POLITICALLY CONTESTABLE RENTS AND TRANSFERS*

by

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ABSTRACT

In empirical analyses the value of a politically allocated prize is typically viewed as a proxy for total outlays made by those contesting the prize. This paper develops a general model of politically contestable rents and transfers which includes asymmetric valuation of prizes. In general, outlays made in contesting a prize are shown to be substantially below the value of the prize. Asymmetric valuation also acts as a barrier to entry. As long as there are asymmetries associated with competition, the number of agents actively contesting a politically allocated rent or transfer is likely to be small.

1. INTRODUCTION

A substantial and historically increasing part of the activity of government has involved the transfer of income from one group in society to another.[1] Government policies can also be the source of rents, secured for example by the industry-specific factors who are the beneficiaries of protection, or by the residual claimants in firms which are the beneficiaries of regulation.[2] The income transfers and the rents which result from government intervention may be contestable via the political allocation mechanism rather than preassigned to designated beneficiaries. If so, the social cost of interventionist policies includes, in addition to the standard resource-misallocation costs reflecting allocative inefficiency, the value of the resources used in seeking to influence the outcome of the political allocation process.

Measurement of the standard allocative inefficiency costs of intervention proceeds in a well-known manner by appropriate evaluation of the Harberger triangles.[3] The informational requirements for computing the triangles are stringent, but not overly so given the requirements of applied economic research. One needs to know market demand and supply elasticities (preferably compensated), and the values of market transactions.

Measuring the additional social loss arising from contestability of politically allocated transfers and rents is more difficult, because of the

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[1] This is reflected in the literature on the growth of government. See for example Harold Demsetz (1982).

[2] Political-support motives suggestively underlie the creation of such rents. In the context of protection see for example Hillman (1982), Baldwin (1982). As Appelbaum and Katz (1987) point out, governments may themselves 'seek rents by creating rents.'

[3] See Harberger (1964).

paucity of direct observations. The activities directed at influencing political allocative decisions are often by their nature undertaken surreptitiously.[4]

However, while the influence-seeking activities may not be directly observable, the value of the politically allocated prize secured by the successful contender may be known. One can accordingly seek a means of inferring the value of the resources allocated to contesting the politically allocated prize from the value of the prize itself. Thus in the early literature beginning with Gordon Tullock (1967) followed by Anne Kreuger (1974) and Richard Posner (1975), the procedure was simply to take the observed value of the political prize as indicative of the value of the resources allocated to the political contest. For example, Posner proposed that the social cost of monopoly power could be measured by the Harberger triangles plus the value of monopoly profits (the prize for achieving monopoly power), and Krueger viewed the cost of protection via an import quota as consisting of the usual production and consumption costs of protection plus the value of the rents secured by acquiring rights of ownership to the quota. The presumption was that there will have been competition for the monopoly power or for the quota rights. With competitive free entry into the contests, identical risk-neutral agents with common valuations of the politically awarded prize would have completely dissipated

[4]Jagdish Bhagwati (1980, 1982) has pointed out that in principle the resources used in such directly unproductive activities may have negative shadow prices, in which case the activities may paradoxically be indirectly productive. This second-best possibility arises when the activities directed at influencing political allocation take place in an equilibrium which deviates from Pareto efficiency because of protectionist policies or regulation which sustains monopoly power. We shall however abstract from such considerations in this paper. Our presumption is that resources allocated to contesting political prizes have more socially productive employment opportunities.

the contested rent or transfer via their allocation of resources to influencing the outcome of the contest, thus making the observed value of the rent or transfer equal to the unobserved social cost of the rent- or transfer-seeking activity.

The presumption of equality between the value of resources expended and the value of the politically allocated prize which is contested offers a direct and tractable means of computing social costs inclusive of lobbying and other rent-seeking activities. The assumptions underlying a prediction of complete rent dissipation may however be inappropriate.

One possible departure is that although contests are competitive, agents contesting the political prize are risk-averse. Hillman and Katz (1984) investigated competitive rent dissipation by risk-averse agents. As is to be expected, rent dissipation in the presence of risk aversion was found to be less than complete since risk-averse expected-utility maximizing agents allocate to a contest less than the expected benefit from participation. However, provided rents were not too large relative to agents' initial wealth, complete rent dissipation was an acceptable approximation in the absence of direct observations on rent-seeking outlays. Risk aversion did not therefore overly compromise the presumption of complete rent dissipation.

Small numbers resulting in strategic rather than competitive contests may be surmised to be a reason for underdissipation of contested rents and transfers. This was confirmed to be generally the case by Gordon Tullock (1980) for a particular type of strategic contest in which the probability of success is a function of the spending levels of rival contenders for the prize. Tullock's contests, which have been influential in directing the course of recent literature,[5] are thus only imperfectly discriminating in that they do not designate the ultimate winner of a contest. Only probabilities of success are established.

An alternative strategic formulation allows perfect discrimination in selecting the winner via the rule that the highest outlay wins the political prize outright. Under this rule for designating the winning contender, the underdissipation presumption suggested by Tullock's contests disappears. Hirshleifer and Riley (1978) in a two-person formulation and Hillman and Samet (1987) more generally for any number of contenders have shown that when the highest outlay wins, rent dissipation is expectationally complete. No matter how many rivals compete for a politically allocated rent or transfer, on average the value of the resources allocated to the contest equals the value of the prize. Hence, in a small-numbers strategic setting with no requirement of a perfectly competitive environment, the presumption returns that the observed value of a politically allocated prize can be taken as indicative of the social cost incurred via the resources allocated to contesting the prize.

The imperfectly and perfectly discriminating models of contestability thus offer quite different predictions on rent dissipation. This paper develops a general model encompassing both types of contests. We introduce the further element of generality of asymmetric evaluation of the contested prize. Previous analyses of contestability have assumed that agents share a common value of the political prize. Yet various circumstances may lead valuations to differ. For example, the political contest may be for the

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[5]See the survey by Robert Tollison (1982). Subsequent elaborations of Tullock's contests include Corcoran and Karels (1985), Higgins, Shughart and Tollison (1985), Appelbaum and Katz (1986a, 1986b, 1987), with commentaries by Tullock (1984, 1985).

designation of the location of a new freeway via the allocation of highway funds; the freeway may yield differently valued benefits in different alternative locations. Or, in a contest for the rights of an import quota, a firm with domestic market power may value an import-restricting quota more highly than an importer with no market power because of the denial of competitive market access made possible by the quota right. In political contests to designate the direction of income transfers, different agents may have different deadweight costs associated with taxes and subsidies, so that valuations of a given monetary transfer may differ.

Asymmetric valuation will be shown to impart a bias towards under-dissipation of contestable rents and transfers. If different agents have different stakes in the outcome of a political contest, it becomes inappropriate to presume that the gain to the ultimate winner is at all an accurate reflection of the value of the resources which have been allocated to influencing the outcome of the contest. With asymmetric valuations, political competition then becomes much less costly in terms of resource use than indicated by observed values of rents and transfers.

The incentives to enter a political contest are fundamentally altered by asymmetric evaluations of the prize. A larger value assigned to the political prize by a rival is a barrier to entry for lower-valuation contenders. Smaller numbers thus enter to actively participate in political contests when valuations are asymmetric than when all potential contenders have the same stake in the outcome. The equilibrium number of active participants is determined endogenously by rivals' evaluations of the politically allocated prize, as is then the number of potential entrants who refrain from active participation and decide to sit the contest out.

We shall distinguish two types of political contests, differentiated by the consequences of refraining from active participation. Contests may be characterized by agents seeking rights to a preexisting rent which exists as the consequence of previous policy decisions. Or alternatively agents may seek to effect a transfer of income in their favor by lobbying for a policy change, such as by seeking protection or regulation. In the quest to establish rights to a given preexisting rent such as for example monopoly profits or quota premia, a nonparticipating individual who ignores the contest loses nothing. There is "nothing ventured and nothing gained." However, this is not so in the case of a contestable transfer. For then an agent who chooses to stay out of a contest may incur a loss if he is obliged to make a payment to finance part of the transfer to the winner. Thus protection transfers income from one group to another, and the losers from a protectionist policy suffer a loss even if they have not been active in countering the protectionist lobbying of the gainers from protection.

We introduce the basic elements of a model of perfectly discriminating contests with asymmetric valuations in section 2. Then in section 3 we investigate perfectly discriminating transfer contests where losers make the transfers which facilitate the prize to the winner. Refraining from active participation therefore does not allow a loss to be avoided. Section 4 considers contestable rents, where all loss can be avoided by choosing not to be an active participant in the political contest. Section 5 allows for uncertainty in the valuations placed on the political prize by rival contenders. Section 6 reformulates Tullock's imperfectly discriminating contests to allow for asymmetric valuations. Section 7 introduces a general stochastic formulation for imperfectly discriminating contests and demonstrates how Tullock's specification is a particular case appropriate in principle only under limited circumstances. Concluding remarks are contained in the final section.

In our subsequent analysis we shall refer to outlays or expenditures made in contesting a rent or transfer rather than to the real resource use which strictly speaking underlies social cost. Both monetary outlays and real resources can in principle be simultaneously used in seeking to influence the outcome of a political contest. The monetary outlays then themselves constitute transfers, which may in turn themselves be contestable. A single politically allocated rent or transfer may thus evoke a number of interrelated contests. Computation of social cost then requires evaluating the resources used in all these contests.[6] Here we forgo this complexity to focus on the relation between the value of outlays made in a contest (monetary or via real-resource allocation) and valuations of the contested prize.

2. A BASIC MODEL

We begin by introducing a basic model describing political contestability of rents and transfers. Let n agents confront the opportunity of influencing the outcome of a political allocation decision. All agents who actively participate in the political contest whether or not they are ultimately successful irretrievably lose the outlays which they make in the attempt to influence the outcome of the contest in their favor.[7] In a model of contestable rents, a prespecified prize is available to the winner of the

[6]For a model of hierarchically structured imperfectly discriminating contests which accounts for resources used in all contests evoked by transfers, see Hillman and Katz (1987).

[7]It is in this basic sense that rent-seeking outlays differ from retained bids in auctions. On the theory of auctions, see for example Riley and Samuelson (1981).

political contest. In a model of contestable transfers, the gain to the ultimate winner is at the expense of the ultimate losers.

Let agent i outlay x_i to influence the outcome of the political contest in his favor. The probability that agent i will be the successful contender is

$$p_i(x) = p_i(x_1, \ldots, x_n)$$

where

$$\sum_{i=1}^{n} p_i(x) = 1$$

and p_i is nondecreasing in x_i and nonincreasing in x_j, $j \neq i$.

Our interest is in inferring from the optimizing behavior of the n contenders the total value of the outlays $\Sigma_{j=1}^{n} x_j$ made in the political contest, given the different valuations $v_i(i=1,\ldots,n)$ which agents assign to winning the contest. We shall assume until indicated otherwise that the individual valuations v_i are known to all agents.

To evaluate the resource cost of political contestability, we require a specification for the probability function $p_i(x)$. Initially we assume that contests are perfectly discriminating. That is, the political process awards the prize to the individual making the greatest outlay in seeking to influence the outcome in his favor. If more than one contender makes the highest outlay, the prize is shared.[8] Formally, the probability of winning is then

[8]Essentially, the contest is to determine the beneficiary of the indivisible prize, and not for a share of the prize. We allow for the possibility of sharing for technical completeness, although in equilibrium sharing does not occur. One agent always has the incentive to outlay slightly more and win. For an analysis of rent seeking where the prizes are shares, see Long and Vousden (forthcoming).

$$(2.1) \qquad p_i(x) = \begin{cases} 0 & \text{if } x_i \text{ is not a maximal element of } \{x_i, \ldots, x_n\}. \\ \frac{1}{m} & \text{if } x_i \text{ is one of } m \text{ maximal elements of } \{x_1, \ldots, x_n\}. \end{cases}$$

While we shall consider complete symmetry as a limiting case, we shall be particularly interested in analysing situations in which the value of the prize is different for different agents.

Our model encompasses a number of seemingly rather different applications. The simplest is that of pure rent seeking where agents contest a prize, but no transfers take place from losers to winners.

Another class of contests is characterized by transfers between agents such that the losers provide the source of the gain to the winners. Again we allow for variations in the value of the prize across contenders. Since an agent who is unsuccessful is taxed and incurs a share of the cost of the project, losers make a transfer payment L_i, as well as possibly having made outlays in unsuccessfully contesting the political prize.[9]

In trying to predict the implications of the model, the following two ideas make useful starting points. First of all, in the early literature on rent seeking (Posner (1975)), an analogy is made with bidding. Consider then open bidding for the prize, as in the usual ascending bid auction. As long as the asking price is below the second highest valuation, at least two agents have an incentive to remain in the bidding. Therefore the bidding continues until the price reaches the second highest valuation, v_2.

[9]See also Gary Becker (1983, 1985) on contestable transfers. Becker presumes that the beneficiaries of the subsidies and those whose taxes finance the redistributive transfers are known. Thus, there is no contest to establish the direction of transfer payments. The focus of Becker's analysis is on the role of deadweight losses in determining the nature and scope of redistributive transfers.

There is a second argument suggesting that total outlays might be v_2. No agent ever has an incentive to spend more than his valuation. Therefore agent 1, by spending just a bit more than v_2 can guarantee himself a payoff of v_1-v_2. The problem for agent 1, however, is that each agent must choose his own outlay prior to observing others' outlays. That is, even though there is complete information about preferences, each agent must try to make an inference as to what his opponents' strategies will be. If agent 1 says he will spend v_2, another agent might reason as follows. "If I were to believe agent 1 there would be no point in competing. My best response would be to stay out of the contest. But, knowing this, agent 1 would then have an incentive to make only a very small outlay and achieve a much larger payoff. Since agent 1 has this incentive, he is just bluffing when he says he will spend v_2."

In game theoretic terms, the strategies chosen by each of the n agents are equilibrium strategies if and only if, for all i, agent i's strategy is his best response given the strategies of the other $n-1$ agents. For our example, each of the other agents' best response to agent 1's strategy of spending v_2 is to stay out of the contest. But then agent 1's best response to agents $2,\ldots,n$ is to spend some very small amount and not v_2. From all this it seems reasonable to conclude that agents other than agent 1 will make positive outlays and that agent 1 will not spend as much as v_2.

The next point to be made is a technical one. In order to write down a mathematical expression for each agent's expected payoff, it is helpful to be able to rule out certain strategies. Specifically, we now argue that no agent will, in equilibrium, ever spend a positive amount β with a strictly positive probability. That is, equilibrium strategies are continuous mixed strategies.

To see this, suppose agent i does spend β with strictly positive probability. Then the probability that a rival agent j beats agent i rises discontinuously as a function of x_j at $x_j = \beta$. Therefore there is some $\epsilon > 0$ such that agent j will bid on the interval $[\beta-\epsilon, \beta]$ with zero probability, for all $j \neq i$. But then agent i is better off spending $(\beta-\epsilon)$ rather than β since his probability of winning is the same, contradicting the hypothesis that $x_i = \beta$ is an equilibrium strategy.

Given this result, it follows immediately that if there are just two agents, they must have the same maximum spending level. For if $\bar{x}$ is agent 1's maximum spending level, agent 2 wins with probability 1 by spending $\bar{x}$ and vice versa.

A similar argument establishes that the minimum spending level is zero for each agent. To see this, suppose to the contrary that agent i spends less than β with zero probability, where $\beta > 0$. Then any spending level between zero and β yields a negative payoff since the probability of winning is zero. Since other agents can always spend zero it follows that no other agent will spend in the interval $(0,\beta)$. But then agent i could reduce his spending level below β without altering the probability of winning, contradicting the hypothesis that agent i could, in equilibrium, do no better than take β as his minimum spending level.

Given these results, if we define $1 - G_i(x_i)$ to be the probability that agent i spends more than x_i, then $G_i(x_i)$ is continuous over $(0,\infty)$. If $0 < G_i(0) < 1$ then agent i spends a strictly positive amount with probability less than 1. His remaining alternatives are to spend zero or stay out of the contest.

Since any transfer payment L_i must be made regardless of agent i's decision, the cost of spending zero does not differ from the cost of remaining inactive.[10] However, if two or more agents spend zero with positive probability then each has a chance of winning. It is important to note that this will not occur, in equilibrium. For if one agent spends zero with positive probability, each of the others can, with an arbitrarily small positive bid increase his win probability and hence his expected payoff, by a finite amount.

The above arguments are summarized as the following Proposition.

<u>Proposition 1</u>:

If agent i is active, his spending level is the realization of a distribution with c.d.f. $G_i(x_i)$ which is continuous over $(0,\infty)$. Moreover,

(a) $G_i(x_i) > 0$ for all $x_i > 0$

(b) With only 2 active agents the maximum spending levels are the same

(c) At most one agent spends zero with strictly positive probability.

3. TRANSFERS

We now consider the simplest case of a contest where a transfer is to take place between two agents and provide a complete characterization of the equilibrium. If agent 1 spends x_1 his expected payoff is

$$(3.1) \quad U_1 = \left\{ \begin{matrix} \text{probability} \\ \text{of winning} \end{matrix} \right\} \left\{ \begin{matrix} \text{value as} \\ \text{winner} \end{matrix} \right\} - \left\{ \begin{matrix} \text{probability} \\ \text{of losing} \end{matrix} \right\} \left\{ \begin{matrix} \text{payment} \\ \text{as loser} \end{matrix} \right\} - x_1$$

$$= p_1 W_1 - (1-p_1) L_1 - x_1$$

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[10] This is to be contrasted with models of competition with entry fees. Then each agent has the option of remaining out of the contest and so achieving a zero payoff.

$$= -L_1 + v_1 p_1 - x_1,$$

where $$v_1 = W_1 + L_1.$$

The amount v_i is the gross value to agent i of winning the competition relative to the option of remaining inactive. Henceforth we shall assume that agents are ranked according to their gross values, that is v_1 exceeds v_2.

Since, by Proposition 1, the probability of a tie is zero for any $x_1 > 0$, agent 1's win probability is $G_2(x_1)$. Agent 1's expected payoff is therefore

$$U_1(x_1) = -L_1 + G_2(x_1)v_1 - x_1 \ . \tag{3.2}$$

Arguing symmetrically, agent 2's expected payoff is

$$U_2(x_2) = -L_2 + G_1(x_2)v_2 - x_2 \ . \tag{3.3}$$

By remaining inactive, agent i loses L_i. Therefore agent i will enter with probability 1 whenever his equilibrium payoff exceeds $-L_i$. To analyze the equilibrium, we first note that, since agent 2 has the option of remaining out of the contest, he will never spend more than his valuation and so earn a return below $-L_2$. Then agent 1 will always enter since, for any small ϵ, he can guarantee himself a return of $v_1 - v_2 - L_1 - \epsilon$ by spending $v_2 + \epsilon$. It follows that $U_1 > -L_1$. Setting $x_1 = 0$ in (3.2) we conclude that $G_2(0) > 0$.

We now show that the equilibrium expected payoff for agent 2 must be $-L_2$. For, if not, setting $x_2 = 0$ in (3.3) we obtain

$$G_1(0) > 0.$$

Moreover, with $U_2 > -L_2$, agent 2 also enters the contest with probability 1. With both $G_1(0)$ and $G_2(0)$ strictly positive and both agents always competing, both agents must spend zero with strictly positive probability. But this contradicts Proposition 1. Then $U_2 = -L_2$ and so, from (3.3)

$$G_1(x_1) = \frac{x_1}{v_2}, \qquad x_1 \in [0,v_2] \tag{3.4}$$

That is, agent 1's equilibrium mixed strategy is to spend according to the uniform distribution over $[0,v_2]$.

Since both agents have the same maximum spending level v_2, we know that v_2 is in support of agent 1's bid distribution. Moreover $G_2(v_2) = 1$. Setting $x_1 = v_2$ in (3.2) we obtain

$$U_1 = G_2(x_2)v_1 - x_2 - L_1 = v_1 - v_2 - L_1 \tag{3.5}$$

Rearranging, it follows that agent 2's equilibrium mixed strategy is

$$G_2(x_2) = \left(1 - \frac{v_2}{v_1}\right) + \frac{x_2}{v_1} = \left(1 - \frac{v_2}{v_1}\right) + \left(\frac{v_2}{v_1}\right)\frac{x_2}{v_2} \tag{3.6}$$

Note that agent 2 makes a strictly positive bid with probability

$$1 - G_1(0) = v_2/v_1 < 1$$

The most natural interpretation of this result is that agent 2 stays out of the contest with probability $(1-v_2/v_1)$ and enters with probability v_2/v_1. From (3.6), conditional upon entering the contest, agent 2 also adopts a uniform mixed strategy over the interval $[0,v_2]$.

To summarize, we have derived the following result.

<u>Proposition 2</u>: Characterization of the Equilibrium

With perfect discrimination and two agents whose gross valuations are v_1 and v_2 $(v_2 \le v_1)$, agent 1 always enters the contest while agent 2 enters with probability v_2/v_1. Conditional upon entry each agent spends according to a uniform mixed strategy over the interval $[0,v_2]$.

Appealing to Proposition 2, we can easily compute the expected spending levels of the two agents. Agent 1's spending is uniformly distributed on $[0,v_2]$ and so his expected spending is $(1/2)v_2$. Conditional upon entering, agent 2's spending is also $(1/2)v_2$. Multiplying the latter by the probability of entry, v_2/v_1, expected total spending is therefore

$$(3.7) \qquad E(x_1+x_2) = \frac{1}{2} v_2 + \frac{1}{2} v_2\left(\frac{v_2}{v_1}\right) = \frac{v_2}{2}\left[1 + \frac{v_2}{v_1}\right]$$

We therefore have

<u>Corollary 2.1</u>: **With perfect discrimination and two agents whose gross valuations are v_1 and v_2 $(v_2 \le v_1)$, expected total spending is**

$$E(x_1+x_2) = v_2\left(\frac{v_1+v_2}{2v_1}\right)$$

For the symmetric case this reduces to the common gross valuation v. (See also Hirshleifer and Riley (1978) or Hillman and Samet (1987) for a discussion of this case). However, under asymmetry, note that expected spending is lower than <u>either</u> of the gross valuations.

We now introduce a third agent with gross valuation v_3 where

$$(3.8) \qquad v_3 \le v_2 \le v_1 .$$

Suppose that agents 1 and 2 continue to act as in the 2 agent case. Then if agent 3 spends x_3 his expected payoff is

$$(3.9) \qquad U_3(x_3) = -L_3 + \text{Prob}\begin{Bmatrix} x_1 \text{ and } x_2 \\ \text{less than } x_3 \end{Bmatrix} v_3 - x_3$$

$$= -L_3 + G_1(x_3)G_2(x_3)v_3 - x_3$$

$$= -L_3 + \left[\left(1 - \frac{v_2}{v_1} \right) + \frac{x_3}{v_1} \right] \frac{v_3}{v_2} x_3 - x_3$$

$$= -L_3 + \frac{v_3}{v_2} x_3 \left[1 - \frac{v_2}{v_1} + \frac{x_3}{v_1} - \frac{v_2}{v_3} \right] .$$

Since agent 3 will never spend more than his gross value v_3, the bracketed expression is no greater than

$$\left(1 - \frac{v_2}{v_3} \right) + \frac{(v_3 - v_2)}{v_1} \quad .$$

By hypothesis, v_3 is no greater than v_2 and so both expressions in parentheses are nonpositive. Thus agent 3 has no spending level which will generate a nonnegative expected payoff. His best response to the strategies of agents 1 and 2 is therefore to be inactive. We have therefore proved

Proposition 3: Equilibrium with more than 2 agents.

With perfect price discrimination and n agents suppose that $v_1 \geq v_2 > v_3 \geq \ldots \geq v_n$. Then if agents 1 and 2 act as if there were no other agents, the other agents have no incentive to compete.

While this Proposition characterizes an equilibrium of the contest, we would like to be able to show that there is no other equilibrium; that is, the only equilibrium strategy of agents 3 through n is to remain inactive.

Proposition 4: Uniqueness

With perfect price discrimination and n agents, suppose that $v_1 \geq v_2 > v_3 \geq \ldots \geq v_n$. Then the unique equilibrium is for the two agents with the highest gross values to compete as if they were the only agents. All other agents remain passive.

<u>Proof</u>: Let $G_i(x_i)$ be the c.d.f. for agent i. For any purely passive agent $G_i(x_i) = 1$ for all $x_i \geq 0$. Suppose that agents other than agent 1 and agent 2 are active and that the highest spending level by these other agents is made by agent m. Let x^* be that spending level. If agent 2 spends x^* his expected payoff is:

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\begin{aligned}
U_2(x^*) &= \prod_{j \neq 2} G_j(x^*)v_2 - x^* - L_2 \\
&\geq \prod_{j \neq m} G_j(x^*)v_2 - x^* - L_2, \quad \text{since} \quad G_2(x^*) \leq G_m(x^*) = 1 \\
&> \prod_{j \neq m} G_j(x^*)v_m - x^* - L_2, \quad \text{since} \quad v_2 > v_m \\
&= [\prod_{j \neq m} G_j(x^*)v_m - x^* - L_m] + L_m - L_2 \\
&= U_m(x^*) + L_m - L_2 \\
&\geq -L_2
\end{aligned}
$$

since any active agent, such as agent m, must be at least as well off as if he were inactive.

We have proved, therefore, that agent 2 is strictly better off entering the contest. It follows that he will do so with probability 1. Moreover, since $U_2 > -L_2$ we have

$$L_2 + U_2(0) = \prod_{j \neq 2} G_j(0)v_2 > 0.$$

In particular it follows that $G_1(0) > 0$.

But exactly the same argument holds for agent 1. That is, agent 1 is strictly better off entering and so enters with probability 1. Moreover $G_2(0) > 0$. Therefore both agents 1 and 2 spend zero with strict probability. But this contradicts part (c) of Proposition 1. It follows that there can be no active agent m, where $m > 2$. Q.E.D.

By way of contrast, if all n agents have the same gross values, that is

$$v_1 = v_2 = \dots = v_n = v$$

there is, for each $m \geq 2$, an equilibrium in which m agents are active and the remainder are inactive.[11] To see this, suppose agents $1,\dots,m$ spend according to the mixed strategy

$$G_j(x) = (x/v)^{\frac{1}{m-1}} \quad j = 1,\dots,m. \tag{3.10}$$

Consider agent i, $i \leq m$. His expected return, if he spends x is

$$U_i(x) = \prod_{j \neq i} G_j(x)v - x - L_i = (x/v)v - x - L_i = -L_i.$$

That is, the mixed strategy given by (3.10) is indeed a best reply. Now consider agent i, $i > m$. If he spends x his expected return is

$$U_i(x) = v(x/v)^{\frac{m}{m-1}} - x - L_i = [(x/v)^{\frac{1}{m-1}}-1]x - L_i$$

$$\leq -L_i, \quad \text{for all } x \in [0,v].$$

Therefore, for each such agent, remaining inactive is also a best reply.

Since the above argument holds for each m between 2 and n, there are $(n-1)$ different types of equilibrium for the symmetric case. In particular, there is an equilibrium in which only two agents are active. It is this equilibrium which the unique asymmetric equilibrium approaches as the difference in gross values declines to zero.

From (3.10), with m active agents the expected value of outlays is

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[11] See Hillman and Samet (1987) for a complete discussion of this case.

$$\sum_{i=1}^{m} E(x_i) = m \int_0^v x dG_1(x) = v$$

That is, regardless of the number of active players, the expected value of outlays precisely equals the common valuation v of the prize.

Asymmetric valuations therefore reduce the total value of outlays. Low valuation agents are discouraged altogether from competing against the two agents with highest valuations. The agent with the second highest valuation is also inhibited in his outlays by the knowledge that there exists an agent with a higher valuation. Agent 2 outlays zero with positive probability, but agent 1 does not win outright without a contest since agent 2 randomizes in a manner which allows for the possibility of a positive outlay. The inhibitions against lower valuation agents competing yield underdissipation with respect to v_2. However, since $v_2 = W_2 + L_2 > v_2 > W_2$, more may be spent than the value assigned to the transfer by agent 2.

4. RENTS

Rent seeking is characterized by $L_i = 0$. The winner does not secure the prize at the expense of the losers. Rather, the quest is to become the beneficiary of a preexisting rent.

Since $L_i = 0$, we have $v_i = W_i$, and our previous results follow for the value of the rent W_i in place of the gross valuation v_i. Only the two agents with the highest valuations W_1 and W_2 $(W_1 > W_2)$ actively compete. The reservation cost of not competing is zero. The expected utility of agent 2 who is indifferent between competing actively and remaining inactive is therefore also zero. From Corollary 2.1 total spending by the two active agents is

$$(4.1) \qquad E(x_1 + x_2) = W_2 \left(\frac{W_1 + W_2}{2W_1} \right) < W_2 < W_1 .$$

5. INFORMATIONAL ASYMMETRY

In the analysis thus far, each agent knows whether or not his opponents have higher or lower valuations. However, agents may be uncertain as to opponents' valuations. Suppose, therefore, that each agent's valuation, v_i, is an independent draw from the cumulative distribution function $F(v)$, $v \in [0,v]$.

We now look for an equilibrium in which each agent's spending level $x_i = B(v_i)$ is an increasing function of his valuation v_i. Suppose agents 2 through n do behave in this way. Then if agent 1 spends

$$\hat{x} = B(\hat{v})$$

his expected payoff is

$$\begin{aligned} U_1 &= v_1 \text{ Prob } \{x_j < \hat{x},\ j \geq 2\} - \hat{x} \\ &= v_1 \text{ Prob } \{B(v_j) < B(\hat{v}),\ j \geq 2\} - B(\hat{v}) \\ &= v_1 \text{ Prob } \{v_j < \hat{v},\ j \geq 2\} - B(\hat{v}) \\ &= v_1 F(\hat{v})^{n-1} - B(\hat{v}) . \end{aligned}$$

Agent 1's best reply is to choose $\hat{v}$ and hence $\hat{x}$ to maximize U_1. That is, $\hat{v}$ must satisfy

$$(5.1) \qquad \frac{\partial U_1}{\partial \hat{v}} = v_1 (n-1) F(\hat{v})^{n-2} F'(\hat{v}) - B'(\hat{v}) = 0 .$$

But, by hypothesis, agent 1's best reply is $x_i = B(v_i)$. Therefore the value $\hat{v}$ which solves equation (5.1) must be equal to v_1.

Since this must hold for all $v_i \in [0,v^*]$ it follows that

$$(n-1)vF(v)^{n-2}F'(v) - B'(v) = 0 \;. \tag{5.2}$$

Integrating

$$x_i = B(v_i) = \int_0^{v_i} vdF^{n-1}(v) \;. \tag{5.3}$$

To take a simple example, suppose v is distributed uniformly on $[0,v*]$. From (5.3)

$$x_i = \int_0^{v_i} v(n-1)F^{n-2}(v)dv = \int_0^{v_i} (n-1)v^{n-1}dv = \frac{(n-1)}{n} v_i^n \;.$$

Taking the expectation over v_i and then summing over i, the expected value of outlays is

$$E\left\{\sum_{i=1}^{n} x_i\right\} = \frac{n-1}{n+1} v* \;. \tag{5.4}$$

In the case of a uniform distribution,

$$\int_0^{v*} vdF^n(v) = \frac{n}{n+1} v* \;.$$

The amount remaining undissipated in the contest is therefore $1/n^{th}$ of the expected gross value.

To understand this result, consider first the two-agent case. If an agent has a high valuation he knows with high probability that his valuation is much greater than that of his opponent. He therefore optimizes by spending much less than he would if he confronted an agent with equal valuation. As a result, expected total outlays are less than the expectation of the high valuation. However, with a large number of agents, each agent anticipates that, even if he has the highest valuation, it is likely that others will have similar valuations. Spending is therefore more aggressive and rent dissipation increases.

More commonly however, informational asymmetries of the type analysed here are likely to be combined with observable differences. From the previous sections we have seen that the latter lead to very small numbers of active agents. Given small numbers, we have seen here that the additional social loss via rent dissipation due to informational asymmetry is likely to be quite significant.

6. IMPERFECTLY DISCRIMINATING CONTESTS

The basic model of politically contestable transfers and rents can be reformulated to portray an imperfectly discriminating contest; that is, a contest where the political process cannot discriminate among contenders to designate the winner with certainty, but rather the outcome of the contest is the assignment to each agent of a probability that he will be the winner. A special case of an imperfectly discriminating contest has been proposed by Gordon Tullock (1980).

Tullock proposed the family of probability functions describing an agent's chance of success in a contest,

$$p_i(x) = \frac{x_i^\alpha}{\sum_{j=1}^{n} x_j^\alpha} \quad , \qquad \alpha > 0 \ .$$

Because of greater analytical tractability, we consider the case in which $\alpha = 1$. There are then constant returns from outlays in the contest.[12] Then

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[12] $\alpha > 1$ implies increasing returns from outlays, in that an extra dollar spent increases an agent's probability of winning by more than the previous dollar. Conversely with $\alpha < 1$ there are decreasing returns.

$$p_i(x) = x_i/s_n(x) \tag{6.1}$$

where

$$s_n(x) = \sum_{j=1}^{n} x_j \ . \tag{6.2}$$

An individual's probability of success in a contest is thus given by the value of his outlay relative to the total value of outlays made.

Tullock assumed that all contenders valued the political prize equally. However, consider the consequences of asymmetric evaluation. Let individual i have a valuation v_i of the prize. To focus on the rent-seeking case (as opposed to transfers) considered by Tullock, assume $L_i = 0$. If x is the vector of agents' outlays, agent i's expected payoff from participation in a contest is

$$U_i(x) = \frac{x_i v_i}{s_n(x)} - x_i \ .$$

In this case an equilibrium in pure strategies does exist. Differentiating with respect to x_i we obtain

$$\frac{\partial U_i}{\partial x_i} = \frac{(s_n - x_i)v_i}{s_n^2(x)} - 1 \quad , \quad i = 1,\ldots,n \tag{6.3}$$

and

$$\frac{\partial^2 U_i}{\partial x_i^2} = \frac{-2(s_n - x_i)v_i}{s_n^3(x)} < 0 \quad , \quad i = 1,\ldots,n.$$

Therefore U_i is concave in x_i and so the first order conditions

$$\frac{\partial U_i}{\partial x_i} = \frac{(s_n - x_i)v_i}{s_n^2} - 1 = 0 \quad i = 1,\ldots,n \tag{6.4}$$

define a global interior maximum of expected utility for agent i.

Suppose that $v_1 \geq v_2 \geq v_3 \dots \geq v_n$ and assume that all n agents actively participate in the contest. Hence $x_i > 0$, $i = 1,\dots,n$. The harmonic mean of the n agents' valuations is

$$(6.5) \qquad \bar{v}_n = \frac{n}{\sum_{j=1}^{n} \frac{1}{v_j}} .$$

From (6.4)

$$s_n - x_i = s_n^2 \left(\frac{1}{v_i} \right).$$

Summing over n

$$ns_n - \sum_{i=1}^{n} x_i = s_n^2 \left(\sum_{i=1}^{n} \frac{1}{v_i} \right) .$$

From the definitions of s_n and $\bar{v}$ it follows that the total value of the outlays made in the contest is

$$(6.6) \qquad s_n = \left(\frac{n-1}{n} \right) \bar{v}_n .$$

Thus, when valuations of the political prize differ, total outlays closely approximate the harmonic mean of individuals' valuations as the number of participants increases.

Tullock investigated the symmetric case where $v_i = v$, $i = 1,\dots,n$. In that case (3.6) reduces to

$$(6.6') \qquad s_n = \left(\frac{n-1}{n} \right) v ,$$

with the common valuation v replacing the harmonic mean of valuations. The same limiting rent-dissipation result emerges, now with respect to the common valuation v.

When evaluations differ, we cannot however presume that all agents will choose to participate actively in a contest. Consider the appearance of an (n+1)th individual whose valuation of the prize is $v_{n+1} < v_n$. Suppose that the prior n individuals believe that individual (n+1) will not actively participate. The n active individuals then maintain their strategies as indicated. To establish whether this is an equilibrium given the appearance of the (n+1)th individual, we need to consider the payoff to entry of the latter individual. Since $U_i(x_i)$ is concave in x_i, whether the (n+1)th individual can increase his payoff by actively participating hinges on the sign of

$$\frac{\partial U_{n+1}}{\partial x_{n+1}}(x_1,\ldots,x_n,0) = \frac{(s_{n+1} - x_{n+1})}{s_{n+1}^2} v_{n+1} - 1$$

$$= \frac{v_{n+1}}{s_n} - 1 .$$

Hence agent (n+1) will remain inactive if and only if $v_{n+1} < s_n$. Substituting from (3.6) indicates that agent (n+1) will remain inactive if and only if $v_{n+1} < (1 - 1/n)\bar{v}_n$.

A rule limiting entry has therefore been established when valuations of the prize differ. The rule is described by:

<u>Proposition 5</u>: When the probability of success in an imperfectly discriminating contest is given by

$$p_i(x) = \frac{x_i}{\sum_{j=1}^{n} x_j}$$

entry will take place until, for some agent (n+1), the valuation of the prize v_{n+1} is lower than $(n-1)/n$ times the harmonic mean of the n higher valuations.

The equilibrium number of contenders is thus established by Proposition 5. With the number of contenders established, rent dissipation is given by (6.6).

To see the strength of the necessary condition limiting entry suppose that, for each j, valuations are geometrically decreasing such that

$$v_{j+1} = \alpha\, v_j, \qquad \alpha \leq 1 . \tag{6.7}$$

Agent (m+1) will not enter if

$$v_{m+1} = \alpha^m v_1 < s_m = \left(\frac{m-1}{m}\right) \frac{m}{\dfrac{1}{v_1} + \dfrac{1}{\alpha v_1} + \ldots + \dfrac{1}{\alpha^{m+1} v_1}} ,$$

which implies

$$\alpha^m < \frac{s_n}{v_1} = (m-1) \frac{\left(\dfrac{1}{\alpha} - 1\right)}{\left(\dfrac{1}{\alpha^m} - 1\right)} \tag{6.8}$$

or

$$1 - \alpha^m < (m-1) \left(\frac{1-\alpha}{\alpha}\right) . \tag{6.9}$$

Using (6.8) and (6.9) the equilibrium number of active agents can be summarized as follows:

m	α_*	s_n/v_1
initial number of active agents	value of α below which there is no further entry	total spending as a fraction of v_1, evaluated at α_*
1	0	
2	.62	.38
3	.81	.53
4	.89	.62
5	.93	.69
8	.98	.78
∞	1	1

Table 1: Active agents with geometrically decreasing valuations $(v_{i+1} = \alpha v_i)$

From Table 1 it is clear that, as in the limiting polar case of perfect discrimination, the number of agents actively participating in Tullock's imperfectly discriminating contest will be small unless valuations are very similar. Given the small numbers of participants, rent dissipation will be very much less than complete.

Alternatively, consider the transfer version of Tullock's imperfectly discriminating contests.[13] Since agent i's probability of success is $p_i = x_i/s_n(x)$, the expected gain from active participation is

$$(6.10) \qquad U^i = \frac{x_i v_i}{s_n(x)} - x_i - L_i .$$

Since L_i is given, the value of x_i which maximizes expected utility

[13] See also Appelbaum and Katz (1986a).

(6.10) in the transfer case is the same as the solution to (6.4).

As in the perfectly discriminating contest, transfers increase dissipation of the contested prize relative to the case of an equally valued contested rent. In particular, this is brought out in the case of symmetric valuation. Then, with $v_i = v$, $L_i = L$, $W_i = W$, $i = 1,\ldots,n$, an agent's outlay in equilibrium is

$$(6.11) \qquad x_i = \left[\frac{n-1}{n^2}\right] v = \left[\frac{n-1}{n^2}\right] (W + L) .$$

Substituting into (6.10), it follows that, in equilibrium,

$$(6.12) \qquad U^i = \frac{1}{n} (W + L) - \left[\frac{n-1}{n^2}\right] (W + L) - L$$

$$= \frac{1}{n^2} (W + L) - L > - L.$$

Thus, all agents actively participate in the contest. If inactive, an agent incurs a loss with certainty of L, an outcome which for finite n can be improved upon by active participation.

Suppose that no deadweight losses are incurred in the transfer. Then

$$(6.13) \qquad W = (n-1)L.$$

Substituting (6.13) into (6.12) reveals that

$$(6.14) \qquad U^i = -L\left(1 - \frac{1}{n}\right) < 0 .$$

That is, although U^i exceeds the certain loss L incurred in the event of non-participation, the expected gain from participation in the contest is nevertheless negative. Thus, as in the perfectly discriminating transfer contest, individuals have no interest in the contest taking place. The very existence of the contest yields an expected net loss.

Now consider the total value of the expenditures made. Aggregating over the individual outlays given by (6.11) yields the total outlays by n participants (all of whom we have established will be active)

$$s_n(x) = \left(\frac{n-1}{n}\right) (W + L) .$$

Substituting from (6.13), it follows that in the absence of deadweight costs of effecting the transfer from the losers to the winner,

$$\begin{aligned} s_n(x) &= \left(\frac{n-1}{n}\right) \left[(n-1)L + L\right] \\ &= (n-1)L \\ &= W. \end{aligned} \tag{6.15}$$

Thus, somewhat remarkably, in the absence of deadweight costs so that the value of the transfer received by the winner is precisely equal to the payments made by the losers, the value of expenditures made in the attempt to influence the outcome of the contest exactly equals the value of the transfer received by the winner.

<u>Transfers to predesignated beneficiaries</u>

Suppose now that the beneficiary of a transfer and the individuals who are to be taxed to pay the transfer are predesignated. Let the proposal be to subsidize agent 1 by \$W paid by the remaining $(n-1)$ agents. The asymmetric valuations are thus: $v_1 = W$ and $v_j = W/(n-1)$, $j \geq 2$. Probabilities that the transfer proposal will be accepted by the political decision mechanism are

$$p_1(x) = \frac{x_1}{x_1 + y} \qquad \text{where } y = \sum_{j=2}^{n} x_j$$

$$p_j(x) = \frac{y}{x_1 + y} , \qquad j \geq 2 .$$

We know that there will be three active agents. However, we can solve for x_1 and y by considering only agents 1 and 2 for whom

$$U_1(x_1) = \frac{W\, x_1}{(x_1 + y)} - x_1$$

$$U_2(x_2) = \frac{W}{(n-1)} \frac{y}{(x_1 + y)} - x_2 .$$

The first order conditions are then

$$\frac{W\, y}{(x_1 + y)^2} - 1 = \frac{W\, x_1}{(n-1)(x_1 + y)^2} .$$

Solving we have

$$y = \frac{W}{n^2} \quad \text{and} \quad x_1 = \left(\frac{n-1}{n^2}\right) W .$$

It follows that agent 1 wins with probability $(n-1)/n$. Total expenditures in this contest are $x_1 + y = W/n$. That is, total expenditures are only 1/nth of the value of the transfer to agent 1.

From this we might conclude that a proposal for such a contest which does reach the agenda will very likely be accepted. Such contests are common, and perhaps characterize the types of contests which do make it to the political agenda. For example, every proposal for protection of a particular industry, considered individually, is of this type. The proposal is to transfer income to individuals whose incomes are tied to the (relative) output price of the industry which is the candidate for protection. The cost of protection is diffusely borne by others in the economy who individually have little incentive to allocate resources to defeat the protectionist proposal, because they bear a small part of the cost of the transfer. Notice that free-rider problems generally associated with collective action are absent here. No public-good attributes have been introduced.

7. TOWARDS A MORE FUNDAMENTAL MODEL OF POLITICAL COMPETITION

In the previous sections we have examined the implications of asymmetry using two simple models. The first model is founded on the assumption that the agent contributing the most will always succeed.

While a useful polar case, it seems more reasonable to model explicitly the uncertainty of the political process. The second imperfectly discriminating model of Tullock can be seen as a preliminary effort along these lines. Tullock's model, however, can only take us so far since the probability of winning is not derived from underlying primitives. In this section we borrow from the literature on tournaments and, in particular, the work of Lazear and Rosen (1981) to build a more fundamental model of political competition. We then use this model to reexamine the issue raised by Tullock (1980) regarding rent dissipation when agents are identical.

The key idea is that the impact of an agent's effort to influence the outcome of the political process is inherently uncertain. If agent i spends x_i dollars the impact

$$\tilde{z}_i = h(x_i, \tilde{\xi}) \tag{7.1}$$

is stochastically increasing in x_i, where the random variable ξ_i is assumed to be independently distributed with cumulative distribution function $G(\xi_i)$. An agent then wins the contest not because he has spent the most but because his spending has the largest political impact.

In the version of the model which is explored here we make the further assumption that higher spending levels have a multiplicative effect on political impact, that is,

$$\tilde{z}_i = A(x_i)B(\tilde{\xi}_i) \quad (7.2)$$

Intuitively, the uncertainty associated with a large expenditure is likely to be considerably higher than with a small expenditure. Therefore an additive structure seems unreasonable. The multiplicative structure is the simplest formulation which allows for the non-linear effect.

Without loss of generality we define the new random variable $\epsilon_i = B(\xi_i)$, with cumulative distribution function $H(\xi_i) = G(B^{-1}(\epsilon_i))$, and rewrite the political impact function as follows

$$\tilde{z}_i = A(x_i)\epsilon_i \quad (7.3)$$

We assume that only a positive political impact can produce a victory. We may therefore assume that ϵ_i has a lower support of zero. In what follows we shall further assume that the cumulative distribution function for ϵ_i, $H(\epsilon_i)$, is strictly increasing and differentiable if and only if $\epsilon_i \in (0,a)$. We use $h(\epsilon_i)$ to denote the density of $H(\epsilon_i)$.

With n active agents, agent i wins if, for all j,

$$\tilde{z}_j < \tilde{z}_i,$$

that is, if

$$A(x_j)\, \epsilon_j < A(x_i)\, \epsilon_i\ , \quad j \neq i$$

or

$$\epsilon_j < \left(\frac{A(x_i)}{A(x_j)}\right)\epsilon_i\ , \quad j \neq i\ .$$

The probability that the random component for ϵ_j for agent j satisfies this last inequality is

$$H\left(\frac{A(x_i)}{A(x_j)}\epsilon_i\right).$$

Therefore, if the realization of the random variable for ϵ_i is $\hat{\epsilon}_i$, agent i wins with probability

$$\prod_{i\neq j} H\left(\frac{A(x_i)}{A(x_j)}\hat{\epsilon}_i\right).$$

Taking the expectations over ϵ_i agent i's probability of winning is

$$p_i(x) = \int_0^a \prod_{i\neq j} H\left(\frac{A(x_i)}{A(x_j)}\epsilon_i\right)h(\epsilon_i)d\epsilon_i . \tag{7.4}$$

For the simplest two agent case the probability of winning for agent 1 is

$$p_1(x_1,x_2) = \int_0^a H\left(\frac{A(x_1)}{A(x_2)}\epsilon\right)h(\epsilon)d\epsilon \tag{7.5}$$

To connect this up with Tullock's work, suppose $n = 2$, $h(\epsilon) = e^{-\epsilon}$, $\epsilon \geq 0$ and $A(x_i) = x_i^{\alpha}$. Substituting these expressions into equation (7.5) we obtain

$$p_1(x_1,x_2) = \int_0^\infty (1-e^{-A_1\epsilon/A_2})\, e^{-\epsilon}d\epsilon$$

$$= 1 - \frac{1}{\frac{A_1}{A_2}+1}$$

$$= 1 - \frac{x_1^{-\alpha}}{x_1^{-\alpha}+x_2^{-\alpha}}$$

$$= \frac{x_1^{\alpha}}{x_1^{\alpha} + x_2^{\alpha}} \quad .$$

However, with three agents, the special case yields

$$p_1(x_1,x_2,x_3) = 1 - \frac{x_1^{-\alpha}}{x_1^{-\alpha}+x_2^{-\alpha}} - \frac{x_1^{-\alpha}}{x_1^{-\alpha}+x_2^{-\alpha}} + \frac{x_1^{-\alpha}}{x_1^{-\alpha}+x_2^{-\alpha}+x_3^{-\alpha}}.$$

Thus, while Tullock's class of probability functions

$$\text{(7.6)} \qquad p_i(x) = \frac{x_i^{\alpha}}{\sum_{j=1}^{n} x_j^{\alpha}}, \quad \alpha > 0,$$

does emerge as a special case of the political competition model when $n = 2$, this is not true for larger n.

One of the intriguing questions raised by Tullock was how much entry would take place in the absence of the asymmetries examined in the previous section. He argued that it would be quite possible to have essentially unlimited entry. However, this conclusion hinged critically upon his assumed form of the probability function. In future work we plan to analyze our conjecture that such a result would be difficult to derive from our more basic stochastic model. Here we present a suggestive example.

Suppose the c.d.f. for ϵ_i, $H(\epsilon_i)$ is uniformly distributed on $[0,1]$. Then, if $x_j = x^*$, $j \neq i$

$$p_i(x) = \int_0^1 \text{Max}\left[1, \frac{A(x_i)}{A(x^*)}\epsilon\right]^{n-1} d\epsilon .$$

For $x_i \geq x^*$, we can rewrite the expression as

$$p_i(x) = \int_0^B \left(\frac{\epsilon}{B}\right)^{n-1} d\epsilon + 1 - B$$

$$= 1 - \left(\frac{n-1}{n}\right) B, \quad \text{where} \quad B = A(x^*)/A(x_i).$$

Then, if $x_j = x^*$, $j = 1,\ldots,n$

(7.7) $$\frac{\partial p_i}{\partial x_i} = \left(\frac{n-1}{n}\right) \frac{A'(x^*)}{A(x^*)}$$

Suppose, as before, that $A(x) = x^{\alpha}$. Then (7.7) becomes

(7.8) $$\frac{\partial p_i}{\partial x_i} = \frac{(n-1)}{n} \frac{\alpha}{x^*}, \quad x_j = x^*, \quad j = 1,\ldots,n.$$

Now consider the equilibrium in the symmetric model with n active agents. Substituting (7.8) into (7.6) we obtain

(7.9) $$x_j = \left(\frac{n-1}{n}\right)\alpha v, \quad j = 1,\ldots,n.$$

But, in the symmetric equilibrium $p_i = 1/n$. Then

$$U_i = vp_i - x_i$$

$$= v\left(\frac{1}{n}\right) - \left(\frac{n-1}{n}\right)\alpha v.$$

Since equilibrium payoffs must be nonnegative, it follows immediately that the number of active agents must satisfy

$$n-1 \le 1/\alpha$$

Moreover, since entry will take place to the point at which one more entrant would generate negative equilibrium payoffs, we also require

$$n > 1/\alpha.$$

Combining these results, it follows that

$$\frac{1}{\alpha} < n \le \frac{1+\alpha}{\alpha}.$$

Therefore, unless the parameter α is very small (reflecting very rapid diminishing returns to expenditures on rent seeking) there will only be a few active agents.

8. Concluding Remarks

The resource costs of political contestability of rents and transfers make for higher social losses due to transfer policies and market intervention by governments than accounted for by deadweight costs alone. But how much does political contestability add to the evaluation of social cost? Past literature addressed this question on the supposition that the contested prize was a preexisting rent which was equally valued by all contenders. Either competitive free entry was assumed to result in complete rent dissipation, or in small-numbers strategic settings contests were imperfectly discriminating in identifying the winning contender.

We have formulated a basic model of contests which are perfectly discriminating and which in the general case encompass payments among agents to finance asymmetrically valued transfers.

Asymmetric valuation has been shown to reduce the cost of political contestability. Free entry into a perfectly discriminating contest does not imply a large number of active contenders when valuations of the political prize differ. Only the two agents with the highest valuations have an incentive to actively contest the prize. The lower-valuation agent is moreover inhibited in his outlays by the knowledge of the higher valuation of his rival. The expected value of outlays made to influence the outcome of a contest is consequently less than the gross value assigned to the prize by the low valuation contender.

The larger the difference between valuations, the smaller are expected total outlays in the contest. On the other hand, in the limiting case of identical valuations, dissipation is expectationally complete. Asymmetric valuation thus acts as a barrier to entry. The social cost of political contestability is reduced by the inhibition on active participation.

Our result that the equilibrium number of active contenders is small -- in general, two -- is consistent with casual observation. In general, the contest is between an incumbent and a challenger. Asymmetric valuation, in terms of gross values, can be associated with the advantage of the incumbent. The challenger values the prize less because of a differential cost disadvantage. The incumbent benefits from reputation, or other advantages of incumbency.

Another source of incumbent advantage is a better understanding of the political system. In the analysis above, the probability of winning is assumed to be a symmetric function of contenders' spending levels. Plausibly this function might be weighted in favor of the incumbent. In this case, even if gross values are identical, the asymmetry survives and so very similar results emerge.

Incumbency is but one source of asymmetric valuation. We have noted other sources, including in particular different deadweight costs associated with transfers.

Uncertainty concerning rivals' valuations has been shown to increase dissipation. Such uncertainty erodes the barrier to entry or inhibition on active participation due to asymmetric valuation.

The consequence of asymmetric valuation is thus reduced dissipation. However, dissipation is increased when the contest involves a transfer rather than allocation of a preexisting rent. In a transfer contest, part of the gross value of the prize is avoidance of payment to the winner and also avoidance of deadweight losses. Agents are therefore not only concerned with winning, but also have the related though separate concern of not losing. Increased dissipation occurs because resources are expended in defensive or preemptive activity.

The expected value of the transfer contest is negative for all agents, with the possible exception of a high-valuation contender. In the limiting case of identical valuations, the transfer contest necessarily has a negative expected value for all agents. Everybody would therefore wish to avoid the contest. This is reflected in overdissipation, given by the value of the losses (transfer to the winner plus deadweight costs) incurred by the losers. Presumably, transfer contests exist only because of asymmetric valuations.

Thus for example in contests to redistribute income via trade policy, the beneficiaries of protection have more to gain from winning the contest than do the more diffused beneficiaries of free trade who are the source of the protectionist income transfer. The higher valuation of the beneficiaries of protection inhibits the activities of the losers from protection, for whom the expected value of the contest is negative. Even if losers refrain from preemptive activity, they still suffer an income loss which includes the burden of the deadweight costs of departure from free trade.

Rent seeking as a quest to influence political allocation of a given preexisting rent has been explained as privately profitable, but socially wasteful. The transfer contest is however also privately unprofitable for all but a high-valuation contender; since even passive behavior yields a loss with certainty.

Finally we have proposed a general stochastic specification of contestability in which the probability of winning is a continuous function of each agent's spending level. In future work we plan to analyze this in some detail. However, from the examples presented here, we conjecture that the central conclusions drawn from the model of perfectly discriminating contests will continue to hold.

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