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# Resource Depletion with Technological Uncertainty and the Rawlsian Fairness Principle

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It is hardly surprising that one of the sharpest controversies arising from the Theory of Justice by John Rawls (1971) is linked to his discussion of the intergenerational distribution of wealth. Despite its continual usage, the standard modified utilitarian approach is really no more than a bastardized version of classical utilitarianism. To quote from Arrow (1973b p. 260):

> The most straightforward utilitarian answer is that the utilities of future generations enter equally with those of the present. But since the present generation is a very small part of the total number of individuals over a horizon easily measurable in thousands of years, the policy conclusion would be that virtually everything should be saved and very little consumed, a conclusion which seems offensive to common sense. The most usual formulation has been to assert a criterion of maximizing a sum of discounted utilities, in which the utilities of future generations are given successively smaller weights. The implications of such policies seem to be more in accordance with common sense and practice, but the foundations of such a criterion seem arbitrary.

An alternative approach, which avoids the asymmetry associated with discounting, is to introduce a social welfare function of the form:

$$W = \sum_{t=1}^{\infty} w(U_t) \qquad w' > 0,\ w'' < 0.$$

By increasing the curvature of $w(\cdot)$ it is possible to weight more heavily those generations with lower utility and thereby reduce the degree of inequality. For example, if $w(U) = \frac{1}{1-r}(U)^{1-r}$, the parameter $r = \frac{-Uw''(U)}{w'(U)}$, is naturally interpreted as the social relative aversion to intergenerational inequality. At least for the standard, neoclassical, one-sector growth model, if $\{U_t^*\}$ maximizes social welfare, the ratio $\inf_t\{U_t^*\}/\sup_t\{U_t^*\}$ is increasing in r and approaches unity as $r \to \infty$. However in the absence of a principle upon which to select the parameter r, this approach is no less arbitrary.

In this unsatisfactory state it was natural that economic theorists would focus upon the application of the Rawlsian principle of justice as "fairness" to the question of intergenerational transfers.

In this context, a natural interpretation is that it would not be fair for the present generation to exploit its position in time, to the relative disadvantage of any of those following. Putting this slightly differently, generation $\tau$'s plan is unjust if the generation would be willing to pay to avoid having to change places with any future generation. Then a plan at time $\tau$ is just only if the associated stream of utilities $\{U_t\}$ satisfies the constraint

$$\inf_{t>\tau}\{U_t\} \geq U_\tau$$

Generation $\tau$, maximizing its own utility subject to the constraint of intergenerational justice, must therefore seek the solution of

$$\text{(1)} \qquad \text{Max}\ \{U_\tau \mid \inf_{t>\tau}\{U_t\} \geq U_\tau\}.$$

In the simplest of neo-classical models, discussed by Arrow (1973a) and Solow (1974) each generation is assumed to live in one period enjoying consumption available in that period. Then, to satisfy the fairness principle, it is obvious that the consumption plan $\{C_t\}$ must be nondecreasing over time. The solution to (1) therefore involves choosing a consumption plan which equalizes consumption levels over time. It is a trivial matter to show that the optimal investment plan is simply to maintain the capital stock at its initial level. Both Arrow and Solow concluded that a welfare criterion which tied an economy so rigidly to the initial capital stock seemed highly unsatisfactory.[1]

In apparent response, Rawls (1974), withdrew any claim that the fairness principle could be applied to the question of inter-generational justice. However, to this author at least, such a retreat was premature. Elsewhere with Phelps (1977), I have argued that the conclusion that Rawlsian justice freezes society essentially into its initial state is based on over-simplified analysis. Only in the absence of an overlap between generations is it impossible for different cohorts to make mutually beneficial trades. Utilizing standard assumptions, it can be shown that such trades lead to a monotonic increase in the capital stock while the utility levels of all generations are equalized.

These results hold even though each generation is egoistic, with tastes only for its own consumption of commodities and leisure. The "just" economy is further freed from its initial position if it is assumed that generation $\tau$ cares also about the preferences of its immediate descendants, who in turn care about generation $\tau + 2$... and so on. Following Koopmans (1960) this is very naturally expressed as:

$$(2) \qquad U_\tau = U(u_\tau, U_{\tau+1}) \qquad \tau = 1, 2 \ldots$$

For simplicity, the additive form of (2) is used throughout, that is

$$(2)' \qquad U_\tau = u_\tau + \left(\frac{1}{1+\rho}\right)U_{\tau+1} \qquad \tau = 1, 2 \ldots$$

$$= \sum_{t=\tau}^{\infty} \left(\frac{1}{1+\rho}\right)^{t-\tau} u_t$$

Applying the fairness principle to generations interconnected by these 'intergenerational ties of sentiment', the following principle emerges for the standard neoclassical model.

If the initial state is such that it is not possible to maintain indefinitely a level of $u_\tau = u(C_\tau)$ greater than the "Golden Rule" level,[2] the justice constraint is not binding. In contrast, if the Golden Rule level of $u_\tau$ is sustainable, the justice constraint is binding. Each generation is obliged to plan a savings policy in such a way as to equalize the level of $u_\tau$, and hence $U_\tau$, at the highest feasible level.

The reasoning is relatively straightforward. In the former case, maximization of $U_1 = \sum_{t=1}^{\infty} \left(\frac{1}{1+\rho}\right)^{t-1} u(C_t)$ requires following a plan which involves a steady increase in consumption levels and hence in the sequence $\{u(C_t)\}$. This in turn implies that the sequence of utility levels $\{U_\tau\} = \{ \sum_{t=\tau}^{\infty} \left(\frac{1}{1+\rho}\right)^{t-\tau} u(C_t)\}$ is strictly increasing. Then maximizing $U_1$ yields an allocation satisfying the fairness principle. If however, the initial state is such that a level of $u_t$ higher than the Golden Rule level is sustainable, maximizing $U_1$ yields a strictly decreasing sequence $\{C_t\}$ and hence a strictly decreasing sequence $\{u(C_t)\}$. Since the asymptotic level of $u_t$ is the Golden Rule level, such a policy cannot be just.

All the above of course depends upon the standard neo-classical assumptions that non-labor inputs are themselves produced goods, and that the economy is "productive." Moreover, the discussion lies within the analytically convenient world of certainty. In the following pages the goal is to examine the implications of Rawlsian justice in a world of social risk. The model analyzed is a simplification of that suggested by Dasgupta and Heal (1974) in their discussion of natural resource depletion. In contrast with the standard neo-classical model, it is assumed that initially there exists only a finite quantity of some resource essential to survival. Social risk is introduced by supposing that at some unknown future date there will be a technological advance which renders the natural resource inessential.

Despite the contrasts between the model discussed below and previously analyzed models, it will be seen that the conclusions are similar. For initial states which are poorly endowed with resources relative to post-innovation states, a generation with intergenerational ties to its immediate descendants has no further obligations toward the future. However, for all initial states which are sufficiently well-endowed with resources, the fairness principle calls for a slower utilization of the natural resource over some initial phase than what would result from maximization of the first generation's utility.

## I. The Structure of the Model and Other Preliminaries

In order to focus upon the essentials without bogging down in what may be analytically impenetrable concessions to reality, we consider the simplest possible model. The discussion in this section follows very closely that of Dasgupta and Heal. At time zero there exists a stock $S_0$ of the only consumable good. There is no production. Writing consumption at time t as C(t) we then have

$$(3) \qquad \dot{S} = -C; \qquad S(t) \geq 0$$

At some time T there will be a technological discovery. All remaining natural stock, S(T), is destroyed in the chain reaction resulting from the discovery. In its place is a new process which yields a perfect substitute at a rate of M units per period.[3]

The date of discovery is uncertain. However, it is believed that the probability that the discovery will take place <u>after</u> any given time T is a differentiable function $\Omega(T)$, where $\Omega'(T) < 0$ and $\lim_{T\to\infty} \Omega(T) = 0$. Unless $\Omega(0) = 1$ there is some finite probability, $1 - \Omega(0)$, that the discovery will never take place.

Each generation cares about its own consumption and the utility level of its immediate descendants according to (2)'. Introducing the expectation operator E, the welfare of generation $\tau$ can then be expressed as

$$(4) \quad V_\tau = E \sum_{t=\tau}^{\infty} (1/1+\rho)^{t-\tau-1} u(C_t).$$

It turns out to be more convenient to work in continuous time. Taking the appropriate limit, generation $\tau$'s utility level becomes:

$$(4)' \quad V(\tau) = E \int_{t=\tau}^{\infty} e^{-\rho(t-\tau)} u(C(t))dt$$

Consider first the utility level of generation T, that is, the generation immediately following the technological break-through. Since there is no longer any uncertainty, V(T) is simply a discounted sum of future levels of u(C(t)). Given that after the break-through the single commodity, produced at a rate M, can only be consumed or stored, it is a straightforward matter to show that maximization of V(T) requires a consumption at a rate of M.[4] Then

$$V^*(T) = \max_{C(t)} V(T) = \int_{T}^{\infty} e^{-\rho\tau} u(M)dt = \rho^{-1} u(M).$$

Of course exactly the same argument can be applied for all generations following the Tth, that is

$$(5) \quad V^*(\tau) = \rho^{-1} u(M), \quad \tau \geq T$$

Thus the policy chosen by generation T, and all generations thereafter, is the solution to (1), that is, it is the just plan. Summarizing we have:

Proposition 1: For all post-discovery generations, the consumption plan yielding greatest utility, $C^*(t) = M$, is the just plan.

Next consider the expected utility maximizing plan for the present generation. Only by solving for this plan will it be possible to see whether it is consistent with the over-riding obligations resulting from adoption of the fairness principle.

Let C(t) be the consumption rate at time t conditional upon no prior technological break-through, as planned by the present generation. If the discovery were to take place at time T, the utility of the latter would be;

$$\int_0^T e^{-\rho t} u(C(t))dt + e^{-\rho T}\rho^{-1}u(M)$$

Since $\Omega(T)$ is the probability of discovery later than time T, $-\Omega'(T)dT$ is the probability of discovery in the interval $[T, T + dT]$. Expected utility at time zero is therefore:

$$(6) \quad V(0) = \int_0^\infty -\Omega'(T)\left[\int_0^T e^{-\rho t} u(C(t))dt + e^{-\rho T}\rho^{-1}u(M)\right]dT + (1-\Omega(0))\int_0^\infty e^{-\rho T}u(C(t))dt$$

Integrating by parts, this can be rewritten as:

$$(6)' \quad V(0) = \int_0^\infty (1-\Omega(0) + \Omega(T))e^{-\rho T}[u(C(T)) - u(M)]dT + \rho^{-1}u(M).$$

It is assumed that

$$(7) \quad u(0) = 0, \lim_{C\to 0} u'(C) = \infty, u'(C) > 0, u''(C) < 0$$

Note that if C(t) maximizes V(0) it must also maximize

$$(8) \quad \int_0^\infty w(t)u(C(t))dt \quad \text{where } w(t) = (1 - \Omega(0) - \Omega(t))e^{-\rho t}$$

But it is well known that the above assumptions ensure that there exists a unique interior solution to the maximization of (8) subject to the 'equation of motion' (3). Moreover the necessary conditions for maximizing (8) are easily seen to be

$$(9) \quad w(t)u'(C(t)) = \text{constant}$$

$$\int_0^\infty C(t)dt = S_0$$

These two conditions completely characterize C(t).

Then V(0) is maximized by choosing a prediscovery consumption plan C(t), satisfying (9). Since w(t) is strictly decreasing in t and u(•) is concave, it follows that the optimal plan C(t) is decreasing in t.

To summarize we have:

Proposition 2: The expected utility of the present generation is maximized by following a declining pre-discovery consumption path which is the solution of

$$\max \int_0^\infty \omega(t)u(C(t))dt \quad \text{s.t.} \quad \int_0^\infty C(t)dt = S_0$$

$$\text{where } \omega(t) = (1 - \Omega(0) + \Omega(t))e^{-\rho t} \quad ,$$

Before turning to a discussion of the just rate of extraction, it should be noted that in general the Strotz-Pollak time inconsistency problem arises. Let $\Omega(\tau + T|\tau)$ be the probability that discovery takes place after time $\tau + T$, conditional upon its not having occurred prior to time $\tau$. The expected utility of generation $\tau$, conditional upon no discovery prior to $\tau$, is then

$$(10) \quad V(\tau) = \int_0^\infty (1 - \Omega(\tau|\tau) + \Omega(\tau+T|\tau))e^{-\rho T}[u(C(\tau+T) - u(M)]dT + \rho^{-1}u(M)$$

From (9) it follows that the first order conditions for the maximization of $V(\tau)$ can be written as

$$(11)\qquad e^{-\rho T}\,\frac{u'(C(\tau+T))}{u'(C(\tau))} = \frac{1}{1 - \Omega(\tau|\tau) + \Omega(\tau+T|\tau)}$$

But (9) must hold at every point in time, and in particular at $t = \tau$ and $t = \tau + T$. Moreover, for consistency between the plans of generation $\tau$ and the present generation both (9) and (11) must be satisfied. Eliminating the intertemporal marginal rate of substitution we thus require

$$(12)\qquad 1 - \Omega(\tau|\tau) + \Omega(\tau+T|\tau) = \frac{1 - \Omega(0) + \Omega(\tau+T)}{1 - \Omega(0) + \Omega(\tau)}$$

There are two rather natural ways in which these conditional probabilities may be defined. The first alternative is to assume that the probability of discovery after calendar time T is unaffected by failure to make the discovery in some intervening sub-interval, that is;

$$(13)\qquad \Omega(\tau+T|\tau) = \Omega(T+\tau)$$

The second alternative is to assume independence with respect to calendar rather than lapsed time. By this it is meant that the probability of discovery over the time interval $[\tau, \tau + T]$ is dependent only upon whether or not discovery was made prior to this interval and not on calendar time t. We must then have

$$(14)\qquad \Omega(\tau+T|\tau) = \Omega(T)$$

It is a relatively straightforward matter to check that for the first alternative, (13), there is no distribution function satisfying (12). Furthermore the only family of distribution functions satisfying (12) and (14) are given by

$$(15)\qquad \Omega(T) = \Omega(0)e^{-\pi T}$$

Since the exponential distribution both avoids time-inconsistency problems and is especially amenable to analysis it is adopted throughout the following discussion. For such a class of distributions allowing for the possibility that the discovery will be made with probability less than one adds little. It is therefore assumed that $\Omega(0) = 1$.

## II. The Just Rate of Resource Depletion.

To see whether the Rawlsian fairness principle obligates the initial generation to consume less of the natural resource requires an examination of the expected utility of future generations. Consider some arbitrary feasible consumption profile $C(t)$. If at time $\tau$, the discovery has not taken place ($\tau<T$), the expected utility of generation $\tau$ is given by (10). Moreover the probability that $\tau$ is less than T is $\Omega(\tau) = e^{-\pi\tau}$. Therefore writing the expected utility of generation $\tau$ from the vantage point of time zero as $EU(\tau)$, we have

$$EU(\tau) = \Omega(\tau)V(\tau) + (1-\Omega(\tau))\rho^{-1}u(M)$$

$$(16) \qquad = \rho^{-1}u(M) + e^{-\pi\tau}\int_0^\infty e^{-(\rho+\pi)t}[u(C(\tau+t)) - u(M)]dt$$

From the previous section, the consumption plan, $C_0(t)$, that maximizes the expected utility of the present generation, must satisfy (9), that is:

$$(17) \qquad e^{-(\rho+\pi)t}u'(C(t)) = u'(C(0))$$

Consider the slope of the expected utility profile, given the consumption plan, $C_0(t)$. Differentiating (16) yields.

$$\frac{d}{d\tau}(EU(\tau)) = -\pi e^{-\pi\tau}\int_0^\infty e^{-(\rho+\pi)t}[u(C_0(\tau+t)) - u(M)]dt$$

$$+e^{-\pi\tau}\int_0^\infty e^{-(\rho+\pi)t}u'(C_0(\tau+t))\frac{d}{d\tau}C_0(\tau+t)dt$$

Rearranging terms and utilizing (17), this derivative can be rewritten as

$$(18) \qquad \frac{d}{d\tau}(EU(\tau)) = -e^{-\pi\tau}\left(\pi\int_0^\infty e^{-(\rho+\pi)t}[u(C_0(\tau+t)) - u(M)]dt + u'(C_0(\tau))C_0(\tau)\right)$$

Since $u(C)$ is concave, and $C_0(t)$ must satisfy (17), $C_0(\tau+t)$ is decreasing in $\tau$. Then the integral in (18) is decreasing in $\tau$. As $\tau \to \infty$, $C_0(\tau+t) \to 0$, hence, the integral approaches $-\frac{u(M)}{p+\pi}$. Moreover, as $\tau \to \infty$, the second term inside the large bracket must approach zero, since for small , $u(0) \simeq u(C) + u'(C)C$, and by assumption $u(0) = 0$. Therefore, for sufficiently large $\tau$, the large bracket of (18) is negative, implying that the expected utility profile is essentially upward sloping.

We now introduce the additional restriction that the elasticity of $u(c)$ is less than unity,[5] that is:

(19) $$\frac{-Cu''(C)}{u'(C)} < 1.$$

In all the discussion that follows it will be assumed that (19) is satisfied. Given this restriction on preferences

$$\frac{\partial}{\partial C}\left(Cu'(C)\right) = u'\left(1 + \frac{Cu''}{u'}\right) > 0$$

and it follows that the second term in the large bracket of (18) is also strictly increasing in $\tau$. We have therefore proved:

**Proposition 3:** The expected utility profile, $EV(\tau)$, resulting from maximizing $EV(0)$, the expected utility of the present generation, is upward sloping for all sufficiently large $\tau$. Moreover if the elasticity of $u(C)$ is less than unity the profile is <u>either</u> upward sloping for all generations or has a unique turning point.

Intuitively, the pure time preference of the present generation leads to a declining consumption profile and, in the absence of a prior discovery, a declining expected utility. For generations sufficiently far into the future, this decline is outweighed by the rising probability of previous discovery.

We next consider the impact of increasing the initial stock $S_0$. Given (17), the consumption profile maximal for the initial generation is strictly increasing in $S_0$. Therefore an increase in $S_0$ increases the two terms inside the large brackets of (18). As $S_0 \to 0$, the first term approaches its lower bound $\frac{-\pi}{\rho+\pi} u(M)$ and the second term approaches zero. Therefore $\frac{d}{d\tau} (EU(\tau))\Big|_{\tau=0}$ is positive for sufficiently small $S_0$. Moreover for sufficiently large $S_0$, both terms in the large bracket of (18) are positive, hence $\frac{d}{d\tau}$ EU($\tau$) is negative. These results can be summarized as:

Proposition 4: The profile of expected utilities associated with the optimal plan for the present generation is upward sloping at $\tau = 0$, if and only if the initial stock of resources is less than some $S_0 = \hat{S}(M,\rho,\pi)$.

Together Propositions 3 and 4 imply:

Proposition 5: Under Rawlsian justice, the present generation have obligations to future generations over and above their own direct concern for their descendants, if and only if the initial resource level is sufficiently large.

Expected utility profiles associated with different initial levels of $S_0$ are depicted in figure 1. As $\tau \to \infty$ $\Omega(\tau) \to 0$, therefore from (16), all such profiles approach asymptotically $\rho^{-1}u(M)$.

[Figure 1]

We next examine the shape of the "just" depletion and expected utility profiles. From (16), for all $\tau$ less than some arbitrary $\bar{\tau}$:

(19) $EU(\tau) - \rho^{-1}u(M)$

$$= e^{-\pi\tau}\{\int_0^{\bar{\tau}-\tau} e^{-(\rho+\pi)t}[u(C(\tau+t)) - u(M)]dt + e^{-\rho(\bar{\tau}-\tau)}[EU(\bar{\tau}) - \rho^{-1}u(M)]\}$$

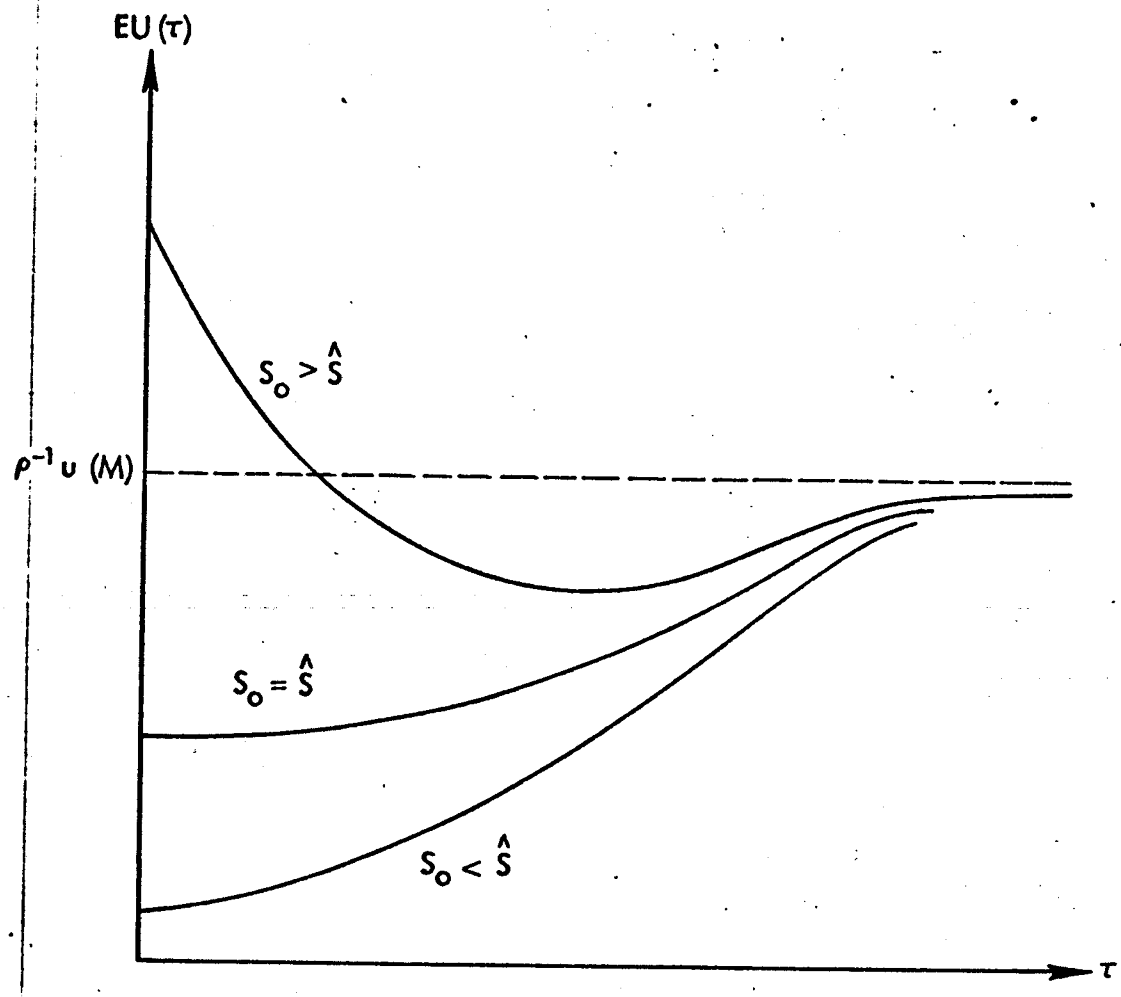


Fig. 1 —Expected utility of future generations as a result of a depletion policy which maximizes the utility of the initial generation

Suppose two consumption plans are identical for all $t < \bar{\tau}$. Then for all generations prior to $\bar{\tau}$, the plan yielding higher expected utility is that which yields the larger expected utility at time $\bar{\tau}$.

At some time $\hat{\tau}$ the "just" planned stock of the resource must equal $\hat{S}$. From Proposition 4, if the expected utility of generation $\hat{\tau}$, $EV(\hat{\tau})$ is maximized, the resulting expected utility profile is upward sloping for all $\tau > \hat{\tau}$. Then any other consumption profile lowers $\inf_{\tau \geq 0} EV(\tau)$, since it lowers the expected utility of all generations $\tau < \hat{\tau}$.

Therefore from time $\hat{\tau}$, when the "just" resource stock $S_*(\hat{\tau}) = \hat{S}$, it is optimal to follow the unconstrained depletion policy satisfying (17).

We now show that for $\tau < \hat{\tau}$ the "just" consumption/depletion plan, $C_*(t)$, must be such as to equalize expected utilities.

Suppose not. Since $EU(0) = \inf_{\tau \geq 0} EU(\tau)$, either the profile is upward sloping for all $\tau$, or there is some interval over which $EU(\tau) > EU(0)$ and $EU(\tau)$ is decreasing with $\tau$.

Consider the former possibility, depicted in figure 2.

[Figure 2]

$EU_0(\tau)$ is the expected utility profile resulting from following a depletion policy $C_0(t)$ which maximizes $EU(0)$. $EU_*(t)$ is the expected utility profile resulting from following the just policy $C_*(t)$. Both $C_0(t)$ and $C_*(t)$ must satisfy the feasibility constraint $\int_0^\infty C(t)dt \leq S_0$. Therefore so must all convex combinations $C_\lambda(t) = \lambda C_0(t) + (1-\lambda)C_*(t)$. Since $u(C)$ is strictly concave, $EU(\tau)$ is strictly concave. Hence:

$$EU_\lambda(\tau) > \lambda EU_0(\tau) + (1-\lambda)EU_*(\tau) \qquad 0 < \lambda < 1$$

$$\geq \inf_{\tau \geq 0}\{\lambda EU_0(\tau) + (1-\lambda)EU_*(\tau)\}$$

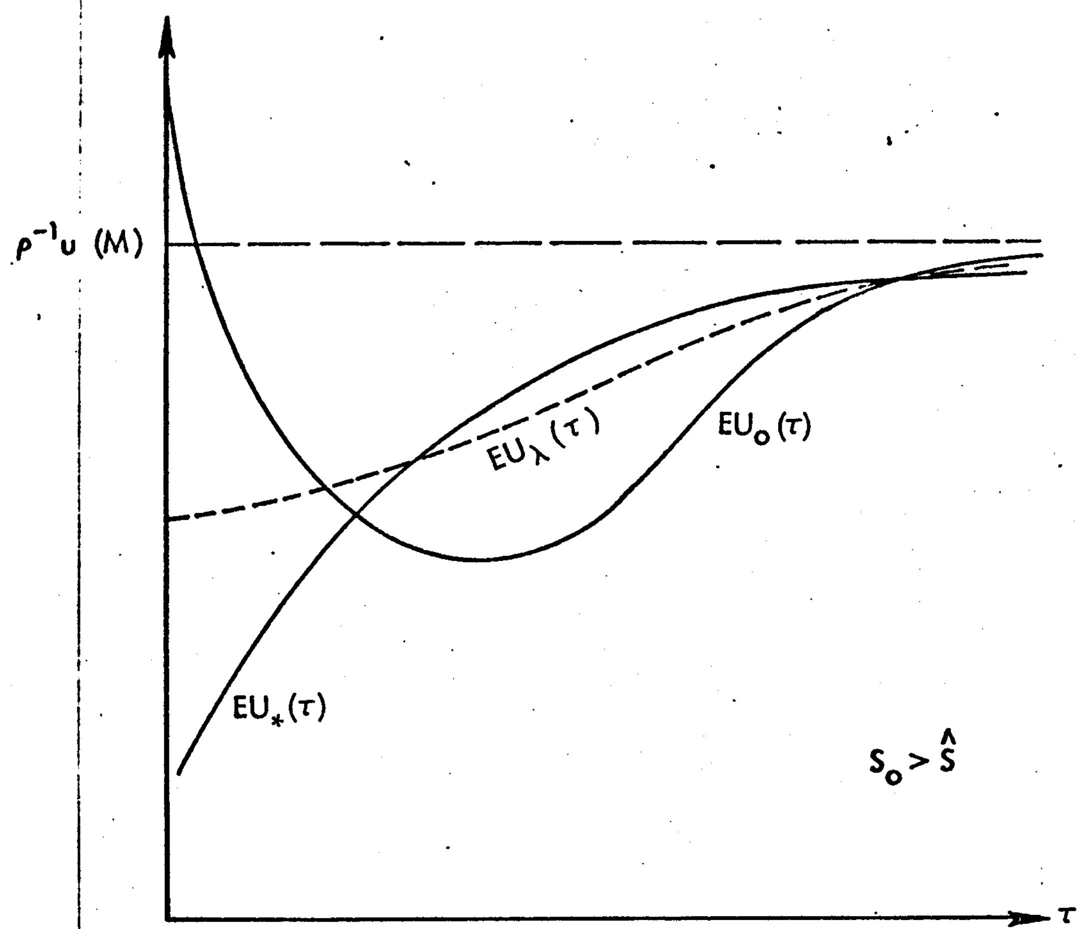


Fig. 2 — Unjust expected utility profiles

From Figure 2 it can be seen that the latter exceeds $\inf_{\tau \geq 0} EU_*(\tau)$ for all $\lambda$ sufficiently close to zero. But this contradicts our assumption that $C_*(t)$ is the "just" policy. A similar argument which considers changes in the consumption path for an interval over which $EU_*(\tau)$ is negatively sloped, rules out this possibility as well. Hence:

Proposition 6: The "just" depletion policy equalizes expected utility until some time $\hat{\tau}$ when $S_*(\hat{\tau}) = \hat{S}(M,\rho,\pi)$. Beyond this time the constraints of intergenerational justice are no longer binding and expected utility rises to its asymptote $\rho^{-1}u(M)$.

We have therefore shown that if the initial stock $S_0$ is sufficiently large, the difference principle imposes an obligation on early generations to reduce their expected utility. However these obligations are only binding until the stock reaches $\hat{S}(M,\rho,\pi)$.

We now consider the effect of parameter changes on this switching level of the resource stock. Recall from (18)

$$\frac{d}{d\tau}(EU_0(\tau)) = -e^{-\pi t}\left(\pi \int_0^{\infty} e^{-(\rho+\pi)t}[u(C_0(\tau+t)) - u(M)]dt - u'(C_0(\tau))C_0(\tau)\right)$$

An increase in M has no effect on the depletion plan, $C_0(t)$, which maximizes EU(0). Therefore if M increases, $\frac{d}{d\tau}(EU_0(\tau))$ increases owing to the increase in u(M). From Proposition 4, $\frac{d}{d\tau}(EU_0(0)) = 0$ if $S_0 = \hat{S}$ and $\frac{d}{d\tau}(EU_0(0))$ is decreasing in S. Then with the post discovery flow equal to $M + \Delta M$ and $S_o = \hat{S}(M,\rho,\pi)$, $\frac{d}{d\tau}(EU_o(o))$ is strictly positive. It follows that $\frac{\partial \hat{S}}{\partial M} > 0$.

While there seems to be no simple way to sign the impact of a ceteris paribus increase in $\rho$ or $\pi$, suppose $\pi$ is increased, and $\rho$ is simultaneously decreased so that $\rho + \pi$ is unchanged. Once again there is no effect on the depletion policy $C_0(t)$. Moreover if $S_0 = \hat{S}$, $\frac{d}{d\tau} EU_0(0) = 0$. Then since the second term in the large brackets is negative the first must be positive. The latter is therefore an increasing function of $\pi$. Thus the compensated increase in $\pi$ results in an increase in $\frac{d}{d\tau} EU_0(0)$. Applying the exactly the same argument as above, we then have $\left.\frac{\partial \hat{S}}{\partial \pi}\right|_{\rho+\pi\ =\ \text{constant}} > 0$.

To summarize we have:

Proposition 7: The level of resources, $\hat{S} = \hat{S}(M,\pi,\rho+\pi)$, below which Rawlsian justice imposes no obligations, is increasing in both M and $\pi$.

While attempts at providing an intuitive explanation of the latter result seem unconvincing, the former is certainly reasonable. An increase in M raises the expected return to later generations more than to earlier generations, since for the former, the probability of prior discovery is higher. This increases the slope of the expected utility profile, $EU_0(\tau)$, and thereby reduces the domain of initial resource levels for which $EU_0(\tau)$ is initially downward sloping.

## III. Concluding Remark

In the previous section it was shown that when there is the prospect of rendering a finite non-renewable resource inessential, the Rawlsian fairness principle operates to reduce expected utility levels of early generations below the maximum that each could potentially attain, if and only if the initial resource stock is sufficiently large.

The discussion would be incomplete without asking what role competitive markets might play in achieving the "just" depletion policy. Specifically, once society has agreed to accept the fairness principle, is it possible for the government to intervene in competitive markets in such a way that if each generation were to maximize its own expected utility, the resulting depletion rate would be the "just" rate?

Suppose at time zero, individuals can trade in contingent futures markets. Let $p(\tau)$ be the price per unit of the exhaustible resource, to be delivered at time $\tau$, conditional upon no discovery prior to this time. Owners of the resource will only be willing to sell the resource at the highest price. Then for positive supply in all periods we require $p(\tau) = p(0)$.

It is easy to check that, given such a price schedule, the initial generation if it ignores the difference principle, will choose a depletion policy which maximizes its expected utility, EU(0).

Then if $S_0 > \hat{S}(M,\pi,\rho+\pi)$ some intervention is clearly necessary in order to maximize EU(0) subject to the constraints of intergenerational justice. Recently, Barro (1974) has pointed out that it is not enough, in a regime of complete markets, for the government to float public debt over an initial interval. Individuals, if they anticipate correctly the profile of government transactions will simply offset the latter by adjusting private saving.

However it does not follow that government is powerless to affect intergenerational transfers. By introducing an appropriate profile of sales taxes, it is able to drive a wedge between producer and consumer prices and thereby achieve any technologically feasible depletion policy.

# FOOTNOTES

[1]Independently, Arrow and also Dasgupta (1974) extended the analysis to allow for individuals who care also about the consumption of the following generation. Both argued that the economy would not move permanently away from the initial capital stock. However, as Riley (1976) has indicated, even in such a model a higher long run capital stock may satisfy the constraints of Rawlsian justice.

[2]That is, the efficient stationary level of u associated with a capital stock $K_\rho$, such that the net marginal product of capital is equal to $\rho$, the private rate of pure time preference. For a more complete discussion see Phelps and Riley (1977).

[3]The destruction of the remaining natural stock greatly simplifies the analysis since all post-innovation decisions are then independent of the time of discovery.

[4]Since generation T discounts future consumption it would prefer to borrow from the future, that is, plan a declining consumption profile c(t). However this is impossible since the consumption good is produced at a fixed rate m. Then the best generation T can do is to consume all that is produced and plan for all future generations to do likewise.

[5]Without this technical assumption it is not possible to derive the strong implications of the following pages. Note that of the class of constant elasticity functions, the assumption that $u(0) = 0$ is satisfied only if (19) is satisfied.

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