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Abstract

This article analyzes the effect of speculation on the economic welfare from various criteria, using a simple Edgeworth box within a three-period Walrasian competition framework. Here "speculation" is defined as a series of transition processes of each agent's spontaneous production of private information, the exchange of commodities based on it under the externality environment, and finally its spillover into public. The methodology and implications are closely related with the concepts of herd behavior by Banerjee (1992), informational externality by Stein (1987), information sharing by Shapiro (1985), and economic value of speculation by Hirshleifer (1971, 1975, 1977). It is explicitly shown that the complete sharing of produced information under externality environment, if not accompanied by almost sure productivity effect, does not necessarily attain the non-negative economic value especially in terms of ex-ante expected utility. The implication is consistent with Aoki (2005). Lastly some criticisms about why this could happen are discussed.

1. Introduction

In this paper, on the basis of the model set in my previous paper ("Models of equilibrium pricing with internalized powers of independent judgment based on autonomy", February 2000), we state a simple model for a speculation mechanism, and analyze its impact on the welfare, and present some problems in the decision-making theory.

According to the information theory, the information is, if produced, supposed to make the prediction regarding the state of the world surer, on the other hand, it is inherently a probabilistic variable, therefore it may be fallible. It is a well-known fact that, given the payoff function exogenously, the information does always have a non-negative economic value.[1-1] Consequently, an agent may be expected to become better off by means of speculation, that is, by producing private information, as defined later in this paper. On the other hand, some related literatures tell us that the existence of informational asymmetry may distort the social welfare and may lead to a negative externality on each agent's welfare. Therefore, the general influence of information on the economic systems is, in spite of historical literatures regarding information and speculation, not necessarily clear.[1-2]

While this paper owes its theoretical foundation to a series of literatures listed in the references, however, the points we carefully consider and newly focus on in this paper are as follows.

[1-1] For the rigorous calculation, see Appendix [A] of Aoki (2000).

[1-2] Aoki (2007) analyzes the effect of information sharing in a vertical market structure with a Bertrand setting, applying some concepts of "shared/private information" equilibria originally developed by Shapiro (1986), and shows that its effect regarding symmetrization, not productivity, under externality environment might not necessarily attain the non-negative economic value.

1. Although it is rather clear that speculation can be explained by speculator's rationality based on her private information, there seems to be no existing literatures, which extract the essence of speculation dynamics within the simplest and classical economic framework, and in addition succeed in deriving some general conclusion regarding welfare analysis, which is not really specific to the model.

2. There seems to be a strong necessity that we discuss more rigorously and explicitly about what the value of information is like over an infinite time horizon, and what speculation eventually brings about in terms of the ex-ante expected welfare including the economic effect for "other" agents. For example, where privately created information spills over perfectly instantaneously, the equilibrium price is to be adjusted also perfectly instantaneously without any transaction of commodities. However, since it does not realistically illustrate what is actually going on in observed financial markets, we need to set at least one period, in which private information itself is not transacted with price and commodities are traded on the basis only of private information, which may be exclusive to that agent of the overall information set of that period. In other words, we need to analyze the externality of information, which is the same concern as in Stein (1987).

3. However, some historical literatures as by Akerlof (1970), already point out that the asymmetry in information may distort the social welfare, so in order to remove the negative effect purely caused by the asymmetry in information and to measure the economic effect of speculation in a true sense, we need to set, after the period processing speculation, another period for adjustment process in which the privately produced information or the true states of the world spill over into public and all agents come to share the common information, which surely enhances the ex-ante welfare for every agents, compared with in the previous period of speculation.

4. We need to describe more carefully whether speculation is economically rational at least for a speculator (an informationally superior agent) or not in the ex-ante sense, even if it is invoked on the "private" or "partial" information, or on the "false" information. Actually we proved that it is, in Proposition 2 and 3.

5. The information as defined in the information theory is "informative" because of the "almost no-correlation" with the existing overall information set, but is also "fallible" because of the very aspect. In this sense, information merely raises a "mean-preserving spread" in terms of prediction, as Hirshleifer (1977, 1975, 1971) expresses it as a "random walk" (Remember that in a risk averse economy, the mean-preserving spread of allocations leads to a decrease in the expected utility.). There seem to be no existing works, which explicitly analyze the economic effect of this aspect, the "fallibility" in information within a competitive, risk averse, dynamic economy, although Sah and Stiglitz (1986, 1985) discuss it

in the context of organizational economics.

In this paper, in order to answer these points, we state a simple model for a speculation mechanism, as an economic process for producing a private information, and analyze its impact on the welfare, and present some problems in the decision-making theory. Specifically, the model is constructed on Walrasian equilibrium transition processes over three periods, and on Shannon's famous definition of "information". This paper is similar to the implications and methodology of Banerjee (1992), in which the mechanism of herd behavior is analyzed from the viewpoints of information revelation and Bayesian inference, and the resulting inefficiency is explicitly stated. In this article we are just trying to be more rigorous by focusing on the role of informational externality, in which the produced information itself is not incorporated within an economic transaction of commodities, by removing the negative effect of informational asymmetry, and by analyzing the impact on the overall exchange economy. For reference, De Long, Shleifer, Summers and Waldmann (1987), Grossman and Stiglitz (1976), Harrison and Kreps (1978), Hong and Stein (1987), Stein (1987) treat informational aspects of speculation, and Harris and Townsend (1981) considers resource allocation under asymmetric information. Some other insightful analyses regarding the economic value of information appear, for example, Bode and Thunik (1998) and Mahieu and Bauer (1998).

The organization of this paper is as follows. Section 2 describes the basic concepts used in this paper. Section 3 explains the three-period setting of the equilibrium transition incorporating the adjustment process of information, and some criteria needed for the welfare analysis, which should be one of the main parts of this paper in section 6. In section 4, we describe a decision-making mechanism for speculation, and prove some fundamental economic features of it, taking the equilibrium price vector exogenously as given. In section 5, we analyze the effect of speculation mainly using an Edgeworth box within a Walrasian competition framework, and in section 6, we summarize all these welfare analysis under the three-period setting.

2. Basic Model

Assume that there exists a world, which incorporates uncertainty. The state of the world is described as a binary random variable, M, so that M=1 or M=0. The true (real) value of M is not revealed as least at the initial period, t=0. The period is described as a discrete number, t=0, 1, 2,…. Two agents, i=1, 2, exchange two commodities, k=1, 2, as Walrasian competitors. Commodity 1 is a riskless asset, and therefore the utility does not depend on the state variable, M. Commodity 2 is a risky asset, so the utility does depend on M. Hence, we assume that agent i has a separable and state-dependent utility function as follows;

$$U_i(\underline{x}^{i,t}) = u_{i1}(x_1^{i,t}) + u_{i2}^j(x_2^{i,t}) \text{ if M=j (j=1,0)} \qquad (2.1)$$

where we denote agent i's bundle vector in equilibrium at the end of period t, by $\underline{x}^{i,t} = (x_1^{i,t}, x_2^{i,t})$, and $x_k^{i,t}$ is an amount of the holdings for commodity k. We also assume that the initial endowment vector of each agent i, which is denoted by $\underline{x}^{i,0} = (x_1^{i,0}, x_2^{i,0})$, is already given. As for the utility functions, we set the following ordinary properties.

(1) Inada conditions or etc:

$$\lim_{x_1^{i,t} \to +0} u_{i1}'(x_1^{i,t}) = +\infty, \ \lim_{x_2^{i,t} \to +0} u_{i2}^j{}'(x_2^{i,t}) = +\infty, \ u_{i1}(0) = 0, \ u_{i2}^j(0) = 0$$

$$\lim_{x_1^{i,t} \to +\infty} u_{i1}'(x_1^{i,t}) = +0, \ \lim_{x_2^{i,t} \to +\infty} u_{i2}^j{}'(x_2^{i,t}) = +0 \qquad (2.2)$$

(2) An increasing and concave (risk averse) function with the commodity holdings:

$$u_{i1}'(x_1^{i,t}) > 0, \ u_{i1}''(x_1^{i,t}) < 0, \ u_{i2}^j{}'(x_2^{i,t}) > 0, \ u_{i2}^j{}''(x_2^{i,t}) < 0 \qquad (2.3)$$

These assumptions just ensure that a short sale (i.e., $x_k^{i,t} < 0$) cannot happen. (Actually, $x_k^{i,t} > 0$ is ensured.) In addition, we assume the following property for some convenience to draw the contract curve in the Edgeworth box, which we will describe later.

(3) Linear relations between states and between commodities:

$$u_{i2}^1(x) / u_{i2}^0(x) = \gamma = const.$$

$$u_{i2}^0(x) / u_{i1}(x) = \gamma' = const. \qquad \text{for all } x > 0 \qquad (2.4)$$

The following common form of the utility functions for both agents satisfies all of (1), (2) and (3), as well as having a constant coefficient of relative risk aversion (CRRA), a, so we adopt this form of functions for computational analysis in this paper.[2-1]

$$u_{i1}(x_1^{i,t}) = \gamma_1 (x_1^{i,t})^{1-a} \text{ where } \gamma_1 > 0.$$

[2-1] Here we need to emphasize that, in this paper, we do not take account of any (positive) endogenous economic effect regarding the profitability of a risky commodity (2), γ_2^j, as discussed in section 7. So, the output results of information produced by agents do not influence, with some economic feedback, the form of each agent's utility, (2.5) at all. Using the notations in (7.1) of footnote 7-3, this is equivalent to $e^{1,1} = e^{0,0} = e^{1,0} = e^{0,1} = 0$.

$$u_{i2}^{j}(x_2^{i,t}) = \gamma_2^j (x_2^{i,t})^{1-a} \text{ if } M = j, \text{ where } \gamma_2^1 > \gamma_2^0 > 0. \quad (2.5)$$

Each agent i has respectively a "subjective" prediction regarding the state of the world, M. We denote it by $\beta_t^i \equiv \Pr ob(M = 1 \mid \Phi_t^i)$, the probability that M=1 conditional on Φ_t^i, where Φ_t^i is an information set which is privately available only for agent i at the end of period t. Apart from her own prediction, β_t^i, each agent i may be able to make use of her inner information production function. We denote the private binary information, which is personally produced by agent i, by D_i, where the output of $D_i (= l)$ takes the binary value, $l = 1,0$. In terms of the information theory, the performance vector, (P_1^i, P_0^i), of this function is defined as follows.[2-2]

$$\Pr ob(D_i = 1 \mid M = 1) \equiv P_1^i$$

$$\Pr ob(D_i = 1 \mid M = 0) \equiv P_0^i \quad (2.6)$$

That is, we denote the probability that $D_i = 1$ is output conditionally on M=j, by P_j^i. We assume that (P_1^i, P_0^i) (i=1,2) are publicly known to both agents i=1,2. In addition, for simplicity, we assume that $P_1^i = 1 - P_0^i \geq 0.5$. Therefore, as $P_1^i (= 1 - P_0^i)$ approaches 1, we expect that a surer prediction can be obtained by producing the private information. The cost of producing information is denoted by C_i in terms of the utility. As long as there is no special comment on it, we assume that $C_i = 0$, that is, the cost of producing information is zero[2-3]. Each agent i, if she produces her private information, D_i, or if she can observe the other agent's disclosed information, D_{-i}[2-4], then she changes the prediction regarding the state of the world, according to the Bayesian inference. In this case, the information set of agent i at period t is represented as $\Phi_t^i = \Phi_{t-1}^i \cup \{D_{i'} = l'\}$ for i'=i, or –i. For example,

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[2-2] This definition was proposed, for the first time, by Shannon ([13]).

[2-3] Strictly speaking, in order to avoid an infinite repetition of producing a private information, that is, exercising speculation, it may be more appropriate to set the cost of producing information at an infinitely small positive number ε, that is, $C_i = \varepsilon > 0$.

[2-4] We denote the other agent by –i. Therefore, -i=2 for i=1 and -i=1 for i=2.

assume that at the beginning of period t, agent i has produced a private information, D_i. Then, as a function of her previous prediction, $\beta_{t-1}^i \equiv \Pr ob(M=1 \mid \Phi_{t-1}^i)$, and the output of $D_i (=l)$, she gets a new prediction as following.[2-5]

If $D_i = 1$, then:

$$\beta_t^i \equiv \Pr ob(M=1 \mid \Phi_t^i) = \Pr ob(M=1 \mid \Phi_{t-1}^i, \{D_i = 1\}) \equiv \tilde{\beta}_t^i (D_i = 1) = \frac{\beta_{t-1}^i P_1^i}{\beta_{t-1}^i P_1^i + (1-\beta_{t-1}^i) P_0^i}$$

If $D_i = 0$, then:

$$\beta_t^i \equiv \Pr ob(M=1 \mid \Phi_t^i) = \Pr ob(M=1 \mid \Phi_{t-1}^i, \{D_i = 0\}) \equiv \tilde{\beta}_t^i (D_i = 0) = \frac{\beta_{t-1}^i (1-P_1^i)}{\beta_{t-1}^i (1-P_1^i) + (1-\beta_{t-1}^i)(1-P_0^i)} \quad (2.7)$$

Here, $\tilde{\beta}_t^i (D_i = l)$ denotes the posterior Bayesian prediction of agent i at period t with the occurrence, $D_i = l$ at period t under the last information set, Φ_{t-1}^i. We also denote the overall collected set of all information, which each agent respectively holds at period t, by $\Phi_t (\equiv \Phi_t^1 \cup \Phi_t^2)$. For example, assume $\Phi_t^1 = \{D_1 = 1\}$ and $\Phi_t^2 = \{D_2 = 0\}$. Then, we have $\Phi_t = \{D_1 = 1, D_2 = 0\}$. At the initial period 0, we assume that Φ_0^i is a null set (denoted by $\{\phi\}$), so that we have $\Phi_0^1 = \Phi_0^2 = \Phi_0 = \{\phi\}$ and $\beta_0^i = \Pr ob(M=1 \mid \Phi_0^i = \{\phi\}) \equiv \alpha = 0.5$, where α is the common value of both agents' initial predictions. Using this overall information set, Φ_t, we can define the "objective" prediction at period t by $\beta_t \equiv \Pr ob(M=1 \mid \Phi_t)$.

At each period t, each agent i behaves herself as a Walrasian competitor, based on her own current prediction, β_t^i, which has just been renewed at the beginning of the period, and taking her last bundle, $\underline{x}^{i,t-1} = (x_1^{i,t-1}, x_2^{i,t-1})$, as her endowment. Then, finally a new Walrasian (competitive) equilibrium is reached, and a new allocation-price pair, $(\underline{x}^t, \underline{q}^t)$,

[2-5] Also see Appendix 2 for more neat calculation. It is easily verified that if $P_1^i (= 1 - P_0^i) = 0.5$, then we have $\tilde{\beta}_t^i (D_i = l) = \beta_{t-1}^i$ for $l = 1, 0$.

where $\underline{x}^t = (\underline{x}^{1,t}, \underline{x}^{2,t})$ is an equilibrium allocation vector, and $\underline{q}^t = (q_1^t, q_2^t)$ (q_k^t for commodity k) is a new equilibrium price vector, are determined at the end of the period t. As explained later in Chapter 4 for the mechanism of speculation, whether each agent should produce a private information or not at the beginning of the period, is to be decided entirely from the viewpoint of her (ex-ante) expected utility. Given her prediction, $\beta_t^i \equiv \Pr ob(M = 1 \mid \Phi_t^i)$, which is calculated conditionally on her available information set, Φ_t^i, agent i's expected utility for holding a bundle vector, $\underline{x} = (x_1, x_2)$, is calculated as follows.

$$\begin{aligned} E\left[U_i(\underline{x}) \mid \Phi_t^i\right] &\equiv u_{i1}(x_1) + \left\{\beta_t^i u_{i2}^1(x_2) + (1-\beta_t^i) u_{i2}^0(x_2)\right\} \\ &= \gamma_1 (x_1)^{1-a} + \{\beta_t^i \gamma_2^1 + (1-\beta_t^i)\gamma_2^0\}(x_2)^{1-a} \end{aligned} \tag{2.8}$$

Here, for the distinct two bundle vectors, $\underline{x} = (x_1, x_2)$ and $\underline{x}' = (x_1', x_2')$, we say that, iff $E\left[U_i(\underline{x}) \mid \Phi_t^i\right] \le E\left[U_i(\underline{x}') \mid \Phi_t^i\right]$, then the bundle $\underline{x}'$ is as least as good as the bundle $\underline{x}$ for agent i, conditionally on the information set, Φ_t^i, or equivalently with the prediction, β_t^i. We denote this by $\underline{x} \preceq_{(\Phi_t^i)} \underline{x}'$, or equivalently by $\underline{x} \preceq_{\beta_t^i} \underline{x}'$.[3] Similarly, we say that, iff $E\left[U_i(\underline{x}) \mid \Phi_t^i\right] < E\left[U_i(\underline{x}') \mid \Phi_t^i\right]$, then the bundle $\underline{x}'$ is better off than the bundle $\underline{x}$ for agent i, conditionally on the information set, Φ_t^i, or equivalently with the prediction, β_t^i, denoting it by $\underline{x} \prec_{(\Phi_t^i)} \underline{x}'$, or equivalently by $\underline{x} \prec_{\beta_t^i} \underline{x}'$.

Since each agent is a Walrasian competitor, takes her last equilibrium bundle, $\underline{x}^{i,t-1} = (x_1^{i,t-1}, x_2^{i,t-1})$, as an endowment, and her current prediction, β_t^i, as given, the demand function and the indirect utility function of agent i at period t are well defined as follows.

$$\underline{x}_D^i(\underline{x}^{i,t-1}, \beta_t^i, \underline{q}) = \underset{\underline{x}=(x_1,x_2)}{\arg\max}\, E\left[U_i(\underline{x}) \mid \Phi_t^i\right] \quad \text{s.t.} \quad \underline{q} \cdot \underline{x} \le \underline{q} \cdot \underline{x}^{i,t-1}$$

[3] This notation is originally and explicitly used in the preliminary version of Aoki (2003).

$$v^i(\underline{x}^{i,t-1}, \beta_t^i, \underline{q}) = \max_{\underline{x}=(x_1,x_2)} E\left[U_i(\underline{x}) \mid \Phi_t^i\right] \quad \text{s.t} \quad \underline{q} \cdot \underline{x} \le \underline{q} \cdot \underline{x}^{i,t-1} \qquad (2.9)$$

where $\underline{q} = (q_1, q_2)$ is a price vector. Therefore, the Walrasian (competitive) equilibrium allocation-price pair $(\underline{x}^t, \underline{q}^t)$ at period t, where $\underline{x}^t = (\underline{x}^{1,t}, \underline{x}^{2,t})$, is also well defined such that:

(1) $\underline{x}^{i,t} = \underline{x}_D^i(\underline{x}^{i,t-1}, \beta_t^i, \underline{q}^t)$ for i=1,2

(2) $\underline{x}^{1,t} + \underline{x}^{2,t} = \underline{x}^{1,t-1} + \underline{x}^{2,t-1}$ (2.10)

Since $(\underline{x}^t, \underline{q}^t)$ is also in a Walrasian equilibrium with the endowment allocation, $\underline{x}^t = (\underline{x}^{1,t}, \underline{x}^{2,t})$, and the current prediction, β_t^i, we have:

$$\underline{x}^{i,t} = \underline{x}_D^i(\underline{x}^{i,t}, \beta_t^i, \underline{q}^t) \quad \text{for i=1,2} \qquad (2.11)$$

We assume that the initial endowment bundle vector of each agent i, $\underline{x}^{i,0} = (x_1^{i,0}, x_2^{i,0})$, satisfies;

$$\underline{x}^{1,0} = (1.0 - s, 1.0 - s) \quad \text{and} \quad \underline{x}^{2,0} = (1.0 + s, 1.0 + s) \quad \text{where} \quad -1 \le s \le 1 \qquad (2.12)$$

so that we have $x_1^{i,0} = x_2^{i,0}$ for i=1,2, and the total endowment vector is always $\underline{w} = \underline{x}^{1,t} + \underline{x}^{2,t} = (2.0, 2.0)$ for all t. For the F.O.C. equilibrium conditions, see Appendix 1.

The assumptions (2.2) and (2.3) assure that the preference of each agent is convex, continuous and monotonic, irrespective of the values of β_t^i (i=1,2), and that the equilibrium allocation-price pair, $(\underline{x}^t, \underline{q}^t)$, is always unique and strictly positive in every element, that is, $(\underline{x}^t, \underline{q}^t) >> 0$. In addition, the form of utility functions represented by (2.4) has a very convenient property for the welfare analysis. That is, although, given these utility functions, the "subjective" contract curve (Pareto set) on the Edgeworth box shifts depending on agents' predictions, β_t^i's, or equivalently on agents' information sets, Φ_t^i's, it is always an identical straight line irrespective of the values of β_t^i, if $\beta_t^1 = \beta_t^2$, that is, if two agents have the same prediction at the period t. See Figure 1. Line OO' is corresponding to this contract curve (a straight line, actually). In other words, whenever the predictions of both agents are the same (i.e., $\beta_t^1 = \beta_t^2$), the equilibrium would be always reached on this straight line. We can also regard the line OO', as an "objective" contract curve, in the sense that, conditionally on the overall, common information set at period t, Φ_t, the contract curve for $\beta_t^1 = \beta_t^2 = \beta_t$ coincides with OO'. The authority, if he knows all output results of both agent's private information, as well as the form of their utility functions, as defined in (2.5), then would get the same line OO', as an "objective" contract curve. In addition, as an extreme case, we get the same line as a "true (real)" contract curve, on the condition that $\beta_t^1 = \beta_t^2 = j$, where the true value of M is M=j (j=1,0). Furthermore, this contract curve is exactly the same as the set of all possible initial endowment allocations, which satisfy (2.12). We define this set as $\underline{X}_p$ (i.e., OO'), that is:

$$\underline{X}_p \equiv \left\{ (\underline{x}_p^1, \underline{x}_p^2) : (\underline{x}_p^1, \underline{x}_p^2) = ((1.0 + s', 1.0 + s'), (1.0 - s', 1.0 - s')), -1.0 \le s' \le 1.0 \right\} \quad (2.13)$$

3. Some simple settings

Now we consider a model of the equilibrium transition over three periods. That is, t=0, 1, and 2. See Figure 2. For convenience and without the loss of generality, we normalize the price of the riskless asset (commodity 1) at 1 for any period. That is, $q_1^0 = q_1^1 = q_1^2 = 1$. At period 0, we assume that the initial endowment allocation, $\underline{x}^{i,0} = (x_1^{i,0}, x_2^{i,0})$, is given by (2.12) (that is, $\underline{x}^{i,0}$ is located on $\underline{X}_p$ (i.e., OO')), and also assume that the allocation-price pair, $(\underline{x}^0, \underline{q}^0)$, is already in a Walrasian equilibrium. From the setting at section 2, we again assume $\Phi_0^1 = \Phi_0^2 = \Phi_0 = \{\phi\}$. At the beginning of period 1, taking her initial endowment (bundle) vector, $\underline{x}^{i,0} = (x_1^{i,0}, x_2^{i,0})$, her initial (prior) prediction, $\beta_0^i (= \alpha)$, and the initial price vector, $\underline{q}^0 = (q_1^0, q_2^0)$, as given[3-1], each agent i judges whether she should produce a private information if she has an inner function for it, and if she does judge she should, then she does produce it, and change her next (posterior) prediction, β_1^i, according to the output of D_i through the Bayesian inference. We call a series of the equilibrium process, which begins by producing a private information, a "speculation", as defined more rigorously in section 4. At this period 1, we assume that agent i cannot observe the other agent's (-i) private information, D_{-i}, but can only make use of her own private information, D_i, if produced. For example, if $D_i(=l)$ is produced, then $\Phi_1^i = \{D_i = l\}$, but if not, then $\Phi_1^i = \{\phi\}$, respectively for i=1,2. Therefore, period 1 is considered to be the period, when each agent behaves herself as a Walrasian competitor, based on her private new information set, Φ_1^i, which would contain only her private information, D_i, if produced. Thus, the next equilibrium allocation-price pair, $(\underline{x}^1, \underline{q}^1)$, is determined at the end of period 1. At period 1, after a private information is produced by some agent, the equilibrium breaks. And, under the situation that the produced information is not disclosed to any other agents, namely, that the deviation (the asymmetry) of information (predictions) exists (i.e., $\beta_1^1 \neq \beta_1^2$), a new equilibrium is reached. Therefore, we may be able to examine the first round impact of speculation by means of analyzing the influence on the welfare at this period.

Thus, at this period, we set the following 2 cases:

<u>Case (A)</u>

Agent 1 has an inner information production function, which is represented by the performance vector, (P_1^1, P_0^1) as of (2.6), but agent 2 does not have it.

In this case, we might be able to say that the economic system is in the informational asymmetry, where agent 1 is information-superior and agent 2 is information-inferior.

<u>Case (B)</u>

Both of the 2 agents do have respectively an identical inner information production function, which is represented by the performance vectors, (P_1^i, P_0^i) for i=1,2 as of (2.6), where

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[3-1] From the assumption, we have $q_1^0 = 1$. From the F.O.C. for the equilibrium (See Appendix 1), assuming that $\underline{x}^{i,0}$ is located on $\underline{X}_p$, necessarily we have $q_2^0 = (\alpha\gamma_2^1 + (1-\alpha)\gamma_2^0)/\gamma_1$.

$P_1^1(=1-P_0^1)=P_1^2(=1-P_0^2)\equiv P_1\equiv 1-P_0$, say.

Although, in Case (B), the economic system is in the informational symmetry, the output of the private information produced by each agent may be distinct with some positive probability, so that each agent's resulting prediction may be different, that is, it may be the case that $\beta_1^1 \neq \beta_1^2$. In this sense, the asymmetry of information may still exist probabilistically in both cases of (A) and (B).

As a next stage, it may seem to us very natural to consider some processes, in which, after period 1, the produced information spills over to other agents, or the true value of the state of the world, m, is exogenously revealed into public, so that each agent comes to hold an identical prediction. Hence, period 2 may be considered to be the period of "the adjustment process" of information. We consider the following 2 cases at this period.

Case (I)

All of the private information, which has been produced at period 1, is entirely disclosed to the other agents, and at period 2 the two agents share the same information set. That is, $\Phi_2^1=\Phi_2^2=\Phi_2$. For example, if $D_1(=l_1)$ and $D_2(=l_2)$ are produced at period 1, then $\Phi_2^1=\Phi_2^2=\Phi_2=\{D_1=l_1, D_2=l_2\}$. For another example, if $D_1(=l_1)$ is produced and D_2 is not at period 1, then we have $\Phi_2^1=\Phi_2^2=\Phi_2=\{D_1=l_1\}$.

Case (II)

The real value of the state of the world, M=j, is exogenously revealed to both two agents at period 2. That is:

$$\beta_2^1(\equiv \Pr ob(M=1\,|\,\Phi_2^1))=\beta_2^2(\equiv \Pr ob(M=1\,|\,\Phi_2^2))=j,$$

where $\Phi_2^1=\Phi_2^2=\Phi_2=\{M=j\}$, if M=j (j=0,1).

Thus, in period 2, we may be able to examine the second impact of speculation.

For the welfare analysis, we consider the following criteria.

(U) Ex-ante Pareto improved? Efficient (optimal)?

In terms of the ex-ante expected utility, based on the initial (ex-ante) overall information set, Φ_0, do the overall possible equilibria allocations in possible equilibria paths attain the Pareto improvement (compared with the initial equilibrium at period 0), or the Pareto optimum?

(V) Ex-ante (expected) social welfare improved? Maximized?

Based on the initial (ex-ante) overall information set, Φ_0, is the ex-ante expected social welfare improved (compared with the initial equilibrium at period 0), or maximized? We adopt the following function as an ex-ante (expected) social welfare;

$$W(\underline{x}^t \mid \Phi_0) = \delta_1 * E\left[U_1(\underline{x}^{1,t}) \mid \Phi_0\right] + \delta_2 * E\left[U_2(\underline{x}^{2,t}) \mid \Phi_0\right] \text{ for t=1,2} \quad (3.1)$$

where the allocation, $\underline{x}^t = (\underline{x}^{1,t}, \underline{x}^{2,t})$, may (or may not) be stochastic variables under Φ_0.[3-2]

(W) Ex-post Pareto efficient (optimal)?

For the equilibrium path which may have actually happened, is the actually realized equilibrium allocation Pareto optimal, under the ex-post (actually realized) overall information set, Φ_1 or Φ_2?

(X) True state-based Pareto improved? Efficient (optimal)?

For each state of the world, M=j (j=1,0), which might be finally revealed to be true, that is, under the overall "true" information set, $\Phi \equiv \{M = j\}$, as $\Phi_2 \equiv \{M = j\}$ of case (II) at period 2, do the overall possible equilibria allocations in all possible equilibria paths attain the Pareto optimum?

(Y) First best Pareto efficient (optimal)?

For the true state of the world, M=j (j=1,0), is the actually realized equilibrium allocation, Pareto optimal under the "true" information set, $\Phi \equiv \{M = j\}$?

(Z) Second best (constrained) Pareto efficient (optimal)?

Can the authority, who knows the form of each agent's utility functions (i.e., (2.5)), but does not care about anything else including their information sets, or their predictions, Φ_t^i's or equivalently their predictions, β_t^i's, attain, by mandatory intervention, the Pareto improved allocation rather than the actually realized equilibrium allocation, whatever the actual state of the world, M=j, is?

In the ex-ante sense, we need to consider two criteria, (U) and (V), because the ex-ante Pareto optimum does not necessarily imply the ex-ante social welfare optimum, while the ex-ante social welfare optimum does imply the ex-ante Pareto optimum. Furthermore, the

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[3-2] We have two settings regarding δ_i's as follows.

1. We set the weight of each agent equally at $\delta_1 = \delta_2 = 0.5$.
2. Another possible setting may be to choose δ_i such that $\delta_i = \lambda_i^{-1} / (\lambda_1^{-1} + \lambda_2^{-1})$, where λ_i is agent i's marginal expected utility of initial income at the initial period 0. For example, assume that the initial equilibrium (and also endowment) allocation is given by (2.12). Then, by simple calculation, we easily get $\delta_1 = (1-s)^a / ((1-s)^a + (1+s)^a)$ and $\delta_2 = (1+s)^a / ((1-s)^a + (1+s)^a)$.

ex-ante social welfare improved does not imply the ex-ante Pareto improved, while the ex-ante Pareto improved does necessarily imply the ex-ante social welfare improved. The welfare analysis and some problems in the decision making theory will be discussed in section 5 and 6.

4. Decision making mechanism for speculation

Now we would dare to repeat that a Walrasian competitor takes her last equilibrium bundle, $\underline{x}^{i,t-1}$, as an endowment, and her current prediction, β_t^i, and the current price vector, $\underline{q}$ (not necessarily the equilibrium price, $\underline{q}^{t-1}$ or $\underline{q}^t$), as given, and furthermore, does not care about the other agents' prediction, β_t^{-i} (or β_{t-1}^{-i}). Then, we define the speculation as an economic behavior, in which some agent produces a private information through her information production function, declares distinct bundles, respectively, depending on the distinct outputs of the produced private information, $D_i = l$. See Figure 3. We denote agent i's bundle pair for speculation by $\underline{x}_s^i(D_i) \equiv \left(\underline{x}_s^i(D_i = 1), \underline{x}_s^i(D_i = 0)\right)$, where $\underline{x}_s^i(D_i = l)$ is a 1x2 vector for $l = 0,1$. In most cases, we simply denote $\underline{x}_s^i(D_i)$ by $\underline{x}_s^i$, as long as there cannot be any confusion. The bundle pair for speculation, $\underline{x}_s^i$, is a 1x4 vector, which designates a desired bundle for each output result, $D_i = l$.

Definition 1: Given the last endowment (and equilibrium) bundle, $\underline{x}^{i,t-1}$, the last prediction, β_{t-1}^i (or equivalently, the last information set, Φ_{t-1}^i), and the last equilibrium price, $\underline{q}^{t-1}$, the *feasible* bundle pair for *rational* speculation of agent i, $\underline{x}_s^{i,t}(D_i) \equiv \left(\underline{x}_s^{i,t}(D_i = 1), \underline{x}_s^{i,t}(D_i = 0)\right)$ at period t, is such that;

$$(1)\ E\left[U_i(\underline{x}^{i,t-1}) \mid \Phi_{t-1}^i\right] \le E\left[U_i(\underline{x}_s^{i,t}(D_i)) \mid \Phi_{t-1}^i\right] - C_i \quad \text{(Rationality constraint)}$$

$$(2)\ \underline{q}^{t-1} \cdot \underline{x}_s^{i,t}(D_i = l) \le \underline{q}^{t-1} \cdot \underline{x}^{i,t-1} \quad \text{respectively for l=1,0}$$

(Feasibility constraint) (4.1)

where D_i is a stochastic binary variable under the last information set, Φ^i_{t-1}, which has a property (2.6).

For some calculation see Appendix 2. Also, for some notations and explanations regarding the preference of the bundles for speculation, see Appendix 3. That is, the economic agent decides to take a plunge into speculation, if and only if a bundle pair for speculation respectively satisfies the budget constraint, given the last equilibrium price, and the (ex-ante) expected utility attained by the bundle pair for speculation is greater than or equal to that attained by keeping the last endowment bundle. Now we claim the following two propositions.

Proposition 1: Assume a bundle pair for speculation at period t, which is defined by:

$$\underset{\sim}{\tilde{x}}{}_s^{i,t} \equiv \left(\underset{\sim}{\tilde{x}}{}_s^{i,t}(D_i=1), \underset{\sim}{\tilde{x}}{}_s^{i,t}(D_i=0)\right) \equiv \left(\underline{x}^i_D(\underline{x}^{i,t-1}, \tilde{\beta}^i_t(D_i=1), \underline{q}^{t-1}), \underline{x}^i_D(\underline{x}^{i,t-1}, \tilde{\beta}^i_t(D_i=0), \underline{q}^{t-1})\right) \tag{4.2}$$

Then, taking the last equilibrium price, $\underline{q}^{t-1}$ as given, $\underset{\sim}{\tilde{x}}{}_s^{i,t}$ attains the *best* ex-ante expected utility under the last information set, Φ^i_{t-1}, among the set of all feasible bundle pair for speculation at period t.

(Proof) Proof is straightforward. The bundle, $\underline{x}^i_D(\underline{x}^{i,t-1}, \tilde{\beta}^i_t(D_i=l), \underline{q}^{t-1})$, is feasible and attains the best ex-post expected utility respectively with the occurrence of $D_i = l$ (l=1,0), based on the posterior prediction, $\tilde{\beta}^i_t(D_i=l)$ (See (2.7)), through the Bayesian inference. Therefore, the overall ex-ante expected utility[4-1], which can be obtained by summing up each ex-post expected utility attained by holding $\underline{x}^i_D(\underline{x}^{i,t-1}, \tilde{\beta}^i_t(D_i=l), \underline{q}^{t-1})$ multiplied by the realization probability that $D_i = l$ occurs (i.e., $\Pr ob(D_i = l \mid \Phi^i_{t-1})$), is also maximized. (Q.E.D.)

Proposition 2: Assume that the cost of producing a private information is zero, i.e., $C_i = 0$. Also assume that the last prediction, β^i_{t-1}, is not 0 or 1, i.e., $0 < \beta^i_{t-1} < 1$. Then, the *best feasible* bundle pair for speculation at period t,

[4-1] See (A2.2) of Appendix 2 for the rigorous calculation,

$\underline{\tilde{x}}_s^{i,t} \equiv \left(\underline{\tilde{x}}_s^{i,t}(D_i=1), \underline{\tilde{x}}_s^{i,t}(D_i=0)\right) \equiv \left(\underline{x}_D^i(\underline{x}^{i,t-1}, \tilde{\beta}_t^i(D_i=1), \underline{q}^{t-1}), \underline{x}_D^i(\underline{x}^{i,t-1}, \tilde{\beta}_t^i(D_i=0), \underline{q}^{t-1})\right)$,

(4.2), is always *rational* for all $P_1^i(=1-P_0^i)$. Furthermore, the expected utility attained by holding $\underline{\tilde{x}}_s^{i,t}$, is increasing with $P_1^i(=1-P_0^i) \geq 0.5$, and equal to that attained by holding the endowment (last) bundle[4-2], $\underline{x}^{i,t-1}$, at $P_1^i(=1-P_0^i)=0.5$.

(Proof) We just sketch the steps for proof.

(1) Define the same indirect utility function as (2.9), but with any arbitrary β, not with the last prediction, β_{t-1}^i, and with the last equilibrium price, $\underline{q}^{t-1}$, not with any arbitrary price, $\underline{q}$:

$$v^i(\underline{x}^{i,t-1}, \beta, \underline{q}^{t-1}) = \max_{\underline{x}=(x_1,x_2)} u_{i1}(x_1) + \left\{\beta u_{i2}^1(x_2) + (1-\beta)u_{i2}^0(x_2)\right\}$$

$$\text{s.t } \underline{q}^{t-1}\underline{x} \leq \underline{q}^{t-1} \cdot \underline{x}^{i,t-1} \qquad (4.3)$$

Then, since the budget constraint does not include β, $v^i(\underline{x}^{i,t-1}, \beta, \underline{q}^{t-1})$ is increasing and convex with β.

(2) From (2.7), we have:

$$\beta_{t-1}^i = \Pr ob(D_i=1 \mid \Phi_{t-1}^i) * \tilde{\beta}_t^i(D_i=1) + \Pr ob(D_i=0 \mid \Phi_{t-1}^i) * \tilde{\beta}_t^i(D_i=0) \quad (4.4)$$

And, $\tilde{\beta}_t^i(D_i=1)$ is increasing with $P_1^i(=1-P_0^i)$, and is equal to β_{t-1}^i at $P_1^i(=1-P_0^i)=0.5$.

(3) The overall (ex-ante) expected utility attained by holding $\underline{\tilde{x}}_s^{i,t}$, is:

$$\Pr ob(D_i=1 \mid \Phi_{t-1}^i) * v^i(\underline{x}^{i,t-1}, \tilde{\beta}_t^i(D_i=1), \underline{q}^{t-1}) + \Pr ob(D_i=0 \mid \Phi_{t-1}^i) * v^i(\underline{x}^{i,t-1}, \tilde{\beta}_t^i(D_i=0), \underline{q}^{t-1})$$

(4.5)

[4-2] Actually, in terms of the (ex-ante) expected utility, we are indifferent between holding $\underline{x}^{i,t-1}$ without speculation, and holding it regardless of the result of a private information, D_i, which has been produced by speculation. This is clear from (4.4), (A2.1) and (A2.2).

Therefore, from (1) and (2), this is increasing with $P_1^i(=1-P_0^i)\geq 0.5$, and equal to the expected utility attained by holding $\underline{x}^{i,t-1}$ at $P_1^i(=1-P_0^i)=0.5$. (Q.E.D.)

Proposition 1 and 2 just say that, assuming the zero cost of producing a private information and the unchanged price vector ($\underline{q}^{t-1}$, in this case), the information is always worth producing and the economic value of information is increasing with the performance of information, $P_1^i(=1-P_0^i)\geq 0.5$. This is an intuitively plausible conclusion. In other words, assuming the zero cost of producing a private information in the Walrasian equilibrium process, each agent always has a "rational" incentive to produce the information, and to declare the "best" feasible bundle pair for speculation.

After the speculation has been made (i.e., after the agent has produced a private information), agent i's prediction changes from β_{t-1}^i to $\widetilde{\beta}_t^i(D_i=l)$, and the price vector, $\underline{q}$, may be expected to begin to move from the last equilibrium price, $\underline{q}^{t-1}$, because the market may not clear anymore. So we assume that the agent would demand the bundle, $\underline{x}_D^i(\underline{x}^{i,t-1},\widetilde{\beta}_t^i(D_i=l),\underline{q})$, as a *Bayesian* competitor, in the sense that this bundle exactly maximizes the agent's (ex-post) expected utility, given the newly revised Bayesian prediction of the agents, $\widetilde{\beta}_t^i(D_i=l)$, and the current price, $\underline{q}$ (not $\underline{q}^{t-1}$). We will review carefully this process in the next section.

5. The effect of speculation under the Walrasian competition

In the course of speculation, it might be rather hard for us to expect that the current price, $\underline{q}$, remains unchanged from $\underline{q}^{t-1}$, until a next new equilibrium is reached. So, it is another problem if the new equilibrium allocation at period t, $\underline{x}^t=(\underline{x}^{1,t},\underline{x}^{2,t})$, would lead to be Pareto-improved as a result of speculation, compared with the last equilibrium allocation, $\underline{x}^{t-1}=(\underline{x}^{1,t-1},\underline{x}^{2,t-1})$. Therefore, in this section, we analyze the effect of speculation on the economy under the framework of the Walrasian competition, assuming that each agent is always a rational speculator as well as being a Bayesian competitor.[5-1] Consider Case (A) at period 1 in the three periods settings as described in section 3, where only agent 1 has an information production function in asymmetry. From the definition of the Walrasian equilibrium, (2.10), we have the following conditions at period 1:[5-2]

$$(1)\quad \underline{x}^{1,1}(D_1 = l) = \underline{x}_D^1(\underline{x}^{1,0}, \beta_1^1 = \tilde{\beta}_1^1(D_1 = l), \underline{q}^1(D_1 = l))$$

$$\underline{x}^{2,1}(D_1 = l) = \underline{x}_D^2(\underline{x}^{2,0}, \beta_1^2 = \beta_0^2(=\alpha), \underline{q}^1(D_1 = l))$$

(Competitive equilibrium condition)

$$(2)\quad \underline{x}^{1,1}(D_1 = l) + \underline{x}^{2,1}(D_1 = l) = \underline{x}^{1,0} + \underline{x}^{2,0} \quad \text{(Market clearing condition)}$$

(5.1)

Since the equilibrium allocation at period 1 depends on the output of $D_1 = l$, we denote it by $\underline{x}^1(D_1 = l) = (\underline{x}^{1,1}(D_1 = l), \underline{x}^{2,1}(D_1 = l))$.

Figure 4(a) shows the equilibrium transition of Case (A) from period 0 to period 1, as a result of speculation. Point A is the equilibrium at period 0, which represents the allocation, $\underline{x}^0 = (\underline{x}^{1,0}, \underline{x}^{2,0})$. A is located on OO', which is the "true" Pareto set, $\underline{X}_p$, because $\beta_1^0 = \beta_2^0 = \alpha = 0.5$. If agent 1 produces the private information, $D_1 = 1$, then, at period 1, the equilibrium shifts to point B, which represents the new allocation with the occurrence of $D_1 = 1$, that is, $\underline{x}^1(D_1 = 1) = (\underline{x}^{1,1}(D_1 = 1), \underline{x}^{2,1}(D_1 = 1))$. On the other hand, if agent 1 produces the private information, $D_1 = 0$, then, at period 1, the equilibrium shifts to C, which represents the new allocation with the occurrence of $D_1 = 0$, that is, $\underline{x}^1(D_1 = 0) = (\underline{x}^{1,1}(D_1 = 0), \underline{x}^{2,1}(D_1 = 0))$. Clearly, B must be located in the core area, which is surrounded by agent 1's indifference curve, $\preceq_{\beta_1^1 = \tilde{\beta}_1^1(D_1 = 1)}$, based on her posterior prediction, $\tilde{\beta}_1^1(D_1 = 1)$, and by that of agent 2, $\preceq_{\beta_1^2 = \beta_0^2(=\alpha)}$, based on her prior prediction,

[5-1] As a matter of course, if she does not incorporate the information production function, or equivalently does incorporate it, whose performance, however, is $P_1^i = 1 - P_0^i = 0.5$, she cannot exercise speculation. Even if she does, she would declare the same bundle (pair) for speculation, irrespective of the output of information, D_i, as the previous bundle.

[5-2] (5.1) is the F.O.C. equilibrium conditions for Case (A), or equivalently for case (a)(i) as defined in (6.1) of section 6. The conditions for other 5 cases in (6.1) are described in Appendix 4.

$\beta_1^2 = \beta_0^2 (= \alpha)$. Similarly, C must be located in the core area, which is surrounded by agent 1's indifference curve, $\preceq_{\beta_1^1 = \tilde{\beta}_1^1 (D_1 = 0)}$, based on her posterior prediction, $\tilde{\beta}_1^1 (D_1 = 0)$, and by that of agent 2, $\preceq_{\beta_1^2 = \beta_0^2 (= \alpha)}$, based on her prior prediction, $\beta_1^2 = \beta_0^2 (= \alpha)$.

Figure 4(b) shows the equilibrium transition of Case (B) from period 0 to period 1, as a result of speculation. If $D_1 = 1$ and $D_2 = 1$, or if $D_1 = 0$ and $D_2 = 0$, then the new equilibrium allocation at period 1 will remain the same as A, which represents the allocation, $\underline{x}^0 = (\underline{x}^{1,0}, \underline{x}^{2,0})$, because $\beta_1^1 = \tilde{\beta}_1^1 (D_1 = l)$ and $\beta_1^2 = \tilde{\beta}_1^2 (D_2 = l)$, where $l = 0,1$, therefore $\beta_1^1 = \beta_1^2$, assuming $P_1^1 = P_1^2$. As shown in Figure 4(b), the equilibrium allocation at period 1 will shift from point A, only if $D_1 = 1$ and $D_2 = 0$, or if $D_1 = 0$ and $D_2 = 1$. If $D_1 = 1$ and $D_2 = 0$, then the equilibrium point shifts at period 1 to point D, which represents the new allocation, $\underline{x}^1 (D_1 = 1, D_2 = 0) = (\underline{x}^{1,1} (D_1 = 1, D_2 = 0), \underline{x}^{2,1} (D_1 = 1, D_2 = 0))$. D must be located in the core area, which is surrounded by agent 1's indifference curve, $\preceq_{\beta_1^1 = \tilde{\beta}_1^1 (D_1 = 1)}$, based on her posterior prediction, $\tilde{\beta}_1^1 (D_1 = 1)$, and by that of agent 2, $\preceq_{\beta_1^2 = \tilde{\beta}_1^2 (D_2 = 0)}$, based on her posterior prediction, $\tilde{\beta}_1^2 (D_2 = 0)$. Similarly, if $D_1 = 0$ and $D_2 = 1$, then the equilibrium point shifts at period 1 to point E, which represents the new allocation; $\underline{x}^1 (D_1 = 0, D_2 = 1) = (\underline{x}^{1,1} (D_1 = 0, D_2 = 1), \underline{x}^{2,1} (D_1 = 0, D_2 = 1))$. E must be located in the core area, which is surrounded by agent 1's indifference curve, $\preceq_{\beta_1^1 = \tilde{\beta}_1^1 (D_1 = 0)}$, based on her posterior prediction, $\tilde{\beta}_1^1 (D_1 = 0)$, and by that of agent 2, $\preceq_{\beta_1^2 = \tilde{\beta}_1^2 (D_2 = 1)}$, based on her posterior prediction, $\tilde{\beta}_1^2 (D_2 = 1)$.[5-3] [5-4]

[5-3] Note that we assume that, at period 1, the output information produced by an agent is not disclosed to the other agent

[5-4] We denote the bundle allocated to agent i (i=1,2) at the allocation points, A, B, C, E,… of the Figures, by A^i, B^i, C^i,…. Using these notations, clearly, $A^i = \underline{x}^{i,0}$, $B^i = \underline{x}^{i,1} (D_1 = 1)$,

For Case (A) at period 1, we claim the following proposition

Proposition 3: (i) Assume Case (A) at period 1. Also, assume that $C_1 = 0$, and $\Phi_0^1 = \Phi_0^2 = \Phi_0 = \{\phi\}$. Then, as a result of speculation caused by agent 1, that is, agent 1's producing a private information, D_1, agent 1 becomes better off in the ex-ante sense that:

$$E\left[U_1(\underline{x}^{1,0}) \mid \Phi_0^1\right] \le E\left[U_1(\underline{x}^{1,1}(D_1)) \mid \Phi_0^1\right](-C_1)$$

or, equivalently $\underline{x}^{1,0} \preceq_{\beta_0^1}^{D_1} \underline{x}^{1,1}(D_1)$ (5.2)

Also, agent 2 becomes worse off in the ex-ante sense that;

$$E\left[U_2(\underline{x}^{2,0}) \mid \Phi_0^2\right] \ge E\left[U_2(\underline{x}^{2,1}(D_1)) \mid \Phi_0^2\right]$$

or, equivalently $\underline{x}^{2,1}(D_1) \preceq_{\beta_0^2}^{D_1} \underline{x}^{2,0}$, (5.3)

where D_1 is a stochastic binary variable under the information set, $\Phi_0 (= \Phi_0^1 = \Phi_0^2)$, which has a property, (2.6).

(ii) In addition to the assumption at (i), also assume that each agent has an identical form of utility functions given by (2.5). Then, the R.H.S. of (5.2) is increasing with regard to $P_1^1 (= 1 - P_0^1)$, and the R.H.S. of (5.2) is decreasing with regard to $P_1^1 (= 1 - P_0^1)$. In other words, as a result of speculation caused by agent 1, the higher $P_1^1 (= 1 - P_0^1)$ will lead to the higher expected utility for agent 1 and the lower expected utility for agent 2.

(Proof) (i) See Figure 4 (a). Point A denotes the equilibrium allocation at period 0, that is, $\underline{x}^o = (\underline{x}^{1,0}, \underline{x}^{2,0})$. Point B denotes the equilibrium allocation at period 1 with the occurrence of $D_1 = 1$, that is;

$$B : \underline{x}^1(D_1 = 1) = (\underline{x}^{1,1}(D_1 = 1), \underline{x}^{2,1}(D_1 = 1)).$$

B must be located in the core area which is surrounded by agent 1's (posterior) indifference

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$C^i = \underline{x}^{i,1}(D_1 = 0)$, $E^i = \underline{x}^{i,1}(D_1 = 0, D_2 = 1)$, and so on. Also, in order to indicate the allocation at each point, for simplicity, we sometimes use the expression like, $A = \underline{x}^0$, $B = \underline{x}^1(D_1 = 1)$, $C = \underline{x}^1(D_1 = 0)$, $E = \underline{x}^1(D_1 = 0, D_2 = 1)$, and so on, although A, B, C, E,… are, precisely speaking, geographical points and not the allocations themselves.

curve, $\preceq_{\beta_1^1=\tilde{\beta}_1^1(D_1=1)}$, and agent 2's last (prior) indifference curve, $\preceq_{\beta_0^2}$. It means that:

$$E\left[U_1(\underline{x}^{1,0}) \mid \Phi_0^1, \{D_1 = 1\}\right] \leq E\left[U_1(\underline{x}^{1,1}(D_1 = 1)) \mid \Phi_0^1\{D_1 = 1\}\right]$$, or equivalently,

$$\underline{x}^{1,0} \preceq_{\beta_1^1=\tilde{\beta}_1^1(D_1=1)} \underline{x}^{1,1}(D_1 = 1) \qquad \text{(A)}$$

On the other hand, although agent 2's prediction at period 1 remains the same as that at period 0, that is, $\beta_1^2 = \beta_0^2$, her "objective" indifference curve shifts to $\preceq_{\tilde{\beta}_1^1(D_1=1)}$ from $\preceq_{\beta_0^2}$ with the occurrence of $D_1 = 1$ at period 1. Since at period 1, agent 2's "objective" indifference curve, $\preceq_{\tilde{\beta}_1^1(D_1=1)}$, is tangent with agent 1's indifference curve, $\preceq_{\beta_1^1=\tilde{\beta}_1^1(D_1=1)}$, B must be located inside A, viewing from O', the original point of agent 2, with respect to $\preceq_{\tilde{\beta}_1^1(D_1=1)}$. Therefore, we have:

$$E\left[U_2(\underline{x}^{2,0}) \mid \Phi_0^2, \{D_1 = 1\}\right] \geq E\left[U_2(\underline{x}^{2,1}(D_1 = 1)) \mid \Phi_0^2\{D_1 = 1\}\right]$$, or equivalently,

$$\underline{x}^{2,1}(D_1 = 1) \preceq_{\tilde{\beta}_1^1(D_1=1)} \underline{x}^{2,0} \qquad \text{(B)}$$

Quite similarly, let point C denotes the equilibrium allocation at period 1 with the occurrence of $D_1 = 0$. Then $C : \underline{x}^1(D_1 = 0) = (\underline{x}^{1,1}(D_1 = 0), \underline{x}^{2,1}(D_1 = 0))$. From the analogy of the above argument, we have:

$$E\left[U_1(\underline{x}^{1,0}) \mid \Phi_0^1, \{D_1 = 0\}\right] \leq E\left[U_1(\underline{x}^{1,1}(D_1 = 0)) \mid \Phi_0^1\{D_1 = 0\}\right]$$, or equivalently,

$$\underline{x}^{1,0} \preceq_{\beta_1^1=\tilde{\beta}_1^1(D_1=0)} \underline{x}^{1,1}(D_1 = 0) \qquad \text{(C)}$$

$$E\left[U_2(\underline{x}^{2,0}) \mid \Phi_0^2, \{D_1 = 0\}\right] \geq E\left[U_2(\underline{x}^{2,1}(D_1 = 0)) \mid \Phi_0^2\{D_1 = 0\}\right]$$, or equivalently,

$$\underline{x}^{2,1}(D_1 = 0) \preceq_{\tilde{\beta}_1^1(D_1=0)} \underline{x}^{2,0} \qquad \text{(D)}$$

From the calculation of Appendix 2 with the results (A), (B), (C) and (D), we get (5.2) and (5.3).

(ii) Abbreviated. (Q.E.D.)

With the last equilibrium price, $\underline{q}^0$, the bundle pair for speculation;

$\underset{\sim}{\tilde{x}}_s^{1,1} \equiv \left(\underset{\sim}{\tilde{x}}_s^{1,1}(D_i = 1), \underset{\sim}{\tilde{x}}_s^{1,1}(D_i = 0)\right) \equiv \left(\underline{x}_D^1(\underline{x}^{1,0}, \tilde{\beta}_t^1(D_1 = 1), \underline{q}^0), \underline{x}_D^1(\underline{x}^{1,0}, \tilde{\beta}_t^1(D_1 = 0), \underline{q}^0)\right)$, is the best, among being feasible and rational for agent 1. See Proposition 1 and 2. Therefore, agent 1 would take a plunge into speculation, and produce a private information. But, afterwards, because of the break of the market clearing condition caused by the change of agent 1's prediction (to the posterior prediction, $\tilde{\beta}_1^1(D_1 = l)$), the current price vector, $\underline{q}$, begins to move and reach the new equilibrium price, $\underline{q}^1$. Proposition 3 says that, even under the new price, $\underline{q}^1$, agent 1, an information superior, and at the same time a positive speculator, would be better off, while agent 2, an information inferior, and a passive speculation taker, would be worse off. Now, we set some parameters at $a = 0.1$, $\gamma_1 = 1.0$, $\gamma_2^1 = 1.5$, $\gamma_2^0 = 0.5$. Define the ex-ante (expected) welfare ratios of agent 1, 2 and the society, R_1, R_2, and R_S, as the ratios of the ex-ante overall expected utility, of the "after-speculation" equilibria allocations to that of the "before-speculation". These ratios are, of course, measured at the initial period 0. See Appendix 5 for the strict definition. Also, set $s = 0$, so that the initial endowment bundle of each agent may be the same. Then, Figure 5(a) shows the ex-ante welfare ratios, R_1, R_2, and R_S, as a function of $P_1^1 (= 1 - P_0^1)$, which well illustrates the result of Proposition 3.

The interesting case arises, when agent 1 behaves herself according to a "rational", but different rule from as a Bayesian competitor. Let us think about the following case. When, at the beginning of period 1, agent 1 produces the private information, $D_1 = l$ (l=0,1), then the agent would hold in mind that the state of the world, m, is exactly l. In other words, her new prediction would be, $\beta_1^1 = D_1 = l$, instead of $\beta_1^1 = \tilde{\beta}_1^1(D_1 = l)$, and her bundle pair for speculation would be:

$$\underline{\hat{x}}_s^{1,1} \equiv \left(\underline{\hat{x}}_s^{1,1}(D_1 = 1), \underline{\hat{x}}_s^{1,1}(D_1 = 0)\right) \equiv \left(\underline{x}_D^1(\underline{x}^{1,0}, \beta_1^1 = 1, \underline{q}^0), \underline{x}_D^1(\underline{x}^{1,0}, \beta_1^1 = 0, \underline{q}^0)\right) \quad (5.4)$$

Furthermore, we assume that, after the price begins to move from $\underline{q}^0$, her demand would be, given any arbitrary price, $\underline{q}$, $\underline{x}_D^1(\underline{x}^{1,0}, l, \underline{q})$ if $D_1 = l$, while agent 2 would demand $\underline{x}_D^2(\underline{x}^{2,0}, \alpha, \underline{q})$, using her previous prediction, $\beta_0^2 = \alpha$. This rule of agent 1 is, in a sense, based on the misperception or the possession that " the real value of the state of the world, M, must be l", while, from the viewpoint of the Bayesian inference, her prediction should be exactly $\widetilde{\beta}_1^1(D_1 = l)$. However, the bundle pair for speculation, $\underline{\hat{x}}_s^{1,1}$, (5.4), might be still feasible and rational, in the sense of Definition 1. See Figure 5 (b) for this case, with the contrast to Figure 5(a). In the region where $P_1^1(=1-P_0^1)$ is smaller than 0.64, $\underline{\hat{x}}_s^{1,1}$ is not rational (feasible, though) for agent 1, so she does not exercise speculation (or does not produce a private information). Therefore, the equilibrium allocation at period 1, $\underline{x}^1$, remains the same as the last allocation, $\underline{x}^0$, and therefore all the ex-ante welfare ratios, R_1, R_2, and R_S are 1's. On the other hand, in the region where P_1^1 is greater than 0.64, $\underline{\hat{x}}_s^{1,1}$ becomes rational, then agent 1 exercises speculation. However, it is noteworthy that, at P_1^1=0.64-0.80, agent 1 herself becomes worse off as well as agent 2, as a result of speculation. Agent 1 becomes better off only in the region where P_1^1 is greater than 0.80. This is a totally different point from Figure 5(a)[5-5], where agent 1 behaves herself as a Bayesian competitor, based on the correctly revised posterior prediction, $\widetilde{\beta}_1^1(D_1 = l)$. This may be considered to be a "rational" overreaction, which should have increased her ex-ante expected utility, given the last equilibrium price, $\underline{q}^0$, but eventually decreased it as a result of the price adjustment through the new equilibrium process. Figure 4(c) illustrates this equilibrium transition from period 0 to period 1, as a result of speculation.[5-6] If agent 1 produces the private information, $D_1 = 1$, then, at period 1, the equilibrium shifts to point F, which represents the new allocation with the occurrence of $D_1 = 1$, that is, $\underline{\hat{x}}^1(D_1 = 1) = (\underline{\hat{x}}^{1,1}(D_1 = 1), \underline{\hat{x}}^{2,1}(D_1 = 1))$. On the other hand, if agent 1 produces the private information, $D_1 = 0$, then, at period 1, the equilibrium shifts to G, which represents the new allocation with the occurrence of $D_1 = 0$, that is, $\underline{\hat{x}}^1(D_1 = 0) = (\underline{\hat{x}}^{1,1}(D_1 = 0), \underline{\hat{x}}^{2,1}(D_1 = 0))$. Clearly, F must be located in the core area,

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[5-5] A similar point is that agent 1's welfare ratio, R_1, is increasing and agent 2's welfare ratio, R_2, is decreasing with P_1^1. But both hold only at $P_1^1 > 0.64$.

[5-6] For the equilibrium F.O.C. conditions of this "rational overreaction" setting, see (A4.1) in Appendix 4.

which is surrounded by agent 1's indifference curve, $\preceq_{\beta_1^1=1}$, based on her "possessed" prediction, $\beta_1^1 = 1$, and by that of agent 2, $\preceq_{\beta_1^2=\beta_0^2(=\alpha)}$, based on her prior prediction, $\beta_0^2(=\alpha)$. Similarly, G must be located in the core area, which is surrounded by agent 1's indifference curve, $\preceq_{\beta_1^1=0}$, based on her "possessed" prediction, $\beta_1^1 = 0$, and that of agent 2, $\preceq_{\beta_1^2=\beta_0^2(=\alpha)}$, based on her prior prediction, $\beta_0^2(=\alpha)$. Note, however, that, as in the figure (actually, for $(0.5 \leq) P_1^1 \leq 0.64$), F might be located outside the core area, which is surrounded by agent 1's "objective" indifference curve, $\preceq_{\tilde{\beta}_1^1(D_1=1)}$, based on her posterior Bayesian prediction, $\tilde{\beta}_1^1(D_1 = 1)$, and by that of agent 2, $\preceq_{\beta_1^2=\beta_0^2(=\alpha)}$, based on her prior prediction, $\beta_0^2(=\alpha)$. Similarly, G might be located outside the core area, which is surrounded by agent 1's "objective" indifference curve based on her posterior Bayesian prediction, $\tilde{\beta}_1^1(D_1 = 0)$, and that of agent 2 based on her prior prediction, $\beta_0^2(=\alpha)$. In this case[5-7], we have for agent 1:

$$F^1 \preceq_{\tilde{\beta}_1^1(D_1=1)} A^1 \quad \text{and} \quad G^1 \preceq_{\tilde{\beta}_1^1(D_1=0)} A^1 \tag{5.5}$$

Now consider agent 2. With the occurrence of $D_1 = 1$, the "objective" indifference curve of agent 2 at period 1 would be $\preceq_{\beta_1^2=\tilde{\beta}_1^1(D_1=1)}$, which is tangent with agent 1's "objective" indifference curve, $\preceq_{\beta_1^1=\tilde{\beta}_1^1(D_1=1)}$. Similarly, with the occurrence of $D_1 = 0$, the "objective" indifference curve of agent 2 at period 1 would be $\preceq_{\beta_1^2=\tilde{\beta}_1^1(D_1=0)}$, which is tangent with agent 1's "objective" indifference curve, $\preceq_{\beta_1^1=\tilde{\beta}_1^1(D_1=0)}$. So, assuming that, as in Figure 4(c), F and G are located inside A, viewing from O', respectively with regard to $\preceq_{\beta_1^2=\tilde{\beta}_1^1(D_1=1)}$

[5-7] Of course, $F^i = \hat{\underline{x}}^{i,1}(D_1 = 1)$, $G^i = \hat{\underline{x}}^{i,1}(D_1 = 0)$, $A^i = \underline{x}^{i,0}$ for i=1,2.

and $\preceq_{\beta_1^1=\tilde{\beta}_1^1(D_1=0)}$. Then we have for agent 2:

$$F^2 \preceq_{\beta_1^2=\tilde{\beta}_1^1(D_1=1)} A^2 \text{ and } G^2 \preceq_{\beta_1^2=\tilde{\beta}_1^1(D_1=0)} A^2 \tag{5.6}$$

Therefore, from (5.5) and (5.6), defining the bundle pairs for speculation;

$$\underline{\hat{x}}^{i,1}(D_1) = (\underline{\hat{x}}^{i,1}(D_1 = 1), \underline{\hat{x}}^{i,1}(D_1 = 0)) = (F^i, G^i) \text{ and}$$

$$\underline{\hat{x}}^{i,0}(D_1) = (\underline{\hat{x}}^{i,0}, \underline{\hat{x}}^{i,0}) = (A^i, A^i), \tag{5.7}$$

then we get:

$$\underline{\hat{x}}^{1,1}(D_1) \preceq_{\beta_0^1}^{D_1} \underline{\hat{x}}^{1,0} \text{ and } \underline{\hat{x}}^{2,1}(D_1) \preceq_{\beta_0^2}^{D_1} \underline{\hat{x}}^{2,1} \tag{5.8}$$

Therefore, we can conclude that both agents might be worse off in terms of the ex-ante expected utility on account of the "rational" speculation caused by agent 1 for a comparatively lower $P_1^1 (= 1 - P_0^1)$.

6. The welfare analysis and some problem in the decision making theory

Now we proceed to the welfare analysis, as in section 5, exactly on the basis of the three period settings described in section 3. In Figure 6[6-1], we show the ex-ante welfare ratios, R_1, R_2, and R_S, as functions of s, which represents the ratio of the economic value of agent 1's initial endowment to agent 2's, by $(1-s)/(1+s)$[6-2], where $-1 \le s \le 1$. The cases we consider are following.

(a)(i): Case (A) at period 1. (The first round effect)
(a)(ii): Case (A) at period 1, and Case (I) at period 2. (The first plus second round effect)
(a)(iii): Case (A) at period 1, and Case (II) at period 2. (The first plus second round effect)
(b)(i): Case (B) at period 1. (The first round effect)
(b)(ii): Case (B) at period 1, and Case (I) at period 2. (The first plus second round effect)
(b)(iii): Case (B) at period 1, and Case (II) at period 2. (The first plus second round effect)
(6.1)

Three cases of Figure 6(a) represent (a)(i), (a)(ii) and (a)(iii) adopting Case (A) at period 1 respectively, and therefore the economic system is in the informational asymmetry. Main

[6-1] As in Figure 5, we set parameters at $a = 0.1$, $\gamma_1 = 1.0$, $\gamma_2^1 = 1.5$, $\gamma_2^0 = 0.5$. We also set the weight of each agent equally at $\delta_1 = \delta_2 = 0.5$, for the ex-ante social welfare ratio, R_S.
[6-2] As s increases, the ratio of agent 1's initial economic value to agent 2's, $(1-s)/(1+s)$, decreases.

features are following. [6-3]

F1. Through (i), (ii) and (iii), R_1 is increasing with respect to s, that is, decreasing with $(1-s)/(1+s)$, the ratio of the economic value of agent 1's initial endowment to agent 2's, and R_1 is always greater than 1. R_2 is not necessarily monotone with s, but always less than 1.

F2. In (i), R_S is always less than 1.

F3. For an arbitrarily given s, each of R_1, R_2, and R_S has the greatest value in (ii), the second in (iii), and the lowest in (i).

Three cases of Figure 6(b) represent (b)(i), (b)(ii) and (b)(iii) adopting Case (B) at period 1 respectively, and therefore the economic system is in the informational symmetry. Main features are following.[6-4]

F4. In (i), R_S is always less than 1.

F5. For an arbitrarily given s, each of R_1, R_2, and R_S has the greatest value in (ii), the second in (iii), and the lowest in (i).

F6. R_1 is greater than R_2 for $0 < s$, and smaller than R_2 for $s < 0$.

F7. Through (i), (ii) and (iii), there does not exist any s such that both R_1 and R_2 are greater than 1.

F8. In (ii) and (iii), R_S can be greater than or smaller than 1 for some s.

With the above results, we claim the following important propositions, keeping the same assumptions as footnote 6-3 for (a)(i), (a)(ii) and (a)(iii), and as footnote 6-4 for (b)(i), (b)(ii) and (b)(iii), respectively. For the proofs and some explanation, see Appendix 6.

Proposition 4: In each of all six cases of the transition processes defined in (6.1), for any s, the overall equilibria allocations, which might be finally attained with a positive probability at the last period (period 1 for (a)(i) and (b)(i), and period 2 for (a)(ii), (a)(iii), (b)(ii) and (b)(iii)), cannot be ex-ante Pareto improved, compared with the initial equilibrium at period 0.

This proposition arises from F1 and F7. It is noteworthy that, even taking account of the adjustment process of information at period 2 (that is, even in (a)(ii), (a)(iii), (b)(ii) or (b)(iii)), or even in the symmetric cases adopting Case (B) at period 1 (that is, even in (b)(i), (b)(ii) or (b)(iii)), the overall equilibria allocations cannot be the ex-ante Pareto improved, that is, at

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[6-3] We are assuming that $0.5 < P_1^1$.

[6-4] We are assuming that $0.5 < P_1^1 = P_1^2 (\equiv P_1) < 1$. That is, both agents have the same performance in producing the information, and they are also fallible.

least one agent will be worse off in terms of the ex-ante expected utility on account of speculation.

Proposition 5: In each of 4 cases of (a)(i), (a)(iii), (b)(i) and (b)(iii), the transition processes defined in (6.1), for any s, the overall equilibria allocations, which might be finally attained with a positive probability at the last period, cannot be ex-ante Pareto efficient, or can never be the ex-ante social welfare maximized.

In Proposition 5, it remains still uncertain that, in (a)(ii) or (b)(ii), the overall equilibria allocations cannot be ex-ante Pareto efficient or ex-ante social welfare maximized. However, the results of our computation as in Figure 6 show the following fact.

Fact 1: For any setting of parameters, a, γ_1, γ_2^1, γ_2^0, and s, the ex-ante expected utility of agent i (i=1,2) attained at period 2, or equivalently the ex-ante welfare ratio, R_i (i=1,2) at the last period, is smaller (worse off) in either (a)(iii) or (b)(iii) than in (a)(ii) or (b)(ii), respectively.[6-5]

As a special example of the symmetric cases, we have an interesting proposition as follows.

Proposition 6: In either of (b)(i), (b)(ii) or (b)(iii) (that is, in the symmetric cases adopting Case (B) at period 1), set $s = 0$, where both agents have equally the same initial endowment. Then, both agents become worse off in terms of the ex-ante expected utility on account of speculation, in the sense that both R_1 and R_2 are less than 1.

This proposition is very interesting, because it shows that the economic value of information can be negative in any possible definition. So far as the ex-ante social welfare improvement is concerned, we claim the following proposition, although we do not arrive at complete proofs for them.

Proposition 7: Assume the case[6-6] that $\delta_1 = \delta_2 = 0.5$. Then, in either (a)(i) or (b)(i), the ex-ante social welfare ratio, R_S, is always less than 1 for any s. In either of (a)(ii), (a)(iii),

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[6-5] The transition of the "objective" prediction in Case (II) from period 1 to 2 is shown as the following formula:

$$\beta_1 = \Pr ob(M = 1 \mid \Phi_1) * 1 + \Pr ob(M = 0 \mid \Phi_1) * 0$$

This implies that the expectation of the true state, which is to be revealed at period 2, is just the current (period 1) objective prediction. This is just like a mean preserving spread transition in terms of prediction. So, under the concave utility, it is easily imagined that the revelation of the true state at period 2, compared with Case (I), may have a negative effect on each agent's ex-ante expected utility, but actually, it cannot be rigorously proved as a general proposition, because each agent's utility is state dependent, and in addition, the equilibria allocations at Case (II) may not be geographically "mean preserving", compared with Case (I). Fact 1 shows, nevertheless, that this is true as long as our computation results are concerned.

[6-6] δ_i is the weight of each agent i in the ex-ante social welfare. See (V) of the welfare criteria in section 3.

(b)(ii) or (b)(iii) (that is, for the cases taking account of the adjustment process of information at period 2), R_S, can be both greater than 1 or less than 1 by choosing some s.[6-7]

Also assume the case that $\delta_i = \lambda_i^{-1}/(\lambda_1^{-1} + \lambda_2^{-1})$, where λ_i is agent i's marginal utility of initial income. Then, in either of (a)(i), (a)(ii), (a)(iii), (b)(i), (b)(ii) or (b)(iii), the ex-ante social welfare ratio, R_S, is always less than 1 for any s.[6-8]

Proposition 8: In either of (b)(i), (b)(ii) or (b)(iii) (that is, in the symmetric cases adopting Case (B) at period 1), the ex-ante welfare ratio, R_i, of agent i who has the less initial endowment is greater than R_{-i} of the other agent –i.

This proposition just restates F6. It implies that, in the symmetric cases, the "poorer" agent will suffer comparatively the less damage in terms of the ex-ante expected utility, than the "richer" agent on account of speculation.

Next, from F3 and F5, we have:

Proposition 9: Among (a)(i), (a)(ii) and (a)(iii) (that is, in the asymmetric cases adopting Case (A) at period 1), for any s, each of R_1, R_2, and R_S has the greatest value in (a)(ii), the second in (a)(iii), and the lowest in (a)(i). Quite similarly, among (b)(i), (b)(ii) and (b)(iii) (that is, in the symmetric cases adopting Case (B) at period 1), for any s, each of R_1, R_2, and R_S has the greatest value in (b)(ii), the second in (b)(iii), and the lowest in (b)(i).

This proposition includes Fact 1. The proof of the part of the proposition, that R_1, R_2, and R_S has the greater value in (ii) than in (i), and the greater value in (iii) than in (i), is clear. See Appendix 6.

One comment on the asymmetric cases (that is, (a)(i), (a)(ii) and (a)(iii)). It is noteworthy that R_2, the ex-ante welfare ratio of the agent, who does not incorporate an information production function, therefore cannot exercise speculation by herself, is not necessarily monotonic with respect to s. For example, in Figure 6(a)(i), R_2 has the lowest value about at $s = -0.3$. This fact does not imply the less s (that is, the less agent 2's initial endowment), the greater damage in welfare for the agent. Specifically, we can regard that the point $s = -0.3$, where agent 2's initial endowment is, in proportion, $1 + s = 0.7$, while that of agent 1 is $1 - s = 1.3$, is the "resonance" point, at which agent 2 suffers the greatest damage in welfare (in terms of the ex-ante expected utility), on account of the speculation caused by agent 1.

[6-7] Our computation results show this part (the first paragraph) of Proposition 7.

[6-8] This part (the second paragraph) of Proposition 7 can be proved immediately from Lemma 1 as described in Appendix 6.

The results we explained so far are regarding the welfare criteria, (U) and (V)[6-9], as defined in section 3. As well as all other criteria ((W), (X), (Y) and (Z)), we summarize all of these results in Table. We will not offer all the proofs of these results, because they are almost clear from the previous arguments. So, we just stay at indicating some interesting points. At first, as to the ex-post Pareto optimality (criterion (W)), it is always achieved at period 2, after the adjustment process of information (i.e., in (a)(ii), (a)(iii), (b)(ii) or (b)(iii)), simply because both agents come to share the same prediction, $\beta_2^1 = \beta_2^2$. As for the true-state basis, (X), neither the Pareto improvement nor the Pareto optimum can be achieved, assuming that the multiple equilibria paths and allocations caused by speculation probabilistically exist. The impossibility of the Pareto improvement is also easily derived, using Lemma 1. That of the Pareto optimum can be proved immediately from considering the unique true state-based utility functions and their concavity, respectively for both agents. The results of the first best Pareto optimality, (Y), can be obtained from the same logic as the ex-post Pareto optimality, (W). The concept of the second best Pareto optimality is useful only at period 1, because at period 2, after the adjustment process of information, the first best Pareto optimum is automatically achieved. See Figure 7 for the case (a)(i) at period 1. Assume that agent 1 produces $D_1 = 1$, and the equilibrium allocation shifts to B at period 1. The shaded area, which is the intersect of the core area for $M = 1$ and for $M = 0$, is exactly the set of the second best Pareto improving allocations, in the sense that every element in this shaded area is Pareto improving regardless of the value of M. On the contrary, if the intersect of these two core areas is empty (that is, these two core areas do not intersect.), then we can conclude that there exists no second best Pareto improving allocation.

Through this paper, we have been assuming the utility functions with very general assumptions as in from (2.1) to (2.4), where each agent has equally the same risk preference for the holding of each commodity with $CRRA = a$. Now we would like to focus on the role of the riskless commodity (commodity 1), which is normally considered to be “money”. Let us consider the following quasi-linear utility function:

$$U_i(\underline{x}^{i,t}) = \gamma_1 x_1^{i,t} + u_{i2}^j(x_2^{i,t}) \quad \text{if M=j (j=1,0)} \qquad (6.2)$$

Here we assume that the slope of the linear part of utility for commodity 1, γ_1 say, is

[6-9] As for the relations between (U) and (V), we just write down the useful relationships, which can be easily derived from the conventional welfare economics theories.

1. Ex-ante social welfare maximum $\Rightarrow$ Ex-ante Pareto optimum
2. NOT Ex-ante Pareto optimum $\Rightarrow$ NOT Ex-ante social welfare maximum
3. Ex-ante Pareto improved $\Rightarrow$ Ex-ante social welfare improved
4. NOT Ex-ante social welfare improved $\Rightarrow$ NOT Ex-ante Pareto improved

constant over agents.

With similar general assumptions:

$$\lim_{x_2^{i,t}\to+0} u_{i2}^{j}{}'(x_2^{i,t}) = +\infty,\ u_{i2}^{j}(0)=0,\ \lim_{x_2^{i,t}\to+\infty} u_{i2}^{j}{}'(x_2^{i,t}) = +0 \qquad (6.3)$$

It is convenient to assume the same form as in (2.5) in the part of $u_{i2}^{j}(x_2^{i,t})$:

$$u_{i2}^{j}(x_2^{i,t}) = \gamma_2^{j}(x_2^{i,t})^{1-a} \text{ if M=j, where } \gamma_2^1 > \gamma_2^0 > 0. \qquad (6.4)$$

The common form of these functions implies that each agent is risk neutral for the part of commodity 1, and risk averse for the part of commodity 2 with $CRRA = a$. We assume that all other settings are exactly the same as so far. See Figure 8(a). Note that every indifference curve of each agent, $U_i(\underline{x}^{i,t}) = x_1^{i,t} + u_{i2}^{j}(x_2^{i,t}) = \overline{U}_i$, say, shifts in parallel along the axis of commodity 1, that is, OO'' or $O'O'''$, whatever the value of each agent's prediction, β_t^i, is. Therefore, in any cases, "subjective", "objective" or "true (real)", the contract curve is always a straight line, which is parallel to OO'' or $O'O'''$. Specifically, at the initial period 0, assuming $\beta_0^1 = \beta_0^2 = \alpha$, the subjective contract curve is shown as a line, $R'R''$, where R' and R'' are the middle points of OO''' and $O'O''$, respectively. Also, the "objective" or "true" contract curve always coincides with $R'R''$, because we should apply $\beta_t^1 = \beta_t^2 = \beta_t$ for "objective", and $\beta_t^1 = \beta_t^2 = M(=j)$ for "true". Next, see Figure 8(b) for Case (A). At period 1, the equilibrium allocation will shift from point A to point B with the occurrence of $D_1 = 1$, and from point A to point C with the occurrence of $D_1 = 0$. So, B is located on the subjective contract curve, $R_1{}'R_1{}''$, say, corresponding to $\beta_1^1 = \widetilde{\beta}_1^1(D_1 = 1)$ and $\beta_0^2 = \alpha$, and, C is located on the subjective contract curve, $R_0{}'R_0{}''$, say, corresponding to $\beta_1^1 = \widetilde{\beta}_1^1(D_1 = 0)$ and $\beta_0^2 = \alpha$. At period 2, however, after the adjustment process of information, the predictions of both agents again coincide (i.e., $\beta_2^1 = \beta_2^2$). So the subjective contract curve comes back to the initial one, $R'R''$, and the every possible equilibrium allocation at period 2 ($\widetilde{B}$, $\widetilde{C}$ or etc.) must be located on it. With such quasi-linear utility, agent i's marginal expected utility of initial income is always $\lambda_i = \gamma_1$ for i=1,2, so that two settings of δ_i's (the weight of each agent on the ex-ante social welfare) in footnote 3-2 coincide with each other at $\delta_1 = \delta_2 = 0.5$. Therefore, as easily seen, the ex-ante social welfare is kept constant at every allocation points on $R'R''$, because each agent's holding of (risky) commodity 2 is constant on $R'R''$, that is, $x_2^{i,t} = 1$ for i=1,2, and the utility of the part of (riskless) commodity 1 is linear and has the same slope for each agent. This implies that, considering the equilibrium process until at period 2 (i.e., the fist plus second round effect), the existence of the risk neutral commodity (1) fully insures the ex-ante social welfare, not each individual ex-ante utility, against speculation, in the sense that the ex-ante social welfare is kept invariant by means of any kind of speculation (i.e., in either case of (a)(ii), (a)(iii), (b)(ii) or (b)(iii)), and as a matter of course, the ex-ante Pareto improvement cannot be still attained. Summarizing these results together, we claim the following proposition.

Proposition 10: Assume the common form of the quasi-linear utility function for each agent, as defined in (6.2) and (6.4). Then, in either case of (a)(ii), (a)(iii), (b)(ii) or (b)(iii), the ex-ante social welfare is kept invariant, but the ex-ante Pareto improvement cannot be attained.

It is interesting to observe that the risk neutrality (linearity) of a riskless commodity completely cancels off, at period 2, the disturbance effect of speculation (a private information), and fully insures the ex-ante social welfare, keeping it invariant, on the other hand, the risk averseness (concavity) of a riskless commodity, with linear relationship between states and commodities, cannot completely cancel off the disturbance effect of speculation, and furthermore always decrease the ex-ante social welfare[6-10].

As for the first round effect at period 1 (i.e., in case (a)(i) or (b)(i)), we would like to comment one point. That is, it is not always assured that the equilibrium holding of (riskless) commodity 1 at period 1 is positive. It implies, for example in Case (A) setting, that when agent 1 has a far larger initial endowment than agent 2, that is, s is close to -1, and furthermore when agent 1 produces $D_1 = 0$, then agent 2's holding of commodity 1 might come easily close to 0 or even be negative, and therefore the cash-in-advance constraint of agent 2 might be easily binding or even violated.

7. Final remarks

The model of this paper is based on a simple exchange economy with no production function, in which two agents exchange two commodities, one risky, and one riskless, they have a common form of the separable, state dependent and concave utility function, and share an identical initial prediction. Some general assumptions like from (2.1) to (2.4) always assure the existence of the Walrasian equilibrium, even when speculation (i.e., the production of information) is in process at period 1. Furthermore, we consider some

[6-10] We are considering the ex-ante social welfare in the second definition of footnote 3-2.

adjustment process at period 2, in which, after speculation, the produced information or the true state of the world spills over into public. In these contexts, the model considered in this paper is very robust against the disturbance (distorted) effect of speculation. Nevertheless, it is noteworthy that, assuming the linear relations between states and between commodities, (2.4), speculation can never attain the ex-ante Pareto improvement, whatever the economic system is in informational asymmetry or symmetry (i.e., in each of 6 cases defined in (6.1)). Furthermore, in some cases (i.e., in either of (a)(i), (a)(iii), (b)(i) or (b)(iii)), the ex-ante Pareto optimum cannot be achieved.

Our definition of "information" as in (2.6), which is well known in the field of engineering, and which, precisely speaking, we should call an "information production function"[7-1], might be rather controversial. For example, see (A2.5):

$$\beta_{t-1}^{i} = \Pr ob(D_i = 1 \mid \Phi_{t-1}^{i}) * \tilde{\beta}_t^{i}(D_i = 1) + \Pr ob(D_i = 0 \mid \Phi_{t-1}^{i}) * \tilde{\beta}_t^{i}(D_i = 0) \quad \text{(A2.5)}$$

(A2.5) just says that the expectation of the future Bayesian predictions is just the current prediction. In this sense, the "information" merely makes the mean preserving spread in terms of prediction. Furthermore, assuming $\alpha = 0.5$, and $P_1^i = 1 - P_0^i$, we have $\Pr ob(D_i = 1 \mid \Phi_0^i) = 0.5$, which means that the information production function (not the information, D_i) itself is useless for predicting in advance whether $D_i = 1$ or $D_i = 0$. Then, a fundamental question arises. Even if D_i, which is not still produced therefore unknown to public, is surely some "information" regarding the profitability of an asset or the productivity of production technology or etc, is D_i really helpful for our economic activity, for example, for innovation or invention in a production process? Can we really call it "information"?

In this regard, Sah and Stiglitz (1986, 1985) make the analogous formulation regarding the judgment (decision-making) of a project as an entire organization, but their implication is more profound than and nontrivially different from ours in the context of the organizational economics. According to them, the accumulation of each agent's independent decision-making heightens the probability that the organization eventually makes a "right" decision[7-2], which surely leads, for example, to the success of a research project or to the technology innovation, or equivalently to the productivity improvement, or to the cost reduction or etc. In these contexts, they implicitly assume "the positive endogenous

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[7-1] It is because (2.6) just describes the performance regarding the correctness or the fallibility of information, D_i, but the information itself is not produced, therefore is not disclosed to any agents.

[7-2] A "right" decision corresponds to $D = 1$ for $M = 1$ or $D = 0$ for $M = 0$, where D denotes the decision of an organization.

economic effect" subsequently brought by a "right" judgment, which consequently enhances the social welfare as well as the economic profit of the organization. This is a decisively important and plausible implication of the organizational economics.

The setting of our model, however, completely ignores such a kind of positive endogenous effects, at least in this paper, and focuses genuinely on one aspect (role) of "information" as purely "predicting" the true state of the world, where the output results of information produced by agents do not have any subsequent and substantial economic effect on the adjustment of production process, on the market innovation or on the restructure of the underlying economic system.[7-3]

In spite of these criticisms, our definition might well capture one aspect of speculation, in the sense that a speculator arouses a new, different, and probabilistic economic action using a private information, at least believing that it would increase her ex-ante individual expected utility even within a multi-agents-interacted economic system. In effect, we succeed in proving, in a simplest pure exchange economy (Edgeworth box), that such a belief can be economically rationalized without assuming any positive endogenous effect, however, that, in terms of the ex-ante welfare, the Pareto improvement cannot be achieved. Furthermore, at least at period 1, each speculator behaves herself on the basis of a private information, which is produced but not still disclosed into public, therefore, in this paper we may be equivalently examining the effect of the negative externality caused by a privately produced information mainly in terms of the ex-ante expected welfare.

The summary of this paper is repeated as follows. In this paper, we construct the model on Walrasian (competitive) equilibrium transition processes over three periods, and on Shannon's famous definition of "information production function". Also we assume that there exist two types of assets (commodities), risky and riskless, each of two agents has an identical, separable and state dependent utility with the same constant relative risk aversion, and all of these assumptions easily enable us to make welfare analysis by Edgeworth box. Some quite general conditions including Inada condition always assure the existence of the equilibrium even when speculation is in process at period 1, and furthermore, we consider some adjustment process at period 2, in which, after speculation,

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[7-3] If we add, in our setting, the endogenous effect of information regarding the profitability of a risky commodity (2), then $u_{i2}^{j}(x_2^{i,t})$ as defined in (2.5) may be roughly rewritten as following.

$$u_{i2}^{j}(x_2^{i,t}) = (\gamma_2^{j} + e^{j,l})(x_2^{i,t})^{1-a} \quad \text{if } M = j \text{ and } D_i = l \qquad (7.1)$$

where $\gamma_2^1 > \gamma_2^0 > 0$ and $e^{1,1} \geq 0, e^{0,0} \geq 0, e^{1,0} \leq 0, e^{0,1} \leq 0$.

The clear positiveness of the endogenous effect holds for the case:

$$e^{1,1} > 0, e^{0,0} > 0, e^{1,0} = 0, e^{0,1} = 0$$

the overall produced information or the true state of the world is revealed into public. In these senses, the model of this paper is very robust against the disturbance effect of speculation.

Nevertheless, specifically we showed in Proposition 4, with the linear relations between states and commodities, that a speculation can never attain ex-ante Pareto improvement, if there exists no positive endogenous economic effect typically represented by learning-by-doing, which might be brought by a “right” judgment of an organization, and would presumably lead to the adjustment of production process, the market innovation, or the restructure of the underlying economic system. This aspect also supports the implication by Aoki (2006), which analyzes the effect of information sharing in a vertical market structure. In these contexts, we equivalently and purely examine the effect of the “negative” externality caused by privately produced information.

The impossibility of Pareto improvement under these simple assumptions (Proposition 4) is a crucial claim newly developed by this paper, and has a quite general economic implication because of the generality and the simplicity of the model. Although Hirshleifer already states that where the economic agents are completely in symmetry in every aspects including the initial endowment, the utility form or the information production ability, an informative signal may make every agent worse off (this statement is equivalent with our Proposition 6, which is shown, however, in an equilibrium eventually attained.), our Proposition 4 always holds regardless of the informational symmetry or asymmetry, regardless of the proportion in each agent’ s initial endowment, or regardless of the high/bad performance (preciseness) of the produced information. It contains also somewhat a different implication from that of Stein (1987), which insists that, “in general, informational externalities can be either positive or negative”. Furthermore, the proof of this proposition explains that such welfare reducing effect of speculation arises from the probabilistic fallibility of each agent’s decision making, rather than from the existence of asymmetric information itself, even though produced information itself is quite informative.

Through Proposition 5 to 8, we examine the same equilibrium transition from different angles, using various welfare criteria, like ex-ante Pareto efficiency (optimality), ex-ante social welfare improvement/maximization, ex-post Pareto optimality, true-state-based Pareto improvement/optimality, first/second best Pareto optimality, and shows some problem in the decision making theory that the Walrasian equilibrium transition, which is probabilistically multiple in paths on account of stochastic nature of produced information, may not necessarily promise the increase or the optimality in welfare also from these criteria.

In addition, Proposition 10 analyzes some decisive role of a riskless asset (money) on the ex-ante expected social welfare, in which its risk averse utility form always cause it decrease in speculation, on the other hand, the risk neutral (linear) form always keeps it invariant.

Main other criticisms of this paper may be categorized into the following four points. First, rigorously speaking, we do not assume that price movements themselves contain some information regarding the predictions of agents in the market, therefore, it should be smoothly and immediately reflected onto the predictions of agents who does not possess the information in advance, while, in our model, in the period of speculation, an informationally inferior agent (non-speculator) is "perfectly insensitive" also to the price movements in disequilibrium at least until the new equilibrium price reaches. However, it is rather easy to generalize this model by constructing the intermediate equilibrium process between "perfectly insensitive" as in this paper and "perfectly sensitive" as an opposite side, and above all, the economic rationality (the positive profitability) in "taking a plunge into speculation one second in advance to someone else even on imperfect information", paradoxically justifies the setting of our paper. Second, the dynamic process in this paper is limited at most over three periods, and the derived equilibrium is, rigorously speaking, not necessarily the sustainable equilibrium calculated over an infinite time horizon. Also as for this point, it is quite possible to derive, keeping the incorporation of the information production mechanism. Although we plan to modify the model in consideration of these two points, it is rather clear that all of the propositions presented in this paper will not be altered at all because of the modification. Specifically, the proof of Lemma 1 shows that Proposition 4 always applies in any modified models, and furthermore that there must exist at least one agent who becomes worse off as long as there exist probabilistically multiple equilibria paths on account of speculation, or as long as at least one equilibrium allocation is, even if it is not probabilistic, distinct from the initial one. Third, our model does not set any production function, although the coefficient of risky asset utility can be regarded as some value representing its productivity. As for this point, further elaborate work is left for future with assuming the endogenous effect of capital accumulation or with assuming increasing-return-to-scale (IRS) technology. However, we have already verified that in a simple model under constant-return-to-scale (CRT) technology and perfect mobility of capital and labor, all of our propositions are not drastically altered although the social welfare itself is kept constant. Fourth criticism is related with the definition of "information" in this paper, which might be quite controversial. For example, instead assume that two-country economy, a developed country with high productivity and a developing country with low productivity, and the information is defined as something, which enhances the low productivity of a developing country for sure, like the intellectual property rights or etc, and which can be easily internalized and transacted with price within the economy. In these settings, needless to comment and as a matter of course, the foreign direct capital flow accompanying such "information" is strongly expected to increase the ex-ante social welfare as well as the welfare of the developing country at least in the long run. On the other hand, repeatedly, the definition of Shannon, as in our model, merely arouses the mean-preserving spread in terms of prediction, however also gives a speculator enough economic incentives, because of no-correlation with the prior information set, to take advantage of the produced information without trading (internalizing) it with market price, which, if it happens, in turn surely leads to the negative externality, as ordinarily observed even in the "mature" financial market like domestic markets in developed countries, where there does not exist any prior inequality in knowledge or any prior asymmetry in information, as well as in the international financial markets.

The model in this paper might have some room for further elaboration and for further extension in various aspects. For example, it would be very helpful to apply this model to a more specific concrete economic system within an open economy framework. Thus, we strongly believe that this paper is rather meaningful as the first and essential step for further analysis.

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Appendix 1

The necessary F.O.C. conditions, in which (2.10) is satisfied, are as following.

$$\frac{q_2^t}{q_1^t} = \frac{\dfrac{\partial E\left[U_1(\underline{x}^{1,t}) \mid \Phi_t^1\right]}{\partial x_2^{1,t}}}{\dfrac{\partial E\left[U_1(\underline{x}^{1,t}) \mid \Phi_t^1\right]}{\partial x_1^{1,t}}} = \frac{\dfrac{\partial E\left[U_2(\underline{x}^{2,t}) \mid \Phi_t^2\right]}{\partial x_2^{2,t}}}{\dfrac{\partial E\left[U_2(\underline{x}^{2,t}) \mid \Phi_t^2\right]}{\partial x_1^{2,t}}} \tag{A1.1}$$

Now each agent's utility function is defined by (2.5). At period 0, also assume that the initial endowment allocation is in equilibrium and given by (2.12) (That is, $\underline{x}^0$ is located on $\underline{X}_p$).

Then, we have:

$$\frac{q_2^0}{q_1^0} = \frac{\beta_0^1\gamma_2^1 + (1-\beta_0^1)\gamma_2^0}{\gamma_1} = \frac{\beta_0^2\gamma_2^1 + (1-\beta_0^2)\gamma_2^0}{\gamma_1} = \frac{\alpha\gamma_2^1 + (1-\alpha)\gamma_2^0}{\gamma_1} \tag{A1.2}$$

Note that $\beta_0^1 = \beta_0^2 = \alpha$, and $x_1^{1,0} = x_2^{1,0}$, $x_1^{2,0} = x_2^{2,0}$. (A1.2) does not depend on s, the parameter which measures the ratio of both agents' economic initial wealth (endowments).

In general, if $\beta_t^1 = \beta_t^2$, then the equilibrium allocation, $\underline{x}^t = (\underline{x}^{1,t}, \underline{x}^{2,t})$, necessarily satisfies $x_1^{1,t} = x_2^{1,t}$ and $x_1^{2,t} = x_2^{2,t}$, so that $\underline{x}^t$ lies on the $\underline{X}_p$ as defined by (2.13).

Furthermore, the following necessary F.O.C. condition holds:

$$\frac{q_2^t}{q_1^t} = \frac{\beta_t^1\gamma_2^1 + (1-\beta_t^1)\gamma_2^0}{\gamma_1} = \frac{\beta_t^2\gamma_2^1 + (1-\beta_t^2)\gamma_2^0}{\gamma_1} \tag{A1.3}$$

Appendix 2

From (2.8), we can rewrite (1) of (4.1) as following:

$$E\left[U_i(\underline{x}^{i,t-1}) \mid \Phi_{t-1}^i\right] \equiv u_{i1}(x_1^{i,t-1}) + \left\{\beta_{t-1}^i u_{i2}^1(x_2^{i,t-1}) + (1-\beta_{t-1}^i)u_{i2}^0(x_2^{i,t-1})\right\} \tag{A2.1}$$

Also,

$$\begin{aligned} &E\left[U_i(\underline{x}_s^{i,t}(D_i)) \mid \Phi_{t-1}^i\right] \\ &= E\left[U_i(\underline{x}_s^{i,t}(D_i = 1)) \mid \Phi_{t-1}^i, \{D_i = 1\}\right] * \Pr ob(D_i = 1 \mid \Phi_{t-1}^i) \\ &+ E\left[U_i(\underline{x}_s^{i,t}(D_i = 0)) \mid \Phi_{t-1}^i, \{D_i = 0\}\right] * \Pr ob(D_i = 0 \mid \Phi_{t-1}^i) \end{aligned}$$

$$= \left[u_{i1}(x_{s1}^{i,t}(D_i = 1)) + \left\{\widetilde{\beta}_t^i(D_i = 1) * u_{i2}^1(x_{s2}^{i,t}(D_i = 1)) + (1 - \widetilde{\beta}_t^i(D_i = 1)) * u_{i2}^0(x_{s2}^{i,t}(D_i = 1))\right\}\right]$$
$$* \Pr ob(D_i = 1 \mid \Phi_{t-1}^i)$$
$$+ \left[u_{i1}(x_{s1}^{i,t}(D_i = 0)) + \left\{\widetilde{\beta}_t^i(D_i = 0) * u_{i2}^1(x_{s2}^{i,t}(D_i = 0)) + (1 - \widetilde{\beta}_t^i(D_i = 0)) * u_{i2}^0(x_{s2}^{i,t}(D_i = 0))\right\}\right]$$
$$* \Pr ob(D_i = 0 \mid \Phi_{t-1}^i)$$

where $\widetilde{\beta}_t^i(D_i = l)$ is defined in (2.7). (A2.2)

In (A2.2), the term, $E\left[U_i(\underline{x}_s^{i,t}(D_i)) \mid \Phi_{t-1}^i\right]$ is the ex-ante expected utility for declaring the bundle pair for speculation, $\underline{x}_s^{i,t}(D_i)$, under the last information set, Φ_{t-1}^i, while the term, $E\left[U_i(\underline{x}_s^{i,t}(D_i = l)) \mid \Phi_{t-1}^i, \{D_i = l\}\right]$, is the ex-post expected utility for each actually realized bundle, $\underline{x}_s^{i,t}(D_i = l)$, with the occurrence, $D_i = l$ as well as the last information set, Φ_{t-1}^i.

We can easily prove that, if $\underline{x}_s^{i,t}(D_i) = \underline{x}^{i,t-1}$ for all $D_i = l (l = 0,1)$, then (A2.1) and (A2.2) are the same.

Also we have:

$$\Pr ob(D_i = 1 \mid \Phi_{t-1}^i) = \beta_{t-1}^i P_1^i + (1 - \beta_{t-1}^i) P_0^i$$

$$\Pr ob(D_i = 0 \mid \Phi_{t-1}^i) = \beta_{t-1}^i (1 - P_1^i) + (1 - \beta_{t-1}^i)(1 - P_0^i),$$

$$\Pr ob(M = 1, D_i = 1 \mid \Phi_{t-1}^i) = \beta_{t-1}^i P_1^i, \quad \Pr ob(M = 1, D_i = 0 \mid \Phi_{t-1}^i) = \beta_{t-1}^i (1 - P_1^i)$$

$$\Pr ob(M = 0, D_i = 1 \mid \Phi_{t-1}^i) = (1 - \beta_{t-1}^i) P_0^i, \Pr ob(M = 0, D_i = 0 \mid \Phi_{t-1}^i) = (1 - \beta_{t-1}^i)(1 - P_0^i)$$

(A2.3)

Furthermore,

$$\widetilde{\beta}_t^i(D_i = l) = \Pr ob(M = 1 \mid \Phi_{t-1}^i, \{D_i = l\}) = \frac{\Pr ob(M = 1, D_i = l \mid \Phi_{t-1}^i)}{\Pr ob(D_i = l \mid \Phi_{t-1}^i)} \quad \text{(A2.4)}$$

Plugging (A2.3) into (A2.4), we get (2.7).

From (A2.4), it is easy to verify (4.4):

$$\beta_{t-1}^{i} = \Pr ob(D_i = 1 \mid \Phi_{t-1}^{i}) * \widetilde{\beta}_t^{i}(D_i = 1) + \Pr ob(D_i = 0 \mid \Phi_{t-1}^{i}) * \widetilde{\beta}_t^{i}(D_i = 0) \quad \text{(A2.5)}$$

Appendix 3

Comparing two distinct bundle pairs for speculation;

$\underline{x}_s \equiv \left(\underline{x}_s(D_i = 1), \underline{x}_s(D_i = 0)\right)$ and $\underline{x}_s' \equiv \left(\underline{x}_s'(D_i = 1), \underline{x}_s'(D_i = 0)\right)$, we write[A3-1]:

$$\underline{x}_s \preceq^{D_i}_{(\Phi_{t-1}^{i})} \underline{x}_s' \text{ or equivalently } \underline{x}_s \preceq^{D_i}_{\beta_{t-1}^{i}} \underline{x}_s',$$

iff $E\left[U_i(\underline{x}_s(D_i)) \mid \Phi_{t-1}^{i}\right] \le E\left[U_i(\underline{x}_s'(D_i)) \mid \Phi_{t-1}^{i}\right]$. (A3.1)

We may interpret this expression as "the bundle pair for speculation, $\underline{x}_s'$, is at least as good as another bundle pair, $\underline{x}_s$ for agent i, under the information set, Φ_{t-1}^{i}, with regard to the information, D_i."

Quite similarly, we write $\underline{x}_s \prec^{D_i}_{(\Phi_{t-1}^{i})} \underline{x}_s'$ or equivalently $\underline{x}_s \prec^{D_i}_{\beta_{t-1}^{i}} \underline{x}_s'$,

iff $E\left[U_i(\underline{x}_s(D_i)) \mid \Phi_{t-1}^{i}\right] < E\left[U_i(\underline{x}_s'(D_i)) \mid \Phi_{t-1}^{i}\right]$, (A3.2)

where we say "the bundle pair for speculation, $\underline{x}_s'$, is better off than another bundle pair, $\underline{x}_s$ for agent i, under the information set, Φ_{t-1}^{i}, with regard to the information, D_i".

We must be very careful about the difference of a "speculation" from a "lottery". In a lottery, which action (the bundle in this case) to take is randomly and independently chosen with some probability. On the other hand, in a speculation, the output of information, D_i, is also a stochastic variable, but has a considerable correlation with the state of the world, M, by which, D_i gives some information about the true state of the world, M. For example, consider the distinct trials of producing information, D_i, and D_i', say, under the information set, Φ_{t-1}^{i}. Then, D_i and D_i' are conditionally independent in the sense that;

A3-1 As a matter of course, the agent who produces a private information, (i of D_i, say.), may be different from the agent who has the underlying utility, (i' of $U_{i'}$, say.), for example, for the case that an agent can observe the output information the other agent has produced. In such cases, we simply describe (A3.1), for example, as:

$$\underline{x}_s \preceq^{D_i}_{(\Phi_{t-1}^{i'})} \underline{x}_s' \text{ or equivalently } \underline{x}_s \preceq^{D_i}_{\beta_{t-1}^{i'}} \underline{x}_s',$$

iff $E\left[U_{i'}(\underline{x}_s(D_i)) \mid \Phi_{t-1}^{i}\right] \le E\left[U_{i'}(\underline{x}_s'(D_i)) \mid \Phi_{t-1}^{i}\right]$

$$\Pr ob(D_i = l, D_i' = l' \mid M = j) = \Pr ob(D_i = l \mid M = j) * \Pr ob(D_i' = l' \mid M = j) \quad \text{(A3.3)}$$

for all l, l', and j. But, in general, it does *not* hold that:

$$\Pr ob(D_i = l, D_i' = l' \mid \Phi_{t-1}^i) = \Pr ob(D_i = l \mid \Phi_{t-1}^i) * \Pr ob(D_i' = l' \mid \Phi_{t-1}^i)$$

On the other hand, for the outputs of distinct trials of a lottery, L_i, and L_i', say, it must always hold that:

$$\Pr ob(L_i = l, L_i' = l' \mid \Phi_{t-1}^i) = \Pr ob(L_i = l \mid \Phi_{t-1}^i) * \Pr ob(L_i' = l' \mid \Phi_{t-1}^i) \quad \text{(A3.4)}$$

for all l, l', and Φ_{t-1}^i.

Since the agent's utility is state dependent, changing the action according to D_i may be able to lead to an increase in the agent's expected utility, even if the utility has a risk averse form, i.e., is a concave function.

Figure 4(a) gives us a good example, which explains equilibrium transitions of Case (A) at period 1. The initial equilibrium point is the allocation A at period 0. At period 1, agent 1 produces her private information, and the equilibrium point moves to the allocation B, if $D_1 = 1$, and moves to the allocation C, if $D_1 = 0$. Let us denote the allocated bundle for agent 1 at A, B and C or etc, respectively by A^1, B^1 and C^1 or etc. Clearly, under the information set, Φ_0^1 (or, equivalently with the prior prediction, β_0^1), $B^1 \prec_{\beta_0^1} A^1$ holds. On the other hand, with the occurrence of $D_1 = 1$ as well as the information set, Φ_0^1, (or, equivalently with the posterior prediction, $\tilde{\beta}_1^1(D_1 = 1)$), we have $A^1 \prec_{\tilde{\beta}_1^1(D_1=1)} B^1$. Quite similarly, while, under the information set, Φ_0^1 (or, equivalently with the prior prediction, β_0^1), $C^1 \prec_{\beta_0^1} A^1$ holds, on the other hand, with the occurrence of $D_1 = 0$ as well as the information set, Φ_0^1, (or, equivalently with the posterior prediction, $\tilde{\beta}_1^1(D_1 = 0)$), we have $A^1 \prec_{\tilde{\beta}_1^1(D_1=0)} C^1$. Since the preference represented by the expected utility, (2.8), shows convexity, a lottery which, for example, comprises the bundle B^1 with any arbitrary probability r, and the bundle C^1 with probability 1-r, denoted by $r \cdot B^1 \oplus (1-r) \cdot C^1$, is worse off than another lottery, which comprises the bundle A^1 with probability 1. That is:

$$r \cdot B^1 \oplus (1-r) \cdot C^1 \prec_{\beta_0^1} 1 \cdot A^1 \tag{A3.5}$$

(A3.5) also holds even when $r = \Pr ob((D_1 = 1 \mid \Phi_0)$.

However, because of the reason described above, defining the bundle pair for speculation;

$$\underline{x}_s' \equiv \left(B^1(D_1 = 1), C^1(D_1 = 0)\right) \text{ and } \underline{x}_s \equiv \left(A^1(D_1 = 1), A^1(D_1 = 0)\right),$$

then we have:

$$\underline{x}_s \prec_{\beta_0^1}^{D_1} \underline{x}_s' \tag{A3.6}$$

Repeatedly, (A3.5) and (A3.6) are not contradictory with each other.

Appendix 4

Consider Case B at period 1 in the three periods setting as described in section 3, where both agents have respectively an identical information production function in symmetry. This is equivalent with the case (b)(i), defined in (6.1). From the assumption, at period 1, the output information produced by an agent i, D_i, is not disclosed to the other agent, -i.

From the analogy with (5.1), we have the equilibrium conditions at period 1:

(1) $\underline{x}^{1,1}(D_1 = l_1, D_2 = l_2) = \underline{x}_D^1(\underline{x}^{1,0}, \widetilde{\beta}_1^1(D_1 = l_1), \underline{q}^1(D_1 = l_1, D_2 = l_2))$

$\underline{x}^{2,1}(D_1 = l_1, D_2 = l_2) = \underline{x}_D^2(\underline{x}^{2,0}, \widetilde{\beta}_1^2(D_2 = l_2), \underline{q}^1(D_1 = l_1, D_2 = l_2))$

(2) $\underline{x}^{1,1}(D_1 = l_1, D_2 = l_2) + \underline{x}^{2,1}(D_1 = l_1, D_2 = l_2) = \underline{x}^{1,0} + \underline{x}^{2,0}$ (A4.1)

And the equilibrium allocation at period 1 is denoted by:

$$\underline{x}^1(D_1 = l_1, D_2 = l_2) = (\underline{x}^{1,1}(D_1 = l_1, D_2 = l_2), \underline{x}^{2,1}(D_1 = l_1, D_2 = l_2)) \tag{A4.2}$$

Here, $\widetilde{\beta}_t^i(D_i = l_i)$ denotes the posterior Bayesian prediction of agent i with the occurrence, $D_i = l_i$ at period t.

Now we just write down the equilibrium conditions at period 2 for the cases (a)(ii), (a)(iii), (b)(ii) and (b)(iii), as defined in (6.1).

<u>Period 2 of case (a)(ii)</u> [A4-1]

[A4-1] As described in (5.1), actually $\underline{x}^{1,1}$ and $\underline{x}^{2,1}$ are the functions of $D_1 = l_1$, so we should write

(1) $\underline{x}^{1,2}(D_1 = l) = \underline{x}_D^1(\underline{x}^{1,1}, \beta_2^1 = \widetilde{\beta}_1^1(D_1 = l), \underline{q}^2(D_1 = l))$

$\underline{x}^{2,2}(D_1 = l) = \underline{x}_D^2(\underline{x}^{2,1}, \beta_2^2 = \widetilde{\beta}_1^1(D_1 = l), \underline{q}^2(D_1 = l))$

(2) $\underline{x}^{1,2}(D_1 = l) + \underline{x}^{2,2}(D_1 = l) = \underline{x}^{1,1}(D_1 = l) + \underline{x}^{2,1}(D_1 = l)$

(A4.3)

Period 2 of case (a)(iii) [A4-2]

(1) $\underline{x}^{1,2}(D_1 = l, M = j) = \underline{x}_D^1(\underline{x}^{1,1}, \beta_2^1 = j, \underline{q}^2(D_1 = l, M = j))$

$\underline{x}^{2,2}(D_1 = l, M = j) = \underline{x}_D^2(\underline{x}^{2,1}, \beta_2^2 = j, \underline{q}^2(D_1 = l, M = j))$

(2) $\underline{x}^{1,2}(D_1 = l, M = j) + \underline{x}^{2,2}(D_1 = l, M = j) = \underline{x}^{1,1}(D_1 = l) + \underline{x}^{2,1}(D_1 = l)$

(A4.4)

Period 2 of case (b)(ii) [A4-3] [A4-4]

(1) $\underline{x}^{1,2}(D_1 = l_1, D_2 = l_2) = \underline{x}_D^1(\underline{x}^{1,1}, \beta_2^1 = \widetilde{\beta}_1^1(D_1 = l_1, D_2 = l_2), \underline{q}^2(D_1 = l_1, D_2 = l_2))$

$\underline{x}^{2,2}(D_1 = l_1, D_2 = l_2) = \underline{x}_D^2(\underline{x}^{2,1}, \beta_2^2 = \widetilde{\beta}_1^2(D_1 = l_1, D_2 = l_2), \underline{q}^2(D_1 = l_1, D_2 = l_2))$

(2)
$$\begin{aligned} &\underline{x}^{1,2}(D_1 = l_1, D_2 = l_2) + \underline{x}^{2,2}(D_1 = l_1, D_2 = l_2) \\ &= \underline{x}^{1,1}(D_1 = l_1, D_2 = l_2) + \underline{x}^{2,1}(D_1 = l_1, D_2 = l_2) \end{aligned}$$

(A4.5)

Period 2 of case (b)(iii) [A4-5]

(1) $\underline{x}^{1,2}(D_1 = l_1, D_2 = l_2, M = j) = \underline{x}_D^1(\underline{x}^{1,1}, \beta_2^1 = j, \underline{q}^2(D_1 = l_1, D_2 = l_2, M = j))$

$\underline{x}^{2,2}(D_1 = l_1, D_2 = l_2, M = j) = \underline{x}_D^2(\underline{x}^{2,1}, \beta_2^2 = j, \underline{q}^2(D_1 = l_1, D_2 = l_2, M = j))$

---

them as $\underline{x}^{1,1}(D_1 = l)$ and $\underline{x}^{2,1}(D_1 = l)$, but we simplifies them just for sparing spaces.

[A4-2] The same as footnote A4-1.

[A4-3] As described in (A4.1), actually $\underline{x}^{1,1}$ and $\underline{x}^{2,1}$ are the functions of $D_1 = l_1$ and $D_2 = l_2$, so we should write them as $\underline{x}^{1,1}(D_1 = l_1, D_2 = l_2)$ and $\underline{x}^{2,1}(D_1 = l_1, D_2 = l_2)$, but we simplifies them just for sparing spaces.

[A4-4] Also, assuming $P_1^1 = P_1^2 (= P_1)$, we have:

$$\widetilde{\beta}_1^1(D_1 = l_1, D_2 = l_2) = \widetilde{\beta}_1^2(D_1 = l_1, D_2 = l_2)$$

[A4-5] Same as footnote A4-1.

(2) $$\underline{x}^{1,2}(D_1=l_1,D_2=l_2,M=j)+\underline{x}^{2,2}(D_1=l_1,D_2=l_2,M=j) = \underline{x}^{1,1}(D_1=l_1,D_2=l_2)+\underline{x}^{2,1}(D_1=l_1,D_2=l_2)$$

(A4.6)

Now the posterior Bayesian prediction of agent i at period t, with the observable outputs, $D_1=l_1,D_2=l_2$, is defined as:

$$\tilde{\beta}_t^i(D_1=l_1,D_2=l_2)\equiv \Pr ob(M=1\mid \Phi_{t-1}^i,\{D_1=l_1,D_2=l_2\}) = \frac{\Pr ob(M=1,D_1=l_1,D_2=l_2\mid \Phi_{t-1}^i)}{\Pr ob(D_1=l_1,D_2=l_2\mid \Phi_{t-1}^i)} \quad \text{(A4.7)}$$

From the analogy with (2.7), if we observe $D_1=1,D_2=1$, for example, then:

$$\tilde{\beta}_t^i(D_1=1,D_2=1)=\frac{\beta_{t-1}^i P_1^1 P_1^2}{\beta_{t-1}^i P_1^1 P_1^2+(1-\beta_{t-1}^i)P_0^1 P_0^2} \quad \text{(A4.8)}$$

If we observe $D_1=0,D_2=0$, for example, then:

$$\tilde{\beta}_t^i(D_1=0,D_2=0)=\frac{\beta_{t-1}^i(1-P_1^1)(1-P_1^2)}{\beta_{t-1}^i(1-P_1^1)(1-P_1^2)+(1-\beta_{t-1}^i)(1-P_0^1)(1-P_0^2)}$$

(A4.9)

For other cases:

$$\tilde{\beta}_t^i(D_1=1,D_2=0)=\frac{\beta_{t-1}^i P_1^1(1-P_1^2)}{\beta_{t-1}^i P_1^1(1-P_1^2)+(1-\beta_{t-1}^i)P_0^1(1-P_0^2)}$$

$$\tilde{\beta}_t^i(D_1=0,D_2=1)=\frac{\beta_{t-1}^i(1-P_1^1)P_1^2}{\beta_{t-1}^i(1-P_1^1)P_1^2+(1-\beta_{t-1}^i)(1-P_0^1)P_0^2} \quad \text{(A4.10)}$$

From these results, as in footnote A4-2, assuming $P_1^1=P_1^2$, we have:

$$\tilde{\beta}_1^1(D_1=l_1,D_2=l_2)=\tilde{\beta}_1^2(D_1=l_1,D_2=l_2)$$

Finally, we describe the equilibrium conditions at period 1 for the "rational overreaction" setting, as explained in footnote 5-4.

<u>The "rational overreaction" setting</u>

(1) $\hat{\underline{x}}^{1,1}(D_1=l)=\underline{x}_D^1(\underline{x}^{1,0},\beta_1^1=D_1(=l),\underline{q}^1(D_1=l))$

$\hat{\underline{x}}^{2,1}(D_1=l)=\underline{x}_D^2(\underline{x}^{2,0},\beta_1^2=\beta_0^2(=\alpha),\underline{q}^1(D_1=l))$

(2) $\hat{\underline{x}}^{1,1}(D_1=l)+\hat{\underline{x}}^{2,1}(D_1=l)=\underline{x}^{1,0}+\underline{x}^{2,0}$

(A4.11)

Appendix 5

Specifically, the ex-ante (expected) welfare ratios, R_1, R_2, and R_S, are defined as following.

Either (a)(i) or (a)(ii) in (5.5)

$$R_1 = \frac{E\left[U_1(\underline{x}^{1,t}(D_1)) \mid \Phi_0\right](-C_1)}{E\left[U_1(\underline{x}^{1,0}) \mid \Phi_0\right]}, \qquad R_2 = \frac{E\left[U_2(\underline{x}^{2,t}(D_1)) \mid \Phi_0\right]}{E\left[U_1(\underline{x}^{2,0}) \mid \Phi_0\right]}$$

$$R_S = \frac{W(\underline{x}^1(D_1) \mid \Phi_0)(-C_1)}{W(\underline{x}^0 \mid \Phi_0)}$$

t=1 for (a)(i) and t=2 for (a)(ii)

(A5.1)

(a)(iii) in (5.5)

$$R_1 = \frac{E\left[U_1(\underline{x}^{1,t}(D_1, M)) \mid \Phi_0\right](-C_1)}{E\left[U_1(\underline{x}^{1,0}) \mid \Phi_0\right]}, \qquad R_2 = \frac{E\left[U_2(\underline{x}^{2,t}(D_1, M)) \mid \Phi_0\right]}{E\left[U_1(\underline{x}^{2,0}) \mid \Phi_0\right]}$$

$$R_S = \frac{W(\underline{x}^1(D_1, M) \mid \Phi_0)(-C_1)}{W(\underline{x}^0 \mid \Phi_0)}$$

t=2

(A5.2)

Either (b)(i) or (b)(ii) in (5.5)

$$R_1 = \frac{E\left[U_1(\underline{x}^{1,t}(D_1, D_2)) \mid \Phi_0\right](-C_1)}{E\left[U_1(\underline{x}^{1,0}) \mid \Phi_0\right]}, \qquad R_2 = \frac{E\left[U_2(\underline{x}^{2,t}(D_1, D_2)) \mid \Phi_0\right](-C_2)}{E\left[U_1(\underline{x}^{2,0}) \mid \Phi_0\right]}$$

$$R_S = \frac{W(\underline{x}^1(D_1, D_2) \mid \Phi_0)(-C_1 - C_2)}{W(\underline{x}^0 \mid \Phi_0)}$$

t=1 for (b)(i) and t=2 for (b)(ii)

(A5.3)

(b)(iii) in (5.5)

$$R_1 = \frac{E\left[U_1(\underline{x}^{1,t}(D_1, D_2, M)) \mid \Phi_0\right](-C_1)}{E\left[U_1(\underline{x}^{1,0}) \mid \Phi_0\right]}, \qquad R_2 = \frac{E\left[U_2(\underline{x}^{2,t}(D_1, D_2, M)) \mid \Phi_0\right](-C_2)}{E\left[U_1(\underline{x}^{2,0}) \mid \Phi_0\right]}$$

$$R_S = \frac{W(\underline{x}^1(D_1, D_2, M) \mid \Phi_0)(-C_1 - C_2)}{W(\underline{x}^0 \mid \Phi_0)} \qquad \text{t=2}$$

(A5.4)

Appendix 6

Proof of Proposition 4

(a)(i): Proposition 3 already proves this case. Agent 2 will be worse off in terms of the ex-ante expected utility. Or, it also suffices to prove that either case, (a)(ii) or (a)(iii), cannot be Pareto improved, because in either case both agents surely will be better off than in case (a)(i).

(b)(i): It suffices to prove that either case, (b)(ii) or (b)(iii), cannot be Pareto improved, because in either case both agents surely will be better off than in case (b)(i).

The proofs of (a)(ii), (b)(ii), (a)(iii) and (b)(iii) can be made comprehensively and rather straightforwardly using Lemma 1, which is described at the last part of this section. But, we dare to leave each proof for each individual case in order to examine what the equilibrium transitions, and the corresponding predictions or the corresponding ex-post utilities are like for each case.

(a)(ii): See Figure A-1. As explained in section 5, point A is the initial equilibrium allocation at period 0. Furthermore, point B is the equilibrium allocation at period 1, with the occurrence, $D_1 = 1$, while point C is the equilibrium allocation with the occurrence, $D_1 = 0$. Now assume that $D_1 = 1$ occurred. Then, at period 2, we have, from the setting of the model, that $\beta_2^1 = \beta_2^2 = \widetilde{\beta}_1^1(D_1 = 1)$, assuming $P_1^1 = P_1^2$. This is also the "objective" prediction based on the overall information set, $\Phi_2 = \Phi_0 \cup \{D_1 = 1\} = \{D_1 = 1\}$. So, as explained in Appendix 1, at that period, the equilibrium will reach point $\widetilde{B}$, which is located on $\underline{X}_p(OO')$ as denoted in Figure A-1. In this case, the ex-post expected utility of each agent is:

$$E[U_i(\underline{x}) \mid \Phi_0, \{D_1 = 1\}] \equiv u_{i1}(x_1) + \left\{\widetilde{\beta}_1^1(D_1 = 1) * u_{i2}^1(x_2) + (1 - \widetilde{\beta}_1^1(D_1 = 1)) * u_{i2}^0(x_2)\right\}$$

(A6.1)

Now we define the associated ex-post expected utility function with $\underline{X}_p$, conditionally on the occurrence, $D_1 = 1$, as following:

$$\widetilde{U}_i^{D_1=1}(X^i) \equiv u_{i1}(x^i) + \left\{\widetilde{\beta}_1^1(D_1 = 1) * u_{i2}^1(x^i) + (1 - \widetilde{\beta}_1^1(D_1 = 1)) * u_{i2}^0(x^i)\right\}$$
$$= \{\gamma_1 + (\widetilde{\beta}_1^1(D_1 = 1) * \gamma_2^1 + (1 - \widetilde{\beta}_1^1(D_1 = 1)) * \gamma_2^0)\}(x^i)^{1-\alpha}$$

(A6.2)

Here $X^i = (x^i, x^i)$ is the bundle allocated to agent i at some arbitrary point X on $\underline{X}_p$. Note that $x^{-i} = 2 - x^i$ for i=1,2. We used the form of the utility functions defined in (2.5). Similarly, with the occurrence, $D_1 = 0$, the equilibrium will reach point $\widetilde{C}$, which is also located on $\underline{X}_p(OO')$ as denoted in Figure A-1. Then, the ex-post expected utility of each agent is:

$$E[U_i(\underline{x}) \mid \Phi_0, \{D_1 = 0\}] \equiv u_{i1}(x_1) + \left\{\widetilde{\beta}_1^1(D_1 = 0) * u_{i2}^1(x_2) + (1 - \widetilde{\beta}_1^1(D_1 = 0)) * u_{i2}^0(x_2)\right\}$$

(A6.3)

Therefore, we can define the associated ex-post expected utility function with $\underline{X}_p$, conditionally on the occurrence, $D_1 = 0$, as following:

$$\widetilde{U}_i^{D_1=0}(X^i) \equiv u_{i1}(x^i) + \left\{\widetilde{\beta}_1^1(D_1 = 0) * u_{i2}^1(x^i) + (1 - \widetilde{\beta}_1^1(D_1 = 0)) * u_{i2}^0(x^i)\right\}$$
$$= \{\gamma_1 + (\widetilde{\beta}_1^1(D_1 = 0) * \gamma_2^1 + (1 - \widetilde{\beta}_1^1(D_1 = 0)) * \gamma_2^0)\}(x^i)^{1-\alpha}$$

(A6.4)

Furthermore, since the initial point A is also located on $\underline{X}_p$, the associated ex-ante expected utility function, which can be obtained at period 0, is defined as;

$$\widetilde{U}_i^{\alpha}(X^i) \equiv u_{i1}(x^i) + \left\{\alpha * u_{i2}^1(x^i) + (1 - \alpha) * u_{i2}^0(x^i)\right\}$$
$$= \{\gamma_1 + (\alpha * \gamma_2^1 + (1 - \alpha) * \gamma_2^0)\}(x^i)^{1-\alpha} \qquad \text{(A6.5)}$$

where $\alpha = \beta_0^1 = \beta_0^2$.

From (A2.5), we have;

$$\widetilde{U}_i^{\alpha}(X^i) = q * \widetilde{U}_i^{D_1=1}(X^i) + (1 - q) * \widetilde{U}_i^{D_1=0}(X^i),$$

where $q = \Pr ob(D_1 = 1 \mid \Phi_0)$. (A6.6)[A6-1]

Note that all of $\widetilde{U}_i^{D_1=1}(X^i)$, $\widetilde{U}_i^{D_1=0}(X^i)$ and $\widetilde{U}_i^{\alpha}(X^i)$ are strictly increasing and concave with regard to $x^i \geq 0$.

Next, see Figure A-2. Since all of A, $\widetilde{B}$ and $\widetilde{C}$ are located on $\underline{X}_p$, we denote each bundle allocated to each agent at these points by $A^i \equiv (a^i, a^i)$, $\widetilde{B}^i \equiv (\widetilde{b}^i, \widetilde{b}^i)$ and $\widetilde{C}^i \equiv (\widetilde{c}^i, \widetilde{c}^i)$ (i=1,2). (A6.7)

Then the ex-ante expected utility for agent i, (i=1,2), which can be obtained by exercising speculation and by assuring, at period 2, the bundle pair for speculation, $\underline{x}^{i,2}(D_1) = (\widetilde{B}^i(D_1 = 1), \widetilde{C}^i(D_1 = 0))$, is;

$$q * \widetilde{U}_i^{D_1=1}(\widetilde{B}^i) + (1-q) * \widetilde{U}_i^{D_1=0}(\widetilde{C}^i), \quad \text{(A6.8)}$$

while that obtained from staying on the initial point A at period 0 is;

$$\widetilde{U}_t^{\alpha}(A^i) = q * \widetilde{U}_i^{D_1=1}(A^i) + (1-q) * \widetilde{U}_i^{D_1=0}(A^i) \quad \text{(A6.9)}$$

Now we need to show that the allocation pair for speculation at period 2, $\underline{x}^2(D_1) = (\widetilde{B}(D_1 = 1), \widetilde{C}(D_1 = 0))$, cannot be ex-ante Pareto improved, compared with the initial allocation, A (or equivalently, the allocation pair for speculation, $(A(D_1 = 1), A(D_1 = 0))$). We take two steps.

1. From the setting regarding each agent's prediction at each period that $\beta_1^1 = \widetilde{\beta}_1^1(D_1 = 1)$ and $\beta_1^2 = \alpha$ at period 1, and $\beta_2^1 = \beta_2^2 = \widetilde{\beta}_1^1(D_1 = 1)$ at period 2, we can conclude both points $\widetilde{B}$ and $\widetilde{C}$ are located on the side of O' on $\underline{X}_p$ (OO'), viewing from point A. Necessarily, we have $a^1 < \widetilde{b}^1$ and $a^1 < \widetilde{c}^1$ for the part of agent 1, and also $a^2 > \widetilde{b}^2$ and $a^2 > \widetilde{c}^2$ for the part of agent 2.
2. Therefore, $\widetilde{U}_1^{D_1=1}(A^1) < \widetilde{U}_1^{D_1=1}(\widetilde{B}^1)$ and $\widetilde{U}_1^{D_1=0}(A^1) < \widetilde{U}_1^{D_1=0}(\widetilde{C}^1)$ for the part of agent 1, and, $\widetilde{U}_2^{D_1=1}(A^2) > \widetilde{U}_2^{D_1=1}(\widetilde{B}^2)$ and $\widetilde{U}_2^{D_1=0}(A^2) > \widetilde{U}_2^{D_1=0}(\widetilde{C}^2)$ for the part of agent 2. So,

[A6-1] Assuming $\alpha = 0.5$, we have $q = 0.5$.

from (A6.7) and (A6.8), we get:

$$q*\widetilde{U}_1^{D_1=1}(\widetilde{B}^1)+(1-q)*\widetilde{U}_1^{D_1=0}(\widetilde{C}^1)>\widetilde{U}_1^{\alpha}(A^1) \text{ and}$$

$$q*\widetilde{U}_2^{D_1=1}(\widetilde{B}^2)+(1-q)*\widetilde{U}_2^{D_1=0}(\widetilde{C}^2)<\widetilde{U}_2^{\alpha}(A^2) \qquad \text{(A6.10)}$$

So, at period 2, agent 2 will be worse off than at period 0, while agent 1 will be better off. That is, the allocation pair for speculation at period 2, $\underline{x}^2(D_1)=(\widetilde{B}(D_1=1),\widetilde{C}(D_1=0))$, cannot be ex-ante Pareto improved, compared with the initial allocation, A. Then the proof is done. (Q.E.D.)

(b)(ii): See Figure A-3 as well as Figure 4(b). From the argument in section 5, we know that the equilibrium allocation point might shift from the initial allocation, A, only in case that $\{D_1=1,D_0=0\}$ or $\{D_1=0,D_0=1\}$ has occurred. In other cases, $\{D_1=1,D_0=1\}$ or $\{D_1=0,D_0=0\}$, the equilibrium will remain at point A. Let $\widetilde{D}$ and $\widetilde{E}$ denote the equilibrium allocation points with the occurrence $\{D_1=1,D_0=0\}$ and $\{D_1=0,D_0=1\}$, respectively. Then, although, the allocations for speculation, unconditionally on Φ_0, are quadruple, where;

$$\underline{x}^2(D_1,D_2)=(A(D_1=1,D_2=1),\widetilde{D}(D_1=1,D_2=0),\widetilde{E}(D_1=0,D_1=1),A(D_1=0,D_2=0)),$$

(A6.11)[A6-2]

we have only to consider, conditionally, the only former two corresponding cases.[A6-3] Define a

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[A6-2] So, the bundles for speculation allocated to each agent i, unconditionally on Φ_0, are also quadruple, that is:

$$\underline{x}^{i,2}(D_1,D_2)=(A^i(D_1=1,D_2=1),\widetilde{D}^i(D_1=1,D_2=0),\widetilde{E}^i(D_1=0,D_1=1),A^i(D_1=0,D_2=0))$$

[A6-3] Strictly speaking, this description needs more neat explanation. Assuming $P_1^1=P_1^2(=P_1)$, the conditional expectation of the Bayesian prediction, $\widetilde{\beta}_1^i(D_1,D_2)$, on Λ or on $\overline{\Lambda}\equiv\{D_1=1,D_0=1\}\cup\{D_1=0,D_0=0\}$, are both α's respectively. That is:

$$\Pr ob(\{D_1=1,D_2=0\}\mid\Lambda)*\widetilde{\beta}_1^i(D_1=1,D_2=0)+ \Pr ob(\{D_1=0,D_2=1\}\mid\Lambda)*\widetilde{\beta}_1^i(D_1=0,D_2=1)=\alpha \quad \text{and}$$

$$\Pr ob(\{D_1=1,D_2=1\}\mid\overline{\Lambda})*\widetilde{\beta}_1^i(D_1=1,D_2=1)+ \Pr ob(\{D_1=0,D_2=0\}\mid\overline{\Lambda})*\widetilde{\beta}_1^i(D_1=0,D_2=0)=\alpha$$

Since, under $\overline{\Lambda}$, the equilibrium allocation does stay at the initial point A, and the conditional expectation of Bayesian prediction on $\overline{\Lambda}$ is, as clear from the second equality, the same as the initial prediction, α, so the conditional expected utility on $\overline{\Lambda}$ is exactly the same as the ex-ante expected set of events, $\Lambda \equiv \{D_1 = 1, D_0 = 0\} \cup \{D_1 = 0, D_0 = 1\}$. Both of these two points, $\widetilde{D}$ and $\widetilde{E}$, are located on $\underline{X}_p$, because $\beta_2^1 = \beta_2^2$ always holds. Then, the allocation pair for speculation, conditionally on Λ, would be, at period 2:

$$\underline{x}^2(D_1, D_2 \mid \Lambda) \equiv (\widetilde{D}(D_1 = 1, D_2 = 0), \widetilde{E}(D_1 = 0, D_1 = 1)). \qquad \text{(A6.12)}$$

Assuming $P_1^1 = P_1^2 (= P_1)$, we have;

for $\{D_1 = 1, D_0 = 0\}$, $\beta_2^1 = \beta_2^2 = \widetilde{\beta}_1^1(D_1 = 1, D_2 = 0) = \widetilde{\beta}_1^2(D_1 = 1, D_2 = 0) = \alpha$ and,

for $\{D_1 = 0, D_0 = 1\}$, $\beta_2^1 = \beta_2^2 = \widetilde{\beta}_1^1(D_1 = 0, D_2 = 1) = \widetilde{\beta}_1^2(D_1 = 0, D_2 = 1) = \alpha$. (A6.13)

Furthermore,

$$\Pr ob(\{D_1 = 1, D_2 = 0\} \mid \Lambda) = 0.5$$
$$\Pr ob(\{D_1 = 0, D_2 = 1\} \mid \Lambda) = 0.5 \qquad \text{(A6.14)}$$ [A6-4]

In either case, the subjective or objective predictions of both agents at period 2 are all the same as the initial prediction, $\beta_0^1 = \beta_0^2 = \alpha$. See Figure A-4. The conditional expected utility of the allocation pair, $\underline{x}^2(D_1, D_2 \mid \Lambda) = (\widetilde{D}(D_1 = 1, D_2 = 0), \widetilde{E}(D_1 = 0, D_1 = 1))$, conditionally on Λ, of each agent is $0.5 * \widetilde{U}_i^\alpha(\widetilde{D}^i) + 0.5 * \widetilde{U}_i^\alpha(\widetilde{E}^i)$ for i=1,2, while that of the initial allocation A is just $\widetilde{U}_i^\alpha(A^i)$. Define $\widetilde{D}^i \equiv (\widetilde{d}^i, \widetilde{d}^i)$ and $\widetilde{E}^i \equiv (\widetilde{e}^i, \widetilde{e}^i)$ (i=1,2). Since $\widetilde{U}_i^\alpha(X^i)$ is strictly increasing and concave with respect to x^i, but strictly decreasing with respect to $x^{-i}(= 2 - x^i)$, there does not exist any point, $\widetilde{A}$, say, where $\widetilde{A}^i \equiv (\widetilde{a}^i, \widetilde{a}^i)$, on $\underline{X}_p$, such that;

$$0.5 * \widetilde{U}_i^\alpha(\widetilde{B}^i) + 0.5 * \widetilde{U}_i^\alpha(\widetilde{C}^i) > \widetilde{U}_i^\alpha(\widetilde{A}^i) \text{ both for i=1,2.} \qquad \text{(A6.15)}$$

---

utility (under Φ_0) of point A. Therefore, we do not have to consider the set, $\overline{\Lambda}$, in order to examine the Pareto improvement.

A6-4 $\Pr ob(\{D_1 = 1, D_2 = 0\} \mid \Phi_0) = \Pr ob(\{D_1 = 0, D_2 = 1\} \mid \Phi_0) = P_1(1 - P_1)$

So, neither the conditional allocation pair on Λ, $\underline{x}^2(D_1, D_2 \mid \Lambda)$, can be ex-ante Pareto improved compared with the initial allocation A, nor, therefore, can the unconditional allocations for speculation, $\underline{x}^2(D_1, D_2)$. (Q.E.D.)

(a)(iii): See Figure A-5(a). We denote each equilibrium allocation at period 2, $\underline{x}^2(D_1, M)$ for each realization of D_1 at period 1 and each realization of M, by $\widetilde{B}_1$, $\widetilde{B}_0$, $\widetilde{C}_1$ and $\widetilde{C}_0$, respectively. That is, the allocations for speculation would be, at period 2:[A6-5]

$$\underline{x}^2(D_1, M) = (\widetilde{B}_1(D_1 = 1, M = 1), \widetilde{B}_0(D_1 = 1, M = 0), \widetilde{C}_1(D_1 = 0, M = 1), \widetilde{C}_0(D_1 = 0, M = 0)) \tag{A6.16}$$

Clearly all of points, $\widetilde{B}_1$, $\widetilde{B}_0$, $\widetilde{C}_1$ and $\widetilde{C}_0$, are located on $\underline{X}_p$, because $\beta_2^1 = \beta_2^2 (= M = j)$.

Now we use the following ex-ante (expected) social welfare;

$$W(\underline{x}^t \mid \Phi_0) = \delta_1 * E\left[U_1(\underline{x}^{1,t}) \mid \Phi_0\right] + \delta_2 * E\left[U_2(\underline{x}^{2,t}) \mid \Phi_0\right] \text{ for t=1,2} \tag{3.1}$$

where $\delta_i = \lambda_i^{-1} / (\lambda_1^{-1} + \lambda_2^{-1})$, and λ_i is agent i's marginal expected utility of initial income at the initial period 0. This definition has some convenient property to examine the Pareto improvement of $\underline{x}^2(D_1, M)$, compared with the initial allocation, $\underline{x}^0 (= A)$. As shown in footnote 3-2, by simple calculation, we easily get $\delta_1 = (1-s)^a / ((1-s)^a + (1+s)^a)$ and $\delta_2 = (1+s)^a / ((1-s)^a + (1+s)^a)$, in which δ_i's do not depend on β_0^i's, assuming $\beta_0^1 = \beta_0^2$. This implies that with any realization of the information, D_i's, or equivalently, with any objective prediction, β_t's, or under any arbitrary overall information set, $\Phi_t (= \Xi, say)$, the *ex-post* (not ex-ante) social welfare is maximized at the initial allocation point, A. That is, $W(A \mid \Xi) \geq W(A' \mid \Xi)$ for any arbitrary information set, Ξ, and for any arbitrary allocation point, A'. Therefore, from (3.1), we have;

[A6-5] We also denote each bundle allocated to agent i at each allocation point as:
$\widetilde{B}_1^i \equiv (\widetilde{b}_1^i, \widetilde{b}_1^i)$, $\widetilde{B}_0^i \equiv (\widetilde{b}_0^i, \widetilde{b}_0^i)$, $\widetilde{C}_1^i \equiv (\widetilde{c}_1^i, \widetilde{c}_1^i)$, $\widetilde{C}_0^i \equiv (\widetilde{c}_0^i, \widetilde{c}_0^i)$ (i=1,2).

$$
\begin{aligned}
&W(\underline{x}^2(D_1,M)\mid\Phi_0)=\\
&z_{11}*W(\widetilde{B}_1\mid\{D_1=1,M=1\})+z_{10}*W(\widetilde{B}_0\mid\{D_1=1,M=0\})\\
&+z_{01}*W(\widetilde{C}_1\mid\{D_1=0,M=1\})+z_{00}*W(\widetilde{C}_0\mid\{D_1=0,M=0\})\\
&\le z_{11}*W(A\mid\{D_1=1,M=1\})+z_{10}*W(A\mid\{D_1=1,M=0\})\\
&+z_{01}*W(A\mid\{D_1=0,M=1\})+z_{00}*W(A\mid\{D_1=0,M=0\})\\
&\le W(A(=\underline{x}^0)\mid\Phi_0)
\end{aligned}
\tag{A6.17}
$$

where $z_{lj}=\Pr ob(D_1=l,M=j\mid\Phi_0)$, and of course, $z_{11}+z_{10}+z_{01}+z_{00}=1$.

Since $\delta_i>0$ for i=1,2, we conclude that, on account of speculation, at least one agent must be worse off at period 2 than at period 0, in terms of the ex-ante expected utility. Then the proof is done.[A6-6] (Q.E.D.)

(b)(iii): See Figure A-5(b). In this case, we define four possible equilibria allocations, $\widetilde{D}_1$, $\widetilde{D}_0$, $\widetilde{E}_1$ and $\widetilde{E}_0$, where the quadruple allocations for speculation, conditionally on $\Lambda\equiv\{D_1=1,D_0=0\}\cup\{D_1=0,D_0=1\}$, at period 2, are described as:

$$
\begin{aligned}
&\underline{x}^2(D_1,D_2,M\mid\Lambda)\equiv(\widetilde{D}_1(D_1=1,D_2=0,M=1),\widetilde{D}_0(D_1=1,D_2=0,M=0),\\
&\widetilde{E}_1(D_1=0,D_2=1,M=1),\widetilde{E}_0(D_1=0,D_2=1,M=0))
\end{aligned}
\quad\text{(A6.18)}
$$
[A6-7]

Similarly as in the proof in (b)(ii) of Proposition 4, we just need to focus on the set, Λ, that is, on these four points of equilibria allocations, $\widetilde{D}_1$, $\widetilde{D}_0$, $\widetilde{E}_1$ and $\widetilde{E}_0$ described in (A6.18).

Clearly, all of points, $\widetilde{D}_1$, $\widetilde{D}_0$, $\widetilde{E}_1$ and $\widetilde{E}_0$, are located on $\underline{X}_p$, because $\beta_2^1=\beta_2^2(=M=j)$. Then, almost the same argument as in the proof of (a)(iii) applies by replacing $\widetilde{B}_1$, $\widetilde{B}_0$, $\widetilde{C}_1$ and $\widetilde{C}_0$, with $\widetilde{D}_1$, $\widetilde{D}_0$, $\widetilde{E}_1$ and $\widetilde{E}_0$, respectively. (Q.E.D.)

As a matter of fact, from the argument made in the proof of (a)(iii), derivatively we have the

---

[A6-6] Actually this argument can be applied for the proofs of (a)(ii) and (b)(ii) (of Proposition 4), as well as (b)(iii).

[A6-7] Therefore, each allocation point is described as:

$$
\begin{aligned}
&\widetilde{D}_1=\underline{x}^2(D_1=1,D_2=0,M=1),\ \widetilde{D}_0=\underline{x}^2(D_1=1,D_2=0,M=0),\\
&\widetilde{E}_1=\underline{x}^2(D_1=0,D_2=1,M=1),\ \widetilde{E}_0=\underline{x}^2(D_1=0,D_2=1,M=0).
\end{aligned}
$$

$\widetilde{D}_1$ and $\widetilde{D}_0$ may be a little confusing notations, but they should be clearly distinguished from D_i's, which are the private information produced by agent i.

following lemma, which can be applied to the comprehensive proof of Proposition 4.

Lemma 1: Define the *ex-post* (expected) social welfare at period t;

$$W(\underline{x}^t \mid \Phi_t) = \delta_1 * E\left[U_1(\underline{x}^{1,t}) \mid \Phi_t\right] + \delta_2 * E\left[U_2(\underline{x}^{2,t}) \mid \Phi_t\right] \text{ for t=1,2} \qquad (3.1)$$

where $\delta_i = \lambda_i^{-1}/(\lambda_1^{-1} + \lambda_2^{-1})$, and λ_i is agent i's marginal expected utility of initial income at the initial period 0. Then, with any realization of the information, D_i's, or equivalently, with any objective prediction, β_t's, or under any arbitrary overall information set, $\Phi_t (= \Xi, say)$, the ex-post social welfare is maximized at the initial allocation point, A.

Proof of Proposition 5

It just suffices to show that the ex-ante Pareto efficiency cannot be attained, because not being ex-ante Pareto efficient necessarily implies not being the ex-ante social welfare maximized. (See footnote 6-6.)

We use the following lemma.

Lemma 2: If with some possible occurrences of D_i's, at least one of the possible equilibria allocations is not the ex-post Pareto optimum, then the overall possible equilibria allocations are not the ex-ante Pareto optimum.

(a)(i) or (b)(i): This is clear. In these transition processes, some or all (that is, at least one) of the possible equilibria allocations are not located on $\underline{X}_p$, which is an identical objective Pareto set for any occurrences of D_i's. So, some of the possible equilibria allocations are not ex-post Pareto optimal. Therefore, from Lemma 2, the overall possible equilibria allocations are not ex-ante Pareto optimal. (Q.E.D.)

(a)(iii): See Figure A-6. We use the same notations as the proof in (a)(iii) of Proposition 4. In addition, define the associated ex-post expected utility functions with $\underline{X}_p$, conditionally on the occurrence, $M = 1$, and on $M = 0$, respectively, as following:

$$\widetilde{U}_i^1(X^i) \equiv u_{i1}(x^i) + u_{i2}^1(x^i) = \{\gamma_1 + \gamma_2^1\}(x^i)^{1-\alpha}$$

$$\widetilde{U}_i^0(X^i) \equiv u_{i1}(x^i) + u_{i2}^0(x^i) = \{\gamma_1 + \gamma_2^0\}(x^i)^{1-\alpha} \qquad \text{(A6.19)}$$

Then, assuming $P_1^1 = P_1^2 (= P_1)$, the ex-ante expected utility of the bundles for speculation,

$\underline{x}^{i,2}(D_1, M)$, is:

$$\Pr ob(D_1 = 1, M = 1 \mid \Phi_0) * \widetilde{U}_i^1(\widetilde{B}_1^i) + \Pr ob(D_1 = 1, M = 0 \mid \Phi_0) * \widetilde{U}_i^0(\widetilde{B}_0^i)$$
$$+ \Pr ob(D_1 = 0, M = 1 \mid \Phi_0) * \widetilde{U}_i^1(\widetilde{C}_1^i) + \Pr ob(D_1 = 0, M = 0 \mid \Phi_0) * \widetilde{U}_i^0(\widetilde{C}_0^i)$$
$$= \alpha P_1 * \widetilde{U}_i^1(\widetilde{B}_1^i) + (1-\alpha)(1-P_1) * \widetilde{U}_i^0(\widetilde{B}_0^i) + \alpha(1-P_1) * \widetilde{U}_i^1(\widetilde{C}_1^i) + (1-\alpha)P_1 * \widetilde{U}_i^0(\widetilde{C}_0^i)$$

(A6.20)

Define two points, H and I, on $\underline{X}_p$, where;

$H^i \equiv (v\widetilde{b}_1^i + (1-v)\widetilde{c}_1^i, v\widetilde{b}_1^i + (1-v)\widetilde{c}_1^i)$ and $I^i \equiv (w\widetilde{b}_0^i + (1-w)\widetilde{c}_0^i, w\widetilde{b}_0^i + (1-w)\widetilde{c}_0^i)$ for i=1,2, and;

$$v \equiv \widetilde{\beta}_1^i(D_1 = 1) = \alpha P_1 / (\alpha P_1 + (1-\alpha)(1-P_1))$$

$$w \equiv \widetilde{\beta}_1^i(D_1 = 0) = \alpha(1-P_1) / (\alpha(1-P_1) + (1-\alpha)P_1). \qquad \text{(A6.21)}$$

Since $\widetilde{U}_i^1(X^i)$ and $\widetilde{U}_i^0(X^i)$ are strictly increasing and concave with respect to x^i, the allocations for speculation; $(H(D_1 = 1, M = 1), I(D_1 = 1, M = 0), H(D_1 = 0, M = 1), I(D_1 = 0, M = 0))$, is at least as good as $\underline{x}^{i,2}(D_1, M)$ for both agents i=1,2. Then the proof is done. (Q.E.D.)

(b)(iii): Similarly as in the proof of (b)(iii) of Proposition 4, we just need to focus on the set, Λ, that is, on the four points of equilibria allocations, $\widetilde{D}_1$, $\widetilde{D}_0$, $\widetilde{E}_1$ and $\widetilde{E}_0$ described in (A6.18). Then, almost the same argument as in the proof of (a)(iii) applies by replacing $\widetilde{B}_1$, $\widetilde{B}_0$, $\widetilde{C}_1$ and $\widetilde{C}_0$, with $\widetilde{D}_1$, $\widetilde{D}_0$, $\widetilde{E}_1$ and $\widetilde{E}_0$, respectively. (Q.E.D.)

Proof of Proposition 6

(b)(i): It suffices to prove that, in either case, (b)(ii) or (b)(iii), this proposition holds, because in either case both agents will be surely better off than in case (b)(i).

(b)(ii): We use the same notations as those used so far. From the assumption, we have $s = 0$, which implies that both agents have initially the same bundle, that is, $\underline{x}^{1,0} = \underline{x}^{2,0} = (1,1)$,

and $A = \underline{x}^0 = (\underline{x}^{1,0}, \underline{x}^{2,0})$. Note that point A is the middle point of $OO'(\underline{X}_p)$. Similarly as in the previous proofs, we just focus on the set, Λ. Since now the economic system is geographically in the complete symmetry, the equilibrium paths;

$$A(=\underline{x}^0) \to D(=\underline{x}^1(D_1 = 1, D_2 = 0)) \to \widetilde{D}(=\underline{x}^2(D_1 = 1, D_2 = 0)) \text{ and,}$$

$$A(=\underline{x}^0) \to E(=\underline{x}^1(D_1 = 0, D_2 = 1)) \to \widetilde{E}(=\underline{x}^2(D_1 = 0, D_2 = 1)), \quad \text{(A6.22)}$$

are geographically in symmetry with regard to the initial point, A. So, using the notations defined in (A6.7), we have:

$A = 0.5 * \widetilde{B} + 0.5 * \widetilde{C}$ so that $a^i = 0.5 * \widetilde{b}^i + 0.5 * \widetilde{c}^i$ (i=1,2).

That is, point A is exactly the middle point of $\widetilde{B}\widetilde{C}$. Now define the ex-ante social welfare:

$$W(\underline{x}^t \mid \Phi_0) = \delta_1 * E\left[U_1(\underline{x}^{1,t}) \mid \Phi_0\right] + \delta_2 * E\left[U_2(\underline{x}^{2,t}) \mid \Phi_0\right] \text{ for t=1,2} \quad (3.1)$$

where $\delta_1 = \delta_2 = 0.5$

Note that in this case, that is, with $s = 0$, two definitions described in footnote 3-2 just coincide with each other. Then, as discussed in the proof of Proposition 4;

$$W(\underline{x}^2(D_1, D_2) \mid \Lambda) = 0.5 * E\left[U_1(\underline{x}^{1,2}(D_1, D_2)) \mid \Lambda\right] + 0.5 * E\left[U_2(\underline{x}^{2,2}(D_1, D_2)) \mid \Lambda\right]$$
$$\leq W(A(=\underline{x}^0) \mid \Lambda) = 0.5 * E\left[U_1(A^1(=\underline{x}^{1,0})) \mid \Lambda\right] + 0.5 * E\left[U_2(A^2(=\underline{x}^{2,0})) \mid \Lambda\right]$$

(A6.23)[A6-8]

where $\Pr ob(\{D_1 = 1, D_2 = 0\} \mid \Lambda) = 0.5$.

From the symmetry, we also have:

$$E\left[U_1(\underline{x}^{1,2}(D_1, D_2)) \mid \Lambda\right] = E\left[U_2(\underline{x}^{2,2}(D_1, D_2)) \mid \Lambda\right] \text{ and,}$$

$$E\left[U_1(A^1(=\underline{x}^{1,0})) \mid \Lambda\right] = E\left[U_2(A^2(=\underline{x}^{2,0})) \mid \Lambda\right] \quad \text{(A6.24)}$$

Therefore, it must hold that, for i=1,2:

$$E\left[U_1(\underline{x}^{1,2}(D_1, D_2)) \mid \Lambda\right] \leq E\left[U_1(A^1(=\underline{x}^{1,0})) \mid \Lambda\right] \quad \text{(A6.25)}$$

Then the proof is done. (Q.E.D.)

(b)(iii): Almost the same argument as in the proof of (b)(ii) applies. (Q.E.D.)

A6-8 Using the same notations as in the proof of (b)(ii) in Proposition 4, we have, for i=1,2: $E\left[U_i(\underline{x}^{i,2}(D_1, D_2)) \mid \Lambda\right] = 0.5 * \widetilde{U}_i^{\alpha}(\widetilde{D}^i) + 0.5 * \widetilde{U}_i^{\alpha}(\widetilde{E}^i)$, and $E\left[U_i(\underline{x}^{i,0}) \mid \Lambda\right] = \widetilde{U}_i^{\alpha}(A^i)$

		(a)(i)	(b)(i)	(a)(ii)	(b)(ii)	(a)(iii)	(b)(iii)
(U)	(1)	N	N	N	N	N	N
	(2)	N	N	U	U	N	N
(V)*	(1)	N	N	D	D	D	D
	(2)	N	N	U	U	N	N
(V)**	(1)	N	N	N	N	N	N
	(2)	N	N	N	N	N	N
(W)		N	N	Y	Y	Y	Y
(X)	(1)	N	N	N	N	N	N
	(2)	N	N	N	N	N	N
(Y)		N	N	Y	Y	Y	Y
(Z)		D	D	Y	Y	Y	Y

Y: Yes. N: No. U: Uncertain (It is not proved.) D: It depends on parameters.

(U)(1): Ex-ante Pareto improved? (U)(2): Ex-ante Pareto optimal?

(V)(1): Ex-ante social welfare improved? (V)(2): Ex-ante social welfare maximized?

(W): Ex-post Pareto optimal?

(X)(1): True state-based Pareto improved? (X)(2): True state-based Pareto optimal?

(Y): First best Pareto optimal?

(Z): Second best Pareto optimal?

*: The weight of each agent is equally set at $\delta_1 = \delta_2 = 0.5$.

**: The weight of each agent is set at $\delta_i = \lambda_i^{-1}/(\lambda_1^{-1} + \lambda_2^{-1})$, where λ_i is agent i's marginal expected utility of initial income at the initial period 0.

Note: We exclude the exceptional cases, in which all equilibrium paths at period 1 or 2 accidentally come back to the initial allocation, A, say, or to the same allocation with each other.

Table

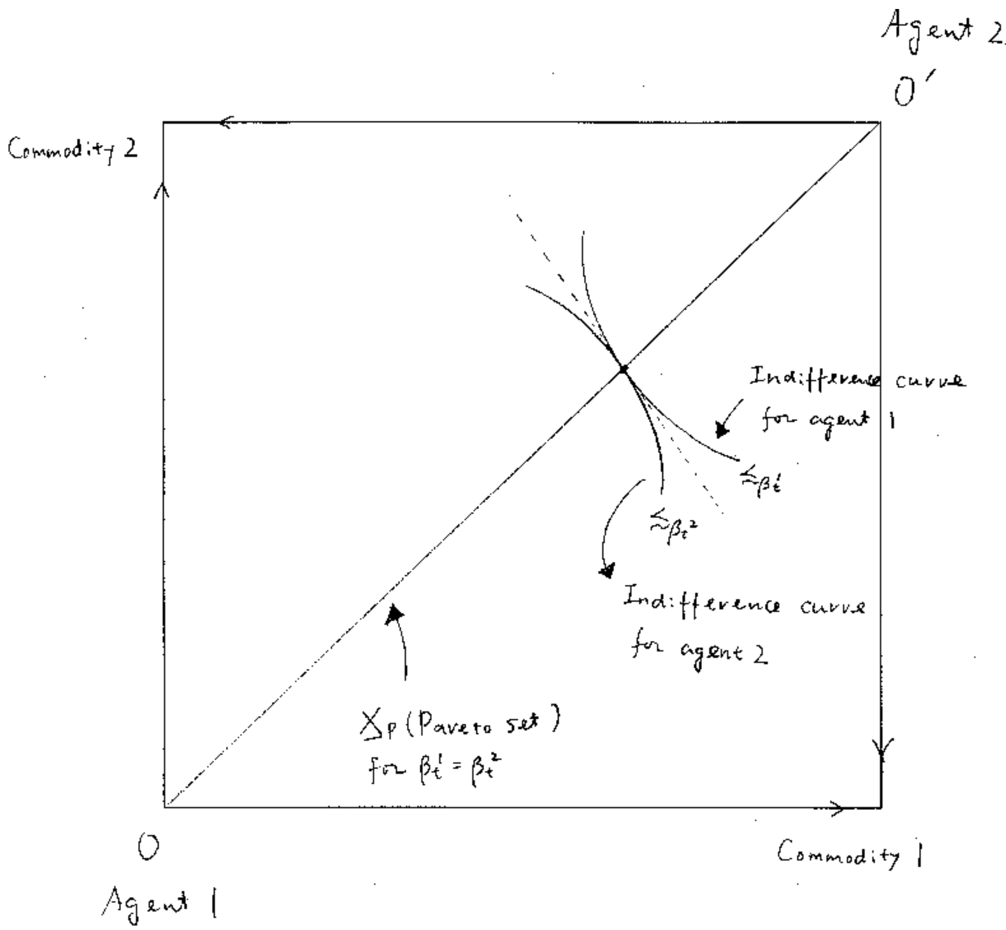


Figure 1

Pareto set on the Edgeworth box in case of $\beta_t^1 = \beta_t^2$

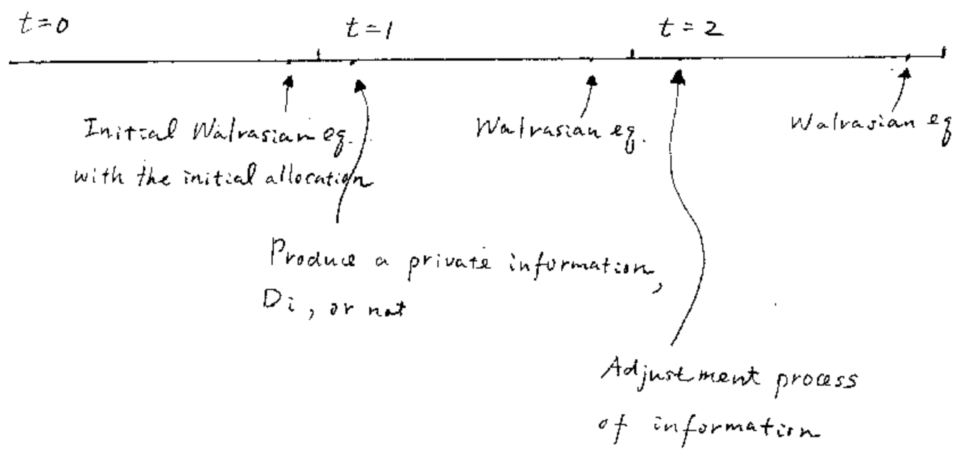


Figure 2

Equilibrium transition model over three periods

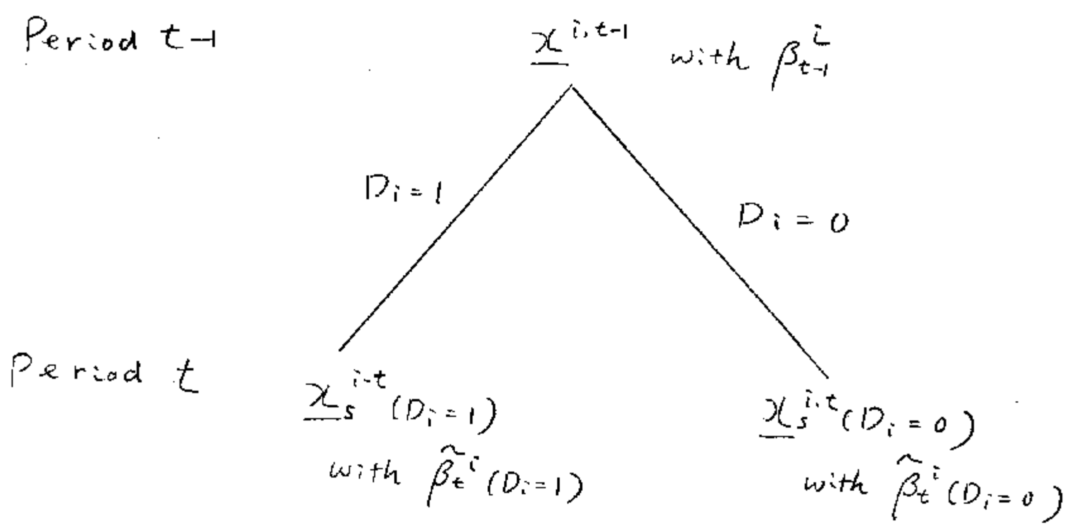


Figure 3

Mechanism of speculation

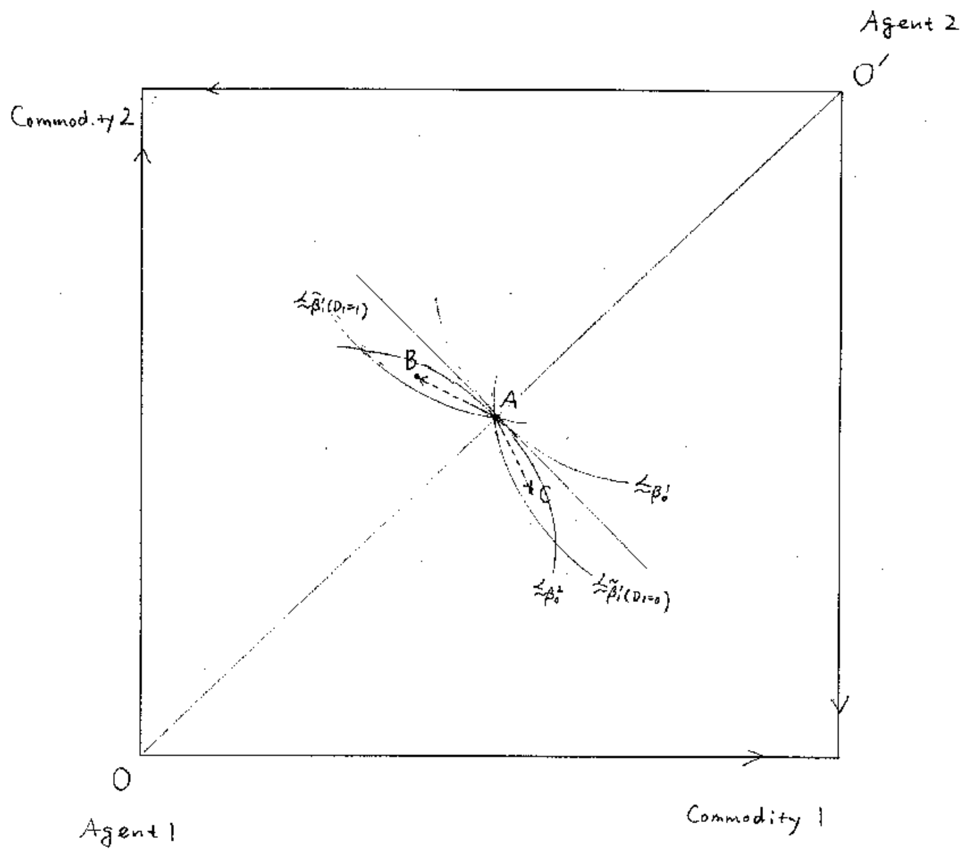
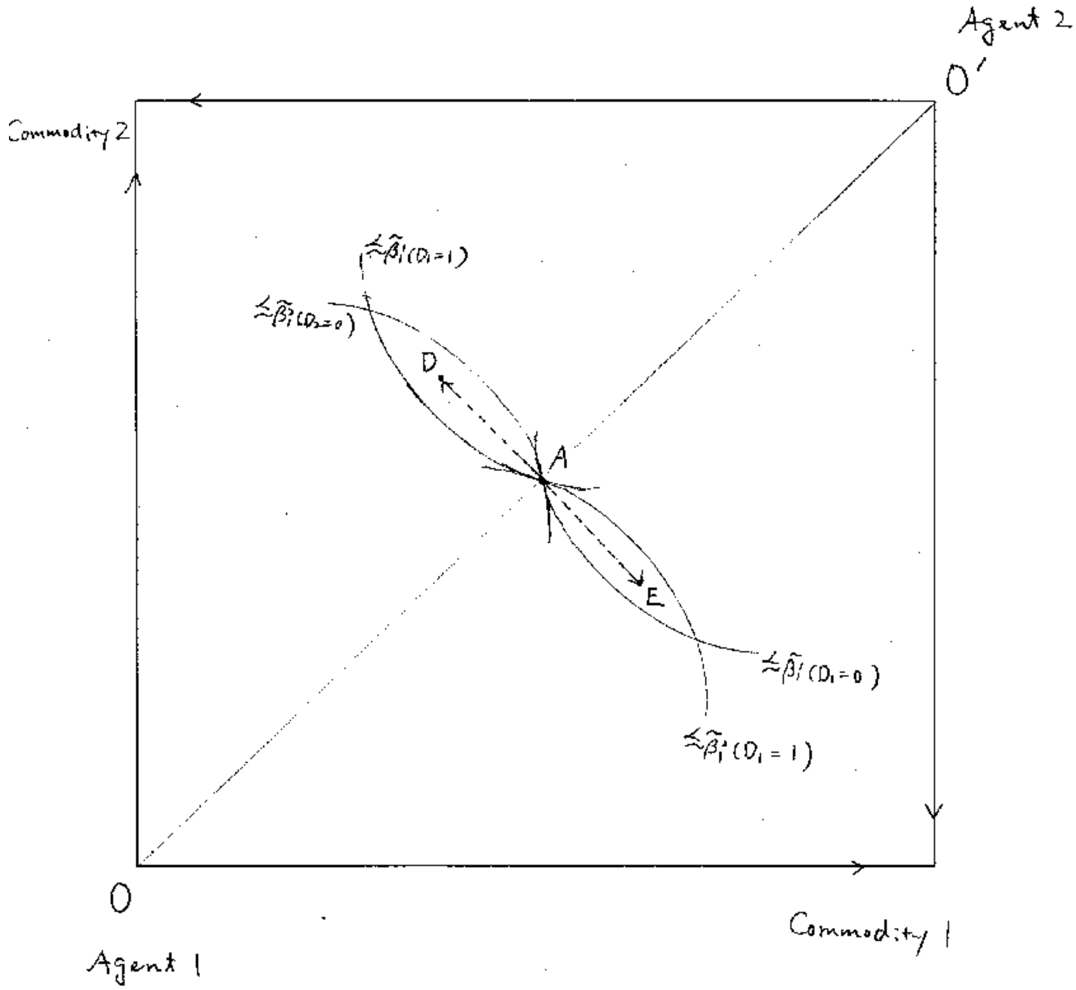


Figure 4(a)



Agent 2
O'
Commodity 2
$\precsim_{\tilde{\beta}_i}(D_1=1)$
$\precsim_{\tilde{\beta}_i}(D_2=0)$
D
A
E
$\precsim_{\tilde{\beta}_i}(D_1=0)$
$\precsim_{\tilde{\beta}_i}(D_1=1)$
O
Agent 1
Commodity 1

Figure 4(b)

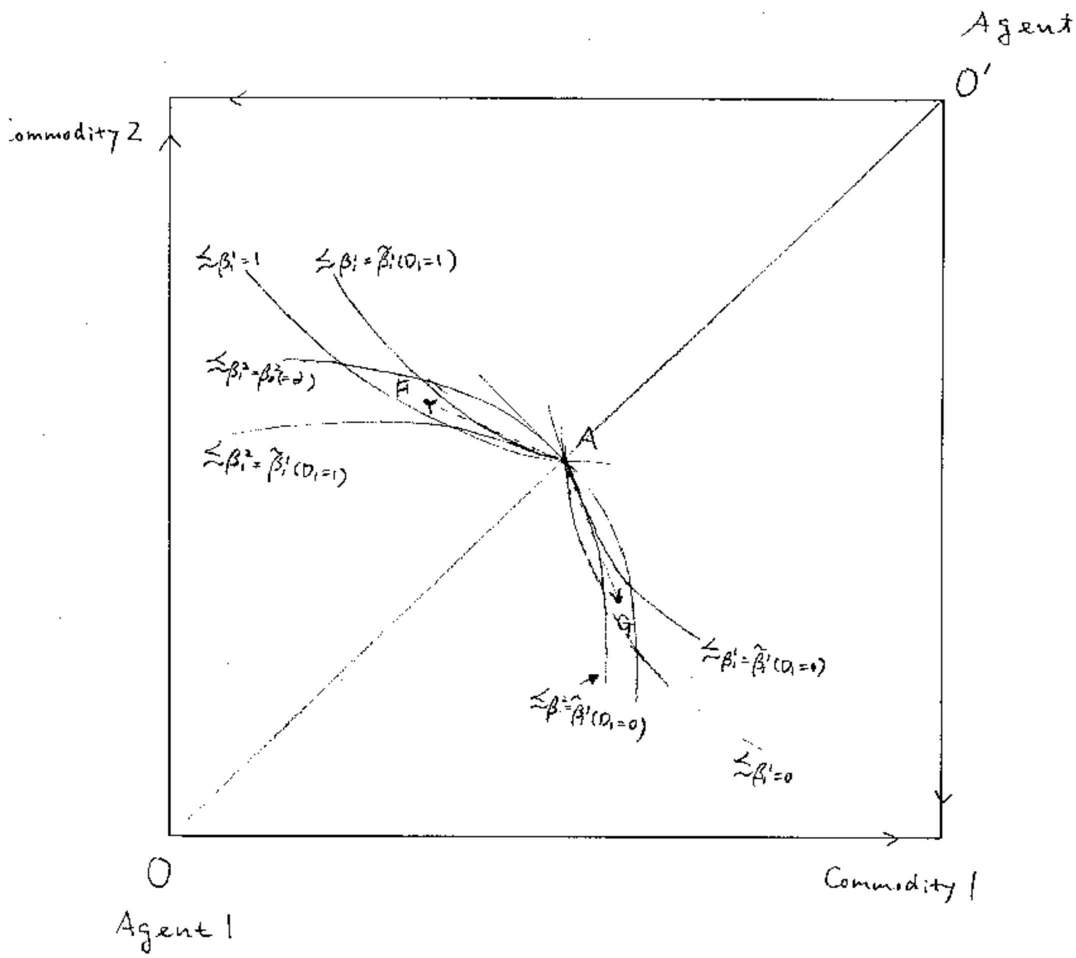


Figure 4(c)

"Rational" overreaction

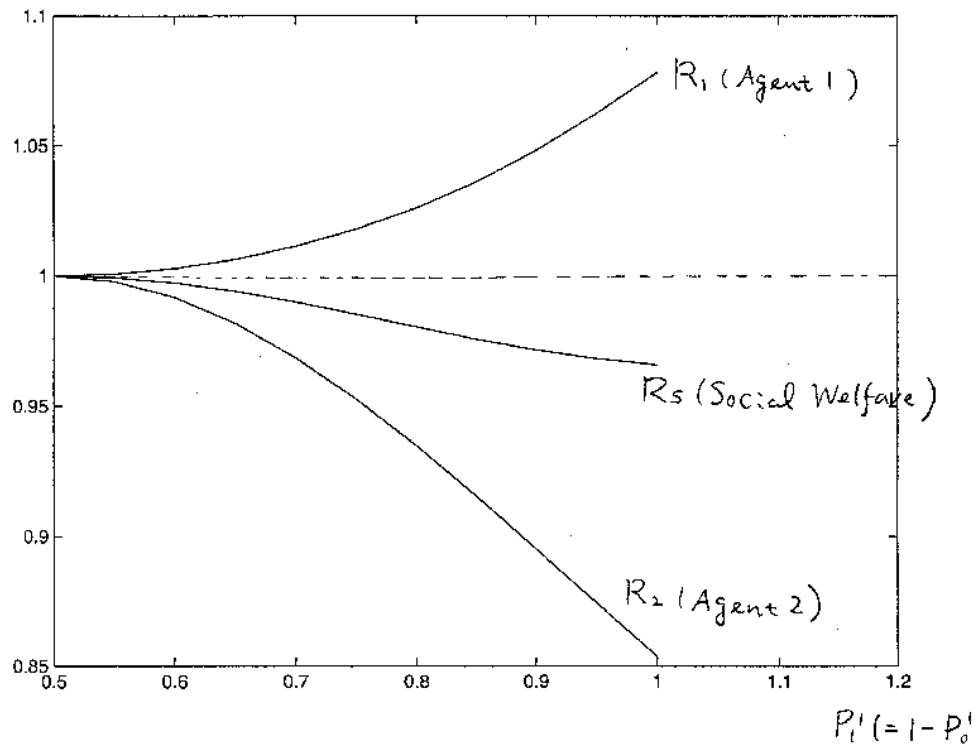


Figure 5(a)

Ex-ante welfare ratios after to before "Bayesian" speculation

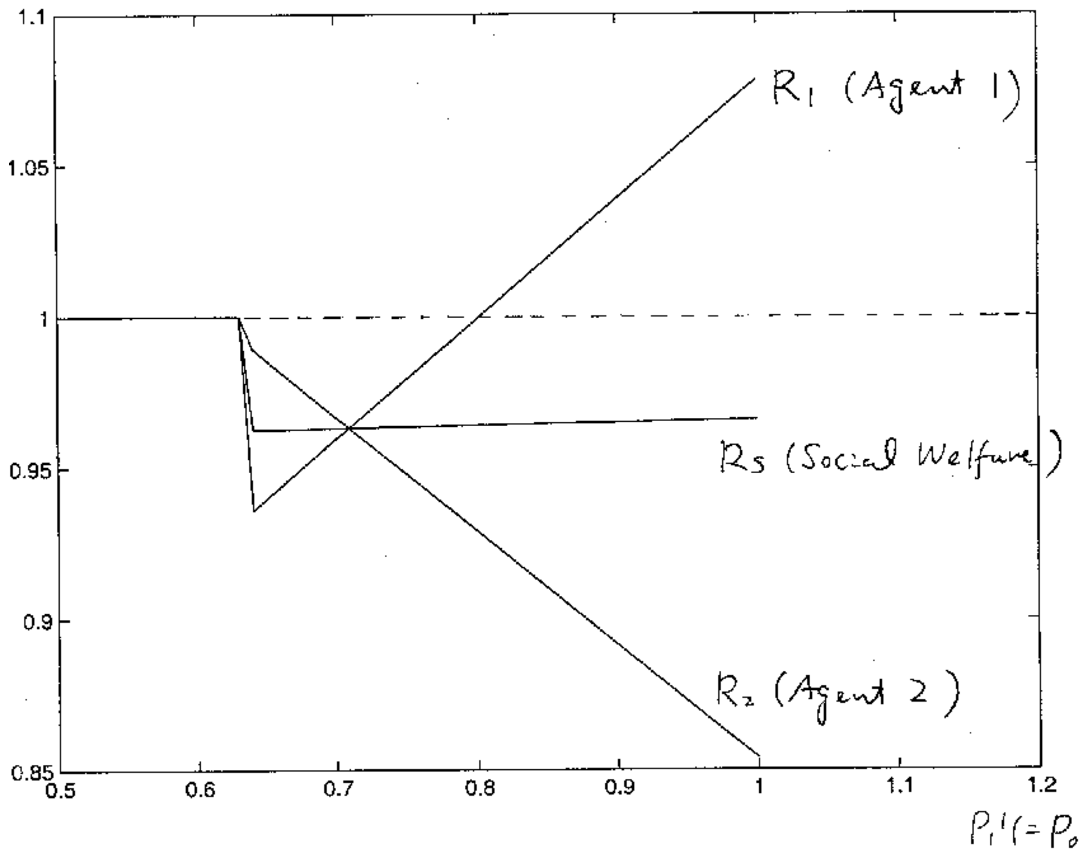


Figure 5(b)

Ex-ante welfare ratios after to before "rational" overreaction

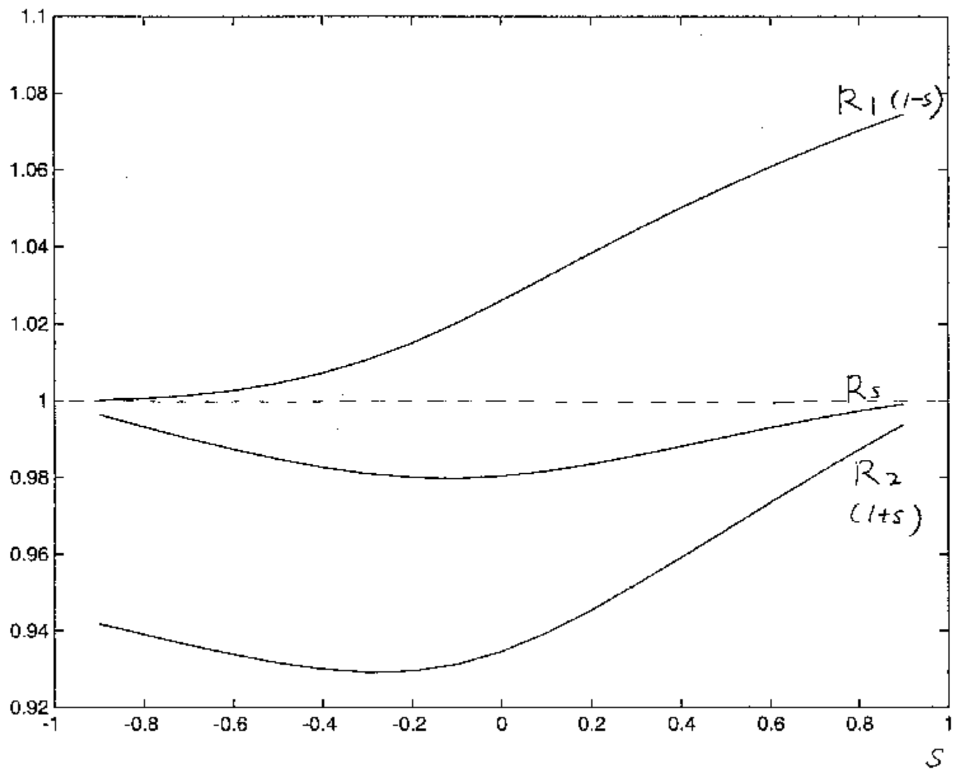

1.1
1.08
1.06
1.04
1.02
1
0.98
0.96
0.94
0.92
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
S
R1 (1-s)
Rs
R2 (1+s)

Figure 6(a)(i)

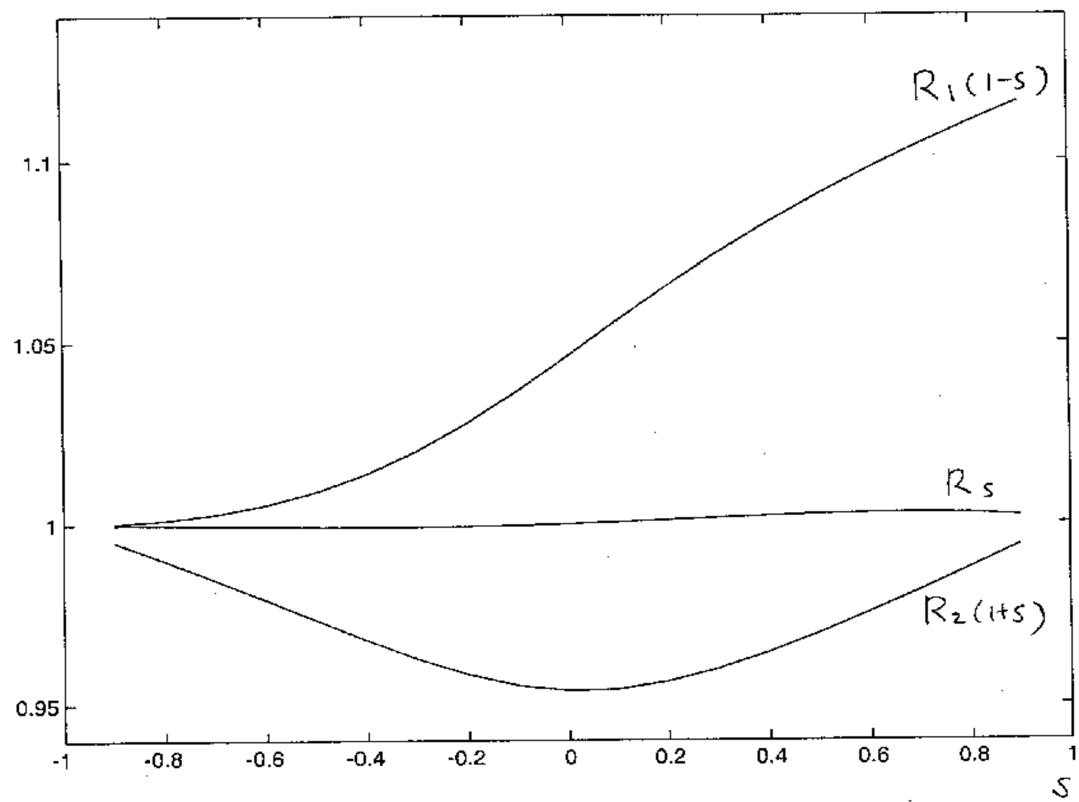


Figure 6(a)(ii)

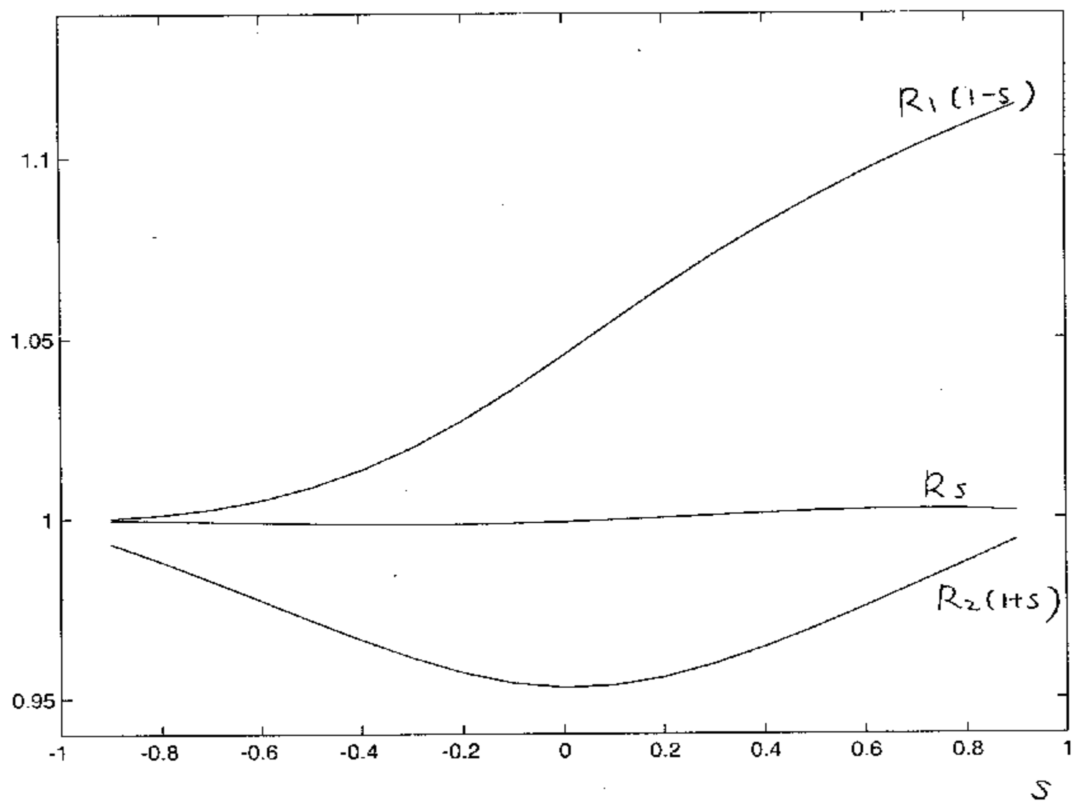


Figure 6(a)(iii)

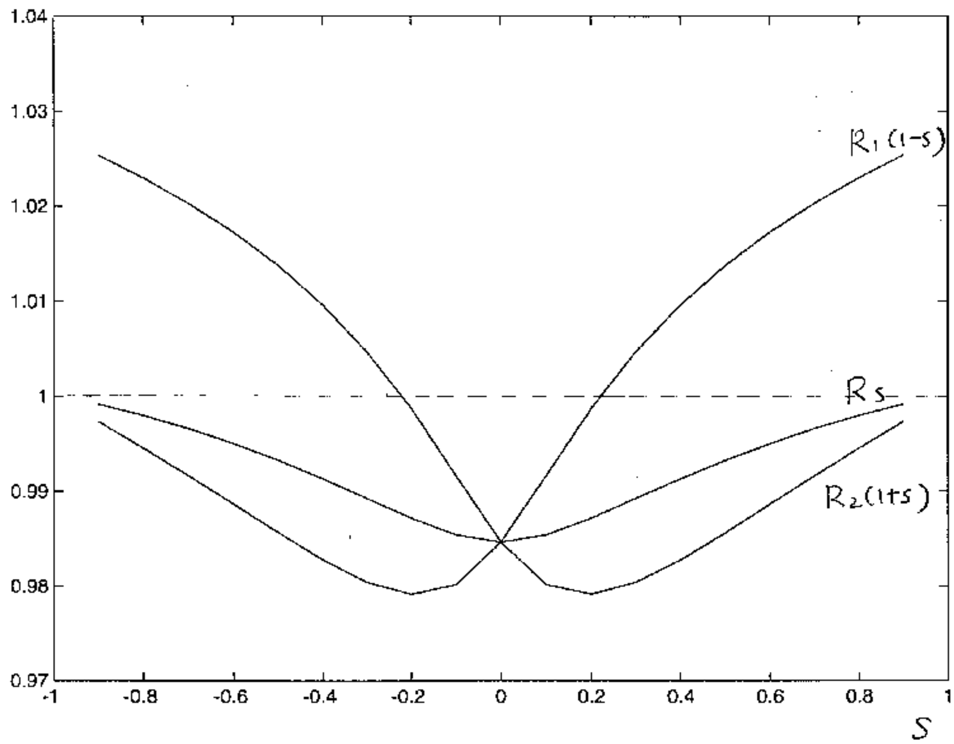

1.04
1.03
1.02
1.01
1
0.99
0.98
0.97
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
R1(1-s)
Rs
R2(1+s)
S

Figure 6(b)(i)

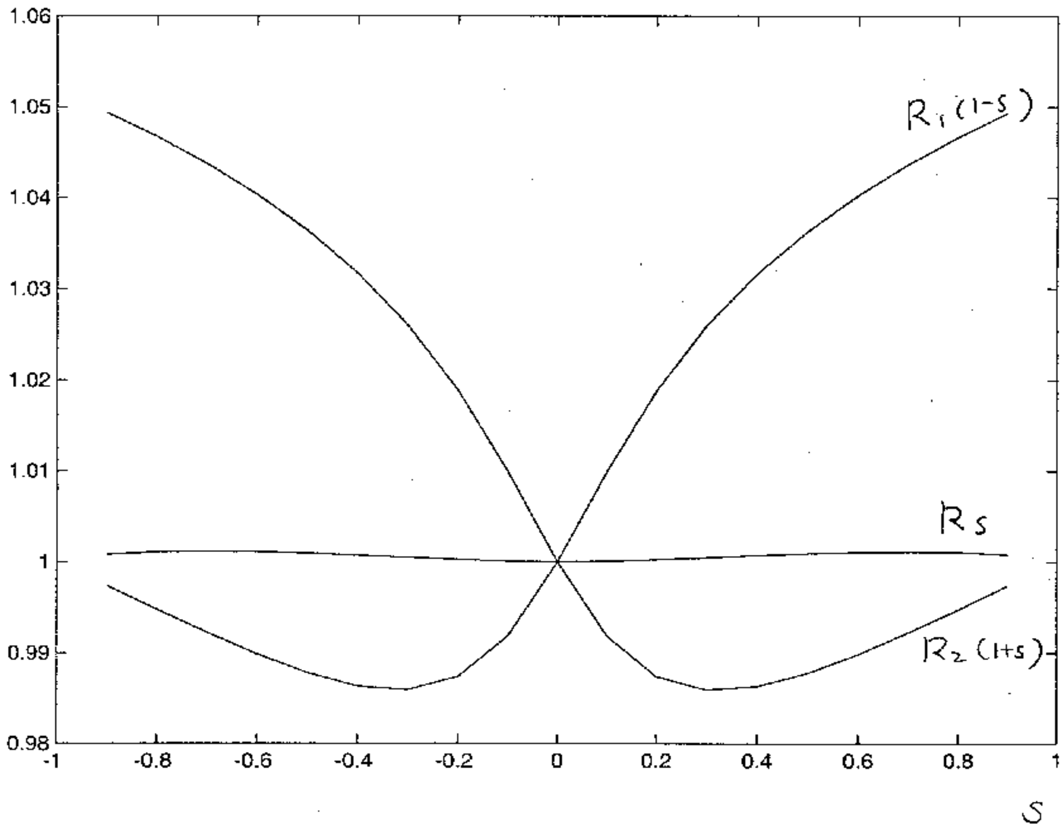

1.06
1.05
1.04
1.03
1.02
1.01
1
0.99
0.98
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
R1 (1-s)
Rs
R2 (1+s)
s

Figure 6(b)(ii)

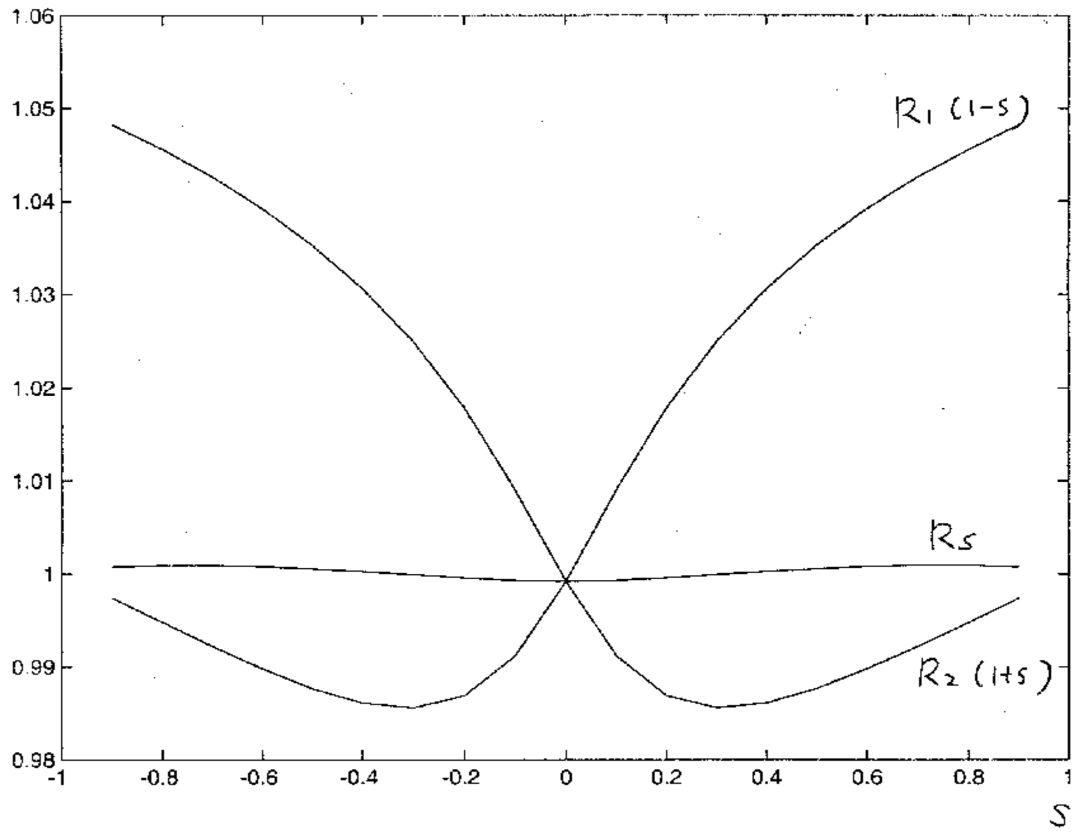


Figure 6(b)(iii)

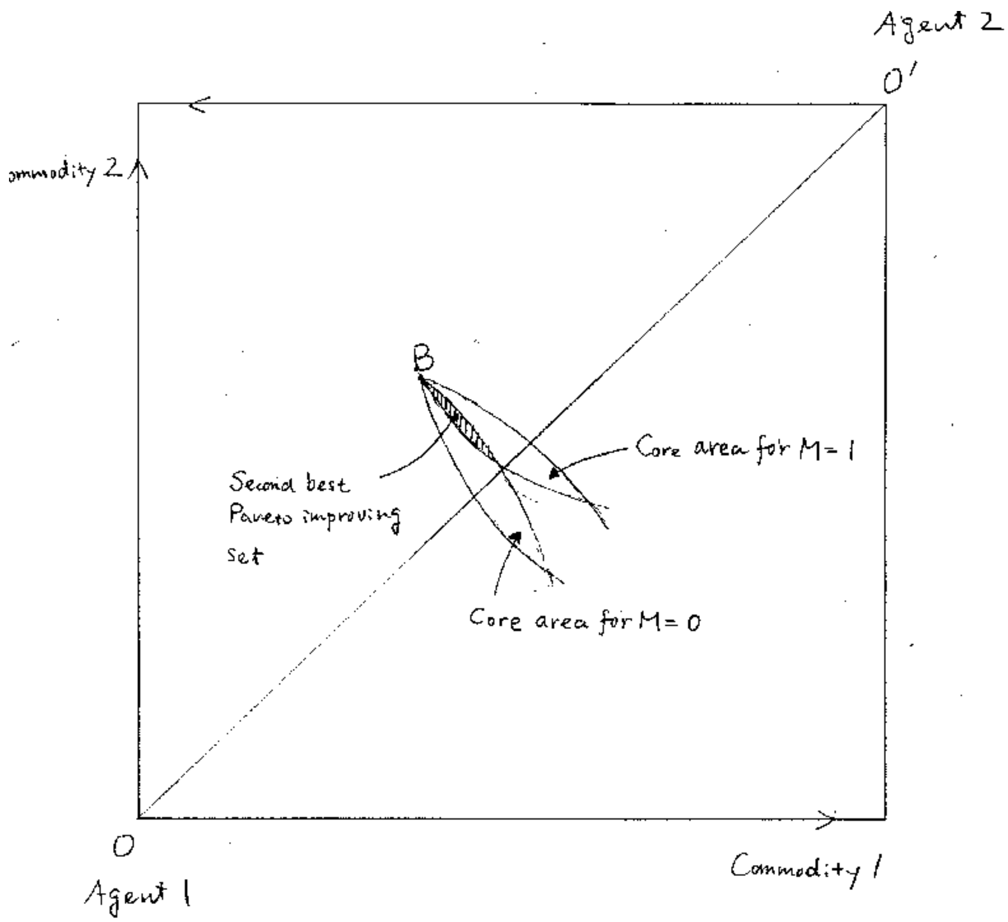


Figure 7

Second best Pareto improving allocations

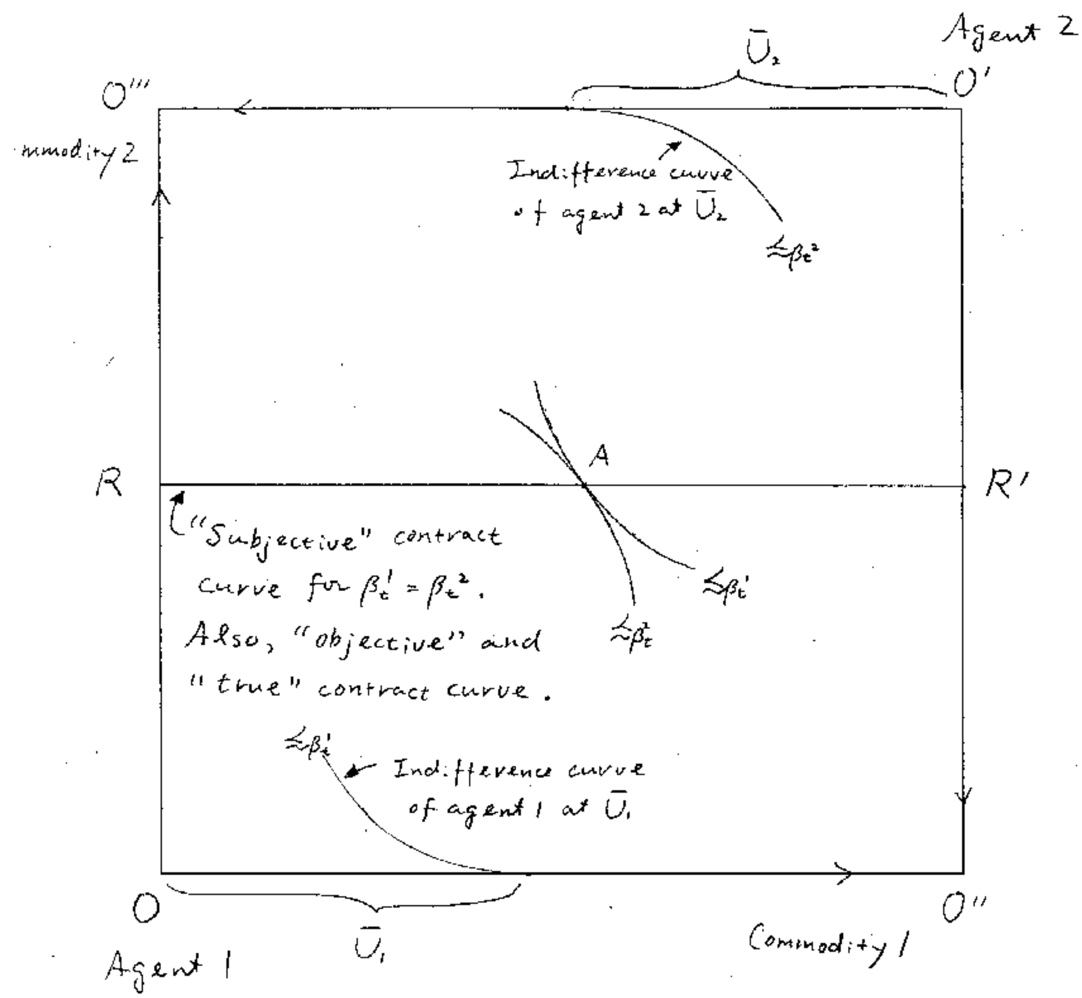


Figure 8(a)

Contract curve with quasi-linear utility

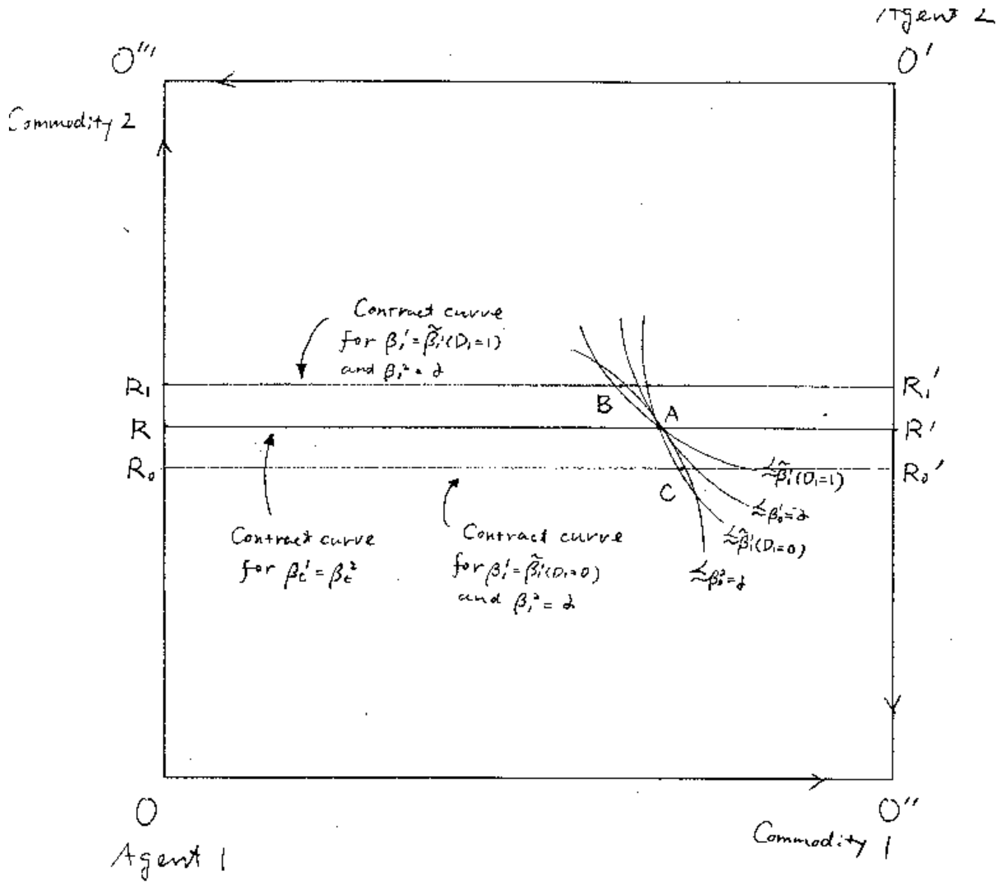


Figure 8(b)

Equilibrium transition with quasi-linear utility

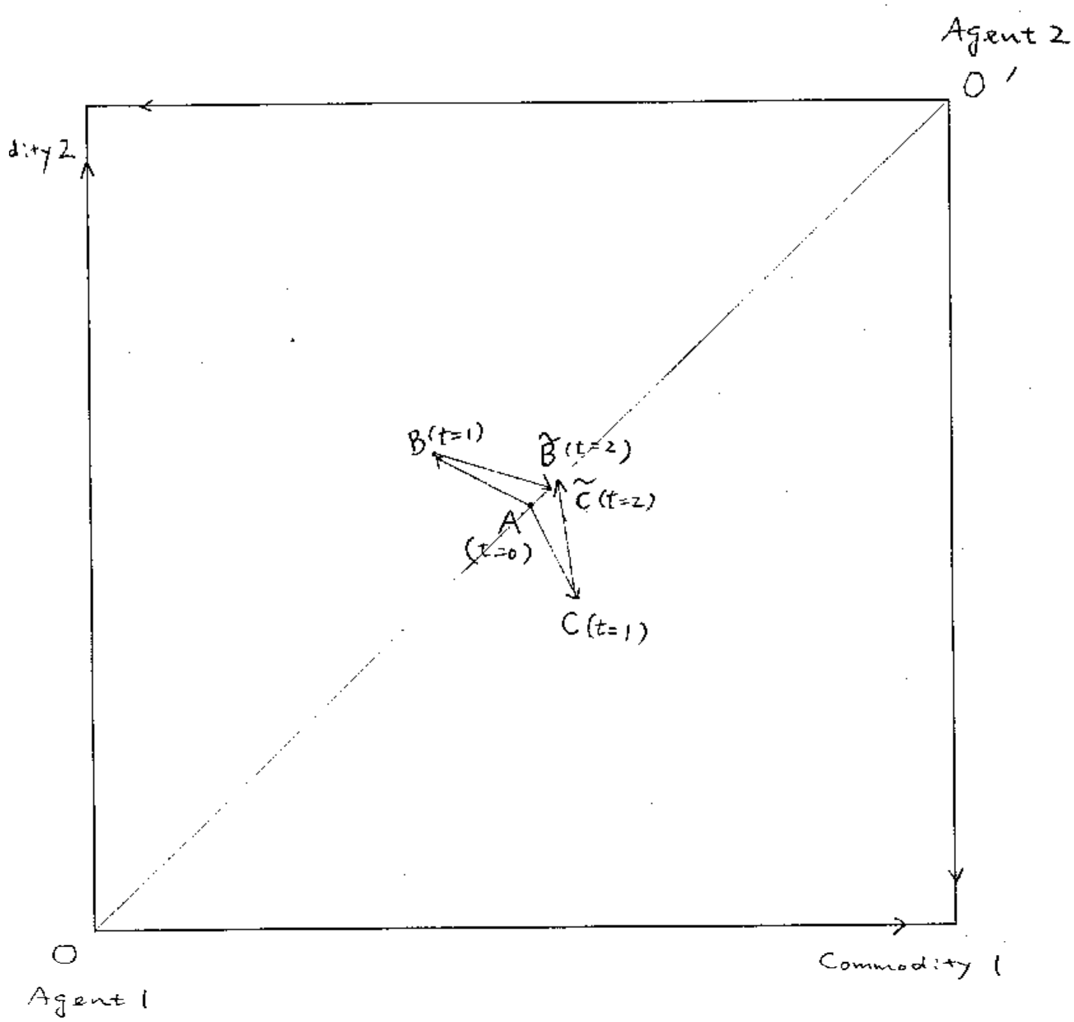


Figure A-1

Equilibrium transition in case (a)(ii)

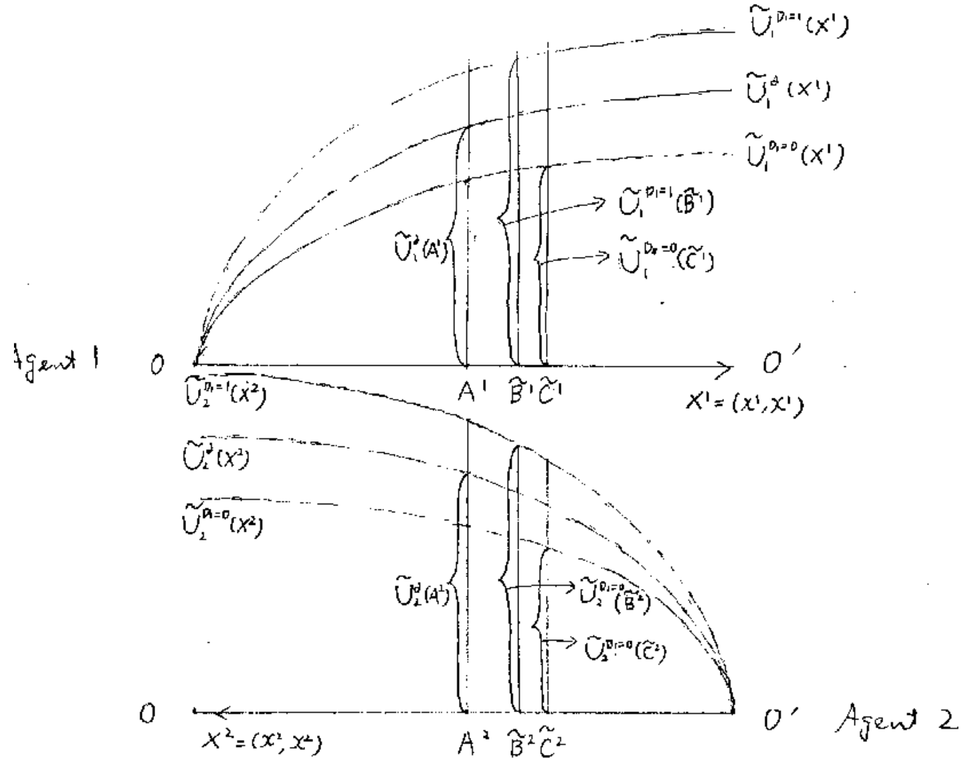


Figure A-2

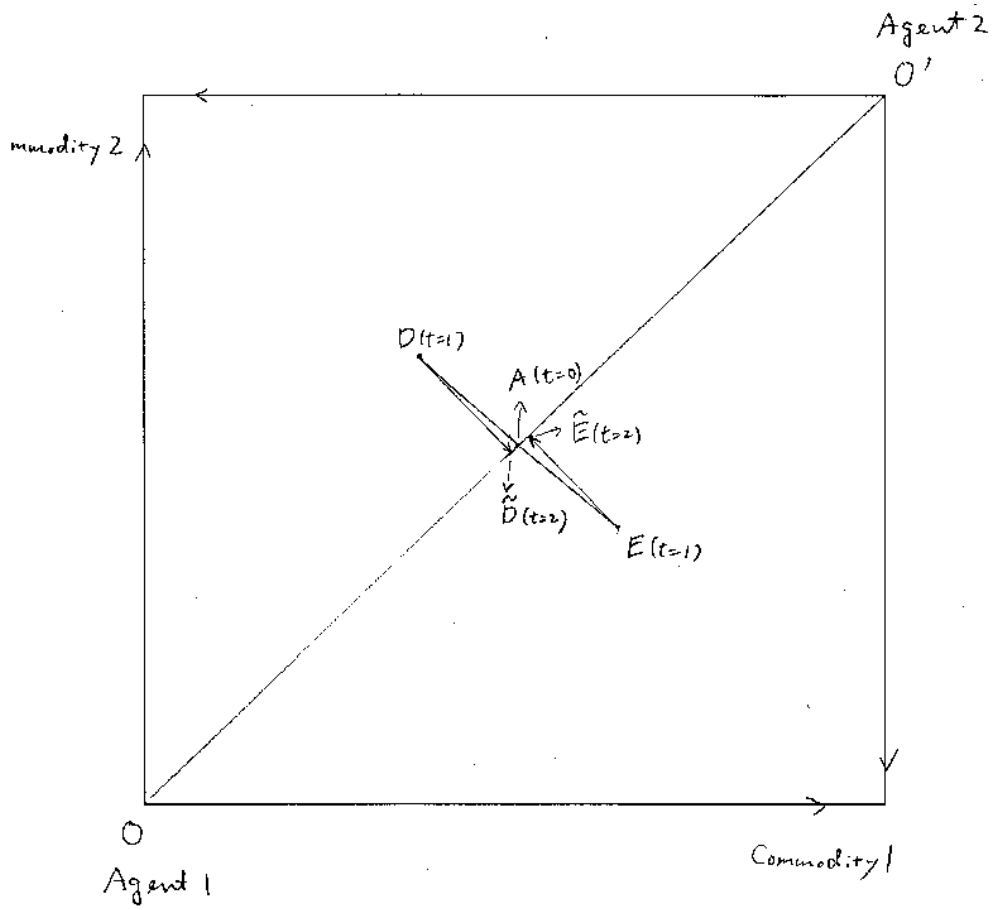


Figure A-3

Equilibrium transition in case (b)(ii)

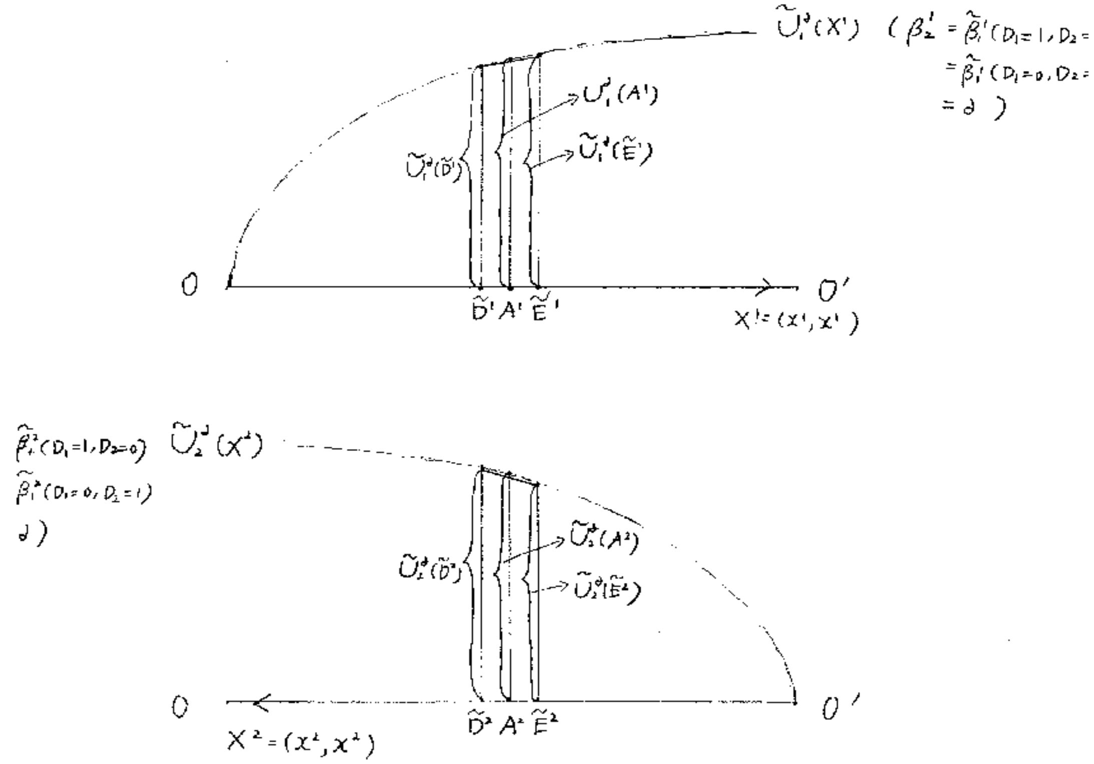


Figure A-4

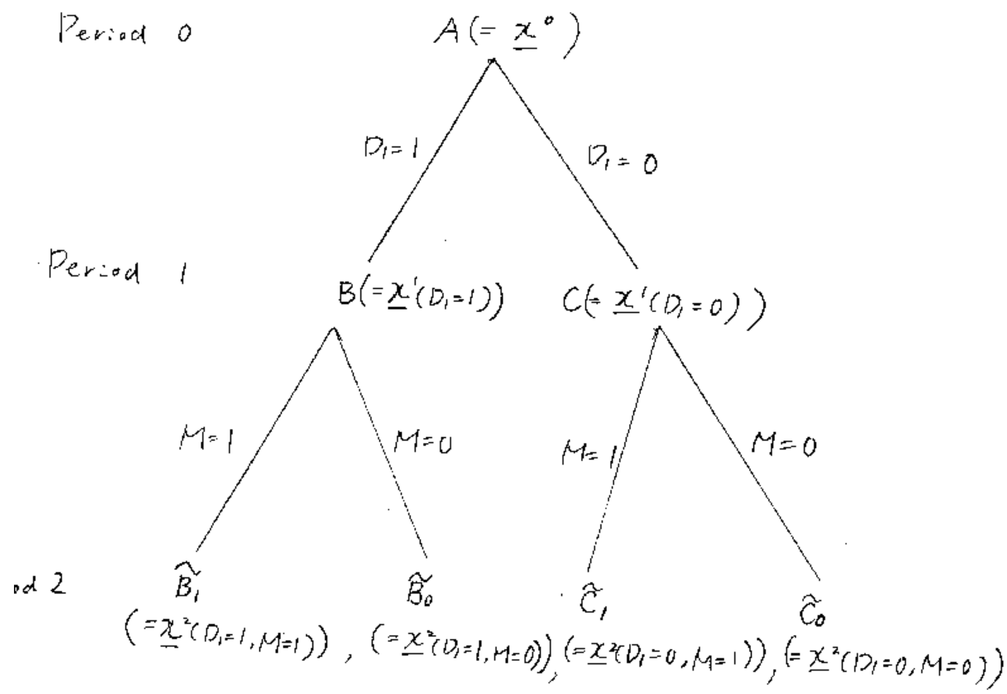


Figure A-5(a)

Equilibrium transition in case (a)(iii)

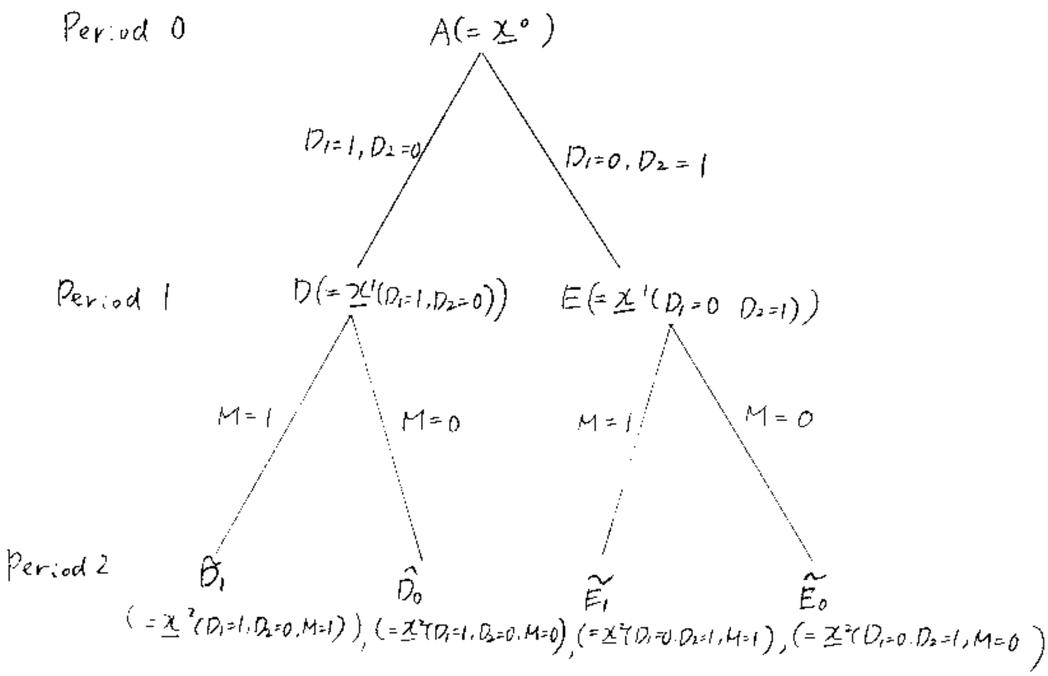


Figure A-5(b)

Equilibrium transition in case (b)(iii)

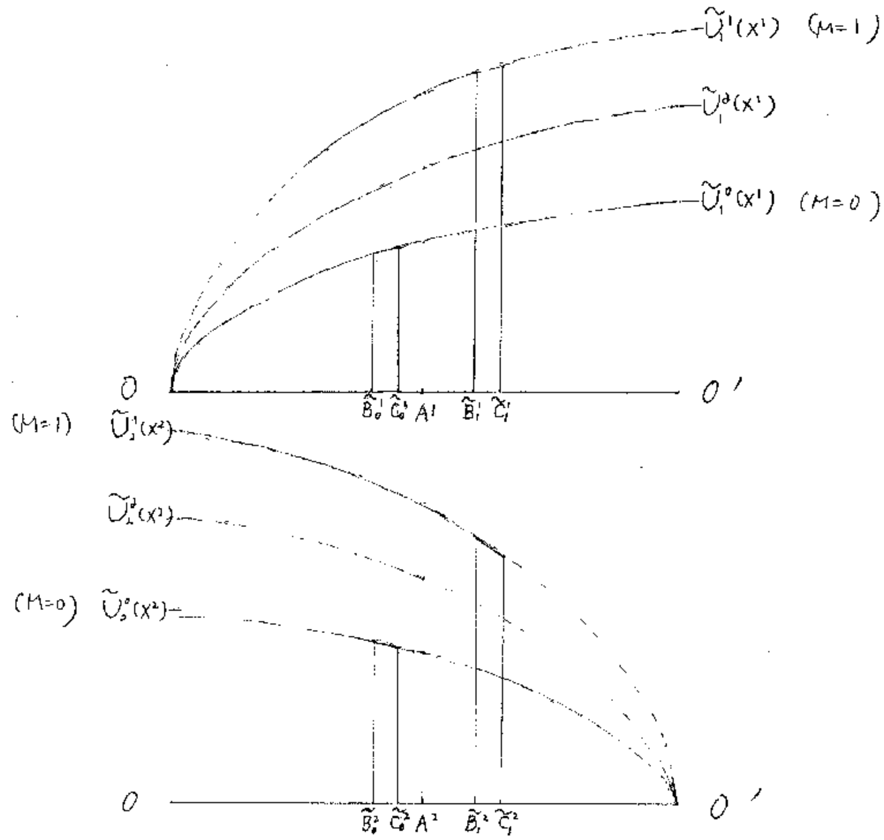

$\tilde{U}_1^1(x^1)$ (M=1)
$\tilde{U}_1^a(x^1)$
$\tilde{U}_1^0(x^1)$ (M=0)
O
O'
$\hat{B}_0^1$ $\hat{C}_0^1$ A^1 $\hat{B}_1^1$ $\hat{C}_1^1$
(M=1) $\tilde{U}_2^1(x^2)$
$\tilde{U}_2^a(x^2)$
(M=0) $\tilde{U}_2^0(x^2)$
O
O'
$\hat{B}_0^2$ $\hat{C}_0^2$ A^2 $\hat{B}_1^2$ $\hat{C}_1^2$

Figure A-6