of the principal for general dominant strategy incentive compatible scheme then it is easy to see, that for a uniform distribution:

$$\frac{\partial V}{\partial \theta^{***}} = -\frac{1}{2}\left(1 - \theta^{***}\right)^2 \left(1 + 2\delta - \theta^{***}\right) < 0$$

Therefore $\theta^{***} = \overline{\theta} = 1$.

Remark 2: We need the values of kinks θ^* and θ^{**} for the optimal mechanism (14).

[37] θ^{**} might be equal to $\overline{\theta}$.

[38] In case $P_1^r \cap P_2^r = \varnothing$ we set up $\theta^{***} = \overline{\theta}$.

They can be determined for general distribution by the first-order conditions:

$$\begin{aligned}\int_{\underline{\theta}}^{\theta^*}\left(\int_{\underline{\theta}}^{\theta_1}\left(\theta^{ast}-\frac{\theta_1+\theta_2}{2}-\delta\right)f\left(\theta_2\right)d\theta_2\right)f\left(\theta_1\right)d\theta_1 &= 0 \quad \text{(A11)}\\ \int_{\theta^{**}}^{\overline{\theta}}\left(\int_{\underline{\theta}}^{\theta^{**}}\left(\theta^{**}-\frac{\theta_1+\theta_2}{2}-\delta\right)f\left(\theta_2\right)d\theta_2\right)f\left(\theta_1\right)d\theta_1 &= 0.\end{aligned}$$

After manipulations we get:

$$\delta=\frac{1}{F\left(\theta^*\right)}\int_{\underline{\theta}}^{\theta^*}F\left(\theta\right)d\theta. \quad \text{(A12)}$$

and

$$\theta^{**}=E_f\left(\theta\right)+\delta\left(1-F\left(\theta^{**}\right)\right)+\int_{\underline{\theta}}^{\theta^{**}}F\left(\theta\right)d\theta-\frac{\int_{\underline{\theta}}^{\theta^{**}}F\left(\theta\right)d\theta}{F\left(\theta^{**}\right)} \quad \text{(A13)}$$

Note that the formula for θ^* is the same as in a single-agent framework. For a uniform distribution, we get some particularly simple expressions (15):

$$\begin{aligned}\theta^* &= 2\delta\\ \theta^{**} &= \frac{1}{2}+2\delta\end{aligned}$$

We can conclude that for $\delta\in\left[\frac{1}{4},\frac{1}{2}\right]$ the optimal dominant strategy mechanism has a form with one cut-off value $\theta^*=2\delta$ and for $\delta\in\left[0,\frac{1}{4}\right]$ there are two cut-off values $\theta^*=2\delta$ and $\theta^{**}=\frac{1}{2}+2\delta$. In order to prove Proposition 7 we have to consider three cases depending on the value of the conflict δ.

Consider the case $\delta\in\left[\frac{1}{4},1\right]$. The cases $\delta\in\left[\frac{1}{6},\frac{1}{4}\right]$ and $\delta\in\left[0,\frac{1}{6}\right]$ are treated similarly. For a coalition between groups, the optimal mechanism yields to the decision-maker:

$$\begin{aligned}V_M(\hat{\theta}_g) &= -\frac{1}{2}\int_0^{\hat{\theta}_g}\int_0^{\hat{\theta}_g}\left(\hat{\theta}_g-\frac{\theta_1+\theta_2}{2}-\delta\right)^2 d\theta_2 d\theta_1\\ &\quad -\int_{\hat{\theta}}^1\left(\int_0^{2\hat{\theta}_g-\theta_1}\left(\hat{\theta}_g-\frac{\theta_1+\theta_2}{2}-\delta\right)^2 d\theta_2\right)d\theta_1\\ &\quad -\int_{\hat{\theta}_g}^1\left(\int_{2\hat{\theta}_g-\theta_1}^{\theta_1}\delta^2 d\theta_2\right)d\theta_1,\end{aligned}$$

where $\hat{\theta}_g$ is the cut-off in the case of a coalition.

Let us consider now the dominant strategy incentive compatible mechanism such that $\theta^*=\hat{\theta}_g$. It yields to the decision-maker an expected payoff

$$\begin{aligned}V_D(\hat{\theta}_g) &= -\frac{1}{2}\int_0^{\hat{\theta}_g}\int_0^{\hat{\theta}_g}\left(\hat{\theta}_g-\frac{\theta_1+\theta_2}{2}-\delta\right)^2 d\theta_2 d\theta_1\\ &\quad -\int_{\hat{\theta}_g}^1\left(\int_0^{\theta_1}\left(\theta_1-\frac{\theta_1+\theta_2}{2}-\delta\right)^2 d\theta_2\right)d\theta_1.\end{aligned}$$

Considering the difference $V_D(\hat{\theta}_g) - V_M(\hat{\theta}_g)$ and taking into account that the first members are the same we have:

$$V_D(\hat{\theta}_g) - V_M(\hat{\theta}_g) = \frac{1}{24}(1-\hat{\theta}_g)^2\left(2\hat{\theta}_g(1+12\delta) - 1 - 7\hat{\theta}_g^2\right) \tag{A14}$$

Using the expression (10) for δ, the sign of (A14) is the same as the sign of the expression:

$$\frac{2\hat{\theta}_g^4 - 16\hat{\theta}_g^3 + 7\hat{\theta}_g^2 + 2\hat{\theta}_g - 1}{2\hat{\theta}_g^2 - 4\hat{\theta}_g + 1}$$

which is positive for $\hat{\theta}_g \in \left[\frac{1}{2}, 1\right]$.[39]

Let us turn to the interest groups. We consider the optimal mechanisms in both cases (coalition and decentralization) for arbitrary $\delta \in [\frac{1}{4}, \frac{1}{2}]$. The expected utility of such a group in a coalition is:

$$\begin{aligned} U_M\left(\hat{\theta}_g(\delta)\right) &= -\frac{1}{2}\int_0^{2\hat{\theta}_g(\delta)-1}\left(\int_0^1\left(\hat{\theta}_g-\theta_1\right)^2 d\theta_2\right)d\theta_1 - \\ &-\frac{1}{2}\int_{2\hat{\theta}_g(\delta)-1}^1\left(\int_0^{2\hat{\theta}_g(\delta)-\theta_1}\left(\hat{\theta}_g(\delta)-\theta_1\right)^2 d\theta_2 + \int_{2\hat{\theta}_g(\delta)-\theta_1}^1\left(\frac{\theta_1+\theta_2}{2}-\theta_1\right)^2 d\theta_2\right)d\theta_1 = \\ &= \frac{1}{6}\hat{\theta}_g(\delta)\left(-1+3\hat{\theta}_g(\delta)-4\hat{\theta}_g^2(\delta)+\hat{\theta}_g^3(\delta)\right) \end{aligned}$$

where $\hat{\theta}_g(\delta)$ is determined by (10).[40] In the case of the decentralized mechanism, we have instead:

$$\begin{aligned} U_D\left(\hat{\theta}_f(\delta)\right) &= -\frac{1}{2}\int_0^{\hat{\theta}_f(\delta)}\int_0^{\hat{\theta}_f(\delta)}\left(\hat{\theta}_f(\delta)-\theta_1\right)^2 d\theta_2 d\theta_1 - \frac{1}{2}\int_{\hat{\theta}_f(\delta)}^1\left(\int_0^{\theta_2}(\theta_2-\theta_1)^2 d\theta_1\right)d\theta_2 = \\ &= -\frac{1}{24}\left(1+3\hat{\theta}_f^4(\delta)\right) \end{aligned}$$

here $\hat{\theta}_f(\delta) = 2\delta$.

It can be shown that the difference between expected utilities: $U_D\left(\hat{\theta}_f(\delta)\right) - U_M\left(\hat{\theta}_g(\delta)\right)$ is positive for $\delta \in [\frac{1}{4}, \frac{1}{2}]$. Therefore for groups $U_M\left(\hat{\theta}_g(\delta)\right) < U_D\left(\hat{\theta}_f(\delta)\right)$.[41] ■

[39] Computations for $\delta \leq \frac{1}{4}$ show that $V_D > V_M$ for $\delta > \delta^* \approx 0.11$ and for $\delta < \delta^*$, $V_M > V_D$.

[40] Equation (11) has three parametrical roots $x_1(\delta)$, $x_2(\delta)$, $x_3(\delta)$. We have to select the one for which $x_i(\frac{1}{2}) = 1$.

[41] We can show that for $\delta > \delta^{**} \in [\frac{1}{6}, \frac{1}{4}]$, $U_D > U_M$ and for $\delta < \delta^{**}$, we have the reverse.