A GENERALIZED MODEL OF INTERNATIONAL PORTFOLIO DIVERSIFICATION: EVALUATING ALTERNATIVE SPECIFICATIONS

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ABSTRACT

In this paper, we synthesize previous theoretical and empirical work on international portfolio diversification by presenting a generalized model and deriving the models in the empirical literature as special cases of it. We then calculate a series of sample portfolios with and without special case assumptions to show how these restrictions systematically bias the estimated portfolio shares. We conclude that a number of puzzling results, such as rejecting mean-variance pricing, may be attributable to model misspecification rather than to the effects of market imperfection.

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I. Introduction

International portfolio diversification has been studied extensively in economics and finance.[1] The aim of most researchers in finance is to determine the relative prices of risky assets, while many economists are more interested in the implications for exchange rate determination. However, the two fields of research share a common framework: the standard model of portfolio choice that has been developed in the domestic finance literature.

Researchers naturally extended the intuitions about domestic portfolio choice to the international setting. This extension complicates the analysis in a number of non-trivial ways. The most obvious of the complications is that assets which are riskless from the point of view of investors in one country are risky from the point of view of investors in another country, who must translate returns into their domestic currency. This complication adds exchange rate risk to the investor's calculus. An even more troublesome complication occurs because investors in different countries calculate expected real returns based on their own national price indices. Hence expectations of real returns are not homogeneous across countries, a result which makes it extremely difficult to draw asset pricing conclusions.

Such complications make the international portfolio problem empirically less tractable than the domestic one. Researchers have found that the imposition of one or more simplifying assumptions restores much of the tractability. For example, if purchasing power parity is assumed, then inflation rates are the same in all countries and expectations are guaranteed to be homogeneous.

Despite the important role played by simplifying assumptions, researchers have paid surprisingly little attention to the question of whether they are supported by the data. Most have been imposed without any prior testing, suggesting an implicit consensus that the assumptions have no significant impact on empirical results. We see this paper as a first step in a systematic examination of the empirical validity of the assumptions most frequently used in the literature. Our conclusions are quite disturbing. We find that some of the most common assumptions create a substantial bias in portfolio shares, where the term 'bias' is not used in the statistical sense but rather to indicate systematic patterns of change.

Our analysis proceeds in three steps. We first present a generalized theoretical model that encompasses the models in the literature. Next, we show how imposing various sets of simplifying assumptions on this model generates the previous models as special cases. Finally, we calculate a number of sample portfolios to illustrate how specific assumptions affect estimated portfolio shares.

Our examination of the generalized model and special cases yields two additional benefits. The first is the synthesis of a wide variety of models from the economics and finance literatures. Because different researchers impose different subsets of simplifying assumptions, the relationships among existing models are hard to discern without the unifying framework of the generalized model.

The second benefit of our analysis is that the comparison of the models sheds light on some puzzling results that have appeared in the empirical literature. These results have led some researchers to reject the mean-variance framework as a good description of international asset choice; others have appealed to market segmentation to explain this rejection. Our results show that these conclusions about market imperfections may not be justified because they are based on models incorporating invalid restrictions.[2]

We wish to note two limitations to our analysis at the outset. First, we do not include any theoretical innovations in the specification of the international portfolio problem, since our intent is to make our model encompassing. Several assumptions that we adopt from the literature may well merit their own investigation--for example, the form of the utility function or the specification of exogenous asset supplies. Second, the aim of our simple empirical analysis is not to provide precise estimates of portfolio shares but rather to evaluate the biases introduced by specific restrictions.

The remainder of the paper is organized as follows. The next section briefly describes the similarities and differences between domestic and international diversification. Intuitions behind the special cases are introduced and the literature is summarized. In Section III we specify the generalized model and derive the expression for the optimal international portfolio from it. Then the key models in the literature are shown to be special cases of the generalized model. Our sample portfolio results are presented in Section IV, where we analyze the nature of the special case biases. Finally, in Section V we discuss the implications of our analysis for future empirical work.

II. Intuitions about International Portfolio Diversification

We begin with intuitions about portfolio diversification from the domestic finance literature. Assuming that investors are risk averse, the motive for diversification is to reduce overall portfolio risk. An optimal portfolio exploits the correlation among returns to hedge against return uncertainty, thus allowing investors to choose portfolios compatible with their risk preferences. Note that an investor who is risk neutral would not diversify risk. S/he would simply hold the asset or the portfolio with the highest expected return, regardless of risk.[3]

In an international setting, we must take account of the fact that assets are denominated in different currencies. In order to compare expected returns on international assets and calculate their contributions to portfolio risk, the investor must first translate the returns into a common currency (which we will call the numeraire) using the appropriate exchange rate. This translation introduces an additional source of uncertainty which the investor seeks to hedge against, namely the risk of changes in the numeraire currency purchasing power of the foreign assets' returns .

In order to understand the implications of purchasing power risk, we again consider the domestic portfolio problem, but now with the addition of uncertain inflation (see Friend, Landskroner and Losq [1976], Solnik [1978], Manaster [1979] and Sercu [1981]). Inflation alters portfolio choice because the investor who maximizes the expected utility of real wealth or consumption must deflate all expected nominal returns by a price index. When returns on different assets are differentially correlated with inflation, the investor creates an additional fund to hedge against the inflation uncertainty.

In an open economy, the consumption bundle typically includes domestic as well as foreign goods, so that the price index is comprised of domestic and foreign prices expressed in the numeraire. The investor is thus exposed to uncertainty about domestic inflation, foreign inflation, and the exchange rate. Therefore, hedging against uncertainty in real returns requires considering the interactions (i.e., correlations or covariances) among four sets of variables: (1) asset returns in the currency of denomination, (2) exchange rates, (3) domestic goods price inflation, and (4) foreign goods price inflation.

Given an individual investor's portfolio solutions, equilibrium asset pricing relations are derived by aggregating individuals' asset demands and equating them to asset supply. The aggregation requires assuming that the individual investor is representative of all investors. Such an assumption is clearly invalid in the international context, since each country's investors deflate by a different price index and hence perceive different real returns. When expectations of real returns differ across countries, asset demands cannot easily be aggregated and it is difficult to obtain tractable expressions for asset pricing (see Williams [1977]).

Many researchers have focused on simplified versions of the generalized model in order to highlight the roles of specific covariances or to permit asset pricing conclusions to be drawn. Table I summarizes the simplifying assumptions made in the empirical literature. We illustrate the ramifications of the various simplifications by considering two examples.

First, it is common among researchers in economics to consider only assets that are 'riskfree,' i.e., assets whose returns in the currency of denomination are non-stochastic. Short-term government bonds are an obvious example. A foreign riskfree asset's return is perceived to be stochastic by the domestic investor because it varies with the exchange rate. Thus, the expression for portfolio shares includes exchange rate and inflation rate variances and their covariances. However, any covariances between asset returns and inflation rates or between asset returns and exchange rates are zero, since returns are not stochastic.[4]

A second simplifying assumption that is often made in the literature is that Purchasing Power Parity (PPP) holds. When PPP holds, currencies have equal inflation rates (measured in the numeraire) and equal purchasing power uncertainty. Therefore, only covariances with the one common rate of inflation need be considered by the domestic investor. Since all investors deflate returns by the same price index, there is no problem with aggregation.

Further simplification can be achieved by combining assumptions. If one assumes both that there are only riskfree assets and that PPP holds, then the only sources of uncertainty are the exchange rate and the common inflation rate. Frankel [1982] carries the simplification one step further by also assuming non-stochastic inflation rates. In his model, therefore, exchange rate variance is the only source of risk.

As shown in Table I, all empirical studies of international diversification to date are based on models that incorporate one or more simplifying assumptions. Thus, they are joint tests of mean-variance optimizing principles and the maintained restrictions. A variety of empirical tests based on these models have failed to support basic intuitions about portfolio behavior or have produced contradictory results.

For example, two puzzles appear in a series of empirical studies based on models that highlight the role of exchange rate uncertainty (Frankel [1982], Frankel and Engel [1984], Engel and Rodrigues [1987], Lewis [1988]). They reject the hypothesis of mean-variance optimization and are unable to reject the unappealing hypothesis that investors are risk neutral, rather than risk averse. As another example, 'portfolio balance' models of the exchange rate implicitly assume international aggregation of asset demands is possible and then express the exchange rate as a function of asset supplies. The statistical support for such models is quite disappointing: coefficients are marginally significant at best (Branson, Halttunen and Masson [1977, 1979]) and of the wrong sign and significant at worst (Frankel [1983]).

Results such as these have left researchers with the unappealing choice of either concluding that international investors do not mean-variance optimize or that there is some other form of market imperfection. In the later category, much interest has focussed on the hypothesis of market segmentation (Rodriguez and Carter [1979], Jorion and Schwartz [1986], Cho, Eun and Senbet [1986]). Our results support an alternative interpretation of the disappointing results in this area, namely that they may be due to model misspecificaton in the form of invalid simplifying restrictions. We now proceed to the derivation of a generalized model that we will use to address the validity of popular empirical specifications.

III. Models of International Portfolio Optimization

A. A General Continuous-Time Model

Because we want our model to encompass others in the literature, we have tried to make it as general as possible. For the present, however, we confine our attention to two countries. The extension to the n-country case is straightforward, but yields no additional intuitions. We assume that international capital markets are perfectly integrated, so that covered interest parity holds: there are no taxes, transactions costs, capital controls, restrictions on short sales or default risks on government bonds.

As has been done in earlier work, we specify a risk-averse investor who maximizes the expected utility of consumption. The investor is assumed to deflate nominal prices by a Cobb-Douglas price index, which is consistent with a utility function in the HARA class. Specifically, since we want to allow for many possible values of the coefficient of relative risk aversion, we have in mind a power utility function, which places minimal restrictions on the range of coefficient values. Since utility is homothetic, expenditure shares are constant within a country as income varies. However, the expenditure shares can differ across countries. There is one representative good or a single composite national output in each country.

The price index for a representative domestic investor is

$$Z = P^{\alpha}P*^{1-\alpha}E^{1-\alpha} \tag{3.1}$$

where, for the domestic consumer, α is the proportion spent on the domestic good whose price is P, and $(1-\alpha)$ is the proportion spent on the foreign good with price P*. The exchange rate is denoted by E. Both E and the prices of the representative goods in the two countries follow arithmetic Brownian motion:

$$dE/E = edt + \sigma_E dz_E \tag{3.2}$$

$$dP/P = pdt + \sigma_P dz_P$$

$$dP^*/P^* = p^*dt + \sigma_P{}^* dz_P{}^*$$

where e is the instantaneous mean change in the exchange rate and σ_E is its standard deviation. Similarly, p (p*) is the instantaneous mean change in the domestic (foreign) output price and σ_P (σ_{P*}) is the standard deviation of that change for the domestic (foreign) country. The terms dz_E, dz_P and dz_{P*} are the associated Weiner processes with expected values of zero and variances of dt. By defining exchange rate changes independently of price changes, we allow for deviations from PPP.[5] The purchasing power of the domestic currency, denoted Q, is the reciprocal of the price index.

Each country has a single domestically riskfree asset (call it a short term government bond) and a single risky asset (call it a stock), both of whose returns are denominated in that country's currency.[6] The change in the value of each asset is also assumed to follow arithmetic Brownian motion:[7]

$$dB/B = idt \tag{3.3}$$

$$dB^*/B^* = i^*dt$$

$$dS/S = sdt + \sigma_S dz_S$$

$$dS^*/S^* = s^*dt + \sigma_S{}^* dz_S{}^*$$

where B (B*) is the value of the domestic (foreign) bond and S (S*) is the value of the domestic (foreign) stock, expressed in the currency of the issuing country ("currency of denomination terms."). The parameters i, i*, s and s* are the constant, instantaneous means of expected returns, σ_S and σ_S* are the standard deviations over time, and dz_S and dz_S* are Weiner processes.[8]

Following Merton [1971], and others, we formally model the investor's problem as choosing portfolio weights (x_i) for i=1,N assets (in our case, N=4) to maximize the utility of consumption over a specified time period and final wealth, which is in the form of a 'bequest.' These weights, which can be zero, positive, or negative, must sum to one. The formal maximization problem is

$$J(W(t),t) = \max_{\underline{x}} E_t \left[\int_t^T U(C,s)ds + I(W(T),T) \right] \quad (3.4)$$

$$\text{s.t. } \Sigma x_i = 1$$

$$J(W(T),T) = I(W(T),T)$$

where U is the utility function, C is consumption, W is wealth, I is the bequest, t is time, and $\underline{x}$ is the vector of portfolio weights for the assets available to the individual. The J function is the solution to the problem, and the second constraint is the necessary boundary condition.

This model is not completely general because it is partial equilibrium; production decisions play no role and asset supplies are exogenous. In addition, the combination of specifying arithmetic Brownian motion for asset returns and a Cobb-Douglas price index implies the 'separation' of the consumption and investment decisions of the representative individual. For all practical purposes, any consumption decision is invariant to the time of the decision because it is a residual choice. Maximizing wealth over time will result in maximizing consumption, since the latter is determined as a fixed percentage of available funds. This structure reduces the multi-period problem to a single-period one. The price paid for this tractability is that, while we can consider the impacts of differences in consumption between countries, we cannot capture the impacts of intertemporal changes in consumption.

To solve the maximization problem, we require an expression for the evolution of W over time. Real wealth in domestic currency terms is

$$W = (B + B^*E + S + S^*E)Q \tag{3.5}$$

To derive its stochastic form requires an expression for the change in purchasing power Q over time, which can be derived by applying Ito's Lemma:

$$dQ = \Big[-\alpha p + (\alpha-1)p^* + (\alpha-1)e + (-\alpha)(\alpha-1)\sigma_{PP*} + (\alpha-1)(-\alpha)\sigma_{EP} \tag{3.6}$$

$$+ (\alpha-1)^2\sigma_{EP*} + \tfrac{1}{2}\Big[(-\alpha)(-\alpha-1)\sigma^2{}_P + (\alpha-1)(\alpha-2)[\sigma^2{}_{P*} + \sigma^2{}_E]\Big]\Big]Qdt$$

$$+ Q\ [(-\alpha)\sigma_P dz_P + (\alpha-1)\sigma_{P*}dz_{P*} + (\alpha-1)\sigma_E dz_E]$$

Thus, the evolution of wealth is given by the stochastic differential of W, which, by Ito's Lemma, is

$$dW = \Big[x_B i + x_B{}^*i^* + x_S s + x_S{}^*s^* + (x_B{}^* + x_S{}^*)e + x_S{}^*\sigma_{S*E} \tag{3.7}$$

$$+ -\alpha p + (\alpha-1)(p^*+e) + (-\alpha)(\alpha-1)\sigma_{PP*} + (\alpha-1)(-\alpha)\sigma_{EP} + (\alpha-1)^2\sigma_{EP*}$$

$$+ \tfrac{1}{2}\ [(\alpha)(\alpha+1)\sigma_P{}^2 + (\alpha-1)(\alpha-2)(\sigma^2{}_{P*} + \sigma_E{}^2)] + (-\alpha)x_s\sigma_{SP} + (-\alpha)x_s{}^*\sigma_{S*P}$$

$$+ (-\alpha)(x_B{}^* + x_S{}^*)\sigma_{EP} + (\alpha-1)x_S\sigma_{SP*} + (\alpha-1)x_S{}^*\sigma_{S*P*} + (\alpha-1)x_S\sigma_{SE}$$

$$+ (\alpha-1)(x_B{}^* + x_S{}^*)\sigma_{EP*} + (\alpha-1)(x_B{}^* + x_S{}^*)\sigma_E{}^2 + (\alpha-1)x_{S*}\sigma_{S*E}\Big]\ WQdt$$

$$+ WQ\Big[x_S\sigma_S dz_S + x_S{}^*[\sigma_{S*}dz_{S*} + \sigma_E dz_E] + x_B{}^*\sigma_E dz_E$$

$$+ (-\alpha)\sigma_P dz_P + (\alpha-1)\sigma_{P*}dz_{P*} + (\alpha-1)\sigma_E dz_E\Big]$$

We incorporate the adding-up restriction on the portfolio weights by setting the weight for domestic bonds x_B equal to one less the other portfolio weights, and presenting only solutions for the remaining weights. In matrix terms, those solutions are given by

$$\underline{x} = [\ x_{B*}\ x_S\ \ x_{S*}\ ]'$$
$$= (1/R)B^{-1}\underline{r} - (1/R)B^{-1}A\underline{\alpha} + B^{-1}A\underline{\alpha} \qquad (3.8)$$

where $R = -(J_{WW}WQ/J_W)$ is the coefficient of relative risk aversion, and the matrices B and A and the vectors $\underline{r}$ and $\underline{\alpha}$ are defined as follows:

$$\underline{r} = \begin{bmatrix} i* + e - i \\ s - i \\ s* + e + \sigma_{S*E} - i \end{bmatrix}$$ is the vector of nominal, numeraire currency asset returns relative to the return on the domestic bond.

$$\underline{\alpha} = \begin{bmatrix} \alpha \\ 1-\alpha \end{bmatrix}$$ is the domestic investor's vector of expenditure shares.

$$A = \begin{bmatrix} \sigma_{EP} & \sigma_{EP*}+\sigma^2_E \\ \sigma_{SP} & \sigma_{SP*}+\sigma_{SE} \\ \sigma_{S*P}+\sigma_{EP} & \sigma_{S*P*}+\sigma_{S*E}+\sigma_{EP*}+\sigma^2_E \end{bmatrix}$$ is the matrix of covariances between relative returns and inflation rates, both measured in the numeraire currency.

$$B = \begin{bmatrix} \sigma^2_E & \sigma_{ES} & \sigma_{ES*}+\sigma^2_E \\ \sigma_{ES} & \sigma^2_S & \sigma_{SS*}+\sigma_{SE} \\ \sigma_{ES*}+\sigma^2_E & \sigma_{SS*}+\sigma_{SE} & \sigma^2_{S*}+2\sigma_{S*E}+\sigma^2_E \end{bmatrix}$$ is the variance-covariance matrix of nominal, numeraire currency returns.

The portfolio defined by $\underline{x}$ can be interpreted as a linear combination of two portfolios:

$$\underline{x} = (1/R)B^{-1}\underline{r} + [1 - (1/R)]B^{-1}A\underline{\alpha} \qquad (3.8')$$

where the first, commonly known as the logarithmic portfolio, is $B^{-1}\underline{r}$ and the second is the minimum variance portfolio (MVP), $B^{-1}A\underline{\alpha}$. The proportion put in each portfolio depends on the degree of relative risk aversion of the individual. The logarithmic portfolio depends on relative asset returns and is the same for all investors; this is the single point that all investors' efficient frontiers share, and it is the optimum portfolio for a logarithmic investor.[9]

Infinitely risk-averse investors ($R = \infty$) would hold only the MVP. Note that the shares in this portfolio are proportional to the expenditure shares. Given that investors are exposed to uncertainty about foreign price changes by virtue of consuming foreign as well as domestic goods, a risk-averse strategy is to choose a combination of assets to hedge that exposure. The MVP differs among investors when expenditure shares differ across countries, i.e., when PPP does not hold.

An additional useful representation of the solution is

$$\underline{x} = (1/R)B^{-1}[\underline{r} - A\underline{\alpha}] + B^{-1}A\underline{\alpha} \qquad (3.8'')$$

where $[\underline{r} - A\underline{\alpha}]$ is the portion of the vector of expected real relative returns that affects portfolio decisions and $B^{-1}A\underline{\alpha}$ is the MVP, as before. The term $B^{-1}(r - A\underline{\alpha})$ is known as the speculative portfolio, and it requires zero net worth: i.e., its portfolio shares sum to zero. The terms in the vector of expected real relative returns that do not enter $[\underline{r} - A\underline{\alpha}]$ are related to the mean and variance of inflation. Since they are the same for all assets, they do not affect choices among assets and, thus, they do not enter the solution for portfolio weights. However, the covariances between real returns and inflation do differ by asset, and they are incorporated in the A matrix.

The representation in equation (3.8") of the solution makes it easy to write the expression for the full vector of portfolio shares (including the benchmark asset), as a function of absolute (as opposed to relative) returns. We denote this full vector $\underline{x}_n$. As shown by, for example, Macedo [1979],

$$\underline{x}_n = [1/r] \begin{bmatrix} \underline{1}'B^{-1}\underline{1} & -\underline{1}'B^{-1} \\ -B^{-1}\underline{1} & B^{-1} \end{bmatrix} \underline{r}_n + \begin{bmatrix} 1-\underline{1}'B^{-1}A\underline{\alpha} \\ B^{-1}A\underline{\alpha} \end{bmatrix} \qquad (3.9)$$

where

$$\underline{r}_n = \begin{bmatrix} i \\ i^* + e \\ s \\ s + e + \sigma_{S*E} \end{bmatrix}$$

is the vector of absolute nominal, numeraire currency returns,

$\underline{1}$ is a (3x1) vector of ones, and the derivation uses the fact that the portfolio shares for the MVP must sum to one while the speculative portfolio shares sum to zero.

B. Special Cases of the General Model

As Table I shows, it has been common in the empirical literature to analyze models that invoke one or more simplifying assumptions. In some cases this has been done to highlight specific sources of risk, particularly those associated with exchange rates. In other cases, the primary goal was to obtain an empirically tractable asset pricing relationship, in the form of an expression for relative asset returns or for the exchange rate. In order to understand what constitutes empirical tractability, we briefly consider the problems faced by an investigator wishing to calculate aggregate asset demands based on expression (3.8).

Suppose, as do Adler and Dumas [1983], that investors within a country are homogeneous, so that (3.8) can be interpreted as giving the portfolio shares for country i investors. In the general model above W is the wealth of the single domestic individual and $\underline{x}$ is that individual's vector of portfolio shares. Here we use W_i to represent the wealth of country i and $\underline{x}_i$ to denote national portfolio shares. Country i's total asset demands are obtained by multiplying each element of $\underline{x}_i$ by W_i

$$\underline{x}_i W_i = [1/R_i] W_i B^{-1} \underline{r} + [1 - (1/R_i)] W_i B^{-1} A \underline{\alpha}_i \tag{3.10}$$

For simplicity we have not included the asset demand for the domestic bond. Note that both expenditure shares and the coefficient of risk aversion are country-specific and that the A and B matrices and the $\underline{r}$ vector are not.[10]

We define the vector of world asset demands as

$$\underline{d} = \begin{bmatrix} \Sigma\, x_{B*,i} W_i \\ \Sigma\, x_{S,i} W_i \\ \Sigma\, x_{S*,i} W_i \end{bmatrix}$$

If we assume a common world measure of risk aversion R -- and no one has attempted to do otherwise -- we have

$$\underline{d} = (1/R) W B^{-1} \underline{r} + [1 - (1/R)]\, B^{-1} A \begin{bmatrix} \Sigma\, \alpha_i W_i \\ \Sigma\, (1-\alpha_i) W_i \end{bmatrix} \tag{3.11}$$

where $W = \Sigma\, W_i$ represents world wealth.

If we wanted to solve for relative asset prices, we would equate the left-hand side of equation (3.11) to asset supplies and solve for $\underline{r}$. While mathematically straightforward, the resulting expression is empirically demanding because we need data on wealth and expenditure shares for all the relevant countries. There are at least three ways to simplify the model and minimize data requirements: assume PPP, optimize over nominal wealth, or assume no risky assets with non-stochastic inflation. We consider each in turn.

The easiest way to avoid these empirical demands is to assume that PPP holds: i.e., in our notation, that P = P*E. When PPP holds, the means and variances of domestic and foreign inflation rates are equal in domestic currency terms, i.e., d(P*E)/P*E = dP/P. As a result, the domestic price index depends only on the (common) rate of inflation, and the expenditure shares are irrelevant:

$$Z = P^{\alpha}(P^*E)^{1-\alpha} = P^{\alpha}(P)^{1-\alpha} = P \tag{3.12}$$

We thus have a simpler expression for the rate of change of domestic purchasing power:

$$\frac{dQ}{Q} = (-p + \sigma^2{}_P)dt - \sigma_P dz_P \tag{3.13}$$

The solution for portfolio shares under PPP is

$$\underline{x}_i = (1/R)B^{-1}\underline{r} + [1-(1/R)]B^{-1}A_1 \tag{3.14}$$

where A_1 is the first column of the original matrix A. Since the expression contains no country-specific terms, aggregate world asset demand is given by

$$\underline{d} = (1/R)WB^{-1}\underline{r} + [1-(1/R)]WB^{-1}A_1 \tag{3.15}$$

The PPP assumption is used widely in international economics because it is theoretically and empirically convenient. Nonetheless, there is a consensus in the literature that PPP does not hold in the short run. Given the abundant evidence against PPP, it is surprising that Frankel ([1982], section III) finds virtually no difference in his results when he relaxes the assumption of PPP by allowing expenditure shares to differ across countries. Similarly, Frankel and Engel ([1984], footnote 14) report that relaxing the PPP assumption does not alter their conclusion that international portfolios do not reflect mean-variance optimizing behavior. However, since both of these papers judge the importance of the PPP assumption in a model that also assumes no risky assets, the effects of assuming only PPP remain to be determined.

A second way to bypass the aggregation problems is to assume that investors maximize the expected utility of nominal, rather than real, wealth. In this case, the expression for portfolio shares reduces to the simple logarithmic portfolio:

$$\underline{x}_i = (1/R)B^{-1}\underline{r}, \tag{3.16}$$

Because investors do not hedge against inflation, the matrix A, which captures the interactions between goods prices and asset returns, does not appear in the solution. Since the logarithmic portfolio is the same for all investors, aggregation poses no problems

$$\underline{d} = (1/R)WB^{-1}\underline{r} \tag{3.17}$$

The solution, in this case, resembles a simple international Capital Asset Pricing Model, because relative asset prices are a function of a single index. Note that this simple structure results only in this very restrictive case.

The usual justification given for invoking the nominal wealth optimization assumption is that inflation rates vary little - relative to exchange rates or stock prices - over the time intervals usually considered (e.g., a month). Evidence that this is the case is reported in Table II below. This same argument is used to justify the assumption of non-stochastic inflation. However, the two assumptions are not equivalent. The nominal wealth assumption removes the entire price index from consideration. The non-stochastic inflation assumption removes only the P and P* parts of the price index, leaving the effects of the exchange rate E and deviations from PPP to be incorporated in the analysis. Thus, the latter assumption is less restrictive than the former. However, the observation that domestic and foreign inflation variation is relatively small compared to other parameters does not guarantee that portfolio shares, which incorporate inflation variance in a non-linear way, will be completely unaffected by the omission of inflation variance terms. A small variance does not necessarily imply a negligable covariance with other variables.

Formally, the effect of assuming non-stochastic inflation is to set the six covariances involving P or P* equal to zero. These covariances all appear in the A matrix. The portfolio shares, given this assumption, are

$$\underline{x}_i = (1/R)B^{-1}\underline{r} + [1-(1/R)]B^{-1}A_2\underline{\alpha}_i \quad (3.18)$$

where

$$A_2 = \begin{bmatrix} 0 & \sigma^2_E \\ 0 & \sigma_{ES} \\ 0 & \sigma^2_E+\sigma_{ES*} \end{bmatrix}$$

Finally, two simplifying assumptions can be combined to simplify the asset pricing problem: no risky assets and non-stochastic inflation. We have already seen the effect of the latter. We consider the impact of the no risky asset assumption, by itself, before considering the consequences of their joint imposition.

As Table I indicates, the assumption of no risky assets is a popular one. Its popularity is largely due to the way in which it highlights the risk associated with exchange rates by simplifying the variance-covariance matrix of asset returns (B). Since assets which are 'riskfree' from the point of view of domestic investors have returns which vary with the exchange rate from the point of view of foreign investors, B will contain only exchange rate variances and covariances.

In our two-country model, the assumption of no risky assets means x_s and x_{s*} are constrained to zero and the B matrix is a scaler. Therefore, the portfolio share vector for the domestic investor is just the scaler

$$x_{B*,i} = (1/\sigma^2_E)\Big[(1/R)[i^* + e - i] + [1-(1/R)][\sigma_{EP}\alpha_i + (\sigma_{EP*}+\sigma^2_E)(1-\alpha_i)]\Big] \quad (3.19)$$

where $(1/\sigma^2_E)$ is the inverse of the B matrix.

When we combine the assumption of non-stochastic inflation with the assumption of no risky assets, the σ_{EP} and σ_{EP*} terms drop out, leaving

$$x_{B*,i} = (1/R)(1/\sigma^2_E)(i* + e - i) + [1 - (1/R)](1-\alpha_i) \qquad (3.20)$$

Thus, in this model exchange rate variance is the sole source of risk. Note that the MVP reduces to the vector of expenditure shares. This happens because the only purchasing power uncertainty that requires hedging is induced by variations in foreign goods prices, which can only come from variations in the exchange rate.

Such a model lends itself easily to an asset pricing solution, even though it contains the expenditure share parameter. To see why this is so, consider the aggregate demand for the foreign bond

$$d_{B*} = (1/R)W(1/\sigma^2_E)(i*+ e - i) + [1-(1/R)]\sum_i W_i(1-\alpha_i) \qquad (3.21)$$

The last term is the sum of country-specific constants. In a regression context, the problem of aggregating across countries then reduces to the simple problem of estimating the intercept (Frankel [1982]). A caveat is in order. While the expenditure shares are constant over time, wealth is not. Therefore, it is not clear that this approach is appropriate for time series estimation.

Before concluding the presentation of special cases, it is worth noting an important measurement issue involving the approximation of rates of return. Since investors care about real rates of return measured in domestic currency terms, Jensen's inequality enters into the calculations. For example, we can approximate the expected real return on a foreign bond by (i*+e-z), where z is the expected rate of inflation. However, the precise measure of expected real return also involves the covariance between the exchange rate and z, i.e., σ_{EP} and σ_{EP*}. Ignoring such second order terms, (e.g., as did Kouri and Macedo [1978], Frankel [1982], and Lewis [1988]), results in the following expression for portfolio shares

$$\underline{x}_i = (1/R)B^{-1}\underline{r} + B^{-1}A\underline{\alpha}_i, \qquad (3.22)$$

where

$$\underline{r} = \begin{bmatrix} i* + e - i \\ s - i \\ s* + e - i \end{bmatrix}$$

Note that the term $(1/R)B^{-1}A\underline{\alpha}_i$ no longer enters, and the returns vector $\underline{r}$ no longer includes the covariance between the foreign stock and the exchange rate σ_{S*E}.

The empirical relevance of these second-order terms has been the topic of a long-standing debate in the exchange rate literature and is usually referred to as Siegel's Paradox (Siegel [1972] and McCulloch [1975]). Some authors suggest the importance of the terms should be minor (e.g., Krugman [1981] and Frankel [1986]), while others disagree (e.g., Sercu [1981] and Branson and Henderson [1985]). However, the empirical evidence to date is not conclusive. While both Frankel ([1982], Appendix 1) and Lewis ([1988], footnote 2) find no difference in results from models with and without the second-order terms, both authors impose additional simplifying assumptions as well. As a result, the empirical impact of the second-order term assumption by itself remains an open question.[11]

IV. An Illustration of the Empirical Effects of Simplifying Assumptions

As we have shown in the previous section, the imposition of the various assumptions and combinations of assumptions leads to simplified expressions for portfolio shares. In all cases, the simplifications involve deleting parameters from the generalized model expression. The obvious question that arises is whether the empirical effects of these restrictions are small enough to ignore in practice. To answer this question, we calculate a series of sample portfolios. We use the results to identify which assumptions cause the largest and smallest changes in portfolio weights and to determine whether restricting the parameters to zero alters the computed shares in predictable ways. We also use the results to shed light on some of the puzzles and controversies in the literature. At the outset, we want to emphasize that this descriptive exercise is intended to illustrate the intuitions from the last section; it should not be interpreted as a complete test of the validity of the simplifying restrictions.

The format of our empirical exercise is quite simple. We assume that investors calculate estimates of the model parameters over a sample period and use them to form a portfolio for the subsequent period. We go through this exercise for the generalized model and for a variety of special case models, which incorporate one or more of the five simplifying restrictions -- PPP, maximization of a function of nominal wealth, non-stochastic inflation, no second-order terms and no risky assets.

Ideally, we would like to have estimates from a generalized world portfolio model to examine the effects of assumptions on portfolio weights. As that is beyond the scope of this paper, we instead calculate our sample portfolios based on the two-country, four-asset model of the previous section. The resulting portfolios will suffice to compare model specifications, although they clearly would be inadequate to represent actual investor behavior.

Since we do not want our conclusions to depend on specific parameter values, we divide our May 1973-August 1987 sample into equal periods of 43 months (1973:5 - 1976:11; 1976:12 - 1980:6; 1980:7 - 1984:1; 1984:2 - 1987:8). We repeat the analysis for each subperiod for three country pairs (U.S.- Germany; U.S.- Japan; U.S.-U.K.). Although the asset menu is unrealistically small, it is important that the model parameters take on realistic values because relative parameter size is a key issue. We therefore estimate the parameters in the returns vector $\underline{r}$ and the covariance matrices A and B using monthly data from International Financial Statistics and Morgan Stanley's Capital International Perspective.[12] Table II shows that the estimates from the four periods and countries provide a large range of values for the model parameters.

There is no statistical theory that provides formal tests for the significance of changes in shares across cases, so we rely on two kinds of descriptive evidence. The first is the changes in the shares themselves -- relative size, direction, and robustness of change. The second is whether investors who formed portfolios based on the generalized model would have earned substantially different returns than investors who formed portfolios based on special cases.

It is easy to assess directions and frequency of changes in the portfolio weights. However, judging whether specific changes are large or small is difficult. It is important to remember that if these shares were weighted by wealth and aggregated into national asset demands, a small alteration in shares might translate into a large absolute change in asset demand. Our intuition is that a change in shares on the order of 0.20 is quite large under any conditions; thinking in terms of national aggregates, even a change of 0.05 might be economically significant. Where share changes of 0.05 or more occur, we will conclude that further investigation of the restriction is warranted. In particular, we think it is important to determine whether changes of similar magnitude occur in models with more assets and countries. We recognize that readers may wish to choose other criteria.

A similar caveat applies to our comparisons of average monthly ex post returns. A 0.001 difference in monthly returns translates into a 4.4 % difference in cumulative returns compounded over any 43-month holding period. This is probably a large enough value to catch the attention of a portfolio manager. We are uncertain about whether it is significant to researchers. However, a difference of 0.005 in monthly returns implies a 23.9 % difference in portfolio returns over 43 months. We are comfortable interpreting this as a large difference. We recognize that we are comparing returns that are unadjusted for risk exposure. However, comparisons of risk across special cases are not meaningful, since the simplified models assume away sources of risk and diversification.

To calculate the sample portfolios we use a coefficient of risk aversion (R) of 4.0 and a domestic expenditure share (α) of 0.7. The resulting portfolio weights are reported in Tables III-A, III-B and III-C. Recall that short sales have not been excluded, so negative weights are possible. The qualitative results are the same for other combinations of R and α values.

As a result of the parameter variation observed in Table II, the portfolios vary from period to period and country to country. However, a number of patterns are strikingly consistent when we compare the nine special case portfolios to the generalized model. For all countries, both the PPP and the nominal optimization assumptions raise the share in domestic bonds and lower the share in foreign bonds. The majority of these changes are at least 0.20 in absolute value. The stock holdings are virtually unchanged. In all but one instance, the assumptions of non-stochastic inflation and no second order terms lower the holdings of domestic bonds and increase foreign bond holdings. These changes range from 0.02 to 0.23 in absolute value. Again, the stock holdings are relatively unaffected. The direction of the changes in bond holdings in the no risky assets case varies, but the changes are almost always large, with many greater than 1.0.

When we combine the no risky assets assumption with PPP or the nominal optimization assumptions, the changes are in the same direction and of approximately the same size as when each assumption is imposed singly. Combining non-stochastic inflation or no second-order terms with the no risky assets assumption produces changes that are similar to, though less consistent than, the results when each is imposed separately.

Before interpreting these results, it is reasonable to ask whether the variation in portfolio shares across cases simply reflects parameter estimation error. In the absence of standard errors for our estimates, we cannot rule out this possibility. However, the consistency of the patterns in share changes across all countries and time periods strongly suggests that the qualitative effects we observe are the effects of the special case assumptions and not simply random estimation error.

In terms of the relative size of effects, the assumption that makes the largest difference is that of no risky assets. This conclusion is supported by changes in both portfolio shares and returns. It is noteworthy that this is the one assumption whose imposition frequently changes a long to a short position, or vice versa. Researchers in finance will not find it surprising that ignoring risky assets has a very large impact, since such assets are central to studies of risk and return. By contrast, researchers in economics frequently omit risky assets from portfolio analysis in order to focus on exchange rate risk. The magnitude of the effects of this simplification suggests this could produce highly misleading results. The effect is only compounded when we combine this assumption with any of the others.

While changes resulting from the imposition of either PPP or nominal optimization are not so dramatic, they are still quite large. This is not surprising given the abundant evidence that PPP does not hold in the short run, and the widely accepted belief that investors are not subject to money illusion. Furthermore, the patterns of changes are consistent with economic intuition. For example, assuming either PPP or nominal optimization removes the exchange rate effects that operate through the price index. Since investors choose portfolios to hedge against price uncertainty in their consumption bundles, excluding foreign price uncertainty (which is, in practice, mostly due to exchange rate uncertainty) reduces the motivation to hold foreign bonds as a diversification tool. Thus, in this case we would expect to see a reduction in foreign bond holdings, as, indeed, we do.

Our observation that assuming PPP substantially alters portfolio shares contrasts with the Frankel [1982] and Frankel and Engel [1984] findings that relaxing the PPP assumption has no substantive effect on their results. Recall that in their models the MVP is effectively ignored. But, as equation (3.14) shows, it is precisely the MVP that captures the impact of the PPP assumption.

Although the imposition of PPP or nominal optimization causes large changes in portfolio shares, the changes in ex post returns are relatively small. It appears that reshuffling the bond portfolio produces only marginal changes in profitability. This is not unexpected given the relatively small differences in bond returns across countries and the likelihood that their risk characteristics are similar.

We are also not surprised that assuming non-stochastic inflation or no second order terms has generally smaller impacts on portfolio shares and returns, because we know that both monthly inflation variance and second order terms are small in absolute size.[13] However, in every period at least one sizeable change in portfolio shares occurs, which suggests that interactions involving these small terms may periodically have large impacts. Thus, it could be a mistake to invoke these assumptions as a general practice. In addition, the finding that second-order terms alone sometimes have non-trivial effects on portfolio shares counters suggestions in the literature that Siegel's Paradox can be ignored.

The special case results also provide a framework for addressing the outstanding puzzles and debates in the literature that we noted earlier. The puzzles of particular interest to us are the weak empirical support for portfolio balance models, the apparent absence of international asset pricing consistent with mean-variance optimization, and the inability of some researchers to reject the hypothesis that investors are risk-neutral.

Consider the standard portfolio balance model, which begins with the specification of asset demand as a function of relative asset returns. The solution to this model expresses the exchange rate as a function of international asset supplies. Our analysis of special cases shows that the only model that produces asset demands of this form is one whose solution includes only the logarithmic portfolio (e.g., assuming nominal wealth optimization), or one in which the MVP reduces to a constant (e.g., assuming no risky assets combined with non-stochastic inflation). Excluding any of the parameters in the MVP clearly causes an omitted variables problem, and therefore we are not surprised to see counterintuitive coefficient estimates in earlier work.

As previously noted, empirical tests of mean-variance optimization have all been based on special case models. Our results clearly show that invoking special case assumptions systematically alters portfolio shares. These biases are carried over to the calculation of aggregate asset demands. Since one test of the model's validity is to compare calculated asset demands with known asset supplies, any effects on asset demands will affect conclusions about mean-variance pricing. If the shares are systematically too big or too small, it would be no surprise to find aggregate asset demands that differ substantially from known asset supplies, as indeed many researchers do. For example, Adler and Dumas [1983] point out that if we looked at only logarithmic portfolios, investors in the aggregate could be found to have negative demands for some assets and demands exceeding 100% of the supplies of others. Furthermore, if a researcher assumes market clearing and solves for asset prices which are based on a simplified model, a potentially incorrect rejection of mean-variance optimization is likely.

In many cases, researchers have been unable to reject the hypothesis that investors are risk-neutral ($R = 0$). Indeed, one of the more perplexing puzzles in the empirical literature arises when the coefficient of risk aversion, R, is jointly estimated with asset pricing relationships. The maximum likelihood estimates of R are clearly unsatisfactory, ranging from unrealistically big values -- $R > 30.0$ in Frankel [1982] -- to unrealistically small values -- $R = -67.0$ in Frankel and Engel [1984]. In these papers, as well as in Engel and Rodrigues [1987] and Lewis [1988], the standard error is so large that it is impossible to reject almost any hypothesized value of R, whether large, small or zero. All of these studies are based on models which effectively have only logarithmic portfolios.

The source of the unsatisfactory R values can now be seen. Suppose one or more assets have logarithmic portfolio shares much larger than 1.0, while other shares are relatively small. Then there are correspondingly large elements in $\underline{d} = (1/R)WB^{-1}\underline{r}$. When we try to fit $(1/R)WB^{-1}\underline{r}$ to actual asset supplies, the best fit will be obtained by making R very large to scale down the dominant elements in $\underline{d}$. Similarly, if $\underline{d}$ contains some large negative demands, the best fit to actual supplies would be obtained by making R negative and sufficiently large to scale down these elements.[14]

V. Conclusions

In this paper we provide evidence that puzzles in the empirical literature on international portfolio behavior may be attributed to the widespread use of simplifying assumptions. We do this by first showing how the empirical models in the literature can be derived as special cases of the solution to a generalized optimal portfolio problem. The framework of an encompassing, generalized model allows us to clarify the relationships among the models that have been studied and to synthesize the economics and finance approaches to international portfolio diversification. This exposition makes explicit the intuition of Stulz [1981], who suggested that empirical rejections of the international portfolio model could be due to invalid simplifying assumptions rather than to investor irrationality or market imperfection.

We then illustrate this intuition by calculating portfolio shares and ex post returns for the generalized and special case models and comparing them over a number of periods and countries. The results of this comparison indicate that the simplifying restrictions systematicaly bias the portfolio shares. Where these biases are large and where there are also large changes in ex post returns, we feel confident in concluding that researchers should be warned against automatically invoking the restrictions.

The most striking result occurs with the assumption of no risky assets, which causes the largest alterations in portfolio shares and returns. It is standard operating procedure for economists to ignore risky assets when analyzing international portfolios or modelling the exchange rate. This has been a convenient way to sidestep the aggregation problems of international asset pricing and to highlight the role of exchange risk. However, our analysis suggests that this shortcut produces misleading results. The other simplifying assumptions that are common in the literature produce lesser but still worrisome changes in portfolio shares and/or return. Furthermore, the results of our empirical exercise provide insight into how rejections of mean-variance optimization, implausible estimates of the coefficient of relative risk aversion, and counterintuitive coefficient signs in exchange rate equations could easily be artifacts of invalid simplifying restrictions.

We think these issues merit further and more formal investigation. Specifically, a research priority should be a comparison of actual asset supplies with the asset demands implied by a generalized model with a more complete menu of assets and countries. We recognize that the problems of measuring world asset supplies will make this a non-trivial endeavor.

FOOTNOTES

1. For surveys of this literature, see Adler and Dumas [1983] and Branson and Henderson [1985]. For seminal theoretical work, see Solnik [1973, 1974] and Stulz [1981].

2. We are not the first to make this point. Stulz [1981] also presents a general model and observes that empirical findings of international market segmentation may be due to incorrect restrictions imposed on the model. We extend Stulz's analyses in two ways. First, we explicitly derive and compare numerous special cases. Second, we examine specific empirical examples, allowing us to confirm Stulz's suspicion that the simplifying assumptions can bias the conclusions. In the appendix of an unpublished dissertation, Meerschwam [1983] and Macedo, Goldstein and Meerschwam [1984] derived a small number of special cases, but did not present empirical estimates for them.

3. This is correct as long as there are only two assets in the world, or if there are restrictions on short sales. However, this result will not generally obtain when there are three or more assets and no restrictions on short sales. Instead, the risk-neutral investor will sell some assets short to 'move up' the investment frontier, thus increasing return beyond the limits of the single asset with the highest return. The result is that the portfolio weights for all assets will, in general, be non-zero.

4. A few papers accomplish the same sort of simplification without excluding risky assets. Kouri [1977] imposes the assumption of zero correlation between asset returns and inflation rates. Solnik [1974] assumes that asset returns (in the currency of denomination) are uncorrelated with exchange rates.

5. As an alternative to specifying an exchange rate process, one could model the deviation from PPP as a stochastic process, as in Kouri [1976].

6. Viewing the exchange rate as the relative price of the two monies, one might consider including the countries' currencies in the menu of assets. However, as illustrated by Kouri [1977], there is no payoff to doing so since the risk attributes of money are the same as those of short bonds. (Money has a non-stochastic nominal return of 0.)

7. The definitions of B, B*, S and S* correspond to those in, for example, Kouri [1977] and Branson and Henderson [1985]. Alternatively, one could express value as price times number of units held for each asset, as did Adler and Dumas [1983], and then specify a stochastic process for asset prices rather than asset values. The two approaches result in the same expression for wealth dynamics.

8. While we have not explicitly included forward foreign exchange contracts in the menu of assets, our analysis can be extended easily to include them. As noted by Solnik [1973] and others, the covered interest parity condition implies that when we know the relative prices of domestic and foreign riskfree assets (our i and i*), we have also priced forward contracts. Taking a position in domestic and foreign bonds is therefore equivalent in all respects to taking a forward currency position.

9. The term logarithmic is potentially misleading. While this portfolio is the solution for an individual with a logarithmic utility function, for which R=1, it is not the case that having R=1 necessarily implies that utility must be logarithmic.

10. An extensive treatment of aggregation issues can be found in Adler and Dumas [1983]. Sercu [1980] shows that portfolio calculations are independent of measurement currency, which means we can assume, without loss of generality, that all countries' investors use the same numeraire.

11. An additional measurement issue arises when a stochastic process is specified for the overall price index rather than for P and P* (see Adler and Dumas [1983]). Although the resulting solution for an investor's portfolio shares looks like the PPP solution (only one inflation rate enters the calculation), in empirical applications it is identical to the generalized model. This is because domestic and foreign goods prices and expenditure shares are imbedded in the price index. We find our disaggregated specification intuitively useful in distinguishing among the effects of the various simplifying assumptions.

12. The IFS series used are: AG (end-of-month exchange rate in dollars per unit of foreign currency), 60B (Federal Funds rate, to represent the riskfree asset's rate of return), and 63 (Wholesale Price Index as a rough proxy for national output prices). The currency of denomination returns on risky assets are calculated from the Capital International Perspective data as the sum of the rate of change of the national stock price index and the dividend yield (dividend/price). Rates of return are calculated using log changes and expressed in per month terms.

13. Inflation variance is generally smaller when measured by the CPI than by the WPI (see Table II). The non-stochastic inflation assumption therefore causes a smaller change in portfolio shares when we use the CPI in our calculations. However, we still conclude that the difference is too big to ignore.

14. The reason that R does most of the adjusting is that the B matrix is constrained to equal the error variance matrix. See Frankel [1982].

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TABLE I

SIMPLIFYING ASSUMPTIONS IN THE EMPIRICAL LITERATURE

STUDY	PPP (1)	Maximize over Nominal Wealth (2)	Non-stochastic Inflation (3)	No Risky Assets (4)	No Second Order Terms (5)	OTHER
Levy-Sarnat (1978)		X				
Kouri-Macedo (1978)				X	X	
Von Furstenberg (1981)				X		No short sales
Frankel (1982)			X	X	X*	
Macedo (1982, 1983)				X		
Adler-Dumas (1983)						Stochastic process for price index, not P, P*
Frankel-Engel (1984)	X*			X		
Macedo-Goldstein-Meerschwam (1984)				X		Only risky asset is gold
Engel-Rodrigues (1987)	X			X		
Lewis (1988)				X	X*	
Bomhoff-Koedjidk (1988)		X				Home investors cannot buy foreign risky assets

* Assumption relaxed in an appendix or footnote, complete results not reported.

TABLE II-A
Parameter Values-Germany

Parameter	Period 1	Period 2	Period 3	Period 4
i*	0.0054	0.0041	0.0068	0.0040
s*	0.0023	0.0043	0.0142	0.0175
e	0.0038	0.0073	-0.0109	0.0102
σ^2S*	0.0027	0.0007	0.0015	0.0038
σ^2E	0.0013	0.0013	0.0007	0.0014
σSS*	0.0010	0.0003	0.0003	0.0006
σES	0.0007	0.0001	0.0002	-.9E-04
σES*	-.6E-04	0.0003	0.0002	-0.0004
σEP	-.7E-04	-.1E-04	-.5E-04	-.2E-04
σEP*	.2E-04	-.3E-04	-.3E-04	-.3E-04
σSP*	.1E-05	.9E-05	-.1E-04	-.2E-04
σS*P	-.1E-04	-.5E-05	-.2E-04	-.7E-04
σS*P*	-.2E-04	-.6E-05	-.4E-04	.2E-04
σ^2P*(WPI)	.4E-04	.1E-04	.1E-04	.1E-04
σ^2P*(CPI)	.1E-04	.8E-05	.7E-05	.5E-05

Notes to TABLE II:
(1) Exchange rates and covariances are defined relative to the U.S. For example, σ_{S*P} = covariance of foreign stock returns with U.S. inflation.
(2) All parameter estimates are based on monthly rates.
(3) The inflation variances calculated from the CPI are included for purposes of comparison. All calculations of portfolio weights and returns were made using covariances with inflation based on the WPI.

TABLE II-B
Parameter Values-Japan

Parameter	Period 1	Period 2	Period 3	Period 4
i*	0.0078	0.0049	0.0060	0.0044
s*	-.2E-04	0.0096	0.0144	0.0252
e	-0.0025	0.0071	-0.0018	0.0116
σ^2S*	0.0023	0.0009	0.0012	0.0030
σ^2E	0.0004	0.0015	0.0013	0.0011
σSS*	0.0010	0.0004	0.0003	0.0007
σES	0.0004	-.8E-05	.6E-04	-0.0001
σES*	0.0002	.8E-05	0.0003	0.0005
σEP	-.8E-04	-.6E-04	-.2E-04	-.4E-04
σEP*	-.8E-04	-0.0001	-.6E-04	-.8E-04
σSP*	--0.0001	.2E-04	-.1E-04	-.3E-04
σS*P	-0.0002	.1E-04	.5E-04	-.4E-04
σS*P*	-0.0002	.3E-04	-.2E-04	-.9E-04
σ^2P*(WPI)	0.0002	0.0001	.2E-04	.4E-04
σ^2P*(CPI)	0.0001	.5E-04	.4E-04	.3E-04

TABLE II-C
Parameter Values-U.K.

Parameter	Period 1	Period 2	Period 3	Period 4
i*	0.0038	0.0046	0.0092	0.0078
s*	-0.0031	0.0202	0.0188	0.0022
e	-0.0096	0.0084	-0.0121	0.0034
σ^2S*	0.0119	0.0032	0.0022	0.0022
σ^2E	0.0006	0.0009	0.0008	0.0014
σSS*	0.0035	0.0010	0.0008	0.0013
σES	0.0001	0.0002	0.0001	.4E-04
σES*	0.0006	.2E-04	0.0002	0.0002
σEP	.4E-05	-.1E-04	-.1E-05	-.1E-04
σEP*	.2E-04	-.4E-06	.2E-04	.3E-05
σSP*	.8E-04	-.2E-04	-.7E-05	-.2E-05
σS*P	-0.0003	.2E-06	.2E-04	.6E-05
σS*P*	0.0002	.3E-04	.2E-04	.2E-04
σ^2P*(WPI)	.6E-04	.3E-04	.1E-04	.8E-05
σ^2P*(CPI)	.8E-04	.7E-04	.3E-04	.2E-04

TABLE II-D
Parameter Values-U.S.

Parameter	Period 1	Period 2	Period 3	Period 4
i	0.0062	0.0071	0.0099	0.0064
s	0.0019	0.0055	0.0122	0.0193
σ^2S	0.0033	0.0017	0.0017	0.0018
σSP	-.0002	-.2E-04	-.7E-05	.2E-04
σ^2P(WPI)	0.0002	.3E-04	.2E-04	.2E-04
σ^2P(CPI)	.1E-04	.1E-04	.1E-04	.5E-05

TABLE III-A

U.S. Investor's Portfolio
Shares and Returns with
Germany as Foreign Country

Model:	General	PPP	Nominal	Non-Stochastic Inflation	No 2nd Order Terms	No Risky Assets	No Risky Assets and PPP	No Risky Assets and Nominal	No Risky Assets and Non-Stoch. Inflat.	No Risky Assets and No 2nd Order Terms
PERIOD 1										
x_B	0.42	0.67	0.60	0.37	0.36	0.19	0.43	0.39	0.16	0.12
x_B*	1.19	0.95	0.98	1.21	1.26	0.81	0.57	0.61	0.84	0.88
xS	-0.54	-0.56	-0.50	-0.50	-0.56					
xS*	-0.07	-0.06	-0.08	-0.08	-0.06					
Returns	0.022	0.021	0.021	0.022	0.022	0.020	0.019	0.019	0.020	0.020
PERIOD 2										
x_B	0.21	0.44	0.42	0.20	0.09	-0.04	0.18	0.17	-0.05	-0.11
x_B*	1.16	0.94	0.94	1.17	1.39	1.04	0.82	0.83	1.05	1.11
xS	-0.30	-0.30	-0.29	-0.29	-0.28					
xS*	-0.07	-0.07	-0.07	-0.07	-0.20					
Returns	-0.006	-0.003	-0.003	-0.006	-0.009	-0.004	-0.001	-0.001	-0.005	-0.00[illegible]
PERIOD 3										
x_B	5.65	5.88	5.83	5.61	5.57	5.56	5.79	5.74	5.52	5.50
x_B*	-7.21	-7.45	-7.40	-7.18	-7.10	-4.56	-4.79	-4.74	-4.52	-4.50
xS	0.65	0.65	0.64	0.64	0.65					
xS*	1.92	1.92	1.93	1.93	1.88					
Returns	0.003	0.001	0.002	0.003	0.003	-0.026	-0.028	-0.027	-0.025	-0.025
PERIOD 4										
x_B	-2.49	-2.27	-2.27	-2.50	-2.56	-0.59	-0.37	-0.38	-0.61	-0.66
x_B*	1.13	0.91	0.91	1.14	1.19	1.59	1.37	1.38	1.61	1.66
xS	1.59	1.60	1.59	1.59	1.59					
xS*	0.76	0.76	0.77	0.77	0.78					

Notes:

1. Shares may not add to 1.0 precisely due to rounding error.
2. The sample periods are each 43 months in length.
   - Period 1 1973:5 - 1976:11
   - Period 2 1976:12 - 1980:6
   - Period 3 1980:7 - 1984:1
   - Period 4 1984:2 - 1987:8
3. Returns are monthly returns realized from holding the given portfolio over the subsequent period.

TABLE III-B

U.S. Investor's Portfolio
Shares and Returns with
Japan as Foreign Country

Model:	General	PPP	Nominal	Non-Stochastic Inflation	No 2nd Order Terms	No Risky Assets	No Risky Assets and PPP	Nominal	Non-Stoch. Inflat.	No 2nd Order Terms
PERIOD 1										
x_B	1.06	1.29	1.15	0.92	1.02	1.49	1.71	1.56	1.34	1.46
x_B*	0.84	0.62	0.70	0.93	0.92	-0.49	-0.71	-0.56	-0.34	-0.46
xS	-0.09	-0.10	-0.07	-0.07	-0.09					
xS*	-0.81	-0.81	-0.78	-0.78	-0.85					
Returns	0.013	0.012	0.013	0.014	0.013	0.014	0.013	0.014	0.015	0.014
PERIOD 2										
x_B	0.58	0.80	0.76	0.53	0.53	-0.02	0.19	0.16	-0.06	-0.08
x_B*	-0.54	-0.75	-0.71	-0.49	-0.49	1.02	0.81	0.84	1.06	1.08
xS	-0.59	-0.60	-0.59	-0.59	-0.60					
xS*	1.55	1.55	1.54	1.54	1.56					
Returns	0.018	0.019	0.019	0.017	0.017	0.006	0.007	0.007	0.006	0.006
PERIOD 3										
x_B	2.46	2.68	2.64	2.42	2.36	1.86	2.08	2.07	1.84	1.79
x_B*	-3.83	-4.06	-3.99	-3.77	-3.67	-0.86	-1.08	-1.07	-0.84	-0.79
xS	0.02	0.02	0.03	0.03	0.03					
xS*	2.35	2.37	2.32	2.32	2.28					
Returns	0.044	0.042	0.042	0.044	0.044	0.001	-0.001	-0.001	0.001	0.002
PERIOD 4										
x_B	-2.67	-2.46	-2.48	-2.70	-2.78	-1.46	-1.24	-1.27	-1.50	-1.52
x_B*	1.03	0.82	0.83	1.06	1.17	2.46	2.24	2.27	2.50	2.52
xS	1.51	1.52	1.51	1.51	1.53					
xS*	1.13	1.13	1.14	1.14	1.08					

TABLE III-C

U.S. Investor's Portfolio
Shares and Returns with
U.K. as Foreign Country

Model:	General	PPP	Nominal	Non-Stochastic Inflation	No 2nd Order Terms	No Risky Assets	No Risky Assets and PPP	No Risky Assets and Nominal	No Risky Assets and Non-Stoch. Inflat.	No Risky Assets and No 2nd Order Terms
PERIOD 1										
x_B	6.71	6.95	6.91	6.69	6.60	6.14	6.37	6.37	6.15	6.06
x_B*	-5.54	-5.76	-5.78	-5.56	-5.43	-5.14	-5.37	-5.37	-5.15	-5.06
xS	-0.41	-0.43	-0.38	-0.38	-0.40					
xS*	0.24	0.24	0.24	0.24	0.23					
Returns	-0.008	-0.010	-0.010	-0.008	-0.008	-0.012	-0.013	-0.013	-0.012	-0.011
PERIOD 2										
x_B	0.25	0.47	0.46	0.23	0.18	-0.89	-0.66	-0.67	-0.90	-0.96
x_B*	0.52	0.29	0.30	0.53	0.59	1.89	1.66	1.67	1.90	1.96
xS	-1.38	-1.38	-1.37	-1.37	-1.38					
xS*	1.61	1.61	1.60	1.60	1.61					
Returns	-0.005	-0.002	-0.002	-0.005	-0.006	-0.014	-0.011	-0.011	-0.014	-0.015
PERIOD 3										
x_B	4.88	5.11	5.10	4.87	4.79	4.58	4.81	4.81	4.59	[illegible]
x_B*	-5.35	-5.58	-5.57	-5.34	-5.24	-3.58	-3.81	-3.81	-3.59	[illegible]
xS	0.04	0.04	0.05	0.05	0.05					
xS*	1.43	1.43	1.42	1.42	1.41					
Returns	0.011	0.010	0.010	0.011	0.012	-0.008	-0.009	-0.009	-0.008	-0.008
PERIOD 4										
x_B	-0.97	-0.75	-0.74	-0.97	-1.07	-0.06	0.17	0.16	-0.07	-0.14
x_B*	-0.10	-0.32	-0.31	-0.09	0.01	1.06	0.83	0.84	1.07	1.14
xS	1.05	1.06	1.05	1.05	1.07					
xS*	1.01	1.01	1.01	1.01	0.98					