Econometric Analysis of Aggregation in the Context of Linear Prediction Models*

by

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Abstract

This paper deals with the problem of aggregation where the focus of the analysis is whether to predict aggregate variables using macro or micro equations. It generalizes the Grunfeld-Griliches prediction criterion to allow for contemporaneous covariances between the disturbances of micro equations, and the possibility of different parameteric restrictions on the equations of the disaggregate model. The paper also develops a formal statistical test of the hypothesis of 'perfect aggregation' which tests the validity of aggregation either through coefficient equality or through the stability of the composition of the regressors across the micro-units over time. The choice criterion and the perfect aggregation test are then applied to employment demand functions for the UK economy disaggregated by 40 industries.

1. Introduction

The problem of aggregation over micro units has been approached in the empirical literature from a number of different view points. In the case of linear models one important issue addressed in this literature is the problem of 'aggregation bias', defined by the deviation of the macro-parameters from the average of the corresponding micro-parameters. [See, for example, Theil (1954), Boot and de Wit (1960), Orcutt, Watts and Edwards (1968), Barker (1970), Gupta (1971), Sasaki (1978), and Winters (1980)].[1] Another closely related issue is the prediction problem originally discussed by Grunfeld and Griliches (1960), where the focus of the analysis is whether to predict aggregate variables using macro or micro equations. Our primary concern in this paper is with the prediction problem in the context of linear models. We present a generalisation of the Grunfeld-Griliches (GG) prediction criterion which allows for contemporaneous covariances between the disturbances of the micro equations, and the possibility of different linear parametric restrictions on the equations of the disaggregate model. We also develop a formal statistical test of the hypothesis of 'perfect aggregation' which, unlike the test proposed by Zellner (1962) in the context of the seemingly unrelated regression model, does not necessitate the requirement that all coefficients across the equations of the disaggregated model are the same. The proposed test allows for the possibility of valid aggregation either through coefficient equality or through the invariance of the composition of the

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(1) On the problem of aggregation across non-linear micro equations see, for example, Ando (1971), Kelejian (1980), Stoker (1984, 1986) and the references cited therein.

regressors across the micro-units over time.

The choice criterion and the test of perfect aggregation developed in the paper are then applied to two alternative specifications of employment functions for the UK economy disaggregated by 40 industries, and for the manufacturing sector disaggregated by 23 industries. As far as the choice criterion is concerned, the empirical results show that for the economy as a whole the disaggregate model fits better than the aggregate specification, while the reverse is true for the manufacturing industries taken as a group. The slightly better fit obtained for the aggregate model in the case of the manufacturing industries should not, however, be taken to mean that there are no aggregation problems at this level. In fact the application of the test of perfect aggregation to the employment functions provides strong evidence in favour of rejecting the hypothesis of perfect aggregation both for the economy as a whole, and for the manufacturing sector. Our results also suggest serious upward bias in the estimates of output and real wage elasticities of aggregate employment demand obtained for the UK in the literature using aggregate relations. The slightly better within-sample performance of the aggregate specification in the case of the manufacturing industries is best interpreted as an indication of the misspecification of the disaggregate equations.

The plan of the paper is as follows. Section 2 sets out the basic econometric framework. Section 3 examines the small sample bias of the GG prediction criterion. Section 4 generalises the basic model so that different specifications for the micro-equations is possible, and derives a goodness-of-fit criterion for discrimination between aggregate and disaggregate models that does not suffer from the small sample problem. Section 5 considers alternative methods of testing for the errors of aggregation, and develops a new test of the hypothesis of perfect aggregation. Section 6 deals with the problem of misspecification of the disaggregate model and the implications that this has for the use of the proposed choice criterion. Section 7 contains a detailed application of the econometric methods developed in the paper to the UK employment functions.

2. The basic econometric framework

We start with the micro-model analysed by Theil (1954), and subsequently by Grunfeld and Griliches (1960), and others, and suppose that the n observations of the m micro-units $\{y_{it},\ i = 1, 2, \ldots, m;\ t = 1, 2, \ldots, n\}$ are generated according to the following linear specifications

$$y_{it} = \sum_{j=1}^{k} \beta_{ij}\, x_{i,jt} + u_{it}, \qquad \begin{array}{l} i = 1, 2, \ldots, m \\ t = 1, 2, \ldots, n \end{array}$$

or in matrix notations (Kloek, 1961)

$$(2.1) \qquad H_d:\ \underset{n\times 1}{\underset{\sim}{y}_i} = \underset{n\times k}{X_i}\ \underset{k\times 1}{\underset{\sim}{\beta}_i} + \underset{n\times 1}{\underset{\sim}{u}_i}, \qquad i = 1, 2, \ldots, m.$$

In the above specification it is assumed that the variations in dependent variables of all micro-units can be explained by means of linear combination of the same set of k explanatory variables. This assumption will be relaxed in the next section.

Writing (2.1) as a System of Seemingly Unrelated Equations (SURE), following Zellner (1962) we have

$$(2.2) \qquad \underset{\sim}{y} = X \underset{\sim}{\beta} + \underset{\sim}{u}$$

where $\underset{\sim}{y} = (\underset{\sim}{y}_1', \underset{\sim}{y}_2', \ldots, \underset{\sim}{y}_m')'$, $\underset{\sim}{\beta} = (\underset{\sim}{\beta}_1', \underset{\sim}{\beta}_2', \ldots, \underset{\sim}{\beta}_m')'$, $\underset{\sim}{u} = (\underset{\sim}{u}_1', \underset{\sim}{u}_2', \ldots, \underset{\sim}{u}_m')'$, and X is an $mn \times mk$ block-diagonal matrix of full column rank with matrix X_i as its ith block. We also make the following assumption:

Assumption 1: The $mn \times 1$ disturbance vector $\underset{\sim}{u}$ is distributed independently of X, has mean zero and the variance matrix $\Omega = \Sigma \otimes I_n$, where $\Sigma = (\sigma_{ij})$, and I_n is the identity matrix of order n.

The problem of aggregation can arise when an investigator interested in the behaviour of the macro-variable $\underset{\sim}{y}_a = \sum_{i=1}^{m} \underset{\sim}{y}_i$, considers the single macro-equation

$$(2.3) \qquad H_a: \underset{n\times 1}{\underset{\sim}{y}_a} = \underset{n\times k}{X_a} \; \underset{k\times 1}{\underset{\sim}{b}} + \underset{n\times 1}{\underset{\sim}{v}_a}$$

where $X_a = \sum_{i=1}^{m} X_i$, instead of the m micro-equations in (2.1). Following Grunfeld and Griliches (1960) we examine the question of whether to predict $\underset{\sim}{y}_a$ using the macro-equation (2.4), or the micro-equation (2.3), or the micro-equations (2.1).

3. The small sample bias of the Grunfeld-Griliches criterion

The GG prediction (or more accurately the within-sample goodness-of-fit) criterion for the discrimination between the disaggregate model, H_d and the aggregate model, H_a can be written as

$$\boxed{\text{Choose } H_d \text{ if } \underset{\sim}{e}_d'\underset{\sim}{e}_d < \underset{\sim}{e}_a'\underset{\sim}{e}_a, \text{ otherwise choose } H_a,}$$

where $\underset{\sim}{e}_d$ and $\underset{\sim}{e}_a$ are the estimates of the errors in predicting $\underset{\sim}{y}_a$ under H_d and H_a respectively. The estimates employed by GG for $\underset{\sim}{e}_d$ and $\underset{\sim}{e}_a$ are based on the Ordinary Least Squares (OLS) method and are given by

$$(3.1) \qquad \underset{\sim}{e}_a = M_a\underset{\sim}{y}_a, \quad M_a = I_n - X_a(X_a'X_a)^{-1}X_a' = I_n - A_a ,$$

and

$$(3.2) \qquad \underset{\sim}{e}_d = \sum_{i=1}^{m} M_i \underset{\sim}{y}_i, \quad M_i = I_n - X_i(X_i'X_i)^{-1}X_i' = I_n - A_i .$$

It is important to note that in general $\underset{\sim}{e}_d$ is not an efficient estimator of $\underset{\sim}{u}_d = \underset{\sim}{y}_a - \sum_{i=1}^{m} X_i \underset{\sim}{\beta}_i$, unless the disturbances of the micro-equations are contemporaneously uncorrelated (i.e. $\sigma_{ij} = 0$, for $i \neq j$), or when X_i can be written as exact linear functions of X_a. The problem of efficient estimation of $\underset{\sim}{\beta}_i$, and hence $\underset{\sim}{u}_d$, and the effect that this has for the GG criterion will be discussed later. For the moment we assume that the GG criterion, as specified above, is applied even in the case where the micro-equation disturbances are contemporaneously correlated, and investigate the small sample bias that such a procedure entails.

Like the justification offered for Theil's $\bar{R}^2$ criterion, the rationale behind the use of the GG criterion must lie in the fact that if the micro-equations are correctly specified, then 'on average' the fit of $\underset{\sim}{y}_a$ from the macro-equation should not be any better than that obtained from the micro-equations. That is we should have

$$(3.3) \qquad E_d(\underset{\sim}{e}_d'\underset{\sim}{e}_d) \leqslant E_d(\underset{\sim}{e}_a'\underset{\sim}{e}_a) \;,$$

where $E_d(\cdot)$ represents the mathematical expectations operator under H_d. However, using (3.1) and (3.2) it is easily seen that[1]

$$E_d(\underset{\sim}{e}_d' \underset{\sim}{e}_d) - E_d(\underset{\sim}{e}_a' \underset{\sim}{e}_a) = -E(\underset{\sim}{\xi}'M_a\underset{\sim}{\xi}) - 2\sum_{s=1}^{k}\sum_{i>j}^{m} \sigma_{ij}\{1 - E(\rho_{s,ij}^2)\},$$

where $\underset{\sim}{\xi} = \sum_{i=1}^{m} X_i \beta_i - X_a \underset{\sim}{b}$, and $\rho_{s,ij}$ is the sth canonical correlation coefficient between the explanatory variables of the ith and the jth micro-equations. Therefore, in general the inequality condition (3.3) need not be satisfied even if H_d is correctly specified. There are, however, two circumstances under which the the GG criterion satisfies the inequality relationship (3.3).

(i) - when X_i can be written as <u>exact</u> linear functions of X_j, for all i and j. In this case $\rho_{s,ij}^2 = 1$, and irrespective of the values of σ_{ij} we have $E_d(\underset{\sim}{e}_d' \underset{\sim}{e}_d) - E_d(\underset{\sim}{e}_a' \underset{\sim}{e}_a) = -E(\underset{\sim}{\xi}'M_a\underset{\sim}{\xi})$.

(ii) - when the micro-disturbances are all contemporaneously uncorrelated ($\sigma_{ij} = 0$, $i \neq j$). In general the direction of the bias involved in the use of the GG criterion in small samples depends on the signs of σ_{ij}, for $i \neq j$.

The finite sample bias in the use of the GG criterion will not disappear even when $\underset{\sim}{\beta}_i$ are estimated efficiently by the SURE method. Consider the simple case where Σ is known. The SURE estimator of $\underset{\sim}{u}_d$,

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(1) In deriving this result we have also made use of the relation $k - \mathrm{Tr}(A_i A_j) = \sum_{s=1}^{k} (1 - \rho_{s,ij}^2) \geqslant 0$. See, for example, Rao (1973, pp. 582-587).

which we denote by $\underset{\sim}{e}_s$, will be

$$\underset{\sim}{e}_s = S(I_{nm} - A)\underset{\sim}{y}$$

where S stands for the n × nm summation matrix

$$(3.4) \qquad S = [I_n \vdots I_n \vdots \dots \vdots I_n] ,$$

and

$$(3.5) \qquad A = X(X'\Omega^{-1}X)^{-1}X'\Omega^{-1}.$$

Under H_d, $\underset{\sim}{e}_s = S(I_{nm} - A)\underset{\sim}{u}$, and hence

$$E_d(\underset{\sim}{e}_s' \underset{\sim}{e}_s) - E_d(\underset{\sim}{e}_a' \underset{\sim}{e}_a) = k\,\sigma_a^2 - E(\underset{\sim}{\xi}'M_a\underset{\sim}{\xi}) - E\{Tr[(X'\Omega^{-1}X)^{-1}X'S'SX]\} ,$$

where $\sigma_a^2 = \sum_{i,j=1}^{m} \sigma_{ij}$. Again leaving the case where X_i are exact linear functions of X_a to one side, the strict inequality $E_d(\underset{\sim}{e}_s'\underset{\sim}{e}_s) \leqslant E_d(\underset{\sim}{e}_a'\underset{\sim}{e}_a)$ holds only in the special case where $\sigma_{ij} = 0$, for $i \neq j$.

4. A generalised goodness-of-fit criterion for discrimination between aggregate and disaggregate models

From the results of the previous section it is now a straightforward matter to derive a choice criterion for discrimination between the disaggregate and the aggregate models that does not suffer from the finite sample bias of the GG criterion. But it is first important to extend the econometric framework of section 2, so that different specifications for the micro-equations can be considered. Such a generalisation is particularly important when the primary purpose of the disaggregation is to achieve a better explanation of the macro-variables. Accordingly, we consider the following specifications for the disaggregate and the aggregate models:

$$\tilde{H}_d : \underset{n\times 1}{\underset{\sim}{y}_i} = \underset{n\times k_i}{X_i} \underset{k_i\times 1}{\underset{\sim}{\beta}_i} + \underset{n\times 1}{\underset{\sim}{u}_i} ; \qquad i = 1, 2, \ldots, m ,$$

$$\tilde{H}_a : \underset{n\times 1}{\underset{\sim}{y}_a} = \underset{n\times k_a}{X_a} \underset{k_a\times 1}{\underset{\sim}{b}} + \underset{n\times 1}{\underset{\sim}{v}_a} ,$$

where $\text{Rank}(X_i) = k_i$, and $\text{Rank}(X_a) = k_a$. In this formulation there are no restrictions on the number of columns of X_i, or what these columns may represent. The micro-equations under $\tilde{H}_d$ can also be viewed as a restricted version of the equations under H_d, with each micro-equation having its own specific linear parametric restrictions. In this way a wide range of different specifications across the micro-equations can be allowed for. The specification of the macro-equation is also generalized so that the investigator can specify a restricted form of the macro-equation defined in (2.3).

Consider now the following 'adjusted' goodness-of-fit criteria for the aggregate and the disaggregate models

$$(4.1) \qquad s_a^2 = \underset{\sim}{e}_a' \underset{\sim}{e}_a / (n - k_a) ,$$

and

$$(4.2) \qquad s_d^2 = \sum_{i,j=1}^{m} \hat{\sigma}_{ij},$$

where

$$(4.3) \qquad \hat{\sigma}_{ij} = \{n - k_i - k_j + \text{Tr}(A_i A_j)\}^{-1} \underset{\sim}{e}_i' \underset{\sim}{e}_j ,$$

with $\underset{\sim}{e}_a$ and $\underset{\sim}{e}_i$ being respectively the OLS residual vectors of the regressions under $\tilde{H}_a$ and $\tilde{H}_d$, and $A_i = X_i(X_i'X_i)^{-1}X_i'$. The use of s_d^2 as a measure of the goodness-of-fit of the disaggregate model is justified on the grounds that it represents an unbiased (and consistent) estimator of $\sigma_a^2 = V(\sum_{i=1}^{m} u_{it})$, the population variance of the error of predicting $\underset{\sim}{y}_a$ from the disaggregate model. It is now easily seen that under $\tilde{H}_d$,

$$(4.4) \qquad E_d(s_d^2) - E_d(s_a^2) = -(n - k_a)^{-1}E(\underset{\sim}{\xi}'M_a\underset{\sim}{\xi}) < 0,$$

where $\underset{\sim}{\xi}$ is now defined by

$$(4.5) \qquad \underset{\sim}{\xi} = \sum_{i=1}^{m} X_i\underset{\sim}{\beta}_i - X_a\underset{\sim}{b} .$$

Therefore, as required we have $E_d(s_d^2) < E_d(s_d^2)$, and unlike the GG criterion, the use of the proposed goodness-of-fit criteria s_a^2 and s_d^2 will 'on average' result in the choice of the disaggregate model in finite samples, assuming, of course, that the disaggregate model is correctly specified. In situations where the disaggregate model fits worse than the aggregate model (i.e. $s_d^2 > s_a^2$), it is likely that the disaggregate model is misspecified. The implications for the above choice criterion when the disaggregate model is subject to errors of specification will be discussed below. Here, for comparison purposes it is worth considering the following decomposition of the s_d^2 criterion.

$$(4.6) \qquad s_d^2 = (n - k_a)^{-1} \underset{\sim}{e}_d' \underset{\sim}{e}_d + (n - k_a)^{-1} \sum_{i=1}^{m} (k_i - k_a)\hat{\sigma}_{ii}$$

$$+ \; 2(n - k_a)^{-1} \sum_{i>j}^{m} \{\Phi_{ij}/(1 - \Phi_{ij})\} \underset{\sim}{e}_i' \underset{\sim}{e}_j \; ,$$

where $\underset{\sim}{e}_d = \sum_{i=1}^{m} \underset{\sim}{e}_i$, and

$$\Phi_{ij} = (n - k_a)^{-1}\{k_i + k_j - k_a - \mathrm{Tr}(A_i A_j)\} \; .$$

The GG prediction criterion focuses on the first term on the right-hand side of (4.6) and ignores the asymptotically negligible second and third terms. The second term represents the contribution to the s_d^2-criterion arising out of the possible differences in the number of estimated coefficients between the aggregate and the disaggregate models. The third term in (4.6) captures the effect of the contemporaneous correlation amongst the disturbances of the micro-equations.

5. Tests of aggregation

In studying the aggregation problem our emphasis so far has been on the model selection procedures. An alternative approach would be to employ classical hypothesis testing procedures and develop a statistical test of the conditions necessary for valid aggregation. In the context of the generalized disaggregate model $\tilde{H}_d$, the necessary condition for perfect aggregation is given by $\underset{\sim}{\xi}=0$, where $\underset{\sim}{\xi}$ is defined in (4.5). Under the hypothesis of 'perfect aggregation'

$$H_{\xi} : \underset{\sim}{\xi} = \sum_{i=1}^{m} X_i \underset{\sim}{\beta}_i - X_a \underset{\sim}{b} = 0 ,$$

it readily follows from (4.4) that $E_d^2(s_d) = E_d(s_a^2) = \sigma_a^2$, and as far as the fit of $\underset{\sim}{y}_a$ is concerned we should not expect to gain from disaggregation.(1)

Before developing a formal test of H_{ξ}, it is important to note that the condition $\underset{\sim}{\xi} = 0$ can be given a meaningful interpretation only in the context of the basic model (2.1) where $\underset{\sim}{\beta}_i$ are of the same dimension and refer to the same type of variables across the micro-equations. In this case the condition $\underset{\sim}{\xi} = 0$ is clearly satisfied under the 'micro-homogeneity' hypothesis,(2)

$$H_{\beta} : \underset{\sim}{\beta}_1 = \underset{\sim}{\beta}_2 = \dots = \underset{\sim}{\beta}_m .$$

This is not, however, the only situation where H_{ξ} holds. Another hypothesis of interest which yields $\underset{\sim}{\xi} = 0$, is the 'compositional stability' hypothesis

$$H_x : X_i = X_a C_i , \qquad i = 1, 2, \dots, m,$$

where C_i are $k \times k$ non-singular matrices of fixed constants, such that $\sum_{i=1}^{m} C_i = I_k$. The 'compositional stability' hypothesis represents a set of restrictions on the joint probability distribution of the regressors and states that the composition of the regressors across

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(1) For the basic disaggregated model (2.1), the hypothesis H_{ξ} is equivalent to the n-covariance condition discussed in Theil (1954) and Lancaster (1966), in the special case where the number of regressors is equal to one.

(2) Notice that this hypothesis can not hold under the generalized disaggregated model $\tilde{H}_d$.

micro-units remain fixed over time. This condition for valid aggregation in linear models has been discussed in the econometric literature by Klein (1953) and Wold and Juréen (1953). Distributional assumptions on the regressors have also been employed in the literature, for example, by Ando (1971), McFadden and Reid (1975), Kelejian (1980), and more recently by Stoker (1984) to connect the aggregate function to the underlying micro-equations in the context of non-linear models. Under H_x, the macro-coefficient vector $\underset{\sim}{b}$, is

defined in terms of the micro-coefficients through the identity $\underset{\sim}{b} = \sum_{i=1}^{m} C_i \underset{\sim}{\beta}_i$. The condition $\xi = 0$ will also be met under the mixed hypothesis(1)

$$H_{\beta x}: \quad X_i = X_a C_i, \qquad i = 1, \ldots, s, \qquad s < m$$

$$\underset{\sim}{\beta}_{s+1} = \underset{\sim}{\beta}_{s+2} = \ldots = \underset{\sim}{\beta}_m = \underset{\sim}{b}_1$$

where in this case $X_a = \sum_{i=1}^{s} X_i$, $\sum_{i=1}^{s} C_i = I_k$ and $\underset{\sim}{b}_i = \sum_{i=1}^{s} C_i \underset{\sim}{\beta}$.

The test proposed by Zellner (1962) for aggregation bias is a test of the micro-homogeneity hypothesis, H_β, and is not necessarily relevant as a test of H_ξ: $\underset{\sim}{\xi} = 0$. The Zellner test can therefore be unduly restrictive. Rejection of H_β does not necessarily imply that the perfect aggregation hypothesis H_ξ should also be rejected. What is needed is a direct test of $\underset{\sim}{\xi} = 0$. In what follows we develop such a test in the case of the basic disaggregated model (2.1) and the

(1) The aggregation condition is also met by an alternative mixed hypothesis where the k regressors X_i can be partitioned into two sub-sets, one of which satisfies the compositional stability hypothesis and the other having an associated parameter vector satisfying the micro-homogeneity hypothesis.

aggregate model (2.3). Although our results can be extended to the generalized model $\tilde{H}_d$, we have chosen not to do this here, since we do not think that the perfect aggregation condition $\xi = 0$ can be given a plausible interpretation under $\tilde{H}_d$. In the case of the generalized model neither the micro-homogeneity hypothesis nor the compositional stability hypothesis can be maintained.

5.1 A test of perfect aggregation: case of known Σ

To help clarify the nature of the test that we are proposing, we first develop the test in the case where Σ, the covariance matrix of the micro-disturbances, is known. A computationally feasible version of the test will be discussed in Section 5.2.

The idea behind the test is straightforward and asks whether the estimator of ξ is significantly different from zero. When Σ is known an efficient estimator of ξ is given by

$$(5.1) \qquad \tilde{\xi} = SX\tilde{\beta} - X_a\hat{b},$$

where $\tilde{\beta}$ and $\hat{b}$ are the SURE and the OLS estimators of the parameters of the disaggregate and the aggregate equations respectively, and S is the summation matrix defined by (3.4). Substituting $\tilde{\beta} = (X'\Omega^{-1}X)^{-1}X'\Omega^{-1}y$, $\hat{b} = (X_a'X_a)^{-1}X_a'y_a$ in (5.1) now yields $\tilde{\xi} = Hy$ where $H = SA - A_aS$. The matrices A_a and A are already defined by (3.1) and (3.5), respectively. On the null hypothesis that $\xi = \sum_{i=1}^{m} X_i\beta_i - X_a b = 0$, we have $\tilde{\xi} = Hu$. Therefore, under the assumption that u is normally distributed with zero means and a known non-singular variance matrix $\Omega = \Sigma \otimes I_n$

(5.2) $$\tilde{\xi}'(H\Omega H')^{-1}\tilde{\xi} \sim \chi^2_n .$$

A necessary condition for $H\Omega H'$ to have a full rank can be obtained in the following manner: since, by assumption Ω is a non-singular matrix then Rank$(H\Omega H')$ = Rank(H). But,

$$\text{Rank (H)} \leqslant \text{Rank (SA)} + \text{Rank } (A_aS) ,$$

$$\text{Rank } (A_aS) = \text{Rank } (A_a) = k ,$$

$$\text{Rank (A)} = \text{Tr(A)} = mk, \quad \text{Rank (S)} = n$$

and

$$\text{Rank (SA)} \leqslant \text{Min}(n, mk) .$$

Consequently, Rank (H) $\leqslant$ k + Min(n, mk), and for matrix H to have full rank equal to n, it is necessary that k + Min(n, mk) $\geqslant$ n, or

(5.3) $$k(m + 1) \geqslant n .$$

This rank condition is clearly satisfied when m is large relative to n/k. But in situations where the number of micro-equations is relatively large, the computational burden of obtaining the SURE estimates, $\tilde{\beta}$ in (5.1) can be considerable. One possibility would be to construct a test of H_ξ based on the OLS estimates of β instead of the SURE estimates. The estimate of ξ based on the OLS estimators is given by

$$\hat{\xi} = \sum_{i=1}^{m} X_i\hat{\beta}_i - X_a\hat{b} = e_a - a_d ,$$

where e_a and e_d are already defined by (3.1) and (3.2), respectively.

Under H_d, and on the assumption that the hypothesis of perfect aggregation H_ξ holds, we have

$$(5.4) \qquad \hat{\underset{\sim}{\xi}} = \sum_{i=1}^{m} (A_i - A_a)\underset{\sim}{u}_i = \sum_{i=1}^{m} H_i \underset{\sim}{u}_i \ .$$

Now assuming that $\underset{\sim}{u}_i$ are normally distributed, then conditional on X_i we have

$$m^{-\frac{1}{2}} \hat{\underset{\sim}{\xi}} | X_i \sim N(0, \Psi_m) \ ,$$

where

$$(5.5) \qquad \Psi_m = m^{-1} \sum_{i,j=1}^{m} \sigma_{ij} H_i H_j' \ .$$

Therefore, assuming that Ψ_m is a non-singular matrix,(1) we arrive at the result

$$(5.6) \qquad m^{-1}(\underset{\sim}{e}_a - \underset{\sim}{e}_d)' \Psi_m^{-1} (\underset{\sim}{e}_a - \underset{\sim}{e}_d) \sim \chi_n^2 \ ,$$

which is the OLS counterpart of (5.2).

5.2 Case of unknown Σ

When $\Sigma = (\sigma_{ij})$ is unknown, it is still possible to obtain an 'approximate' test of the perfect aggregation hypothesis by replacing σ_{ij} in (5.2) or (5.6) with their SURE or the OLS estimates. Here, we focus on the latter and consider testing H_ξ by means of the statistic

$$(5.7) \qquad \underset{\sim}{a}_m = m^{-1}(\underset{\sim}{e}_a - \underset{\sim}{e}_d)' \hat{\Psi}_m^{-1} (\underset{\sim}{e}_a - \underset{\sim}{e}_d) \ ,$$

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(1) Notice that a necessary condition for Ψ_m to be invertible is given by (5.3).

where

$$(5.8) \qquad \hat{\Psi}_m = m^{-1} \sum_{i,j=1}^{m} \hat{\sigma}_{ij} H_i H_j ,$$

$$(5.9) \qquad \hat{\sigma}_{ij} = \{n - 2k + \mathrm{Tr}(A_i A_j)\}^{-1} e_i' e_j .$$

We shall refer to a test of H_ς based on (5.7) as the perfect aggregation test, or the a-test for short.

The exact distribution of the a_m statistic under H_ς is no longer a χ^2_n, and unfortunately does not lend itself to a simple derivation either. But it is possible to approximate the distribution of a_m by means of a 'suitable' limiting distribution. The usual asymptotic theory where the limiting distribution is obtained by letting n, the sample size, tend to infinity is clearly not applicable here. A relevant asymptotic framework for testing the hypothesis of perfect aggregation is to allow the level of disaggregation, m, to increase without a bound, while keeping the sample size n, fixed. The idea of expanding the micro units to obtain distribution properties in an aggregate framework is not particularly novel and has been used by Powell and Stoker (1985), and Granger (1987). In our application of the large m-asymptotics we make the following assumptions:

Assumption 2: The average matrix $\bar{X}_m = m^{-1}X_a$, and the aggregate projection matrix $A_a = \bar{X}_m(\bar{X}_m'\bar{X}_m)^{-1}\bar{X}_m'$, converge (in probability) to finite limits.

Assumption 3: The elements of the disaggregate projection matrices, $A_i = X_i(X_i'X_i)^{-1}X_i'$ remain bounded in absolute value as $m \to \infty$. Notationally we write $|A_i| < P < \infty$.

Assumption 4: The elements of the variance matrix $\Sigma = (\sigma_{ij})$ remain bounded as $m \to \infty$. Namely, $|\sigma_{ij}| < \tau^2 < \infty$, $\forall i,j$.

Assumption 5: the variance matrix Ψ defined by

$$\Psi_m = m^{-1} \sum_{i,j=1}^{m} \sigma_{ij} H_i H_j$$

tends to a non-singular matrix Ψ, as $m \to \infty$.

When assumptions 1-5 hold, it seems reasonable to suppose that the distribution of $\underline{a}_m$, on the null hypothesis of perfect aggregation will tend towards a χ^2_n as $m \to \infty$. Although, at this stage we are not able to present a proof of this proposition in its present form, we can nevertheless offer the following less general theorem.

Theorem: consider the disaggregate model (2.1) and suppose that the standardised micro-disturbances $u_{it}/\sqrt{\sigma_{ii}}$ are identically distributed, independently both across time periods and across equations, with zero means, unit variances and finite third order moments. Then conditional on X, and under assumptions 2-5, the statistic

$$\underline{a}_m = (\underset{\sim}{e}_a - \underset{\sim}{e}_d)'\left(\sum_{i=1}^{m} \hat{\sigma}_{ii} H_i^2\right)^{-1}(\underset{\sim}{e}_a - \underset{\sim}{e}_d) ,$$

will be asymptotically distributed as a χ^2_n variate on the null hypothesis of perfect aggregation (i.e. $\underset{\sim}{\xi} = 0$), as $m \to \infty$.

(See Appendix A for a proof)

It is worth noting that the above theorem is applicable even when micro-equations contain lagged dependent variables, macro-variables, or other common variables such as an intercept term or a time trend.[1]

6. Disaggregation and specification error

The model selection criterion and the aggregation test developed in this paper are based on the assumption that the disaggregate model is correctly specified. In reality, however, both the disaggregate and the aggregate models may suffer from errors of specification, with the latter also being subject to the additional problem of aggregation error. In such a circumstance the issue of whether disaggregation is useful for the study of macro-phenomena and the extent of the gain that may be expected from disaggregation depends very much on the relative importance of the two types of errors of specification and aggregation. In this section the implications that errors of specification may have for the use of our proposed choice criterion will be examined.

Let the correctly specified disaggregate model be

$$\underset{n\times 1}{\underset{\sim}{y}_i} = \underset{n\times k_i}{X_i}\ \underset{k_i\times 1}{\underset{\sim}{\beta}_i} + \underset{n\times s_i}{W_i}\ \underset{s_i\times 1}{\underset{\sim}{\gamma}_i} + \underset{n\times 1}{\underset{\sim}{u}_i}, \qquad i = 1, 2, \ldots, m \tag{6.1}$$

which in a stacked form can also be written as

---

1. Here it is assumed that the micro-disturbances, u_{it} are serially uncorrelated; otherwise the estimates can be badly biased if lagged values of the dependent variable are included in the disaggregate equations.

(6.2) $\quad \underset{\sim}{y} = X\underset{\sim}{\beta} + W\underset{\sim}{\gamma} + \underset{\sim}{u}$,

where X is now an $mn \times \tilde{k}$ ($\tilde{k} = \Sigma_{i=1}^{m} k_i$) block diagonal matrix with X_i as its ith block, $\underset{\sim}{\gamma} = (\underset{\sim}{\gamma}_1', \underset{\sim}{\gamma}_2', \ldots, \underset{\sim}{\gamma}_m')'$, and W is an $mn \times \tilde{s}$, ($\tilde{s} = \Sigma_{i=1}^{m} s_i$) block-diagonal matrix with W_i on its ith block. The other notations are as in relation (2.2). Suppose now that a researcher misspecifies this model by omitting the variables in W, and continues to employ the model selection criterion based on s_a^2 and s_d^2, defined by (4.1) and (4.2) respectively. Clearly, the result $E_d(s_d^2) \leqslant E_d(s_a^2)$, which provided the rationale for the choice criterion, need no longer hold.

Stacking the OLS residuals $\underset{\sim}{e}_i = M_i\underset{\sim}{y}_i$ in the vector $\underset{\sim}{e} = (\underset{\sim}{e}_1', \underset{\sim}{e}_2', \ldots, \underset{\sim}{e}_m')'$, s_d^2 can also be written as $s_d^2 = \underset{\sim}{e}'L\underset{\sim}{e}$, in which $L = (\Lambda \otimes I_n)$, and Λ is an $m \times m$ matrix with a typical element equal to $[Tr(M_iM_j)]^{-1}$. Now under the correctly specified model (6.2),

$$\underset{\sim}{e} = M\underset{\sim}{y} , \qquad M = I_{mn} - X(X'X)^{-1}X' ,$$

$$= MW\gamma + Mu .$$

Hence

(6.3) $\quad E_d(s_d^2 | X, W) = \sigma_a^2 + \underset{\sim}{\gamma}'W'MLMW\underset{\sim}{\gamma}$.

Since in general L may not be a positive semi-definite matrix, without further information about the nature of the specification error, it will not be possible to say whether misspecification leads to an upward or a downward bias in the application of the choice criterion. Expanding (6.3) in terms of the misspecification of the individual micro-equations we have

$$(6.3)' \quad E_d(s_d^2|X, W) = \sigma_a^2 + (n - k_a)^{-1} \sum_{i=1}^{m} \underset{\sim}{d}_i'\underset{\sim}{d}_i + 2 \sum_{i>j}^{m} \{\underset{\sim}{d}_i'\underset{\sim}{d}_j/Tr(M_iM_j)\} ,$$

where $\underset{\sim}{d}_i = M_iW_i\underset{\sim}{\gamma}_i$, and $Tr(M_iM_j) = n - k_i - k_j + Tr(A_iA_j)$. The direction of the bias resulting from misspecification clearly depends on the sign of the cross-equation terms $\underset{\sim}{d}_i'\underset{\sim}{d}_j$, $i \neq j$, and their quantitative importance relative to the equation-specific terms $\underset{\sim}{d}_i'\underset{\sim}{d}_i$. In practice, however, it is reasonable to expect that $E_d(s_d^2) > \sigma_a^2$.

Now turning to the s_a^2 criterion, under (6.1) we obtain

$$(6.4) \quad E_d(s_a^2|X, W) = \sigma_a^2 + (n - k_a)^{-1}\underset{\sim}{\xi}'M_a\underset{\sim}{\xi} \geq \sigma_a^2 ,$$

where

$$(6.5) \quad \underset{\sim}{\xi} = \sum_{i=1}^{m} X_i\underset{\sim}{\beta}_i + \sum_{i=1}^{m} W_i\underset{\sim}{\gamma}_i = \underset{\sim}{\xi}_a + \underset{\sim}{\xi}_s .$$

Comparing (6.3) and (6.4) it is clear that in general it is not possible to say whether $E_d(s_a^2)$ exceeds $E_d(s_d^2)$. The result depends on the relative importance of the specification error and the aggregation error for the explanation of the macro-variable y_a. In their work, Grunfeld and Griliches (1960), consider a special case of some interest where there are micro-specification errors that cancel out in the aggregate. In the context of model (6.1) this can arise either when there are, for example, errors of measurement in the micro-variables that cancel out exactly in the aggregate[1] (i.e.

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(1) The problem of measurement errors in a disaggregate model in the special case where $m = k = 2$ is discussed by Aigner and Goldfeld (1974).

$\underset{\sim}{\xi}_s = \sum_{i=1}^{m} W_i \underset{\sim}{\gamma}_i = 0$), or when the micro-specification errors involve omission of macro-variables already included in the aggregate model,[1] (i.e. $M_a \underset{\sim}{\xi}_s = 0$). In such a case, using (6.4), we have

$$E_d(s_a^2 | X, W) = \sigma_a^2 + (n - k_a)^{-1} \underset{\sim}{\xi}_a' M_a \underset{\sim}{\xi}_a ,$$

and only aggregation errors ($\underset{\sim}{\xi}_a \neq 0$) cause the expectations of s_a^2 to exceed the true error variance of the aggregate model. However, even in this special case it is not possible to say whether it is better to use the aggregate model. The answer still depends on the relative importance of the micro-specification errors in the disaggregate model and the aggregation error in the aggregate model for the explanation and prediction of macro-behaviour. The issue of whether one should choose the aggregate or the disaggregate model cannot be resolved by *a priori* reasoning alone and has to be settled with respect to particular problems and in the context of specific models.

7. Applications: employment demand functions in the UK

In this section the methods described in the preceding sections will be applied to the annual estimates of disaggregate and aggregate

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(1) It is beyond the scope of the present paper to go into the reasons for the importance of macro-variables in the explanation of micro-behaviour. In general they may arise because individual micro-behavioural relations are not independent but are influenced or constrained by outcomes (or expectations of outcome) of the market as a whole.

employment demand functions for the UK economy. Although, our emphasis will be on the aggregation problem, it is hoped that the disaggregate results are of some interest in their own right.

Our empirical analysis is based on the Cambridge Growth Project Databank and uses a consistent set of data on man hours (EH_i), outputs (Y_i), and real product wages (W_i) across 41 industry groups. Details of the data and the sources are given in the data appendix. For the employment equation at the industry level we have adopted the following fairly general log-linear dynamic specification

$$
\begin{aligned}
(7.1) \qquad LEH_{it} = {} & \beta_{i1}/m + \beta_{i2}(T_t/m) + \beta_{i3}LEH_{i,t-1} + \beta_{i4}LEH_{i,t-2} \\
& + \beta_{i5}LY_{it} + \beta_{i6}LY_{i,t-1} + \beta_{i7}LW_{it} + \beta_{i8}LW_{i,t-1} \\
& + \beta_{i9}(SLYT_t/m) + \beta_{i,10}(SLYT_{t-1}/m) + u_{it}, \\
& \qquad i = 1, 2, 3, 5, 6, \ldots, 41, \\
& \qquad t = 1956, 1957, \ldots, 1984,
\end{aligned}
$$

where

LEH_{it} = log of man-hours employed in industry i at time t,

T_t = time trend ($T_{1980} = 0$),

LY_{it} = log of industry i output at time t,

LW_{it} = log of average real wage rate per man-hours employed in industry i at time t,

$$SLYT_t = \sum_{\substack{i=1 \\ i\neq 4}}^{41} LY_{it} .$$

Industry 4 (Mineral oil and natural gas) is excluded from the analysis, on the grounds that output and employment in this industry have been negligible before 1975.

The above specification for the employment demand function can be justified theoretically when employment decisions are made at the industry level by cost minimizing firms with identical production functions and the same given demand and factor price expectations. In this framework the inclusion of lagged employment variables can be justified on the grounds of inertia in revision of expectations, adjustment costs involved in hiring and firing of workers, or aggregation over different labour types. (See, for example, Sargent, 1978, and Nickell, 1984). The variable $SLYT_t$, which measures the level of aggregate output (in logs) is a proxy measure intended to capture changes in demand expectations arising from the perceived interdependence of the demand in economy by the firms in the industry.(1) The time trend is included in the specification in order to allow for the effect of neutral technical progress on the labour productivity.(2) Ideally, we would have liked to avoid using a simple time trend as a proxy for the trend productivity. But, unfortunately direct reliable observations on technical change, especially at the

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(1) Apart from the aggregate variable $SLYT_t$, the employment function (7.1) is similar to the equations estimated by Peterson (1988), as a part of the Cambridge Multisectoral Dynamic Model of the UK economy. (See Barker and Peterson, 1988).

(2) Notice that, for the ease of comparison of the aggregate and the disaggregate parameter estimates the time trend, and the aggregate output variable that are common to all the micro-equations are specified in the 'average' form. Clearly this has no effect on the overall fit of the equations for a fixed level of disaggregation.

industry level are not available.[1] The use of time trends in regression equations with non-stationary variables also poses a number of important econometric problems and, as shown by Nelson and Kang (1983), Mankiw and Shapiro (1985, 1986), and Durlauf and Phillips (1986), can result in biased inferences.[2] In view of these measurement and econometric problems it is not clear how one should proceed to allow for trend changes in labour productivity on employment demand functions.[3] Here, in the absence of direct measures of trend productivity at the industry level we estimate (7.1) with a time trend, but also briefly report on the effects of omitting the time trends.[4]

For the aggregate employment function we adopted the following dynamic specification

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(1) In their work on aggregate employment demand function Layard and Nickell (1985, p. 168) use a production function approach to obtain an index of labour-augmenting technical progress as a 'residual'. This approach requires time series data on capital stock and the share of capital which are not readily available at the industry level. Moreover, since their measure of technical progress is constructed using actual employment, including it as a regressor in the employment demand function can lead to biased estimates.

(2) Notice, however, that in the case of the test of perfect aggregation where the test is justified asymptotically for a fixed sample size but with an increasing number of micro-units, the inclusion of time trends in the micro-equations does not affect the validity of the test.

(3) However, see Harvey et al. (1986) where a stochastic specification (a random walk with a drift) is advanced for trend productivity. In their formulation T_t is modelled as $T_t = a + T_{t-1} + \epsilon_t$, where ϵ_t is a white-noise process.

(4) The effect of replacing the time trend by other proxies such as distributed lag functions of gross investment as a way of modelling endogenous technical change à la Kaldor (1956) is discussed in Lee et al. (1988).

$$(7.2) \quad SLET_t = b_1 + b_2T_t + b_3SLET_{t-1} + b_4SLET_{t-2} + b_5SLYT_t + b_6SLYT_{t-1} + b_7SLWT_t + b_8SLWT_{t-1} + u_t, \quad t = 1956, 1957, \ldots, 1984,$$

where

$$SLET_t = \sum_{\substack{i=1 \\ i\neq 4}}^{41} LEH_{it}, \quad \text{and} \quad SLWT_t = \sum_{\substack{i=1 \\ i\neq 4}}^{41} LW_{it} .$$

Here we are assuming that the purpose of the study is to explain $SLET_t$, which is the sum of the logarithms of industry employment (in man-hours). This is clearly different from the more usual practice of specifying aggregate employment functions in terms of the logarithm of the sum of industry employment. For our purposes the specification (7.2) has the advantage that it fits directly within the theoretical framework of the paper, and as is pointed out, for example, by Lovell (1973), it also satisfies the Klein-Nataf consistency conditions. A theoretical analysis of the alternative methods of aggregating micro-specifications such as (7.1), and an econometric investigation of the relative merits of such aggregation methods is beyond the scope of the present paper.

7.1 Results for the economy as a whole

The estimates of the unrestricted version of the industry demand functions (7.1) for the 40 industry groups 1, 2, 3, 5, 6, ..., 41, over the sample period 1956-84 are set out in Table 1. The estimates of the standard errors of the regression coefficients are given in the brackets. The Table also includes the adjusted multiple correlation

Table 1*

Disaggregate employment demand functions (unrestricted)
(1956-1984)

	Industry groups	INPT/40	T/40	LY_{it}	$LY_{i,t-1}$	$LEH_{i,t-1}$	$LEH_{i,t-2}$	LW_{it}	$LW_{i,t-1}$	$SLYT_t/40$	$SLYT_{t-1}/40$	$\hat{\sigma}$	$\bar{R}^2$	LLF	DW	$\chi^2_{SC}(2)$
1.	Agriculture,	56.4355	0.0264	0.2538	0.1743	0.5162	0.0152	-0.4196	-0.0073	-0.2408	-0.1762	0.0148	0.9981	87.21	1.9633	0.93
	forestry and fishing	(80.5624)	(0.1580)	(0.1722)	(0.1926)	(0.2165)	(0.1479)	(0.0899)	(0.1315)	(0.1091)	(0.1358)					
2.	Coal Mining	-46.3870	-0.3462	0.2753	-0.4222	1.1160	-0.1770	-0.2095	-0.0670	0.0901	0.0709	0.0159	0.9986	85.11	1.6715	1.07
		(60.2401)	(0.0742)	(0.0362)	(0.0600)	(0.1186)	(0.1076)	(0.0356)	(0.0615)	(0.1188)	(0.1315)					
3.	Coke	-334.1719	-1.4903	-0.0190	0.5195	0.1201	-0.1380	-0.3478	0.0845	0.9039	0.2270	3.3499	0.9763	54.48	2.4823	0.05
		(89.0939)	(0.3678)	(0.1169)	(0.2007)	(0.1636)	(0.1091)	(0.0762)	(0.0998)	(0.3324)	(0.3801)					
4.	Mineral Oil and	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	Natural Gas															
5.	Petroleum Products	-438.0316	-0.7008	0.3845	-0.4844	0.5455	0.1274	-0.2769	-0.0819	0.7001	0.7214	0.0580	0.9136	47.53	1.9463	0.68
		(286.8173)	(0.2349)	(0.3395)	(0.3300)	(0.2211)	(0.2427)	(0.1085)	(0.1570)	(0.7530)	(0.7879)					
6.	Electricity Etc.	91.2396	-0.2074	0.2053	0.2029	1.1513	-0.6629	-0.1465	-0.0588	-0.1838	-0.1144	0.0205	0.9855	77.73	2.0344	0.41
		(69.2323)	(0.2179)	(0.2592)	(0.3127)	(0.2277)	(0.2021)	(0.0937)	(0.1039)	(0.2523)	(0.2579)					
7.	Public Gas Supply	-103.6365	-0.4472	-0.1297	0.1423	0.4114	0.1617	-0.2605	0.1188	0.2816	0.3240	0.0329	0.9707	63.97	2.2360	2.99
		(116.3665)	(0.2799)	(0.2341)	(0.2165)	(0.2288)	(0.1842)	(0.0962)	(0.1212)	(0.2945)	(0.2682)					
8.	Water Supply	42.9711	0.0492	0.6751	-0.8120	0.8817	-0.0969	-0.3652	0.3937	-0.5947	0.7993	0.0448	0.9149	55.07	1.8170	0.90
		(76.4264)	(0.3303)	(0.5110)	(0.5967)	(0.2256)	(0.1862)	(0.1664)	(0.1806)	(0.4200)	(0.3770)					
9.	Minerals and Ores	197.0285	-0.0037	0.2785	0.0091	0.4998	0.1898	-0.1741	0.0437	-0.5639	-0.0421	0.0342	0.9722	62.89	2.0457	0.72
		(119.2942)	(0.1311)	(0.1428)	(0.1517)	(0.2283)	(0.1899)	(0.0814)	(0.0971)	(0.4006)	(0.3775)					
10.	Iron and Steel	-349.9418	-0.9245	0.1361	0.0248	0.4620	-0.0029	-0.4526	0.0666	0.9679	0.2003	0.0279	0.9925	68.77	1.9940	0.85
		(110.1168)	(0.3780)	(0.1031)	(0.0976)	(0.2354)	(0.1512)	(0.1087)	(0.1491)	(0.3587)	(0.4190)					
11.	Non-Ferrous Metals	-58.6696	-0.4029	0.1387	-0.2285	1.3339	-0.5140	-0.0696	0.0623	0.8235	-0.4050	0.0251	0.9863	71.84	2.2252	1.69
		(38.5798)	(0.2149)	(0.1359)	(0.1414)	(0.1682)	(0.1418)	(0.0549)	(0.0592)	(0.2501)	(0.2939)					
12.	Non-Metallic	-389.8347	-0.5395	0.3985	-0.2419	0.5945	0.0854	-0.3173	-0.1961	0.4744	0.4906	0.0179	0.9933	81.72	2.3446	3.62
	Mineral Products	(116.1591)	(0.2589)	(0.1690)	(0.1680)	(0.2070)	(0.1785)	(0.1212)	(0.1348)	(0.3233)	(0.3838)					
13.	Chemicals and	-159.1132	-0.0882	0.0983	0.1058	0.2440	0.2676	-0.2988	-0.1215	0.3032	0.2011	0.0158	0.9789	85.22	2.2664	5.28
	Manmade Fibres	(71.0291)	(0.1744)	(0.1607)	(0.1596)	(0.2123)	(0.1780)	(0.0923)	(0.1146)	(0.3051)	(0.2983)					
14.	Metal Goods	-31.2352	-0.2375	0.2866	0.0653	0.6460	-0.0954	-0.1761	0.0345	0.3585	-0.2214	0.0207	0.9858	77.48	2.1717	5.66
		(54.8630)	(0.2189)	(0.1359)	(0.1618)	(0.2287)	(0.1537)	(0.1164)	(0.1288)	(0.3055)	(0.3215)					
15.	Mechanical	-131.8896	0.1976	0.5334	-0.2266	0.4505	-0.1080	-0.2575	-0.3748	-0.0908	0.5055	0.0144	0.9912	87.94	2.0471	3.81
	Engineering	(56.4251)	(0.1567)	(0.1187)	(0.1038)	(0.2068)	(0.1457)	(0.1193)	(0.1400)	(0.1860)	(0.1974)					
16.	Office Machinery	-397.5477	0.2032	0.2159	-0.0564	1.0066	0.1224	-0.6515	-0.1085	0.3853	-0.0189	0.0303	0.9166	66.37	1.7819	2.05
	etc.	(147.5041)	(0.3070)	(0.0947)	(0.1499)	(0.2501)	(0.2832)	(0.1613)	(0.2020)	(0.2938)	(0.3014)					
17.	Electrical	11.0264	-0.3711	0.4461	-0.2587	1.0101	-0.2373	-0.4120	0.4282	0.1741	-0.1295	0.0179	0.9678	81.67	2.2173	2.73
	Engineering	(35.8630)	(0.2243)	(0.1248)	(0.1853)	(0.2172)	(0.1463)	(0.1355)	(0.1440)	(0.2172)	(0.2073)					
18.	Motor Vehicles	-210.7119	-0.2799	0.5063	-0.3633	0.8391	-0.1636	-0.0568	-0.1675	0.5151	0.1640	0.0192	0.9867	79.67	2.3117	2.34
		(61.6410)	(0.1336)	(0.0718)	(0.1230)	(0.1883)	(0.0946)	(0.1000)	(0.1043)	(0.2088)	(0.2397)					
19.	Aerospace	200.2420	-0.7488	0.0710	0.0468	0.5991	-0.3688	-0.0085	-0.1422	-0.2225	-0.2250	0.0284	0.9864	68.28	2.0370	1.17
	Equipment	(98.6992)	(0.1808)	(0.0821)	(0.0806)	(0.2238)	(0.2001)	(0.0921)	(0.0879)	(0.2480)	(0.2600)					
20.	Ships and Other	-159.2346	0.2741	0.6650	-0.3609	1.1618	-0.1246	-0.0186	-0.0490	0.6358	-0.5276	0.0302	0.9840	66.47	2.2101	2.48
	Vessels	(63.9773)	(0.2704)	(0.1660)	(0.1540)	(0.2077)	(0.2075)	(0.0808)	(0.0921)	(0.2322)	(0.2384)					
21.	Other Vehicles	-127.8464	-0.4292	0.2594	0.0648	0.8089	-0.0679	-0.1583	0.0360	0.1202	0.1451	0.0258	0.9969	71.09	1.9994	1.32
		(84.5230)	(0.2006)	(0.1060)	(0.1115)	(0.2445)	(0.2152)	(0.0656)	(0.0669)	(0.1989)	(0.2126)					

Table 1* (continued)

Disaggregate employment demand functions (unrestricted)
(1956-1984)

Industry groups	INPT/40	T/40	LY_{it}	$LY_{i,t-1}$	$LEH_{i,t-1}$	$LEH_{i,t-2}$	LW_{it}	$LW_{i,t-1}$	$SLYT_t/40$	$SLYT_{t-1}/40$	$\hat{\sigma}$	$\bar{R}^2$	LLF	DW	$\chi^2_{SC}(2)$
22. Instrument	-102.7548	-0.2292	-0.0485	-0.1541	0.6941	-0.1319	-0.2264	0.1475	0.5565	0.2893	0.0246	0.9311	72.37	1.7674	0.81
Engineering	(86.1619)	(0.1516)	(0.2108)	(0.1699)	(0.2304)	(0.2037)	(0.1196)	(0.1373)	(0.2681)	(0.3691)					
23. Manufactured Food	-209.2242	-0.4900	0.7273	0.0656	0.2394	0.2501	-0.1577	-0.0753	0.0340	0.0808	0.0174	0.9816	82.49	1.7024	4.77
	(138.5611)	(0.2336)	(0.3465)	(0.2334)	(0.2303)	(0.1746)	(0.0869)	(0.1211)	(0.1695)	(0.1486)					
24. Alcoholic Drinks	101.6376	-0.6943	0.5685	-0.0005	0.8466	-0.2612	-0.0878	0.0948	-0.2328	-0.2029	0.0282	0.9152	68.45	2.0663	2.53
etc.	(121.3637)	(0.3435)	(0.4859)	(0.4296)	(0.2624)	(0.2854)	(0.1068)	(0.1051)	(0.4551)	(0.4300)					
25. Tobacco	-300.3142	-0.2605	0.7063	-0.2275	0.8345	0.4788	0.0238	-0.0569	-0.6541	0.8452	0.0496	0.8803	52.10	2.3769	5.28
	(126.6923)	(0.4339)	(0.4676)	(0.5169)	(0.2947)	(0.3493)	(0.0730)	(0.0907)	(0.5131)	(0.4385)					
26. Textiles	-127.5754	0.1293	0.4654	-0.1376	0.5979	0.0464	-0.4302	0.0292	0.1772	-0.0316	0.0186	0.9979	80.59	2.2309	2.33
	(62.4817)	(0.5631)	(0.1663)	(0.1515)	(0.2123)	(0.1390)	(0.0970)	(0.1446)	(0.3237)	(0.2984)					
27. Clothing and	-59.4303	-0.1340	0.4585	0.0217	0.5867	-0.0346	-0.4004	0.1067	-0.0342	0.0194	0.0118	0.9981	93.85	2.0674	0.26
Footwear	(26.3298)	(0.2073)	(0.1154)	(0.1572)	(0.2160)	(0.1428)	(0.0833)	(0.1092)	(0.1568)	(0.1683)					
28. Timber and	33.7633	-0.5183	0.2993	-0.0213	0.3362	-0.1005	-0.2491	0.1186	0.1765	0.1864	0.0140	0.9859	88.71	1.9694	1.53
Furniture	(44.1269)	(0.1714)	(0.0895)	(0.1248)	(0.2314)	(0.1190)	(0.0853)	(0.0892)	(0.2467)	(0.2876)					
29. Paper and Board	-17.8043	-0.2271	0.5324	0.3381	0.1236	0.1281	-0.2353	-0.0844	-0.1617	-0.0889	0.0200	0.9921	78.36	2.2984	4.73
	(37.0048)	(0.1687)	(0.1528)	(0.1761)	(0.2513)	(0.1320)	(0.0722)	(0.1120)	(0.3221)	(0.2940)					
30. Books etc.	106.4095	0.0419	0.3518	-0.0926	1.2912	-0.6039	-0.0640	-0.0395	-0.1222	-0.1749	0.0124	0.9296	92.29	2.2104	3.23
	(41.1675)	(0.0468)	(0.1167)	(0.1381)	(0.2215)	(0.1718)	(0.0611)	(0.0649)	(0.1968)	(0.1937)					
31. Rubber and Plastic	-124.2511	-0.6223	0.1943	-0.0152	0.4938	0.1046	-0.2650	0.2121	0.5905	-0.0211	0.0176	0.9811	82.18	2.1726	7.97
Products	(52.6895)	(0.2588)	(0.2485)	(0.2093)	(0.2227)	(0.1469)	(0.1171)	(0.1475)	(0.4566)	(0.3819)					
32. Other Manufactures	202.2515	-0.4204	0.1757	0.1393	0.5916	-0.0512	0.0166	0.1451	0.3999	-0.7045	0.0134	0.9921	90.08	1.9558	0.90
	(80.7245)	(0.1658)	(0.0858)	(0.1148)	(0.1755)	(0.0985)	(0.0939)	(0.0967)	(0.1678)	(0.1813)					
33. Construction	84.3552	-0.0906	0.3618	-0.2488	1.1406	-0.3369	-0.2854	0.3684	0.1373	-0.2078	0.0174	0.9709	82.53	1.5347	1.84
	(48.6187)	(0.0723)	(0.1301)	(0.1453)	(0.1571)	(0.1199)	(0.1122)	(0.1105)	(0.1798)	(0.1962)					
34. Distribution etc.	111.7644	0.5543	0.0254	0.5177	0.6887	-0.1943	-0.2772	-0.1398	-0.0449	-0.5522	0.0142	0.9587	88.44	2.2835	2.14
	(46.6165)	(0.2857)	(0.1910)	(0.2715)	(0.2281)	(0.1634)	(0.1209)	(0.1452)	(0.1750)	(0.2230)					
35. Hotels and	131.7171	0.1650	0.2394	0.1746	0.6033	-0.1807	-0.3824	0.2140	-0.0718	-0.1783	0.0209	0.9077	77.22	1.9588	0.30
Catering	(132.1092)	(0.1157)	(0.2469)	(0.3314)	(0.2542)	(0.2362)	(0.1426)	(0.1300)	(0.2039)	(0.1910)					
36. Rail Transport	9.2868	-0.0880	0.0969	0.3117	0.8301	-0.0399	-0.0821	0.0953	-0.0469	-0.1113	0.0253	0.9952	71.59	1.9033	3.80
	(135.1609)	(0.1796)	(0.1543)	(0.1906)	(0.2087)	(0.2156)	(0.1427)	(0.1388)	(0.2911)	(0.2803)					
37. Other Land	141.1240	-0.4169	0.0524	0.1751	0.9730	-0.5159	-0.0200	0.0269	0.1628	-0.1715	0.0170	0.9724	83.11	2.4060	5.29
Transport	(74.6539)	(0.1293)	(0.1457)	(0.1818)	(0.2248)	(0.2020)	(0.0599)	(0.0638)	(0.1645)	(0.1545)					
38. Sea, Air and Other	8.3655	-0.1128	0.3135	-0.3582	1.1912	-0.4875	-0.2868	0.1941	-0.1525	0.4478	0.0216	0.9254	76.26	2.2168	1.04
	(132.0896)	(0.1340)	(0.1866)	(0.1944)	(0.1884)	(0.2454)	(0.1370)	(0.1291)	(0.2147)	(0.2436)					
39. Communications	72.2461	-0.4312	0.6876	-0.5043	0.7816	-0.2629	-0.1278	0.2150	0.0045	0.2006	0.0178	0.9392	81.85	2.3222	2.15
	(57.0163)	(0.2596)	(0.2987)	(0.2480)	(0.1689)	(0.1616)	(0.0851)	(0.0879)	(0.2004)	(0.1648)					
40. Business Services	294.6267	0.1653	0.1874	0.0981	0.5952	-0.3759	0.0312	-0.0237	0.0360	-0.2212	0.0139	0.9929	88.96	2.1808	1.79
	(131.8402)	(0.3506)	(0.1452)	(0.1485)	(0.2703)	(0.2248)	(0.0839)	(0.0783)	(0.1145)	(0.1257)					
41. Miscellaneous	41.4060	0.2004	0.2764	-0.2928	0.8375	0.0023	-0.1728	0.0470	-0.0622	0.0803	0.0240	0.9429	73.10	1.6954	2.48
Services	(241.2260)	(0.3980)	(0.2071)	(0.2149)	(0.2620)	(0.2789)	(0.1434)	(0.1433)	(0.2152)	(0.1956)					

* For source of data see the Appendix. Standard errors in brackets

$\hat{\sigma}$ is equation standard errors

LLF is the maximized value of the log-likelihood function.

$\bar{R}^2$ is adjusted multiple correlation coefficient,

DW is the Durbin-Watson statistic

$\chi^2_{SC}(2)$ is the Lagrange multiplier test against second order residual serial correlation.

coefficient ($\overline{R}^2$), the equations' standard errors ($\hat{\sigma}$), the maximized values of the log-likelihood function (LLF), the Durbin-Watson statistic (DW), the Lagrange multiplier statistic for testing against second order residual autocorrelation ($\chi^2_{SC}(2)$).

The results are in general quite satisfactory: the equations fit reasonably well, and the value of $\overline{R}^2$ for the majority of the industries is well above 0.95. Only in the case of the tobacco industry does it fall below 0.90. With the exception of the estimates for industry 31 (Rubber and Plastic Products), the results do not show significant evidence of residual serial correlation. The parameter estimates, when statistically significant have signs that are a *priori* plausible. The short run elasticities of employment with respect to real wages and output are generally well determined and have the correct signs. The (current) real wage variable is significant at the five percent level in 23 out of the 40 industry groups, and the (current) output variable is significant in 17 of the industries. Notice also that the few incorrectly signed estimates obtained for the real wage and the output variables are not statistically significant, even at the 10 percent level of significance using a one-tailed test. Overall the results provide further evidence in support of the view that both the demand and the product wage variables are significant determinants of changes in employment, although, as is already stressed by Peterson (1987), in the case of most industries changes in demand have been historically more important than changes in product wages in the explanation of employment changes.

As far as the time trends are concerned they are significant at the five percent level only in 12 of the industry estimates, and there are no cases where the coefficient of the time trend is positive and statistically significant. In fact omitting the time trend variable from the analysis in general proved to have only a marginal effect on the coefficient estimates and the significance of the real wage and the output variables.[1]

The results in Table 1 are, however, subject to two important shortcomings: in many cases they seem to be over-parameterized, and the estimates for the industries 16 (Office Machinery etc.), 20 (Ships and other Vessels) and 25 (Tobacco) are unstable.[2] To deal with these shortcomings we estimated a restricted version of the industry employment functions by imposing suitable linear restrictions on the coefficients of (7.1). The coefficient estimates of this 'restricted' specification and their estimated standard errors are summarized in Table 2. The chi-squared statistics for testing the validity of the restrictions together with a number of important diagnostic statistics for tests of misspecification arising from residual serial correlation, functional form, non-normal errors, and heteroscedasticity are given in Table 3. These results are generally more satisfactory than the unrestricted versions. The parameter restrictions cannot be rejected, and only in the case of a very few of the industries do diagnostic statistics indicate that the regression equations are likely to be

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(1) The effects of omitting the time trend variable on the coefficient estimates were particularly marked only in the case of industries, 2, 3, 5, 10, 24, 32 and 37.

(2) The autoregressive parts of the regressions for these three industries have unstable roots.

Table 2

Disaggregate employment demand functions (restricted)
(1956-1984)

Industry groups	INPT/40	T/40	LY_{it}	$LY_{i,t-1}$	$LEH_{i,t-1}$	$LEH_{i,t-2}$	LW_{it}	$LW_{i,t-1}$	$SLYT_t/40$	$SLYT_{t-1}/40$
1. Agriculture,	52.1517	-	0.2687	0.1752	0.5312	-	-0.4211	-	-0.2437	-0.1729
forestry and fishing	(64.9478)		(0.1375)	(0.1121)	(0.0589)		(0.0821)		(0.0981)	(0.1088)
2. Coal Mining	41.2000	-0.3502	0.2734	-0.4181	1.1604	-0.2648	-0.2018	-	-	-
	(14.3416)	(0.0670)	(0.0345)	(0.0589)	(0.0811)	(0.0765)	(0.0296)			
3. Coke	-351.5712	-1.3100	-	0.6330	-	-	-0.3005	-	1.0448	-
	(44.6561)	(0.1752)		(0.1471)			(0.0418)		(0.1564)	
4. Mineral Oil and	-	-	-	-	-	-	-	-	-	-
Natural Gas										
5. Petroleum Products	-70.7959	-0.5087	0.3640	-	0.5185	-	-0.3144	-	-	-
	(71.7711)	(0.1297)	(0.1324)		(0.1348)		(0.0869)			
6. Electricity Etc.	18.5225	-	0.1614	-	1.2739	-0.5958	-0.1732	-	-	-
	(14.6976)		(0.0798)		(0.1744)	(0.1563)	(0.0687)			
7. Public Gas Supply	-47.1096	-0.6014	-	0.0611	0.4191	-	-0.1507	-	0.5379	-
	(97.2188)	(0.1995)		(0.0659)	(0.1524)		(0.0496)		(0.1827)	
8. Water Supply	8.1676	-	0.6536	-0.6536	0.8112	-	0.4027	0.4027	-0.6415	0.7906
	(18.9241)		(0.4042)	(0.4042)	(0.0785)		(0.1086)	(0.1086)	(0.3085)	(0.3064)
9. Minerals and Ores	172.9158	-	0.2655	-	0.6931	-	-0.1494	-	-0.5337	-
	(79.1246)		(0.1265)		(0.0790)		(0.0622)		(0.2560)	
10. Iron and Steel	-349.9558	-0.9045	0.1083	-	0.4978	-	-0.3873	-	1.1803	-
	(58.8686)	(0.2732)	(0.0893)		(0.0832)		(0.0777)		(0.2928)	
11. Non-Ferrous Metals	-84.8257	-0.5749	0.1817	-0.3091	1.2461	-0.4796	-0.0756	0.0756	0.5854	-
	(30.7245)	(0.1517)	(0.1286)	(0.1273)	(0.1458)	(0.1229)	(0.0481)	(0.0481)	(0.1789)	
12. Non-Metallic	-280.5702	-0.3729	0.3101	-	0.6919	-	-0.2356	-0.2214	0.5170	-
Mineral Products	(60.6439)	(0.2148)	(0.1511)		(0.0877)		(0.1075)	(0.0959)	(0.2901)	
13. Chemicals and	-125.0557	-	-	-	0.6205	-	-0.2810	-	0.6049	-
Manmade Fibres	(23.8339)				(0.0693)		(0.0337)		(0.0773)	
14. Metal Goods	-32.2448	-0.1231	0.4365	-	0.5798	-	-0.1671	-	-	-
	(25.5280)	(0.0976)	(0.0444)		(0.0542)		(0.0817)			
15. Mechanical	-149.7049	-	0.4122	-0.1779	0.3215	-	-0.3100	-0.2725	-	0.6080
Engineering	(38.5546)		(0.0584)	(0.0977)	(0.1093)		(0.0868)	(0.1104)		(0.1488)
16. Office Machinery	-3.4674	-	0.1694	-0.1694	1.2748	-0.3244	-0.3884	0.3123	-	-
Etc.	(22.7537)	-	(0.0865)	(0.0865)	(0.2004)	(0.1800)	(0.1379)	(0.1344)		
17. Electrical	2.5709	-0.3785	0.5239	-0.2827	0.9582	-0.1929	-0.4143	0.4027	-	-
Engineering	(32.7774)	(0.2110)	(0.0757)	(0.1276)	(0.1935)	(0.1228)	(0.1259)	(0.1295)		
18. Motor Vehicles	-184.6112	-0.2365	0.4908	-0.3811	0.9237	-0.1783	-	-0.1843	0.5856	-
	(50.0625)	(0.1093)	(0.0629)	(0.1093)	(0.1610)	(0.0897)		(0.0713)	(0.1774)	
19. Aerospace	200.3920	-0.6788	0.0732	-	0.7560	0.4659	-	-0.1252	-	-
Equipment	(53.1219)	(0.1586)	(0.0654)		(0.1659)	(0.1440)		(0.0674)		
20. Ships and Other	-0.7667	-	0.4809	-0.4809	1.4717	-0.4717	-	-	0.5103	-0.5103
Vessels	(0.3086)		(0.1171)	(0.1171)	(0.1543)	(0.1543)			(0.2000)	(0.2000)
21. Other Vehicles	-132.1537	-0.4754	0.3130	-	0.7270	-	-0.1432	-	-	0.2845
	(54.3892)	(0.1730)	(0.0729)		(0.0884)		(0.0462)			(0.1069)

Table 2 (continued)

Disaggregate employment demand functions (restricted)
(1956-1984)

Industry groups	INPT/40	T/40	LY_{it}	$LY_{i,t-1}$	$LEH_{i,t-1}$	$LEH_{i,t-2}$	LW_{it}	$LW_{i,t-1}$	$SLYT_t/40$	$SLYT_{t-1}/40$
22. Instrument	-11.3576	-0.3580	0.3611	-	0.5319	-	-0.2624	-	-	-
Engineering	(44.4947)	(0.1353)	(0.1005)		(0.1253)		(0.1134)			
23. Manufactured Food	-172.1572	-0.4510	0.6697	-	0.3177	0.2237	-0.1962	-	-	0.1157
	(76.0519)	(0.1973)	(0.1734)		(0.1742)	(0.1560)	(0.0645)			(0.1233)
24. Alcoholic Drinks	-15.1802	-0.4844	0.2933	-	0.7283	-	-0.0945	0.0591	-	-
etc.	(73.4889)	(0.1411)	(0.1167)		(0.1239)		(0.0919)	(0.0882)		
25. Tobacco	-213.3698	-0.3959	0.7424	-	0.7367	0.2633	-	-	-	-
	(80.8449)	(0.1161)	(0.2840)		(0.2225)	(0.2225)				
26. Textiles	-68.1499	-	0.5278	-0.1236	0.5880	-	-0.3428	-	-	-
	(10.0202)		(0.0546)	(0.0754)	(0.0600)		(0.0465)			
27. Clothing and	-68.9489	-	0.4514	-	0.5364	-	-0.3756	-	-	-
Footwear	(11.9600)		(0.0372)		(0.0411)		(0.0284)			
28. Timber and	60.3105	-0.3017	0.3769	-	0.4312	-	-0.2460	0.1493	-	-
Furniture	(20.9479)	(0.0788)	(0.0365)		(0.0582)		(0.0662)	(0.0740)		
29. Paper and Board	-44.7394	-0.3259	0.4680	0.1585	0.3644	-	-0.2503	-	-	-
	(13.2869)	(0.1040)	(0.0652)	(0.0925)	(0.0842)		(0.0433)			
30. Books etc.	58.9250	-	0.2973	-0.2575	1.4842	-0.7029	-0.0454	-	-	-
	(20.8186)		(0.0583)	(0.0592)	(0.1686)	(0.1518)	(0.0482)			
31. Rubber and Plastic	-64.4432	-0.3192	0.5398	-0.1401	0.6844	-	-0.1820	-	-	-
Products	(14.2846)	(0.1872)	(0.0588)	(0.0963)	(0.0818)		(0.1007)			
32. Other Manufactures	60.3555	-0.3233	0.2345	-	0.6028	-	-	-	0.4274	-0.4274
	(20.0274)	(0.0653)	(0.0435)		(0.0933)				(0.1287)	(0.1287)
33. Construction	7.2409	-	0.5490	-0.4527	1.0813	-0.2453	-0.4434	0.3376	-	-
	(20.5598)		(0.0828)	(0.0863)	(0.1559)	(0.1135)	(0.0822)	(0.1096)		
34. Distribution etc.	109.9863	0.3892	-	0.5034	0.5641	-	-0.3036	-	-	-0.5578
	(43.3346)	(0.2057)		(0.1964)	(0.0884)		(0.1187)			(0.1655)
35. Hotels and	-58.7494	-	0.3544	-	0.7096	-	-0.3876	0.1959	-	-
Catering	(44.4425)		(0.1150)		(0.1022)		(0.1191)	(0.1094)		
36. Rail Transport	-65.1073	-	-	0.4070	0.8047	-	-0.0729	-	-	-
	(26.2802)			(0.0979)	(0.0532)		(0.0531)			
37. Other Land	146.4317	-0.4542		0.2451	0.9023	-0.4855	-	-	-	-
Transport	(37.8129)	(0.1047)		(0.0701)	(0.1931)	(0.1838)				
38. Sea, Air and Other	48.5900	-0.1921	0.1924	-	1.1919	-0.5542	-0.0853			
	(104.9126)	(0.1054)	(0.1634)		(0.1741)	(0.2189)	(0.0683)			
39. Communications	14.3221	-0.6566	0.9014	-0.4533	0.8261	-0.2785	-0.1686	0.1565	-	-
	(41.3966)	(0.2354)	(0.1808)	(0.1966)	(0.1727)	(0.1579)	(0.0822)	(0.0807)		
40. Business Services	209.6513	-	0.3108	-	0.6781	-0.3104	-	-	-	-0.1633
	(49.1545)		(0.0718)		(0.1759)	(0.1680)				(0.0486)
41. Miscellaneous	-39.9043	-	0.2123	-	0.8264	-	-0.1408	-	-	-
Services	(33.3057)		(0.0790)		(0.0970)		(0.0747)			

* For source of data see the data appendix. The standard errors are in brackets. The relevant summary, and diagnostic statistics are given in Table 3.

Table 3

Summary and Diagnostic Test Statistics for the Restricted Employment Equations
(1956-1984)

	Industry groups	$\bar{R}^2$	χ^2_r	$\hat{\sigma}$	$\chi^2_{SC}(1)$	$\chi^2_{FF}(1)$	$\chi^2_N(2)$	$\chi^2_H(1)$
1.	Agriculture, forestry and fishing	.9983	0.04(3)	0.0137	0.01	7.25	0.39	2.25
2.	Coal Mining	.9986	3.72(3)	0.0158	0.84	0.91	0.32	0.05
3.	Coke	.9771	5.20(5)	0.0449	0.24	0.67	0.27	1.87
4.	Mineral Oil and Natural Gas	-	-	-	-	-	-	-
5.	Petroleum Products	.9178	4.89(5)	0.0566	0.48	0.01	1.83	0.85
6.	Electricity Etc.	.9876	2.19(5)	0.0190	0.17	1.26	0.18	0.12
7.	Public Gas Supply	.9719	3.97(4)	0.0322	1.29	0.00	4.86	1.42
8.	Water Supply	.9279	0.73(4)	0.0412	1.67	0.00	0.47	1.05
9.	Minerals and Ores	.9760	2.40(5)	0.0318	1.36	0.16	32.70	0.00
10.	Iron and Steel	.9933	2.49(4)	0.0265	0.08	0.19	1.42	0.43
11.	Non-Ferrous Metals	.9864	2.63(2)	0.0250	0.01	3.47	0.20	1.89
12.	Non-Metallic Mineral Products	.9935	3.44(3)	0.0177	1.11	0.23	0.76	3.15
13.	Chemicals and Manmade Fibres	.9795	6.27(6)	0.0156	3.51	1.80	0.96	1.14
14.	Metal Goods	.9877	2.37(5)	0.0192	0.09	0.27	0.38	1.00
15.	Mechanical Engineering	.9913	3.64(3)	0.0143	0.093	0.10	0.02	0.73
16.	Office Machinery etc.	.8922	10.47(4)	0.0345	0.05	2.68	7.24	5.05
17.	Electrical Engineering	.9698	1.02(2)	0.0173	0.33	0.11	2.19	2.29
18.	Motor Vehicles	.9874	1.29(2)	0.0186	1.55	8.92	3.89	0.01
19.	Aerospace Equipment	.9878	2.21(4)	0.0268	0.90	0.30	1.81	1.30
20.	Ships and other Vessels	.9817	9.70(6)	0.0323	0.45	0.61	0.40	4.46
21.	Other Vehicles	.9973	1.69(4)	0.0241	0.01	0.81	0.17	0.04
22.	Instrument Engineering	.9250	7.92(5)	0.0257	0.47	3.07	0.01	0.84
23.	Manufactured Food	.9837	0.85(3)	0.0164	1.69	2.78	1.33	4.38
24.	Alcoholic drinks etc.	.9232	2.56(4)	0.0269	1.32	0.02	0.94	2.06
25.	Tobacco	.8796	7.09(6)	0.0497	0.25	8.22	0.65	7.62
26.	Textiles	.9981	3.18(5)	0.0175	0.05	4.46	0.74	5.09
27.	Clothing and Footwear	.9984	3.76(6)	0.0110	0.36	1.92	0.62	0.03
28.	Timber and Furniture	.9864	4.24(4)	0.0138	0.00	2.43	1.34	0.30
29.	Paper and Board	.9927	2.86(4)	0.0192	1.09	1.33	1.74	4.41
30.	Books etc.	.9306	4.69(4)	0.0123	1.70	0.01	0.14	0.44
31.	Rubber and Plastic Products	.9818	4.73(4)	0.0173	0.21	1.59	0.96	1.03
32.	Other Manufactures	.9917	7.18(5)	0.0137	0.37	0.21	1.12	0.00
33.	Construction	.9689	5.54(3)	0.0179	5.00	2.34	1.62	1.00
34.	Distribution etc.	.9580	5.44(4)	0.0143	0.49	0.02	0.94	2.06
35.	Hotels and Catering	.9169	3.49(5)	0.0198	0.58	1.88	0.45	0.63
36.	Rail Transport	.9960	2.36(6)	0.0230	0.28	0.00	1.27	1.98
37.	Other Land Transport	.9747	4.04(5)	0.0163	0.02	1.23	0.64	2.71
38.	Sea, Air and Other	.9155	7.87(4)	0.0229	0.27	4.31	0.39	2.75
39.	Communications	.9351	4.42(2)	0.0184	1.56	0.48	0.14	1.81
40.	Business Services	.9940	1.81(5)	0.0128	0.98	2.01	1.98	0.17
41.	Miscellaneous Services	.9512	3.21(6)	0.0222	0.06	0.47	0.39	1.91

Notes:

χ^2_r is the chi-squared statistic for the test of r linear restrictions on the parameters of unrestricted employment equations (see Table 1). The value of r is given in brackets after the statistic.

$\chi^2_{SC}(1)$ is the first order LM test of residual serial correlation.

$\chi^2_{FF}(1)$ is Ramsey's RESET test of order 1.

$\chi^2_N(2)$ is a test of normality of the errors.

$\chi^2_H(1)$ is a heteroscedasticity test of order 1.

$\hat{\sigma}$ is equations' standard errors.

$\bar{R}^2$ is the adjusted multiple correlation coefficient.

The underlying regressions and the test statistics reported in this table are computed on Data-FIT package. For details of relevant algorithms and references - see Pesaran and Pesaran (1987).

misspecified.[1] Also note that the restricted estimates for the industries 16, 20 and 25 are no longer unstable, although the equations for the latter two industries are specified in first differences and do not possess long run solutions. The long run elasticities of employment with respect to output and real wages for the 38 industries that do have long run solutions are displayed graphically in figures 1 and 2, respectively.

Although there is still a great deal more room for improving the results by, for example, including 'industry specific' variables in the employment demand functions, we believe that the results obtained so far provide a reasonable basis for the application of the methods developed in this paper to the restricted and unrestricted disaggregate results and those that can be obtained by the direct estimation of the aggregate specification (7.2). For the unrestricted estimate of (7.2) we obtained

$$\begin{aligned}(7.3)\qquad SLET_t = & \underset{(51.47)}{-136.50} - \underset{(0.0861)}{0.0217}\, T_t + \underset{(0.2274)}{0.5862}\, SLET_{t-1} + \underset{(0.1833)}{0.0819}\, SLET_{t-2} \\ & + \underset{(0.0670)}{0.4817}\, SLYT_t + \underset{(0.1253)}{0.0088}\, SLYT_{t-1} - \underset{(0.0799)}{0.3508}\, SLWT_t \\ & - \underset{(0.0955)}{0.0334}\, SLWT_{t-1} + \hat{u}_{tT}\,,\end{aligned}$$

$LLF = -7.77,\quad \bar{R}^2 = 0.9958,\quad \hat{\sigma} = 0.3717,\quad DW = 2.06,\quad n = 29,$

$\chi^2_{SC}(1) = 0.88,\quad \chi^2_{FF}(1) = 0.6279,\quad \chi^2_{N}(2) = 4.86,\quad \chi^2_{H}(1) = 1.48.$

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(1) The results in Table 2 are also of some interest in so far as they show evidence of significant aggregate output effects on employment demand at the industry level. See footnote 2 on p. 26.

Figure 1

Histogram of the long run elasticities of employment with respect to output in different industries

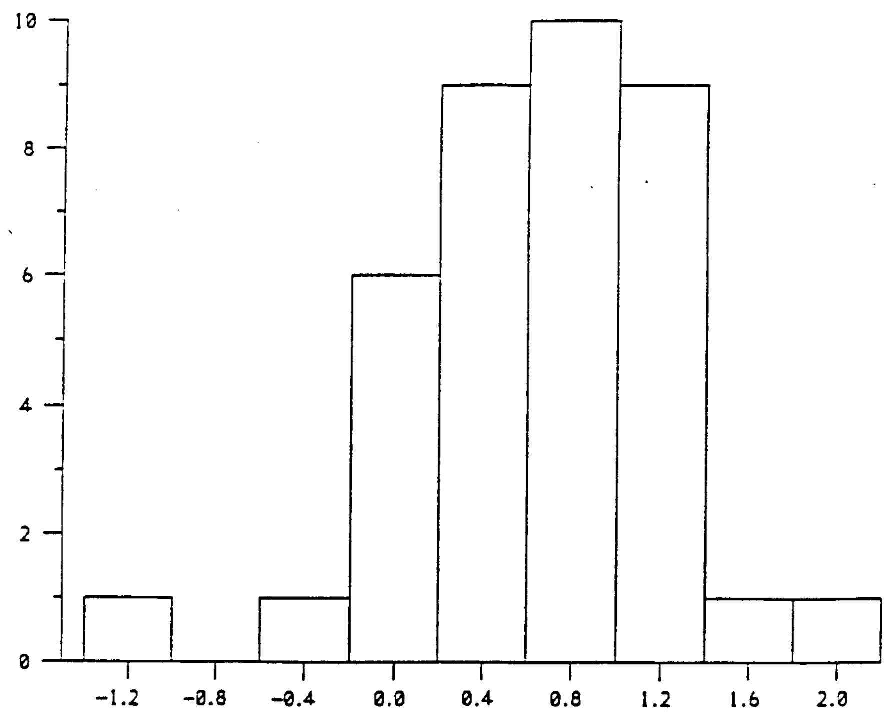
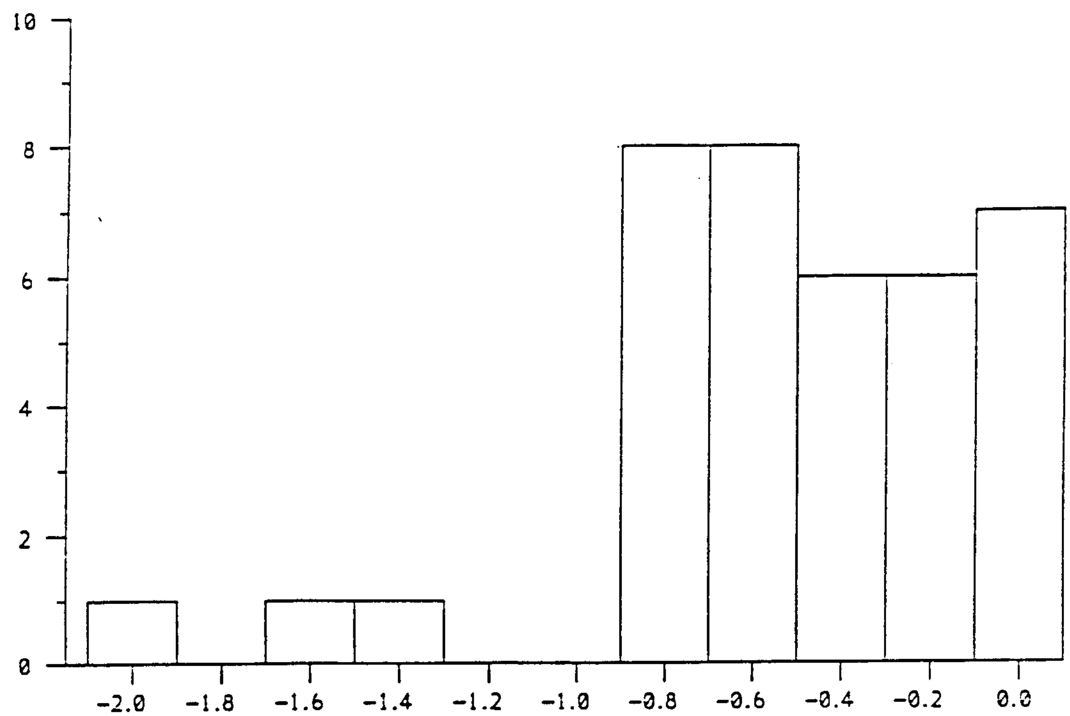


Figure 2

Histogram of the long run elasticities of employment with respect to real wages in different industries



The notations are as before, and the test statistics χ^2_{SC}, χ^2_{FF}, χ^2_N, and χ^2_H are already defined at the foot of table 3. This aggregate specification passes all the tests and has reasonable short run and long run properties. However, it is again over-parameterized. The coefficients of T_t, $SLET_{t-1}$, $SLYT_{t-1}$ and $SLWT_{t-1}$ are statistically insignificant whether considered singly or jointly. The chi-squared statistic for the joint test of zero restrictions on the coefficients of these variables was equal to 0.53. So we also estimated the following restricted version of (7.2)

$$\text{(7.4)} \quad SLET_t = \underset{(15.22)}{-134.07} + \underset{(0.0417)}{0.6956}\, SLET_{t-1} + \underset{(0.0457)}{0.4611}\, SLYT_t - \underset{(0.0354)}{0.3718}\, SLWT_t + \hat{u}_{tT}$$

LLF = -8.39, $\bar{R}^2 = 0.9963$, $\hat{\sigma} = 0.3481$, DW = 2.27, n = 29,

$\chi^2_{SC}(1) = 0.65$, $\chi^2_{FF}(1) = 0.86$, $\chi^2_N(2) = 5.53$, $\chi^2_H(1) = 2.20$.

The coefficient estimates are all well determined and imply long run elasticities of aggregate employment with respect to output and real wages of 1.52 and -1.22, respectively.[1] The long run real wage elasticity is only marginally different from the value of -0.92 reported recently by Layard and Nickell (1985, p. 177) for the UK. This similarity is especially striking considering the differences that exist between the two analyses as far as the aggregation procedure, the specification of employment function, and the estimation periods are concerned.

We are now in a position to compare the disaggregate and the aggregate results. As far as the in-sample 'predictive' performance of the aggregate and the disaggregate models are concerned, we computed the s_d^2 criterion [as defined by (4.2)] for the unrestricted and the restricted versions of the disaggregate model. These were 0.1091 and 0.1035 respectively, thus providing evidence of a slightly

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(1) To check for the possible effect of the simultaneous determination of output, employment, and real wages on the OLS estimates, we also estimated (7.4) by the instrumental variable method using $\underset{\sim}{z}_t = \{1, SLET_{t-1}, SLET_{t-2}, SLYT_{t-1}, SLYT_{t-2}, SLWT_{t-1}, SLWT_{t-2}\}$ as instruments. We obtained the following results:

$$SLET_t = \underset{(20.70)}{-137.01} + \underset{(0.0569)}{0.6840}\, SLET_{t-1} + \underset{(0.0708)}{0.4745}\, SLYT_t - \underset{(0.0540)}{0.3830}\, SLWT_t + \tilde{u}_{tT},$$

$\bar{R}^2 = 0.9963$, $\hat{\sigma} = 0.3487$, $DW = 2.25$, $n = 29$,

$\chi^2_{SC}(1) = 0.56$, $\chi^2_{FF}(1) = 0.07$, $\chi^2_N(2) = 4.15$, $\chi^2_H(1) = 2.18$,

which differ only marginally from the OLS results. In fact the Wu-Hausman statistic (T_2 statistic in Wu (1973)), for the test of the 'exogeneity' of $SLYT_t$ and $SLWT_t$ in (7.4), using $\underset{\sim}{z}_t$ as the instruments, was equal to 0.112, which is well below the 5 percent critical value of the F distribution with 2 and 23 degrees of freedom.

better fit for the restricted version of the disaggregate model.(1) The value of the goodness-of-fit criterion for the aggregate equations (7.3) and (7.4) were equal to 0.1359 and 0.1153, respectively. These results are summarized in Table 4, where the uncorrected GG criterion (the first term on the right hand side of (4.8)) is also reported in brackets. On the basis of the proposed choice criterion the restricted as well as the unrestricted versions of the disaggregate model are preferable to the aggregate equation. The computation of the statistic for the test of perfect aggregation defined by (5.8) also provided additional support in favour of the disaggregate model. In the case of the unrestricted version the value of this test statistic was equal to 81.66, which is approximately distributed as a χ^2_{29}, thus firmly rejecting the hypothesis of perfect aggregation. This is also clearly reflected in the estimates of the long run elasticities obtained from the disaggregate and the aggregate results. For example, concentrating on the restricted versions of the employment functions, the long run elasticity of aggregate employment with respect to output based on the disaggregate results (Table 2), turned out to be equal to 0.724 as compared with the figure of 1.52 obtained using the aggregate

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(1) Notice that in general there is no reason to believe that the restricted model should perform better than the unrestricted model as far as the s_d^2 criterion is concerned. Although it is true that the imposition of statistically 'acceptable' linear restrictions on the parameters of the micro equations, such as omitting one or more variables from the micro equations whose t- or F- values are less than unity, lowers the estimates of σ_{ii}, the same is not true of the estimates of the contemporaneous covariances, σ_{ij}, $i \neq j$. As a result the effect of restrictions on

$$s_d^2 = \sum_{i=1}^{m} \hat{\sigma}_{ii} + 2 \sum_{i>j} \hat{\sigma}_{ij}$$

will, in general, be ambiguous.

Table 4

Relative predictive performance of the aggregate and the disaggregate employment functions
(1956-1984)

	Aggregate equations		Disaggregate equations[3]	
	Unrestricted[1]	Restricted[2]	Unrestricted	Restricted
All industries[4]	0.1382	0.1201	0.1091 (0.0846)	0.1035 (0.0859)
Manufacturing[5]	0.0492	0.0433	0.0506 (0.0439)	0.0455 (0.0373)

1. See equations (7.3) and (7.5).

2. See equations (7.4) and (7.6).

3. See the results in tables 1 and 2.

4. Excluding Industry 4, Mineral Oil and Natural Gas.

5. Industries 10 to 32 inclusive.

Bracketed figures refer to the degrees-of-freedom uncorrected measure of the choice criterion, given by the first term in the expression for s_d^2 defined in (4.6).

specification (7.4).[1] Similarly the long run elasticity of aggregate employment with respect to real wages based on the disaggregate results was equal to -0.4551 as compared with the estimate of -1.22 based on the aggregate specification (7.4). These results clearly suggest the existence of important upward bias in the estimates of output and real wage elasticities of employment demand obtained in the literature using an economy wide aggregate specification.

7.2 Results for the manufacturing industries

Having rejected the aggregate employment function in favour of the disaggregate model, the question of what the appropriate level of disaggregation should be naturally arises. One possibility would be to repeat the above analysis for all possible levels of disaggregation. Here in the way of illustration we only consider the problem in the case of the manufacturing industries. The disaggregate results for this industry grouping are given by the industries labelled 10 to 32 inclusive in Tables 1 and 2. We also obtained the following estimates of the unrestricted and the restricted employment demand functions for the manufacturing sector as a whole:

$$(7.5) \qquad SLEM_t = \underset{(22.19)}{-65.58} - \underset{(0.0565)}{0.0039}\, T_t + \underset{(0.2211)}{0.7491}\, SLEM_{t-1} - \underset{(0.1655)}{0.0162}\, SLEM_{t-1}$$

$$+ \underset{(0.0531)}{0.4933}\, SLYM_t - \underset{(0.1170)}{0.0897}\, SLYM_{t-1} - \underset{(0.0659)}{0.2979}\, SLWM_t$$

$$- \underset{(0.0837)}{0.0148}\, SLWM_{t-1} + \hat{u}_{tM},$$

---

(1) The estimates of the long run elasticities for the disaggregate model were computed from the simple averages of the micro coefficients.

$LLF = 7.20, \quad \bar{R}^2 = 0.9968, \quad \hat{\sigma} = 0.2218, \quad DW = 2.21, \quad n = 29,$

$\chi^2_{SC}(1) = 4.56 \quad \chi^2_{FF}(1) = 0.01, \quad \chi^2_N(2) = 0.24, \quad \chi^2_H(1) = 1.70,$

and

$$(7.6) \qquad SLEM_t = \underset{(6.65)}{-66.82} + \underset{(0.0417)}{0.7407}\, SLEM_{t-1} + \underset{(0.0434)}{0.4004}\, SLYM_t$$

$$+ \underset{(0.0551)}{0.0906}\, \Delta SLYM_t - \underset{(0.0343)}{0.3137}\, SLWM_t + \hat{u}_{tM},$$

$LLF = 7.13, \quad \bar{R}^2 = 0.9972, \quad \hat{\sigma} = 0.2080, \quad DW = 2.19, \quad n = 29,$

$\chi^2_{SC}(1) = 0.34, \quad \chi^2_{FF}(1) = 0.00, \quad \chi^2_N(2) = 0.21, \quad \chi^2_H(1) = 1.47,$

where $SLEM_t = \sum_{i=10}^{32} LEH_{it}, \quad SLYM_t = \sum_{i=10}^{32} LY_{it}, \quad SLWM_t = \sum_{i=10}^{32} LW_{it}.$

The restricted version (7.6) clearly cannot be rejected against the unrestricted version (7.5).(1) In this application the values of the goodness-of-fit criterion (s_d^2) for the unrestricted and the restricted models were 0.0506 and 0.0455 respectively, indicating that the restricted version of the disaggregate model has a better in-sample performance in so far as predicting the aggregate employment variable $SLEM_t$ is concerned. The goodness-of-fit criterion for the aggregate specifications (7.5) and (7.6) are given by 0.0492 and 0.0433 respectively. (See also Table 4). Hence on the basis of the choice criterion, for the manufacturing industries the

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(1) We also estimated the restricted version (7.6) by the IV method using $\underset{\sim}{z}_t = (1, SLEM_{t-1}, SLEM_{t-2}, SLYM_{t-1}, SLYM_{t-2}, SLWM_{t-1}, SLWM_{t-2})$ as instruments and obtained very similar results.

aggregate models give marginally a better fit than either of the disaggregate models. This, of course, does not mean that the aggregate model is not subject to the aggregation error problem. In fact the application of the test of perfect aggregation to this example resulted in the value of 69.92 for the $\underline{a}_m$ statistic which is well in excess of the 5% critical value of the χ^2 distribution with 29 (= n) degrees of freedom.

The rejection of the perfect aggregation hypothesis is also reflected in the large differences that exist between the estimates of the long run real wage and output elasticities of the manufacturing employment based on the disaggregate and the aggregate results. In the case of the restricted models, the estimates of the long run real wage elasticity based on the aggregate and the disaggregate models were -1.21 and -0.509, respectively. The corresponding figures for the long run real output elasticities were 1.54 and 0.763, respectively. The better performance of the aggregate model should be interpreted as an important indication that the disaggregate employment functions are misspecified. This suggests the need for a much more detailed analysis of employment demand at the industry level, which may involve including 'industry specific' variables in employment equations, experimenting with a different choice of functional forms across industries, or searching for new industry-specific explanatory variables, or even compiling a more reliable set of micro-data.

8. Concluding remarks

In this paper our primary concern has been with the problem of choice between macro and micro regression equations for the purpose of predicting macro variables. The test of perfect aggregation developed in the paper also addresses the macro prediction problem; although as our application to the UK employment demand functions shows, it has some bearing on the problem of aggregation bias as well. In using the goodness-of-fit criterion and the test of perfect aggregation it is, however, important to note that these methods, like most other methods of inference in econometrics, suffer from the fact that they may have little to say on the validity of the aggregation conditions outside the estimation period. In the case of aggregation across micro units this problem is especially serious as the extension of the results of aggregation tests to the post estimation period requires stability of the micro-coefficients as well as the stability of the industrial composition of the economy.

Appendix A

A proof of the asymptotic validity of the proposed test of perfect aggregation

In this appendix we provide a proof of the theorem stated in the paper. Let

$$\text{(A1)} \qquad \underset{\sim}{g}_m = \left(\sum_{i=1}^{m} \hat{\sigma}_{ii} H_i^2\right)^{-1/2} (\underset{\sim}{e}_a - \underset{\sim}{e}_d) ,$$

where $\underset{\sim}{e}_a$, $\underset{\sim}{e}_d$, H_i and $\hat{\sigma}_{ii}$ are already defined in the text. For convenience we reproduce them here

$$\underset{\sim}{e}_a = [I_n - X_a(X_a'X_a)^{-1}X_a']\underset{\sim}{y}_a = (I_n - A_a)\underset{\sim}{y}_a ,$$

$$\underset{\sim}{e}_d = \sum_{i=1}^{m} M_i \underset{\sim}{y}_i ; \quad M_i = I_n - X_i(X_i'X_i)^{-1}X_i' = I_n - A_i,$$

$$\hat{\sigma}_{ii} = \underset{\sim}{y}_i' M_i \underset{\sim}{y}_i/(n - k) ,$$

$$H_i = A_i - A_a.$$

Then the test statistic in the theorem can be written as

$$\text{(A2)} \qquad \underline{a}_m = \underset{\sim}{g}_m' \underset{\sim}{g}_m .$$

Consider now the probability limit of $\hat{\Psi}_m = m^{-1} \sum_{i=1}^{m} \hat{\sigma}_{ii} H_i^2$, as $m \to \infty$. Under (2.1) we obtain

$$\text{(A3)} \qquad \hat{\psi}_m = [m(n-k)]^{-1} \sum_{i=1}^{m} (\underset{\sim}{u}_i' M_i \underset{\sim}{u}_i) H_i^2 .$$

But since M_i is an idempotent matrix of rank n - k, we can also write

$$\text{(A4)} \qquad \sigma_{ii}^{-1} \underset{\sim}{u}_i' M_i \underset{\sim}{u}_i = \sum_{t=1}^{n-k} \epsilon_{it}^2 , \qquad i = 1, 2, \ldots, m$$

where ϵ_{it} represent scalar random variables distributed independently across i and t with zero means and unit variances. Substituting (A4) in (A3) yields

$$\text{(A5)} \qquad \hat{\psi}_m = (n-k)^{-1} \sum_{t=1}^{n-k} \left\{ \sum_{i=1}^{m} \sigma_{ii} \epsilon_{it}^2 H_i^2 / m \right\} .$$

But, noting that $H_i = A_i - A_a$, we have

$$\text{(A6)} \qquad m^{-1} \sum_{i=1}^{m} \sigma_{ii} \epsilon_{it}^2 H_i^2 = f_m A_a + F_m - F_m A_a - A_a F_m ,$$

where

$$f_m = m^{-1} \sum_{i=1}^{m} \sigma_{ii} \epsilon_{it}^2 ,$$

$$F_m = m^{-1} \sum_{i=1}^{m} \sigma_{ii} \epsilon_{it}^2 A_i ,$$

Now under assumption 4 it readily follows that

$$\underset{m\to\infty}{\text{plim}}(f_m) \leq \tau^2 \underset{m\to\infty}{\text{plim}}(m^{-1} \sum_{i=1}^{m} \epsilon_{it}^2) ,$$

and since ϵ_{it} are identically and independently distributed random variables, then by the law of large numbers $m^{-1}\sum_{i=1}^{m}\epsilon_{it}^{2}\overset{p}{\to}1$, and

$$\text{(A7)}\qquad \underset{m\to\infty}{\text{plim}}(f_m) \leqslant \tau^2 < \infty .$$

Similarly, under assumptions 3 and 4 we have

$$\text{(A8)}\qquad \underset{m\to\infty}{\text{plim}}(F_m) \leqslant \tau^2 P < \infty ,$$

where P is already defined by assumption 3. The results (A7) and (A8) establish the existence of the probability limits of f_m and F_m, as $m \to \infty$, and this in turn establishes (using (A6) and noting that by assumption 2 matrix A_a has a finite limit as $m \to \infty$) that

$$\underset{m\to\infty}{\text{plim}}(m^{-1}\sum_{i=1}^{m}\sigma_{ii}\epsilon_{it}^{2}H_i^2) = \lim_{m\to\infty}(m^{-1}\sum_{i=1}^{m}\sigma_{ii}H_i^2) .$$

Using this result in (A5) we finally obtain

$$\text{(A9)}\qquad \hat{\Psi}_m = m^{-1}\sum_{i=1}^{m}\hat{\sigma}_{ii}H_i^2 \overset{p}{\to} \lim_{m\to\infty}(m^{-1}\sum_{i=1}^{m}\sigma_{ii}H_i^2) = \Psi .$$

Therefore, asymptotically we have(1)

$$g_m \overset{a}{\sim} \Psi^{-\frac{1}{2}}m^{-\frac{1}{2}}(\underset{\sim}{e}_a - \underset{\sim}{e}_d) .$$

But under (2.1) on the assumption that H_g: $\sum_{i=1}^{m} X_i\beta_i = X_a\underset{\sim}{b}$ holds

(1) Note that by assumption 5, matrix Ψ is non-singular.

$$m^{-1/2}(\underset{\sim}{e}_a - \underset{\sim}{e}_d) = m^{-1/2}\sum_{i=1}^{m} H_i\underset{\sim}{u}_i \ .$$

Hence

(A10) $$\underset{\sim}{g}_m \overset{a}{\sim} m^{-1/2}\sum_{i=1}^{m} \underset{\sim}{z}_i \ ,$$

in which

$$\underset{\sim}{z}_i = \sigma_{ii}^{1/2}\, \Psi^{-1/2} H_i\underset{\sim}{v}_i \ ,$$

and $\underset{\sim}{v}_i = \underset{\sim}{u}_i/\sqrt{\sigma}_{ii}$. We now show that under the assumptions of the theorem, as $m \to \infty$, the sum $\underset{\sim}{s}_m = m^{-1/2}\sum_{i=1}^{m} \underset{\sim}{z}_i$ tends to a multivariate normal distribution with mean zero and the covariance matrix I_n; an identity matrix of order n. For this purpose it is sufficient to demonstrate that for any fixed vector $\underset{\sim}{\lambda} = (\lambda_1, \lambda_2, \ldots, \lambda_n)'$, the limiting distribution of $\underset{\sim}{\lambda}'\underset{\sim}{s}_m$ is $N(0, \underset{\sim}{\lambda}'\underset{\sim}{\lambda})$.

Let

(A11) $$d_m = \underset{\sim}{\lambda}'\underset{\sim}{s}_m = m^{-1/2}\sum_{i=1}^{m} w_i \ ,$$

in which

(A12) $$w_i = \sigma_{ii}^{1/2}\, \underset{\sim}{\lambda}'\Psi^{-1/2} H_i\underset{\sim}{v}_i \ , \qquad i = 1, 2, \ldots, m$$

is now a scalar random-variable. We have, for all i,

$$E(w_i) = 0 \ ,$$

$$V(w_i) = \sigma_{ii} \, \lambda' \Psi^{-1/2} H_i^2 \Psi^{-1/2} \lambda .$$

Setting $\mu = \Psi^{-1/2}\lambda$, then

(A13) $$C_m^2 = \sum_{i=1}^{m} V(w_i) = \mu' \left(\sum_{i=1}^{m} \sigma_{ii} H_i^2 \right) \mu .$$

Denoting the (t, t′) element of matrix H_i by $h_{i,tt'}$, we also have [using (A12)]

$$w_i = \sigma_{ii}^{1/2} \sum_{t'=1}^{n} \left(\sum_{t=1}^{n} \mu_t \, h_{i,tt'} \right) v_{it'} .$$

Therefore, since by assumption Ψ is non-singular and $h_{i,tt'}$ are bounded in absolute value for all i, then

$$|w_i| \leqslant n\kappa\sigma_{ii}^{1/2} \left| \sum_{t'=1}^{n} v_{it'} \right| ,$$

where $|\mu_t h_{i,tt'}| < \kappa < \infty$. Consequently

$$E|w_i|^3 \leqslant n^3\kappa^3\sigma_{ii}^{3/2} \; E\left| \sum_{t=1}^{n} v_{it} \right|^3 .$$

However, since the random variables v_{it} are i.i.d. with finite third order moments, then $E\left| \sum_{t=1}^{n} v_{it} \right|^3 \leqslant n\theta^3$, where $\theta^3 = E|v_{it}|^3$, and

(A14) $$E|w_i|^3 \leqslant n^4\kappa^3\theta^3\sigma_{ii}^{3/2}.$$

We are now in a position to apply the Liapunov Central Limit Theorem to the sum d_m defined by (A11).[1] Setting

$$B_m^3 = \sum_{i=1}^{m} E|w_i|^3 ,$$

then using (A14) it follows that

$$B_m^3 \leqslant (n^4\kappa^3\theta^3) \sum_{i=1}^{m} \sigma_{ii}^{3/2},$$

which together with (A13) yields[2]

$$\lim_{m\to\infty} \left[\frac{B_m}{C_m} \right] \leqslant \left\{ \frac{n^{4/3}\kappa\theta}{(\underset{\sim}{\lambda}'\underset{\sim}{\lambda})^{1/2}} \right\} \lim_{m\to\infty} m^{-1/2}\left[\sum_{i=1}^{m} \sigma_{ii}^{3/2} \right]^{1/3} .$$

But under assumption 4

$$\lim_{m\to\infty} m^{-1/2}\left[\sum_{i=1}^{m} \sigma_{ii}^{3/2} \right]^{1/3} \leqslant \lim_{m\to\infty} (m^{-1/6}\tau) = 0 ,$$

and for a fixed n, we have $\lim(B_m/C_m) = 0$, as $m \to \infty$; and the condition of the Liapunov theorem will be met. Hence

$$\underset{\sim}{g}_m \overset{a}{\sim} \underset{\sim}{s}_m \overset{a}{\sim} N(0, I_n) .$$

Now using (A2) we have

$$\underset{\sim}{a}_m = \underset{\sim}{g}_m'\underset{\sim}{g}_m \overset{a}{\sim} \chi_n^2 .$$ Q.E.D.

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(1) See, for example, Rao (1973, p. 127)

(2) Notice that $\lim_{m\to\infty}\{\underset{\sim}{\mu}'(m^{-1} \sum_{i=1}^{m} \sigma_{ii}H_i^2)\underset{\sim}{\mu}\} = \underset{\sim}{\mu}'\Psi\underset{\sim}{\mu} = \underset{\sim}{\lambda}'\underset{\sim}{\lambda}$.

Data Appendix: Data Sources and Definitions

The data used in the empirical analysis in section 7 are annual observations on 41 industry groups for the UK obtained from the Cambridge Growth Project Databank. The data on industry man-hours, employment, wages and salaries and employers' contributions were originally provided by the Institute for Employment Research at the University of Warwick. The data on industry output were obtained from the Central Statistical Office's commodity flow accounts adjusted for our industrial classification. The data on producer price indices of industry output were obtained from a number of published sources including the Department of Trade and Industry's publication 'British Business', the CSO's publications, the 'Annual Abstract of Statistics' and the 'Monthly Digest of Statistics', and the Department of Energy's 'Energy Trends'.

Some of the 41 industry groups are identical to the 'groups' distinguished in the 1980 Standard Industrial Classification. However in view of the significant differences between them in a large number of cases, the groups are listed in Table B1, using as a reference the Division, Class or group of the 1980 Standard Industrial classification. In the analysis of the manufacturing sector groups 10 to 32 inclusive are included.

For empirical estimation, the man-hours employed (EH_i) are defined as a product of the actual hours worked per week and the numbers employed in each of 41 industries, including self employed ('000s) in these industries. Industry output (Y_i) is gross value added by industry in 1980 prices (£m). Average real wage rate (W_i) is a measure of the real product wage by industry. It is obtained by first deflating an industry's total labour costs including both employees' wages and salaries and employers' national insurance contributions (£m) by the price index of industry output (1980 = 1.00). This is then divided by the man-hours employed in that industry to obtain the average real wage rate.

All the data are annual covering the period 1954-1984 with both the aggregate and disaggregate equations estimated over the period 1956-1984. These data, and the computer programmes used both in estimation and in the computation of the choice criterion and the statistics for the test of perfect aggregation, are available on request from the authors.

Table A

Classification of industry groups

(in terms of the 1980 Standard Industrial Classification)

Industry	Division, class or group
1. Agriculture, forestry and fishing	0
2. Coal Mining	1113, 1114
3. Coke	1115, 1200
4. Mineral oil and Natural gas	1300
5. Petroleum Products	140
6. Electricity etc.	1520, 1610, 1630
7. Public Gas Supply	1620
8. Water Supply	1700
9. Minerals and Ores n.e.s.	21, 23
10. Iron and Steel	2210, 2220, 223
11. Non-Ferrous Metals	224
12. Non-Metallic Mineral Products	24
13. Chemicals and Manmade Fibres	25, 26
14. Metal Goods n.e.s.	31
15. Mechanical Engineering	32
16. Office Machinery etc.	33
17. Electrical Engineering	34
18. Motor Vehicles	35
19. Aerospace equipment	3640
20. Ships and other vessels	3610
21. Other Vehicles	3620, 363, 3650
22. Instrument Engineering	37

Table A (continued)

23. Manufactured food	41, 4200, 421, 422, 4239
24. Alcoholic drinks etc.	4240, 4261, 4270, 4283
25. Tobacco	4290
26. Textiles	43
27. Clothing and footwear	45
28. Timber and furniture	46
29. Paper and Board	4710, 472
30. Books etc.	475
31. Rubber and Plastic Products	48
32. Other Manufactures	44, 49
33. Construction	5
34. Distribution etc.	61, 62, 63, 64, 65, 67
35. Hotels and Catering	66
36. Rail transport	71
37. Other land transport	72
38. Sea, air and other	74, 75, 76, 77
39. Communications	79
40. Business Services	81, 82, 83, 84, 85
41. Miscellaneous Services	94, 98, 923, 95, 96, 97.

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