

THE OPTIMAL ROLE OF THE GOVERNMENT  
FOR A COMPETITIVE ECONOMY  
CONTAINING MONEY

By

Earl A. Thompson

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Containing Money

by Earl A. Thompson<sup>\*</sup>  
U.C.L.A.

A. The Initial policy question and its current answer

The initial question of this paper is concerned with optimal monetary policy in a full employment economy. In particular: What set of monetary policies will achieve a Pareto optimum in a full competitive equilibrium of an economy containing an initially non-interest-bearing, exogenously controlled, money supply?

A rather elegant answer to this question appears to be developing. Namely, if Pareto optimality is to characterize any competitive equilibrium, then the holding of money, an initially non-interest-bearing medium of exchange, must be somehow financially rewarded with the money rate of interest (e.g., Samuelson [15], Friedman [7]). There are simple monetary policies which yield this proposed optimum. One such policy is the direct payment of interest on money. Another is the continual manipulation of the aggregate, nominal money supplies, and thereby the expected price levels, in such a way that the free market's money rates of interest are always equal to zero.

At first glance, the economic reasoning which supports this answer appears to be straightforward. The current benefit accruing to a rational

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individual who holds an extra dollar in his existing cash balance must be equal to the value of the current flow which the individual is sacrificing in order to hold the extra dollar, the money rate of interest. Assuming the cost to society of an individual's having an extra dollar in his current cash balance is zero, the current return to the individual's holding of an extra dollar in his cash balance exceeds the social cost by the money rate of interest. Only by rewarding money holding with the money rate of interest can monetary policy achieve an equality between the marginal returns and marginal social costs of holding money.<sup>(1)</sup>

B. The empirical inappropriateness of this reasoning

But the above argument makes the empirically unreasonable assumption that a perfectly competitive, no-transaction-cost economy is achievable with proper monetary policy.

Making room for an economy with unavoidable transaction costs in the above argument requires more than minor rephrasings. One difficulty which immediately arises is that in an economy with transaction costs in equilibrium, money rates of interest on different assets are generally different so that the previously proposed answer would give us no guide as to which interest rate, if any, is the one relevant to the correct answer. Another difficulty is that since rewarding the holding of money keeps money out of the hands of these individuals who otherwise would have sold nonmonetary assets to the money holders, thereby altering the pattern of asset ownership, it is not at all clear that the policy implied by the proposed answer produces a Pareto optimal distribution of asset ownership in an economy with transaction costs. Still another difficulty is that, since money is not fundamentally the result of one's direct utility for money but rather yields

benefits by producing something of value for the money holder, and since the private product of an asset to its holder is not necessarily equal to the social product of his holding the asset in the presence of transaction costs, the implied equality of these products in the proposed answer may not be true. These difficulties can be met only if there is an explicit specification of: an individual's relevant alternatives to holding money, the effects on others of an individual's holding money, and the nature of the product created for an individual by his holding money. In other words, the formal structure of the problem forces us to specify a general equilibrium model in which transaction costs enter in a sufficiently explicit manner for us to determine all of the real effects of rewarding the holding of money.

### C. Outline and Summary of Main Results

Section I presents a positive theory of general equilibrium in an economy which includes equilibrium transaction costs. Section II presents specializations of the model of Section I in order to derive the economic effects of rewarding the holding of money. The special models are somewhat more general than the original model of Baumol [2] and its recent generalization by Feige and Parkin [6], and are also more complete in that they can be used to determine the effects on others of inducing an individual to hold more money.

Once the general equilibrium effects of rewarding the holding of money are determined, there is still a problem of describing a "social optimum" in a world complicated by transaction costs. Paretian optimality conditions are specified in Section III. In Section IV, these conditions are applied to the model developed in Sections I and II so as to describe a Pareto

optimal monetary policy for an economy in full competitive equilibrium. In this statically optimal system, the reward to money holdings is equal to or less than the initial rate on time deposits and is always sufficiently large to obliterate everyone's purchases of time deposits.

We shall find an alternative, less centralized monetary system which will also generate the same equilibrium allocation of resources in our model. This specification permits a simple and complete description of the optimal role of the government for any competitive economy with transaction costs in full equilibrium. This specification is the central theoretical result of the paper.

I. THE ECONOMIC THEORY APPROPRIATE TO AN ECONOMY WITH UNAVOIDABLE TRANSACTION COSTS AND THE ASSUMPTIONS OF OUR ANALYSIS

A. The meaning of transaction costs

A transaction or exchange is defined as any transfer of property rights between individuals. Transaction costs are the sum of contract, search, bargaining, and calculation costs. Contract costs are the joint losses to transacting individuals which result from the initial lack of perfect information of the parties who protect property rights regarding the existence and nature of the exchange agreement and the performance of the parties according to the agreement. These include legal fees, court battles, and joint surplus losses resulting from the prohibitive costs of inserting or enforcing certain conditions in an exchange. Search costs are the joint losses to society which result from the lack of initial information of some of the individuals concerning the available exchange offers in the economy. These include advertising costs, shopping costs, and surplus losses due to an individual's rational failure to discover

better exchange offers. Bargaining costs are the joint losses to transacting individuals resulting from the lack of initial information concerning the terms of an actual exchange. Calculation costs are simply the costs of an individual's computing his personal optimal program of purchases and sales.

We assume the absence of calculation costs. We shall also assume a competitive general equilibrium so that market prices for each particular kind of good are costlessly known and constant to each individual. This latter assumption implies the absence of search and bargaining costs. Consequently, contract costs are the only transaction costs which are considered in this paper. These costs are still quite a large part of empirically observed transaction costs. They include, for example, the joint wealth losses to transacting parties because of well-known malincentives involved in fixed rental agreements, constant percentage piecerates or quotas for workers, budgets or profit-sharing for managers, and cost-plus contracts.

Transaction costs do not include the costs of protecting the initial distribution of property rights. These protection costs are assumed to be identically zero. Allowing such costs would immediately admit an inefficiency in a purely competitive equilibrium because individuals have a private advantage in devoting resources to claiming and protecting rights to a given price of property (i.e., the total revenue from the property) which exceeds the social advantage of such resource-using activities (i.e., the consumer surplus gained by price-rationing the benefits of the property).

#### B. Describing a competitive economy with transaction costs

An economy without transaction costs in equilibrium can be completely

described in terms of consumer benefits and productive services and their respective flow prices (or "rentals") over time. Current benefits and service prices in a competitive equilibrium in such an economy can always be found by simply equating the current market demand and supply for each benefit and service. The equilibrium amounts of future services and benefits (perhaps contingent on certain technological states of nature) depend upon the time preference and productivity between current benefits and future benefits. The equilibrium distribution of given amounts of rights to future services and benefits depends only on the initial distribution of wealth and the relative attitudes toward risk bearing between individuals. Capital goods need not be explicitly referred to even though they are the physical sources of consumer benefits and productive services. Bonds and conditional claims are merely contracts through which exchanges of present for future benefits or services are made, and, as long as the implications of such exchanges are recognized (see Thompson [16]), these contracts, like capital goods, need not be part of the description of the equilibrium. Making the technological role of capital goods and the stock of bonds explicit in an economy without transaction costs is at best redundant and typically constitutes an unnecessary specialization of the general model.

In contrast, an economy with transaction costs cannot generally be described without making reference to capital goods and bonds. Once unavoidable transaction costs are introduced, markets for capital goods will generally replace the markets for benefits or services in a nontrivial manner. In general, some sequences of separate exchanges of rights to use flows of services or benefits are not worth the sequence of transaction costs required to make such agreements even though exchanges of the capital goods which

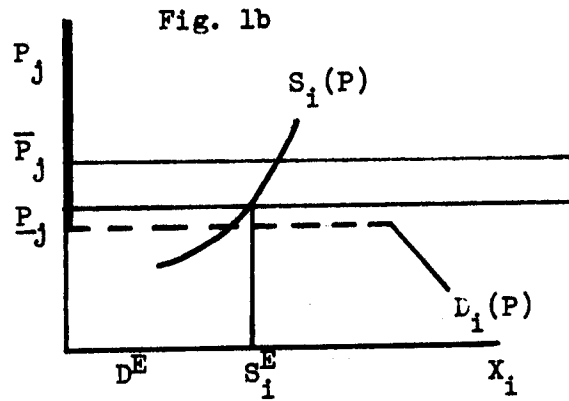
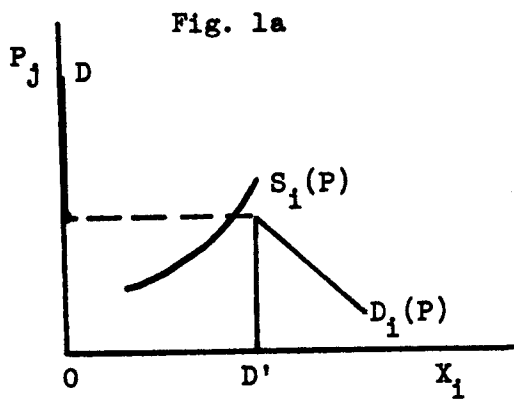
generate the flows are worth the capital-good-transaction costs. Similarly, markets for capital goods, bonds, and insurance policies will replace the markets for all future services and benefits (perhaps conditional upon future states of nature) in a nontrivial fashion. Thus, we cannot generally describe the equilibrium in an economy with transaction costs without making explicit the capital goods and future contracts, and also the corresponding transaction costs. We shall call any property right -- whether a service right, a leasehold, an ownership right to a present or future capital good, or a conditional claim -- an asset.

We assume that all assets are pure private goods so that neither non-pecuniary externalities nor collective-type goods can appear in our economy. The purpose of this assumption, like the assumption of pure competition, is to prevent nonmonetary sources of inefficiency from unnecessarily complicating the analysis. (2)

Individuals are each assumed to be maximizing a continuous intertemporal utility function subject to a non-empty, compact benefit constraint set formed by combining a standard budget constraint for each point in time, a constraint expressing the cost of each transaction, and a production feasibility set. It follows that desired purchase and sales correspondences exist for each asset. And it follows from budget constraint at each point in time that Walras' Law and zero-order-homogeneity with respect to accounting prices are satisfied at each point in time. We wish to apply the well-known existence theorem of Gale ([8], Theorem 2) to demonstrate the existence of an equilibrium for each point in time for given expectations of future events and prices. But to do so, we require, in addition to Walras' Law and



zero-order-homogeneity, the continuity of excess demand correspondences and the convexity of the set of excess demands that exist at a given set of prices. The usual device generating these conditions is the convexity of preference and production possibilities sets (Arrow-Debreu). But this device will not work in an economy with overhead transaction costs. With overhead transaction costs, the set of individual excess demands at a given set of prices is not a convex set. Without convex excess demands at given prices, there is generally no way to prove that a competitive equilibrium exists, as is illustrated in Fig. 1a. But we shall also assume the presence of competing trading specialists, who each have insignificant overhead transaction costs and are willing to buy, sell, and store any amount of each asset. The prices at which these specialists trade are determined by transaction costs and their expected future prices and technology.



$X_i$  = quantity of good  $i$ ;  $S_i$  = quantity supplied of  $i$ ;  $D_i$  = quantity demanded;  $P_j$  = the price of good  $j$ ;  $S_i^E$  = equilibrium supply of  $i$  given that the suppliers of  $i$  are also suppliers of  $j$ ;  $\bar{P}_j$  = buyer's price of  $j$ ;  $\underline{P}_j$  = seller's price of  $j$ ;  $(S_i^E - D_i^E)$  = specialist demand in equilibrium.

The existence of these traders assures us that the set of aggregate excess demands at some set of prices is convex and contains zero. This is illustrated

in Fig. 1b and is sufficient for the existence of an equilibrium at any point in time.

C. The optimality question.

For the optimality theorem which is the main subject of this paper, we add the assumption that each individual is locally nonsatiated in that in any neighborhood of a feasible consumption bundle, there is always a preferred current consumption bundle which contains more of all consumption goods, where all prices are non-negative and at least one consumption good price is strictly positive. While this additional assumption is sufficient for the Pareto optimality of a standard competitive equilibrium, it is definitely not sufficient, as we shall soon see, for Pareto optimality in a competitive equilibrium containing transaction costs.

We shall be concerned with the Pareto optimality of a full competitive equilibrium.

D. Full equilibrium in an economy with transaction costs.

A full general equilibrium is said to occur when there is a time-ordered set of desired purchase-sale equilibria, each corresponding to a distinct technology, in which there is perfect knowledge of current price offers, the future price offers that would result under each of the possible future technologies, and the joint probability distribution of future technologies. Consequently, in full equilibrium, individuals all know the probability distribution of market prices, i.e. relative prices between assets of agreed upon physical and legal characteristics; they cannot hold differing beliefs concerning such prices. <sup>(3)</sup>

Nevertheless, individuals may still have differing probability distributions in equilibrium to the extent that they have differences in information

concerning contract-relevant physical and legal properties of particular assets. These information differences are the only source of transaction costs that occur in a full competitive equilibrium.

E. The assumption of a strong law of demand.

Aggregate market behavior is assumed to follow a "strong" law of demand. Namely, if an asset's current market price were to drop and individual wealths were reduced so as to leave everyone at his original utility level, the buyers in the aggregate would want to buy more -- not just the same amount -- of the asset and the sellers would want to sell less; similarly, if the value of a current market information advantage were to rise, and wealth effects removed, individuals would produce more of this type of market information. The purpose of assuming a strong law of demand will be specified when it is employed.

F. The specification on feasible and actual government policy.

Different systems of property rights -- and thus different responses of other individuals to the behavior of each individual -- generally imply different aggregate transaction costs and therefore generally different sets of individual utilities. In other words, for every different method of defining and protecting property rights, there is a distinct equilibrium allocation of resources.

We shall assume that there is an economic agent called the government. We assume that the government has already set up a system of defining and protecting property rights which determines, given rational individual responses to the system, the transaction costs associated with each possible transaction. In particular, we shall assume that the legal and administrative structure (which determines the procedures whereby an individual, say by restrictions on

the contract information which he can gather and distribute, is induced to incur certain contract costs) is predetermined to be that structure which minimizes aggregate transaction costs for any given set of property transactions (i.e., any given set of transactions other than those which are solely for the creation or enforcement of contracts or the creation of contract information). In addition, all governmental administrative costs induced by a transaction are assumed to be included in contract costs by means of a user-tax on the transaction. This determines a legal structure and a structure of taxes on transactions for any given set of transactions leaving us with a narrower problem, one of achieving an optimal set of transactions.

It should be pointed out that the policies which the government pursues in order to carry out this role of determining the cost of any given transaction is very broad in scope, apparently much broader in scope than the policies which economists traditionally recommend in the presence of monopoly and nonpecuniary externalities. The government may reduce the costs of a given private transaction by outlawing blackmail or fraudulent misrepresentation in the transaction, behavior which would have both sides rationally devoting real resources to the production of information advantages which create or prevent mere redistribution between the parties in the transaction. The government may reduce the sum of transaction costs by forcing certain transactions that would have taken place anyway (such as land transfers via land appropriation and sale for "urban renewal" and such as the replacement of tax financing with lending via debt financing), thereby eliminating some expenditures of real resources on the production of market information, which has purely redistributive effects as it benefits the information producer at the cost of the other party in the transaction (such as information produced by a land buyer about what he is really getting for his money and such as information produced by a private lender

about the ability of the borrower to default by "skipping out").

By so determining the cost of each transaction, the government helps to determine the equilibrium quantity of each type of transaction. The government may also affect equilibrium quantities by (1) the additional taxing (or subsidizing) of transactions, (2) governmental production, or (3) governmental control of the monetary sector. Regarding the first of these policies, we shall initially assume that there are no taxes (or subsidies) on transactions additional to the transaction taxes described above (those which were just sufficient for transactors to internalize the government's real administrative costs induced by the transaction). Regarding the second policy, we assume that the government directly transacts in assets by producing and distributing any asset in full equilibrium whenever this can be done profitably in real terms (given the costs of each transaction as determined by the governmental policy role described above) and therefore more cheaply in real terms than can private enterprise. Under this policy, the government may obviate certain private transactions by itself engaging in production. While fire protection is an important example for substantially noncollective goods produced by a government, the most obvious examples are found in the production of several kinds of collective goods such as national defense, bridges, and weather information. Such collective-type goods may be produced by the efficient government, not because of any free market under- or over-production of these goods (which would merely dictate a subsidy or tax policy), but because the private costs of erecting barriers, collecting, and excluding nonpayers (costs which are avoided when the government tax-finances and freely distributes its output of the collective good) may exceed the wastes due to misdirected incentives involved in the government's non-private-property reward structure. (See Thompson [17]).

It is assumed that individuals assume that their purchases and sales have

no effect on governmental production or the property rights system. The support for this assumption is the same as for the conventional assumption that individuals assume that their purchases and sales have no effect on equilibrium prices. It is also assumed that any taxes or subsidies that arise (other than the above mentioned transaction taxes and any user charges for government supplied private goods) are monetary lump sums. These assumptions rule out social inefficiencies resulting from (a) spreads between buyer's and seller's prices which are not justified by real transaction costs and (b) purchases or sales which redistribute from others via induced changes in government policy.

The third type of additional governmental policy affecting the equilibrium quantities of private transactions, the government's control over the monetary system, will be allowed to vary. Initially (in Section II) we shall consider a "centralized" banking system (described in the following subsection) similar to the familiar, textbook model of the current U.S. system. Then (in Section IV) we shall alter key variables within the system in order to generate a Pareto optimal competitive equilibrium. In the process of altering these variables, we shall examine the competitive equilibrium with a "decentralized" banking system, one with monetary policy guided only by the other roles of the government specified in this section. We shall see that this generalized competitive equilibrium is also Pareto optimal.

We shall thereby have proved our central result: Pareto optimality in a full competitive equilibrium with transaction costs is achieved by a government which (a) minimizes transaction costs for a given set of property transactions and (b) engages in transactions when and only when this can be done profitably in terms of real benefit-cost analysis.

#### G. The financial structure of the economy.

We shall consider an economy originally composed of any number of kinds

of real assets and a complete Gurley-Shaw [9] array of financial assets: primary (including governmental) debt, time deposits with financial institutions, and money.<sup>(4)</sup> Money is assumed to be used for every purchase. It consists of currency, which is issued by the government, plus demand deposits, which are issued by banks. We assume, until Section IV, that there is no direct interest paid on currency, a currency reserve requirement on demand deposits, and a prohibition of interest payments on demand deposits. The entire structure of this original economy is subject to change by the government's achievement of a Pareto optimal policy.

The individuals in our world are either bankers (who own banks), nonbank financiers (who own financial intermediaries), or nonfinanciers. A nonbank financier does not issue money but specializes his nonconsumptive behavior in transacting in financial assets. Financial assets are defined as assets yielding direct money returns while real assets are assets yielding commodity returns, which only afterwards may be converted into money returns. Financial assets have the property that the money value of the return is the same regardless of the owner of the asset.

Positive costs exist for private transfers of every asset with the possible exception of money. This is an implication of the existence of money rather than an independent assumption, for the asset (or assets) which end up being the medium of exchange, which we are taking to be money as defined above, is that commodity which has the lowest costs of transfer (see, for example, Brunner and Meltzer [4]). Since private producers of any asset typically receive some benefit out of producing and then withholding or concealing some information about the properties of the asset and since a lesser amount of such benefits would appear if the producer had to share profits with a consumer cooperative in the form of a government, it is not surprise that the assets which have the lowest cost of

transfer, and thus serve as the medium of exchange, are assets which either are produced by the government (e.g., currency) or have their physical properties rigidly controlled by the government (e.g., demand deposits).<sup>(5)</sup>

H. Summary of the specifications and the problem.

To briefly summarize the specifications introduced in this Section: We shall consider: a complete, Gurley-Shaw-array of private good-assets in a competitive economy admitting a strong law of demand, positive costs of all transactions, specialist traders with no overhead transaction costs, and a property rights system which determines the level of private costs associated with every possible private transaction so that aggregate transactions costs are minimized for any complete set of property transactions. Money is initially non-interest bearing and there is a currency reserve requirement on private issues of money (i.e., demand deposits). This economy always has a current competitive equilibrium, a set of current market prices for which no excess demands exist given the expected joint probability distribution of all future events and market prices of each individual. The problem is to determine a set of government policies for which a full competitive equilibrium (a current equilibrium in which the joint probability distribution of all future events and prices is known by everyone) in our economy is a Pareto optimum under the conventional assumption of local nonsatiation.

Some additional, probably inessential, assumptions will be introduced in the following section.

II. THE EFFECTS OF REWARDING THE HOLDING OF MONEY

A. The compensated demand for money.

As has long been recognized, the private holding of positive money balances means the existence of unsynchronized streams of future payments and receipts.<sup>(6)</sup>



Clearly, if all of these unsynchronized streams of future payments and receipts were taken as exogenously given, an individual's money holdings at any date would then automatically equal the original level plus the cumulative excess of total receipts over total payments. Therefore, if we are to consider variations in an individual's choice among alternative levels of cash holdings, we must set up the analysis so that the individual varies his purchases or sales of at least one type of nonmonetary asset.

When the current reward to holding money rises,<sup>(7)</sup> and there is a compensating decrease in wealth, individuals will hold more money and less of a certain class of nonmonetary assets. That is, whenever individuals are presented with an additional current reward per dollar held but also lose lump sums that pull them back to their original utility levels, they desire to currently hold greater amounts of money and lesser amounts of some other assets. These latter assets will be called money-substitutes (following standard, Hicksian terminology).

We assume that there are no money-complements (such as safes and cookie jars). Subsection D below presents a special case of our general model which derives this assumption).

B. The nature of the effects created by rewarding the current holding of money.

We divide continuous time up into a sequence of nonintersecting, connected intervals called "periods". Within each period, the technology is known over the whole period from the beginning of that period. We define the "money rate of interest" on an asset to be the monetary value of real returns on the asset during a unit period plus the market price appreciation of the asset over the period divided by its initial money price. These rates are generally different on different assets, depending on the degree to which the different assets have different transaction costs.

An individual may be either a current purchaser or a current seller of money-substitutes (if he is neither, he does not affect this analysis). The rational purchaser of money-substitutes in the initial equilibrium has decreased his initial cash balance until the present value of the interest return to holding these assets rather than money is equal to the present cost of the transactions involved in purchasing and later disposing of these assets.<sup>(8)</sup> And the rational seller of money-substitutes has decreased his cash balance below its maximum potential level by selling money-substitutes only up to the point that the additional transaction cost involved in delaying his sale of money-substitutes is just offset by the interest return to holding onto the money-substitutes until the later selling date occurs. Thus, the product to the money-holder created by inducing him to hold an extra dollar in his current cash balance, the private return to holding an extra dollar rather than buying more or selling less of a money-substitute, consists of all transaction costs that would otherwise have been incurred, either in purchasing and later reducing his stock of money-substitutes by a dollar or in selling the dollar's worth of money-substitutes at a less opportune, future date, costs which equal the yield on money-substitutes.<sup>(9)</sup> (This yield includes the value of contract information that the financiers create for their customers and is net of any direct interest paid on money.)

The other effects of rewarding the holding of money, those which do not directly affect the individuals as holders of larger cash balances, all stem from the lower cash receipts of the sellers of money-substitutes.

C. The two components of the benefits of holding money.

Transacting into and out of money-substitutes may involve doing business with financiers, specialists in these assets. These specialists, as creators of or dealers in money-substitutes, have superior knowledge concerning contracts in money-substitutes. They use their knowledge superiority to advantage by

giving their less informed customers lower effective returns than they would if the nonspecialists knew as much about the money-substitute contracts as the specialists. The less informed individuals cannot, when purchasing money-substitutes, expect to escape these lower total benefits with successful demands for price discounts that reflect the true magnitude of the implicit deception concerning the contract. This is because a nonspecialist does not know the appropriate adjustment factor: so when the discount that a nonspecialist requests from a specialist is not below that which a specialist could get, the specialist will refuse the customer's offer; specialists consider a discounted price only when the requested discount is smaller than that which he would have to give an equally informed individual. The resulting expected loss to the non-financiers will consequently appear to the nonfinancier as a cost of transacting. Non-financiers deal with financiers because the contract costs in doing so are even higher in dealing with other nonfinanciers.

Recapitulating, the product to the money-holder created by rewarding his holding of money, his avoided net cost of transacting in money-substitutes, is the sum of the direct expenditures on such exchanges and the expected cost of being deceived by specialists in money-substitutes.

By our assumption of a strong law of demand, specialists do not respond inelastically to inelastically endowed information advantages; specialists incur costs in exploiting or creating their information advantages.<sup>(9)</sup> On the margin, the value of an information advantage to a specialist is equal to his cost of exploiting or creating his information advantage.<sup>(10)</sup> Hence, the interest return on a money-substitute is equal to the real resource expenditures induced by spending an extra dollar on a money-substitute and selling it at the most opportune times in the future.

We can define the "social value of money balances" to be the savings in

real transaction costs in money-substitutes generated by the holding of an extra dollar. This establishes the part of the standard answer to our introductory question that tacitly assumes that the real "social value" of money is equal to the private value of money, the value of the foregone interest returns to the money holders. But we must keep in mind that the effects of inducing additional money-holding also include all of the effects of having the creators of money-substitutes with lower cash receipts. All of these effects then determine the "social cost of real money balances". But there is no general equivalence between the social cost of creating real money balances and the cost of producing nominal money (which we are assuming to be zero). The original problem thus becomes a problem of specifying the "social cost of creating real money balances." (In section IV, we shall see that Pareto optimality indeed requires the equating of what we have described as the social value and social cost of creating real money balances.)

D. The identification of money-substitutes.

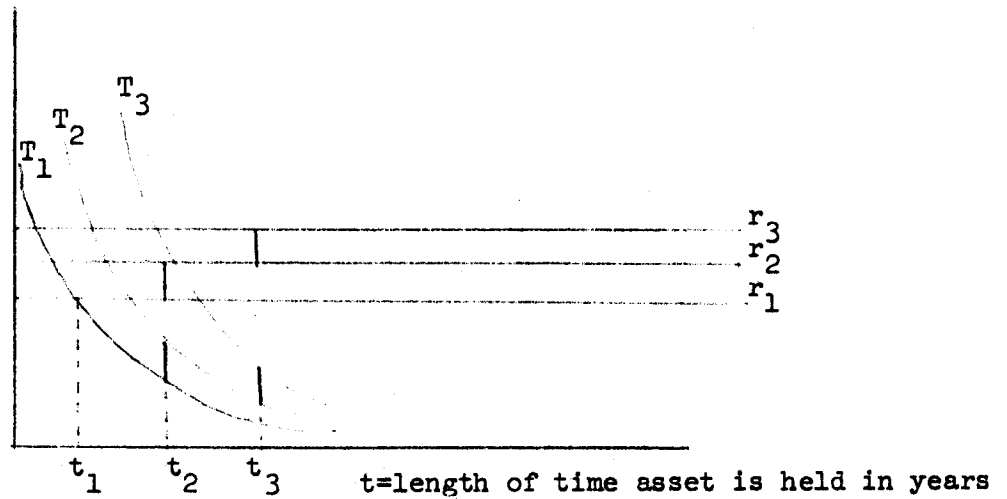
To obtain some clue as to the identity of the assets which serve as money-substitutes, we shall temporarily consider a special case of our general model above. It is only meant to be suggestive, as none of its assumptions are necessary for our conclusion. Our conclusion will be that an asset serving an individual as a money-substitute is a nonmonetary asset which, for that individual, has the lowest associated transaction costs and therefore the lowest yield and highest rate of turnover. Before using this conclusion, we shall digress, in Section E, to note several interesting empirical implications of this special case.

For the special economy treated in this subsection, we assume that all transaction costs are overhead purchasing costs (i.e., all resale costs and variable transaction costs are zero), that all individuals possess perfect certainty of all their future streams of payments and receipts, that all assets are long-lived,

that the various asset-yields are constant over time.

Each currently purchased asset is going to be held for a certain length of time. The transaction (purchase) cost per length of time held of an asset is, of course, inversely proportional to the length of time the asset is held. Given the money rates of interest on nonmonetary assets, i.e., their nominal yields minus any possible nominal yield on money, and the purchasing costs of the various assets, and given the optimal times in the future at which each asset purchased now is going to be sold, Figure 1 describes the current purchase choices of a rational individual.

Fig. 1



$T_i$  is the purchasing cost per length of time held, in years, and  $r_i$  is the annual money rate of interest on the  $i^{\text{th}}$  nonmonetary asset ( $i = 1, 2, 3$ ). The  $T_i(t)$  curves are all rectangular hyperbolas and cannot touch one another. We need consider only those assets that may be rationally purchased. Hence, none of the assets we are considering can have a greater transaction cost than some other asset without also having a greater yield.

That is, the yield-rank of an asset is identical to the transaction cost-rank of the asset. As is easily seen from the graph, for assets which are going to be held for less than  $t_1$  years (where  $T_1(t) = r_1$ ), the only asset worth holding is money; for any nonmonetary asset, the annual purchase cost exceeds the rate of interest. For assets held slightly longer than  $t_1$  years, the annual money rate of interest on one asset, namely asset 1, exceeds its annual transaction cost so that this asset will be held rather than money. For assets held slightly longer than the  $t$  corresponding to the intersection of  $T_2(t)$  and  $r_2$ , it will still pay to hold asset 1, because the excess of yield over annual transaction cost is greater for that asset than it is for asset 2. Once the length of time that an asset is held exceeds the point  $t_2$ , the point at which the excesses of annual yields over respective annual transaction costs are equal for assets 1 and 2, asset 2 is chosen over asset 1. The analysis is similar for asset 3. [At the point  $t=t_3$ ,  $r_3-r_2 = T_3(t) - T_2(t)$ ]. Putting this description of rational behavior in converse fashion: if a rational individual chooses to hold some money, he will hold it for less than  $t_1$  years; if he purchases some of asset 1, he will hold it for from  $t_1$  to  $t_2$  years; if he purchases some of asset 2, he will hold it for from  $t_2$  to  $t_3$  years. And if he purchases asset 3, he will hold it for  $t_3$  years or more.

Therefore, the asset which is most easily marketable receives the highest liquidity premium, and has the highest rate of turnover. At the other extreme, the asset which is least marketable has the highest rate of return and the lowest rate of turnover.

It seems quite apparent from casual empirical knowledge that, in the present day United States, time deposits with financial institutions and

treasury bills would represent asset 1, bonds (with yields adjusted for default risk) would represent asset 2, and real assets (including common stock) would represent asset 3. We shall assume this to be the case.

We can now determine which of the assets are money-substitutes. Suppose the holding of money is granted a slight reward. Then the  $r_1$ 's, the levels of the horizontal lines in Fig. 1, all shift equally downward by this rate of reward (either because a new, direct yield on money is sacrificed by the holding of a nonmonetary asset or because there is a price-level deflation which, at the same allocation of real resources, reduces all nominal yields on nonmonetary assets equally so as to keep the real yields on all nonmonetary assets constant.) Since  $t_2$  and  $t_3$  are each determined by equating a difference between interest rates with a difference in transactions costs per unit of time, they are not altered by a shift down in all of the interest rates by a constant amount. But  $t_1$ , which is determined by the intersection of  $r_1$  and  $T_1(t)$ , is obviously increased by the reduction in interest rates. Hence given the initial pattern of future receipts and expenditures on assets 2 and 3, which serves to maintain the original holding periods of currently purchased assets,<sup>(11)</sup> the correct amount of money held will increase and the current amount of only asset 1 held will decrease.<sup>(12)</sup> Removing the wealth effects of the reduced real expenditures on transactions (effects which would alter future expenditures on bonds or real assets) and keeping the receipts of the financier selling asset 1 constant, there will indeed be no change in future purchases and sales of bonds and real assets. Therefore asset 1, time deposits with financial institutions and treasury bills, serve as the one and only money-substitutes in the original economy.

E. Some Empirical Implications of the Special Model

The special model in the above subsection easily generates some fairly interesting, testable implications. For one, the model, as well as the general theory used in this paper, implies (as long as the asset purchases of the dealers in money-substitutes are maintained) that an anticipated deflation will decrease (increase) the equilibrium real rates of interest if and only if the production process of transacting in money-substitutes is factor-intensive in durable (perishable) factors, assuming no effect of wealth on time preferences. This can be contrasted to the hypotheses of Mundell [13] and Phelps [14].

For another implication, since financial assets tend to be created so that their lifetimes approximate their holding periods in order to obviate what would otherwise be high reselling costs to nonspecialists, financial assets which are observed to have relatively long lives will also be observed to have relatively high yields. Evidence for the existence of this kind of "liquidity premium" is found in Kessel [10]. Thus, our "explanation" of normal backwardation is that since financial assets with relatively long maturities are going to be held by their purchasers for relatively long periods of time, they must have relatively high transactions costs and therefore relatively high yields. If long maturities yielded what shorts did, they would necessarily have similarly low transaction costs and thus would not be held for a long period of time.

Finally, there is an implication regarding the effect of the growth of financial intermediation through lower real costs of financial intermediation. A result of such a shift in transaction costs is a lower interest rate on long term bonds and real assets and a higher interest rate on



deposits in financial institutions. This latter increase serves to steepen the slope of the  $T_1(t)$  curve at the post shift equilibrium. Assuming a periodic distribution of net expenditures over time (where the period is a small fraction of  $t_1$ ), ignoring the effect of changes in the real rate on expenditures streams, and supplying individuals with the post-shift money demands, movements in  $t_1$  imply proportional movements in the desired stock of money. A steeper slope of the  $T_1(t)$  curve then implies a steeper demand for money function. Hence, under suitable conditions, our special model implies that the demand for money becomes steeper as the cost of intermediation falls. This implication runs counter to the conjecture popularized by Gurley-Shaw.

## II Necessary and Sufficient Conditions for a Pareto Optimum

We now state three obviously **necessary** conditions for Pareto optimal current decisions. The first is that a system of property rights be chosen among the feasible alternatives such that there is a minimum of aggregate transactions costs <sup>(13)</sup> for a given allocation of real resources to present consumption and real production. By keeping the allocation of resources to present consumption and real production the same, we are holding constant the utility levels of all individuals. Nevertheless, the real resources saved by an improvement under the first condition could be used in such a way as to benefit at least one person without harming anyone else (say, for the production of leisure). In other words, any change based upon the first condition satisfies a Pareto Condition. <sup>(14)</sup> The tricky part of applying the first condition lies in changing the process of making transactions, while, at the same time, adjusting the new economy in such a way that the equilibrium allocation of resources toward the consumption and production or real assets remains the same at the same distribution of utility. This condition for Pareto optimality applies immediately, however, to economic activities which only alter the equilibrium allocation of resources to

nontransaction activities through lump sum redistributions of wealth. Such activities should be -- and, like the crimes of blackmail and extortion often are -- effectively outlawed, because, whatever the distribution of wealth, these activities represent wasted transaction costs; the costs of blackmail and extortion are 100 percent transaction costs because they are made solely in order to transfer property rights. (15)

The first condition for Pareto optimality is not implied by our assumption that the government minimizes aggregate transaction costs for any given set of property transactions. Transactions for the consumption or production of consumables can conceivably stay constant while property transactions can still vary. This can be done by varying transactions made in financial assets. Our prior assumption on the behavior of the government is strong but does not guarantee quantities of transactions in financial assets such that there is a minimum of transaction costs for a given set of transactions in real assets, transactions determining the allocation of resources to consumption and the production of future consumables.

The second condition for a Pareto optimal allocation of resources is that each individual has maximum utility for a given allocation of resources to transactions activities. This condition is trivially satisfied in that it is immediately implied by our rationality assumption.

The third necessary condition for Pareto optimality determines an optimal division of resources between transactions and of real production consumption activities. It is simply that there is no alternative set of transactions in real assets that satisfies a Pareto Condition.

These three necessary conditions, taken together with an initial distribution of resources, are also sufficient to determine a Pareto optimal

current allocation of resources. For, given an initial distribution of resources and of information, the first condition determines an optimal cost of making any transaction involving real assets, the second condition determines the value of making the transaction by attaching individual benefits to the various real assets, and, using these two conditions, the third condition determines reallocations until further transactions in real assets can no longer be made that will satisfy a Pareto condition. When such a situation is achieved, there is a Pareto optimum.

If there were no transaction costs, these three conditions would degenerate into the second and third conditions. The second would still be trivially satisfied by our rationality assumption. The third, which is equivalent to the familiar definition of Pareto optimality, would of course, be satisfied in a competitive equilibrium containing no transaction costs.

Since the second condition is trivially satisfied under our rationality assumption, we can concentrate on the first and third condition.

#### IV. ACHIEVING A PARETO OPTIMAL GENERAL EQUILIBRIUM

##### A. Satisfying the first condition for Pareto optimality

Utilizing the first condition for Pareto optimality and the theory of the product created by rewarding the holding of money which preceded it, the following is clear: If there is a virtually costless method of maintaining the original competitive equilibrium's allocation of resources to production and consumption, then a government policy of rewarding the holding of money until no money-substitutes are purchased is necessary for an optimum in our competitive equilibrium. This policy is not restricted to

the original money-substitute. Once the original money-substitute is driven out of existence, succeeding assets which may serve as money-substitutes must be considered. We shall consider our money-substitutes one at a time.

1. First, suppose that a reward to the holding of money is introduced which is just sufficient to reduce to zero the desired purchases of a purchaser and the desired holdings of a non-financier seller of the original money-substitutes, time deposits and treasury bills. This reward may be introduced either through an expected deflation or the direct financial payment of interest on money. As a result of this policy, all money rates of interest on nonmonetary assets will be reduced by the same amount until it pays to hold all nonmonetary assets for such a long time that it never pays an original purchaser to purchase the original money-substitute, or an original seller to delay his sales of the original money-substitute, at the original interest rate differentials. [On Figure 1, this means that  $r_1$  is reduced to  $T_1(t_2)$  and  $r_2$  to  $T_1(t_2) + r_2 - r_1 = T_2(t_2)$ ].

This clearly saves the economy transactions costs involved in either creating, purchasing and later disposing of these assets or of disposing of the existing assets of this type at later, less convenient dates. The problem now is to specify a government policy which will maintain the allocation of resources to the production and consumption of real assets. Except for the positive wealth effects of the transaction costs they save, nonfinanciers maintain their original purchases and sales of primary debt and real assets. But ex-creators of the money-substitutes no longer have sufficient funds to make their previous purchases of primary debt and real assets. The dollar amounts of these previous purchases by the financial institutions must somehow be maintained, otherwise the allocation of resources to the production

and consumption of real assets may be altered as a result of the decrease in their purchases of primary debt and real assets. One method of maintaining these purchases is to loosen sufficiently the currency reserve requirements on the demand deposits of banks (and any possible portfolio restrictions) on banks so that banks find it profitable to acquire the financial assets and lending facilities of the financial intermediaries and offer demand deposits in exchange for their own time deposits to willing ex-owners of time deposits. The financial assets and asset purchasing facilities of the financial intermediaries are therefore acquired before the exit of these institutions raises real rates of interest by cutting back the demand for primary debt and real assets. (16) And treasury bills are monetized into currency and demand deposits in proportions that insure the maintenance of the original price level. Under this policy individuals would end up holding additional money instead of the original time deposits and treasury bills and would face the same price level and relative prices of the original, nonliquid assets. They would save the real expenses of transacting into and out of the original money-substitutes and of selling these assets later at higher marginal transaction costs. The economy would also save those resources that were spent previously by financial institutions in creating and servicing savings deposits and by the government in floating and servicing treasury bills.

2. As argued above, the new money-substitutes will be the most liquid of all the remaining nonmonetary assets, i.e., primary debt (including government bonds). A further reward to the holding of money will indeed save some individuals the transaction costs of buying and later selling their new money-substitutes. But in order to maintain the previous allocation of resources to real asset production and consumption activities, these assets must be

bought by someone; and there is a transaction cost involved in this alternative.

A further reward to the holding of money satisfies our first Pareto condition if and only if, on the new margin, the real transaction costs saved by the individuals who are purchasing the new money-substitutes are above the real transaction costs of those who would alternatively purchase these assets. Consider banks as the alternative purchasers of these assets. A bank's marginal cost of purchasing primary debt is equal to the difference between the original interest rate on bonds and the original interest rate on time deposits. This difference equals the difference between the new rate on bonds and that interest rate on the original money-substitute just sufficient to induce an individual to switch money-substitutes. But a nonbank purchaser of the new money-substitute has a marginal cost of purchasing equal simply to the new rate of interest on bonds.

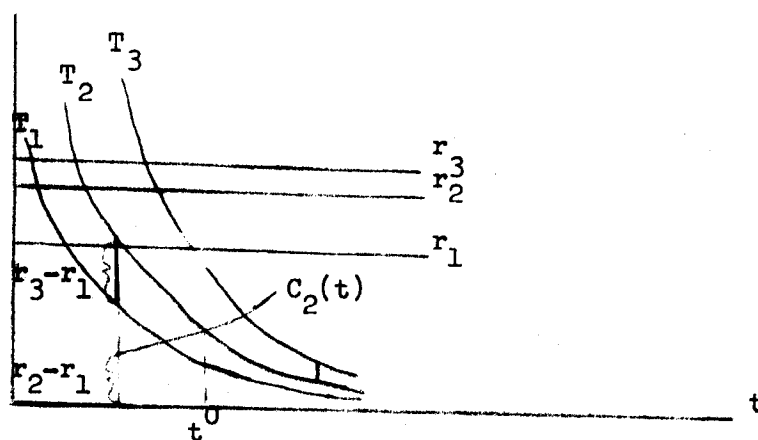
In terms of Figure 1: A bank's transaction costs of holding an extra dollar's worth of the new money-substitute is  $(r_2 - r_1)$ , which equals  $T_2(t_2) - T_1(t_2)$ ; and an individual's transaction cost saved when he is induced to hold an extra dollar instead of his new money-substitute is  $T_2(t_2)$ . Thus, the bank has a lower transaction cost of buying bonds than does an individual, and a further reward to the holding of money will further reduce transaction costs. The allocation of resources to the consumption and production of real assets can be maintained by again lowering the reserve requirements (and removing possible portfolio restrictions) on the banks so that they are able to make the bond purchases which would otherwise be made by other sectors of the private economy.

We shall assume throughout that an optimum is reached in which some bonds are still purchased by nonbanks. Therefore, obtaining a minimum sum of bank

and nonbank bond purchasing costs for a given, total number of bond purchases (and thus a given allocation of resources to the consumption and production of real assets at the same distribution of utility) implies an equality between bank and nonbank marginal costs of bond purchasing, the latter being those transactions costs incurred by nonbanks in putting an extra dollar into bonds rather than holding it as a cash balance.

In terms of our special model, this optimum can be visualized by (1) superimposing onto Figure 1 a nondecreasing, continuous curve, starting at  $t-t_2$ , labeled  $C_2(t)$ , with  $C_2(t_2) = r_2 - r_1$ , the curve representing the banks' marginal cost of lending corresponding to the amount of lending that they would be doing in place of nonbanks if the latter held a dollar at most  $t$  years before spending it, and (2) observing the intersection of this curve with  $T_2(t)$ . This is illustrated in Figure 2, where the socially optimal maximal holding time of money by nonfinanciers is given by  $t^0$  and the optimal level of interest rates is where  $r_1 = C_2(t^0)$ .

Fig. 2



The policy, of rewarding the holding of money and simultaneously reducing

the reserve requirement and loosening portfolio restrictions so as to maintain the original purchases of bonds, ceases to be possible once the reserve requirement becomes an ineffective constraint on bank lending; for then, any further lowering of the reserve requirement would not increase the bank's purchases of bonds.<sup>(17)</sup> The first condition for Pareto optimality is satisfied when this occurs because, with no constraint on reserve requirements, the marginal cost of bank lending is simply equal to the money rate of interest on bonds which is also the nonbank cost of lending. (In contrast, in our original economy, the interest rate on bonds exceeded the marginal cost of lending as the reserve requirement was an effective constraint on bank lending.)

In the special case of a constant industry cost of bank lending, i.e., equal resource proportions between bank and nonbank lending, this optimal policy rewards the holding of money at a rate equal to the original yield on the original money-substitutes, i.e., time deposits and treasury bills. This has the effect of equating the constant cost of bank lending ( $r_2 - r_1$ ) with the marginal cost of nonbank lending ( $r_2$  minus any direct reward to holding money.) Since this makes the net return to holding time deposits zero, desired purchases (and delayed sales) of the original money-substitutes are zero in this optimum. When there is an increasing industry cost of bank lending, the optimum reward to holding money is somewhat below the original rate on the original money-substitute; but it is still above that which would just do away with the desired purchases (and delayed sales) of money-substitutes by all nonfinanciers. That is, while the first stage of the policy does not necessarily eliminate everyone's purchases of the original money-substitutes from the economy, the second stage does. The following is a demonstration of this proposition: When resource proportions in lending between nonfinanciers and



financiers are not equal, then lending costs to banks rise because of the shift from nonfinancier to bank lending. The lending costs of financial intermediaries also rise to the same extent. Letting the direct payment of interest on money be zero, this correspondingly increases the optimal rate of interest on primary debt relative to the optimal rate in the case of constant lending costs (as is obvious in Figure 2). The increase in lending costs also lowers the intermediary demand price for time deposits by an equal magnitude. Since the reduction in the demand price for time deposits due to the higher lending costs is equal to the increase in the optimal rate of interest on primary debt, the demand price for time deposits remains at zero even though there are increasing costs of bank lending.

### 3. The decentralized alternative.

The only remaining independent government constraint on the banks is the restriction that banks are not free to set competitive rates on demand deposits. But, the above-achieved equality between the marginal cost of lending and the interest rate on bonds implies that the banks would not alter their behavior if they were allowed to freely compete for deposits. An extra demand deposit would not be lent out, so that banks have no incentive to raise the rate paid on these deposits. And since an intramarginal deposit yielded positive marginal profit as the interest rate then exceeded (by some small amount) the marginal cost of lending, banks would have no incentive to reduce their rate paid on deposits. In other words, when the reward to holding money is sufficiently high for the reserve requirement constraint to become redundant, the restriction on the payment of interest on demand deposits also becomes redundant. Hence, the government can also achieve the first optimality condition by completely freeing the banking system of its independent constraints

and paying currency holders the same rate that banks pay holders of demand deposits. The difference between these two systems, both of which generate the same, full equilibrium, is that in one the government determines the rate of reward to holding money while in the other the competing banks determine it. The former is thus "centralized" relative to the latter. The "decentralized" system is achieved by simply omitting any independent role of the government in controlling the monetary system.

The only controls on the banking system in the decentralized case are those derivable from (1) the minimization of transactions for any given set of transactions, and (2) government production under benefit-cost criteria. In satisfying the first role, the government may produce currency, control private money creation, and insure demand deposits. In fulfilling the second role, the government pays the market rate of interest on the money which it creates, i.e., currency, and may provide direct governmental lending and clearing-house services if such operations would yield positive profit to a private firm if the private firm had the same administrative costs as the government.

In either a decentralized optimum or a centralized system in which a direct financial yield is paid on money, the government's optimal rate of expansion of its produced currency supply is indeterminate in our model because the financial yield on money rises with the rate of inflation. An optimal inflation rate would have to be determined on the basis of a transactions cost model which is more general than that which we are employing in this paper. Such a model might, for example, contain different real costs of producing monies with different denominations.

Further, a more general model would permit differential costs of effecting

the centralized and decentralized systems. With a decentralized system, there are administrative transactions costs of the banks paying of interest on demand deposits and of the government's operation of a "stamped money" program (see Keynes [12]) to pay interest on currency. These same costs may still be present in a centralized system, in which case the decentralized system would probably be superior because the payment to money holding in the centralized system is granted by some rough consensus of the entire population to individuals who do not know the social optimum and would have little incentive to so vote even if they did. In contrast, in the decentralized system in full equilibrium, no knowledge of the social optimum is required, and the reward structure to the decision makers' actual behavior already satisfies the first optimality condition.

But the government could also achieve the first condition for a Pareto optimum by running an expected deflation sufficient to obviate the payment of interest on money. While involving the same kind of costs as the government's determining and enforcing of an optimal payment to holding money and while adding the heavy cost of teaching everyone to adjust to the deflationary price expectations, this policy would at least save the transactions and governmental administrative costs of paying interest on money. The choice between optimal deflation and decentralization is not at all clear on the full equilibrium level with which we deal in this paper.

Since the only element of increased transaction costs once the decentralized system is in effect is the cost of paying interest on money, the nature of this cost should be examined in order to improve subjective judgments as to its importance. Now the transaction cost of a bank's payment of interest on money in equilibrium is comprised of computing the interest on

the individuals' deposit balances, say at the end of each day, and crediting the individuals' accounts, say at the end of each year with the compounded accumulation of these amounts. These costs of multiplication, addition and recording are currently being incurred by financial intermediaries and appear, because of the use of computers, to be sufficiently low to be quite safely ignored. Apparently, however, if the government paid interest on the day-to-day currency-holdings of individuals, it would be extraordinarily expensive under current technology. For individuals would have to demonstrate the extent of their daily currency balances to the government and, under current technology, doing so would apparently involve all individuals taking daily trips with stacks of currency to governmentally administered offices. But suppose the government paid interest on currency only at the end of each year. Then the costs of such a stamped money operation, even under current technology, would be reasonably low. Banks, realizing that they would be swamped with withdrawals at the end of the year if they did not pay the annual interest on deposits held at the end of the year, would replace their within-year interest payments with end-of-year payments. The policy of paying interest on money only annually does not have the effect of removing the within-year reward to holding money. For, with constant intra-year returns on nonmonetary assets and constant transaction costs, regardless of currency supplies, the prospect of the interest return on money at the end of the year will make all nonmonetary asset prices gradually fall from the beginning to the end of the year at a rate equal to the yield paid on holding money. Prices will jump at the outset of each year, with the beginning-of-year to beginning-of-year changes in prices determined by the long term inflationary or deflationary forces such as changing currency supplies.

Since the effecting of this policy is not really without administrative cost, we should obtain some idea of the actual magnitude of the increase in real wealth resulting from the policy. To do this, we first recall that the interest which an individual receives on his original money-substitutes is the minimum marginal savings in real resources to the economy. Then we assume, for the sake of a reasonably conservative calculation, that: (1) The aggregate quantity of the original money-substitutes would have been at least constant over time, and (2) The average interest reward to the holding of a money-substitute ( $r_1 t_a$ , where  $t_a$  is the average time a money-substitute is held) exceeds the average transaction cost involved in holding the money-substitute ( $T_1(t_1)t_1$ , or  $r_1 t_1$ ) by no more than one-half the transaction cost. [In terms of our illustrative model, this means that the average time that a money-substitute is held is no more than 50 percent larger than the minimum time it is held ( $\frac{t_a - t_1}{t_1} \leq \frac{1}{2}$ )], and (3) The marginal rates of time preference which are used to capitalize the annual savings in real resources spent on transactions are no more than twice the original rates of interest paid on the original money-substitutes. The second assumption implies that the average real resource savings is at least one-half the interest return on the money-substitutes. The third assumption implies that the real capital value of resource savings brought about by a permanent aggregate holding of an extra real dollar is at least fifty cents. Putting these assumptions together, a minimum gain in aggregate real wealth resulting from the optimal policy -- which rewards the holding of money sufficiently to squeeze the original money-substitutes out of the economy while "monetizing" these money-substitutes and thereby keeping the price level constant -- is one-quarter of the total original market value of the savings deposits and treasury bills

in the economy. For the United States, this gain in real wealth turns out to be over ninety-five billion dollars worth of real wealth! (18)

4. With minimum aggregate transaction costs assumed to exist for any complete set of property transactions, and with minimum aggregate costs of transactions in financial assets for a given set of transactions in real assets achieved by the above policy, there is a minimum aggregate cost of the given set of transactions in real assets. Hence, we have satisfied our first condition for Pareto optimality.

B. Satisfying the final condition for Pareto optimality

The original economy, altered by only those programs which enable the economy to satisfy the first condition for Pareto optimality, also satisfies the third and final condition, which states that there is no reallocation of real assets that makes at least one individual better off without harming anyone else. To see this, first note that we may now treat transaction costs as privately incurred, unavoidable real costs of transferring goods from one real use to another; this follows from (1) the assumed taxation of transactions according to the real costs the government incurs in redefining property rights, so that all real transaction costs are private transaction costs, (2) the assumed strong law of demand which implies that all transaction costs are real costs, and (3) the above-achieved minimization of transaction costs for any allocation of resources to the production and consumption of real assets.

Next, note that each individual's optimizing problem can now be described as a maximization of utility for current and prospective future consumption goods subject to (a) a cash flow constraint for each period, given initial endowments, prices, and profits, and (b) a transactions cost constraint for each period, expressing the real cost of each possible transaction. This follows from the above-achieved satisfaction of the first condition for Pareto optimality combined with our assumptions that all general taxes are lump sum taxes and that individuals assume that they are too insignificant to have any effect on government production or the system of property rights.

Once in this form, the individual optimization problem is a stochastic dynamic programming problem. Following Bellman's optimality principle, we consider the last period first. There is then no future and thus no technological uncertainty to consider.<sup>(19)</sup> Pareto optimality of our competitive equilibrium in the last period for given allocations in the previous periods is easily established by a variant of Arrow's classic proof [1] of the Pareto optimality of a competitive equilibrium without transaction costs. In particular: Suppose that the equilibrium in the last period is not a Pareto optimum. Then there is a feasible allocation of resources in the last period which differs from an equilibrium but which makes someone better off without making anyone else worse off. The existence of locally nonsatiated, rational consumers facing parametric prices, the absence of consumption externalities, and the unavailability of transaction costs implies that the allocation hypothesized to be Pareto superior would require a greater value of consumption for an individual who benefits in the allocation and no smaller value of consumption for anyone else when the new allocation is evaluated at equilibrium prices. So the aggregate value of consumption in the new allocation at equilibrium prices

exceeds the equilibrium value. But aggregate profits are already maximal over the feasible production set at equilibrium prices. So the hypothesized allocation is infeasible, which is a contradiction. Now, consider the next to last period. Prospective consumption for the last period is determined by the savings of each real asset in the prior period. So we may write the next-to-last period's utility for then-current consumption and prospective, last-period consumption as a function of consumption and savings of real assets in the next-to-last period. This is maximized subject to an income constraint stating that the value of consumption and savings is the value of output in the next-to-last period and real-asset transactions cost constraint containing initial endowments in the next-to-the-last period. This does not deny that individual's hold cash or bonds as stores of value; rather, it converts these assets into their debt-equivalents of real assets in the last period, appropriately reducing the real assets of the creditors and suppliers of cash in the last period so that the net effect obeys a real asset income constraint for each individual. It is crucial here that we have already achieved minimal transaction costs for any given set of transactions in real assets. Now, suppose that there is an alternative, feasible set of consumption and productions in the next-to-last period such that at least one individual is better off and no one else is worse off. Then, as above, the value of present and prospective future consumption evaluated at equilibrium prices must be higher for at least one individual and no lower for anyone else. This implies that profits evaluated at equilibrium prices be higher than in equilibrium, which again is a contradiction as it implies that the alternative allocation is infeasible. This same procedure can be applied to all periods back to the present.



C. SUMMARY

1. A Pareto optimum is achieved in a competitive equilibrium containing transaction costs but no externalities when the government defines property rights so as to minimize aggregate transaction costs for each set of transactions and uses real benefit-cost calculations to make production decisions. On one hand, this result permits a more pervasive influence of government than one obtains from the standard view of the economic of private goods, where only monopoly and externalities rationalize government intervention. On the other hand, it places rather severe limitations on the government policies which can be rationalized in the presence of contract costs. In particular, in a competitive equilibrium containing the market imperfection of positive contract costs, the efficient government applies no taxes or subsidies on property transactions (transactions not involving the sale of contract services or contract information) other than user taxes to collect the marginal cost of government-provided legal services. The efficient government can only alter the legal system, restrict transactions in contract services, or fairly compete with the private sector by producing substitute goods when it is profitable in terms of real benefit-cost analysis. The result does not hold once we allow positive costs of property protection or externalities, but it still serves to define the efficient government response to informational imperfections leading to unavoidable contract costs.

2. In the general money model of this paper, a Pareto optimum is achievable by increasing the reward to holding money (either by (a) directly paying interest on money, (b) announcing, and subsequently carrying out, future reductions in the stock of currency) and lifting the reserve requirement and portfolio restrictions on banks to increase the supplies of demand

deposits and currency to match the increase in demand for demand deposits and currency induced by the increased reward to money. The increase in the reward to holding money should be halted once the loosening of the restrictions on banks has no effect on the money supply.<sup>(20)</sup> The optimal reward to holding money is equal to, or less than the original rate of interest on time deposits but is sufficiently large that no time deposits are purchased in the optimum. The government can achieve this optimum by simply freeing the banking system of reserve requirements and restrictions on the payment of interest on money and annually paying tax-financed interest on currency held on a given day of the year at a rate equal to the annual free market rate paid on demand deposits.

3. Finally, in the most specific money model used in this paper, we found that:

(a) The only money substitutes are assets with the lowest transaction costs so that the demand for money is directly affected by this one interest rate.

(b) There is a rising term structure of interest rates because when bond issue that is going to be held for a relatively long time must have a relatively high transaction cost and thus a relatively high yield,

(c) The demand for money steepens as the real cost of financial intermediation falls.

(d) There is no systematic effect of secular inflation on real interest rates, the actual effect being positive if and only if the activity of purchasing money-substitutes is capital intensive.

FOOTNOTES

- (1) The same answer can be formulated in more familiar analytical terms as an external diseconomy in the spending of money: If an individual spends a one dollar gift from the government rather than adding it to his cash balance, he and his trading partner are not substantially benefited by this marginal transaction. But the bystanding moneyholders, those not directly involved in the expenditure of the dollar, all suffer a real loss, because the expenditure raises the price level and lowers their real cash holdings. (True, the price of real assets rises, but this merely redistributes wealth from net monetary creditors to net monetary debtors). Disregarding distribution effects on the pattern of demand and assuming no money illusion, since the price level rises to the extent that aggregate real cash balances remain at the preshift level, the innocent bystanders lose, in total, the monetary services that the grantee-spender gained, the services of a dollar. Therefore, since the current cost of this external diseconomy in the spending of money is the money rate of interest, the recipient of the dollar gift should be taxed for his spending--or compensated for his hoarding--at an ad valorem rate equal to the money rate of interest.
- (2) While the gathering of information regarding the nature of a contract or a particular product in a transaction creates private transaction costs, the gathering of information which would alter market prices, such as inventions and weather forecasts, does not create what we are calling "transaction costs." If the latter type of information is produced and withheld from the public for a certain length of time (in order to reap a speculative gain), there would be different expectations of market prices between individuals, and therefore the

economy would be out of equilibrium. If, on the other hand, the use of this type of information were immediately sold at positive prices by a system of patents, it would mean that the information is a collective-type good. Since we are assuming in this paper that there is both a full general equilibrium and no collective-type goods, our model formally excludes the production of the type of information which alters market prices. The reason we have assumed the absence of both differences in market price information and collective-type goods is that a freely competitive economy would not be optimal if either were to exist (see Thompson [18] and [20], respectively).

- (3) Some severe and unfamiliar misallocations resulting from the phenomenon of differing subjective probability distributions on future market prices between individuals are pointed out in Hirshleifer [10], Thompson [18], [19], and in footnote #2 above.
- (4) These categories of assets are mutually exclusive. Following Gurley-Shaw, "primary debt" includes all debt other than time deposits.
- (5) Similarly, having assumed that the government has already defined and protected private property in a way which minimizes aggregate transaction costs for any complete set of transactions for nontransactions purposes, we would also have no reason to exclude a governmental policy of deposit insurance, as the policy has the direct effect of reducing the information costs incurred in making and accepting demand deposits.
- (6) If there were never any transaction costs, any deficit in an individual's stream of net payments would be met by an instantaneous sale

of bonds to the rest of the economy, which initially received the surplus corresponding to the individual's deficit. Consequently, given that asset markets are equilibrating, there would always be an exact synchronization between total payments and total receipts on all nonmonetary assets. Thus there would be no demand for money balances if there were never any transaction costs and asset markets were equilibrating.

When transaction costs do exist, instantaneous borrowing typically is not the cheapest way to finance all deficits; and instantaneous lending typically is not the best use of all surpluses. For example, an individual's aggregate transactions costs typically are reduced by financing some deficits with previous borrowing or hoarding (either of which would also have served to finance previous deficits). And his aggregate transactions costs also are typically reduced by accumulating some surpluses from the sale of real assets before going to the market to buy bonds. In this way, positive transaction costs typically prevent exact synchronization of total payments and receipts and thereby lead to positive money balances.

- (7) This reward takes the form of either decreases in the future price levels due to decreases in the future money supplies or a direct payment of money interest on money balances. In case of the latter, the government must finance its interest payments on currency with taxes. If the government tried to finance interest payments on currency with newly printed currency rather than lump sum taxes, it would find that interest rates would always rise to the full extent of the percentage

increase in the currency supply so that it would be impossible for their interest payment to match their interest liability. The only interest payment which currency creation can finance is the increase in interest due to the currency creation itself.

- (8) We are therefore assuming that each individual has perfect foresight concerning his future transactions for the time interval from the current date to the next date at which he will transact in money substitutes. So the length of the period is the maximal length of time any individual will hold a money substitute in our initial equilibrium. Subsection D below, argues for the acceptability of this assumption.
- (9) We shall note below (in footnote 20) the peculiar implications of having all information advantages inelastically endowed and unaugmented when there are also no direct expenditures in making contracts. If some informational advantages were inelastically supplied and exploited -- but some others were not, or contract costs existed -- then our central results below would not hold. In this case, a smaller reward to the holding of money would be optimal, because part of the cost of transacting to the poorly informed individuals would represent a non-real-resource cost. (For the same argument in a different context, see Thompson [19].)
- (10) Part of a specialist's marginal cost of creating or exploiting an information advantage may be transaction costs which themselves are the costs of being relatively poorly informed. But these costs, in turn, would be information advantages to other sellers, advantages which are

also not inelastically created or responded to. Obviously, this starts a possibly infinite regression. Assuming that informational transaction costs are not arbitrarily close to the entire cost of producing informational advantages in some positive fraction of transactions in the sequence, the farther we go from the initial transaction, the smaller the fraction of the original transaction cost represented by non-real-resource costs, and the part of the cost of the transaction ultimately represented by a non-real-resource cost is zero. This, of course, applies to all transactions -- not just transactions in money substitutes.

- (11) Since reselling costs are zero, resales of asset 1 occur only when the individual's ~~cash~~ has run out, and the receipts from the sale will be immediately used to purchase assets 2 or 3. Thus, with given future purchases and sales of assets 2 and 3, resales of still-purchased units of asset 1 are unaffected by an increase in the reward to holding money.
- (12) Note that a given individual may not be purchasing any of asset 1. In this case, that individual does not change his holdings of asset 1 or money. Nor does he use any other asset for a money-substitute. In other words, the asset with the lowest transaction cost and interest rate is the only asset that can serve as a money-substitute for any individual, whether or not the individual ever buys this asset.
- (13) The aggregate of real transaction costs, when several types of resources are devoted to transaction activities, are dependent upon the weights on resources used in computing these costs. Our weights are the marginal productivities of the respective resources in saving some numeraire resource used in all transactions (say, leisure time).

- (14) We have assumed that the government's administrative cost of effecting this change, if the change is possible, is zero.
- (15) Examples of these activities which have been allowed to run rampant in the U.S. economy are property ownership exchanges which do not alter the use of the property but are privately advantageous only because the buyer is more bullish regarding the property than the seller. Such "purely speculative" transactions are most familiar in markets for raw land, that will be used by neither the buyer nor the seller, and previously issued bonds and stocks, that are either nonvoting or held for a period too short to allow any owner to affect the decisions of the company. This speculation problem does not arise in our model because differences in price expectations are not consistent with a full general equilibrium.
- (16) We are using here the assumption that the cost of asset purchasing to some banks is no greater than the cost of asset purchasing to the financial intermediaries once the rights to the asset purchasing property of the financial intermediaries are transferred to the banks and central bank portfolio restrictions with respect to the purchase of these assets are lifted.
- (17) This reserve requirement may be as low as zero, but, since reserves do serve as a source of cash in case of a sudden reduction of deposits, it may easily be positive.
- (18) This figure is based upon the January 31, 1968 statistics in the Federal Reserve Bulletin, April 1968. The breakdown is as follows:



Savings deposits in banks .....	185.634	Billions
Savings deposits in savings and loan assn.....	124.113	"
Treasury bills.....	<u>72.905</u>	"
Total.....	382.652	Billions of Dollars

- (19) The absence of future also implies that the demand for money at any positive price is zero at the end of the last period. Since a zero price of money in the last trade would also make money worthless as a store of value, (and thus as a medium of exchange) in each of the preceding trades, the supply of money at the last trade must be zero in our model. This makes money a purely intermediate good and, while there is an equilibrium with a zero price of money in our model, as there is nothing which guarantees that it is positive, there is also an equilibrium with a positive price of money. The former equilibrium has simply been assumed away.
- (20) A theoretically interesting singularity appears when we drop the assumption of the strong law of demand and assume the costs of transacting into and out of money-substitutes to be entirely wealth transfers to natural specialists in money-substitutes. Then, while a reward to the holding of money equal to that value described above is still optimal, any smaller reward - positive, zero, or negative - to the holding of money by these individuals is also optimal. The reason is simply that there are no real resources spent on transacting in money-substitutes in this extreme case. Our optimality conditions are satisfied identically for any reward to money holding. Rewarding the holding of money merely redistributes wealth away from the natural specialists in money-substitutes.

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