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Abstract

A one-dimensional, three-component, fluid model has been employed to investigate the existence of a speed cut-off point for antiforce breakdown waves. The term antiforce wave is used to identify breakdown waves for which the electric field force on electrons is in the opposite direction of wave propagation. The electron fluid-dynamical equations for antiforce waves are different from those of proforce waves. This presentation will address the difference in the set of equations for proforce and antiforce waves and the method of integration of the set of equations through the dynamical transition region for antiforce waves. Also, for antiforce waves, the existence and approximate value of a speed cut-off point will be discussed.

Introduction

The basic set of equations in the fluid model consists of the equations of conservation of mass, momentum, and energy, coupled with Poisson's equation. The three equations of conservation of mass, momentum, and energy are in Eulerian form and were adopted by Fowler (1964). Also, the wave is considered to be a shock front driven by the electron gas pressure. The shock front is followed by a dynamical transition region in which a neutral cold gas entering from the front is turned into a partially ionized hot gas.

For antiforce waves, the net electric field (applied plus space charge field) is in the direection of wave propagation. Therefore, the electric field force on electrons is in the opposite direction of wave propagation. However, the electron temperature and therefore fluid pressure are assumed to be large enough to provide the driving foree. The problem is assumed to be one-dimensional and time independent with the wave propagating along the *x*-axis. In the wave frame, the frame whose origin is located at the shock front, the electrons, ions, and neutral particle velocities and number densities are time independent.

Model and Theory

Consider the breakdown wave as an infinite plane wave traveling in the positive x direction with a speed V. Due to the absence of observed Doppler shift, in the wave frame, the frame which the wave front is considered to be stationary, the ions and neutral particles will have a velocity of – V, and the wave will extend from 0 to – ∞. The shock front at x = 0 divides the neutral particles in front of the wave from the three component gas (electrons, ions, and neutral particles) behind the wave.

In gas electrical discharge, the wave front is followed by a dynamical transition region. In this region the net electric field (applied plus space charge field), starting from its value E_0 at the wave front, falls to zero at the trailing edge of this region. This region, which is somewhat thicker than a Debye length, is referred to as the sheath region. At the end of the sheath region the electrons come to rest relative to ions and neutral partieles. These physical conditions will be the guiding tool in solving the set of electron fluid-dynamical equations. Since 1964, Fowler's (1964) set of electron fluid dynamical equations has been improved. The set of equations which have proven to be suceessfull in the case of proforce waves was completed by Fowler et al. (1984). Their set of equations include equations of conservation of mass, momentum, and energy. Coupled with Poisson's equation and in one dimension, they are, respectively

$$\frac{d(nv)}{dx} = \beta n, \tag{1}$$

$$\frac{d}{dx}\left\{ nmv(v - V) + nkT_e \right\} = - enE - Kmn(v - V), \tag{2}$$

$$\frac{d}{dx}\left\{ nmv(v - V)^2 + nkT_e\,(5v - 2V) + 2e\phi nv + \epsilon_0 VE^2 - \frac{5nk^2\,T_e}{mk}\frac{dT_e}{dx} \right\}$$

$$= - 3\left(\frac{m}{M}\right) nkKT_e - nmK\left(\frac{m}{M}\right)(v - V)^2, \tag{3}$$

$$\frac{dE}{dx} = \frac{en}{\epsilon_0}\left(\frac{v}{V} - 1 \right). \tag{4}$$

The symbols n, v, m, e, and T_e represent electron number density, velocity, mass, charge, and temperature inside the sheath, and k, K, β, ϕ, V, M, E_0, x are Boltzman constant, elastic collision frequency, ionization frequency, ionization potential, wave velocity, neutral particle mass, electric field at the wave front, and position inside the sheath, respectively.

For antiforce waves, Sanmann and Fowler (1975) considered the wave to be charaeterized by a weak discontinuity at its front. That is, the electron temperature and number density derivatives were considered to change discontinuously; however, the variables themselves changed continuously. For examplee, his conditions on electron number density, n, at the wave front were n=0 and $\frac{dn}{dx} \neq 0$. Application of Sanmann's approximate method of solutions and initial conditions reflecting a weak discontinuity to the completed set of electron fluid dynamical equations did not bear fruit; therefore, we had to consider an alternate approach.

In our attempts to solve the set of equations, assuming a strong discontinuity (a shock front), has proven to be successful. That is, at the wave front the variables such as electron temperature and number density change discontinuously. Sanmann and Fowler (1975) used variables suggested by Shelton and Fowler (1968) to reduce the set of equations to nondimensional form. However, these variables lead to a contradiction in sign for the variables. Therefore, we will choose a slightly different set of variables, and they are

$$\eta = \frac{E}{E_0}, \quad \xi = -\frac{eE_0}{mV^2}x, \quad \psi = \frac{v}{V}, \quad \theta = \frac{kT_e}{2e\phi}, \quad \nu = \frac{2e\phi}{\epsilon_0 E_0^2}n,$$

$$\omega = \frac{2m}{M}, \quad \mu = \frac{\beta}{K}, \quad \alpha = \frac{2e\phi}{mV^2}, \quad \kappa = -\frac{mVK}{eE_0}$$

In the above equations, ν, ψ, θ, μ, κ, η, and ξ are the dimensionless electron concentration, electron velocity, electron temperatue, ionization rate, elastic collision frequency, electric field, and position inside the sheath, respectively. All the above nondimensional quantities, including κ, are intrinsically positive. Introducing the above dimensionless variables in the equations 1-4 results in

$$\frac{d(\nu\psi)}{d\xi} = \kappa\mu\nu, \tag{5}$$

$$\frac{d}{d\xi}\left\{\nu\psi(\psi-1) + \alpha\nu\theta\right\} = \nu\eta - \kappa\nu(\psi-1), \tag{6}$$

$$\frac{d}{d\xi}\left\{\nu\psi(\psi-1)^2 + \alpha\nu\theta(5\psi-2) + \alpha\nu\psi + \alpha\eta^2 - \frac{5\alpha^2\nu\theta}{\kappa}\frac{d\theta}{d\xi}\right\} = -\omega\kappa\nu\left\{3\alpha\theta + (\psi-1)^2\right\}, \tag{7}$$

$$\frac{d\eta}{d\xi} = \frac{\nu}{\alpha}(1-\psi). \tag{8}$$

Solution of the Equations

At the shock front, the electron velocity, v, is not as high as the wave velocity, V. Therefore, the value of the nondimensional electron velocity, v, will be less than one. All varaibles are intrinsically positive; therefore, at the wave front Poisson's equation will yield a positive electric field derivative ($\frac{d\eta}{d\xi} > 0$). This indicates that, traversing the sheath, at first the nondimensional electric field value will increase from its initial value of 1. However, gradually the electrons reach speeds larger than those of ions and neutral particles. This results in ψ values larger than one, and therefore, a negative value of electric field derivative ($\frac{d\eta}{d\xi} < 0$). Thus, the electric field value starts decreasing. Since a contained volume of plasma cannot support an electric field, as one approaches the trailing edge of the sheath the electric field value must approach zero ($\eta \rightarrow 0$). Approaching the end of the sheath, due to collisions with heavy particles, the electrons slow down to speeds equal to those of heavy particles. The dimensionless electron velocity value, therefore, must approach unity at the end of the sheath ($\psi \rightarrow 1$).

Integrating the electron fluid dynamical equations through the sheath region, the physical conditions at the end of the sheath, referred to in the above paragraph, will become the guiding tool. For given values of α and κ, a combination of initial electron number density, υ_1, and initial electron velocity, ψ_1, are selected. Then, for such a combination, the equations are numerically integrated and variations of the variables through the sheath region are observed. κ is called the wave constant and determines the relation between the laboratory wave speed and the initial value of the electric field. Changes in the value of κ are utilized for dramatic impact in the process of numerical integration. Additionally, the values of υ_1 and ψ_1 are altered to achieve a solution by trial and error.

Through the sheath region, the electron fluid dynamical equations have successfully been integrated for six values of α (0.01, 0.05, 0.1, 0.25, 1, 2). α = 0.01 represents a fast wave speed (V = 3 x 10^7 m/s) and α = 2 represents a slow wave speed (V = 2 x 10^6 m/s). Figure 1 is a plot of the electric field, η, as a function of electron velocity, ψ, inside the sheath for all six values of α. The graphs show that for all six values of α the solutions to the electron fluid dynamical equations conform to the expected physical conditions at the end of the sheath. To achieve successful integration for different values of α, the following values of κ and initial electron velocity and electron density had to be utilized:

$\alpha = 0.01, \ \kappa = 1.3, \ \nu_1 = 0.886$, and $\psi_1 = 0.645$

$\alpha = 0.05, \ \kappa = 0.6, \ \nu_1 = 0.853$, and $\psi_1 = 0.801$

$\alpha = 0.1, \ \kappa = 0.477, \ \nu_1 = 0.801$, and $\psi_1 = 0.924$

$\alpha = 0.25, \ \kappa = 0.3883, \ \nu_1 = 0.985$, and $\psi_1 = 0.96$

$\alpha = 1, \ \kappa = 0.22, \ \nu_1 = 0.9$, and $\psi_1 = 0.94$

$\alpha = 2, \ \kappa = 0.16, \ \nu_1 = 0.93094$, and $\psi_1 = 0.97$

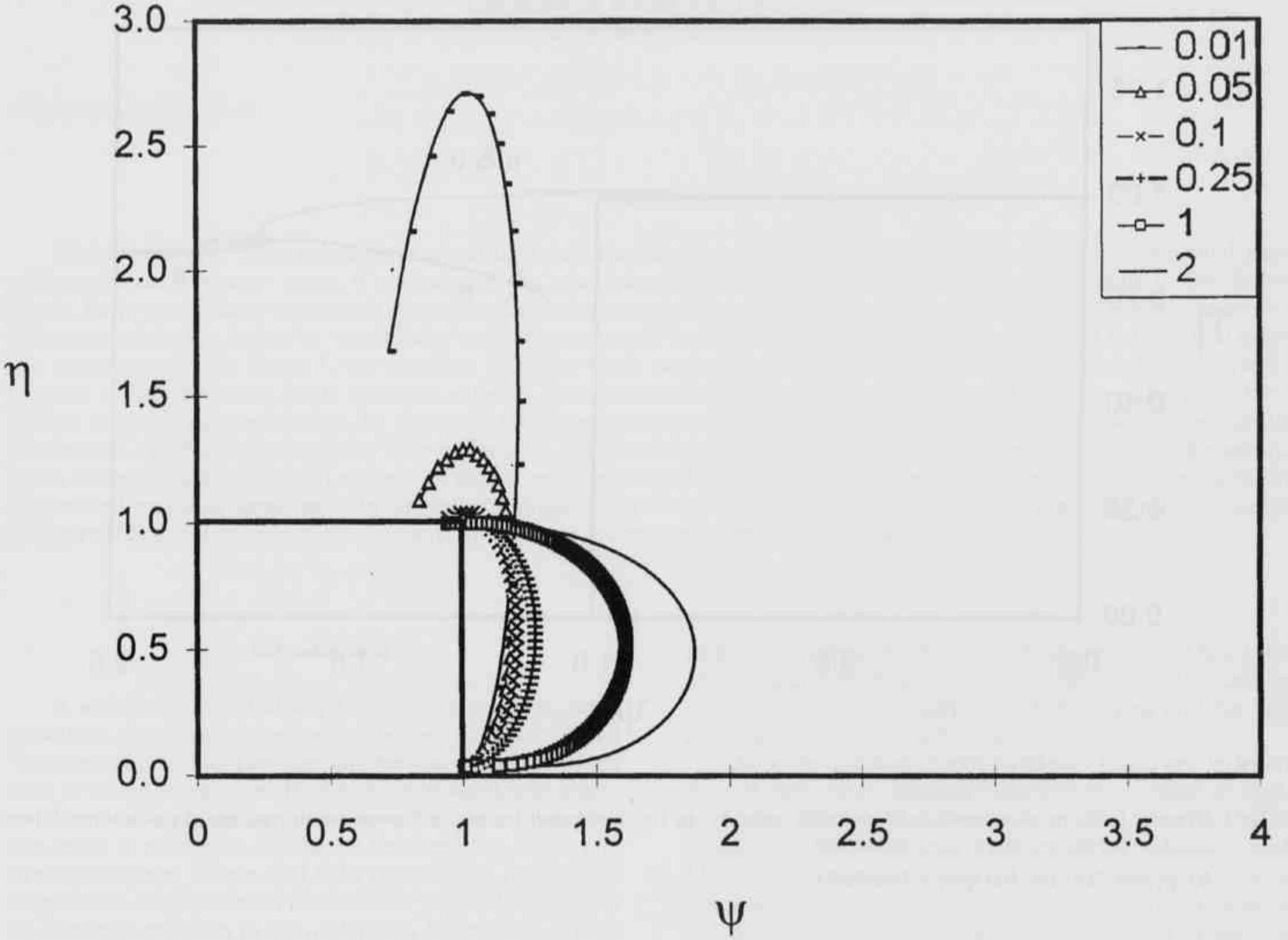

Fig. 1. Electric field, η, as a function of electron velocity, ψ, for six different values of α. α = 0.01, 0.05, 0.1, 0.25, 1, and 2.

As the value of α increases, the wave speed decreases and the numerical integration of the electron fluid dynamical equations becomes more difficult. Solution for $\alpha = 2$ required long hours of computer work and analysis. A great deal of time was spent trying to achieve solution for $\alpha = 4$; however, there seems to be no solution for $\alpha = 4$. Therefore, there seems to be a cut-off point for values of α which allow successful integration of the set of equations ($\eta \rightarrow 0$, $\psi \rightarrow 1$). That is, there seems to be a cut-off point for wave speeds. Figure 2 shows graphs of electric field, η, as a function of electron velocity, ψ, for two sets of variables. A slight variation of υ_1 results in two different paths, neither is an acceptable solution.

Conclusions

For antiforce waves and for six different values of wave speeds, the electron fluid dynamical equations have successfully been integrated. The integration of the equations become more difficult as the breakdown wave speed decreases. There seems to be a cut-off point in the value of α beyond which successful integration of the equations is not possible. That is, there seems to be a cut-off point for wave speeds.

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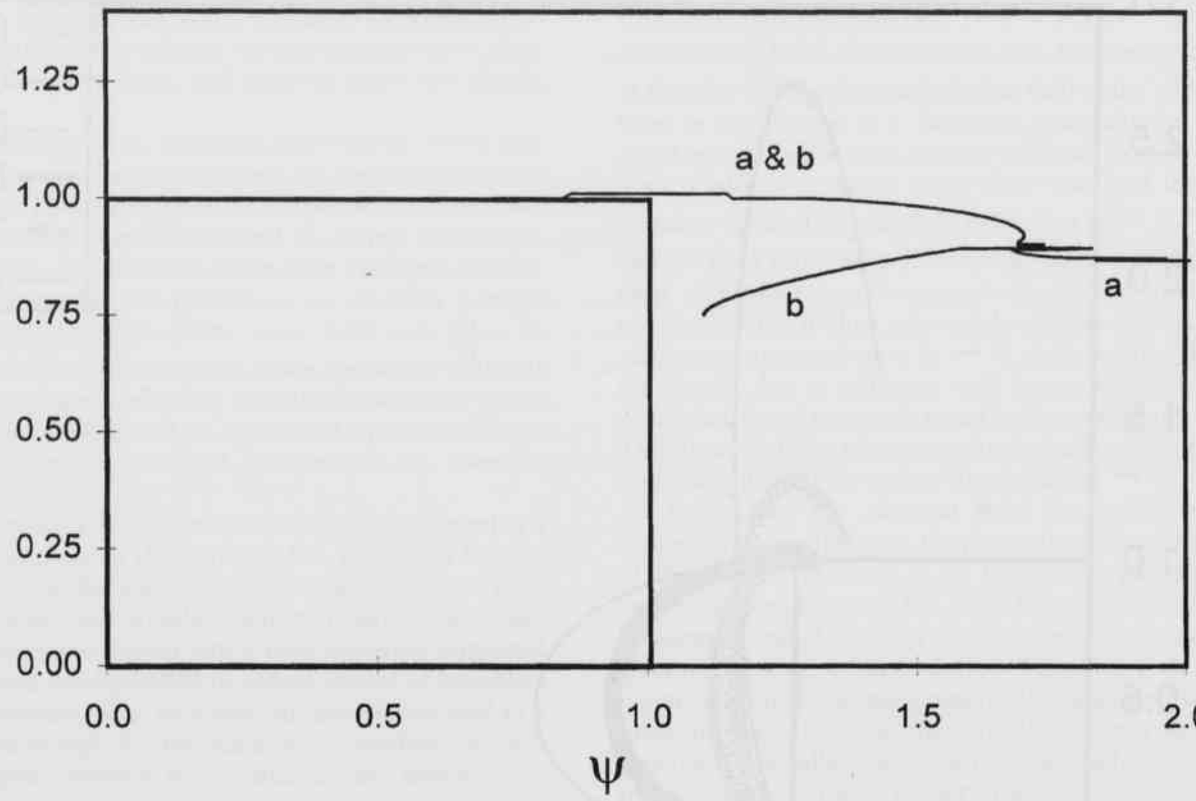

Fig. 2. Electric field, η, as a function of electron velocity, ψ, for $\alpha = 4$ and for two different combinations of parameter values.
a) $\psi_1 = 0.710,\ \kappa = 0.33,\ \nu_1 = 0.398502$
b) $\psi_1 = 0.710,\ \kappa = 0.33,\ \nu_1 = 0.398501$

Literture Cited

Fowler, R. G. 1964, Electrons as a hydrodynamical fluid. ADV. in Electronics and Electron Phys. 20:1-58.

Fowler, R. G. M. Hemmati, R. P. Scott, and **Parsenajadh.** 1984. Electric breakdown waves: Exact numerical solutions. Part 1. Phys. Fluids. 27:1521-1526.

Sanmann, E. E. and **R. G. Fowler.** 1975. Structure of electron fluid dynamical plane waves. Phys. Fluids. 18:1433-1438.

Shelton, G. A. and **R. G. Fowler** 1968. Nature of electron-fluid-dynamical waves. Phys. Fluids. 11:740-746.