



Munich Personal RePEc Archive

## **Monetary policy rules and indterminacy**

Sosunov, Kirill and Khramov, Vadim

New Economic School, Higher School of Economics

June 2008

Online at <http://mpra.ub.uni-muenchen.de/11996/>

MPRA Paper No. 11996, posted 07. December 2008 / 21:42

ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ  
ВЫСШАЯ ШКОЛА ЭКОНОМИКИ

*Vadim Khramov, Kirill Sosunov*

MONETARY POLICY RULES  
AND INDETERMINACY

Препринт WP12/2008/01  
Серия WP12

Научные доклады лаборатории  
макрэкономического анализа

Москва  
ГУ ВШЭ  
2008

Редактор серии WP12

«Научные доклады лаборатории макроэкономического анализа»

Л.Л. Любимов

K42 Khramov V., Sosunov K. **Monetary Policy Rules and Indeterminacy**: Working paper WP12/2008/01. — Moscow: State University — Higher School of Economics, 2008. — 24 p.

In recent papers it is shown that in the presence of price stickiness, investment and capital accumulation activity, active monetary policy (MP) rules can lead to indeterminacy under various assumptions about the structure of the model.

We analyze the conditions for real indeterminacy to occur in the model with capital accumulation. The key assumption is that we add response to output to the monetary policy rule. In our paper we show that adding Current or Expected Output to MP rule substantially changes the conditions for real indeterminacy to occur.

In contrast to some existing research we show that under current-looking with respect to output MP rules indeterminacy is almost impossible; under forward-looking with respect to output MP rules indeterminacy is almost impossible under active MP rules and very likely to occur under passive MP rules. We also show that stability conditions are almost not sensitive to changes in capital share in output and aggregate markup. We provide the nominal determinacy analysis and show that active and forward-looking MP rules with respect to output give better results in stabilizing the economy.

УДК 330.11  
ББК 65.012.2

*Keywords:* Indeterminacy, Monetary Policy Rules, Capital Accumulation Activity.

*JEL codes:* E52, E31, E22.

*Vadim Khramov* — Laboratory for Macroeconomics Analysis, State University — Higher School of Economics, Moscow, Russia; E-mail: vkhramov@mail.ru

*Kirill Sosunov* — Laboratory for Macroeconomics Analysis, State University — Higher School of Economics, Moscow, Russia; E-mail: ksosunov@nes.ru

**Храмов В., Сосунов К. Монетарные правила и множественность равновесий:** Препринт WP12/2008/01. М.: Изд. дом ГУ ВШЭ, 2008. — 24 с. (in English)

В недавних исследованиях было показано, что в условиях жесткости цен при накоплении капитала активная монетарная политика может приводить к множественности равновесий при разных вариантах калибровки моделей экономики.

Мы анализируем условия, при которых может возникать множественность равновесий в моделях с накоплением капитала. Ключевым отличием нашей работы является добавление реакции процентной ставки на выпуск в монетарном правиле. В работе показано, что добавление реакции на текущий или ожидаемый будущий выпуск в экономике значительно изменяет условия устойчивости, при которых может возникать множественность равновесий.

В противовес к существующим исследованиям мы показываем, что при монетарном правиле, в котором процентная ставка реагирует на текущий выпуск, множественность равновесий невозможна при реалистичных параметрах экономики; при наличии реакции на будущий выпуск множественность равновесий вероятна только при пассивных монетарных правилах.

Препринты ГУ ВШЭ размещаются на сайте:  
<http://new.hse.ru/C3/C18/preprintsID/default.aspx>.

© V. Khramov, 2008  
© K. Sosunov, 2008  
© Оформление. Издательский дом ГУ ВШЭ, 2008

## Introduction<sup>1</sup>

Since John Taylor (1993) estimated the Federal Reserve monetary policy rule, there has been an increasing interest in building the model and the explanation of the efficiency of “active monetary policy rules”, that is a policy that respond to an increase in inflation more than one-to-one increase in nominal interest rate. The basic idea is that in order to get a unique stable solution, policymakers should use active monetary policy taking into account current inflation (Kerr and King, 1996) or future inflation expectations (Bernanke and Woodford, 1997; Clarida, Gali and Gertler, 2000).

The empirical confirmation was recently got in the paper Clarida, Gali and Gertler (2000). Authors estimate monetary policy rules in the US for the different periods of time and show that in the US monetary policy was passive during Pre-Volker period (1960–1979) and became active during Volcker-Greenspan period (1979–1996), arguing that active monetary policy rule gave much better results in stabilizing the economy.

The equilibrium determinacy under active monetary policy rules is showed in Kerr and King (1996), Rotemberg and Woodford (1998), Christiano and Gust (1999). But there are papers arguing that such policy cannot be sustainable.

In recent papers (especially, Duport, 2001; Carlstrom and Fuerts, 2005; Kurozumi and Zandweghe, 2007; and Benhabib and Eusepi, 2005; Sveen and Weinke, 2005; Kurozumi, 2006; Huang, Meng, 2007; Xiao, 2008) it is shown that in the presence of price stickiness, investment and capital accumulation activity, active monetary policy rules can lead to indeterminacy under various assumptions about the structure of the economy. In contrast to previous models, where the interest rate affects output only through the consumption and saving decision of the household, in these papers *investment decision* of the firm is added. And that is the *crucial assumption* that changes the stability structure of the model and makes indeterminacy likely to occur.

In seminal paper Duport (2001) builds a continuous-time model with a quadratic nominal price adjustment cost and endogenous capital accumula-

<sup>1</sup> The authors would like to thank Oleg Zamulin (New Economic School) and Dmitriy Stolyarov (University of Michigan) for valuable comments and suggestions.

tion. The author implies monetary policy only as a function of inflation and shows that the dynamics of the model demonstrates sunspot-equilibria under active monetary policy. The key channel of the indeterminacy is in the endogenous capital accumulation, especially in the no-arbitrage condition between the real return on bonds and the real return on capital. This implies that capital rental rate increases when monetary policy responds by increasing real interest rate to higher inflation, increasing the cost of renting capital and leading to cost-push inflation. So that inflation expectations become self-fulfilled.

The idea of endogenous capital accumulation gave birth to large academic research on whether interest rate rules may lead to local real indeterminacy of equilibrium. Carlstrom and Fuerts (2005) construct a discrete-time version of Duport's (2001) model and analyze necessary conditions for local indeterminacy under current-looking and forward-looking monetary policy rules only with respect to inflation, but not output. In endogenous capital accumulation environment real indeterminacy is very likely to occur, and in particular, they show that all forward-looking interest rate rules are subject to indeterminacy.

Kurozumi and Zandweghe (2007) modify Carlstrom and Fuerts (2005) model by introducing more complicated MP rule that responds also to output and contains interest rate smoothing. They show that nominal indeterminacy conditions are sufficiently dependent on the model calibration parameters, and sunspot equilibria is less likely to occur as long as the policy response to expected future inflation is sufficiently strong.

Kurozumi (2006) provides the indeterminacy analysis using different timings of money balances of the utility function. Huang, Meng (2007) add quadratic price adjustment cost to the utility function and show capital accumulation activity leads to macroeconomic instability. Xiao (2007) analyses indeterminacy conditions under increasing returns to scale. King and Wolman (2004) construct the model with dynamic complementarity between forward-looking private agents and a discretionary monetary authority and show how discretionary monetary policy can lead to multiple equilibria. Benhabib, Eusepi (2005) and Sveen, Weinke (2005) analyze the influence of including capital into the model on the firm's pricing decisions through the cost channel.

In the case of indeterminacy one can not exactly construct predictable monetary policy due to the fact that the economy is driven by the sunspot-shocks, and each trajectory converges to steady-state. Is this again a "peril of Taylor rules"? Was the Volcker-Greenspan-Bernanke monetary policy driven by the stochastic shocks? This is key questions and the field our paper contributes to.

Neither Duport's (2001), Carlstrom and Fuerts (2005) nor most recent papers do not introduce current or future expected output to MP rule and provide the real indeterminacy analysis and policy implications. We show that these conditions are quite significant.

We calibrate the standard version of Carlstrom and Fuerts (2005) model. The key difference is that we add response to output to the monetary policy rule. In contrast to Kurozumi and Zandweghe (2007) we analyze not nominal, but *real* indeterminacy conditions. Also we found that the calibration of model in Kurozumi and Zandweghe (2007) is inconsistent because of the incorrectly defined steady state shares of consumption and investment in output. In our paper we correct this problem and show that adding Current or Expected Output to MP rule substantially changes the conditions for real indeterminacy to occur.

In contrast to some existing research we show that under current-looking with respect to output MP rule real indeterminacy is almost not likely to occur under passive or active MP; under forward-looking with respect to output MP rule real indeterminacy is not likely to occur under active and likely to occur under passive MP rules.

We also show that monetary policy that takes into account output usually leads to nominal indeterminacy: situation at which the inflation rate is always indetermined. In this case monetary policy seems rather ineffective.

So, we provide the next step of our analysis: the nominal determinacy conditions that lead the economy to unique inflation rates. We show that active monetary policy that takes into account *current* output leads to nominal determinacy; under forward-looking monetary policy rules with respect to output nominal determinacy is impossible. The normative result is that active and forward-looking MP rules with respect to output give better results in stabilizing the economy.

The outline of the paper is as follows. In Section 2 we briefly discuss the sticky price model with capital accumulation in discrete time (as in Carlstrom and Fuerts (2005)) and construct monetary policy rule with interest rate smoothing and response to output. In Section 3 we analyze the dynamic properties of the log-linearized around the steady-state model. In Section 4 we provide the real indeterminacy analysis for current and forward-looking monetary policy rules with respect to output. In Section 5 we provide the analysis of nominal determinacy conditions and the efficiency of monetary policy. In Section 5 we conclude.

## 2. The Model

We follow Yun (1998), Carlstrom and Fuerst (2005) and construct the model of general equilibrium with sticky prices in discrete time. We also introduce monetary policy rule as in Kurozumi and Zandweghe (2007).

### 2.1. Households

The economy consist of large number of households, those seek to maximize their life-time utility function:

$$\sum_{t=0}^{\infty} \beta^t U(C_t, \frac{M_{t+1}}{P_t}, 1 - L_t) \quad (1)$$

where  $\beta$  is the discount rate,  $C_t$  is consumption,  $M_{t+1}$  nominal money holding and the beginning of the period  $(t+1)$ ,  $\frac{M_{t+1}}{P_t}$  is real money balances<sup>2</sup>,  $(1 - L_t)$  – leisure amount.

We specify the utility function separable by leisure in the following form:

$$U(C_t, \frac{M_{t+1}}{P_t}, 1 - L_t) = \ln C_t + \psi \ln \frac{M_{t+1}}{P_t} + \frac{(1 - L_t)^{1-\chi}}{1-\chi} \quad (2)$$

Each households maximizes (1) by  $C_t$ ,  $M_{t+1}$  and  $L_t$  subject to intertemporal budget constraint:

$$\begin{aligned} M_{t+1} + B_t + P_t C_t + P_t K_{t+1} &= \\ = M_t + T_t + B_{t-1} R_{t-1} + P_t \{w_t L_t + [r_t + (1 - \delta)]K_t\} + \Pi_t \end{aligned} \quad (3)$$

At the beginning of each period  $t$  household have  $M_t$  cash balances and  $B_{t-1}$  nominal bonds. Household starts the period  $t$  by trading bonds and receiving a lump-sum monetary transfer  $T_t$  from the government. Household receives interest payment on bonds  $B_{t-1}$  with gross interest rate  $R_{t-1}$  and spend money on new bonds  $B_t$ . Households also receive real factor payments from labor market  $w_t L_t$ , capital market  $[r_t + (1 - \delta)]K_t$  and get firms's profits  $\Pi_t$ , spending

<sup>2</sup> As in recent papers we introduce end-of-period money holdings to be consistent with Dupor's continuous-time analysis (for discussion see Carlstrom, Fuerst (2005)).

money on capital in the new period  $K_{t+1}$  and current consumption  $C_t$  at current prices  $P_t$ .

Later we introduce utility function linear by labor (by putting  $\chi = 0$ ) and analyze the case of infinite labor supply elasticity<sup>3</sup>.

The first order conditions for the household's maximization problem are the following:

$$\frac{U_c}{U_L} = -\frac{1}{w} \quad (4)$$

$$U_c(t) = \beta \{U_c(t+1)[r_{t+1} + (1 - \delta)]\} \quad (5)$$

$$\frac{U_L(t)}{P_t} = \beta R_t \left( \frac{U_c(t+1)}{P_{t+1}} \right) \quad (6)$$

$$\frac{U_m(t)}{U_c(t)} = \frac{R_t - 1}{R_t} \quad (7)$$

Equation (4) is standard consumption-labor optimal choice with respect to their prices. Equation (5) is Euler equation of consumption dynamics. Equation (6) is the Fisher equation that connects inflation and interest rates. Equation (7) is the money demand.

### 2.2. Firms

Firms are monopolistic competitors in the intermediate good market. The final output  $Y_t$  is produced from intermediate goods  $y_t(i)$  by Dixit-Stiglitz (1977) technology:

$$Y_t = \left\{ \int_0^1 [y_t(i)]^{\frac{\eta-1}{\eta}} di \right\}^{\frac{\eta}{\eta-1}} \quad (8)$$

The corresponding demand for intermediate good possesses constant price elasticity  $\eta$ :

<sup>3</sup> Carlstrom and Fuerst (2000, 2001) show that the indeterminacy conditions are independent of labor supply elasticity.

$$y_t(i) = Y_t \left( \frac{P_t(i)}{P_t} \right)^{-\eta} \quad (9)$$

where  $P_t(i)$  is the price of intermediate good and  $P_t$  is the price of final good.

Production function of each firm exhibits constant returns to scale:

$$f(K, L) = K^\alpha L^{1-\alpha} \quad (10)$$

The first order conditions for cost minimization problem are:

$$r_t = z_t f_K(K_t, L_t) \quad (11)$$

$$w_t = z_t f_L(K_t, L_t) \quad (12)$$

where  $z_t$  is the marginal costs of production.

Taking into account the Cobb-Douglas production function (10), the FOC's take the form:

$$r_t = \alpha z_t Y_t / K_t \quad (13)$$

$$w_t = (1 - \alpha) z_t (Y_t / K_t)^{\frac{-\alpha}{1-\alpha}} \quad (14)$$

We use Calvo (1983) staggered pricing model at the economy level. Each period fraction  $(1 - \nu)$  of firms gets a signal to set a new price. So, each firm maximizes the sum of discounted profits taking into account the probability of changing its price. The optimizations problem of such firm takes the form:

$$\sum_{j=0}^{\infty} \left( \frac{\nu}{\prod_{i=0}^j R_i} \right) \left[ \left( \frac{P_t(i)}{P_t} \right)^{-\eta} Y_t \left( \frac{P_t(i)}{P_t} - z_t \right) \right] \rightarrow \max_{P_t(i)} \quad (15)$$

The profit maximization conditions give a New Keynesian Phillips Curve (see Galf, Gertler (1998) and Clarida, Gali, Gertler (1999) for details), which is discussed later.

### 2.3. Monetary policy rule

According to the empirical estimates of Clarida, Gali, Gertler (1999) we assume that the monetary policy should react to inflation and output using interest rate smoothing. We follow Kurozumi and Zandweghe (2007) and use a generalized version of a monetary policy rule:

$$R_t = (R_{t-1})^{\phi_R} \left[ R \left( \frac{E_t \pi_{t+1}}{\pi} \right)^{\phi_\pi} \left( \frac{E_t Y_{t+j}}{Y} \right)^{\phi_Y} \right]^{(1-\phi_R)} \quad (16)$$

where  $R$ ,  $\pi$ ,  $Y$  are the steady state values of the interest rate, inflation and output.  $\phi_\pi$  and  $\phi_Y$  are elasticity parameters of the interest rate with respect to the expected inflation and output correspondingly. By introducing  $j = 0$  we get current-looking MP rule with respect to output; and  $j = 1$  — forward-looking MP rule with respect to output. We also use interest rate smoothing with weights  $\phi_R$  and  $(1 - \phi_R)$  to previous period interest rate and response to inflation and output correspondingly. If we use  $\phi_R = 0$  and  $\phi_\pi = 0$  we get standard Carlstrom and Fuerst (2005) model in discrete time, and the analog of continuous time Duport's (2001) model.

We say that monetary policy rule is active, if the nominal interest rate increases more that one-to-one with inflation ( $\phi_\pi > 1$ ), otherwise we call it passive.

Money supply is endogenous in this model and as in recent papers we assume that Ricardian equivalence holds. So we do not analyze the hidden government budget constraint and the equation for the evolution of government debt. Indeterminacy analysis for non-Ricardian fiscal policy is presented at Benhabib, Schmitt-Grohe, Uribe (2001a).

## 3. Dynamics of the model

The dynamics of the model is presented by the system of the log-linearized FOCs for households, firms and the monetary policy rule around the steady-state:

$$R_t - \pi_{t+1} = -(E_t C_{t+1} - C_t), \quad (17)$$

$$R_t - \pi_{t+1} = [1 - \beta(1 - \delta)](E_t z_{t+1} + EY_{t+1} - K_{t+1}), \quad (18)$$

$$C_t = z_t + \frac{\alpha}{1-\alpha} (K_t - Y_t), \quad (19)$$

$$K_{t+1} = (1-\delta)K_t + \delta I_t, \quad (20)$$

$$Y_t = s_C C_t + s_I I_t, \quad (21)$$

$$\pi_t = \beta E_t \pi_{t+1} + \lambda z_t. \quad (22)$$

$$R_t = \varphi_R R_{t-1} + (1-\varphi_R)(\varphi_\pi E_t \pi_{t+1} + \varphi_Y E_t Y_{t+j}). \quad (23)$$

Equation (17) is the Euler equation for households' dynamic optimization problem. Equation (18) is the Fisher relation between the nominal interest rate, expected future and real interest rate, where the last is determined in the production sector. Equation (19) is the wage-equilibrium relation of the log-linearized equation (4) and (14). Equation (20) is the capital accumulation relation with depreciation rate  $\delta$ . Equation (21) is the division of the output between consumption and investment with shares in output  $s_C, s_I$  correspondingly dependent on the parameters of the model and calculated at the steady-state. Equation (22) is New Keynesian Phillips Curve derived from Calvo (1983) staggered-pricing model, where  $\lambda = \frac{(1-\nu)(1-\beta\nu)}{\nu}$  is the real marginal elasticity of inflation. Equation (23) is the log-linearized monetary policy rule (16).

#### 4. Indeterminacy analysis

In this part of the paper we provide the indeterminacy analysis for the current- and forward-looking monetary policy rules with respect to output.

Equations (17)–(23) are the first-order dynamic system of equations. The variables  $\pi_t, C_t, Y_t$  are non-predetermined, and  $K_t, R_{t-1}$  are predetermined. According to Blanchard-Kahn (1980) conditions we get indeterminacy if the number of stable roots (eigenvalues of the matrix of the system which are less than one) is more than two in our case (number of predetermined variables). In this case we get a *nominal indeterminacy* as inflation rate is still indetermined, simply because there exists an infinite number of initial price levels consistent with a perfect-foresight equilibrium.

We are interested in a *real indeterminacy*, situation in which the behavior of one or more real variables is not pinned down by the model and leads to multiple (sunspot) equilibria<sup>4</sup>. In order to get real indeterminacy we need one more stable root in addition to Blanchard-Kahn (1980) conditions.

In our case equations (17)–(23) represent the system for five variables, the three of them are non-predetermined, and two are predetermined.

The stability conditions are as follows:

1. In the case of two stable roots (out of five possible) we get nominal determinacy;

2. In the case of three stable roots we get nominal indeterminacy;

3. In the case of four or five stable roots we get real indeterminacy.

Interest rate smoothing leads to the case that variable  $R_{t-1}$  is predetermined.

As  $R_{t-1}$  appears only in the MP rule equation, it always leads to additional stable root in the system. Interest rate smoothing assumption does not influence the stability conditions. So, during numerical simulations we simply use  $\varphi_R = 0$ .

We are mainly interested in the elasticities of interest rate with respect to expected inflation ( $\varphi_\pi$ ) and output ( $\varphi_Y$ ) in the intervals  $\varphi_\pi = [0.5; 2.5]$  and  $\varphi_Y = [0.2; 0.7]$ <sup>5</sup>.

We analyze the stability of the model under the baseline calibration used in recent papers (Table 1).

Table 1. Baseline calibration of the model

Parameters of the model		Calibration value
$\beta$	Discount factor	0.99
$1-\nu$	Fraction of firms that get a signal to set a new price	0.477
$\alpha$	Capital share	1/3
$\psi$	Degree of non-separability of utility function between consumption and real balances	1
$\delta$	Depreciation rate of capital	0,02
$\varphi_R$	Interest rate smoothing parameter	0

<sup>4</sup> The detailed analysis of conditions for nominal and real indeterminacy is presented in Benhabib, Schmitt-Grohe, Uribe (2001a).

<sup>5</sup> According to simple Taylor rule and empirical estimates of Clarida, Gali and Gertler (2000).



Table 1

Parameters of the model		Calibration value
$\varphi_\pi$	Elasticity of interest rate with respect to expected inflation	0.5
$\varphi_Y$	Elasticity of interest rate with respect to output	1.5
$\frac{P}{z} = \frac{\eta}{\eta-1}$	Steady state markup	1.1
$\chi$	Inverse labor supply elasticity	0

**4.1. Current-looking monetary policy rule with respect to output**

Under various calibrations of the model we analyze the conditions for the real indeterminacy to occur. Under current-looking monetary policy rule with respect to output the monetary policy rule takes the form:

$$R_t = \varphi_R R_{t-1} + (1 - \varphi_R)(\varphi_\pi E_t \pi_{t+1} + \varphi_Y Y_t) \tag{23}$$

Under baseline calibration we show that real indeterminacy is almost impossible (Fig. 1A) if monetary policy reacts to output deviations. There is only a small area of indeterminacy region under active monetary policy and very small (even zero) response to output. This result is very similar to Duport (2001), Carlstrom and Fuerts (2005) who used monetary policy rule without response to output. But the addition of response to output to MP rule leads to real indeterminacy in contrast to the results of recent papers.

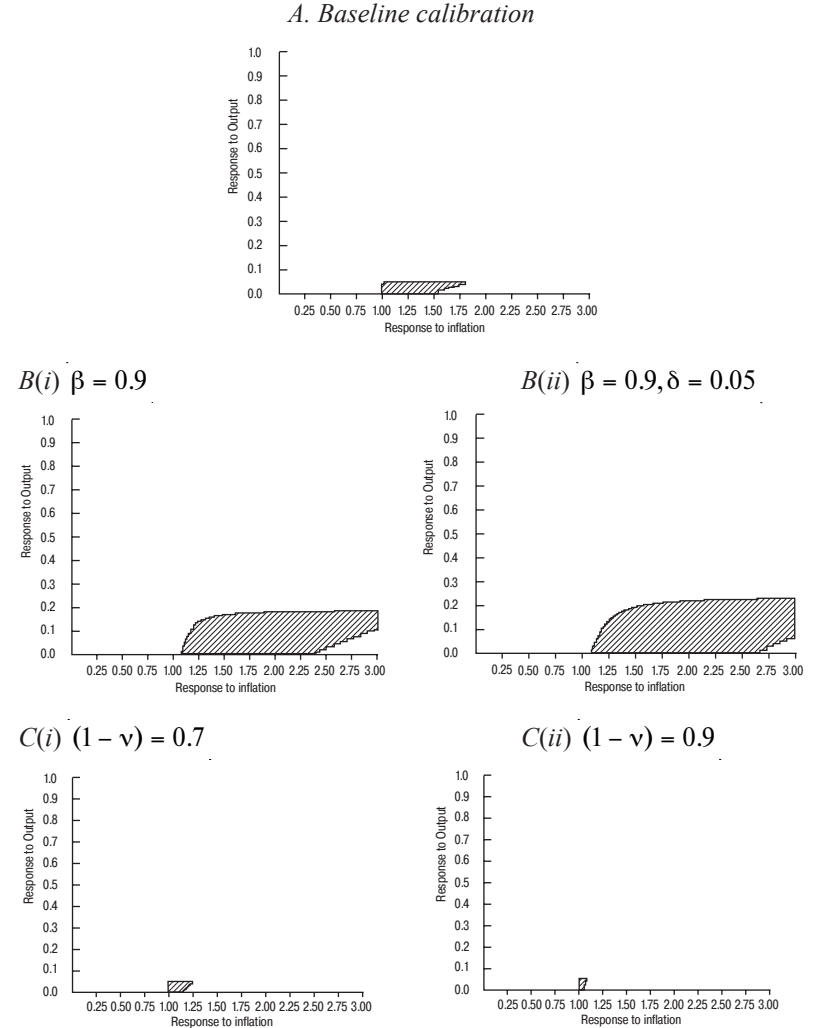
We also analyze the robustness of our results by using different calibrations for the discount factor, capital share, fraction of firms that get a signal to set a new price, depreciation rate of capital.

Changes in the discount factor ( $\beta$ ) and the depreciation rate of capital ( $\delta$ ) dramatically increase the real indeterminacy area (Fig. 1B(i) and 1B(ii)). This result is quite predictable: it is due to cost channel of inflation — the main channel of indeterminacy in recent papers — investment activity in the standard sticky price models.

Changes in fraction of firms that get a signal to set a new price ( $1 - \nu$ ) decreases the indeterminacy area (Fig. 1C(i) and 1C(ii)). When prices are almost flexible ( $1 - \nu = 0.9$ ) there is almost no real indeterminacy regions in the model.

Our model shows that real indeterminacy regions are almost not sensitive to changes in capital share ( $\alpha$ ) and the equilibrium markup ( $\frac{\eta}{\eta-1}$ ). A related

paper by Huang and Meng (2007) has the opposite finding, perhaps, due to different assumptions about the price-adjustment process.



**Fig. 1.** Real Indeterminacy regions (shaded area) under current-looking monetary policy rules with respect to output



#### 4.2. Forward-looking monetary policy rule with respect to output

Under forward-looking monetary policy rule with respect to output the monetary policy rule takes the form:

$$R_t = \varphi_R R_{t-1} + (1 - \varphi_R)(\varphi_\pi E_t \pi_{t+1} + \varphi_Y E_t Y_{t+1}). \quad (23)$$

Under baseline calibration we show that real indeterminacy is almost impossible under active monetary policy rules; and there is a large indeterminacy region under passive monetary policy rules (Fig. 2A). This result is opposite to the results of the basic models with investment (Duport (2001), Carlstrom and Fuerts (2005) and others) that do not take into account a response to future output.

Under very small response of monetary policy to output real indeterminacy is possible under active monetary policy rules and not possible under passive (Fig. 2A). But the results change dramatically if the response to future output in monetary policy rule increases: indeterminacy becomes almost impossible under active monetary policy rules and very possible under passive. So, the main results of recent papers are mainly based on small indeterminacy region under active monetary policy rules and very small response to output.

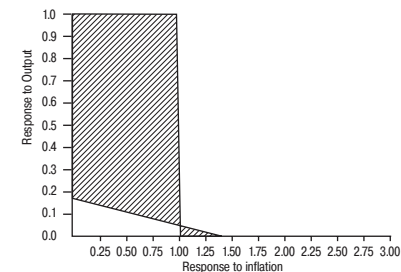
Changes in the discount factor ( $\beta$ ) and the depreciation rate of capital ( $\delta$ ) substantially influence the real indeterminacy area (Fig. 2B(i) and 2B(ii)). This cost channel of inflation leads to additional indeterminacy area under active monetary policy due to dramatic increase of equilibrium interest rate in the economy.

Changes in the fraction of firms that get a signal to set a new price ( $1 - \nu$ ) decreases the indeterminacy area (Fig. 2C(i) and 2C(ii)). When prices are almost flexible ( $1 - \nu = 0.9$ ) there is almost no real indeterminacy in the model.

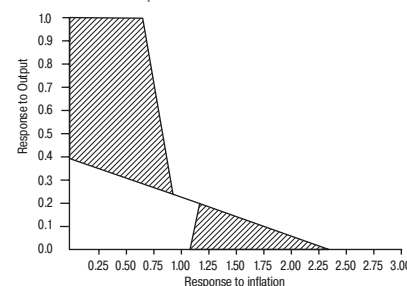
Our model shows that real indeterminacy regions are very small sensitive to changes in the capital share ( $\alpha$ ) and the equilibrium markup ( $\frac{\eta}{\eta - 1}$ ) under

forward-looking monetary policy rule with respect to output. This result is opposite to the result of Huang, Meng (2007).

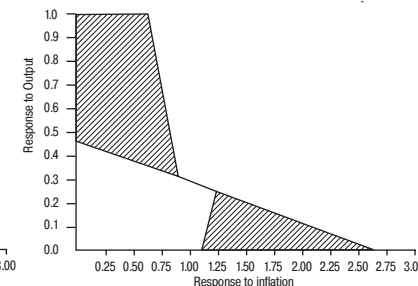
A. Baseline calibration



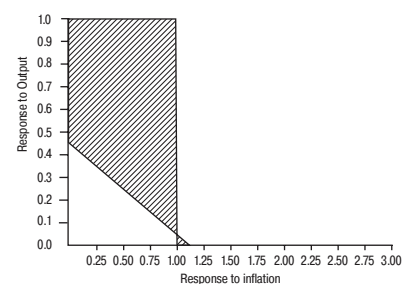
B(i)  $\beta = 0.9$



B(ii)  $\beta = 0.9, \delta = 0.05$



C(i)  $(1 - \nu) = 0.7$



C(ii)  $(1 - \nu) = 0.9$

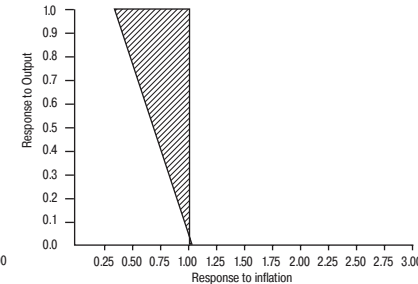
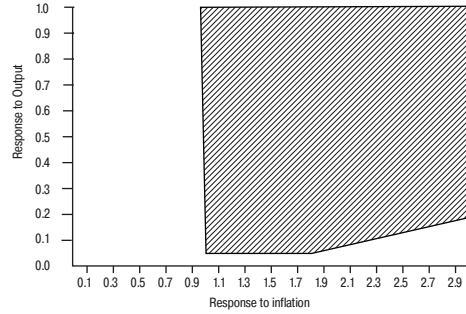
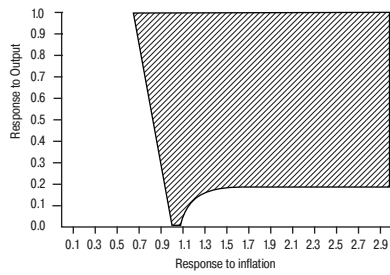


Fig. 2. Real Indeterminacy regions (shaded area) under forward-looking monetary policy rules with respect to output

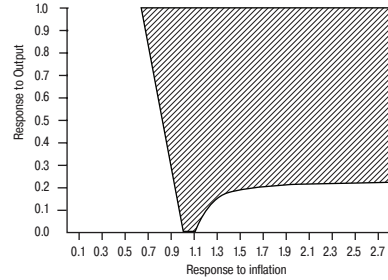
A. Baseline calibration



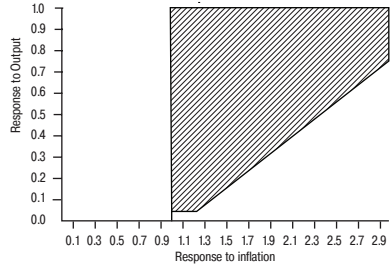
B(i)  $\beta = 0.9$



B(ii)  $\beta = 0.9, \delta = 0.05$



C(i)  $(1 - \nu) = 0.7$



C(ii)  $(1 - \nu) = 0.9$

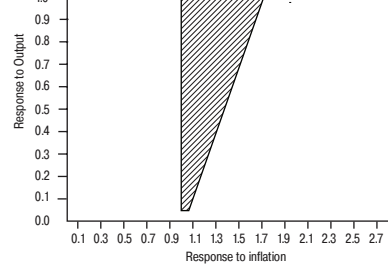
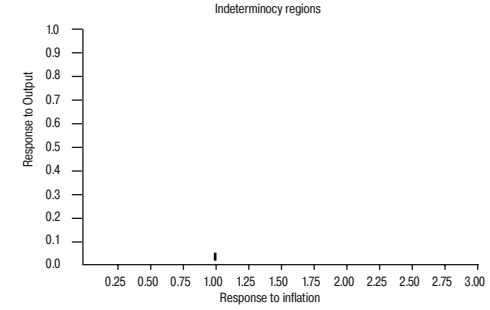
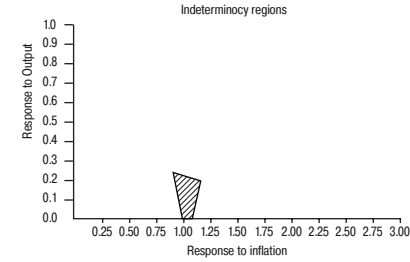


Fig. 3. Nominal determinacy regions (shaded area) under current-looking monetary policy rules with respect to output

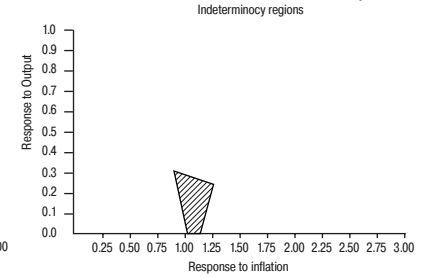
A. Baseline calibration



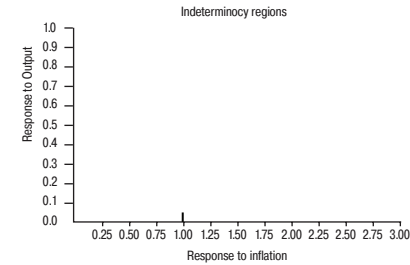
B(i)  $\beta = 0.9$



B(ii)  $\beta = 0.9, \delta = 0.05$



C(i)  $(1 - \nu) = 0.7$



C(ii)  $(1 - \nu) = 0.9$

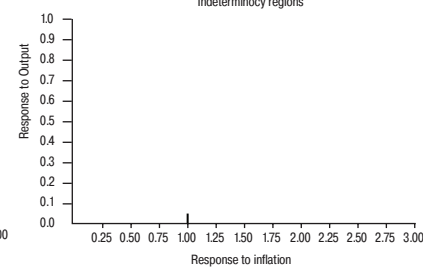


Fig. 4. Nominal determinacy regions (shaded area) under forward-looking monetary policy rules with respect to output

## 5. Nominal Determinacy Analysis

In our paper we analyze the conditions for real indeterminacy to occur in the model with capital accumulation. The key assumption is that we add response to output into the monetary policy rule. In the case of real indeterminacy, nominal indeterminacy always exists. In this case neither inflation nor price level can be determined. The result seems to be quite counterintuitive, but under nominal and real indeterminacy models with monetary policy rules do not determine inflation rates definitely.

In this part of our paper we analyze the conditions not for real, but even for *nominal determinacy* to occur so that inflation rates to be definitely determined. Only in this case monetary policy gives good results in stabilizing inflation rates.

As it was mentioned before equations (17)–(23) are the first-order dynamical system of five equations and the stability conditions are as follows:

1. In the case of two stable roots (out of five possible) we get nominal determinacy;
2. In the case of three stable roots we get nominal indeterminacy;
3. In the case of four or five stable roots we get real indeterminacy.

### 5.1. Nominal Determinacy Analysis under Current-looking monetary policy rule with respect to output

Under baseline calibration we show that nominal determinacy is possible under active and even small response to output monetary policy rules; under passive monetary policy rules nominal determinacy is impossible (Fig. 3).

Changes in the discount factor ( $\beta$ ) and the depreciation rate of capital ( $\delta$ ) influence the nominal determinacy area such that under passive monetary policy rules and high response to output nominal determinacy become possible (Fig. 3B(i) and 3B(ii)).

When prices are almost flexible the range of parameters for nominal determinacy to occur becomes much smaller (Fig. 3C(i) and 3C(ii)).

### 5.2. Nominal Determinacy Analysis under Forward-looking monetary policy rule with respect to output

Under forward-looking monetary policy rules with respect to output nominal determinacy is almost impossible (Fig. 4). Even significant changes in discount factor ( $\beta$ ) and depreciation rate of capital ( $\delta$ ) do not substantially increase nominal determinacy areas (Fig. 3B(i) and 3B(ii)). Flexibility of prices

and other parameters do not have substantial influence on the determinacy areas (Fig. 3C(i) and 3C(ii)).

### 5.3. Nominal Determinacy Analysis: main results

We show that under capital accumulation monetary policy that takes into account *expected future* output can not lead economy to unique inflation rates under realistic calibration of the model: inflation rates are always indetermined.

But active monetary policy with response to *current* output leads to nominal determinacy. If output is not taken into account the result are quite opposite: even real indeterminacy is possible (as in Duport (2001), Carlstrom and Fuerts (2005)).

The normative result is that active and forward-looking monetary policy rules with respect to output give better results in stabilizing the inflation.

## 6. Concluding remarks

We analyze the conditions for real indeterminacy to occur in the models with capital accumulation. The key assumption is that we add response to output into the monetary policy rule. In our paper we show that adding Current or Expected Output to the MP rule substantially changes the conditions for real indeterminacy to occur. We also provide the nominal determinacy analysis and show that active and forward-looking monetary policy rule with respect to output give better results in stabilizing the economy.

In contrast to some existing research we show that:

1. Under Current-looking with respect to output MP rule
  - a. Real indeterminacy is almost not likely to occur under passive or active MP rules;
  - b. Real indeterminacy areas are almost not sensitive to changes in the markup and the share of capital in output;
  - c. Real indeterminacy areas are very sensitive to changes in the discount and depreciation rates;
  - d. Under more flexible prices real indeterminacy is less likely to occur;

2. *Under Forward-looking with respect to output MP rule*
  - a. Real indeterminacy is not likely to occur under active and likely to occur under passive MP rules;
  - b. Real indeterminacy areas are almost not sensitive to the markup changes and the share of capital in output;
  - c. Real indeterminacy areas are very sensitive to changes in the discount and depreciation rates;
  - d. Under more flexible prices real indeterminacy is less likely to occur;
3. *Active MP rules that take into account current output usually leads to nominal determinacy;*
4. *Under forward-looking MP rules with respect to output nominal indeterminacy is impossible.*

## References

Benhabib J., Eusepi S. (2005) The design of monetary and fiscal policy: a global perspective // *Journal of Economic Theory* 123, 40–73.

Benhabib J., Schmitt-Grohe S., Uribe M. (2001a) Monetary policy and multiple equilibria // *Amer. Econ. Rev.* 91 (2001a), 167–185.

Benhabib J., Schmitt-Grohe S., Uribe M. (2001b). The Perils of Taylor Rules // *Journal of Economic Theory* 96, 40–69

Bernanke B., Woodford M. (1997) Inflation forecasts and monetary policy, *J. Money, Credit, Banking* 24, 653–684.

Blanchard O.J., Kahn C.M. (1980) The solution of linear difference models under rational expectations // *Econometrica* 48 (5), 1305–1311.

Calvo G.A. (1983) Staggered prices in a utility-maximizing framework // *Journal of Monetary Economics*, 12, 383–398.

Carlstrom C.T., Fuerst T.S. (2000) Forward vs. backward-looking Taylor rules, Federal Reserve Bank of Cleveland working paper.

Carlstrom C.T., Fuerst T.S. (2001) Real indeterminacy in monetary models with nominal interest rate distortions // *Rev. Econ. Dynam.* 4, 767–789.

Carlstrom C.T., Fuerst T.S. (2005). Investment and interest rate policy: a discrete-time analysis // *Journal of Economic Theory* 123, 4–20.

Christiano L., Gust C. (1999) Taylor Rules in a Limited Participation Model. NBER Working Paper 7017.

Clarida R., Gali J., Gertler M. (1999) The science of monetary policy: a new Keynesian perspective // *Journal of Economic Literature*. Vol. XXXVII. P. 1661–1707.

Clarida R., Gali J., Gertler M. (2000) Monetary policy rules and macroeconomic stability: evidence and some theory, *Quart. J. Econ* 115 147–180.

Dixit A., Stiglitz J. (1977) Monopolistic competition and optimum product diversity // *American Economic Review* 67, 297–308.

Dupor B. (2001) Investment and interest rate policy // *Journal of Economic Theory* 98, 81–113.

Galf J., Gertler M. (1998) Inflation dynamics: A structural econometric analysis // *Journal of Monetary Economics*. Vol. 44. Issue 2. October 1999. P. 195–222.

Huang K., Meng Q. (2007) Capital and macroeconomic instability in a discrete-time model with forward-looking interest rate rules // *Journal of Economic Dynamics & Control* 31, 2802–2826.

Kerr W., King R. (1996) Limits on interest rate rules in the IS-LM model, Federal Reserve Bank of Richmond Economic Quarterly, Spring, 1996.

King R.G., Wolman A.L. (2004) Monetary discretion, pricing complementarity, and dynamic multiple equilibria // *Quarterly Journal of Economics* 119, 1513–1553.

Kurozumi T. (2006) Determinacy and expectational stability of equilibrium in a monetary sticky-price model with Taylor rules // *Journal of Monetary Economics* 53, 827–846.

Kurozumi T., Zandweghe W. (2007) Investment, interest rate policy, and equilibrium stability // *Journal of Economic Dynamics and Control*, doi:10.1016/j.jedc.2007.06.006.

Rotemberg J., Woodford M. (1998) An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy: Expanded Version. NBER Technical Working Paper No. 233.

Sveen T., Weinke L. (2005) New perspectives on capital, sticky prices, and the Taylor principle // *Journal of Economic Theory* 123, 21–39.

Taylor J.B. (1993) Discretion versus policy rules in practice' Carnegie-Rochester Conference Series on Public Policy 39. P. 195–214.

Xiao W. (2008) Increasing Returns and the design of interest rate rules. *Macroeconomic Dynamics*, 12, 2008, 22–49.

Yun T. (1996) Nominal price rigidity, money supply endogeneity, and business cycles, *J. Monet. Econ.* 37 (2) 345–370.

*Препринт WP12/2008/01*  
*Серия WP12*  
*Научные доклады лаборатории макроэкономического анализа*

В. Храмов, К. Сосунов

**Монетарные правила и множественность равновесий**

(на английском языке)

Публикуется в авторской редакции

Зав. редакцией *А.В. Заиченко*  
Технический редактор *Ю.Н. Петрина*

ЛР № 020832 от 15 октября 1993 г.  
Отпечатано в типографии ГУ ВШЭ с представленного оригинал-макета.  
Формат 60×84 <sup>1</sup>/<sub>16</sub>. Бумага офсетная. Тираж 150 экз. Уч.-изд. л. 1,2.  
Усл. печ. л. 1,4. Заказ № . Изд. № 854.

ГУ ВШЭ. 125319, Москва, Кочновский проезд, 3  
Типография ГУ ВШЭ. 125319, Москва, Кочновский проезд, 3  
Тел.: (495) 772-95-71; 772-95-73

## Notes

---

---