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Abstract

Two- and three-dimensional coordinate systems are fundamental to most quantitative mapping applications. The Geodetic, Universal Transverse Mercator (UTM), and State Plane systems have traditional roles in various science, surveying, and government agency engineering applications. The coordinates of three-dimensional Geodetic system are latitude, longitude, and height above ellipsoid (HAE). Because of its ability to cope with the intrinsically three dimensional character of the earth's surface, the Geodetic system is capable of supporting precise relative positioning and very high accuracy computations of distance between any two positions on or near the earth's surface. The two-dimensional UTM and State Plane systems are extremely useful for the local horizontal positioning and scaling required for paper maps of county-size land areas. In the two plane systems, horizontal distance computation is a very straightforward application of the distance formula (analytic geometry) based on the Pythagorean theorem. Although precision line- and geodesic- distance formulas based on geodetic coordinates are more complex, useful horizontal distance estimates are easily derived from the latitudes and longitudes of two positions. This paper examines this premise for Arkansas. The approach to estimating horizontal distances utilizes an application of the distance formula in conjunction with an assumed constant distance/unit latitude of 30.8 m (arc sec)$^{-1}$. A linear regression equation is used to represent distance/unit longitude as a function of latitude in Arkansas. The approximation math is extremely simple, and the process as a whole is equivalent to a portable coordinate projection.

Introduction

Longitude λ and latitude ϕ are two of the three coordinates of a geodetic coordinate system. The third is height above ellipsoid, h or HAE. Due in part to their non-linear relationships, to more familiar plane distance and direction variables, interpretations of λ and ϕ can require the aid of a geodetically referenced map, a three-dimensional model (globe), or specialized computer software. In spite of their native three-dimensional positioning roles, λ and ϕ can be used for horizontal positioning operations without explicit reference to HAE. For example λ and ϕ tics appear along the neat lines of certain Arkansas Highway and Transportation Department (AHTD) maps and of United States Geological Survey (USGS) topographic maps of various scales. When they are used in a paper map context, some authors (for example, Verbyla, 1995) refer to latitude and longitude as geographic coordinates.

Plane coordinates, such as the Easting (E) and Northing (N), of the Universal Transverse Mercator (UTM) Zone 15 or of the Arkansas North Zone (AN) or Arkansas South Zone (AS) State Plane coordinate systems, are much easier to manage in basic field positioning operations than are λ and ϕ. First, the plane coordinates directly express horizontal distances east (E) or north (N) of the particular plane system's origin. Secondly, the horizontal distance separating two positions can be obtained with a calculator that supports arithmetic and square root operations. However, users are compelled to accept a given system's N-axis alignment, and the alignment does not agree, necessarily, either with the local geodetic north, with magnetic north, or with a direction that might provide a particular advantage in a given project, such as a straight road or field boundary. Finally, relating E and N of one of the plane systems to λ and ϕ requires the use of intervening projection tables or specialized computer software, such as National Geodetic Survey's (NGS) NADCON program or the Army Corps of Engineers CORPSCON program.

With today's widespread availability and use of global positioning systems (GPS) and geographic information systems (GIS), geodetic coordinates are more easily observable by a broad range of scientists, government agency personnel, and the general public (Featherstone and Langley, 1997; Hurn, 1989; Teague et al., 1999). Precise projection formulas commonly built into GPS receiver firmware can provide rapid conversion between the geodetic and either AN, AS, or UTM Zone 15 coordinates in the field. In spite of this it is likely that situations will arise in which a custom plane coordinate system that can be referenced to the geodetic system would provide an advantage in certain field operations. One example is where differential GPS (DGPS) equipment is available for one-time geodetic referencing of two or three semi-permanent markers at a particular site, but where it is desired to carry out future positioning operations with traditional distance and direction measurement devices. The λ and ϕ values could be calculated for positions initially labeled according to their measured distances east and north of a geodetically referenced origin and without the necessity of an intervening map or specialized software. Also, a computationally lean procedure for estimating the horizontal distance, d(A, B), between A = (ϕ_1, λ_1) and B = (ϕ_2, λ_2) would facilitate rapid comparison of the relative locations of two positions with known λ and ϕ values.

The approach taken here is to use an empirical d (A, B) formula that is appropriate for Arkansas's λ and ϕ ranges. The proposed d (A, B) computation depends directly on the differences, $\Delta\lambda = \lambda_1 - \lambda_2$ and $\Delta\phi = \phi_1 - \phi_2$. It requires the use of three empirical constants and the plane Euclidean distance formula (Thomas, 1968). Horizontal distance in the E direction depends on $\Delta\lambda$ through a fitted linear equation expressing the distance per unit difference in longitude, $s_\lambda(\phi)$, as a function of latitude. Horizontal distance in the N direction depends on $\Delta\phi$ by an averaged constant value of the distance per unit latitude, s_ϕ.

One objective of this manuscript is to present the above mentioned mathematically simple expressions for $s_\lambda(\phi)$, s_ϕ, and d (A, B). A second objective is to estimate an upper bound for relative error when d (A, B) is used to approximate the ellipsoidal distance, d*, between A and B, with HAE = 0. A third objective is to show that the empirical formulas lead to a coordinate projection. Finally, examples are given to illustrate application of the empirical formulas.

Theory

Development of an accurate paper map of a portion of the earth's surface depends on the use of a map datum that is associated with a particular reference ellipsoid model. The datum provides the information necessary to reference three-dimensional position coordinates of the selected ellipsoid to the physical earth. A map- or coordinate- projection is then used to calculate two-dimensional coordinates that best represent desired geometric properties of the region of interest (Snyder, 1987; Featherstone and Langley, 1997; Bomford, 1962). Many currently available maps in the United States are based on the NAD – 27 horizontal datum, which in turn is referenced to the Clarke 1866 ellipsoid. The more recent GRS – 80 ellipsoid is the reference for maps utilizing the NAD – 83 horizontal datum (Stem, 1990; Dewhurst, 1990). A GPS receiver's position computation is carried out with three-dimensional coordinates of the WGS – 84 geodetic system. WGS – 84 utilizes an ellipsoid model essentially identical to GRS – 80 (Snyder, 1987; Langley, 1998).

Various definitions are given for a map- or coordinate-projection (Snyder, 1987; Bomford, 1962); however, the effective definition of a particular projection (Bomford, 1962) is a pair of mathematical formulas for mapping the ellipsoid model surface into the plane:

$$N = f_1(\phi, \lambda) \text{ and } E = f_2(\phi, \lambda). \quad (1)$$

The projection's convergence, γ, is the angle between a projected meridian and the N grid line of the plane coordinate system, and the tangent of the convergence is given by the equation

$$\tan\gamma = -(\partial E/\partial\phi)/(\partial N/\partial\phi). \quad (2)$$

The scale of the projection along a meridian is (Snyder, 1987)

$$h = [(\partial E/\partial\phi)^2 + (\partial N/\partial\phi)^2]^{1/2}/\rho, \quad (3)$$

while along a parallel, the scale is

$$k = [(\partial E/\partial\phi)^2 + (\partial N/\partial\lambda)^2]^{1/2}/[\nu\cos\phi]. \quad (4)$$

In the two latter equations the variables ρ and ν are the principle radii of curvature of the reference ellipsoid (Bomford, 1962)

$$\rho = \alpha(1 - e^2)/(1 - e^2\sin^2\phi)^{3/2} \text{ and} \quad (5)$$

$$\nu = \alpha/(1 - e^2\sin^2\phi)^{1/2}, \quad (6)$$

where α and e are the ellipsoid's equatorial radius and eccentricity, respectively. For the GRS-80 ellipsoid, e^2 = 0.0066943800, and α = 6,378,137 m (Snyder, 1987). If h = k, the projection is orthomorphic (or conformal) – i.e. the scale at a given point is independent of direction. The radius of a parallel is $\nu\cos\phi$, so arc length along a parallel, and corresponding to a one arc-second difference in longitude, is

$$s_\lambda^* = \pi\nu\cos\phi/648000. \quad (7)$$

Also, for small latitude differences, $\Delta\phi$, arc length, s_ϕ^*, along a meridian is closely approximate by $\rho\Delta\phi$ (Bomford, 1962). Therefore arc length corresponding to a one arc-second difference in latitude along a meridian may be expressed:

$$s_\phi^* = \pi\rho/648000. \quad (8)$$

The empirical projection considered here is based on the following pair of equations:

$$E = [a\phi_0) + b\phi](\lambda - \lambda_0) \quad (9)$$

$$N = c\cdot(\phi - \phi_0) \quad (10)$$

Equations (9) and (10) define a coordinate projection by virtue of (1) above. The origin (0,0) of the E - N grid has geodetic coordinates λ_0 and ϕ_0, both constants in (9) and (10).

The distance in the plane between two projected positions $A'(E_l, N_l)$ and $B'(E_2, N_2)$ is

$$d = (\Delta E^2 + \Delta N^2)^{1/2}, \tag{12}$$

where $\Delta E = E_2 - E_l = (a + b\,\phi_2)(\lambda_2 - \lambda_0) - (a + b\,\phi_1)(\lambda_1 - \lambda_0)$ and $\Delta N = N_2 - N_l = c \cdot (\phi_2 - \phi_1) = c \cdot \Delta\phi$.

A useful approximation to the Easting difference, ΔE, is

$$\Delta E^* = (a + b\phi_a)(\lambda_2 - \lambda_1), \text{ where } \phi_a = 0.5 \cdot (\phi_2 - \phi_1) \tag{13}$$

The difference between ΔE and ΔE^* is $b \cdot \Delta\phi \cdot (\lambda_a - \lambda_0)$, where $\lambda_a = 0.5 \cdot (\lambda_2 + \lambda_1)$. Subsequently, equation (12) with substitution of ΔE^* for ΔE will be used to approximate the ellipsoidal distance, d^*, between (ϕ_1, λ_1) and (ϕ_2, λ_2)

Methods

Eight values of d^* were calculated using a computer program "invers3d.exe". The program was obtained from National Geodetic Survey's (NGS) web site. The eight calculated d^* values correspond to the ellipsoidal distance between two positions having the same λ but with a difference in ϕ of $\Delta\phi = \phi_1 - \phi_2 = 0.0166667$ deg (1 min). All eight d^* calculations utilized $\lambda = -93.0^o$; the eight ϕ_1 values used were 33.0, 33.5, 34.0, 34.5, 35.0, 35.5, 36.0, and 36.5 deg. Each of the resulting d^* values was divided by 60 to obtain corresponding s^*_ϕ, and these six s^*_ϕ values were averaged to obtain $s^*_\phi = 30.81$m sec^{-1}. Similarly, in order to obtain the empirical linear expression for s_λ, eight new (ϕ, d^*) pairs were generated with the "invers3d.exe" program. The eight ϕ values were the ϕ_1 values used to generate $s\phi$. However, in this case the d^* values were computed using $\Delta\lambda = 1.0^o$, with $\lambda_1 = -93.5^o$. Each of the resulting d^* values was divided by 3600 to obtain a corresponding s^*_λ. A linear equation was fitted (Fig. 1) to the resulting set of (ϕ, s^*_λ) pairs. The fitted equation is

$$s_\lambda(\phi) = a' - b'\phi = 25.966 - 0.3066(\phi - 33) \quad R^2 = 0.9998 \tag{14}$$

Consistent with units of s_ϕ, the values of a' and b' in (14) yield s_λ in units of m sec^{-1}. The coefficient c in (10) is now set to $c = 3600\ s_\phi = 110916$ m deg^{-1}. The coefficients a and b of (9) are determined from a' and b' so that the value of E in (9) depends on $s\lambda$ evaluated at $0.5 \cdot (\phi + \phi_0)$:

$$b = 1800 \cdot b' = -551.7 \text{ m deg}^{-2};$$

$$a(\phi_0) = 3600 \cdot (a' + 0.5\ b'\phi_0) = (129890 - 551.7\phi_0) \text{ m deg}^{-1}.$$

Expressions for h and k were derived by substituting partial derivatives of the E and N expressions of equations (9) and (10) into equations (3) and (4), respectively. Derived

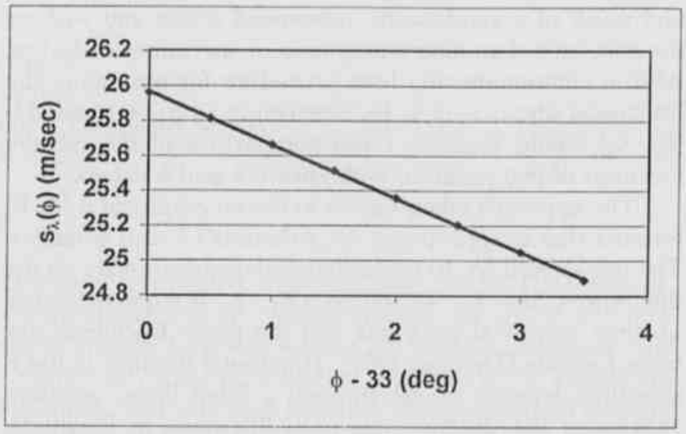

Fig. 1. Graph of calculated $s\lambda$ (ϕ) values (diamond symbols) and of the fitted linear equaiton.

expressions for h and $\tan\gamma$ were found to depend on $\lambda - \lambda_0$, whereas k depends on both ϕ and ϕ_0. The equation defining the tangent of the convergence is $\tan\gamma = -b \cdot (\lambda - \lambda_0) / c$. Values of $s_\lambda(\phi)$, ρ, ν, h, $a(\phi_0)$, $a + b\phi_0$, k, $\tan\gamma$, γ, and the difference h - k, were calculated for several values of ϕ, ϕ_0, λ, and λ_0, with ϕ in the range 33^o to 37^o. The calculations were based on equations (14),(5),(6), and the derived expressions for h, k, and $\tan\gamma$. The value of γ was determined as $\tan^{-1}(\tan\gamma)$.

An upper bound was estimated for the relative error in d as an estimator of d^*, the ellipsoidal distance between two positions. Let $\varepsilon_\lambda = |s_\lambda(\phi) - s^*_\lambda| / s^*_\lambda$ where s^*_λ is the ellipsoidal distance per second of longitude difference that is estimated by $s_\lambda(\phi)$, according to equation (14). The relative error in calculated d that is due solely to ε_λ can be estimated. Similarly, the relative error in d, that is due to relative error ε_ϕ in s_ϕ, can be estimated, and the joint contribution of ε_ϕ and ε_λ is approximately

$$\varepsilon_d \cong \varepsilon_\phi + \varepsilon_\lambda \tag{15}$$

where $\varepsilon_d = |d - d^*|/d^*$. In deriving (15) it was assumed that the calculation of d utilizes (12) with the earlier mentioned substitution (13) of ΔE^* for ΔE. Equation (15) only suffices for errors due to the empirical representations of $s_\lambda(\phi)$ and s_ϕ. It does not include the effects of the failure of the plane distance formula to precisely represent geodesic or normal section lengths (Bomford, 1962) over large distances. Two independent estimates were made for the maximum values of both ε_ϕ and ε_λ over the range $33 \le \phi \le 36.5$. Values of ε_ϕ and ε_λ were calculated for each of the eight ϕ_1 values used to fit the empirical expressions for $s_\lambda(\phi)$ and s_ϕ. The maximum of the eight values of ε_ϕ thus calculated, together with the maximum of the eight values of ε_λ (Table 1) thus calculated, were substituted into equation (15), with the result that

Table 1. $s_\lambda(\phi)$, s_ϕ, ε_λ, and ε_ϕ values used to calculate ε_{dmax}.

Latitude	$s_\lambda(\phi)$		Difference	ε_λ	s_ϕ		Difference	ε_ϕ
deg	Calculated	Equation	m sec^{-1}		Calculated	Constant	m sec^{-1}	
33.0	25.959	25.966	0.0069	2.65E-04	30.807	30.810	0.0032	1.03E-04
33.5	25.812	25.813	0.0010	4.00E-05	30.809	30.810	0.0007	2.16E-05
34.0	25.662	25.660	0.0028	1.11E-04	30.812	30.810	0.0018	5.95E-05
34.5	25.511	25.506	0.0048	1.86E-04	30.814	30.810	0.0043	1.41E-04
35.0	25.358	25.353	0.0047	1.86E-04	30.817	30.810	0.0068	2.22E-04
35.5	25.203	25.200	0.0027	1.09E-04	30.820	30.810	0.0095	3.08E-04
36.0	25.045	25.047	0.0011	4.56E-05	30.822	30.810	0.0120	3.89E-04
36.5	24.886	24.893	0.0069	2.79E-04	30.825	30.810	0.0145	4.70E-04
Estimated Maximum Value			0.0069	2.79E-04			0.0145	4.70E-04

$\varepsilon_{dmax} \cong 7.5 \times 10^{-4}$. An independent estimate of maximum ε_d used s^*_λ, s^*_ϕ, and $s_\lambda(\phi)$ values that were based on equations (7), (8), and (14), respectively, and on the constant s_ϕ. In this case ε_ϕ and ε_λ values were calculated at 0.1° intervals over the range $33 \leq \phi \leq 33.6$. The calculated maximum value for ε_{dmax} again was 7.5×10^{-4}, provided $33 \leq \phi \leq 33.5$. With $\phi = 33.6$ included, the maximum relative error increased to 8.2×10^{-4}.

Equations (12) and (13) were used to estimate distances, d, between positions having relatively large separations. Positions selected for this test were those having whole degree values for both λ and ϕ over the ranges $33 \leq \phi \leq 37$, and $-95 \leq \lambda \leq -90$, or 30 positions in all. This led to 435 distinct d values, with 29 non-zero distances from any one position to other positions. Corresponding exact ellipsoidal distances (d^*) were calculated using the imbedded computational procedure, "ReturnGeodesicDistance", of the GIS program, ArcView 3.2. A few of the d^* values generated with the ArcView procedure were compared to values computed with the earlier mentioned NGS program, "revers3d.exe", and agreement was excellent. Absolute error and relative error, $\varepsilon_d = |d - d^*| / d^*$ were calculated for each of the 435 d and d^* pairs. Also, maximum values of the absolute error and relative error (Fig. 2.) were determined for each of the 30 base positions.

In a second test, side length and corner angle properties of reverse projected rectangles were calculated using ellipsoidal distance and azimuth procedures imbedded in ArcView 3.2. The purpose of the test was to observe the side length and corner angle distortions of small rectangular grids that are reverse-projected to the reference ellipsoid surface. The test utilized six reverse-projections of 16 plane rectangles, each having a different L X W, or different rotation in the plane with respect to its lower left corner. The four L X W combinations were 1 X 0.2, 1 X 0.7, 10 X 2, and

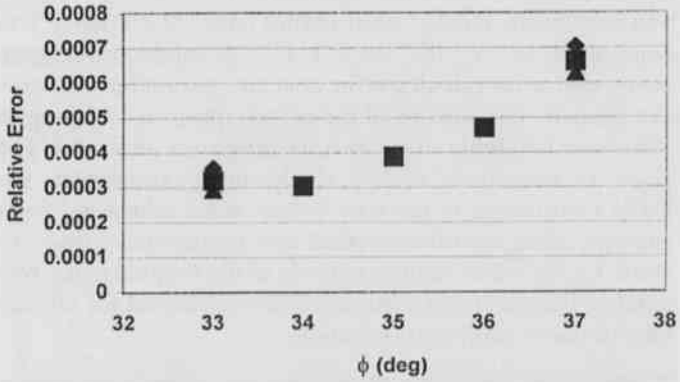

Fig. 2. Maximum relative error for the distance calculation versus latitude of base position. Diamond - $\lambda_B = -90$ and -95. Square - $\lambda_B = -91$ and -94. Triangle - $\lambda_B = -92$ and -93.

10 X 7 km. The four rotation angles were 0, 15, 50, and 75°. Plane coordinates assigned to the lower left corner of each rectangle were $(E_0, N_0) = (0, 0)$, which serves as a grid origin. Coordinates (E_i, Ni), i = 1, 2, 3, for the remaining corners were assigned according to rectangle dimensions and rotation angle (example in Table 2). Reverse projection of a rectangle was accomplished by assigning geodetic coordinates (ϕ_0, λ_0) to the grid origin, $(E_0, N_0) = (0,0)$. Equations (9) and (10) then were solved for (ϕ_i, λ_i) in terms of ϕ_0, λ_0, E_i, Ni, to complete the reverse projection of the remaining three corners. The reverse projection of the rectangles was carried out with $\lambda_0 = -90°$ or $-93°$ and with $\phi_0 = 33°$, 35°, or 37°. Side lengths of the reverse-projected rectangles were calculated by applying the ArcView 3.2 ".ReturnGeodesicDistance" procedure to adjacent corner positions. These calculated dimensions were subtracted

Table 2. Plane coordinates and parameters for calculating geodetic coordinates of the 1 X 0.2 km rectangle with 75 deg rotation with calculated corner angle and side length discrepancies

Side or Corner #	E_0 (m)	N_0 (m)	$\Delta\phi$ (deg)	ϕ_i (deg)	ϕ_a (deg)	s_λ (m sec-1)	$\Delta\lambda$ (deg)	λ_i (deg)	A - 90 (deg)	L - L_0 (m)
0	0.00	0.00	0.00000	37	0.00000	---------	0.0000	-93	0.034	0.48
1	258.50	966.10	0.00871	37.0087	37.00436	24.738	0.0029	-92.9971	0.034	0.11
2	65.60	1017.80	0.00918	37.0092	37.00459	24.738	0.0007	-92.9993	0.032	0.48
3	-193.00	51.70	0.00047	37.0005	37.00023	24.740	-0.0022	-93.0022	0.032	0.09

from corresponding original side lengths to determine a side-length discrepancy, $| L_{i'} - L_{i0} |$, i = 0, 1, 2, 3. Corner angles for the reverse-projected rectangles were determined by first calculating the forward and reverse azimuths, at each corner position. Then the difference $A_i = | az_{i,i+1} - az_{i,i-1} |$ was calculated. Finally, each corner angle discrepancy was determined as $| A_i - 90|$, i = 0, 1, 2, 3. Absolute and relative errors also were calculated for area and perimeter. The corner geodetic coordinates of the reverse-projected rectangles with base longitude, 93°, were re-projected into the UTM Zone 15 coordinate system. Following re-projection, the UTM coordinates of the base corner were subtracted from corresponding coordinates of all four reprojected corners to yield (E_i, N_i) values similar to those of the original plane rectangles. Displacement distances were calculated for all corners of the re-projected rectangles.

Discussion

Although it is beyond the scope of this manuscript to completely characterize the empirical projection, a few values of h, k, and γ were examined. Both the convergence γ and the scale factor h were relatively insensitive to variations in λ for the tests that were run, as well as to variations in ϕ. The scale factor, k, on the other hand was found to be extremely sensitive to variations in ϕ from the base latitude, ϕ_0 (Table 3). For $\phi = \phi_0$, the calculated convergence ranged from 0 to 0.29° as $\lambda - \lambda_0$ ranged from 0 to 1.0°. As $\phi = \phi_0$ increased from 33° to 37°, h decreased from 1.0001 to 0.9994, for $\lambda - \lambda_0$ = 0.01, 0.1, or 1.0. The scale factor, k, exhibited a minimum value of approximately 0.9998 for $\phi \cong$ 35°, and maximum values of 1.0003 for $\phi = \phi_0 = 33°$ and

Table 3. Principle radii of curvature and example projection parameters

$\lambda - \lambda_0 = 0.2$			$\phi_0 =$	35	$a(\phi_0) =$	110581	
ϕ (deg)	$s_\lambda(\phi)$ (m sec^{-1})	$\rho(\phi)$ (m)	ν (m)	h (rad-1)	a+bf (m deg-1)	k (rad-1)	h - k (rad-1)
34.6	25.475	6354357	6384479	1.00010	91492	0.9975	2.62E-03
34.7	25.445	6354869	6384651	1.00002	91437	0.9981	1.96E-03
34.8	25.414	6355385	6384823	0.99994	91381	0.9986	1.30E-03
34.9	25.383	6355904	6384997	0.99986	91326	0.9992	6.37E-04
35	25.353	6356427	6385172	0.99978	91271	0.9998	-3.29E-05
35.1	25.322	6356953	6385348	0.99970	91216	1.0004	-7.08E-04
35.2	25.291	6357482	6385526	0.99961	91161	1.0010	-1.39E-03
35.3	25.261	6358015	6385704	0.99953	91105	1.0016	-2.07E-03
35.4	25.230	6358551	6385883	0.99945	91050	1.0022	-2.76E-03
$\tan\gamma =$	0.001	γ (deg) =	0.000995				

1.0006 for $\phi = \phi_0 = 37^o$. By comparison, the UTM Zone 15 system's scale factor (k = h) has its minimum value, 0.9996, along the central meridian $\lambda = -93^o$. The highest values of k for UTM Zone 15 occurs along the boundary meridians $\lambda = -90^o$ and $\lambda = -96^o$, where k is approximately 1.0006 at $\phi = 33^o$ and 1.0005 at $\phi = 37^o$. The UTM system's convergence is 0^o along the central meridian and approximately 0.5^o along the -92^o meridian for ϕ between 33^o and 37^o. Returning to the empirical projection with $\phi_0 = 35^o$, k was found to increase from 0.998 to 1.002, as ϕ increases from 34.6^o to 35.4^o (Table 3), thus exhibiting nearly an order of magnitude greater variation over this limited range of ϕ than does the UTM Zone 15 scale factor over the entire state of Arkansas. The change in k between $\phi = 34.9^o$ and 35.0^o, equivalent to approximately 11 km north - south distance, is only 0.0006.

The distance absolute errors for the 6 X 5 array of base positions were found to be equal on a pair-wise basis for the -92 and -93, the -91 and -94, and the -90 and -95 degree meridian pairs. Absolute errors ranged from slightly greater than 50 meters to approximately 315 meters. The greatest absolute error, 312.8m, was associated with the separation distance between $(\lambda_1, \phi_1) = (-90, 37)$ and $(\lambda_2, \phi_2) = (-95, 37)$. The second highest absolute error, 235.5 m, was associated with distance between (-90, 36) and (-95, 37), and between (-90, 37) and (-95, 36). Thus the two highest absolute errors occurred with position pairs having at least one member-outside the fitting range for $s\lambda(\phi)$ and s ϕ, and also having a large east-west separation. In spite of this the relative error in calculated distance did not exceed the estimated upper bound, $\varepsilon_{dmax} = 7.5 \times 10^{-4}$. Where both position latitudes were $\leq 36^o$, the relative errors were lower (Fig. 2). Similarly, in the rectangle test, maximum calculated $| L_i, - L_{i0} | / L_{i0}$ was below ε_{dmax}, for $\phi_0 = 33$ (6.9×10^{-4}) and for $\phi_0 = 35$ (6.8×10^{-4}), but for $\phi_0 = 37$, maximum calculated $| L_i, - L_{i0} | / L_{i0}$ was 1.1×10^{-3} (Fig. 3), or 11 m in 10 km. Likewise, the maximum corner angle discrepancy was significantly larger for $\phi_0 = 37$ (Fig. 4) than for $\phi_0 = 33$ or $\phi_0 = 35$. Generally the corner angle discrepancy was lower for the 1 X 0.2 km and 1 X 0.7 km rectangles (0.002^o to 0.010^o) than for the 10 X 2 km and 10 X 7 km rectangles (0.007^o to 0.031^o).

Other rectangle properties followed a pattern different from the one pointed out for the side length and corner angle errors. Maximum relative error in calculated area was 8.0×10^{-4} for $\phi_B = 33^o$, 8.0×10^{-4} for $\phi_B = 35^o$, and 5.9×10^{-4} for $\phi_B = 37^o$. Maximum relative error in calculated perimeter was 3.4×10^{-4} for $\phi_B = 33^o$, 4.3×10^{-4} for $\phi_B = 35^o$, and 3.9×10^{-4} for $\phi_B = 37^o$. Finally, corners of rectangles re-projected into the UTM system along the -93^o meridian had maximum corner position displacements from the original corner positions as follows: 11.2, 7.3, and 15.8m for the large rectangles with base latitude, 33, 35, and 37, respectively; 0.8, 0.3, and 1.0m for the small rectangles in the same base latitude order. In all cases the maximum displacement was associated with the upper right corner of the rectangle with 0^o rotation.

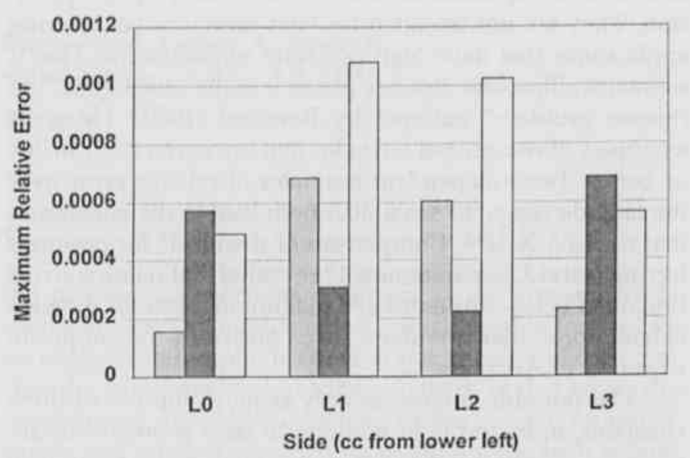

Fig. 3. Maximum relative length discrepancy for the four rectangle sides, L0, L1, L2, and L3 over all side length combinations and rotations of the rectangles. Left column - ϕ_B = 33. Center column - ϕ_B = 35. Right column - ϕ_B = 37.

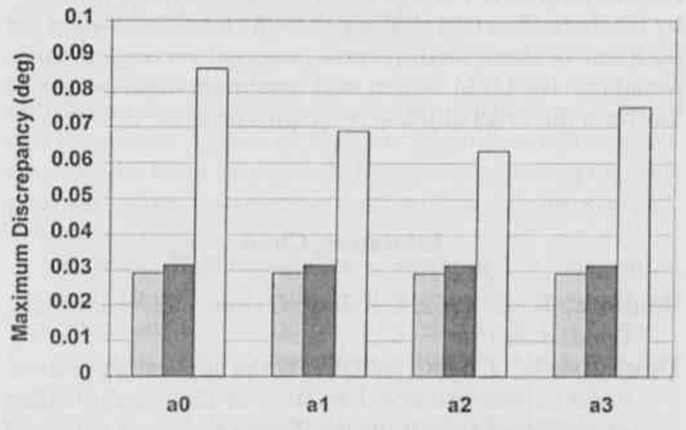

Fig. 4. Maximum corner angle discrepancy for the four rectangle corners, a0, a1, a2, and a3 over all side length combinations and rotations of the rectangles. Left column - ϕ_B = 33. Center column - ϕ_B = 35. Right column - ϕ_B = 37.

Summary

A simple formula for calculating ellipsoidal distance, d, and a portable but empirical projection based on equations (9) and (10) were presented and tested. Both are intended for field or other applications that tolerate relatively low precision. They are not intended for land survey or engineering applications that have high precision requirements. That d estimates ellipsoidal distance places it in the category of "the reverse problem" outlined by Bomford (1962). He gives examples of recognized formulas that are correct to 1 in 10^7 or better. Two independent estimates of relative error over the latitude range $33 \leq \phi \leq 36.5$ both lead to the conclusion that $\varepsilon_d \leq 7.5 \times 10^{-4}$. Comparison of d with d* for positions having ϕ and λ, separations $\leq 1^o$ revealed that relative errors remained below this estimated maximum, even for distance calculations that involved two positions on opposite extremes of Arkansas.

The portable projection only requires the use of three constants, a, b, and c, in addition to base geodetic coordinates (ϕ_0, λ_0), in order to develop a small plane coordinate grid that is referenced to the geodetic system. The grid size would be limited by the precision requirements of a given project. The empirical projection's scale factor, k, is extremely sensitive to ϕ different from ϕ_0. An example (Table 3) suffices to demonstrate the sensitivity. However, differences in k were small for a more limited range of ϕ, not exceeding a 0.1^o departure from ϕ_0. The corner angles in reverse-projected 1 X 0.7 km rectangles differed from 90^o by no more than 0.01^o. Along the -93^o meridian the corner positions of these small reverse-projected rectangles reprojected into the UTM system with maximum displacement of 1m from the original plane rectangle corners.

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