Difficulties with Testing for Causation*

by

Rodney L. Jacobs, Edward E. Leamer,
and Michael P. Ward

Department of Economics
University of California
Los Angeles, California 90024

Discussion Paper Number 117
May 1978

Preliminary Report on Research in Progress
Not to be quoted without permission of the author.
*Research reported herein was partly supported by NSF Grant SOC76-08863.

I. Introduction

According to traditional wisdom "correlation does not imply causation." Casual reading of a seminal paper by Sims (1972) and an earlier paper of Granger (1969) has left many economists with the contrary impression that observed correlations can be used to infer the direction of causation. This, we believe, represents a major misinterpretation of Granger and Sims and of subsequent work. We offer in this paper a pedagogical comment.

Section II of the paper presents a simple two equation model through which we interpret the causality results. We discuss three definitions of causality which involve zero restrictions on parameters of either the structural or reduced form equations. We demonstrate that only one of these definitions of causality is testable. The testable definition, which we call "informativeness," does not coincide with the intuitive notions of causality which most individuals entertain. The testable definition merely implies that one variable is useful or informative in predicting another. Section III considers the forward-backward regressions proposed by Sims as a test of causality and demonstrates that they provide the same information as do the coefficients of the reduced form equations. Section IV considers the effect of specification error and illustrates that any specification error, no matter how small, renders the causality test uninterpretable.

II. Testing Causality in an Underidentified Model

In this section we consider a closed model consisting of two time series variables x and y. The question posed is the extent to which y affects x. In this context there are two hypotheses which are of interest. First, can x be considered an exogenous variable for the purpose of modeling y. More precisely, will a least squares regression of y on x yield consistent estimates of the structural parameters. The second hypothesis is a considerably stronger conjecture about the lack of feedback from y to x. Does y exert _any_ inflence, either directly (contemporaneously) or indirectly (with a lag), on x? A negative answer to this question insures that x is statistically exogeneous. Unfortunately, without _a priori_ information neither of these hypotheses can be tested. Furthermore, what has come to be called the "test for causality" is only weakly associated with these hypothesis.[1/] To make these points explicit we consider the following simple structural model:

$$y_t = \theta x_t + \beta_{11} y_{t-1} + \beta_{12} x_{t-1} + \varepsilon_{1t} \tag{1}$$

and

$$x_t = \gamma y_t + \beta_{21} y_{t-1} + \beta_{22} x_{t-1} + \varepsilon_{2t}, \tag{2}$$

where ε_{1t} and ε_{2t} are independent, serially uncorrelated random variables with zero means and variances σ_1^2 and σ_2^2 respectively. The reduced form of this structural system is given by

$$(3) \qquad \begin{pmatrix} y \\ x \end{pmatrix}_t = [\pi]\begin{pmatrix} y \\ x \end{pmatrix}_{t-1} + \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}_t ,$$

where $[\pi]$ is the matrix

$$[\pi] = (1-\theta\gamma)^{-1}\begin{bmatrix} \beta_{11} + \theta\beta_{21} & \beta_{12} + \theta\beta_{22} \\ \gamma\beta_{11} + \beta_{21} & \gamma\beta_{12} + \beta_{22} \end{bmatrix}$$

and

$$\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}_t = (1-\theta\gamma)^{-1}\begin{bmatrix} 1 & \theta \\ \gamma & 1 \end{bmatrix}\begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix}_t$$

In this system there are three hypotheses which describe the extent to which y influences x. These hypotheses are:

H_1: $\gamma = \beta_{21} = 0$. This is the hypothesis that the disturbance in the y equation is never transmitted to x. We will refer to H_1 as the hypothesis that "y does not cause x." By this we mean that a policy which controlled y by selecting the error ε_{1t} could not have any impact on the x variable.

H_2: $\gamma = 0$. This is the hypothesis that the current disturbance in the y equation does not effect current x. The hypothesis H_2 is more difficult to name and we shall settle with "x is contemporaneously exogenous." This means, among other things, that a least squares estimation of equation (1) will yield consistent estimates of θ, β_{11} and β_{12}.

H_3: $\gamma\beta_{11} + \beta_{21} = 0$. This is the hypothesis that an optimal prediction of x does not depend on y. Referring to equation (3) we see that this hypothesis implies that a particular coefficient (π_{21}) of the reduced form is zero. We will refer to H_3 as the hypothesis that "y is not informative about future x." It should be noted that H_3 is often referred to as the hypothesis that y does not cause x in "Granger's" sense.

The usefulness of the data in shedding light on the validity of these three hypotheses can be ascertained by inspection of the reduced form parameters. These reduced form parameters are the only parameters which can be estimated from the data. The structural parameters can be estimated only to the extent that they can be uniquely determined from the reduced form coefficients. As it turns out, the structural model is not identified and none of its parameters can be estimated. It is, therefore, not possible to estimate either γ or β_{21}, and it is not possible for the data to reveal the extent to which y causes x in the sense of hypothesis of H_1.

The data can be used to estimate the reduced form parameter $\pi_{21} = (1-\theta\gamma)^{-1}(\gamma\beta_{11} + \beta_{21})$ and, therefore, to test the informativeness hypothesis that $\pi_{21} = 0$. If it is discovered that π_{21} is not zero, so that y is informative about x, we would also conclude that y causes x. This occurs because γ and β_{21} cannot both be equal to zero if π_{21} is not zero. If it is discovered that $\pi_{21} = 0$ so that y is not informative about x we cannot also conclude that y does not cause x. This is because $\gamma\beta_{11} + \beta_{21} = 0$ does not imply that both γ and β_{21} are equal to zero.

Although the informativeness hypothesis $\gamma\beta_{11} + \beta_{21} = 0$ may seem an unlikely outcome if γ and β_{21} are both non-zero, Sargent (1976) discusses an interesting example where this outcome occurs. Assume that the series y is a policy instrument used to control the variability of x. If the economy generates reactions to shocks in y according to equation (1), then the policy maker can minimize the variance of x by selecting y such that the systematic part of equation (2) is zero, namely $0 = \gamma y_t + \beta_{21}y_{t-1} + \beta_{22}x_{t-1}$. Thus the policy maker will select equation (1) to be $y_t = (-\beta_{21}y_{t-1} - \beta_{22}x_{t-1})/\gamma$. Note in particular that the policy maker will set β_{11} equal to $-\beta_{21}/\gamma$, which is exactly the constraint necessary for y not to be informative about x. In this context, a test of the informativeness of y is properly thought to be a test of the intentions and computing ability of the policy maker. Clearly, the test has nothing to do with the hypothesis that y does not cause x. In fact, evidence that y is not informative about x might be regarded as evidence in favor of the hypothesis that y causes x, since the authorities would have little incentive to select the reactive policy $\beta_{11} = -\beta_{21}/\gamma$ unless y did cause x.

Given the above relationship between causality and informativeness, how should we interpret a t-statistic on π_{21}? If the t is low, the data favor the hypothesis that "y is not informative about x" but do not contain useful information about the hypothesis "y does not cause x." If the t is high, the data cast doubt on the sharp hypothesis that "y is not informative about x" and also on the hypothesis that "y does not cause x." However, the data have nothing to say about the neighboring hypothesis that "y has only an infinitesimal effect on x." Even if γ and β_{21} are very small, π_{21} can take on any value. Because of this discontinuity in the evidential content of a large t, the usefulness of the causality test is restricted to the unlikely circumstances when only the sharp hypothesis is of interest. In particular, the slightest misspecification will greatly reduce the value of the test (see Section IV).

Since the hypothesis of exogeneity involves only the structural restriction that $\gamma = 0$ and no restrictions on the reduced form, this hypothesis is not testable. Although the informativeness hypothesis is testable, it is not sufficient to eliminate the simultaneous equation bias in an estimate of equation (1). To prove this point, define

$$r = \frac{\mathrm{cov}(u_1,u_2)}{\mathrm{Var}(u_2)} = \frac{\gamma\sigma_1^2 + \theta\sigma_2^2}{\gamma^2\sigma_1^2 + \sigma_2^2}$$

and rewrite the reduced form equation for y_t as

(4) $$y_t = \pi_{11}y_{t-1} + \pi_{12}x_{t-1} + ru_{2t} + u_{1t} - ru_{2t}.$$

By using the reduced form equation for x_t to eliminate ru_{2t} we obtain

(5) $$y_t = rx_t + (\pi_{11} - r\pi_{21})y_{t-1} + (\pi_{12} - r\pi_{22})x_{t-1} + u_{1t} - ru_{2t}.$$

The error term of this equation is independent of u_{2t} by construction and is, therefore, uncorrelated with x_t. The coefficients of x_t, y_{t-1} and x_{t-1} in equation (5) would, therefore, be obtained from a least squares regression of y_t on these variables. Equation (5) is identical in form to equation (1) so that least squares applied to the first structural equation would yield

$$\text{(6)} \qquad \text{Plim}(\hat{\theta}) = r = \theta + \frac{\gamma\sigma_1^2(1-\theta\gamma)}{\gamma^2\sigma_1^2 + \sigma_2^2},$$

$$\text{(7)} \qquad \text{Plim}(\hat{\beta}_{11}) = \pi_{11} - r\pi_{21} = \beta_{11} - \frac{\gamma\sigma_1^2(\beta_{11}\gamma + \beta_{21})}{\gamma^2\sigma_1^2 + \sigma_2^2}$$

and

$$\text{(8)} \qquad \text{Plim}(\hat{\beta}_{12}) = \pi_{12} - r\pi_{22} = \beta_{12} - \frac{\gamma\sigma_1^2(\gamma\beta_{12} + \beta_{22})}{\gamma^2\sigma_1^2 + \sigma_2^2} .$$

The asymptotic biases in equations (6) through (8) are, of course, zero if $\gamma = 0$ so that x_t is exogenous. If $\gamma \neq 0$ but y is not informative about x, ($\pi_{21} = \beta_{11}\gamma + \beta_{21} = 0$) the bias on β_{11} is zero but the remaining parameters are biased. As a result, the informativeness hypothesis is not a useful indicator of simultaneous equation bias.

These results demonstrate that if we accept the hypothesis that $\pi_{21} = 0$, least squares estimation of equation (1) may still be subject to simultaneous equation bias. If we reject the hypothesis that $\pi_{21} = 0$, least squares estimation of equation (1) may still yield consistent estimates of its parameters. All that is required for consistent estimates is that $\gamma = 0$. This restriction cannot be tested but must be imposed by <u>apriori</u> considerations.

III. Two Sided Regressions as Tests for Causality

While the simple structural model of the previous section clarifies the inferential limitations of causality tests, such models are rarely used in actual tests for causality. Instead, a non-restrictive time series relationship between x and y is posited and the data are then used to causally order the two series. The test format is to estimate a regression in which current y is the dependent variable and in which past, current and future x are the independent variables. This regression forms the basis of the causality test suggested by Sims:

> "...y can be expressed as a distributed lag function of current and past x with a residual which is not correlated with any values of x, past or future, if, and only if, y does not cause x in Granger's sense.
>
> We can always estimate a regression of y on current and past x. But only in the special case where causality runs from x to y can we expect that no future values of x would enter the regression if we allowed them. Hence, we have a practical statistical test for unidirectional causality: regress y on past and future values of x, taking account by generalized least squares or prefiltering of the serial correlation.... Then if causality runs from x to y only, future values of x in the regression should have coefficients insignificantly different from zero as a group. (p. 545, 1972).

In this section we show that the Sims test is a test of informativeness ($\pi_{21} = 0$) and is not a test for exogeneity or causality.

Suppose that π_{21} equals zero so that y is not informative about x. Given this restriction, the structural model of equations (1) and (2) becomes

$$z_t = \theta x_t + \beta_{12} x_{t-1} + \varepsilon_{1t} \tag{9}$$

and

(10) $$x_t = \gamma z_t + \beta_{22}x_{t-1} + \varepsilon_{2t},$$

where $z_t = y_t - \beta_{11}y_{t-1}$. Equations (9) and (10) provide two independent predictors of z_t given by

(11) $$\hat{z}_t = \theta x_t + \beta_{12}x_{t-1}$$

and

(12) $$\hat{z}_t = \frac{1}{\gamma}x_t - \frac{\beta_{22}}{\gamma}x_{t-1}.$$

These predictors are unbiased with variances σ_1^2 and σ_2^2/γ^2 respectively. The minimum variance linear combination of these two predictors becomes

(13) $$\hat{z}_t = \frac{\sigma_2^2(\theta x_t + \beta_{12}x_{t-1}) + \sigma_1^2\gamma(x_t - \beta_{22}x_{t-1})}{\sigma_2^2 + \gamma^2\sigma_1^2},$$

with a prediction error of $v_t = (\sigma_2^2\varepsilon_{1t} - \gamma\sigma_1^2\varepsilon_{2t})/(\sigma_2^2 + \gamma^2\sigma_1^2)$. By construction, the prediction error v_t is independent of $u_{2t} = (\gamma\varepsilon_{1t} + \varepsilon_{2t})/(1 - \theta\gamma)$ which is the reduced form disturbance for x_t given by equation (2). Since u_{2t} and v_t are uncorrelated it follows that the best predictor of z_t (for $\pi_{21} = 0$) can be expressed as a function of current and past x with an error that is uncorrelated with any x. It remains to demonstrate that this is also true for the best predictor of y_t.

We first substitute $y_t - \beta_{11}y_{t-1}$ for z_t in the expression $z_t = \hat{z}_t + v_t$ to obtain

(14) $$y_t - \beta_{11}y_{t-1} = \alpha_1 x_t + \alpha_2 x_{t-1} + v_t,$$

where $\alpha_1 = (\sigma_2^2\theta + \sigma_1^2\gamma)/(\sigma_2^2 + \gamma^2\sigma_1^2)$ and $\alpha_2 = (\sigma_2^2\beta_{12} - \sigma_1^2\gamma\beta_{22})/(\sigma_2^2 + \gamma^2\sigma_1^2)$. Using repeated substitution to eliminate lagged values of y_t yields

$$(15) \qquad y_t = \alpha_1 \sum_{i=0}^{\infty} \beta_{11}^i x_{t-i} + \alpha_2 \sum_{i=0}^{\infty} \beta_{11}^i x_{t-i-1} + \sum_{i=0}^{\infty} \beta_{11}^i v_{t-i}.$$

For any pair of indices, t and t', we have already shown that x_t and $v_{t'}$ are uncorrelated for $t' \geq t$. We now note that the reduced form expression for x_t (with $\pi_{21} = 0$) is

$$(16) \qquad x_t = \pi_{22} x_{t-1} + u_{2t}$$

so that a stationary x_t can be written as an infinite distributed lag on past u_{2t}. Since $v_{t'}$ and u_{2t} are independent for all index pairs (t,t'), the same is true for x_t and $v_{t'}$.

The preceeding paragraph established that the error term in equation (15) is uncorrelated with either past, current or future x. A regression of y_t on past, current and future x would, therefore, lead to zero coefficients on future values of x when π_{21} equals zero. The test for causality proposed by Sims is, thus, a test of the informativeness hypothesis.

It might be presumed that the test proposed by Sims is exactly equivalent to a direct test of the informativeness hypothesis using the estimated reduced form. However, this equivalency only holds asymptotically. Note that there is only a single coefficient, π_{21}, that is the subject of the testing procedure. The appropriate statistic, assuming normality, is a t-test of the reduced form coefficient. The Sims procedure, though, would require an F-test with as many degrees of freedom in the numerator as there are future unconstrained x-coefficients in the regression of y on all x's. Without imposing the implied restrictions on these future coefficients, the F-test will have lower power than the simple t-test of the reduced form.

The Sims test for causality is often incorrectly viewed as a test for exogeneity. That is, it is viewed as a test which indicates whether equation (1) could be consistently estimated by ordinary least squares. This use of the test can be attributed to Sims:[2/]

> "In time-series regressions it is possible to test the assumption that the right-hand side variable is exogenous; thus the choice of "direction of regression" need not be made entirely on a priori ground." (p. 550, 1972a).

As we have shown, this interpretation of the test is incorrect. Moreover, as a test for exogeneity it can lead to two types of error. It can reject exogeneity when the variable is exogeneous and it can accept exogeneity when the variable is truely endogeneous.

IV. Causality and Specification Error

In Section II we demonstrated that the hypothesis "y is not informative about x" is testable since it implies a zero restriction on a particular reduced form coefficient. The hypothesis "y does not cause x" is only indirectly testable. It is not possible to accumulate evidence in favor of this hypothesis without recourse to special prior information. It is possible only to cast doubt on the one-way causality hypothesis. However, in this section we demonstrate that statistical rejection of the one-way causality hypothesis need <u>not</u> be interpreted as casting doubt on the hypothesis that y does not cause x. The hypothesis rejection may be attributed instead to slight specification errors which bias zero coefficients away from zero.

It is fair to say that any model used by an economist to analyze nonexperimental data is misspecified. At best it can be argued that if the specification error is small, then inferences drawn from the data set are only negligibly affected. For example, a well known result for the estimation of regression coefficients is that the bias is a continuous linear function of the specification error. Thus if the specification error is small the parameter bias will be correspondingly small. As we have discussed above, the test of causality does not have this continuity property. The one-way causality hypothesis is the sharp hypothesis that a certain reduced form coefficient is identically zero. The slightest specification error will imply that this hypothesis will almost certainly be rejected if the sample size is large. Because of the dependence of the test of a sharp hypothesis on a correct model specification, it is our contention that sharp hypotheses cannot be sensibly tested with economic data. We should instead concentrate on the composite hypothesis that parameters are close to zero. However, in this case the neighborhood hypothesis that γ and β_{21} are small is not testable because the model is underidentified. In the remainder of this section we consider two specific examples of specification error for which the "causality" test results are misleading.

For the first case we focus on the effect of observation errors on the test for causality. We assume that the observed variables are

$$\tilde{y}_t = y_t + v_t$$

and

$$\tilde{x}_t = x_t + u_t,$$

where u_t and v_t are independent white noise processes with zero means and variances σ_u^2 and σ_v^2 respectively. Least squares estimates of the reduced form model of Section II would yield

$$(17) \quad \text{Plim}(\hat{\pi}) = \begin{bmatrix} \pi_{11} + \dfrac{\pi_{12}\sigma_{12}\sigma_u^2 - \pi_{11}\sigma_{22}\sigma_v^2}{\bar{\sigma}} & \pi_{12} + \dfrac{\pi_{11}\sigma_{12}\sigma_v^2 - \pi_{12}\sigma_{11}\sigma_u^2}{\bar{\sigma}} \\ \pi_{21} + \dfrac{\pi_{22}\sigma_{12}\sigma_u^2 - \pi_{21}\sigma_{22}\sigma_v^2}{\bar{\sigma}} & \pi_{22} + \dfrac{\sigma_{21}\sigma_{12}\sigma_v^2 - \pi_{22}\sigma_{11}\sigma_u^2}{\bar{\sigma}} \end{bmatrix}$$

where σ_{ij} are elements of the covariance matrix of $\tilde{y}$ and $\tilde{x}$ and $\bar{\sigma} = \sigma_{11}\sigma_{22} - \sigma_{12}^2$. If π_{21} equals zero, then observation errors will bias $\hat{\pi}_{21}$ away from zero. For large samples, we would always reject the true hypothesis that y is not informative about x. As π_{21}' is biased away from zero the observation errors could also bias a non-zero π_{12} towards zero.[3/] In this case, the causality test could detect "one way causality" but in the wrong direction.

The remaining example suggests that the causality results can be dominated by a left out variable that influences x and y with a different lag structure. We illustrate this point with the model

(18) $$y_t = \beta_{11}y_{t-1} + \gamma_1 z_t + \varepsilon_{1t}$$

and

(19) $$x_t = \beta_{22}x_{t-1} + \gamma_2 z_{t-1} + \varepsilon_{2t}$$

where y and x are independently caused by z_t and z_{t-1} respectively. We assume that z_t is a known white noise process with variance σ_z^2 which is excluded from the regressions. If we test for causality between y and x by regressing these variables on past y and x we obtain

(20) $$\text{Plim}(\hat{\Pi}) = \begin{bmatrix} \beta_{11} & 0 \\ \dfrac{\gamma_1\gamma_2\sigma_z^2\sigma_{xx}}{\sigma_{xx}\sigma_{yy}-\sigma_{xy}^2} & \beta_{22} - \dfrac{\gamma_1\gamma_2\sigma_z^2\sigma_{xy}}{\sigma_{xx}\sigma_{yy}-\sigma_{xy}^2} \end{bmatrix}$$

where the σ_{ij} are elements of the covariance matrix of x and y. The causality test would indicate a one way causal system in which y causes x. This result occurs because the left out variable influences y before it has an impact on x, so that lagged values of y are important in explaining x.

V. Concluding Remarks

We have discussed three hypotheses which describe the extent to which y influences x. These hypotheses were the hypothesis of causality in which disturbances in y are not transmitted to x, the hypothesis of exogeneity in which the current disturbance in y is not transmitted to current x, and the hypothesis of informativeness in which an optimal prediction of x does not depend on y. Only the latter hypothesis of informativeness, which entails a zero restriction on a parameter of the reduced form, turns out to be completely testable. If there is no specification error, it is possible to gather evidence against the causality hypothesis but not in favor of it. However, evidence which apparently casts doubt on the causality hypothesis can be attributed to the slightest misspecification. We also demonstrated that the Sims test for causality is actually a test of the informativeness hypothesis and is not a test for exogeneity or causality as is generally believed.

REFERENCES

Granger, C.W.J., "Investigating Causal Relations by Econometric Models and Cross-Spectral Methods," Econometrica, July 1969, 37, pp. 424-438.

Sargent, Thomas J., "A Classical Macroeconometric Model for the United States," Journal of Political Economy, April 1976, 84, pp. 207-37.

Sims, Christopher A., "Money, Income, and Causality," AER, September 1972, 62, pp. 540-552.

__________, "Exogeneity and Causal Ordering in Macroeconomic Models," in New Methods in Business Cycle Research: Proceedings From a Conference, October 1977, Federal Reserve Bank of Minneapolis.

FOOTNOTES

[1]This point is clearly appreciated by both Granger and Sims. This is also true of other points made later in the paper.

[2]We do not mean to imply that Sims views the test as a test for exogeneity. As Sims (1977) has subsequently pointed out, this is a problem of semantics.

[3]If $\pi_{12} < 0$, for instance, then the bias term is unambiguously positive and $\hat{\pi}_{12}$ is biased in the direction of zero.